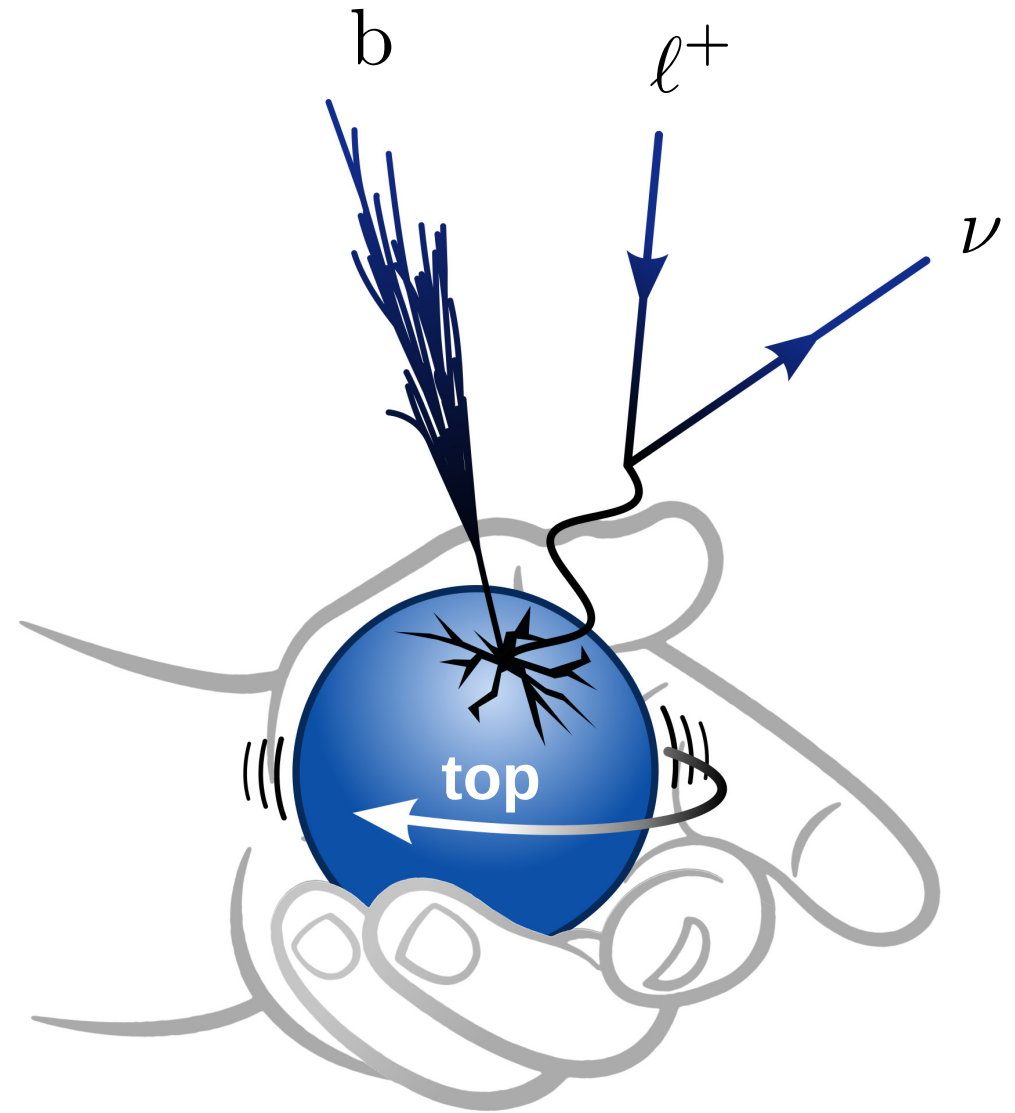


Top quark polarization measurements

Matthias Komm



Overview

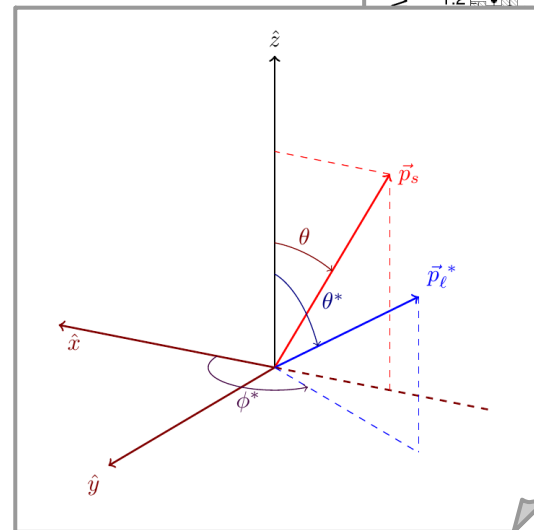
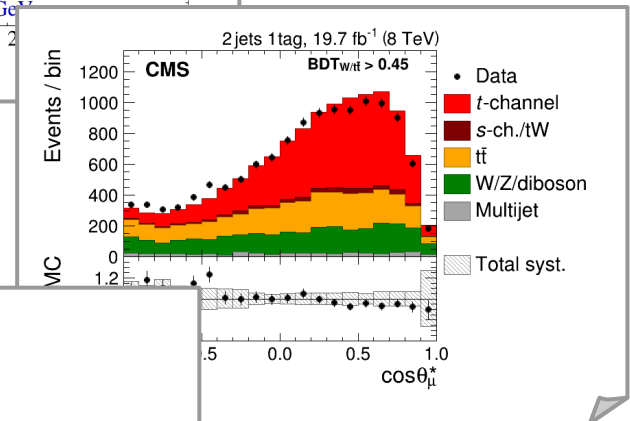
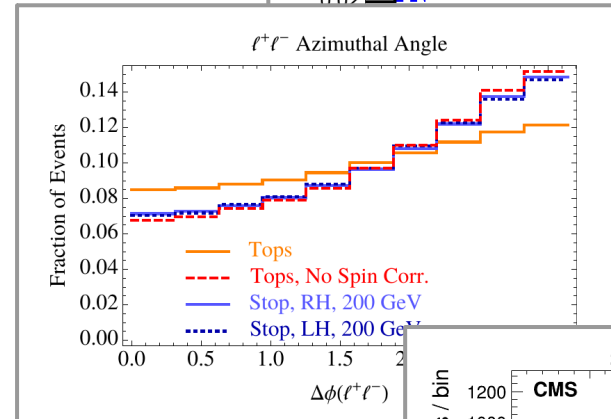
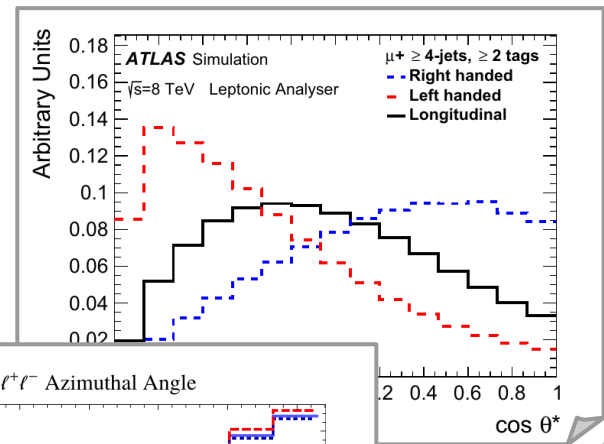
- top quark properties
 - coupling structure
 - spin analyzers
 - EFT & anomalous couplings
 - W helicity fractions

- W helicity fraction measurements by ATLAS & CMS
 - Eur. Phys. J. C 77 (2017) 264
 - Phys. Lett. B 762 (2016) 512

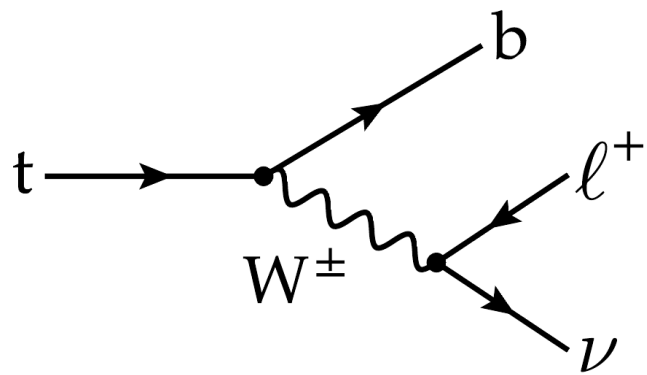
- spin correlation in $t\bar{t}$ production by ATLAS ATLAS-CONF-2018-027

- single top quark polarization in t-channel by CMS JHEP 04 (2016) 073

- Wtb vertex analysis in t-channel by ATLAS JHEP 12 (2017) 017



Top quark properties



- top quark = heaviest particle of the SM:
 $m_{\text{top}} = 172.4 \pm 0.5 \text{ GeV} > m_{\text{H}} > m_{\text{W}}$ [1]
- only quark which decays via electroweak interactions to on-shell W bosons
 - short life time: $1/\Gamma_{\text{top}} = 10^{-25} \text{ s} < 1/\Lambda_{\text{QCD}}$
 - no hadronization, retains spin coherence
 - can study “free quark” properties
- decays almost exclusively to bW due to CKM matrix $V_{\text{tb}} \approx 1$

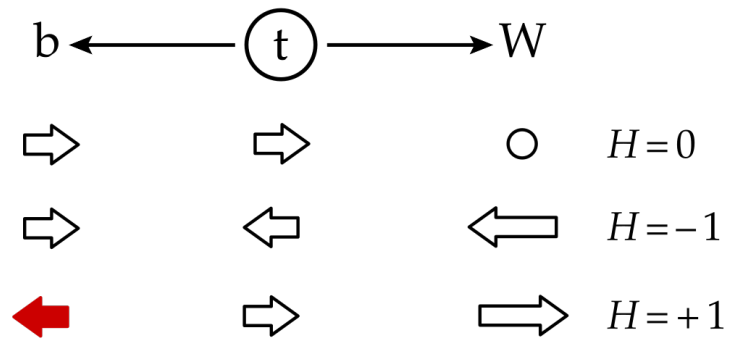
➤ coupling structure

$$\mathcal{L}_{Wtb}^{\text{SM}} \propto \bar{b} \left(\begin{array}{|c|} \hline \gamma_{\mu} \\ \hline \text{V} \\ \hline \end{array} - \begin{array}{|c|} \hline \gamma_{\mu} \gamma_5 \\ \hline \text{A} \\ \hline \end{array} \right) t W^{\mu}$$

- “vector” – “axialvector” structure
- only left-handed fermions can couple to W bosons

influences angular distributions

e.g. W boson helicity



suppressed!

[1] Phys. Rev. D 93, 072004 (2016)

Spin analyzers & EFT

- top quark “spin-analyzing power” [1]
 - dimensionless quantity $\alpha_X \in [-1; 1]$
 - degree of polarization wrt. top decay product in top quark rest frame
 - for top antiquark: $\alpha_X(t) = -\alpha_{\bar{X}}(\bar{t})$
 - charged lepton direction ideal probe of top quark spin due to W helicity interference

Decay product X	spin-analyzing power α_X	
	LO	NLO
ℓ^+	1.00	0.998
ν	-0.32	-0.33
\bar{q}' (down-type)	1.00	0.93
q (up-type)	-0.32	-0.31
b	-0.41	-0.39
W^+	0.41	0.39

- characterize deviations from SM coupling structure
 - effective field theory approach

$$\mathcal{L}^{\text{eff}} = \mathcal{L}_{(4)}^{\text{SM}} + \frac{1}{\Lambda} \sum_i c_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} O_i^{(6)} + \mathcal{O}\left(\frac{O^{(7)}}{\Lambda^3}\right)$$

- recast some dim.-6 effective operators into anomalous Wtb couplings

$$\mathcal{L}_{Wtb}^{\text{eff}} = -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu \left(\underbrace{V_L P_L}_{\text{SM}} + V_R P_R \right) t W_\mu^- - \frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{m_W} (g_L P_L + g_R P_R) t W_\mu^- + \text{h.c.}$$

anomalous vector- and tensor-like couplings
(in SM $V_L = V_{tb}$, $V_R = g_L = g_R = 0$)

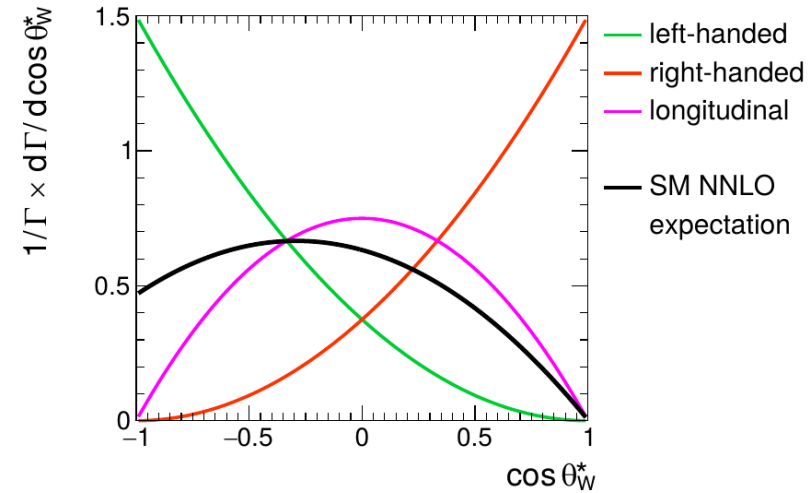
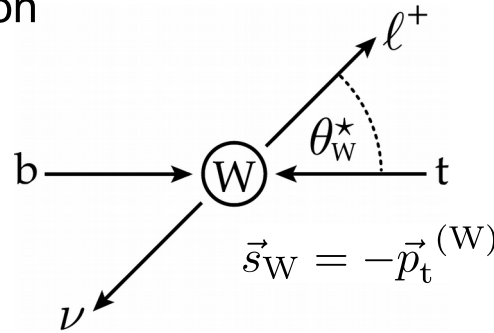
- ➔ spin-analyzing powers can be parametrized as $\alpha_X(V_L, V_R, g_L, g_R)$
- ➔ note: for single top production 4-fermion operator also important (missing here)

[1] Nucl.Phys. B840 349-378, 2010

W boson helicity fractions

- approximate W boson spin with reversed top quark direction in W boson rest frame

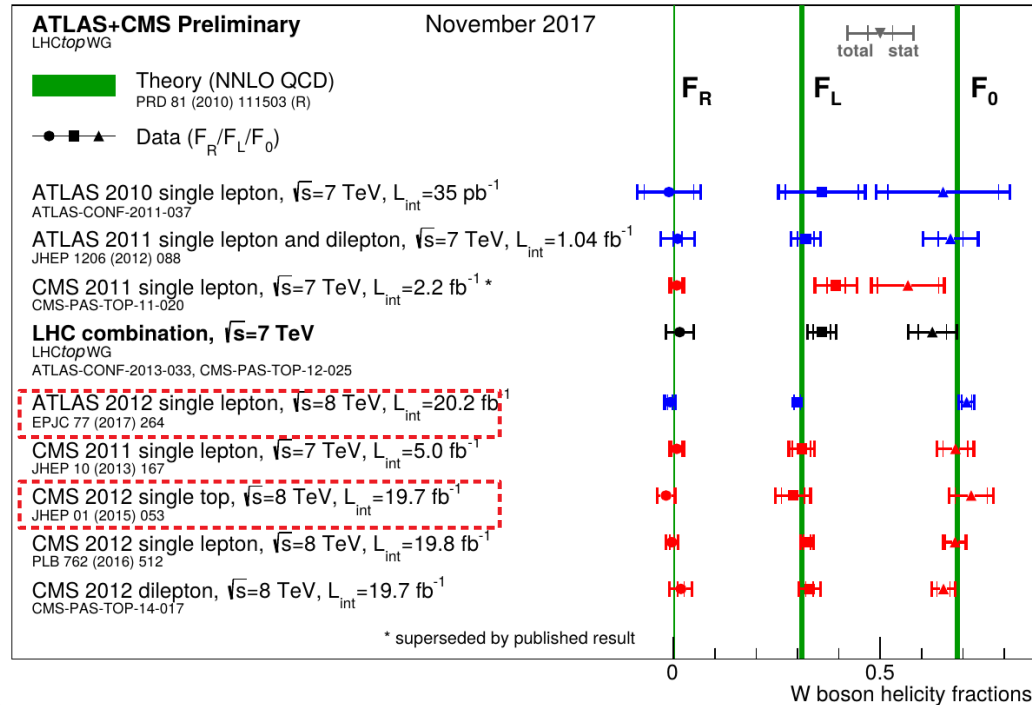
$$\cos \theta_W^* = \frac{\vec{s}_W \cdot \vec{p}_\ell^{(W)}}{|\vec{s}_W| \cdot |\vec{p}_\ell^{(W)}|}$$



- helicity fractions

$$\frac{d\Gamma}{\Gamma \cdot d \cos \theta_W^*} = \frac{3}{8} (1 - \cos \theta_W^*)^2 F_L + \frac{3}{8} (1 + \cos \theta_W^*)^2 F_R + \frac{3}{4} \sin^2 \theta_W^* F_0$$

- normalized as: $F_L + F_R + F_0 = 1$
- SM NNLO predictions [1]
- $F_L = 0.687$, $F_0 = 0.311$, $F_R = 0.002$
- can be measured in any top quark production process!
- $t\bar{t}$ ideal since low background & high stat.
- single top t-channel also possible
- overall analysis strategies very similar across process/experiment



[1] Phys. Rev. D 81, 111503

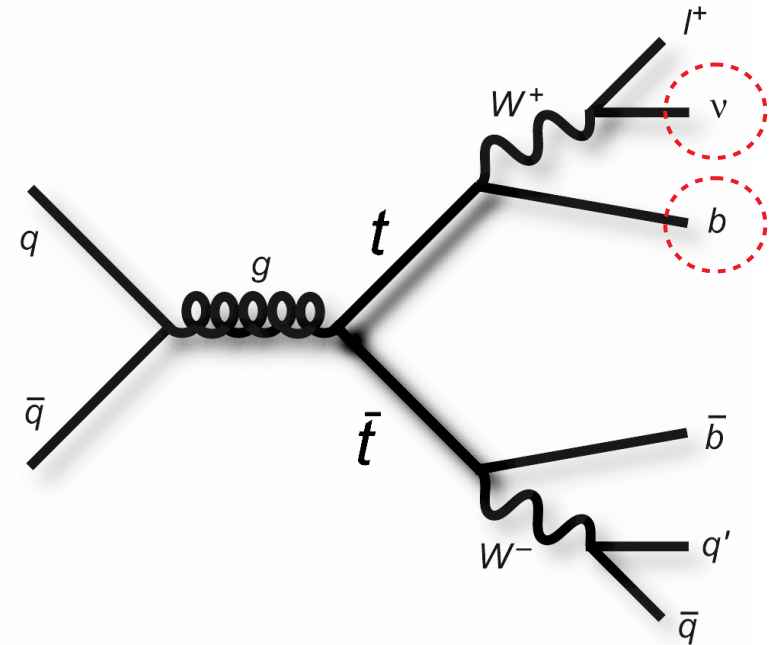
Typical analysis strategy

➤ event selection targeting semileptonic $t\bar{t}$ decay

- 1 lepton (electron or muon)
- at least 4 jet; 2 are b-tagged
- missing transverse energy

➤ kinematic fit

- utilize W boson & top quark mass constraints
- estimate unknown p_z component of neutrino from E_T^{miss}
- find jet-quark assignment for calculating $\cos \theta_W^*$

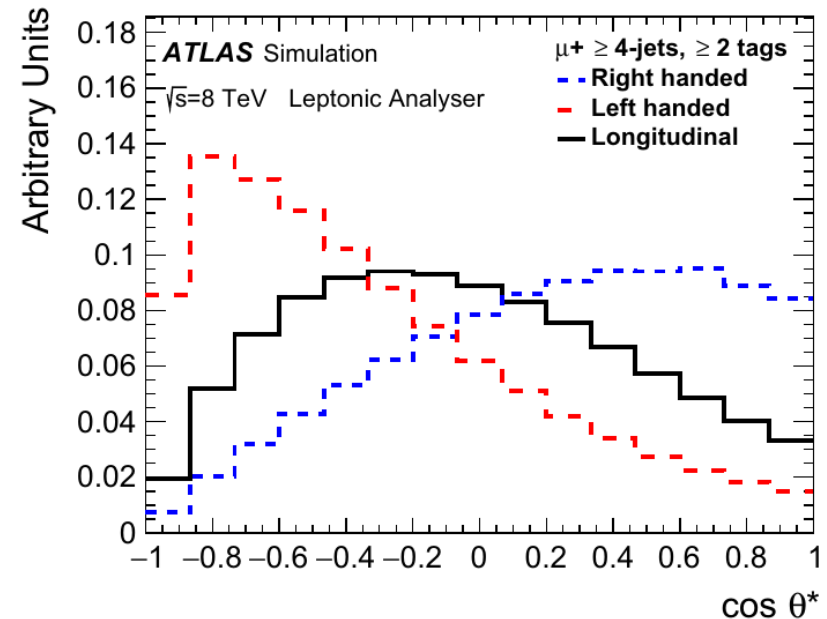


➤ create 3 templates from SM sample

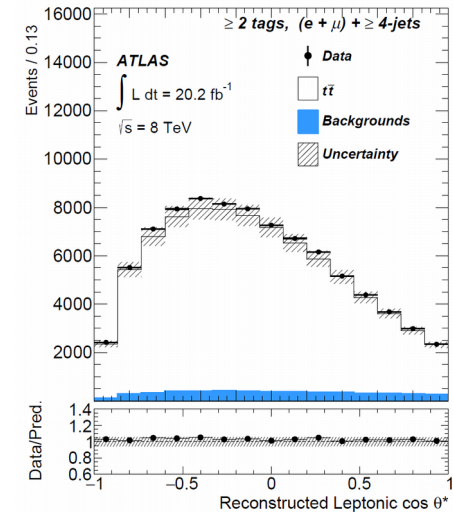
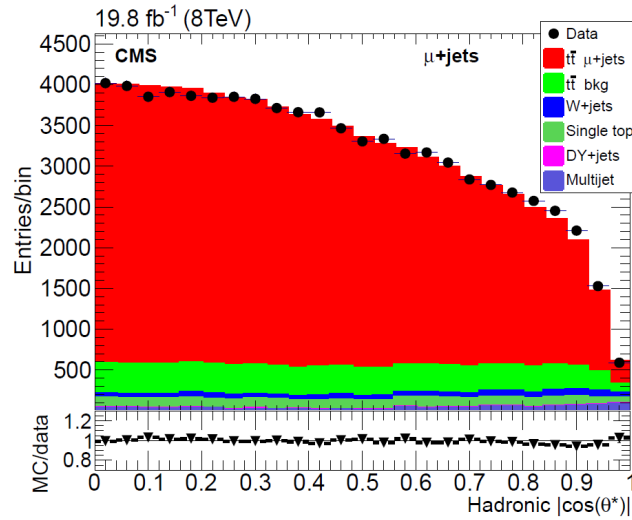
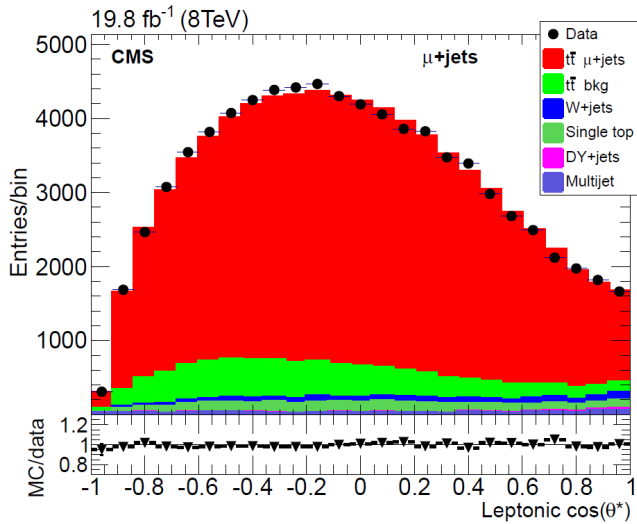
- event weight based on angle at generator level

$$\frac{\frac{3}{8}F_L(1 - \cos \theta_{\text{gen}}^*)^2 + \frac{3}{4}F_0 \sin^2 \theta_{\text{gen}}^* + \frac{3}{8}F_R(1 + \cos \theta_{\text{gen}}^*)^2}{\frac{3}{8}F_L^{\text{SM}}(1 - \cos \theta_{\text{gen}}^*)^2 + \frac{3}{4}F_0^{\text{SM}} \sin^2 \theta_{\text{gen}}^* + \frac{3}{8}F_R^{\text{SM}}(1 + \cos \theta_{\text{gen}}^*)^2}$$

- most backgrounds are unaffected by weights except single top quark production

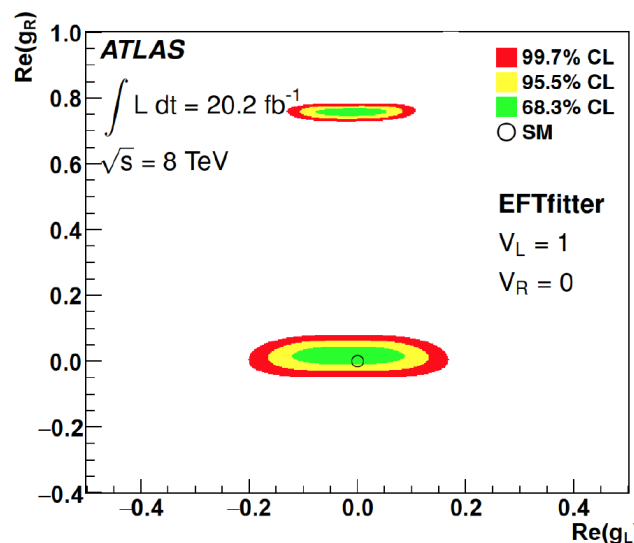
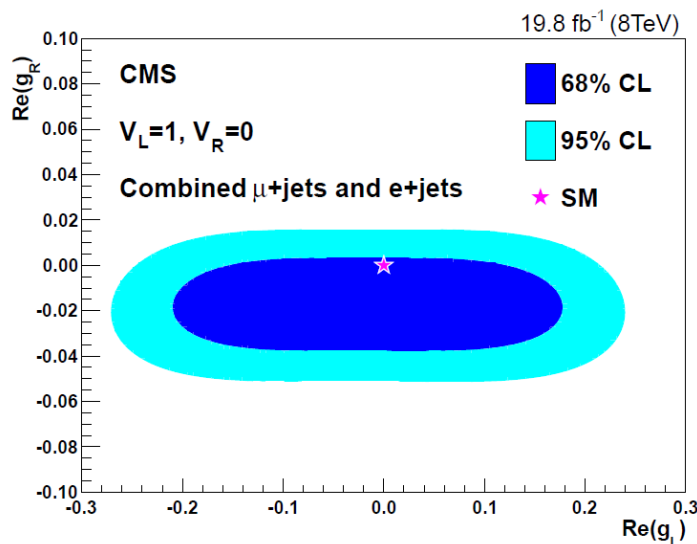


measure fractions through template-based ML fit to reconstructed helicity angle



can also reconstruct angle from $t \rightarrow bq\bar{q}'$; but only $|\cos\theta_{W}^*|$ since down-type quark cannot be distinguished from up-type quark ($\alpha_u \neq \alpha_d$)

limits on anomalous tensor-like left- and right-handed couplings ($V_L = 1, V_R = 0$)



→ consistent with SM expectations

Spin correlation in $t\bar{t}$ production

➤ no net polarization of top quarks in pair production ...

... BUT, top quark spins are correlated!

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_X d\cos\theta_{\bar{X}'}} = \frac{1}{4} (1 + C \alpha_X \alpha_{\bar{X}'} \cos\theta_X \cos\theta_{\bar{X}'})$$

$$C \equiv \frac{\sigma(t_R \bar{t}_R) + \sigma(t_L \bar{t}_L) - \sigma(t_R \bar{t}_L) - \sigma(t_L \bar{t}_R)}{\sigma(t_R \bar{t}_R) + \sigma(t_L \bar{t}_L) + \sigma(t_R \bar{t}_L) + \sigma(t_L \bar{t}_R)}$$

– difference in angles

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\varphi_{X\bar{X}'}} = \frac{1}{2} (1 + D \alpha_X \alpha_{\bar{X}'} \cos\varphi_{X\bar{X}'})$$

– NLO SM prediction [1]

$$C = 0.326 \pm 0.012, D = -0.237 \pm 0.07$$

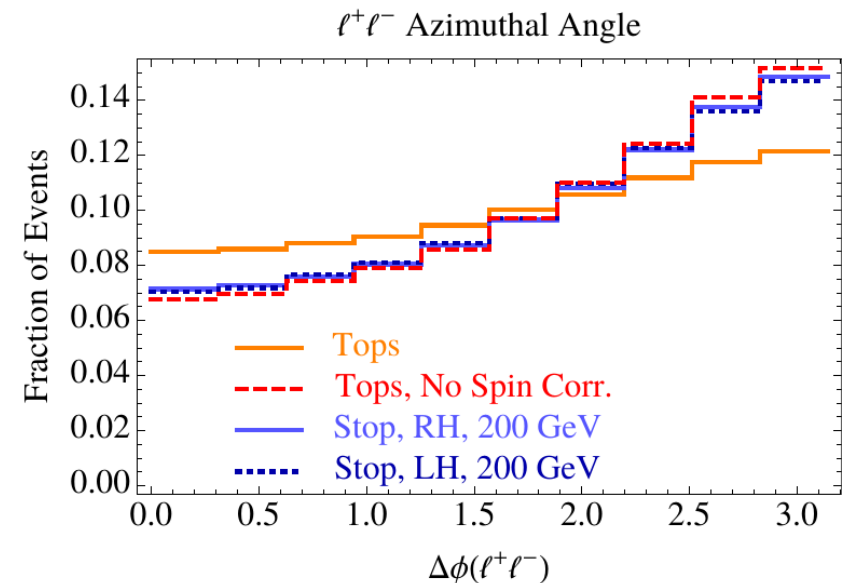
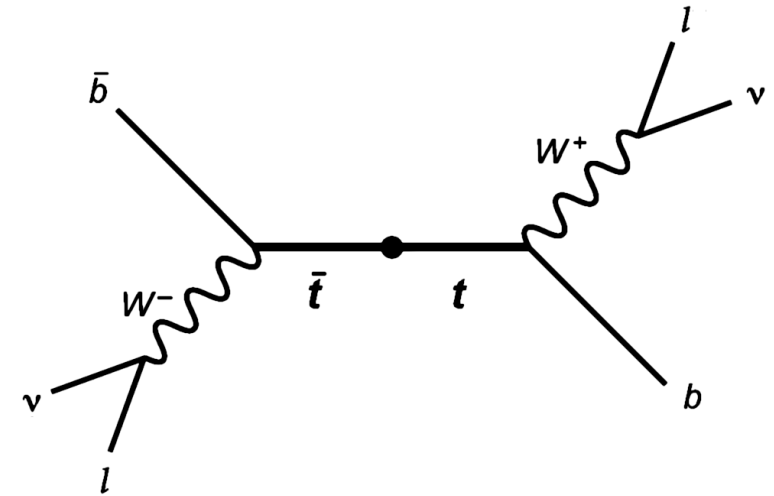
– spin correlation depend on $m_{t\bar{t}}$

➔ overall experimentally challenging since event needs to be fully reconstructed and jets/leptons correctly assigned to the corresponding top quark

➤ proposal: $\Delta\phi_{\ell^+\ell^-}$ in lab frame [2]

– reconstruction of $t\bar{t}$ system not required

– less affected by smearing/finite resolution of detector & reconstruction



[1] Nucl.Phys. B690 81-137, 2004

[2] Phys.Rev.D81:074024,2010

Recent ATLAS measurement

- event selection
 - 1 muon & 1 electron of opposite charge
 - 2 jets of which at least 1 is b-tagged
 - missing transverse energy

- analysis strategy
 - unfold reconstructed angle to parton/particle level (later more) using iterative Bayesian D'Agostini unfolding method [1]

$$\frac{d\sigma_{t\bar{t}}}{dX^i} = \frac{1}{\mathcal{L} \cdot \Delta X^i \cdot \epsilon_{\text{eff}}^i} \cdot \sum_j R_{ij}^{-1} \cdot f_{\text{acc}}^j \cdot (N_{\text{obs}}^j - N_{\text{bkg}}^j)$$

- extracting degree of correlation through template-based fit: $n_i = f_{\text{SM}} \cdot n_{\text{spin}} + (1 - f_{\text{SM}}) \cdot n_{\text{nospin}}$

➤ results

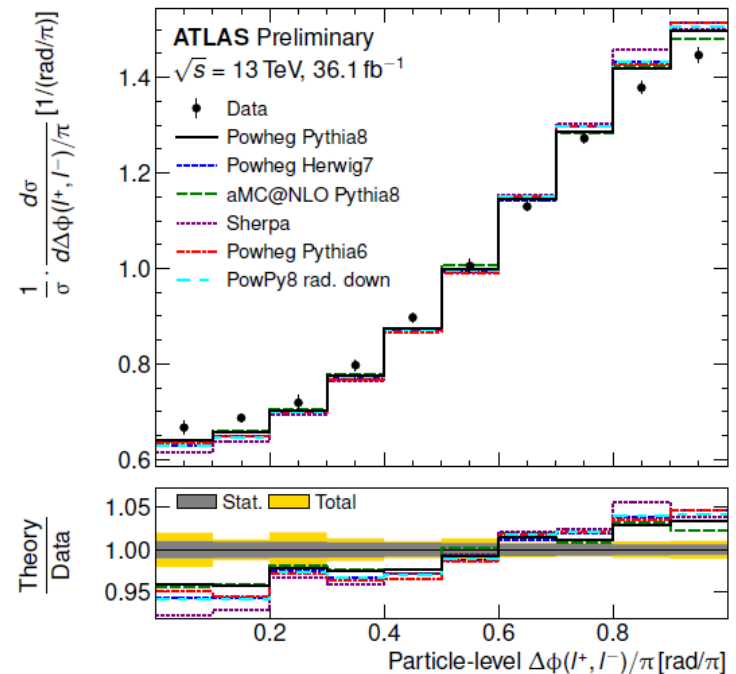
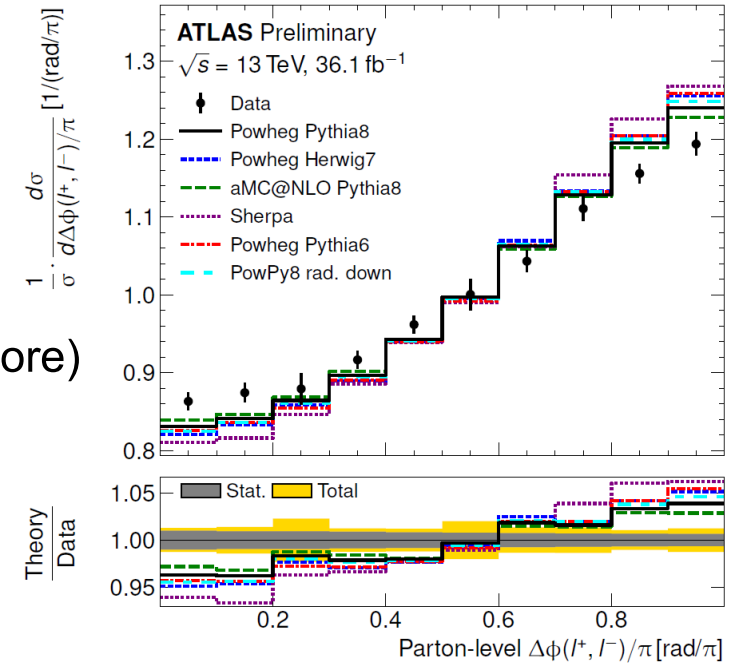
Region	f_{SM}
$m_{t\bar{t}} < 450 \text{ GeV}$	$1.11 \pm 0.04 \pm 0.13$
$450 < m_{t\bar{t}} < 550 \text{ GeV}$	$1.17 \pm 0.09 \pm 0.14$
$550 < m_{t\bar{t}} < 800 \text{ GeV}$	$1.60 \pm 0.24 \pm 0.35$
$m_{t\bar{t}} > 800 \text{ GeV}$	$2.2 \pm 1.8 \pm 2.3$

inclusive

$1.250 \pm 0.026 \pm 0.063$

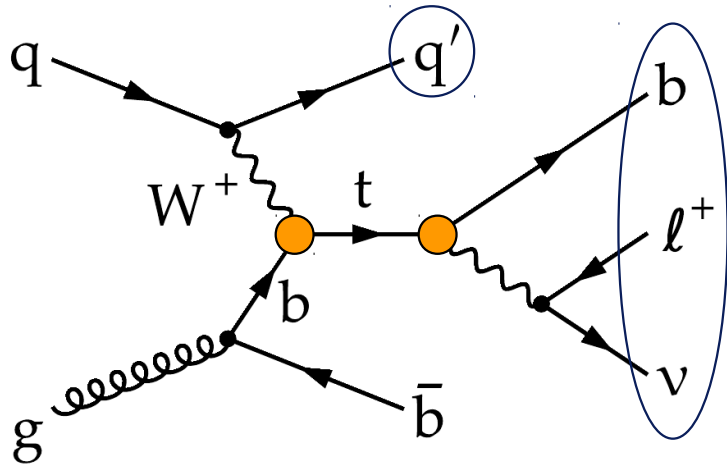
significance: 3.2 σ

[1] Nucl.Instr.Meth. A362 (1995) 487-498

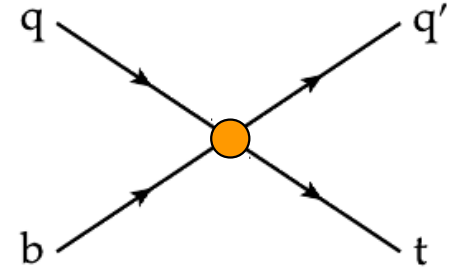


Polarization in single top production

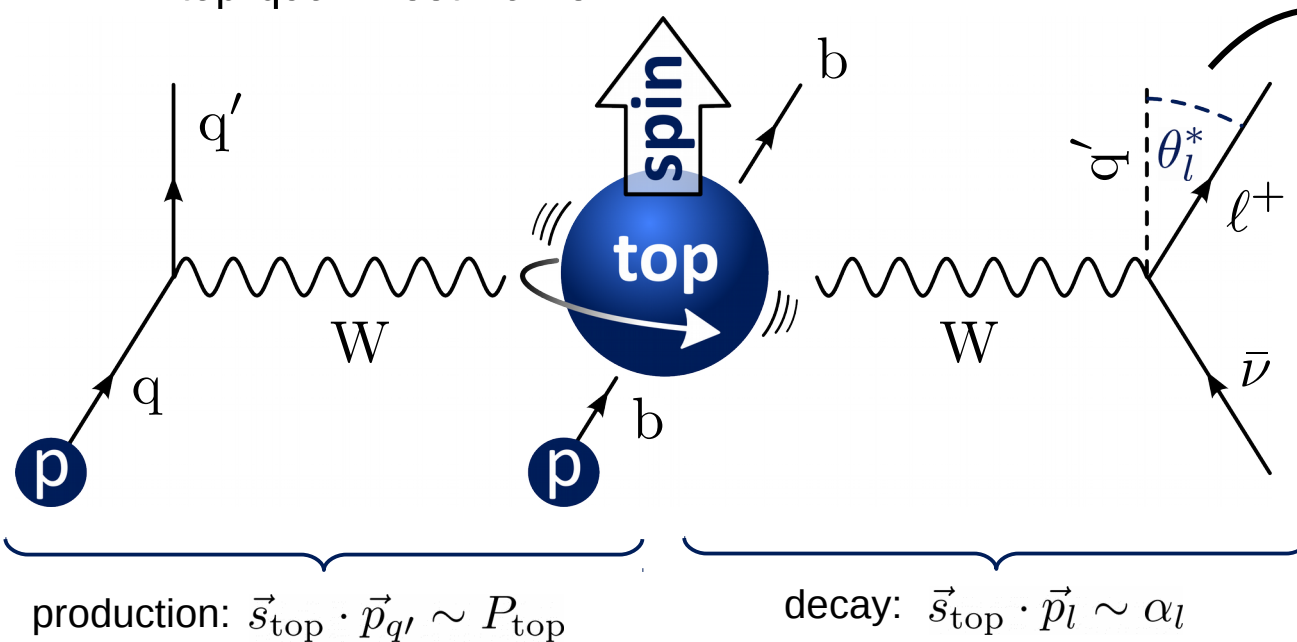
- Wtb coupling structure in t-channel



- in SM, top quark spin aligned with spectator quark q'
- in addition to anomalous Wtb couplings 4-fermion operator can also contribute to production side



- observable: **spin asymmetry** (sensitive to polarization)
 - in top quark rest frame

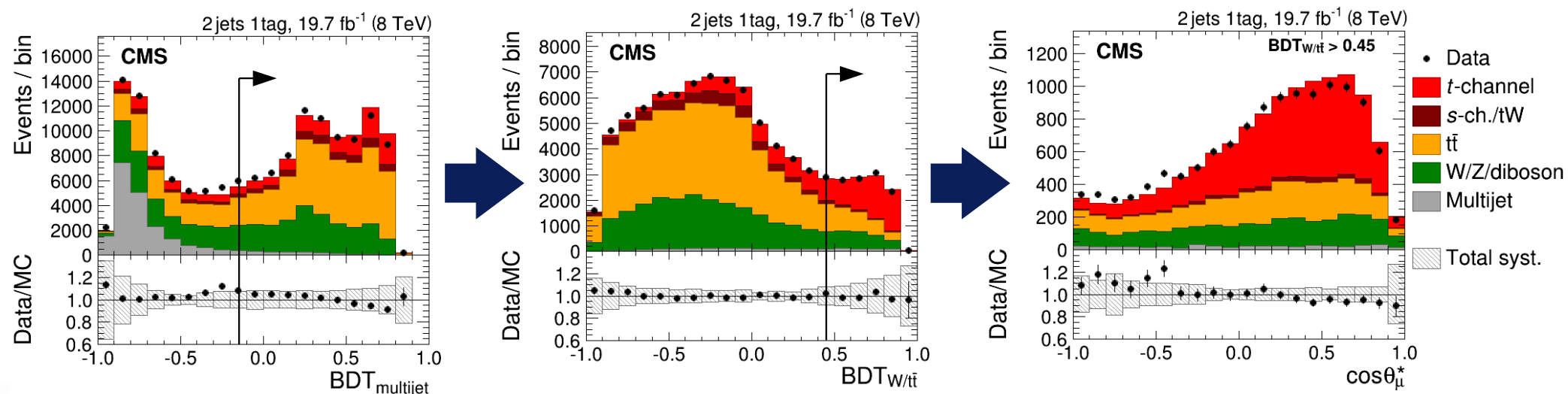


$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_l^*} = \frac{1}{2} \left(1 + \alpha_l P_t \cos \theta_l^* \right)$$

$$A_l = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} = \frac{1}{2} P_t \cdot \alpha_l$$

Spin asymmetry by CMS

- event selection
 - 1 muon, 2 jets with one b-tag, missing transverse energy
 - only ~10% of selected events from t-channel single top production
 - large background from W+jets, multijet, & $t\bar{t}$
 - use MVA to define signal-enriched phase space
- analysis strategy
 - 1st BDT for rejecting multijet events
 - 2nd BDT for rejecting remaining backgrounds from signal
 - training observables need to be carefully chosen to be uncorrelated wrt. polarization angle; otherwise result may be biased towards SM expectation
 - neutrino p_z reconstructed from lepton momentum & E_T^{miss} using W boson mass constraint
 - perform template-based fit to discriminants for estimating background contributions
 - apply successive selection on BDT discriminants → signal-enriched phase space (S/B=~1)



Intermezzo: Unfolding

➤ general problem

- given a reconstructed distribution (e.g. $\cos \theta_\ell^*$), how to compare to theoretical predictions?
- problem can be modeled as

$$\underbrace{f(y)}_{\text{reco.}} = \int \underbrace{A(y)\epsilon(y)R(y,x)}_{\text{detector}} \cdot \underbrace{g(x)}_{\text{true}} dx$$

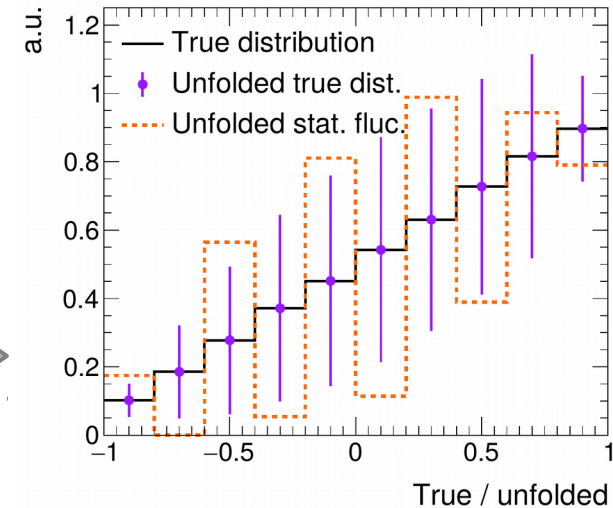
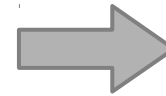
detector acceptance, efficiency & resolution “known” from simulation

- quantized as histogram: $\vec{y} = \tilde{\mathcal{R}} \cdot \vec{x}$, $\tilde{\mathcal{R}} = \mathcal{A} \cdot \mathcal{E} \cdot \mathcal{R}$

- simple inversion of response matrix

$$\vec{x} = (\tilde{\mathcal{R}})^{-1} \vec{y} = \left(\mathcal{U} \cdot \underbrace{\mathcal{S}}_{\text{diagonal}} \cdot \mathcal{V} \right)^{-1} \vec{y} = \mathcal{V}^{-1} \cdot \begin{pmatrix} \frac{1}{s_{11}} & \dots & 0 \\ 0 & & \frac{1}{s_{nn}} \end{pmatrix} \cdot \mathcal{U}^{-1} \vec{y}$$

→ statistical fluctuations are amplified if s_{ii} are small



➤ regularized unfolding (aka “TUnfold” method) [1]

- rewrite unfolding as minimization problem

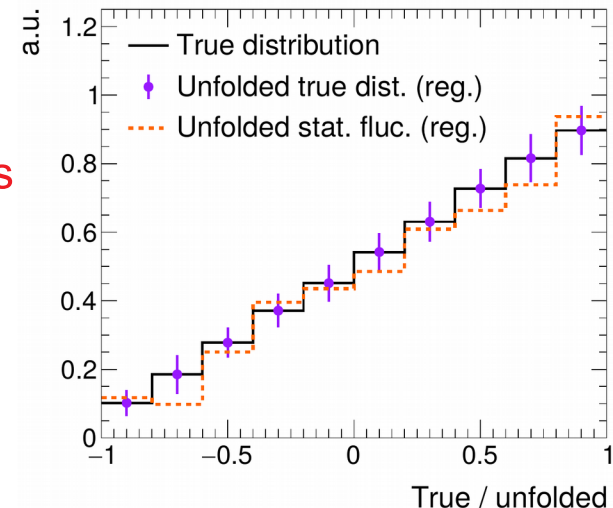
$$\chi^2 = (\vec{y} - \tilde{\mathcal{R}} \vec{x})^T \mathcal{V}^{-1} (\vec{y} - \tilde{\mathcal{R}} \vec{x}) + \mathfrak{R}_1 + \mathfrak{R}_2 \quad \text{penalty terms}$$

- suppress oscillating solutions: $\mathfrak{R}_1 = \tau^2 \vec{x}^T \mathcal{L}^T \mathcal{L} \vec{x}$ 2nd derivatives

→ regularization strength τ needs to be optimized

→ typically: optimize for minimal correlations between bins

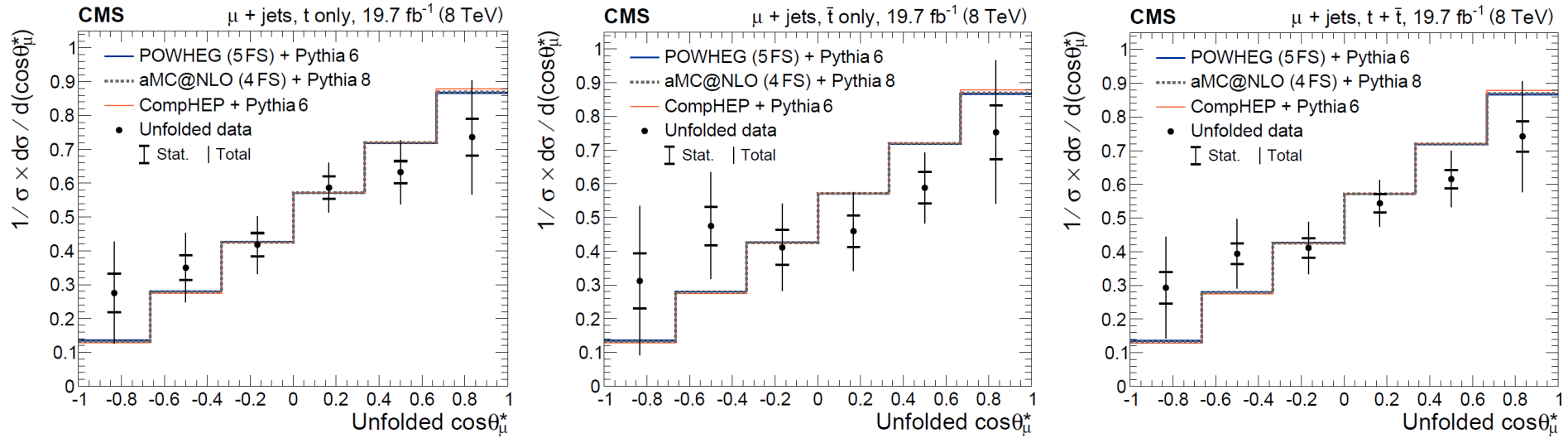
- match overall selection efficiency: $\mathfrak{R}_2 = \lambda \left(\sum y_i - \vec{\epsilon} \cdot \vec{x} \right)$



[1] JINST 7 (2012) T10003

Spin asymmetry: Results

- unfolded distributions of polarization angle at parton level



- measured spin asymmetries

- separately for top quarks/antiquarks events

$$A_{\mu}(t) = 0.29 \pm 0.03 \text{ (stat)} \pm 0.10 \text{ (syst)} = 0.29 \pm 0.11,$$

$$A_{\mu}(\bar{t}) = 0.21 \pm 0.05 \text{ (stat)} \pm 0.13 \text{ (syst)} = 0.21 \pm 0.14,$$

} consistent within uncertainties

- combination

$$A_{\mu}(t + \bar{t}) = 0.26 \pm 0.03 \text{ (stat)} \pm 0.10 \text{ (syst)} = 0.26 \pm 0.11, \longrightarrow \text{compatible with } A_{\text{PowHeg}}^{\text{SM}} = 0.44 \text{ within } 2.0 \sigma$$

- 2-bin cross check w/ unregularized unfolding

$$A_{\mu}(t + \bar{t}) = 0.28 \pm 0.03 \text{ (stat)} \pm 0.1 \text{ (syst)} = 0.28 \pm 0.12$$

Wtb analysis in t-channel

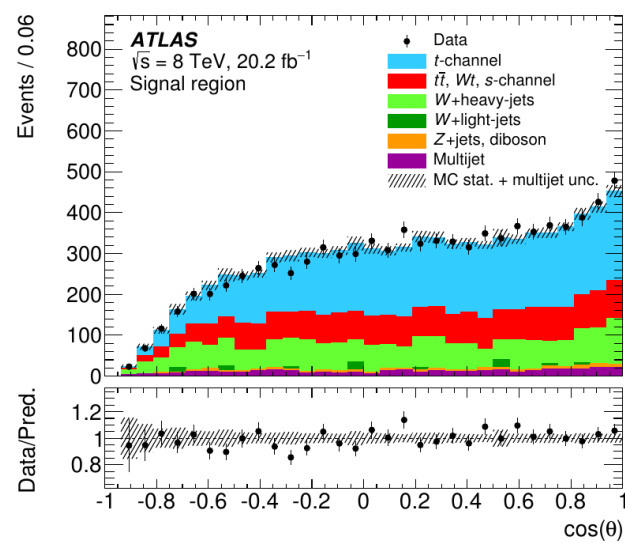
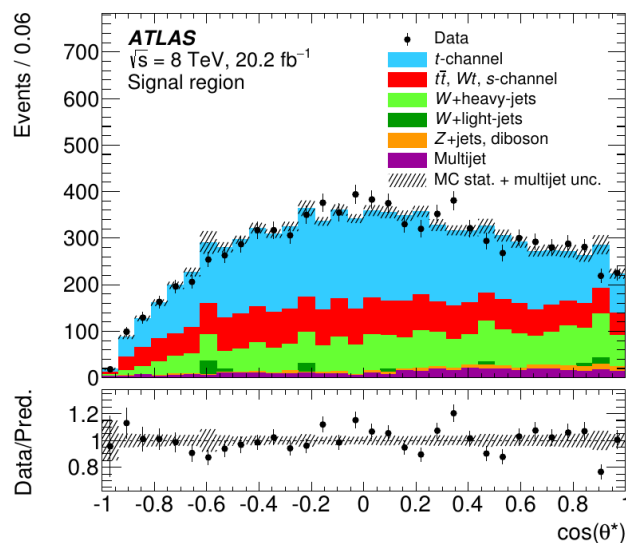
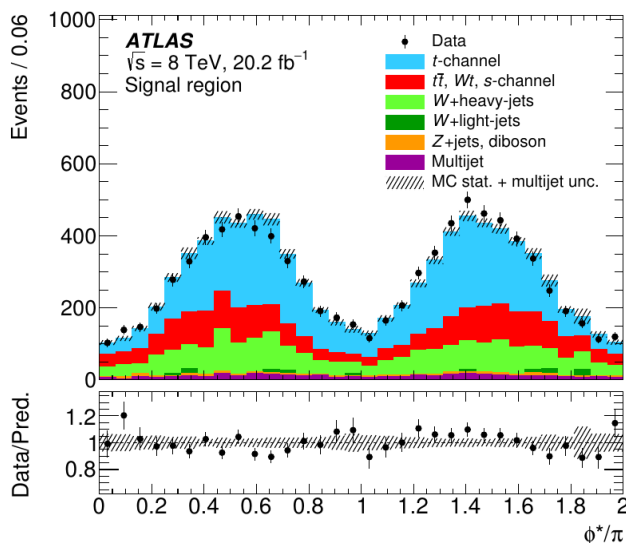
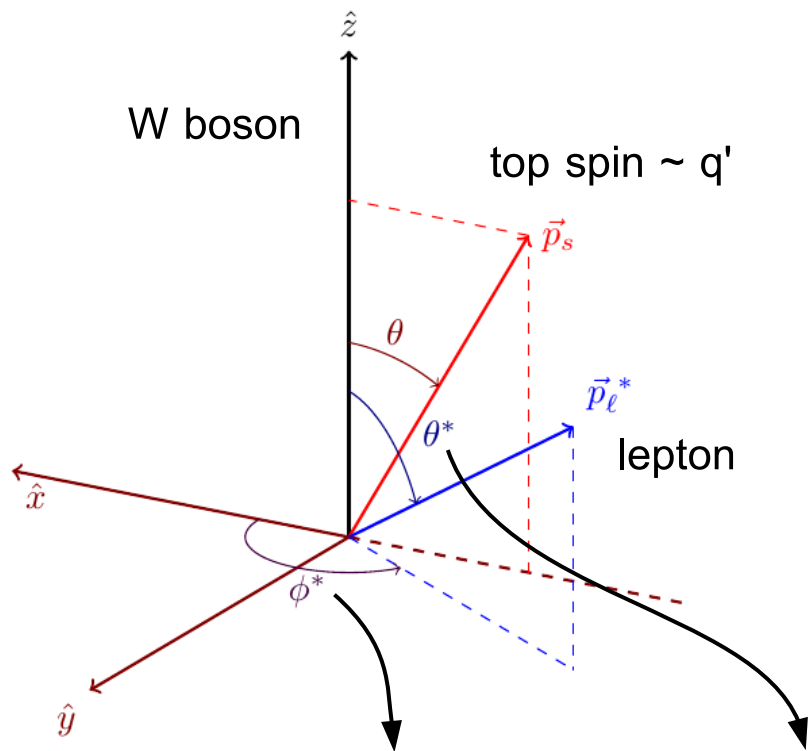
➤ idea

- use triple angular distribution to set limits on anomalous Wtb couplings and polarization
- decompose distributions into spherical harmonics

$$\rho(\theta, \theta^*, \phi^*; P) = \frac{1}{N} \frac{d^3 N}{d(\cos \theta) d\Omega^*}$$

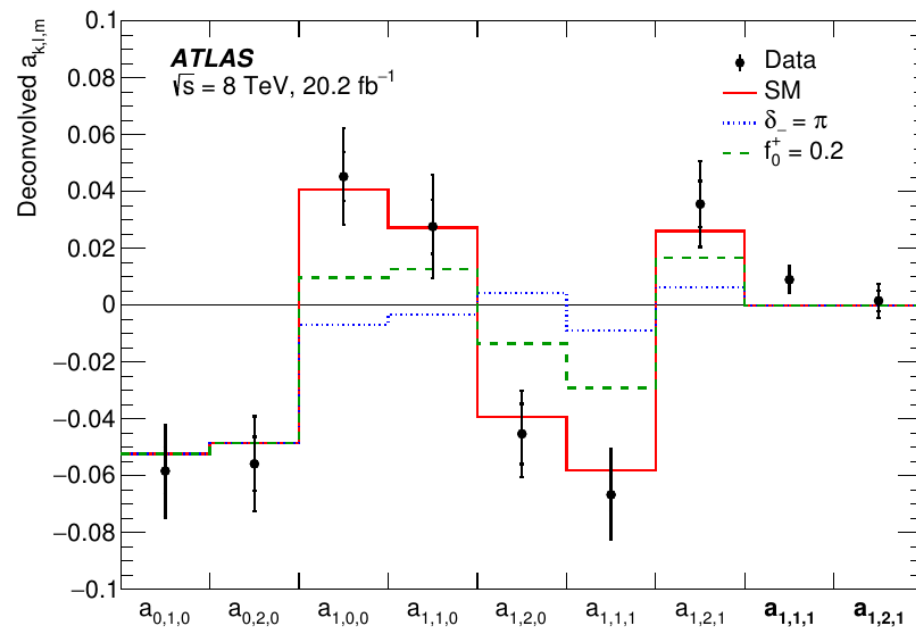
$$= \sum_{k=0}^1 \sum_{l=0}^2 \sum_{m=-k}^k a_{k,l,m} M_{k,l}^m(\theta, \theta^*, \phi^*)$$

- magnitudes $a_{k,l,m}$ depend on generalized helicity fractions $\vec{\alpha} = (f_1, f_1^+, f_0^+, \delta_+, \delta_-)$ & polarization P
- select events with $|\eta_{j'}| > 2$, $H_T > 195$ GeV & top mass window → signal-enriched phase space

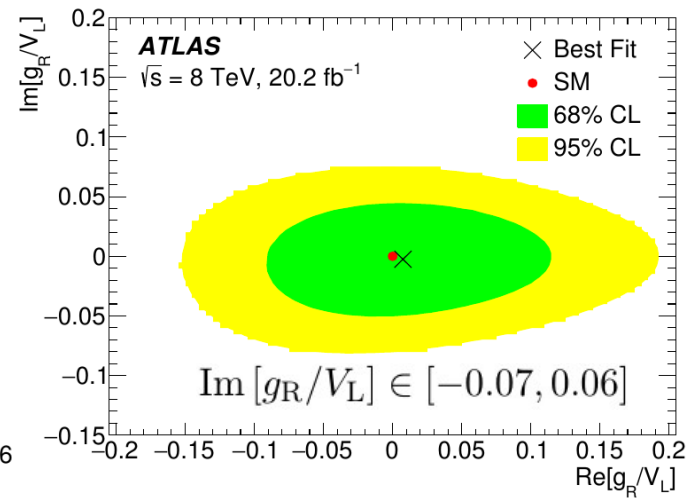
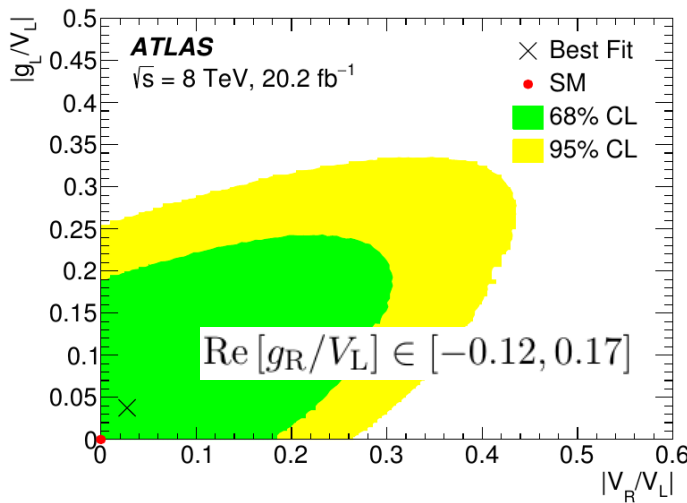
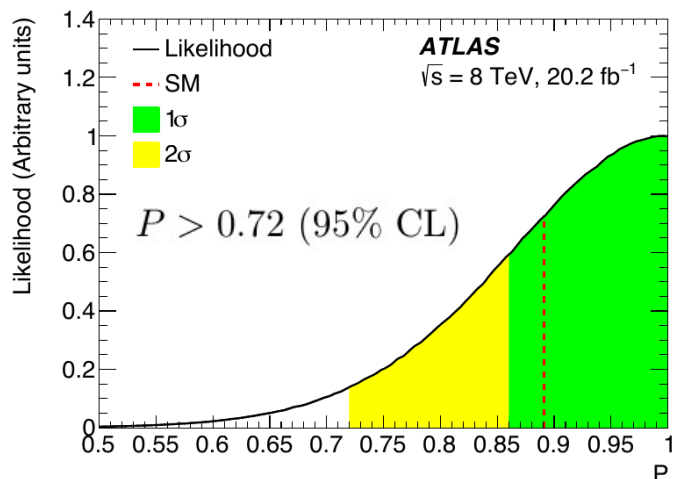


Wtb analysis: Results

- analysis strategy
 - estimate coefficients in reconstructed distributions; subtract background contributions
 - translate coefficients to parton level through analytical unfolding
 - derive limits on anomalous couplings and top quark polarization



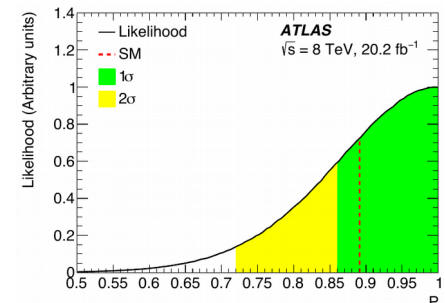
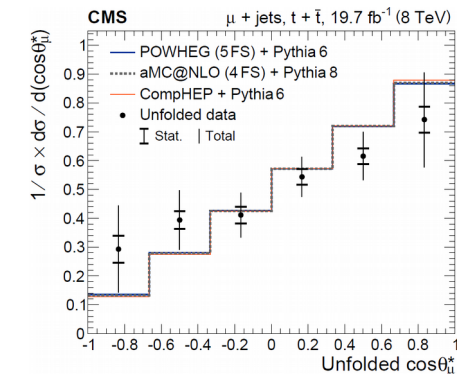
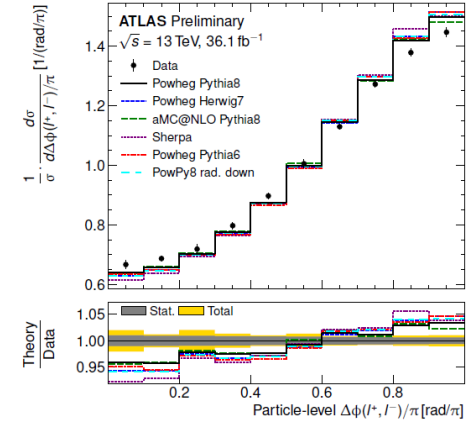
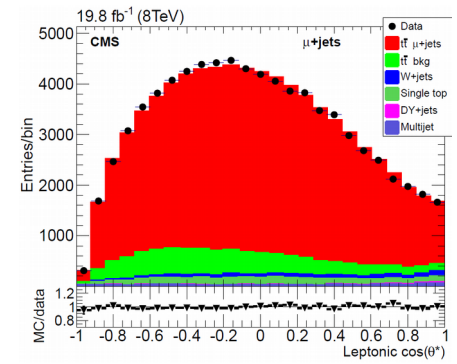
limits on anomalous couplings



→ results in agreement with SM prediction

Conclusion

- top quark properties
 - V-A coupling structure dictates angular distributions
 - spin analyzing power α_X quotes alignment of top quark spin with decay product X
- W helicity fraction measurements by ATLAS & CMS
 - reweight $t\bar{t}$ SM to 3 helicity fractions F_+, F_0, F_-
 - perform template-based fit to reconstructed helicity angle
- spin correlation in $t\bar{t}$ production by ATLAS
 - observable $\Delta\phi_{\ell^+\ell^-}$ in lab frame does not require full reconstruction
 - unfold reconstructed distribution to parton/particle level
- single top quark polarization in t-channel by CMS
 - polarized top quark in production wrt. spectator quark direction
 - unfolded angle to parton level \rightarrow spin asymmetry: $A = \frac{1}{2}\alpha_\ell P_t$
- Wtb vertex analysis in t-channel by ATLAS
 - triple angular analysis of polarization angles in production/decay
 - estimate and unfold angular moments to parton level
 - set limits on anomalous Wtb couplings & polarization



Backup

W helicity measurement

ATLAS, Eur. Phys. J. C 77 (2017) 264

➤ ATLAS

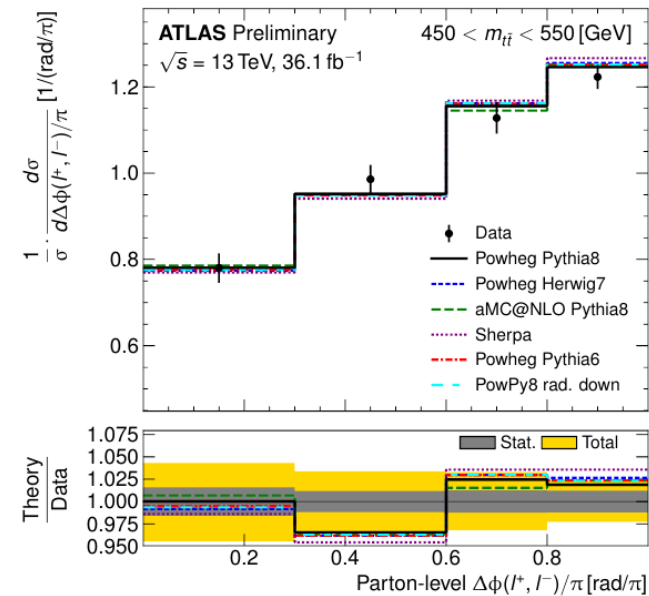
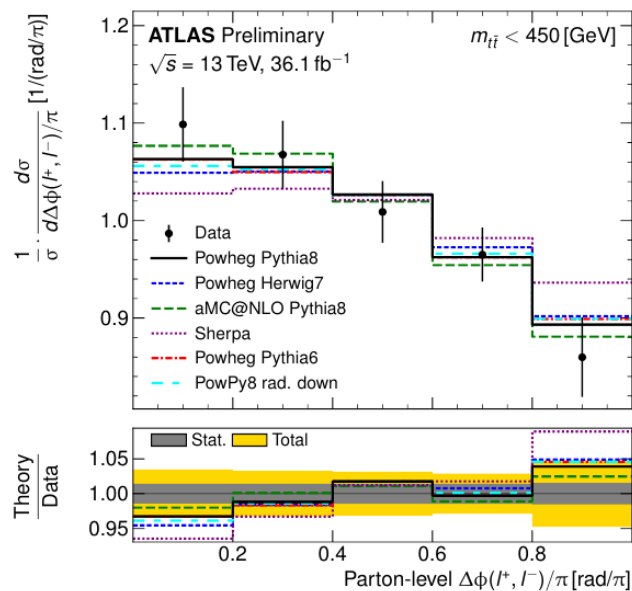
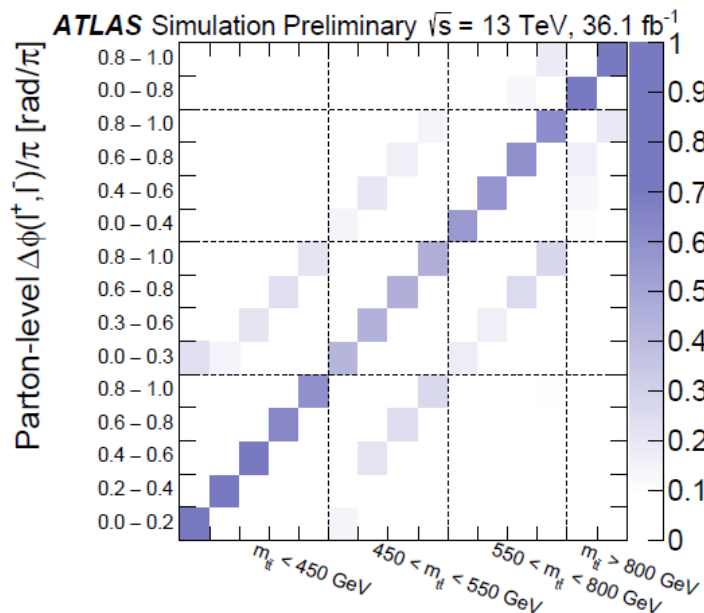
Uncertainty	Leptonic, ≥ 2 b -tags			Hadronic, 1 + ≥ 2 b -tags		
	F_0	F_L	F_R	F_0	F_L	F_R
Reconstructed objects						
Electron	+0.0028	+0.0018	+0.0011	+0.0025	+0.0028	+0.0051
	-0.0030	-0.0020	-0.0011	-0.0021	-0.0038	-0.0058
Muon	+0.0024	+0.0013	+0.0010	+0.0026	+0.0046	+0.0072
	-0.0029	-0.0015	-0.0015	-0.0037	-0.0035	-0.0072
Jet energy scale	+0.0063	+0.0028	+0.0037	+0.0069	+0.012	+0.014
	-0.0033	-0.0025	-0.0014	-0.0070	-0.008	-0.005
Jet energy resolution	+0.0062	+0.0048	+0.0072	+0.027	+0.033	+0.057
	-0.0059	-0.0018	-0.0067	-0.031	-0.041	-0.071
Jet vertex fraction	+0.0036	+0.0019	+0.0017	+0.013	+0.0012	+0.011
	-0.0017	-0.0013	-0.0006	-0.009	-0.0046	-0.005
Jet reconstruction efficiency	+0.0002	<0.0001	+0.0002	+0.0008	+0.0004	+0.0011
	-0.0002	<0.0001	-0.0002	-0.0008	-0.0004	-0.0011
b -tagging	+0.0017	+0.0012	+0.0011	+0.029	+0.013	+0.034
	-0.0021	-0.0013	-0.0012	-0.031	-0.014	-0.035
Sum reconstructed objects	+0.010	+0.0064	+0.0085	+0.043	+0.038	+0.069
	-0.008	-0.0044	-0.0072	-0.045	-0.044	-0.080
Signal modelling						
Showering and hadronisation	± 0.0019	± 0.0019	± 0.0037	± 0.015	± 0.001	± 0.014
ME event generator	± 0.0025	± 0.0032	± 0.0057	± 0.016	± 0.024	± 0.040
ISR/FSR	± 0.0033	± 0.0058	± 0.0034	± 0.018	± 0.039	± 0.057
PDF	± 0.0033	± 0.0042	± 0.0009	± 0.0010	± 0.0020	± 0.0020
Top quark mass	± 0.0017	± 0.0050	± 0.0033	± 0.0033	± 0.0100	± 0.0068
Sum signal modelling	± 0.0058	± 0.0094	± 0.0082	± 0.028	± 0.047	± 0.072
Method uncertainty						
Template statistics	± 0.0091	± 0.0056	± 0.0044	± 0.0076	± 0.016	± 0.016
Total uncertainty						
Total systematic	+0.015	+0.013	+0.013	+0.052	+0.063	+0.100
	-0.014	-0.012	-0.012	-0.054	-0.067	-0.110
Stat. + bkg. norm	± 0.012	± 0.008	± 0.006	± 0.010	± 0.021	± 0.022

W helicity measurement

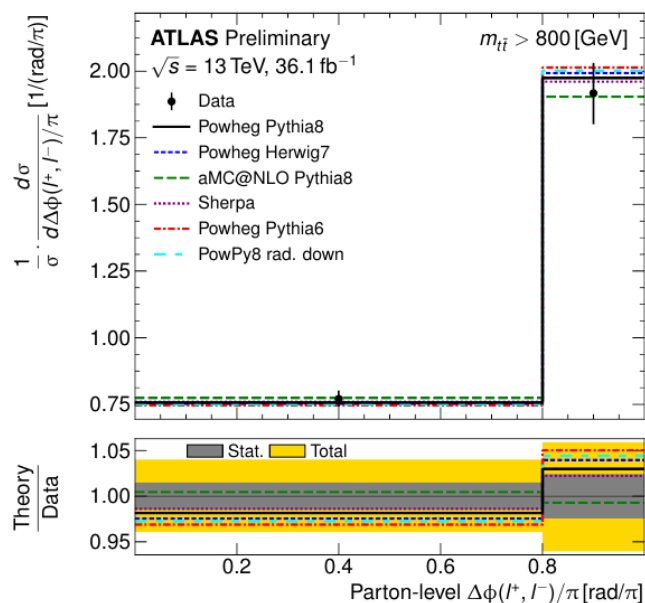
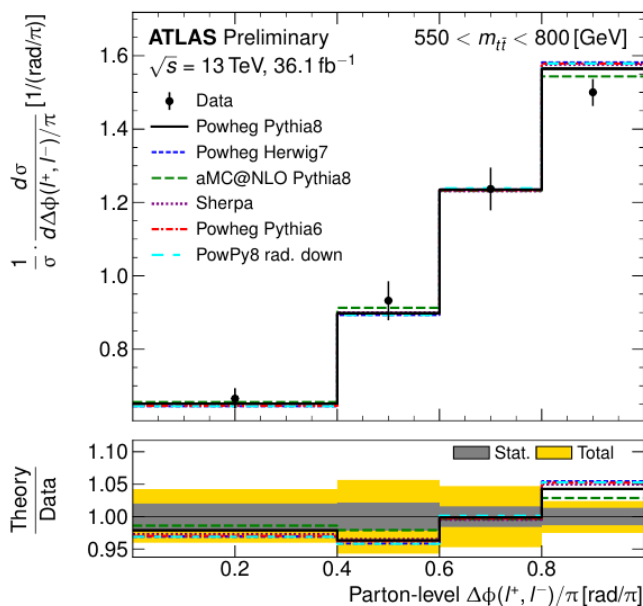
➤ CMS

	e+jets		μ +jets		ℓ +jets	
	$\pm \Delta F_0$	$\pm \Delta F_L$	$\pm \Delta F_0$	$\pm \Delta F_L$	$\pm \Delta F_0$	$\pm \Delta F_L$
JES	0.004	0.003	0.005	0.003	0.005	0.003
JER	0.001	0.002	0.004	0.003	0.003	0.003
b tagging eff.	0.001	$<10^{-3}$	0.001	$<10^{-3}$	0.001	$<10^{-3}$
Lepton eff.	0.001	0.002	0.001	0.001	0.001	0.001
Single top normal.	0.002	$<10^{-3}$	0.003	0.001	0.003	0.001
W+jets bkg.	0.008	0.001	0.007	0.001	0.007	0.001
DY+jets bkg.	0.002	$<10^{-3}$	0.001	$<10^{-3}$	0.001	$<10^{-3}$
Multijet bkg.	0.023	0.007	0.007	0.003	0.008	0.001
Pileup	0.001	0.001	$<10^{-3}$	$<10^{-3}$	0.001	$<10^{-3}$
Top quark mass	0.012	0.008	0.010 (*)	0.008 (*)	0.010	0.007
$t\bar{t}$ scales	0.011	0.008 (*)	0.014	0.007 (*)	0.012	0.007
$t\bar{t}$ match. scale	0.011 (*)	0.007 (*)	0.010	0.007	0.009	0.007
$t\bar{t}$ MC and hadronisation	0.015	0.009	0.005	0.003	0.006	0.004
$t\bar{t}$ p_T reweight	0.011	0.010	$<10^{-3}$	0.001	$<10^{-3}$	0.002
Limited MC size	0.002	0.001	0.002	0.001	0.002	0.001
PDF	0.004	0.001	0.002	0.001	0.002	0.001
Total	0.037	0.020	0.024	0.014	0.023	0.014

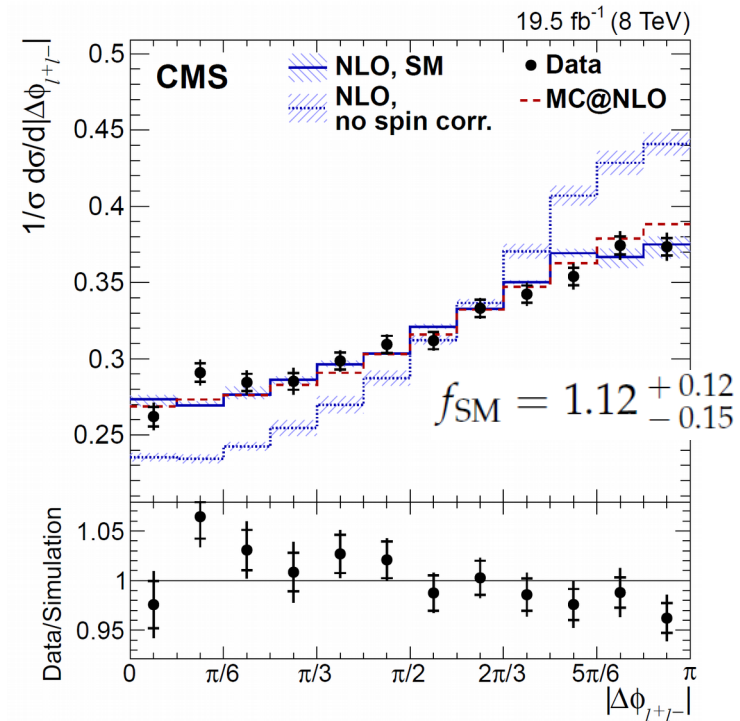
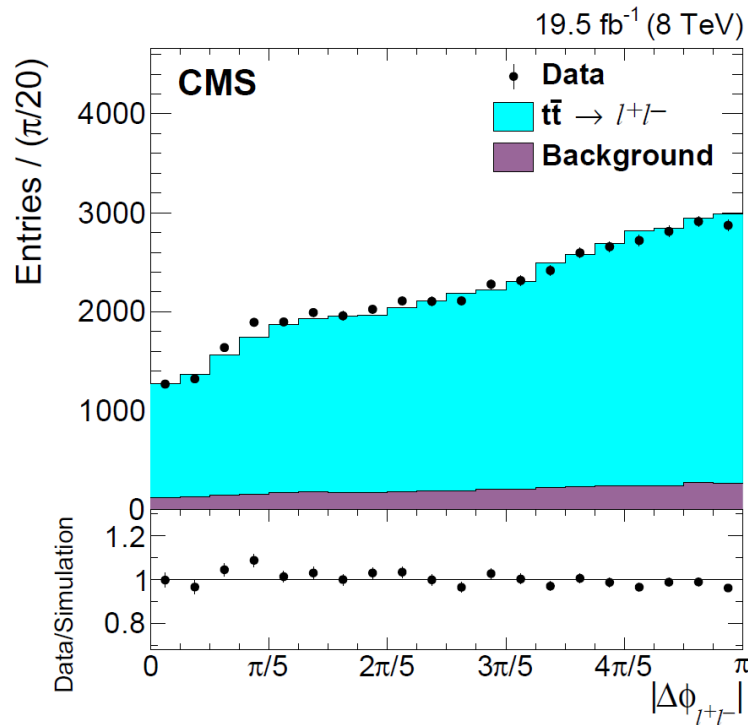
Spin correlations wrt. $m_{t\bar{t}}$



Detector-level $\Delta\phi(I^+, I^-)/\pi$ [rad/ π]



Spin correlations by CMS



	$\delta A_\mu(t)/10^{-2}$	$\delta A_\mu(\bar{t})/10^{-2}$	$\delta A_\mu(t + \bar{t})/10^{-2}$
Statistical	3.2	4.6	2.6
ML fit uncertainty	0.7	1.2	0.6
Diboson bkg. fraction	<0.1	<0.1	<0.1
Z/ γ^* +jets bkg. fraction	<0.1	<0.1	<0.1
s-channel bkg. fraction	0.3	0.2	0.2
tW bkg. fraction	0.1	0.7	0.2
Multijet events shape	0.5	0.7	0.5
Multijet events yield	1.9	1.2	1.7
b tagging	0.7	1.2	0.9
Mistagging	<0.1	0.1	<0.1
Jet energy resolution	2.7	1.8	2.0
Jet energy scale	1.3	2.6	1.1
Unclustered E_T	1.1	3.3	1.3
Pileup	0.3	0.2	0.2
Lepton identification	<0.1	<0.1	<0.1
Lepton isolation	<0.1	<0.1	<0.1
Muon trigger efficiency	<0.1	<0.1	<0.1
Top quark p_T reweighting	0.3	0.3	0.3
W+jets W boson p_T reweighting	0.1	0.1	0.1
W+jets heavy-flavour fraction	4.7	6.2	5.3
W+jets light-flavour fraction	<0.1	<0.1	0.1
W+jets $\cos\theta_\mu^*$ reweighting	2.9	3.4	3.1
Unfolding bias	2.5	4.2	3.1
Generator model	1.6	3.5	0.3
Top quark mass	1.9	2.9	1.8
PDF	0.9	1.6	1.2
t-channel renorm./fact. scales	0.2	0.2	0.2
$\bar{t}\bar{t}$ renorm./fact. scales	2.2	3.4	2.7
$\bar{t}\bar{t}$ ME/PS matching	2.2	0.5	1.6
W+jets renorm./fact. scales	3.7	4.6	4.0
W+jets ME/PS matching	3.8	3.0	3.4
Limited MC events	2.1	3.2	1.8
Total uncertainty	10.5	13.8	10.5

Source	Helicity parameters		Coupling ratios	
	$\sigma(f_1)$	$\sigma(\delta_-)/\pi$	$\sigma(\text{Re}[g_R/V_L])$	$\sigma(\text{Im}[g_R/V_L])$
Statistical	0.022	0.013	0.030	0.027
Jets	0.029	0.007	0.039	0.009
Leptons	0.014	0.002	0.017	< 0.001
E_T^{miss}	< 0.001	< 0.001	< 0.001	< 0.001
Generator	0.027	0.006	0.030	0.010
Parton shower and hadronisation	0.004	0.003	< 0.001	0.003
PDF variations	0.008	0.004	< 0.001	< 0.001
Background normalisation	< 0.001	< 0.001	< 0.001	< 0.001
Multijet normalisation	< 0.001	< 0.001	< 0.001	< 0.001
W +jets shape	0.015	0.005	0.007	0.009
Luminosity	< 0.001	< 0.001	< 0.001	< 0.001
MC sample sizes	0.009	0.006	< 0.001	0.013
Other	< 0.001	< 0.001	< 0.001	< 0.001
Total systematic uncertainty	0.044	0.010	0.061	0.017
Total	0.049	0.017	0.068	0.032

LO Spin analyzing power

Nucl.Phys. B840 349-378, 2010

➤ dependence on anomalous Wtb couplings

$$\begin{aligned} a_{\ell^+} = & [|V_L|^2 - |V_R|^2] (1 + x_W^2 - 2x_W^4) + 2 [|g_L|^2 - |g_R|^2] \left(1 - \frac{x_W^2}{2} - \frac{x_W^4}{2} \right) \\ & - 12x_W^2 x_b \operatorname{Re} [V_L V_R^* + g_L g_R^*] - 6x_W \operatorname{Re} [V_L g_R^* + V_R g_L^*] (1 - x_W^2) \\ & + 6x_W x_b \operatorname{Re} [V_L g_L^* - V_R g_R^*] (1 + x_W^2) + 12x_W^2 [|V_R|^2 - |g_R|^2] \\ & + 6 \frac{M_W}{|\vec{q}|} x_W \log \frac{E_W + |\vec{q}|}{E_W - |\vec{q}|} [|g_R|^2 - x_W^2 |V_R|^2 + 2x_W x_b \operatorname{Re} V_R g_R^*] , \end{aligned}$$

$$\begin{aligned} a_0 = & [|V_L|^2 + |V_R|^2] (1 + x_W^2 - 2x_W^4) + 2 [|g_L|^2 + |g_R|^2] \left(1 - \frac{x_W^2}{2} - \frac{x_W^4}{2} \right) \\ & - 12x_W^2 x_b \operatorname{Re} [V_L V_R^* + g_L g_R^*] - 6x_W \operatorname{Re} [V_L g_R^* + V_R g_L^*] (1 - x_W^2) \\ & + 6x_W x_b \operatorname{Re} [V_L g_L^* + V_R g_R^*] , \end{aligned}$$

LO W helicity fractions

➤ dependence on anomalous Wtb couplings

$$\Gamma_0 = \frac{g^2 |\vec{q}|}{32\pi} A_0, \quad \Gamma_{\pm} = \frac{g^2 |\vec{q}|}{32\pi} \left(B_0 \pm 2 \frac{|\vec{q}|}{m_t} B_1 \right)$$

$$\begin{aligned} A_0 = & \frac{m_t^2}{M_W^2} [|V_L|^2 + |V_R|^2] (1 - x_W^2) + [|g_L|^2 + |g_R|^2] (1 - x_W^2) \\ & - 4x_b \operatorname{Re} [V_L V_R^* + g_L g_R^*] - 2 \frac{m_t}{M_W} \operatorname{Re} [V_L g_R^* + V_R g_L^*] (1 - x_W^2) \\ & + 2 \frac{m_t}{M_W} x_b \operatorname{Re} [V_L g_L^* + V_R g_R^*] (1 + x_W^2), \end{aligned}$$

$$\begin{aligned} B_0 = & [|V_L|^2 + |V_R|^2] (1 - x_W^2) + \frac{m_t^2}{M_W^2} [|g_L|^2 + |g_R|^2] (1 - x_W^2) \\ & - 4x_b \operatorname{Re} [V_L V_R^* + g_L g_R^*] - 2 \frac{m_t}{M_W} \operatorname{Re} [V_L g_R^* + V_R g_L^*] (1 - x_W^2) \\ & + 2 \frac{m_t}{M_W} x_b \operatorname{Re} [V_L g_L^* + V_R g_R^*] (1 + x_W^2), \end{aligned}$$

$$\begin{aligned} B_1 = & - [|V_L|^2 - |V_R|^2] + \frac{m_t^2}{M_W^2} [|g_L|^2 - |g_R|^2] + 2 \frac{m_t}{M_W} \operatorname{Re} [V_L g_R^* - V_R g_L^*] \\ & + 2 \frac{m_t}{M_W} x_b \operatorname{Re} [V_L g_L^* - V_R g_R^*], \end{aligned}$$

$$\begin{aligned}
 \varrho(\theta, \theta^*, \phi^*; P) &= \frac{1}{N} \frac{d^3 N}{d(\cos \theta) d\Omega^*} = \frac{1}{8\pi} \left\{ \frac{3}{4} \left| A_{1, \frac{1}{2}} \right|^2 (1 + P \cos \theta) (1 + \cos \theta^*)^2 \right. \\
 &\quad + \frac{3}{4} \left| A_{-1, -\frac{1}{2}} \right|^2 (1 - P \cos \theta) (1 - \cos \theta^*)^2 \\
 &\quad + \frac{3}{2} \left(\left| A_{0, \frac{1}{2}} \right|^2 (1 - P \cos \theta) + \left| A_{0, -\frac{1}{2}} \right|^2 (1 + P \cos \theta) \right) \sin^2 \theta^* \\
 &\quad - \frac{3\sqrt{2}}{2} P \sin \theta \sin \theta^* (1 + \cos \theta^*) \operatorname{Re} \left[e^{i\phi^*} A_{1, \frac{1}{2}} A_{0, \frac{1}{2}}^* \right] \\
 &\quad \left. - \frac{3\sqrt{2}}{2} P \sin \theta \sin \theta^* (1 - \cos \theta^*) \operatorname{Re} \left[e^{-i\phi^*} A_{-1, -\frac{1}{2}} A_{0, -\frac{1}{2}}^* \right] \right\} \\
 &= \sum_{k=0}^1 \sum_{l=0}^2 \sum_{m=-k}^k a_{k,l,m} M_{k,l}^m(\theta, \theta^*, \phi^*),
 \end{aligned}$$