



The importance of being polarized

A. Ballestrero

in collaboration with

E. Maina G. Pelliccioli

Torino University and INFN



Polarization and VBS

Polarized cross sections definition

Resonant contributions and approximations

$\cos^2\theta^*$ distributions and legendre analysis.

Cuts , interferences and legendre failure

Fit to distributions

A more realistic case: WZ

Next steps

Conclusions



The relevance of weak bosons polarizations effects is documented by

TH studies which start from LEP2 times (Gounaris et al Int.J.Mod.Phys A8(1993))
and continue at LHC e.g. Bern .. Phys.Rev.D84(2011), Stirling..EPJ Web. Conf. 49 (2013),
Belyaev.. Jhep 1308(2013), Aguilar..Phys.Rev.D93(2016)

and

by several measurements performed which start at CDF and regard at LHC
polarization in W and Z production, ttbar events, WZ, WZ in boson fusion ...
(ATLAS and CMS)

Importance of polarization in VBS comes from
gauge cancellations in longitudinal polarized amplitudes

Longitudinal cross sections depend on the way EWSB
is realized

Important for searches of
deviations from the SM and hints of New Physics

A quick reminder:



EWSB gives mass to W, Z. Massive vector bosons have three physical polarization states.

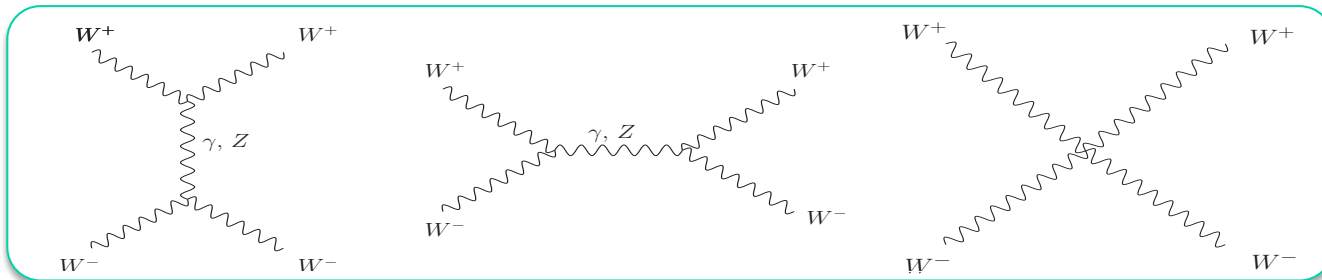
$$\varepsilon_{L/R}^\mu = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) \quad \varepsilon_0^\mu = (\kappa, 0, 0, E)/\sqrt{Q^2} \quad E \gg M_W \quad \varepsilon_0^\mu \approx p_W^\mu/M_W \quad p_W^\mu = (E, 0, 0, \kappa)$$

0 = longitudinal

Longitudinal W+W- -> W+W- scattering

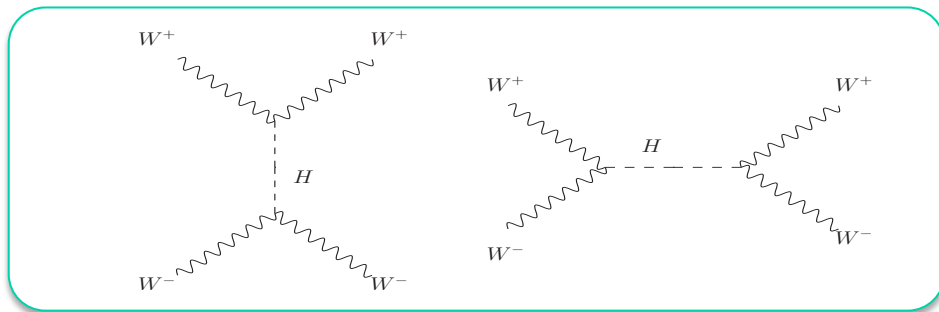
$$\varepsilon_0^{W^+} \cdot \varepsilon_0^{W^-} \propto p^{W^+} \cdot p^{W^-} = s \quad \implies \quad D_i \propto s^2$$

Longitudinal components:
single diagram $\propto s^2$



cancellations:

$$\sum \propto s^1$$



SM Higgs contributions

$$\sum \propto s^1$$

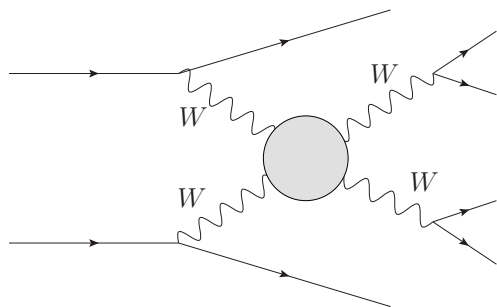
cancellations between the two groups:

$$\sum_{\text{tot}} \propto s^0$$

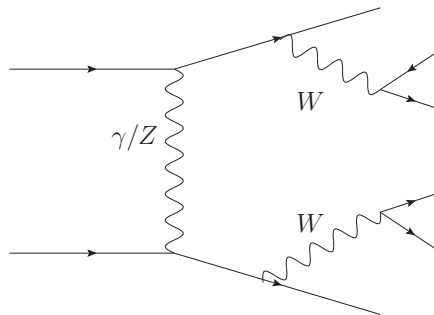
The same for other weak bosons, not for ZZ->ZZ



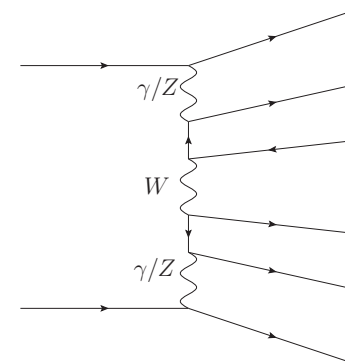
Full process : 3 types of contributions



Resonant signal



Resonant background



Non resonant

$$A_{FULL} = A_{RES} + A_{NONRES}$$

A_{NONRES} Necessary for gauge invariance! Huge cancellations
Numerically relevant in some phase space regions

Boson polarization well defined for on shell W's and Z's (NWA)
But Breit Wigner modulation is lost and difficult to compare with data.

The only alternative: consider only resonant contribution A_{RES}

- How to define polarization for off shell vector bosons?
- How to cope with gauge invariance?

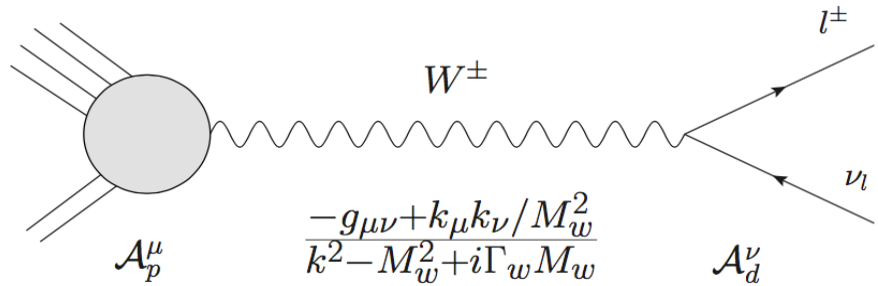
In any case polarization defined only in some approximation !

And results depend on the reference system in which you define polarization vectors



Polarization for off shell contributions

The propagator of an off shell decay



can be decomposed in a sum of polarization vectors ϵ_λ

$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2} = \sum_\lambda \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*}$$

The amplitude becomes the sum of polarized amplitudes:

$$A_f = \sum_\lambda \frac{A_p^\mu \epsilon_\mu^\lambda \epsilon_\nu^{\lambda*} A_d^\nu}{k^2 - M_W^2 + i\Gamma_W M_W} = \sum_\lambda A_f^\lambda$$

The substitution: $\sum_\lambda \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*} \rightarrow \epsilon_\lambda^\mu \epsilon_\lambda^{\nu*}$ defines the various (λ) polarized amplitudes

No need to be on-shell

The cross section contains the sum of polarized amplitudes + interferences

$$\underbrace{|A_f|^2}_{\text{coherent sum}} = \underbrace{\sum_\lambda |A_f^\lambda|^2}_{\text{incoherent sum}} + \underbrace{\sum_{\lambda \neq \lambda'} A_f^{\lambda*} A_f^{\lambda'}}_{\text{interference term}}$$

Interferences are present also on shell (NWA) !

other methods in MC ?



Considering only resonant contribution may lead to big errors
and violates gauge invariance

$$pp \rightarrow jj e^- \bar{\nu}_e \mu^+ \nu_\mu \mathcal{O}(\alpha_{EM}^6)$$

$$|\eta_j| < 5, p_t^j > 20 \text{ GeV}, M_{jj} > 600 \text{ GeV}, |\Delta\eta_{jj}| > 3.6$$

$$M_{WW} > 300 \text{ GeV}$$

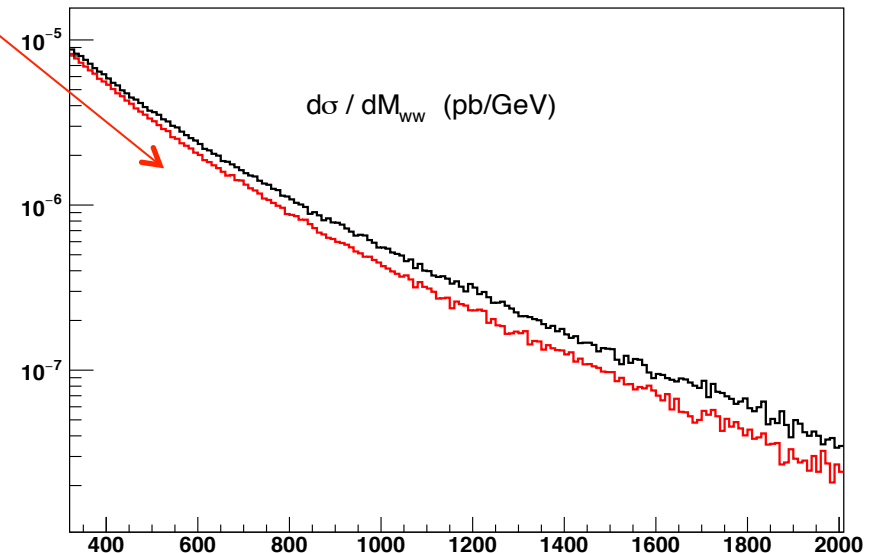
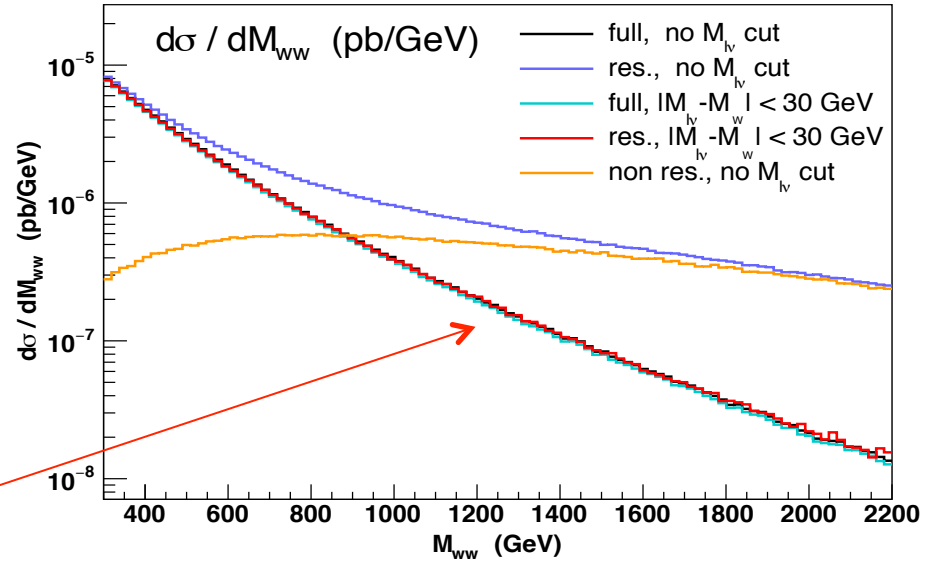
Cuts on invariant mass
of the decay products
seem to solve the problem
as the resonant contributions
dominate

$$|M_{l\nu} - M_W| < 30 \text{ GeV}$$

$$|\eta_j| < 5.5, p_t^j > 10 \text{ GeV}$$

But is cut dependent
Differences 5-10% in cross sections

And dangerous in some regions
when cancellations are important





On shell projection (OSP)

In computing the amplitudes of resonant contributions, one can project (in the numerator) the four momenta of the decay particles on shell

$$\mathcal{A}_f = \sum_{\lambda} \frac{\mathcal{A}_{p,RES}^{\mu}(p, k) \varepsilon_{\mu}^{\lambda} \varepsilon_{\nu}^{\lambda*} \mathcal{A}_d^{\nu}(k, q)}{k^2 - M_W^2 + i\Gamma_W M_W} + \mathcal{A}_{NONRES} \quad \Rightarrow \quad \sum_{\lambda} \frac{\mathcal{A}_{p,RES}^{\mu}(p, k_{OSP}) \varepsilon_{\mu,OSP}^{\lambda} \varepsilon_{\nu,OSP}^{\lambda*} \mathcal{A}_d^{\nu}(k_{OSP}, q_{OSP})}{k^2 - M_W^2 + i\Gamma_W M_W}$$

kind of On shell production X decay modulated by Breit Wigner with all exact spin correlations

If applied to both the two bosons (OSP2) the procedure is **gauge invariant** (Ward id.)

provided $\Gamma_W, \Gamma_Z \rightarrow 0$ in $\mathcal{A}_{p,RES}^{\mu}$ and $\cos \theta_W, \sin \theta_W$ (no complex mass)

Similar to DPA Denner, Dittmaier, Roth, Wackerth NP B587(2000)67

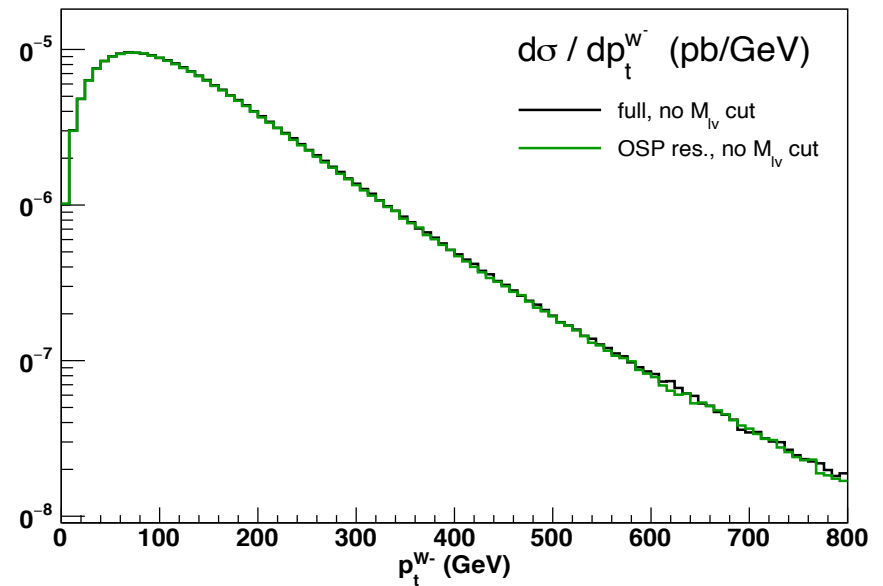
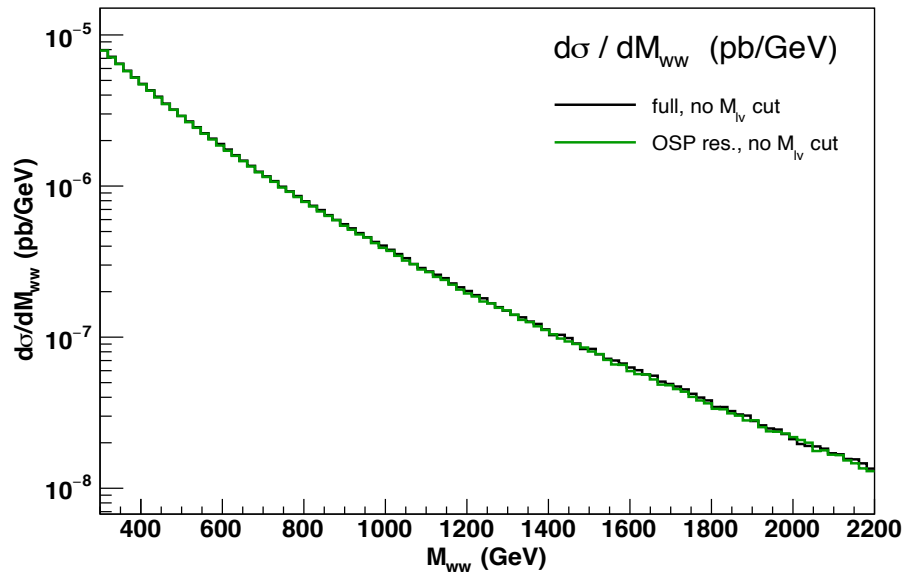
Not uniquely defined. To fully specify Phantom conserves:

1. the total four-momentum of the WW system;
2. the direction of the two W bosons in the WW center of mass frame;
3. the direction of each charged lepton in his W center of mass frame.

Applicable only for $M_{WW} > 2 M_W$



OSP2 : excellent agreement for W+W- without cut on decay invariant mass



OSP1 : projection for only one boson. $pp \rightarrow W/Z + X$
 can be used for single polarization/resonant studies
 gives good results but the other boson width $\neq 0$: not gauge invariant

Not uniquely defined: In PHANTOM conserves:

- X four momentum

- WZ 3-mom in lab

- The direction of W/Z decay in W/Z rest frame

- * Modifies initial parton 4-mom to conserve 4-mom



The ZZ case

ZZ -> ZZ processes behave for OSP similar to normal resonant contributions and without cut on $l^+ l^-$ inv. mass are very different from FULL

different cuts around the Z pole

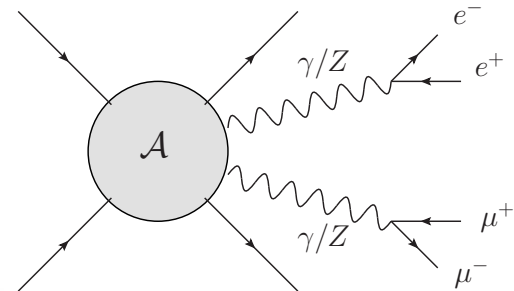
cut	FULL	RES OSP	RES NO OSP
$u u \rightarrow u u e^- e^+ \mu^- \mu^+ (4Z)$			
no cut	44.79	13.02	13.18
30 GeV	17.76	12.64	12.66
5 GeV	10.09	9.55	9.53

pb^{-8}

$m(l^+ l^-) > 40 \text{ GeV}$

Contributions missing :

When selecting ZZ resonant diagrams, gamma contributions are discarded



In fact



The ZZ case

different cuts around the Z pole

cut	FULL	RES OSP	RES NO OSP
$u u \rightarrow u u e^- e^+ \mu^- \mu^+ (4Z)$			
no cut	44.79 (13.34)	13.02	13.18
30 GeV	17.76 (12.66)	12.64	12.66
5 GeV	10.09 (9.52)	9.55	9.53

pb^{-8}

$m(l^+ l^-) > 40 \text{ GeV}$

numbers in () obtained with gamma-lepton coupling = 0

OSP2 satisfies Ward Id

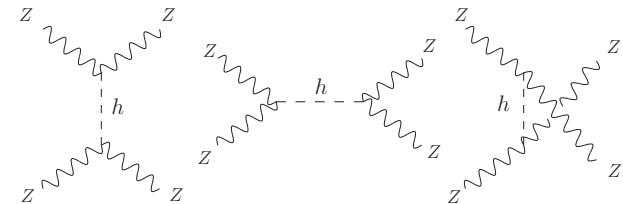
Doubts about gauge inv of a theory with Z on shell ?

Similitude between OSP and NO OSP results

ZZ->ZZ has

No unitarity/gauge cancellations

Different from WW->WW WW->ZZ and WZ->WZ



different behaviour for WW-> ZZ

cut	FULL	RES OSP	RES NO OSP
$u s \rightarrow d c e^- e^+ \mu^- \mu^+ (2W2Z)$			
no cut	267.30	248.60	324.38

pb^{-8}



In practice the above ZZ problem not so relevant as

ZZ production in VBF at LHC includes both

ZZ \rightarrow ZZ and WW \rightarrow ZZ (largely dominant)

cut	FULL	RES OSP	RES NO OSP
$p p \rightarrow j j e^- e^+ \mu^- \mu^+$			
no cut	6.399 (4Z: 0.075)	5.895 (4Z: 0.030)	7.543 (4Z: 0.030)
30 GeV	5.844 (4Z: 0.036)	5.714 (4Z: 0.029)	5.764 (4Z: 0.029)
5 GeV	4.321 (4Z: 0.023)	4.305 (4Z: 0.022)	4.315 (4Z: 0.022)

different cuts
around the
Z pole

pb^{-5}

**it is evident that for ZZ final state
strong cuts on $l^+ l^-$ invariant masses are necessary**



ZW

pp -> jj e+ e- μ+ ν_μ

$M_{jj} > 500 \text{ GeV}$, $|\Delta\eta_{jj}| > 2.5$, $p_t^j > 20 \text{ GeV}$, $|\eta_j| < 5$, $|M_{\ell+\ell-} - M_Z| < 15 \text{ GeV}$, $M_{WZ}^{\text{true}} > 250 \text{ GeV}$
 no p_t^{miss} , η_ℓ, p_t^ℓ cuts

	SM
full unpol.	3.980
OSP 1 unpol.	3.969
OSP 2 unpol.	3.939

OSP1
single (W) resonant

Cross sections in 10^{-4} pb: a cut on the true M_{WZ} is understood.

$M_{jj} > 500 \text{ GeV}$, $|\Delta\eta_{jj}| > 2.5$, $p_t^j > 20 \text{ GeV}$, $|\eta_j| < 5$, $|M_{\ell+\ell-} - M_Z| < 15 \text{ GeV}$ $M_{WZ} > 250 \text{ GeV}$,
 $|\eta_\ell| < 2.5$, $p_t^\ell > 20 \text{ GeV}$, $p_t^{\text{miss}} > 40 \text{ GeV}$

	SM TRUE	SM RECO
full unpol.	1.364	1.385
OSP 1 unpol.	1.355	1.377
OSP 2 unpol.	1.342	-

RECO :
reconstructed p_z neutrino
forcing $m(\mu+ \nu_\mu) = m_W$

different methods.
(for details see Ezio's talk)

Cross sections in 10^{-4} pb: a cut on the M_{WZ} is understood (true or reconstructed).

Good agreement

Conclusion: OSP2 for WW cuts for ZZ OSP1 and cut for ZW



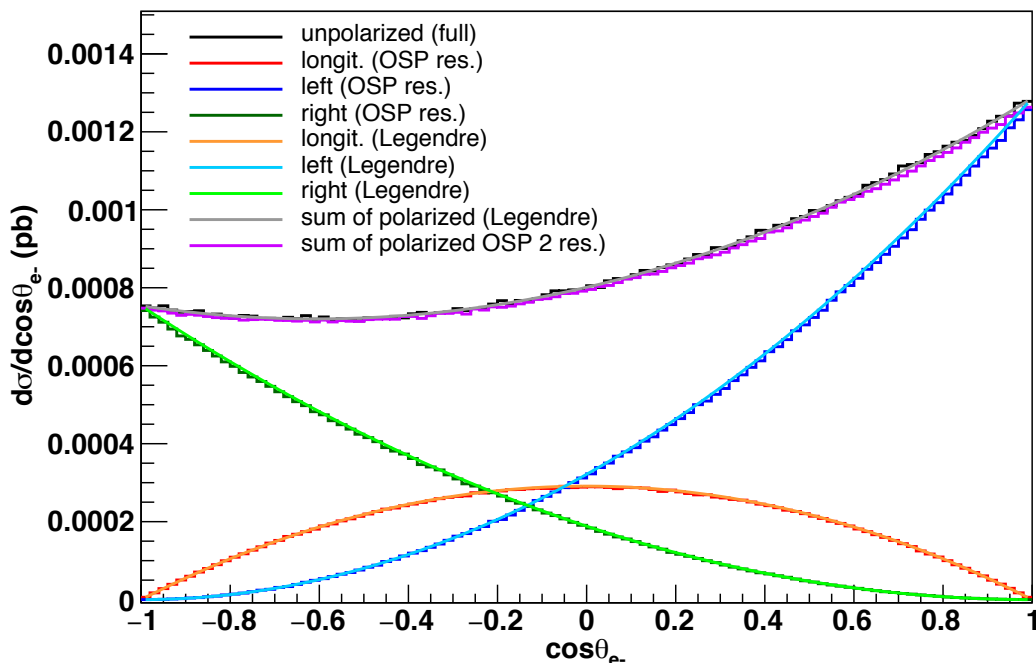
Distributions of the decay polarized amplitudes in Boson rest frame angles

0 = longitudinal $\mathcal{A}_d^0 = ig \sqrt{2} E \sin \theta$ $\mathcal{A}_d^{R/L} = -ig E (1 \mp \cos \theta) e^{\pm i\phi}$

Integrating over the full Φ range interferences disappear.

$$\frac{1}{\frac{d\sigma(X)}{dX}} \frac{d\sigma(\theta, X)}{d \cos \theta dX} = \frac{3}{8} (1 \mp \cos \theta)^2 f_L(X) + \frac{3}{8} (1 \pm \cos \theta)^2 f_R(X) + \frac{3}{4} \sin^2 \theta f_0(X)$$

$d\sigma / d\cos\theta_{e^-}$ (pb), $M_{\text{ww}} > 300$ GeV



no left cuts

polarized components can be extracted from full differential angular distribution by a projection on first 3 Legendre polynomials

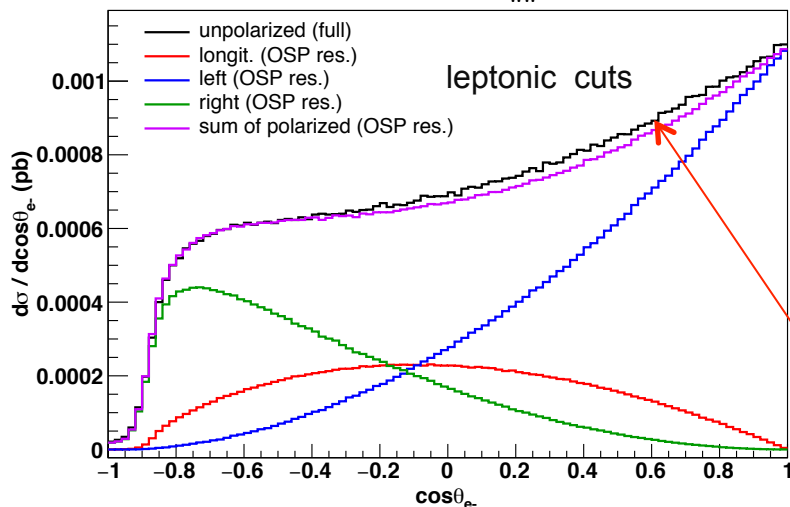
Legendre analysis can be used to test MC

In realistic situation cuts on lepton variables prevent their use for determining polarized components



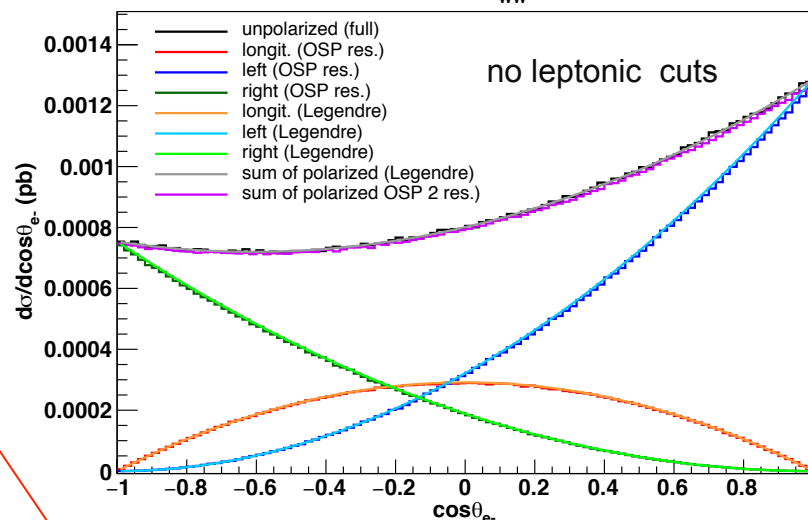
lept cuts

SM: $d\sigma / d\cos\theta_{e^-}$ (pb), $M_{\text{ww}} > 300$ GeV

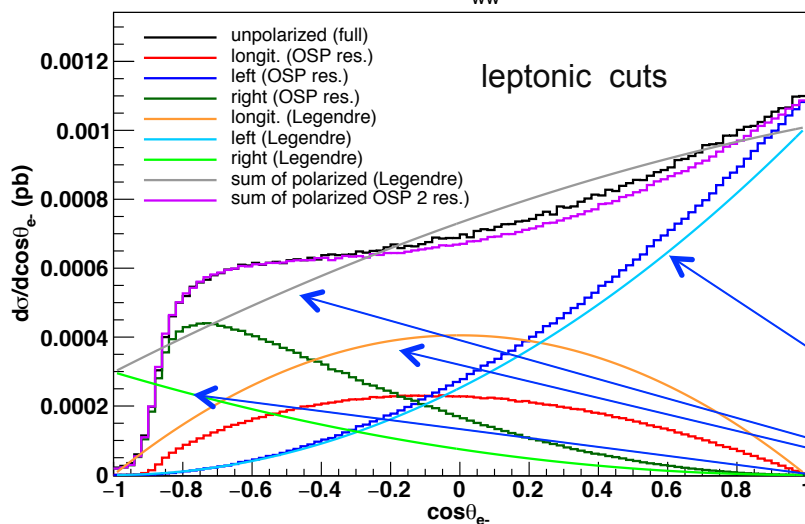


no lept cuts

$d\sigma / d\cos\theta_{e^-}$ (pb), $M_{\text{ww}} > 300$ GeV



$d\sigma / d\cos\theta_{e^-}$ (pb), $M_{\text{ww}} > 300$ GeV



leptonic cuts:

$$p_t^e > 20 \text{ GeV}, \quad |\eta^e| < 2.5.$$

interferences (small)

Pols are affected differently by cuts Mainly at $\vartheta = \pi$

different shapes



failure of legendre method !!

how determine pol xsects from full?



Measured angular distribution can be fitted to a linear combination of MC normalized shapes to obtain the various polarizations.

Verified that this procedure works well for the SM:

starting from generated full angular distributions we reproduce well polarized cross sections and angular distributions.

Fitted results are much better if one uses also the shape of the interference.

Suppose measured distributions correspond to a BSM different from SM

Do we have to repeat the analysis for each model separately?

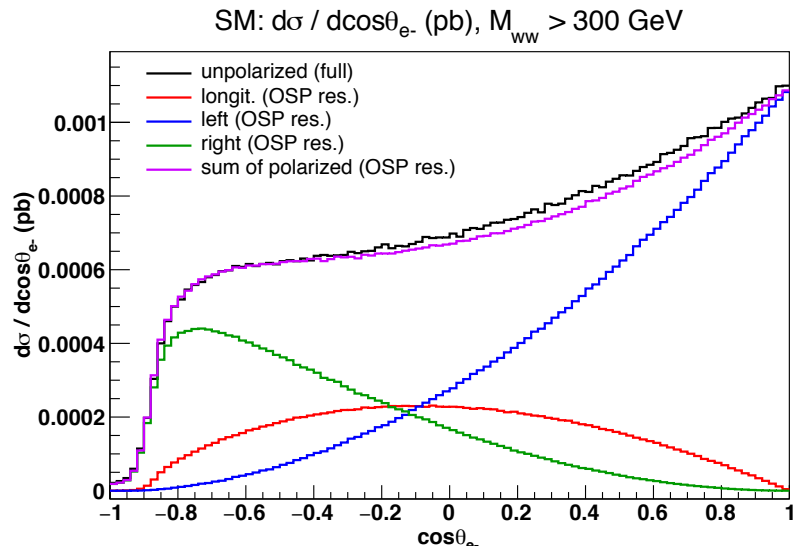
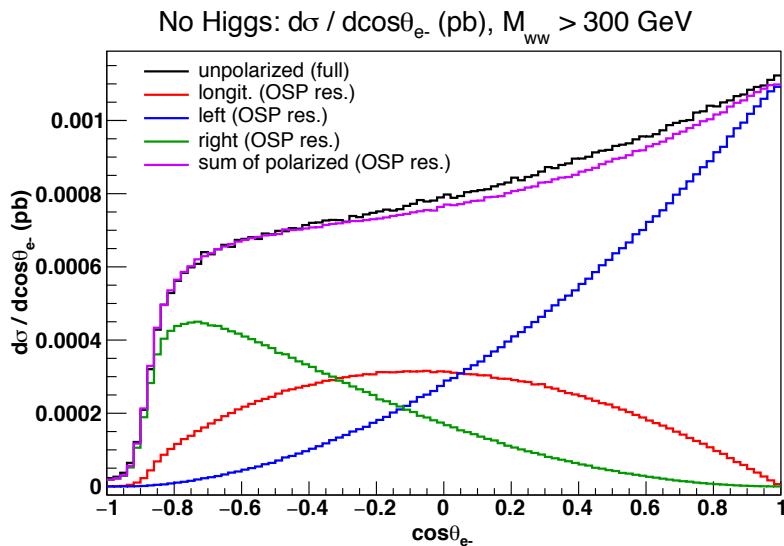
Or the shapes are similar for all models and we can trust the fit made with SM ones?

If not, what is the uncertainty of the fit results?

Tested this possibility for WW fitting Higgsless model results with SM shapes

Higgsless model: SM with $m_h \rightarrow \infty$, no cancellation of terms $\propto s$ in VBS

Unphysical but maximizes differences compared to SM



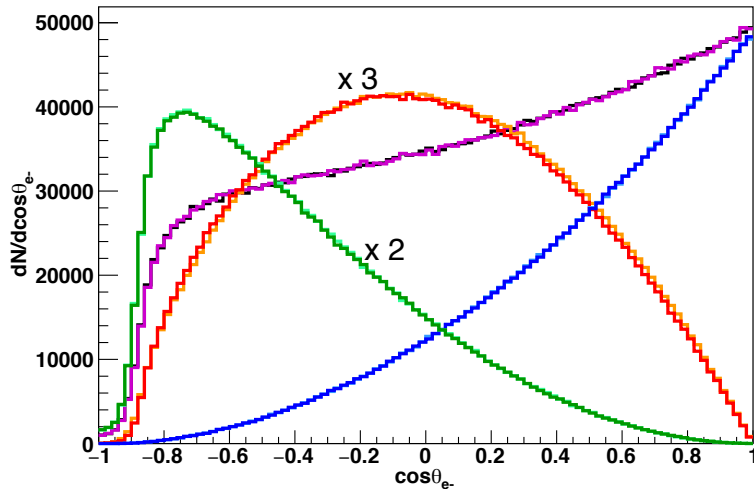


W+W-

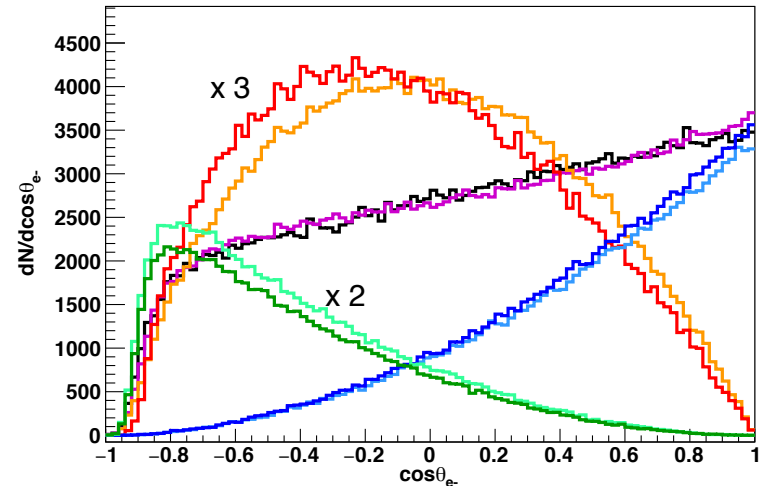
no p_tmiss cut

$p_t^\ell > 20 \text{ GeV}, |\eta^\ell| < 2.5$

SM fit of $\cos\theta_{e^-}$ distributions, $M_{W^+W^-} > 300 \text{ GeV}$



SM fit of $\cos\theta_{e^-}$ distributions, $M_{W^+W^-} > 1000 \text{ GeV}$



Light colours: singly polarized noH Dark colours: fit of exact noH using SM shapes

red longitudinal green right blue left black unpolarized

	Long.	L	R	Int.
no Higgs	0.266	0.481	0.231	0.022
Fit	0.265	0.481	0.230	0.024
$M_{WW} > 1000 \text{ GeV}$				
no Higgs	0.347	0.448	0.173	0.031
Fit	0.344	0.467	0.161	0.029

fractions of the various polarizations

Fit uses SM shapes

← reasonable agreement



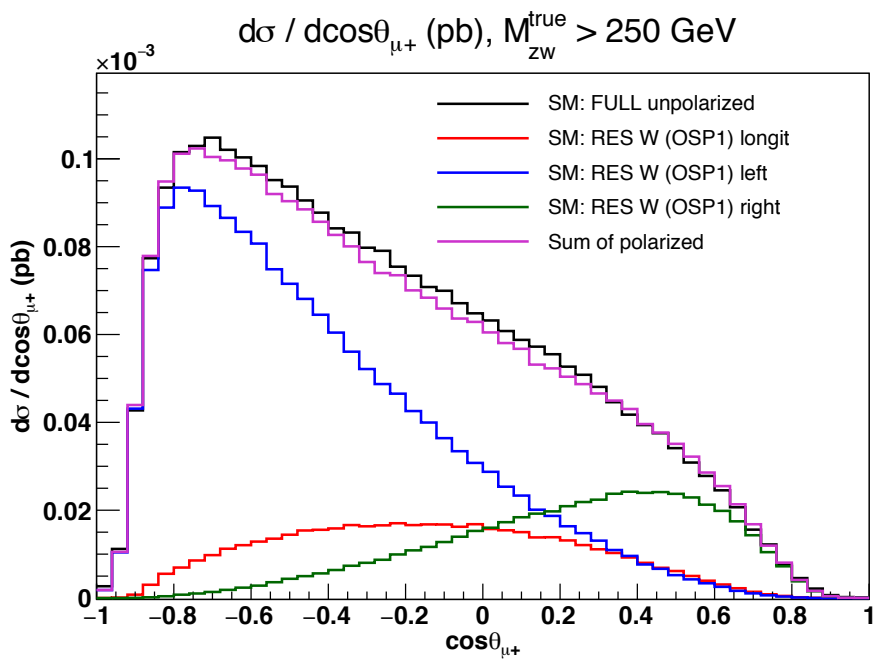
A more realistic case: WZ

pp -> jj e+ e- μ+ ν_μ

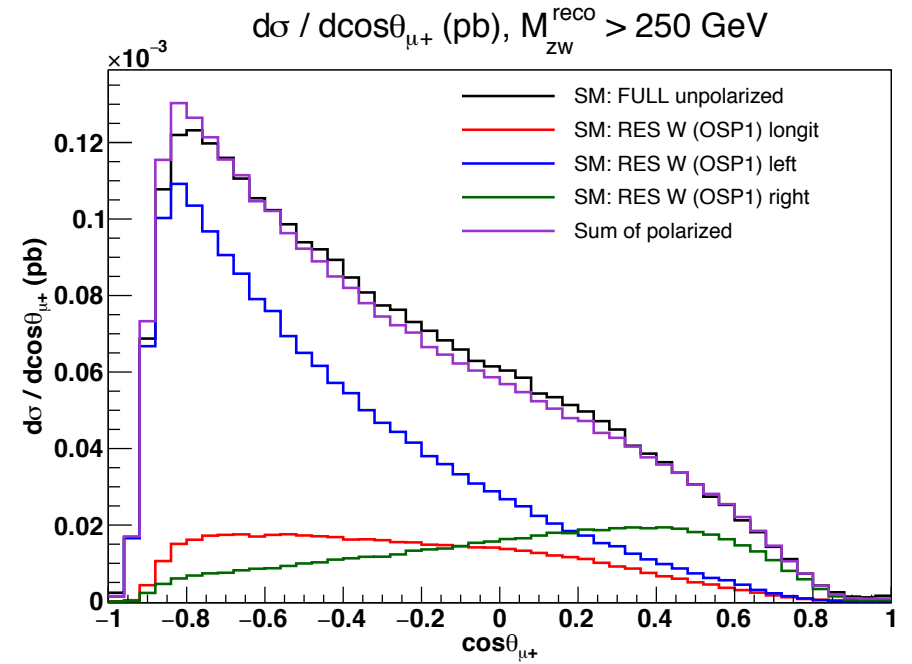
(for ZZ see Ezio's talk)

OSP1 for W resonant cut m(e+ e-) ± 15 GeV around Z pole

RECO : reconstructed p_z neutrino forcing m (μ+ ν_μ) = m_W



red longitudinal
green right
blue left
black unpolarized
true neutrino



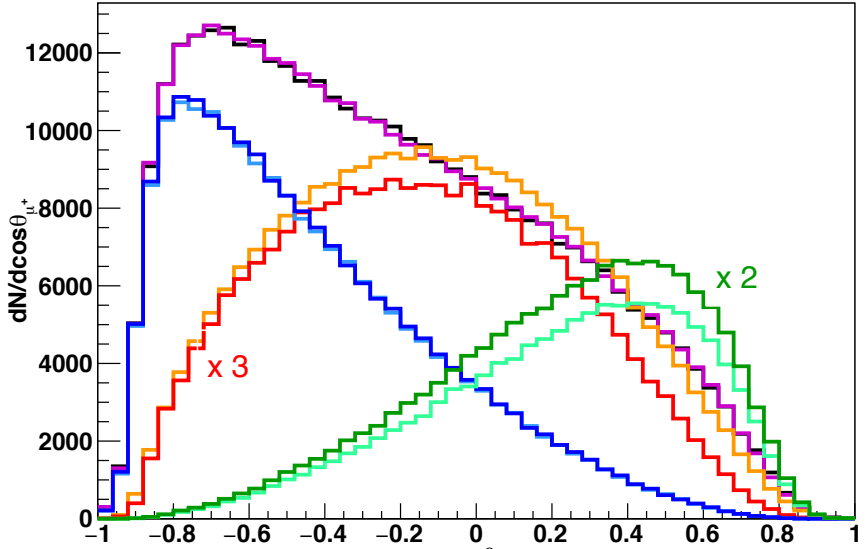
red longitudinal
green right
blue left
black unpolarized
neutrino reconstructed

Reconstruction changes the shapes
it remains a big difference among polarized distributions
one can still use cos θ for fitting the different contributions from the total.
(it works for SM)



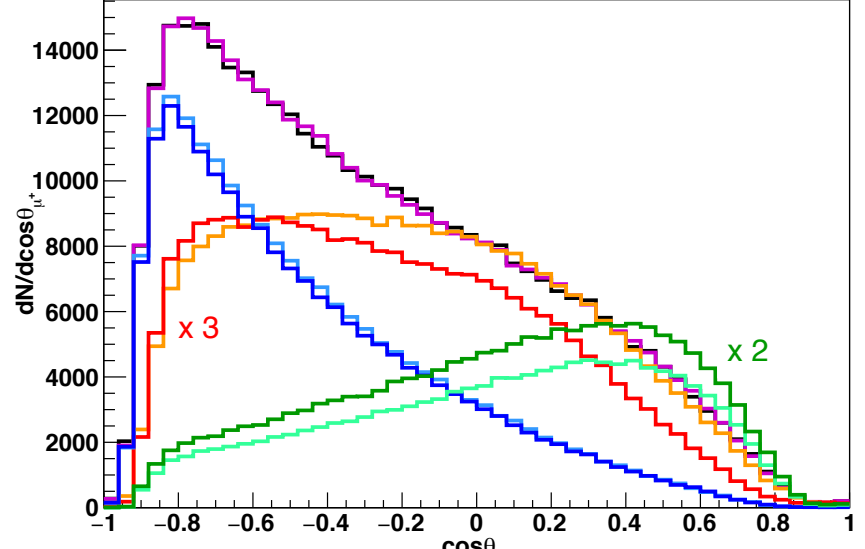
model (in)dependence

Fit of NoHiggs $\cos\theta_{\mu^+}$ distribution with SM templates, no ν reconstruction



Light colours: singly polarized noH

Fit of NoHiggs $\cos\theta_{\mu^+}$ distribution with SM templates, yes ν reconstruction



Dark colours: fit of exact noH using SM shape

	Long.	L	R	Int.
no Higgs	0.256	0.548	0.173	0.024
Fit	0.223	0.552	0.205	0.019

true neutrino

	Long.	L	R	Int.
no Higgs	0.261	0.544	0.179	0.017
Fit	0.231	0.530	0.225	0.015

neutrino reconstructed

Not very accurate: for polarized cross sections errors of ~10%
Is this a superior limit due to extreme no Higgs model?

Not only a neutrino reconstruction problem



- ◆ Double polarized cross sections
 - Double longitudinal cross sections are more sensitive to alternative EWSB
 - Fit to 2- dimensional distributions $(\theta_e \theta_\mu)$
 - their (in)dependence on the model to be studied
- ◆ Polarization in semi-leptonic channel
 - WW and ZW contributions
- ◆ Distributions in leptonic WW events
 - reconstructing W rest frame with 2 neutrinos (and θ distributions).
 - finding alternative variables which discriminate among polarizations
 - several studies ongoing
- ◆ Polarization at $\alpha_s^2 \alpha_{em}^4 + \alpha_{em}^6$
 - signal or background ? needed in any case
- ◆ Qcd corrections
 - should not be difficult for fully leptonic
 - they do not modify the shapes of the distributions
- ◆ EW corrections
 - is it possible? are they relevant given the approximation in the definition?



It is extremely important to define in a consistent way the polarized cross section and to find a method to measure them in a model independent way

A lot of work is ahead of us , both TH and EXP

I hope the present workshop will be the starting point of a true wide collaborations among different groups, TH and EXP for VBS polarization physics.

I am looking forward to tomorrow MC talks because

- The methods employed should be discussed and clarified among MC's authors
- It is essential that the results are validated by several MC's

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- It would be important for me to understand how the polarization measurements (WZ, ttbar ..) have been performed in some details
 - e.g. in ttbar, reconstruction of the 2 neutrinos how much does it affect distributions?
 - in WZ how is it accounted for possibility of other models?
 - how the templates are used for measurement ?
 - what about "uncut " θ distributions?



BACKUP

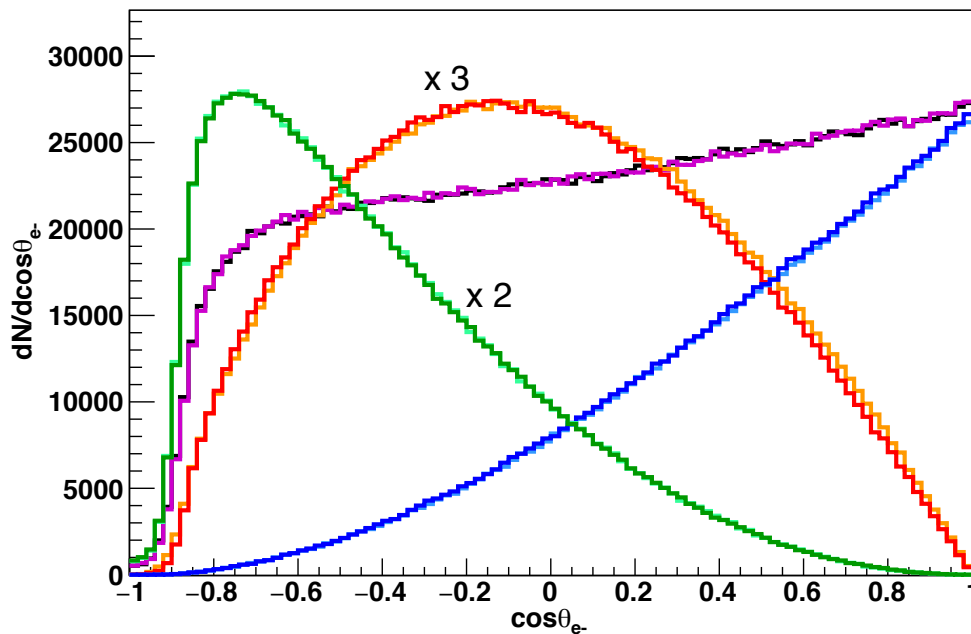


W+W-

with $p_{tmiss} > 40$ GeV

$p_t^\ell > 20$ GeV, $|\eta^\ell| < 2.5$

SM fit of $\cos\theta_{e^-}$ distributions, $M_{ww} > 300$ GeV, full set of lepton cuts



Light colours: singly polarized noH Dark colours: fit of exact noH using SM shapes

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