Multi-particle simulation code for IBS

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CONTENTS

- Introduction
- Conventional Calculation of IBS
- SIRE structure
- Comparison with conventional theories
- Results of simulations
- Conclusions

Introduction: Motion in the DR

The motion of the particles in the CLIC damping rings can be expressed through 3 invariants (and 3 phases).

Transversal invariants:

$$\epsilon_x(i) = \beta_x \left(x_i' - D'\frac{\Delta p_i}{p}\right)^2 + 2\alpha_x \left(x_i' - D'\frac{\Delta p_i}{p}\right)\left(x_i - D\frac{\Delta p_i}{p}\right) + \gamma_x \left(x_i - D\frac{\Delta p_i}{p}\right)^2$$

$$\epsilon_z(i) = \beta_z {z'_i}^2 + 2\alpha_z z_i z_i' + \gamma_z {z_i}^2$$

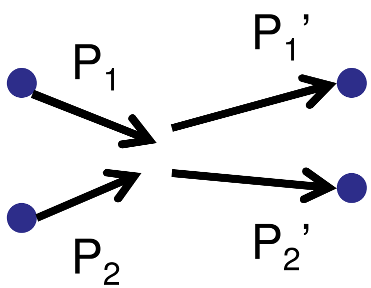
Longitudinal invariant:

$$\epsilon_s(i) = \left(\frac{\Delta p_i}{p}\right)^2 + \frac{(2\pi)^2 \nu_s^2}{(\alpha - \frac{1}{\gamma^2})^2 C^2} {\Delta s_i}^2 \qquad i = 1, \ldots, Num.Part.$$

Emittance: $$\tilde{\epsilon}_k = \frac{1}{2N} \sum_{i=1}^{N} \epsilon_k(i) \qquad k = x, z, s$$

Introduction: Intra-Beam Scattering in DR

IBS is the effect due to multiple Coulomb scattering between charged particles in the beam:



$$\delta\epsilon_k(i)$$

$$\delta\tilde{\epsilon}_k = \frac{1}{2N}\sum_{i=1}^{N}\delta\epsilon_k(i)$$

Evolution of the emittance:

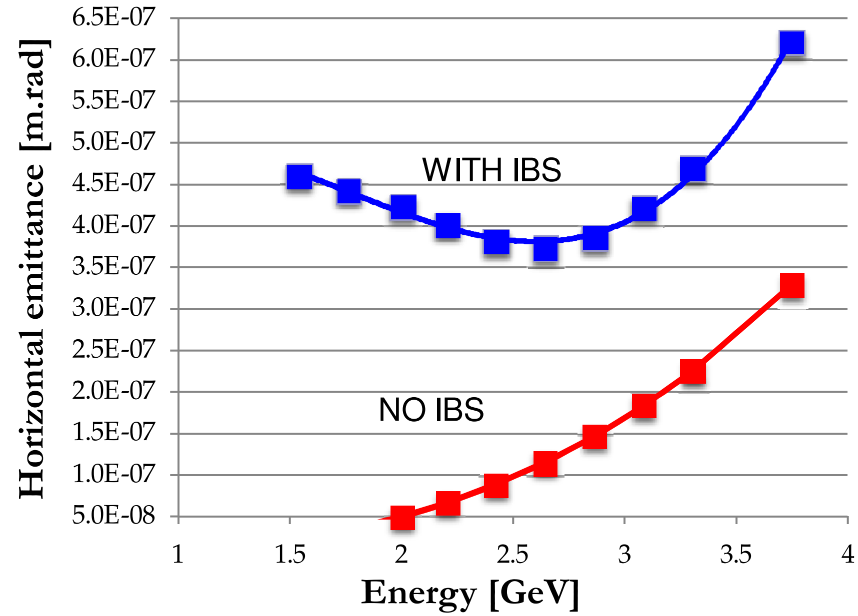
$$\frac{d\tilde{\epsilon}_k}{dt} = -\frac{1}{\tau_k}\left(\tilde{\epsilon}_k - \epsilon_k^0\right) + \frac{\tilde{\epsilon}_k}{T_k}$$

Radiation Damping, Quantum Excitation, IBS

T_k IBS Growth Times

T_k contain the effect of all the scattering processes in the beam at a certain time.

Y. Papaphilippou, et al. EPAC08



Introduction: Conventional theories of IBS

Conventional IBS theories in Accelerator Physics (Bjorken-Mtingwa, Piwinski) derive T_k by the formula:

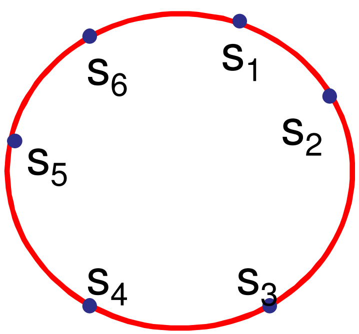
$$\frac{1}{T_k} = \frac{1}{\epsilon_k}\frac{1}{2N}\int d^3x d^3p_1 d^3p_2 d^3p_1' d^3p_2' \, \rho(x,p_1)\rho(x,p_2)|M|^2 \left[\epsilon_k(x,p_1') - \epsilon_k(x,p_1)\right] \frac{\delta^{(4)}(p_1' + p_2' - p_1 - p_2)}{(2\pi)^2}$$

1. The particle distribution is inserted from outside the theory.
2. The integral is too complicate.

In practise, the integral has been solved only for Gaussian particles distribution.
In this case the formulas for the growth times reduce to:

$$\frac{1}{T_k} = \frac{{r_0}^2 cN(\log)}{8\pi\gamma^4\beta^3\epsilon_x\epsilon_z\epsilon_s}\int_0^\infty d\lambda \frac{\lambda^{1/2}}{|L+\lambda I|^{1/2}} \left\{ TrL^{(k)} Tr(L+\lambda I)^{-1} - 3\, TrL^{(k)}(L+\lambda I)^{-1} \right\}$$

Growth rates are calculated at different points of the lattice and then averaged over the ring:



$$\frac{1}{T_k} = \sum_{i=1}^{M} \frac{S_{i+1} - S_i}{C} \frac{1}{T_k^i} \qquad S_{M+1} = C \qquad S_1 = 0$$

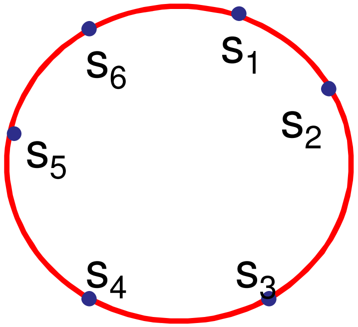
IBS studies for the CLIC Damping Rings

Goals:

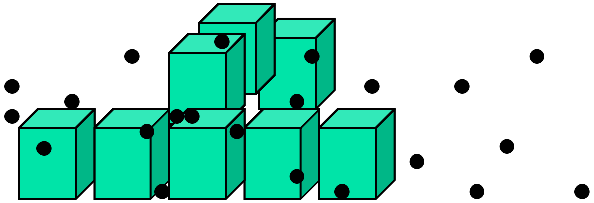
1. Follow the evolution of the particle distribution in the DR (we are not sure it remains Gaussian).
2. Calculate IBS effect for any particle distribution (in case it doesn't remain Gaussian).

Development development of a tracking code computing the combined effect of radiation damping, quantum excitation and IBS during the cooling time in the CLIC DR.
(Software for IBS and Radiation Effects)

Software for IBS and Radiation Effects



- The lattice is read from a MADX file containing the Twiss functions.
- Particles are tracked from point to point in the lattice by their invariats (no phase tracking up to now).
- At each point of the lattice the scattering routine is called.



- 6-dim Coordinates of particles are calculated.
- Particles of the beam are grouped in cells.
- Momentum of particles is changed because of scattering.
- Invariants of particles are recalculated.

- Radiation damping and excitation effects are evaluated at the end of every loop.

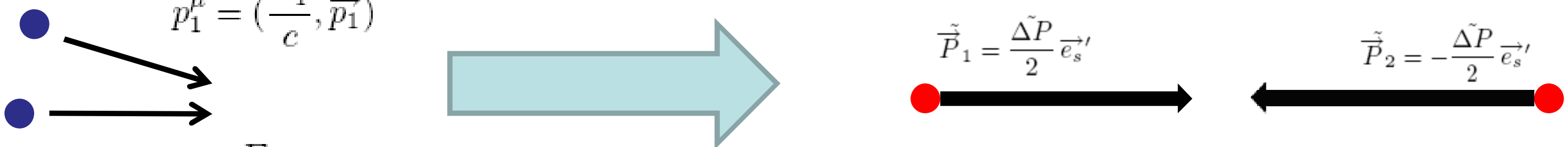
- A routine has also been implemented in order to speed up the calculation of IBS effect.

Zenkevich-Bolshakov algorithm (from MOCAC)

Laboratory Frame:

Relativistic Center of Mass Frame:

$$p_1^\mu = \left(\frac{E_1}{c}, \overrightarrow{p_1}\right)$$



$$\vec{\tilde{P}}_1 = \frac{\Delta\tilde{P}}{2}\vec{e_s}' \qquad \vec{\tilde{P}}_2 = -\frac{\Delta\tilde{P}}{2}\vec{e_s}'$$

$$p_2^\mu = \left(\frac{E_2}{c}, \overrightarrow{p_2}\right)$$

$$\overrightarrow{P_1} = P_0\left\{x_1'\overrightarrow{e_x} + z_1'\overrightarrow{e_z} + (1+\delta_1)\overrightarrow{e_s}\right\}$$

$$\overrightarrow{P_2} = P_0\left\{x_2'\overrightarrow{e_x} + z_2'\overrightarrow{e_z} + (1+\delta_2)\overrightarrow{e_s}\right\}$$

$$\delta\vec{\tilde{P}}_1(\vec{\tilde{P}}_2) = \vec{\tilde{P}}_1{}' - \vec{\tilde{P}}_1 = \frac{\Delta\tilde{P}}{2}\left\{\sqrt{\frac{4\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3}L_c\Delta t}\,(\cos\Xi\,\vec{e_x}' + \sin\Xi\,\vec{e_z}') - \left(\frac{2\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3}L_c\Delta t\right)\vec{e_s}'\right\}$$

$$\vec{\tilde{P}}_2' = -\vec{\tilde{P}}_1'$$

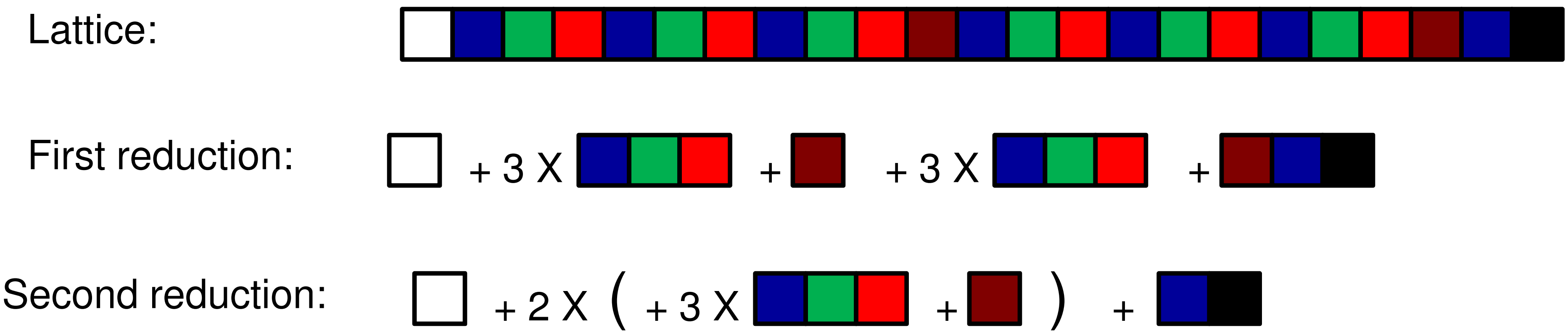
(P.R. Zenkevich, O. Boine-Frenkenheim, A. E. Bolshakov, *A new algorithm for the kinetic analysis of inta-beam scattering in storage rings*, NIM A, 2005)

Radiation damping and quantum excitation are calculated with the formula:

$$\Delta\epsilon_k(i) = -\left(\epsilon_k(i) - 2\epsilon_k^0\right)\frac{\Delta T}{\tau_k} \qquad i = 1,\ldots,N_{part}$$

Lattice Recurrences

Elements of the lattice with twiss functions differing of less than 10% are considered equal.

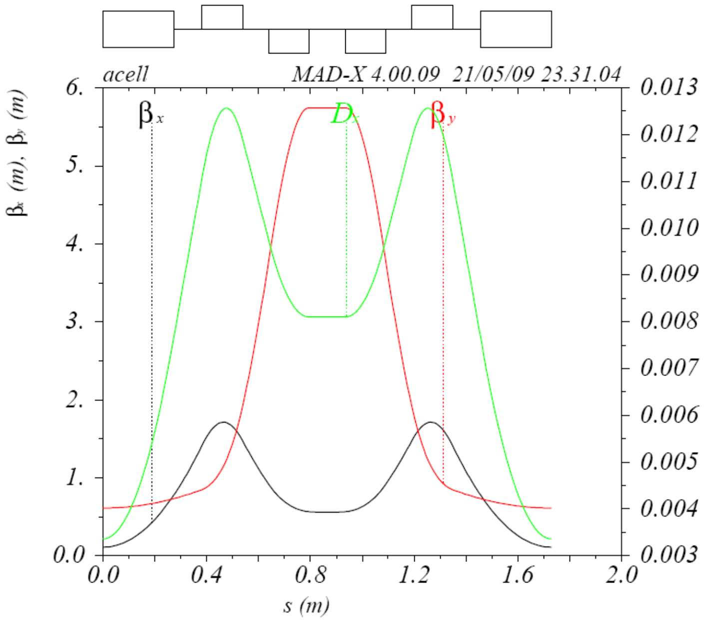


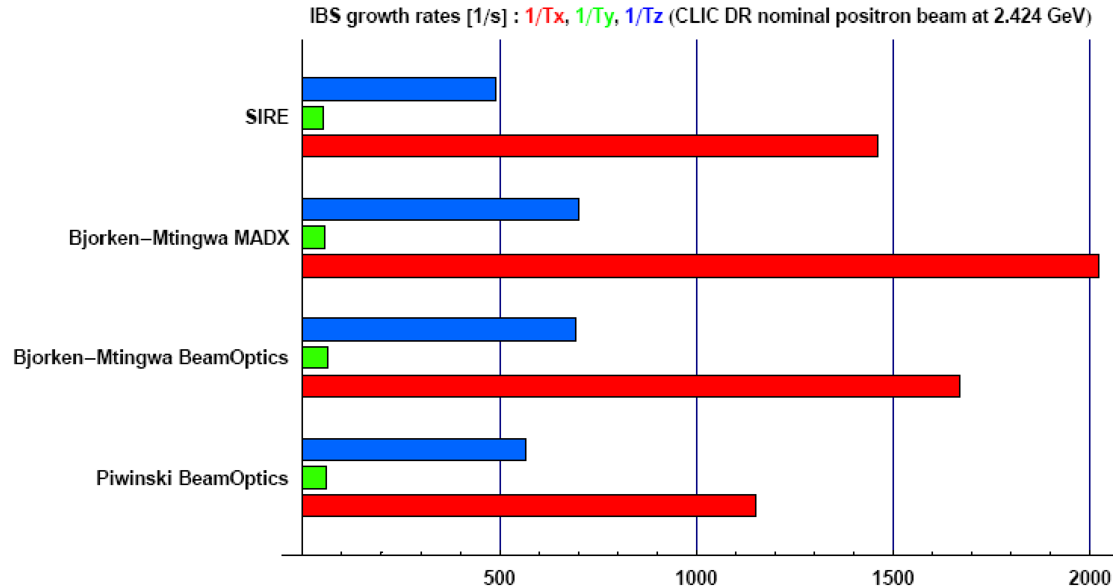
$$\frac{\Delta\epsilon}{\epsilon} = \frac{\Delta t_w}{T_w} + \frac{\Delta t_b}{T_b} + \frac{\Delta t_g}{T_g} + \frac{\Delta t_r}{T_r} + \frac{\Delta t_b}{T_b} + \frac{\Delta t_g}{T_g} + \cdots = \frac{\Delta t_w}{T_w} + \frac{6\Delta t_b}{T_b} + \frac{6\Delta t_g}{T_g} + \frac{6\Delta t_r}{T_r} + \frac{2\Delta t_{br}}{T_{br}} + \frac{\Delta t_b}{T_b} + \frac{\Delta t_{bl}}{T_{bl}}$$

CLIC DR LATTICE: 14400 elements → 420 elements

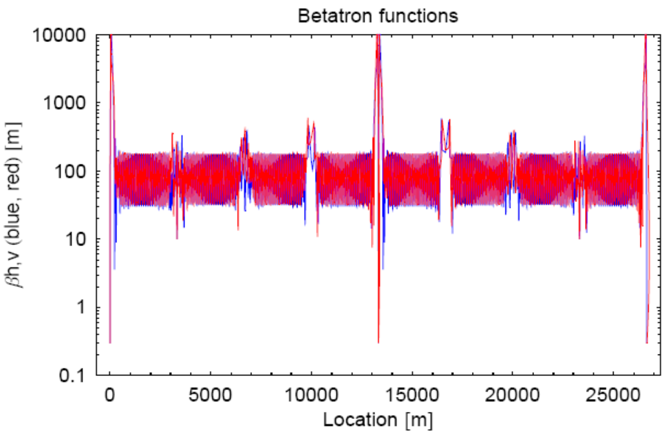
SIRE: Benchmarking (Gaussian Distribution)

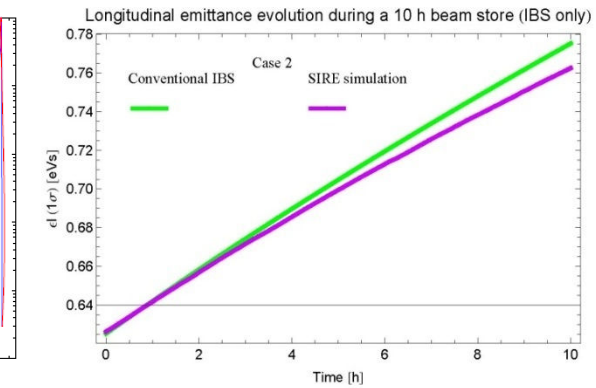
Arc cell of the CLIC DR (old parameters)

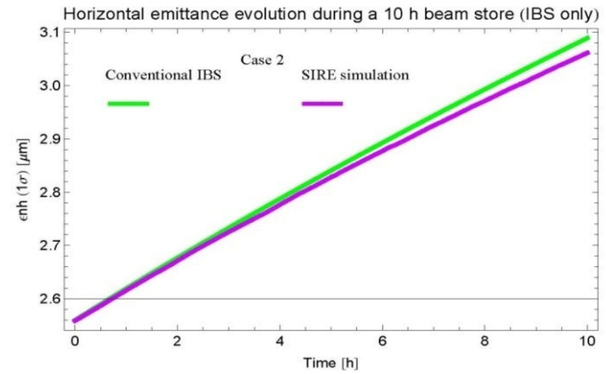


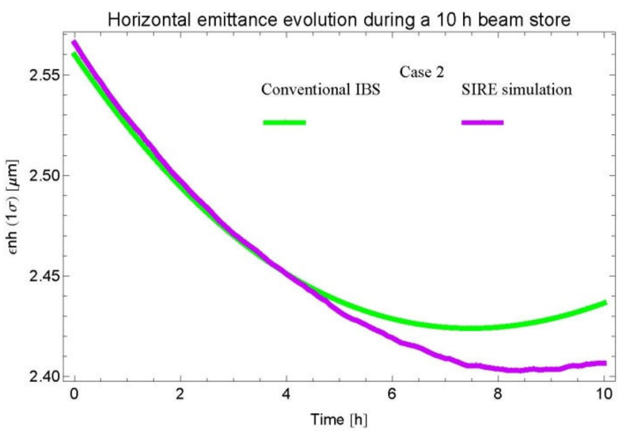


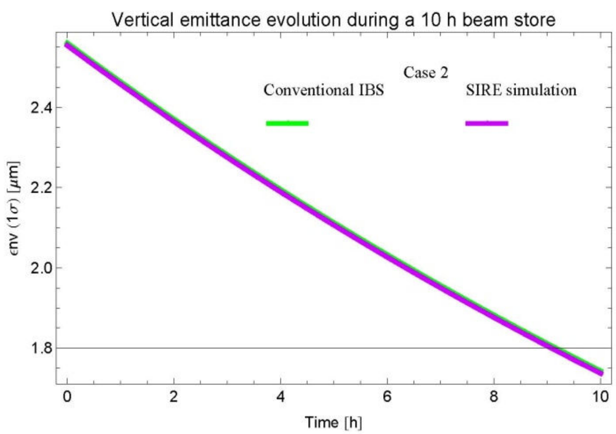
SIRE: Benchmarking (Gaussian Distribution) on LHC

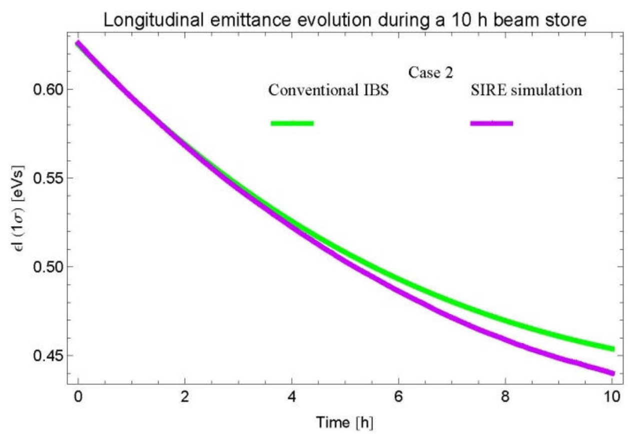






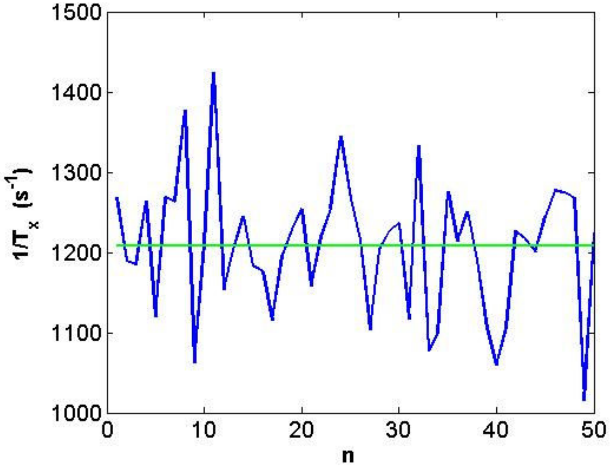


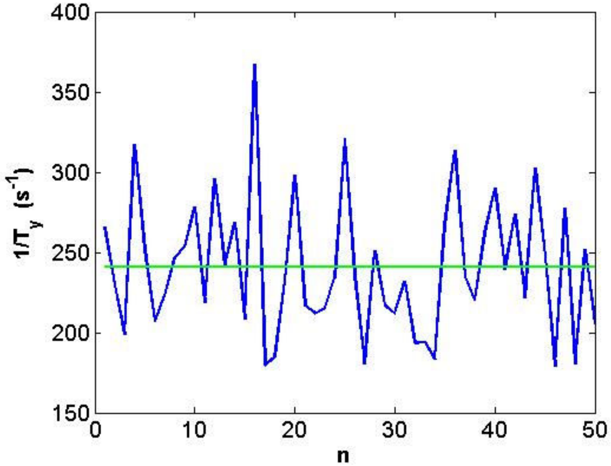


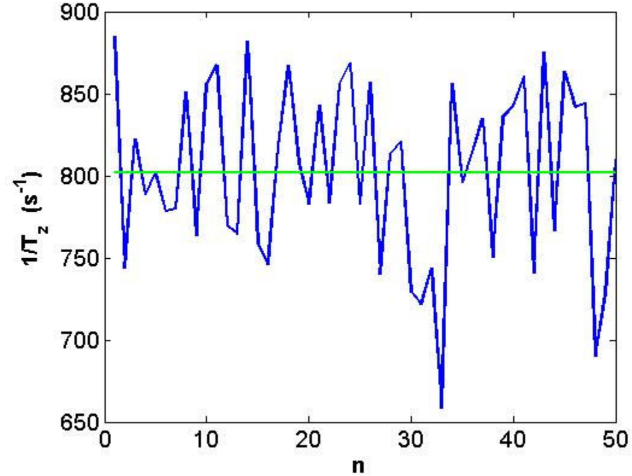


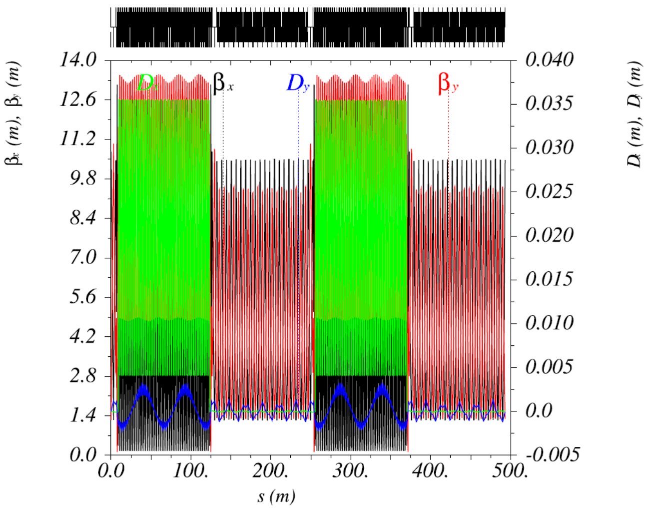
SIRE: Benchmarking (Gaussian Distribution) on CLIC DRs

Intrinsic random oscillations in SIRE



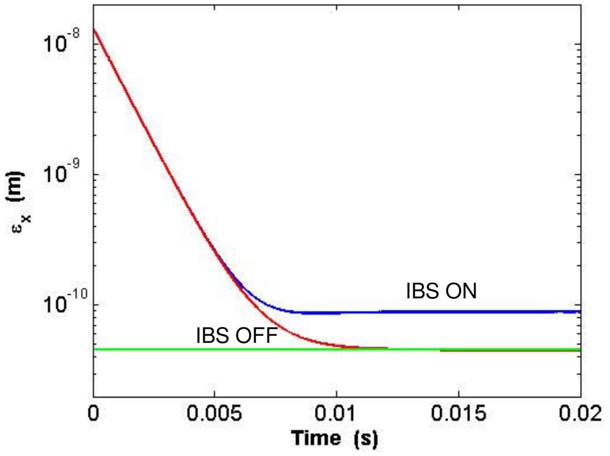


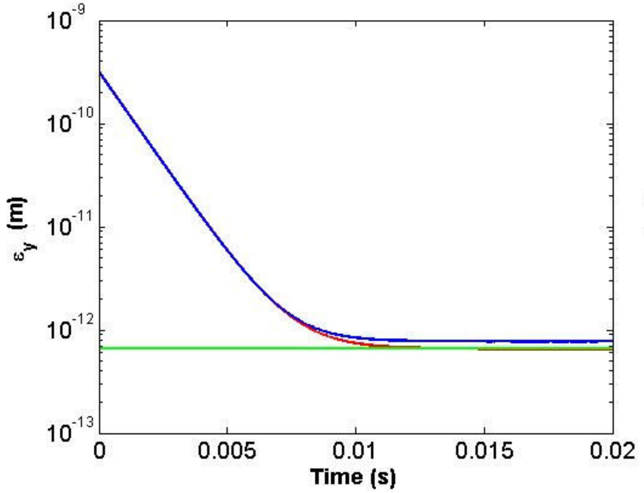


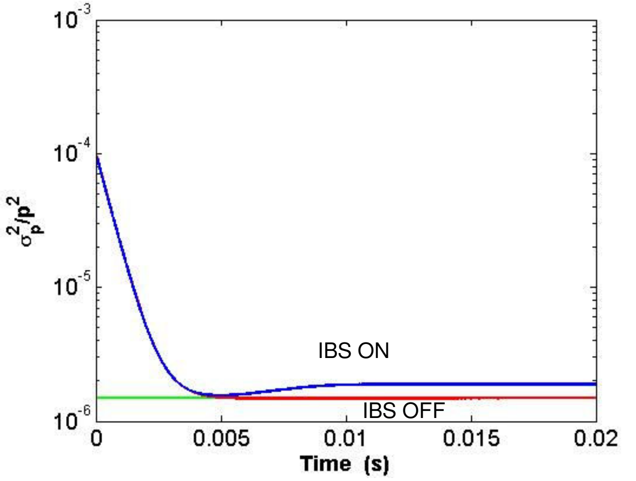


	$1/Tx\ (s^{-1})$	$1/Ty\ (s^{-1})$	$1/Tz\ (s^{-1})$
MADX (B-M)	2007.29	1485.97	1096.57
SIRE (compressed)	1207.96	240.69	802.08
SIRE (not compressed)	1188.98	252.99	811.21
Mod. Piwinski	546.54	354.13	383.50

Simulation of the CLIC Damping Rings:







Beam parameters

	$\gamma\varepsilon_x$ (m)	$\gamma\varepsilon_y$ (m)	ε_z (eV m)
Injection	74.14e-6	1.76e-6	130589
Extraction	497.95e-9	4.33e-9	3729.98
Equilibrium (NO IBS)	254.315e-9	3.668e-9	2914.52

Conclusions

- A new code to investigate IBS effect in the CLIC damping rings is being developed:
  - Benchmarking with conventional IBS theories gave good results after some parameters training.
  - Calculation of the evolution of emittances gives reliable results.
  - Refinement of quantum excitation routine will be implemented in order to give reliable particle distribution evolution.
  - Vertical dispersion will be added soon.

Simulations of the CLIC DR damping will continue.

THANKS.

The End