



W Reconstruction in semi/full-leptonic Vector Boson Scattering

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Formula Derivation for a process with one ν

$$m_W^2 = (p_\mu + p_\nu)^2 \quad \text{ultra relativistic limit} \quad \xrightarrow{m \rightarrow 0} \quad 2p_\mu p_\nu;$$

- Let's solve for the longitudinal component of the neutrino $p_{\nu L}$;

$$\underbrace{(p_{lL}^2 - E_l^2)}_a p_{\nu L}^2 +$$

$$\underbrace{(m_W^2 p_{lL} + 2p_{lL} \vec{p}_{lT} \vec{p}_{\nu T})}_b p_{\nu L} +$$

$$\underbrace{\frac{m_W^4}{4} + (\vec{p}_{lT} \vec{p}_{\nu T})^2 + m_W^2 \vec{p}_{lT} \vec{p}_{\nu T} - E_l^2 \vec{p}_{\nu T}^2}_c = 0;$$

$$\boxed{p_{\nu L_{1,2}} = \frac{-b \pm \sqrt{\Delta}}{2a}} \quad \text{where} \quad \Delta = b^2 - 4ac$$

As a second order parametric equation, Δ determines the number of solution and their nature.

$m_W \Rightarrow$ **fixed value** (80.385 GeV)

- if $\Delta > 0 \Rightarrow$ 2 solutions (+/-)
- if $\Delta < 0$, from the formula:

we have two working options, we choose the first one

$$\Delta(p_L) \begin{cases} \text{put } \Delta = 0 \\ m_W = m_{WT} \Rightarrow \text{correct } m_W \text{ with transverse mass} \end{cases}$$

VBS Semileptonic channel

PHANTOM PARAMETER for the production

- semi-leptonic: $pp \rightarrow jjjj\mu^+\nu_\mu$
- full-leptonic:
 $pp \rightarrow jj\mu^+\nu_\mu e^+\nu_e$
- Parton level events
- MC generator: Phantom
- events generated with
NNPDF30_nnlo_as_0118
- CALCULATION TYPE: α_e^6
- SCALE CHOICE: (invariant
mass of the 2 central jets and
of 2 leptons)/ $\sqrt{2}$

Kinematical cuts:

- $p_{\top}^{\ell} > 20$ GeV
- $|\eta^{\ell}| < 3$
- $p_{\top}^{\text{min}} > 30$ GeV
- $|\eta_j| < 5.4$
- $p_{\top}^{\text{miss}} > 20$ GeV
- $m_{jj} > 500$ GeV
- $\Delta R_{j\ell} > 0.3$

Selection criteria

- Selection 1: $p_{\nu L} > 50 \text{ GeV}$
- Selection 2: $p_{\nu L} a/b > -0.5 \rightarrow$ choosing larger* solution
- Selection 3: $\vec{p}_{\nu} \cdot \vec{p}_W < 5000 \text{ GeV}^2$
- Selection 4: $\vec{p}_{\nu} \cdot \vec{p}_W a/b < 25 \text{ GeV}$
- Combined:
 - Selection of solutions with $p_{\nu L} < 50 \text{ GeV}$
 - If both solutions lie above 50 GeV, Selection 3 is applied
 - solution with lower value of the $|\vec{p}_{\nu} \cdot \vec{p}_W a/b|$ is taken.

$$*p_{\nu L} a/b = -\frac{1}{2} \pm \frac{\sqrt{\Delta}}{2b}$$

Selection efficiencies

- **Efficiencies:** count number of "correct solutions"
 - correct solution: the one which lies closest to the truth
 - wrong solution: the one which lies further from to the truth
- Solutions with negative discriminant are not taken into account

Fixed m_W (on-shell)

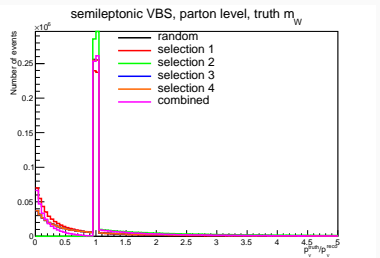
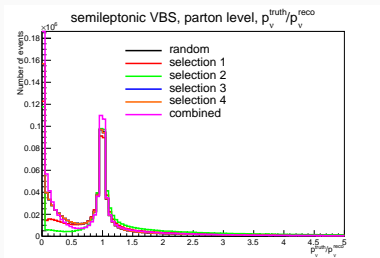
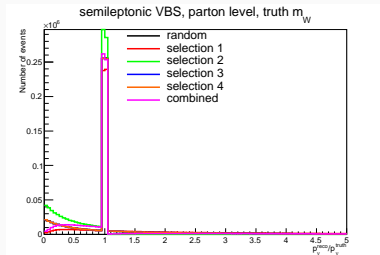
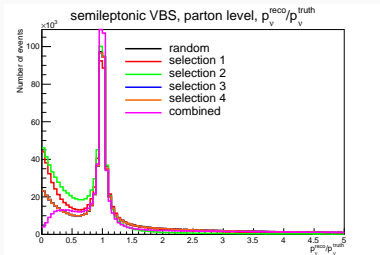
- Random: 49.9 %
- Selection 1: 56.3 %
- Selection 2: 61.3 %
- Selection 3: 46.0 %
- Selection 4: 52.6 %
- Combined selection 44.6 %

Truth m_W (off-shell)

- Random: 50.0 %
- Selection 1: 53.7 %
- Selection 2: 58.4 %
- Selection 3: 48.1 %
- Selection 4: 51.4 %
- Combined selection 46.4 %

It turns out that efficiencies is not always a good measure for selection performance

Selection criteria



■ Fixed m_W

■ Truth m_W

Ljubljana variable

In order to visualize the whole range of relative error we introduced a variable

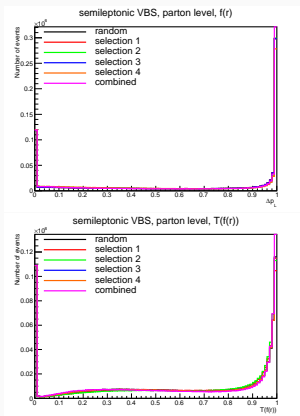
$$f(r) = \frac{2}{\frac{1}{r} + r}; \text{ where } r = \frac{p_L^{reco}}{p_L^{truth}}$$

Properties:

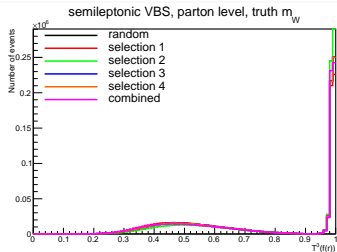
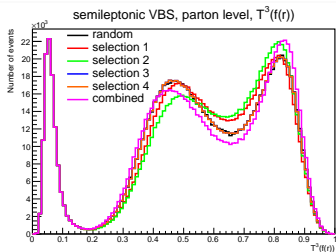
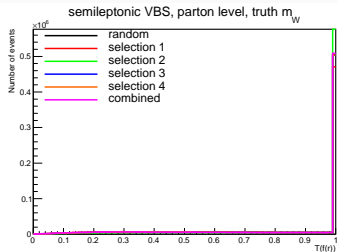
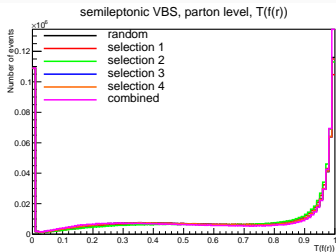
- wrong solutions: $f(r) \rightarrow 0$
- correct solutions: $f(r) \rightarrow 1$

In order to enhance separation between selection criterion we need a phase space transformation T :

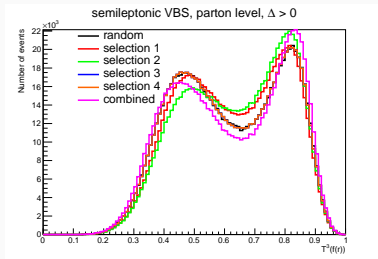
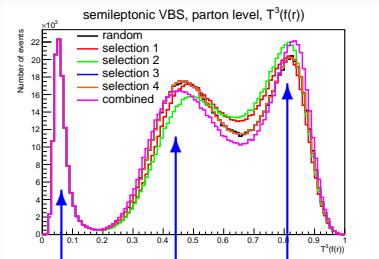
- $T(x) = \arcsin(2x - 1)/\pi + \frac{1}{2}$; where $x = f(r)$



Ljubljana variable



Ljubljana variable



$\Delta < 0$

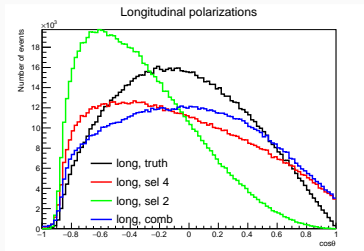
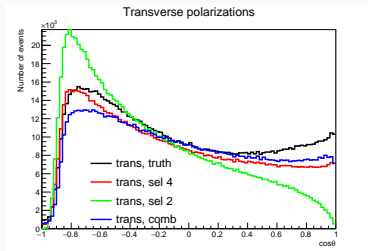
$\Delta > 0$, "wrong"

$\Delta > 0$, "correct"

- first peak are the events with negative discriminant
- second peak are wrong reconstructed events with positive discriminant
- third peak are correctly reconstructed events

semileptonic angular distributions

$\cos\theta$ distribution of the charged lepton in the W reference frame.



Combined selection criterion gives angular distribution closer to truth.

VBS Fully leptonic channel

Reco of neutrino momentum in $pp \rightarrow jj e^+ \nu_e \mu^+ \nu_\mu$

- 8 unknown parameters (2 x neutrino four momentum)
- 6 equations:
 - $\vec{p}_T^{\nu_\mu} + \vec{p}_T^{\nu_e} = \vec{p}_T^{\text{miss}} \quad (2x)$
 - $(p^\ell + p^\nu)^2 = m_W^2 \quad (2x)$
 - $p_\nu^2 = 0 \quad (2x)$
- Remaining 2 equations:
 1. Setting some parameters to fixed values - for example:
 $M_{WW}^2 = (p_e + p_{\nu_e} + p_\mu + p_{\nu_\mu})^2$ and $M_{\nu\nu}^2 = (p_{\nu_e} + p_{\nu_\mu})^2$, M_{WW} and $M_{\nu\nu}$ are fixed numbers.¹
 2. Using of *MT2*-Assisted On-Shell (MAOS) quantities, i.e. minimization of the transverse masses of the lepton-neutrino pairs.¹
 3. Other ideas ??

¹arXiv:hep-ph/0603011, Higgs spin analysis in Collins-Soper frame using opening angles of different-flavour final state leptons

¹arXiv:0908.0079

2. Reco with MAOS quantities - Equations

- MAOS estimations $\vec{p}_T^{\nu e'}$ and $\vec{p}_T^{\nu \mu'}$ for neutrinos transverse momenta can be obtained by minimizing the function $f(\vec{p}_1, \vec{p}_2) = \max\{M_T^{W_1}, M_T^{W_2}\}$, constrained by a bond $\vec{p}_1 + \vec{p}_2 = \vec{p}_T^{\text{miss}}$, where

$$M_T^{W_1} = 2(|\vec{p}_T^\mu| |\vec{p}_1| - \vec{p}_T^\mu \cdot \vec{p}_1), \quad M_T^{W_2} = 2(|\vec{p}_T^e| |\vec{p}_2| - \vec{p}_T^e \cdot \vec{p}_2)$$

- Minimum of the function f defines quantity M_{T2} :

$$M_{T2} \equiv \min_{\vec{p}_1 + \vec{p}_2 = \vec{p}_T^{\text{miss}}} f(\vec{p}_1, \vec{p}_2) = f|_{\vec{p}_T^{\nu e'}, \vec{p}_T^{\nu \mu'}} \quad (1)$$

- p_L are then determined from the m_W constraints
- Solution of the problem (1), under assumption $p_T^{WW} \sim 0$ (approximate solution):

$$\vec{p}_T^{\nu e'} = -\vec{p}_T^\mu \quad \vec{p}_T^{\nu \mu'} = -\vec{p}_T^e$$

2. Reco with MAOS quantities - Equations

Exact solution:

- $\min \left[\max \{ M_T^{W_1}, M_T^{W_2} \} \right]$ can always lie only on the intersection of $M_T^{W_1}$ and $M_T^{W_2}$. \Rightarrow Additional bond: $M_T^{W_1} = M_T^{W_2}$

- It follows that:

$$\Rightarrow 2(|\vec{p}_T^\mu||\vec{p}_1| - \vec{p}_T^\mu \cdot \vec{p}_1) = 2(|\vec{p}_T^e||\vec{p}_2| - \vec{p}_T^e \cdot (\vec{p}_T^{\text{miss}} - \vec{p}_1))$$

$$|\vec{p}_T^\mu||\vec{p}_1| - \vec{p}_T^{\ell\ell} \cdot \vec{p}_1 + \vec{p}_T^e \cdot \vec{p}_T^{\text{miss}} = |\vec{p}_T^e| \sqrt{|\vec{p}_T^{\text{miss}}|^2 - 2\vec{p}_T^{\text{miss}} \cdot \vec{p}_1 + |\vec{p}_1|^2}$$

$$(|\vec{p}_T^\mu||\vec{p}_1| - |\vec{p}_T^{\ell\ell}||\vec{p}_1| \cos \varphi + \vec{p}_T^e \cdot \vec{p}_T^{\text{miss}})^2 =$$

$$|\vec{p}_T^e|^2 |\vec{p}_T^{\text{miss}}|^2 - 2|\vec{p}_T^e|^2 |\vec{p}_T^{\text{miss}}||\vec{p}_1| \cos(\varphi + \varphi_0) + |\vec{p}_T^e|^2 |\vec{p}_1|^2$$

- φ_0 - angle between \vec{p}_T^{miss} and $\vec{p}_T^{\ell\ell}$

$$\rightarrow \text{Parameter of the equation: } \varphi_0 = \arccos \left(\frac{\vec{p}_T^{\ell\ell} \cdot \vec{p}_T^{\text{miss}}}{|\vec{p}_T^{\ell\ell}||\vec{p}_T^{\text{miss}}|} \right)$$

- φ - angle between $\vec{p}_T^{\ell\ell}$ and \vec{p}_1 ;

2. Reco with MAOS quantities - Equations

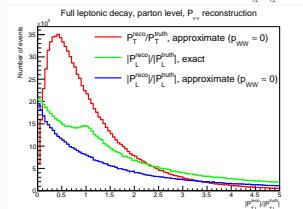
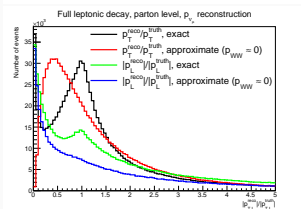
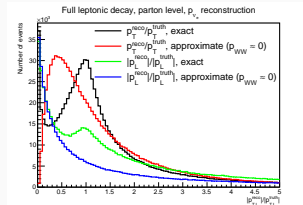
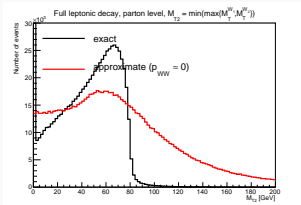
$$\begin{aligned}
 & \underbrace{(|\vec{p}_T^\mu|^2 + |\vec{p}_T^{\ell\ell}|^2 \cos^2 \varphi - 2|\vec{p}_T^\mu||\vec{p}_T^{\ell\ell}| \cos \varphi - |\vec{p}_T^e|^2)}_{f(\varphi)} |\vec{p}_1|^2 + \\
 & \underbrace{(2(|\vec{p}_T^\mu| - |\vec{p}_T^{\ell\ell}| \cos \varphi) \vec{p}_T^e \cdot \vec{p}_T^{\text{miss}} + 2|\vec{p}_T^e|^2 |\vec{p}_T^{\text{miss}}| \cos(\varphi + \varphi_0))}_{g(\varphi)} |\vec{p}_1| + \\
 & \underbrace{(\vec{p}_T^e \cdot \vec{p}_T^{\text{miss}})^2 - |\vec{p}_T^{\text{miss}}|^2 |\vec{p}_T^e|^2}_c = 0
 \end{aligned}$$

- Equation of the intersection curve in parametric form - x-axis of coordinate system coinciding with the $\vec{p}_T^{\ell\ell}$ direction;

$$|\vec{p}_1| = \frac{-g(\varphi) \pm \sqrt{g(\varphi)^2 - 4cf(\varphi)}}{2f(\varphi)}, \quad \vec{p}_2 = \vec{p}_T^{\text{miss}} - \vec{p}_1$$

- Minimum of M_{T2} on the intersection curve can be found numerically;
- The following plots are produced by evaluating M_{T2} in 2000 points;

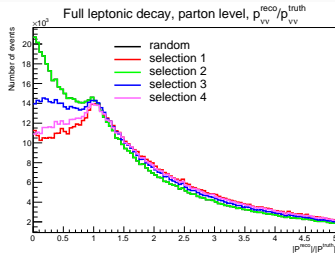
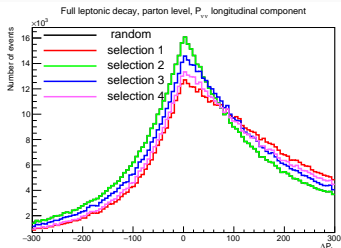
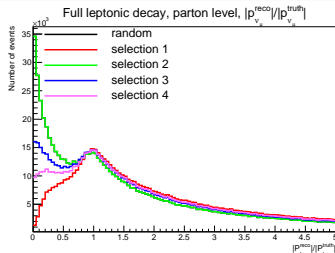
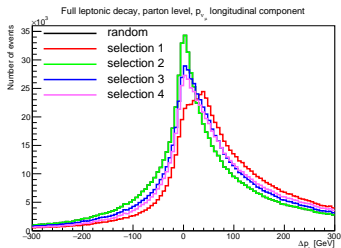
Results - random solution choice



- Exact solution performs better as we have peak at 1.
- M_{T_2} has a sharp edge at m_W

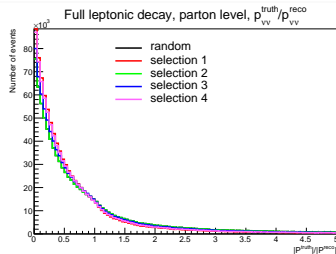
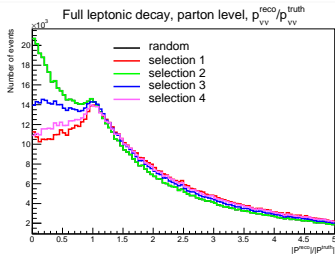
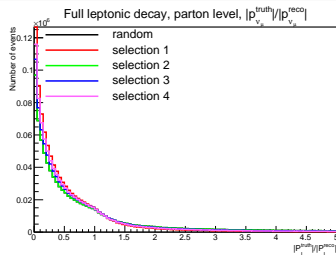
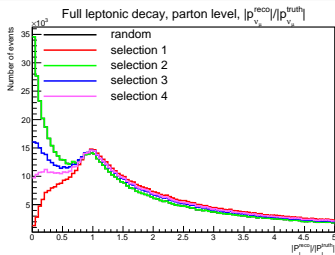
Performance of different selections

Error and relative error in the reconstruction of longitudinal component for ν_μ and $\nu\nu$ pair momenta.



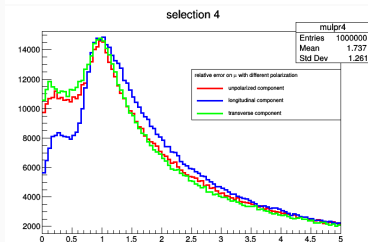
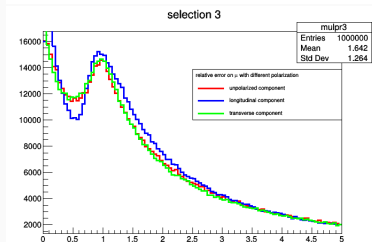
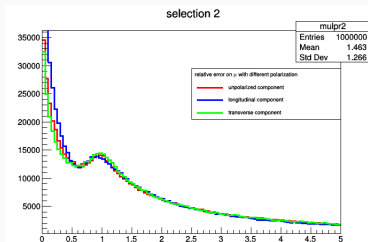
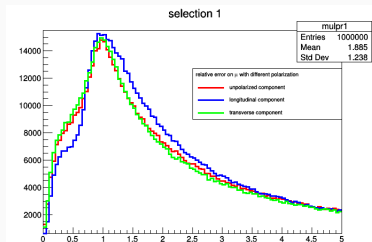
Results - random solution choice

Some selections improve p^{reco}/p^{truth} ratio but degrade p^{truth}/p^{reco}



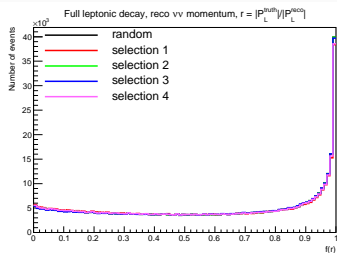
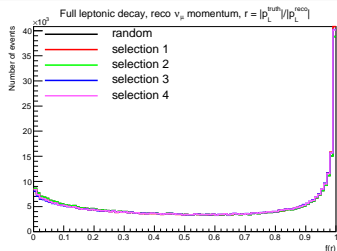
Polarization effect on selection criteria

Effect of polarization on reconstruction is negligible



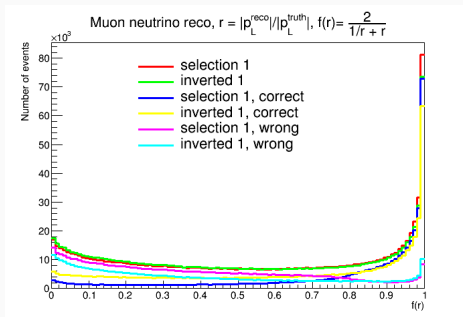
Efficiencies:

- Random: 50.0 %
- Selection 1: 42.2 %
- Selection 2: 50.0 %
- Selection 3: 48.2 %
- Selection 4: 45.1 %



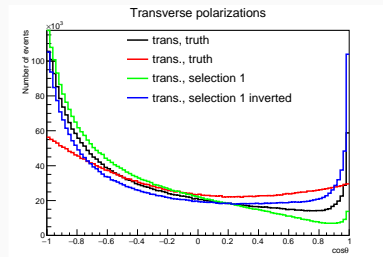
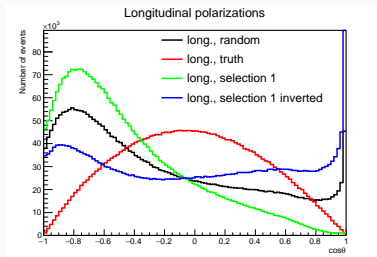
Selection performance

- inverted selection 1 has better efficiency
- inverted selection 1 has worst relative error
- inverted selection 1 has selects better in case of events with bad reconstruction



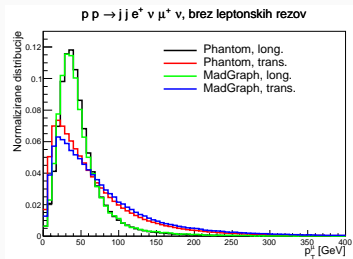
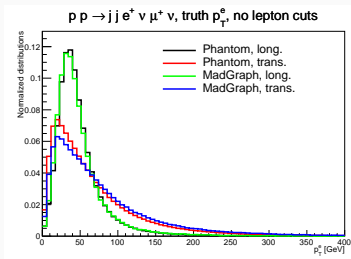
Angular distributions with selection criteria

- We are not able to reconstruct truth angular distribution accurate enough.
- inverted selection 1 increase discriminating power for separation of polarization



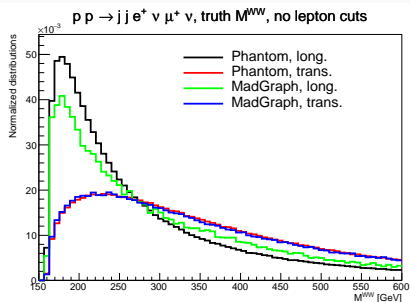
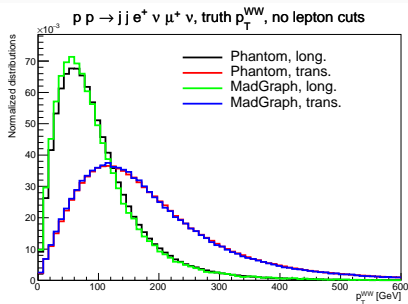
Phantom to MadGraph comparison

Phantom to MadGraph comparison



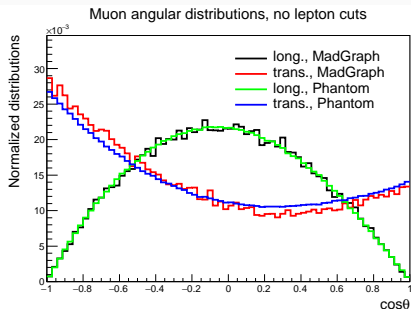
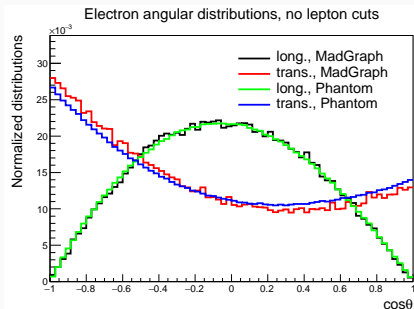
- in MG5 the generation is carried out in 2 steps
 1. polarized $pp \rightarrow W^+ W^+ jj$
 2. decay of the W with MadSpin
- we modified matrix.f file in MadGraph distribution so we produce $W_L W_L$ and $W_T W_T$ pairs
- we used MadSpin in the "bridge" mode using helicity-classified decays
- we needed to fix some python technicalities

Phantom to MadGraph comparison



Phantom to MadGraph comparison

- Good agreement can be observed between the two MC generator



Conclusions

CONCLUSIONS: WW reconstruction

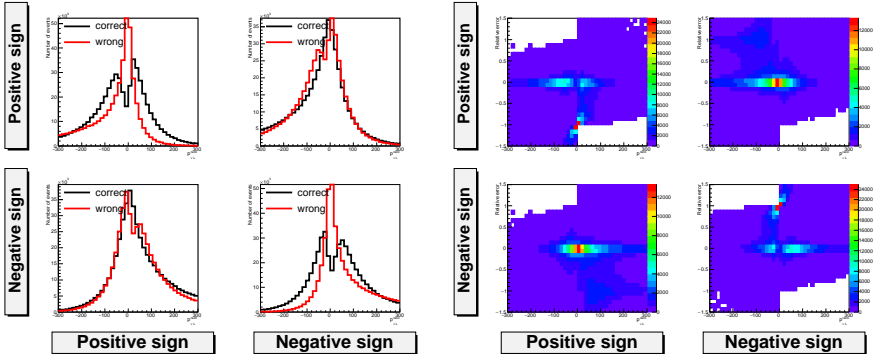
- we proposed different approach for evaluate selection criterion
- it is not clear that the use of selection criteria improves performance of reconstruction algorithm
- the use of alternative reconstruction algorithm can be tested in fully-leptonic channel
- we carried out the first comparison of polarized VBS samples between MadGraph and Phantom

References

- [1] arXiv:hep-ph/0603011
- [2] arXiv:0801.3359 [hep-ph]
- [3] arXiv:1710.09339
- [4] arXiv:1205.2484
- [5] arXiv:hep-ph/9406381
- [6] arXiv:hep-ph/9406381
- [7] arXiv:0908.0079
- [8] B. Hoonhout, K. Oussoren, S. Bentvelse: *Higgs spin analysis in Collins-Soper frame using opening angles of different-flavour final state leptons* (link)

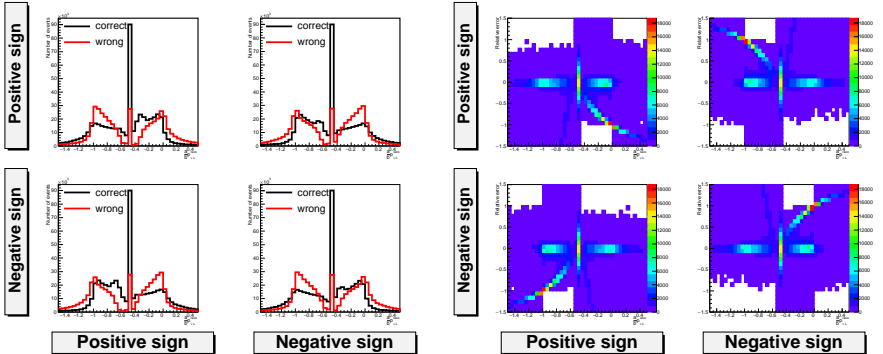
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Criteria Selection 1 at Parton Level



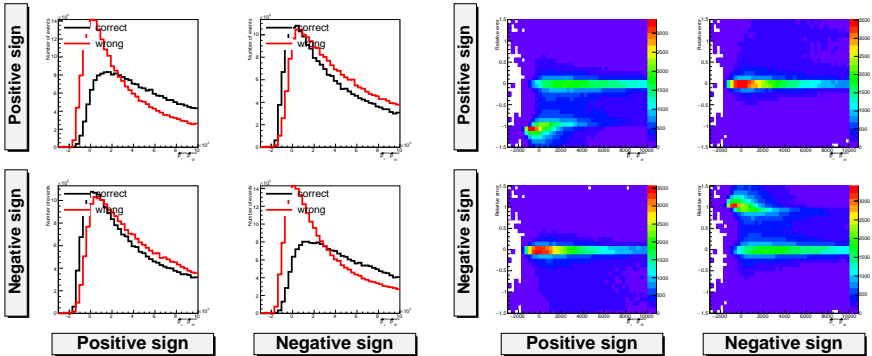
Selection 1 - algorithm discards the solutions which have absolute value smaller than 50 GeV, if both solutions lie under or above 50 GeV, random solution is chosen.

Criteria Selection 2 at Parton Level



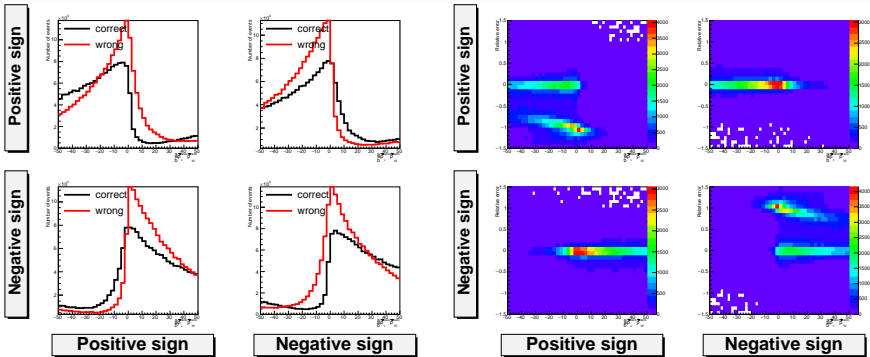
selection 2 - algorithm discards all the solutions for which $-p_L * a/b < 0.5$. If both solutions pass or don't pass this criterium, random solution is chosen.

Criteria Selection 3 at Parton Level



Selection 3 - if the scalar product of the reconstructed neutrino three-momentum (solution is taken for the longitudinal component) with the reconstructed W three-momentum is smaller than 2500GeV^2 , the solution is discarded. If both solutions lie under or above 2500GeV^2 , random solution is chosen.

Criteria Selection 4 at Parton Level



selection 4- if the value of the scalar product of the reconstructed neutrino three-momentum (solution is taken for the longitudinal component) with the reconstructed Ws three-momentum (each threaded separately), multiplied by a/b , is smaller than 30 GeV or larger than 25 GeV, the solution is discarded. a and b are the parameters of the