Plasma wakefields via modulation (and not only)

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- Utility criterion for proton drivers
- Problem of hosing
- Control of instability modes
- Quadrupole focusing

Utility criterion for proton drivers

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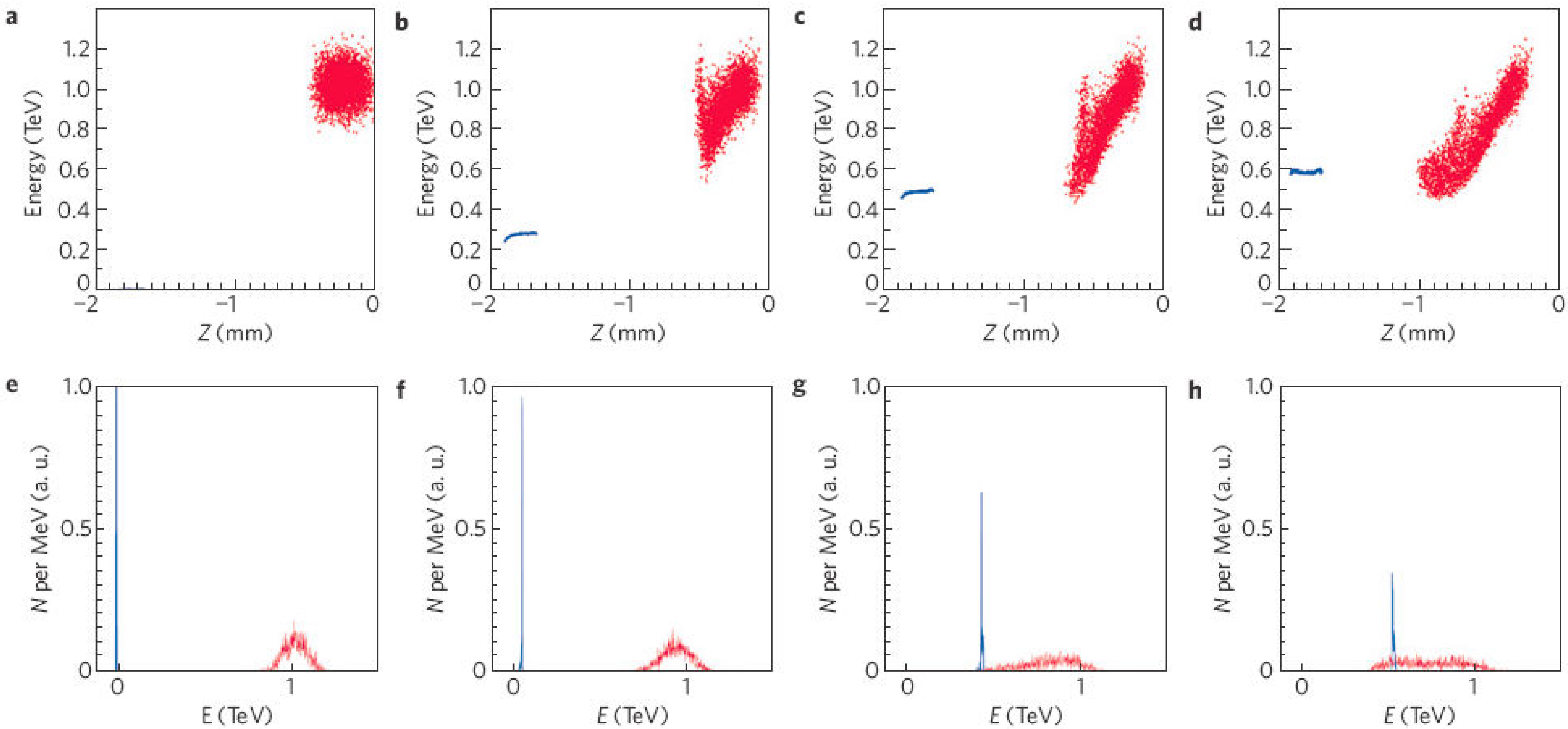


In simulations of short proton drivers (1 TeV), the energy gain of electrons was limited to ~0.6 TeV. Acceleration stopped by driver elongation.

Utility criterion for proton drivers

Assume all fields $\sim \alpha E_0 = \alpha m c \omega_p / e$,

then the driver depletion length $L_0 \sim \dfrac{W_P}{e \alpha E_0}$,

and the energy of slowest driver particles $W \approx W_P (1 - L/L_0)$.

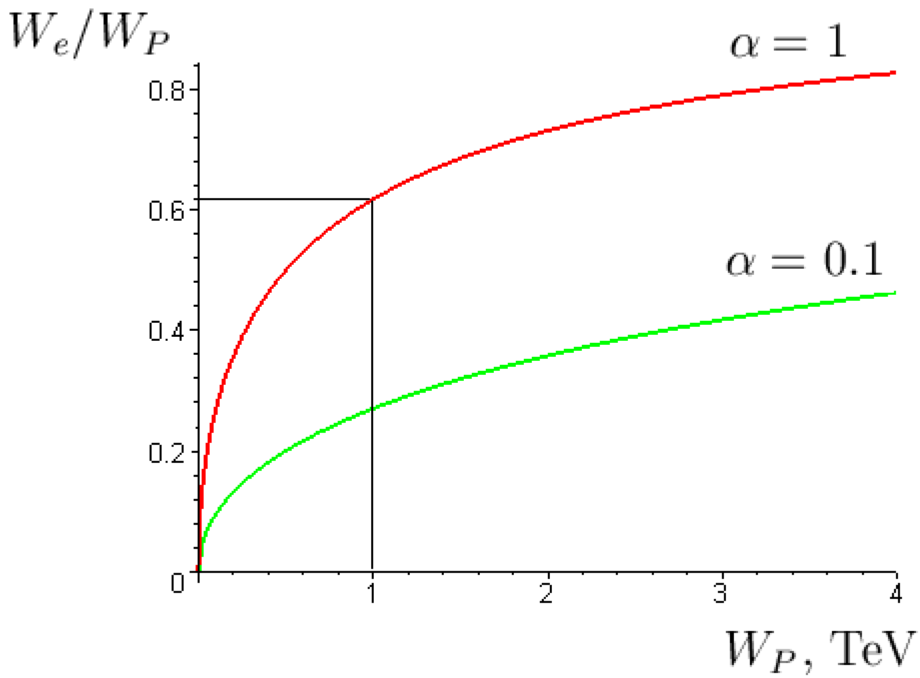
Their velocity $v \approx c \left(1 - \dfrac{1}{2\gamma_0^2 (1 - L/L_0)^2} \right)$.

The wakefield drops if $\dfrac{c}{\omega_p} \sim \displaystyle\int_0^L \dfrac{(v_0 - v)}{c} dL = \dfrac{L^2}{2\gamma_0^2 (L_0 - L)}$,

whence $L \sim L_0 f(A_w)$ and $W_e \sim W_P f(A_w)$,

$$A_w = \frac{2\gamma_0^2 c}{L_0 \omega_p} = \frac{2\alpha\gamma_0 m}{m_P} \approx \frac{\alpha W_P}{1\ \mathrm{TeV}}, \quad f(x) = \frac{-x + \sqrt{x^2 + 4x}}{2}.$$

Utility criterion for proton drivers



Simulations: $W_e \approx 0.6$ TeV
for $W_P = 1$ TeV
$\Downarrow$
missed numerical factor ≈ 1

Energy gain of the witness is comparable with the energy of proton driver only for the full-amplitude wave and multi-TeV driver energy.

Equally applicable to multi-bunch excitation!

PS: Wp=24 GeV, α=0.01, We=0.4 GeV

SPS: Wp=450 GeV, α=0.01, We=30 GeV

Problem of hosing: evolution of ideas

Short bunch is necessary, but difficult to produce => multibunch excitation

Bunch-to-bunch distance is too short for RF => let plasma wave make it

We cannot directly create strong plasma wave for modulation => try to harness an instability (transverse two stream)

First 3d run by A.Pukhov shows no hosing for PS beam => optimization with fast 2d axisymmetric code (results to be reported)

3d run with optimum parameters (PS beam): there is hosing, and wave amplitude 10 times lower => we have the problem of hosing and must (and possibly can) control the instability mode

Problem of hosing: run with no hosing

VLPL3D simulation

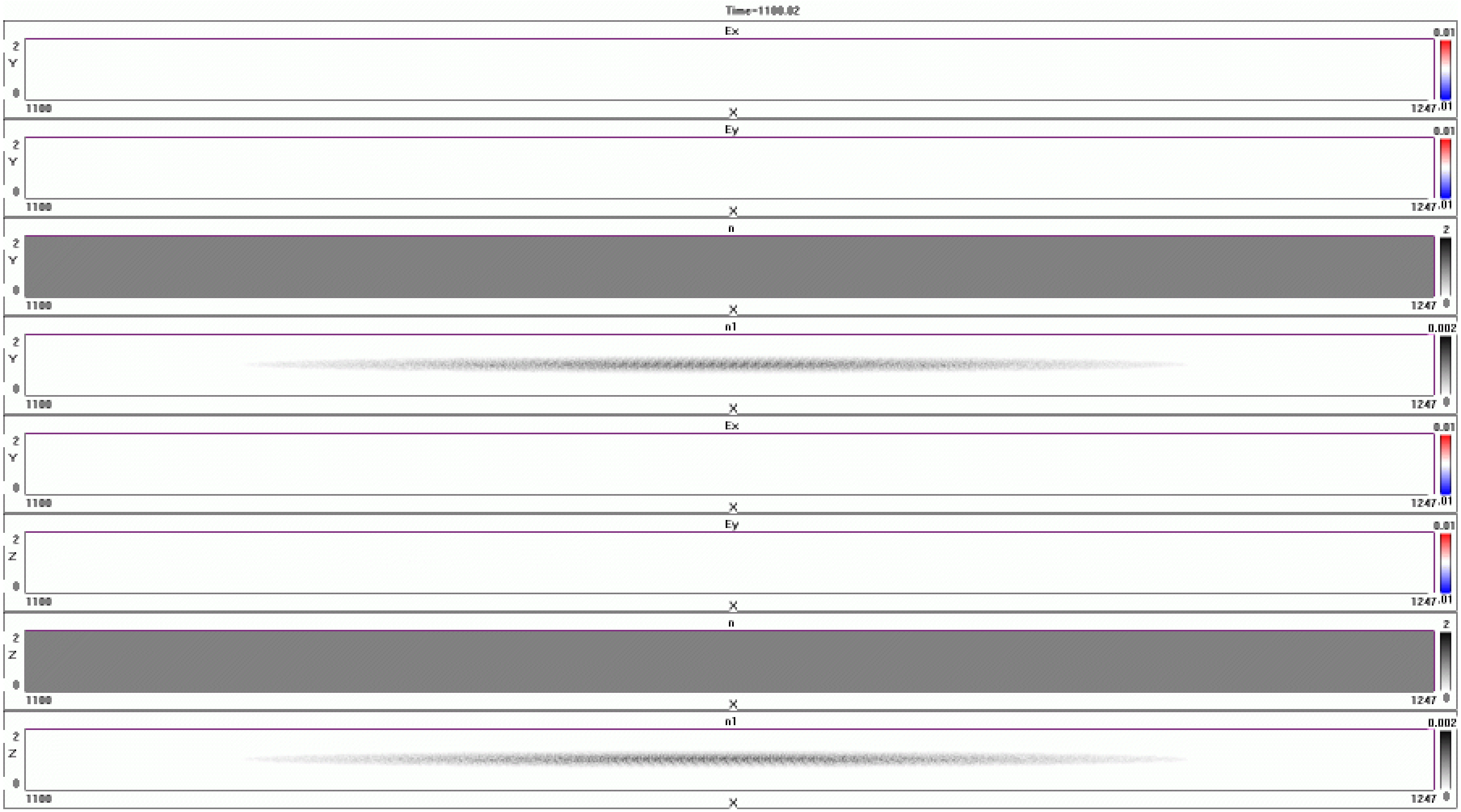
PS-beam:

10cm,10e11 p

Plasma: 10e13 1/cc

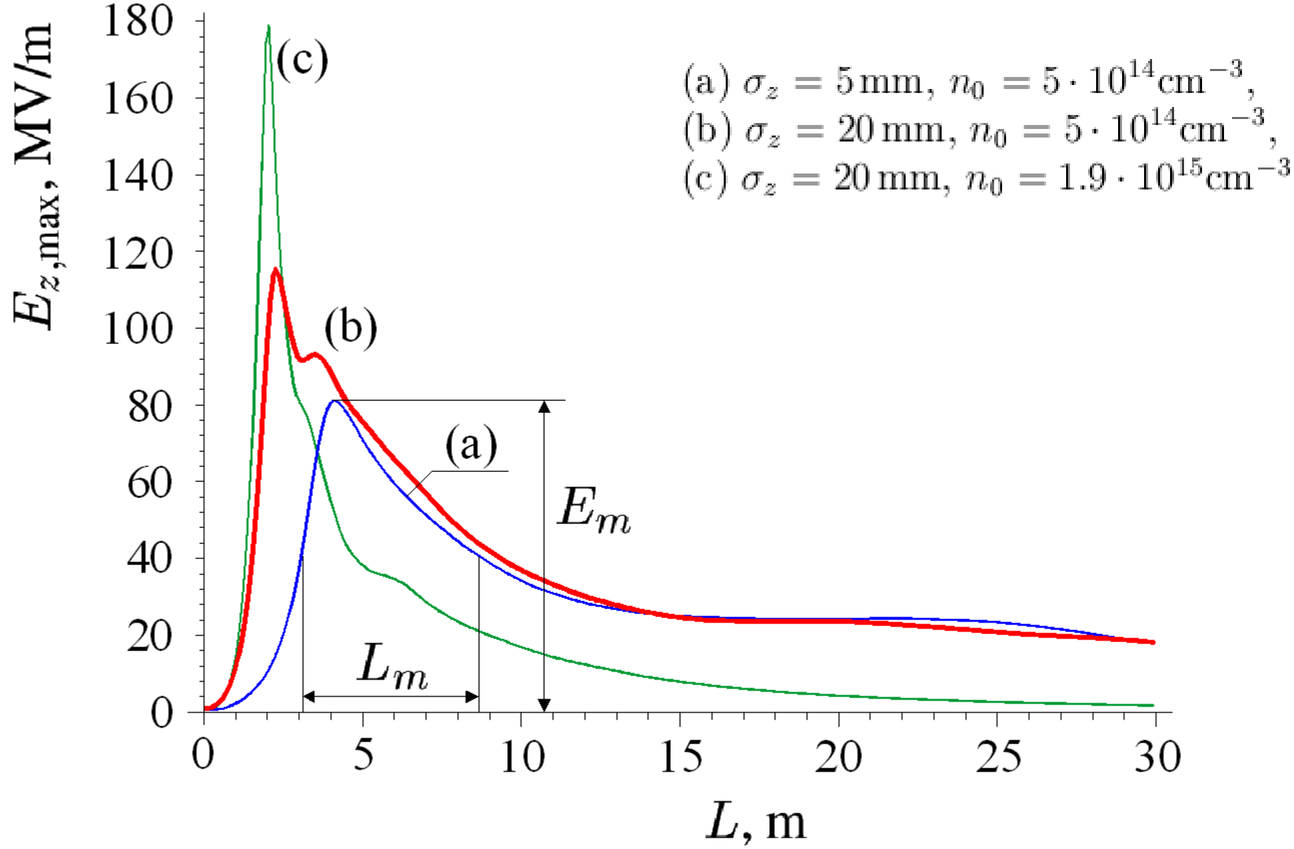
Problem of hosing: run with hosing

3d simulation of PS beam compressed to σ_z=5cm in 5 10^{14} cm^{-3} plasma (made by Alexander Pukhov): maximum field ~ 10 MV/m



Problem of hosing: optimized 2d runs

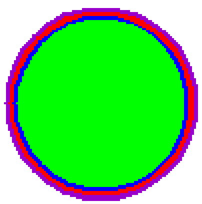
2d simulations of PS beam: maximum field ~ 100 MV/m



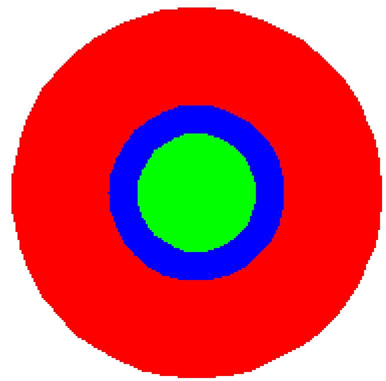
=> The same instability, but another mode, produces much lower fields

Problem of hosing: explanation of lower fields

Front view:

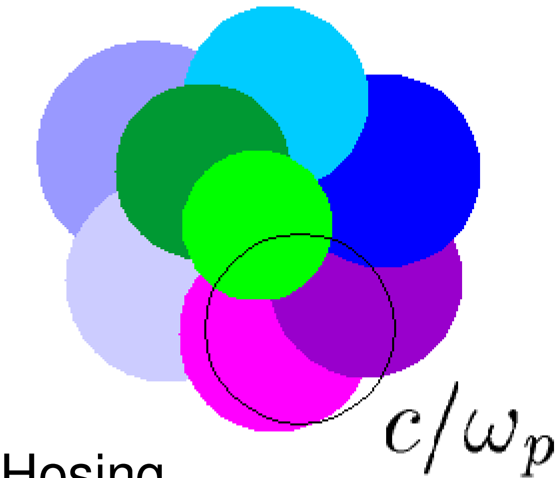


Original beam



Axisymmetric mode

Half of the beam contributes to on-axis field excitation



Hosing mode

Small fraction of the beam contributes to the field at a given point

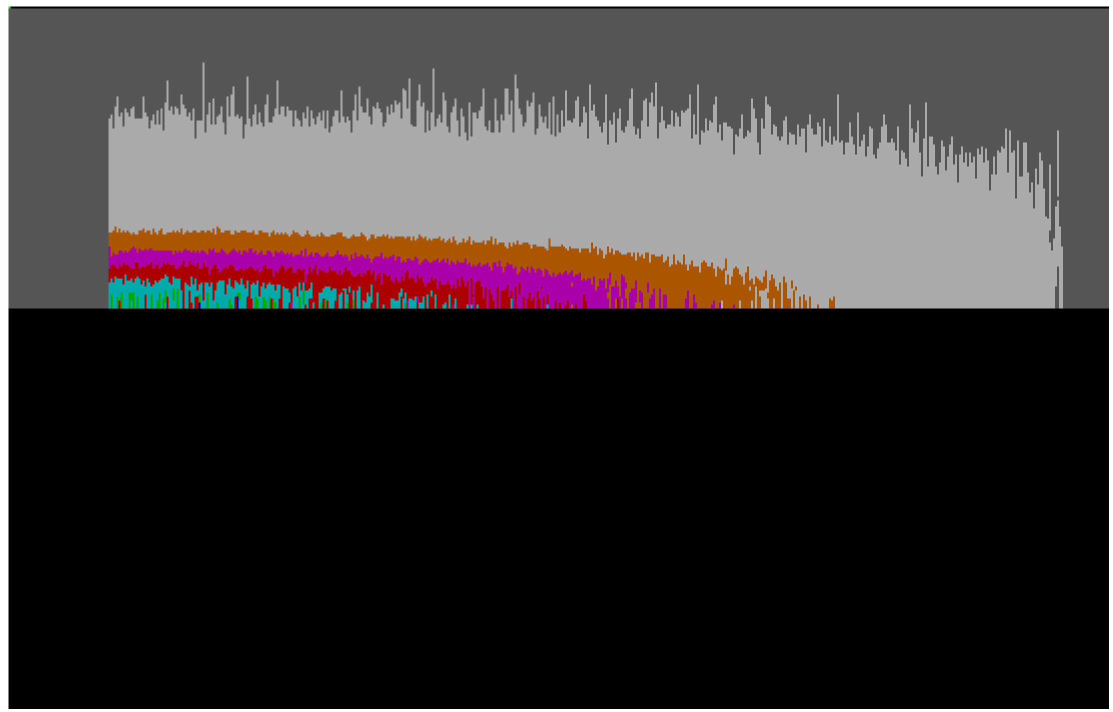
=> reason to avoid the hosing

Problem of hosing: one more reason to avoid

the effect can be understood by the example of 500 MeV electron beams and axisymmetric modes

Axisymmetric modes first modulate the beam, then destroy it

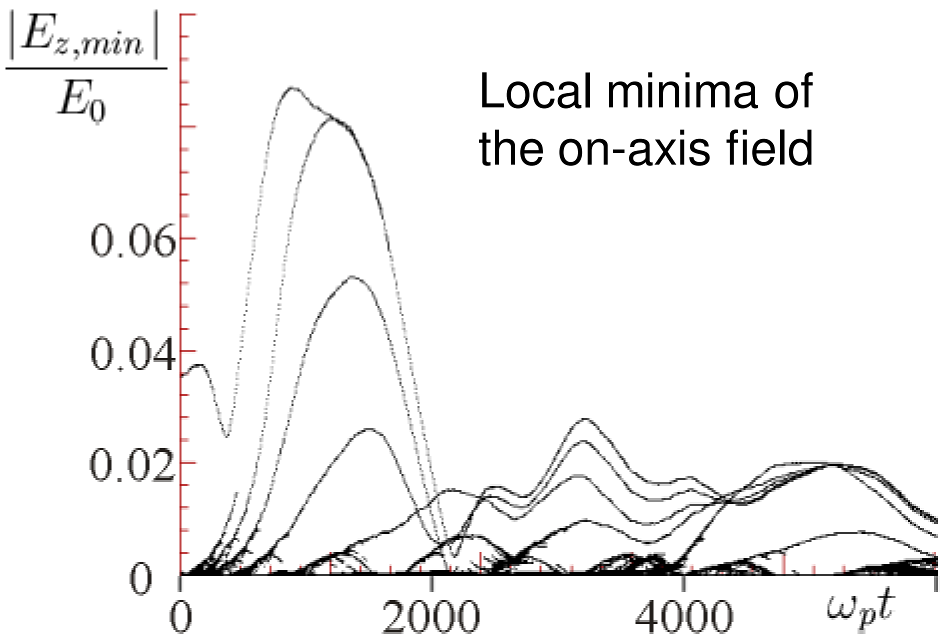
beam density

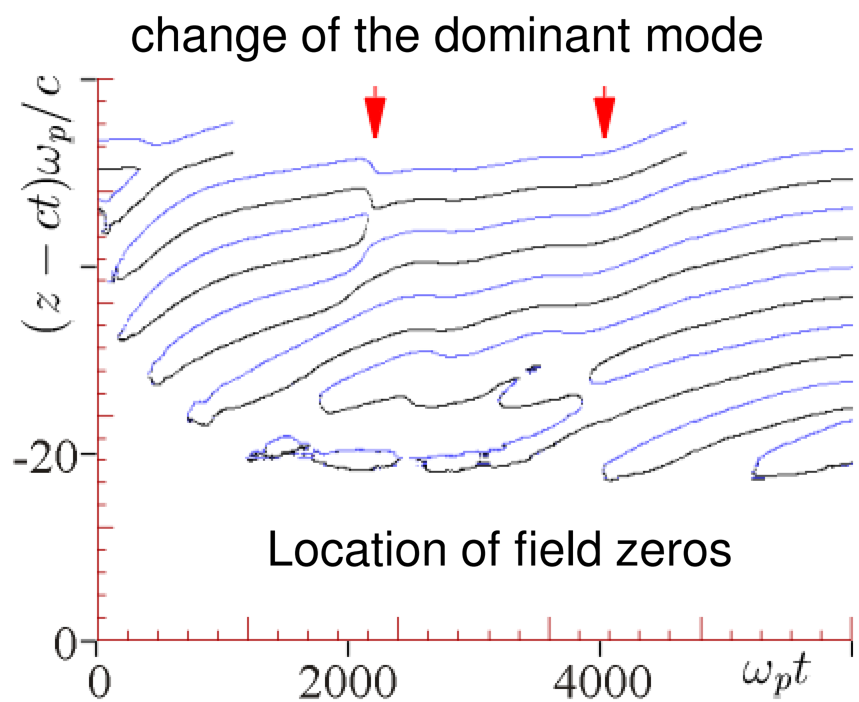


on-axis electric field and beam density

Problem of hosing: modes in competition

The reason for destruction is simultaneous growth of several modes



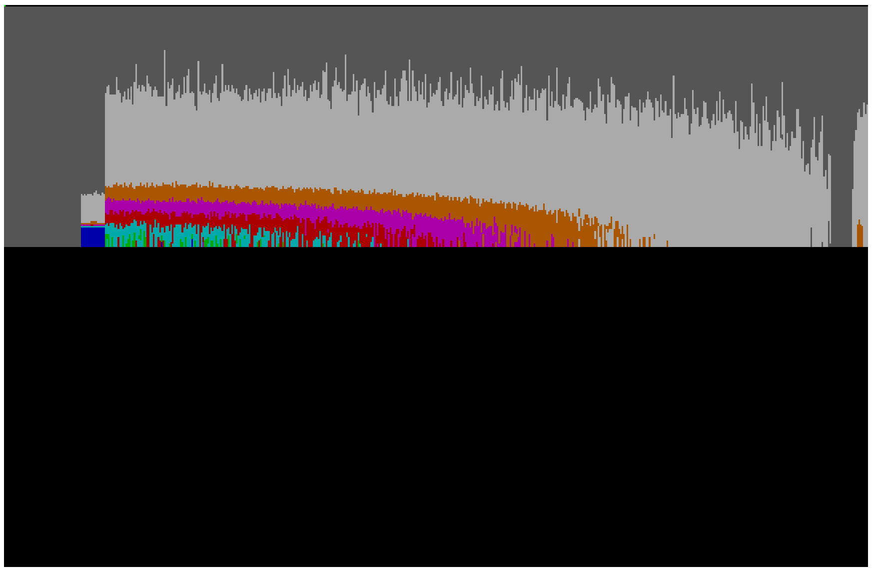


=> the less modes the better

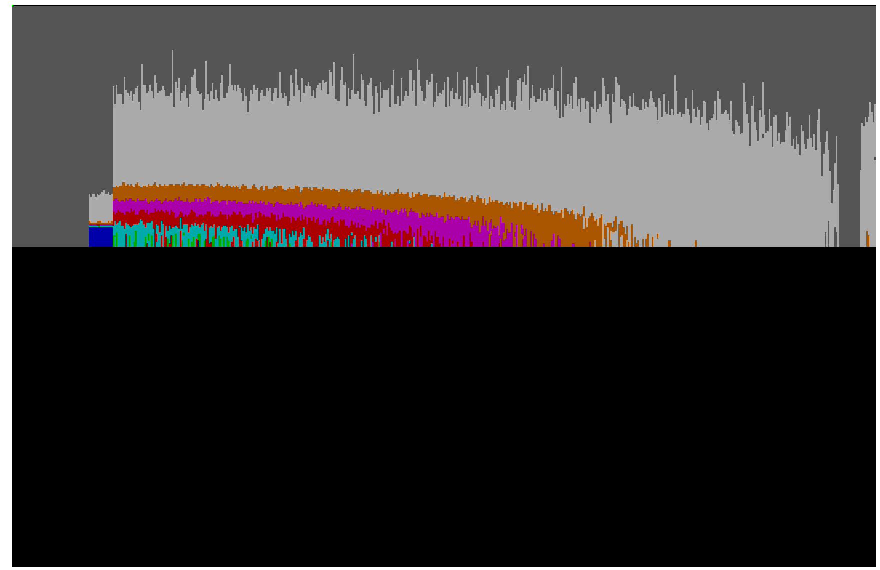
=> at least two reasons (lower fields, more competing modes) to avoid the hosing

Control of instability modes: a precursor

We can seed the proper mode by a precursor



Small electron precursor (1/150 of the total charge) seeds the instability mode and transforms the long electron beam into a bunch train

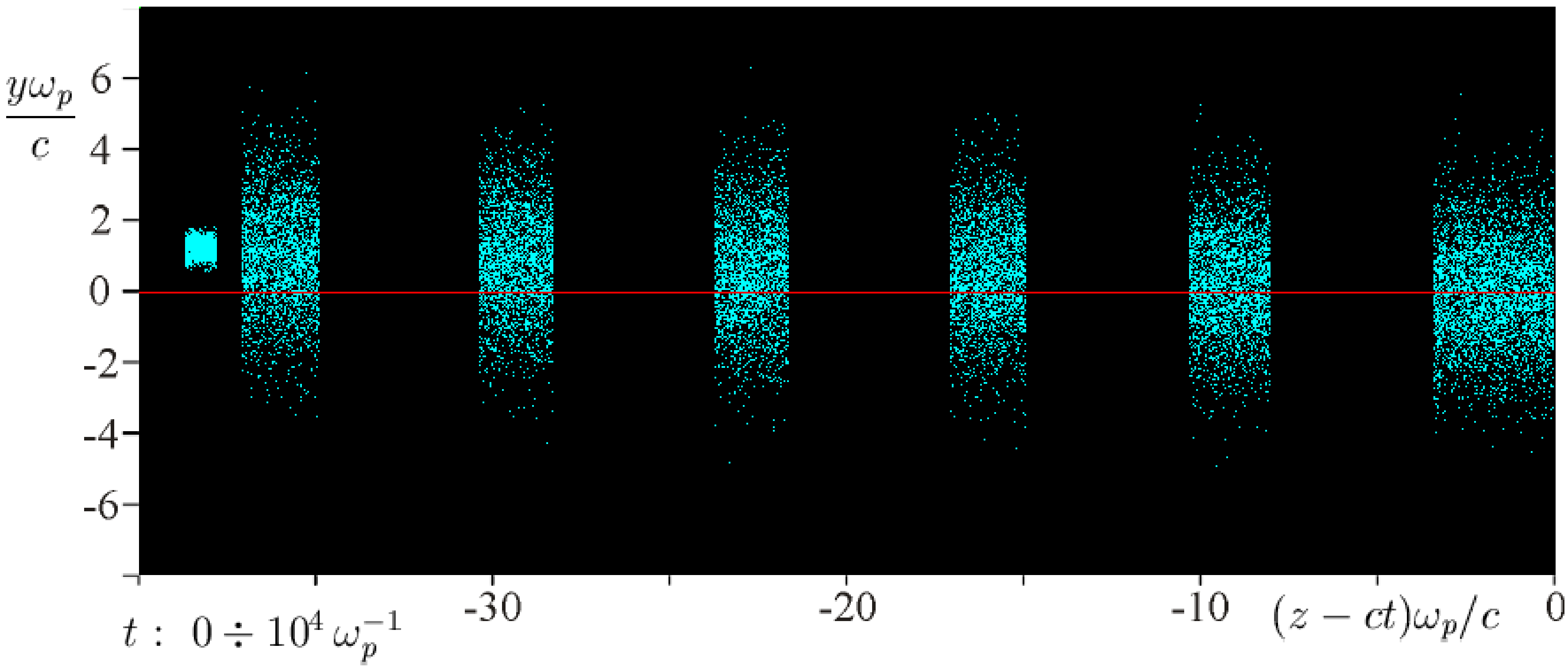


A precursor with 1/200 of the total charge is not sufficient to suppress other modes, the beam is destroyed

(K.V.Lotov, Instability of long driving beams in plasma wakefield accelerators, Proc. 6th European Particle Accelerator Conference (Stockholm, 1998), p.806-808)

Control of instability modes: goal

The train of microbunches is stable!



Even initially titled bunch train does not suffer from hosing (2d plane simulation)

Proper seeding can open a way to the single-mode instability and transformation of the long beam into a bunch train

Control of instability modes: laser beam

Laser: $P_m = 100\,\mathrm{TW}, \ \sigma_z = 15\,\mu\mathrm{m}\ (50\,\mathrm{fs}), \ \lambda = 810\,\mathrm{nm} = 2\pi c/\omega$

Strength parameter (Rayleigh diffraction): $a^2 = a_m^2 \dfrac{\sigma_0^2}{\sigma_r^2} e^{-r^2/\sigma_r^2 - z^2/\sigma_z^2}$

$$\sigma_r = \sigma_0 \sqrt{1 + l^2/z_R^2}, \qquad z_R = \frac{\omega \sigma_0^2}{c}, \qquad a_m^2 = \frac{4 P_m e^2}{m^2 c^3 \omega^2 \sigma_0^2}$$

Radial wakefield (linear): $E_r = \dfrac{2\pi n_p c}{\omega_p} \displaystyle\int_z^\infty \dfrac{\partial a^2(r, z')}{\partial r} \sin \dfrac{(z - z')\omega_p}{c}\, dz',$

maximum: $E_{rm}(r, l) = \dfrac{m c \omega_p}{e} \cdot \dfrac{a_m^2 \sigma_0^2}{\sigma_r^2} \cdot \dfrac{\sqrt{\pi} r \sigma_z}{\sigma_r^2} \exp\left(-\dfrac{\sigma_z^2 \omega_p^2}{4c^2} - \dfrac{r^2}{\sigma_r^2} \right).$

(De)focusing strength: $S = \displaystyle\lim_{r \to 0} \dfrac{e E_{rm}}{r} = \dfrac{a_m^2 \sigma_0^2 \cdot m c \omega_p \cdot \sqrt{\pi} \sigma_z}{\sigma_r^4} \exp\left(-\dfrac{\sigma_z^2 \omega_p^2}{4c^2} \right)$

Control of instability modes: laser beam

Scale of the radial push (at radius c/ω_p):

$$\Delta p_r = \frac{c}{\omega_p} \int_{-\infty}^{\infty} \frac{S\,dl}{c} = mc \frac{\pi\sqrt{\pi}\, a_m^2 \sigma_z z_R}{2\sigma_0^2} \exp\left(-\frac{\sigma_z^2 \omega_p^2}{4c^2}\right).$$

Assume $\sigma_z \ll c/\omega_p$: $\Delta p_r = mc \cdot \frac{P_m}{P_1} \cdot \frac{\sqrt{\pi}\lambda\sigma_z}{\sigma_0^2}$, $\qquad P_1 = \frac{mc^3}{r_e} \approx 9\,\mathrm{GW}.$

For $\sigma_0 = c/\omega_p$, $n_p = 5 \cdot 10^{14}\,\mathrm{cm}^{-3}$ $(c/\omega_p \approx 0.24\,\mathrm{mm})$: $\qquad \Delta p_r \approx 2\,\mathrm{MeV}/c$

SPS beam focused to 0.24 mm: $\qquad \delta p_r \sim 15\,\mathrm{MeV}/c$

available lasers are too weak for direct beam modulation, and maybe too weak even for seeding the instability

Control of instability modes: sharp beam front

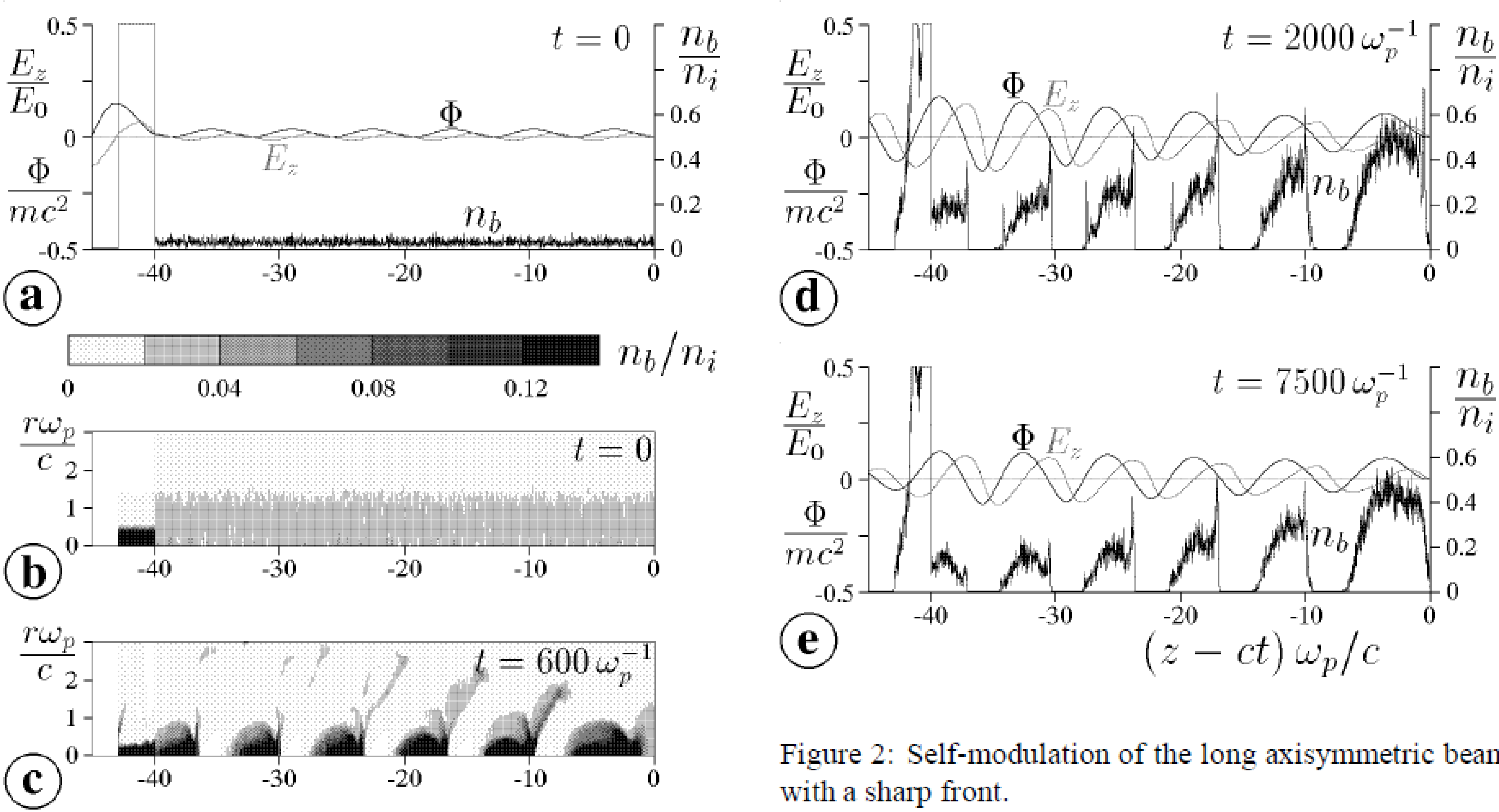
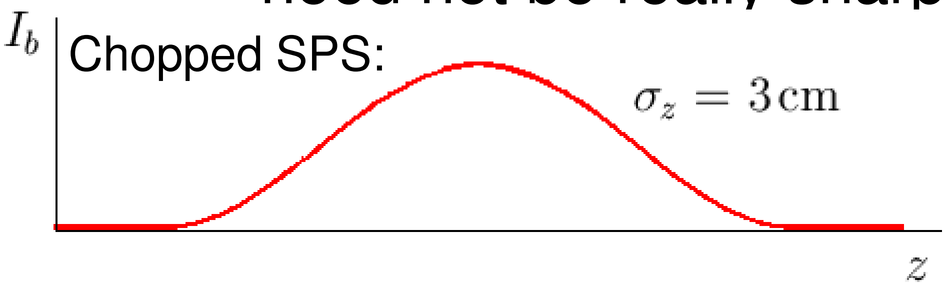
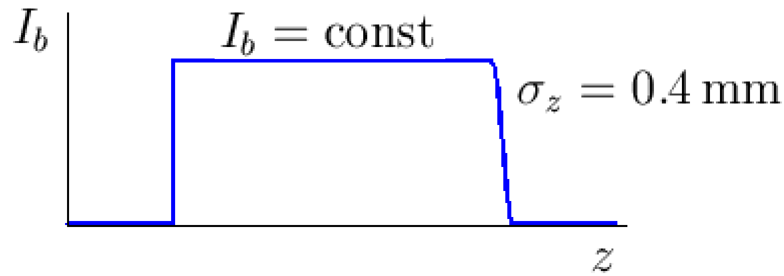


Figure 2: Self-modulation of the long axisymmetric beam with a sharp front.

(K.V.Lotov, Instability of long driving beams in plasma wakefield accelerators, Proc. 6th European Particle Accelerator Conference (Stockholm, 1998), p.806-808)

Control of instability modes: sharp beam front need not be really sharp



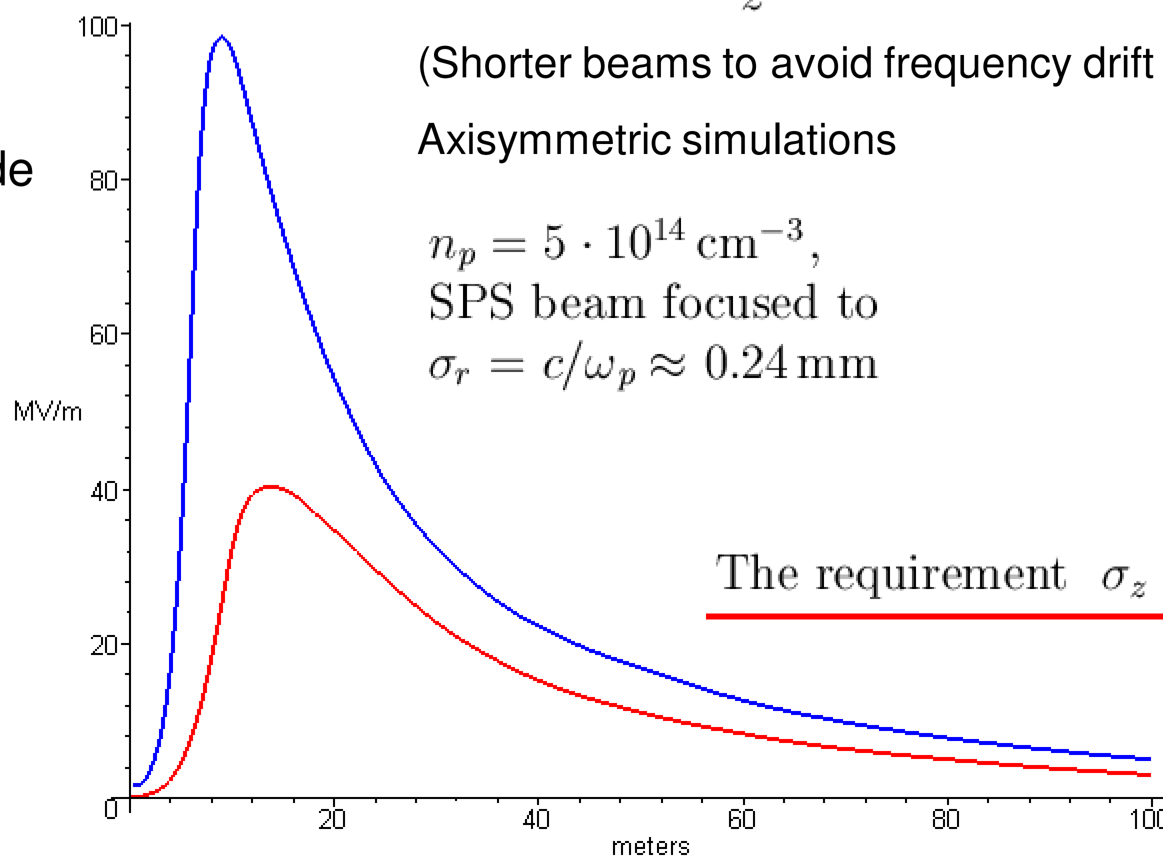


(Shorter beams to avoid frequency drift due to ion motion etc.)

Axisymmetric simulations

$n_p = 5 \cdot 10^{14}\,\mathrm{cm}^{-3}$,
SPS beam focused to
$\sigma_r = c/\omega_p \approx 0.24\,\mathrm{mm}$

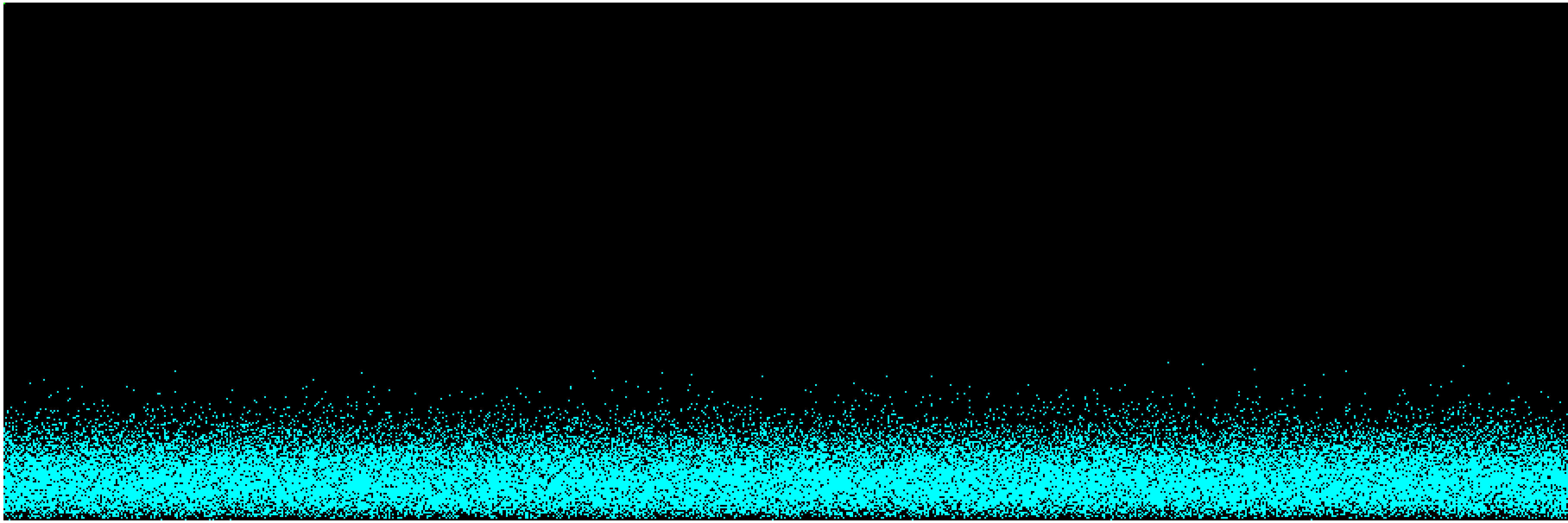
field amplitude

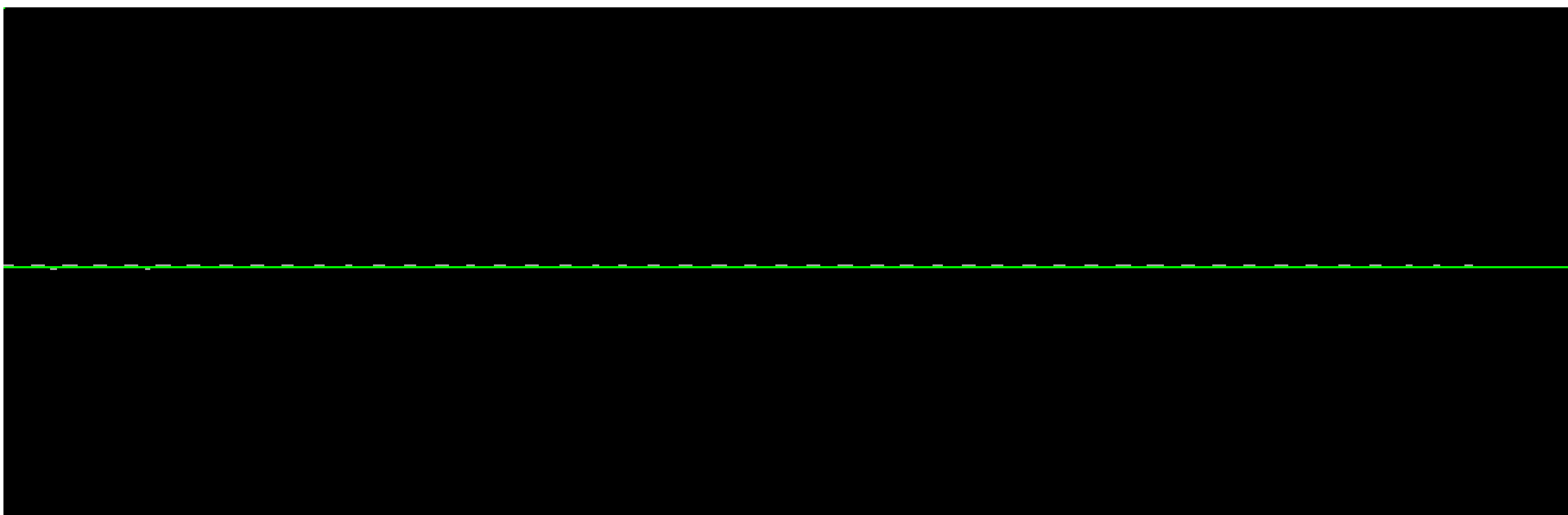


propagation distance

The requirement $\sigma_z \lesssim c/\omega_p$ is relaxed!

Control of instability modes: sharp beam front





Control of instability modes: sharp beam front

Seeding the wave with the beam front is probably a way to control the instability,

but it may be technically challenging to seed only the axisymmetric mode

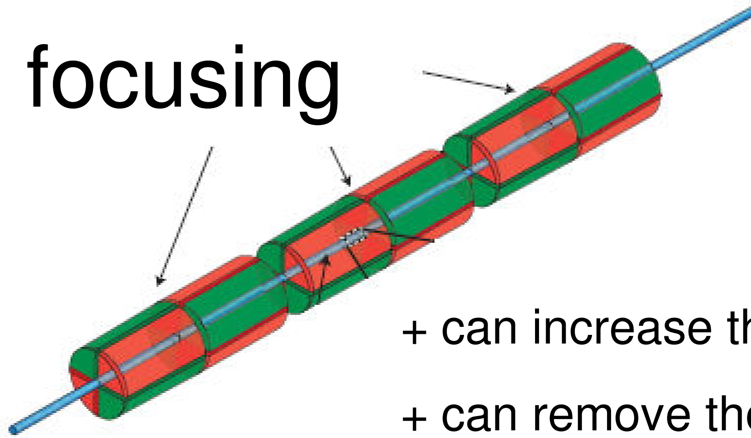
Assume it is possible, then

the high-field distance is limited by the beam divergence,

defocused protons form the diverging cone ($2\ 10^{-4}$ rad) which is 1 cm in radius after 50 m of propagation

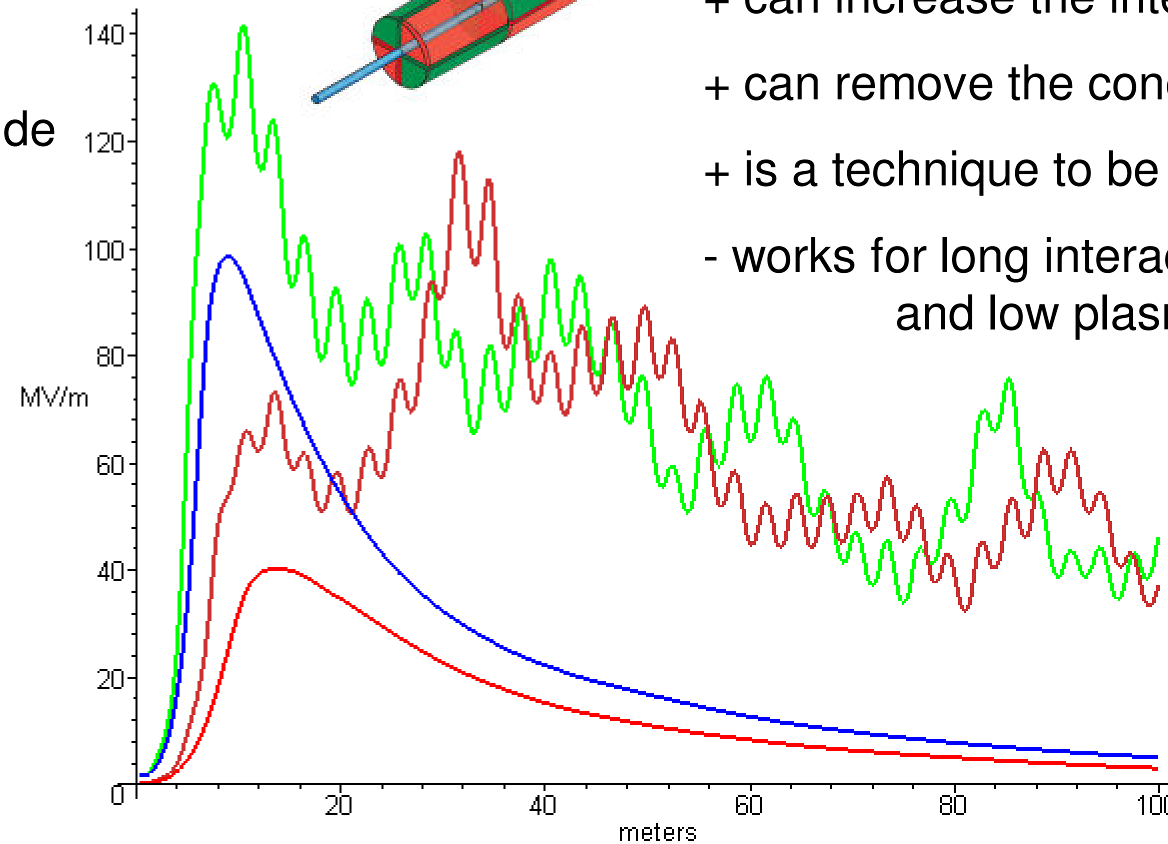
K.Lotov, Budker INP, presented at PPA-2009 (CERN)

Quadrupole focusing



+ can increase the interaction distance,
+ can remove the cone of defocused protons,
+ is a technique to be tested for the future,
- works for long interaction distances (SPS) and low plasma densities

field amplitude



no seed, no quads

seed, no quads

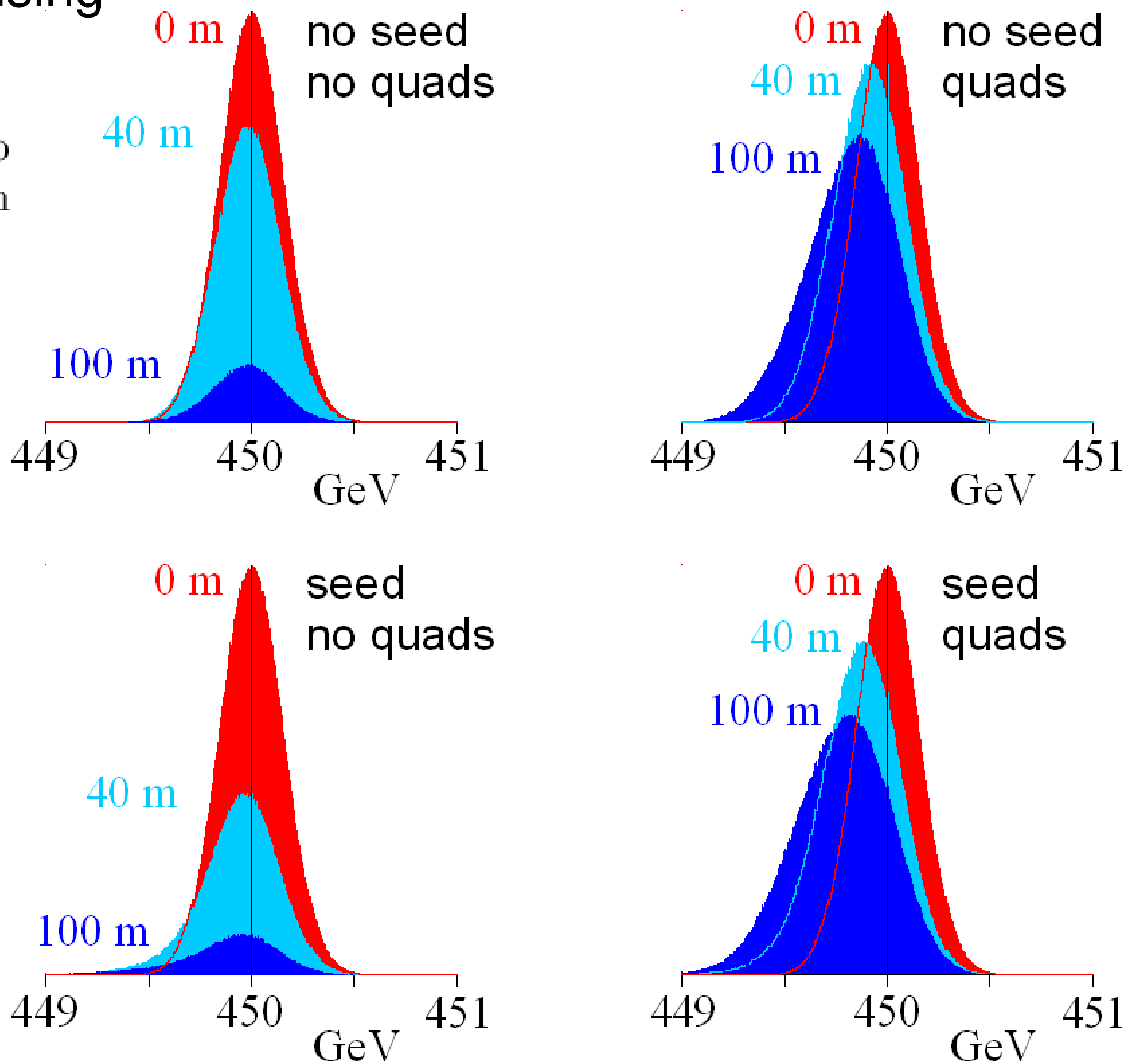
no seed, quads

seed, quads

propagation distance

Quadrupole focusing

$n_p = 5 \cdot 10^{14}\,\mathrm{cm}^{-3}$,
SPS beam focused to
$\sigma_r = c/\omega_p \approx 0.24\,\mathrm{mm}$



Quadrupole focusing

... is good for wakefields