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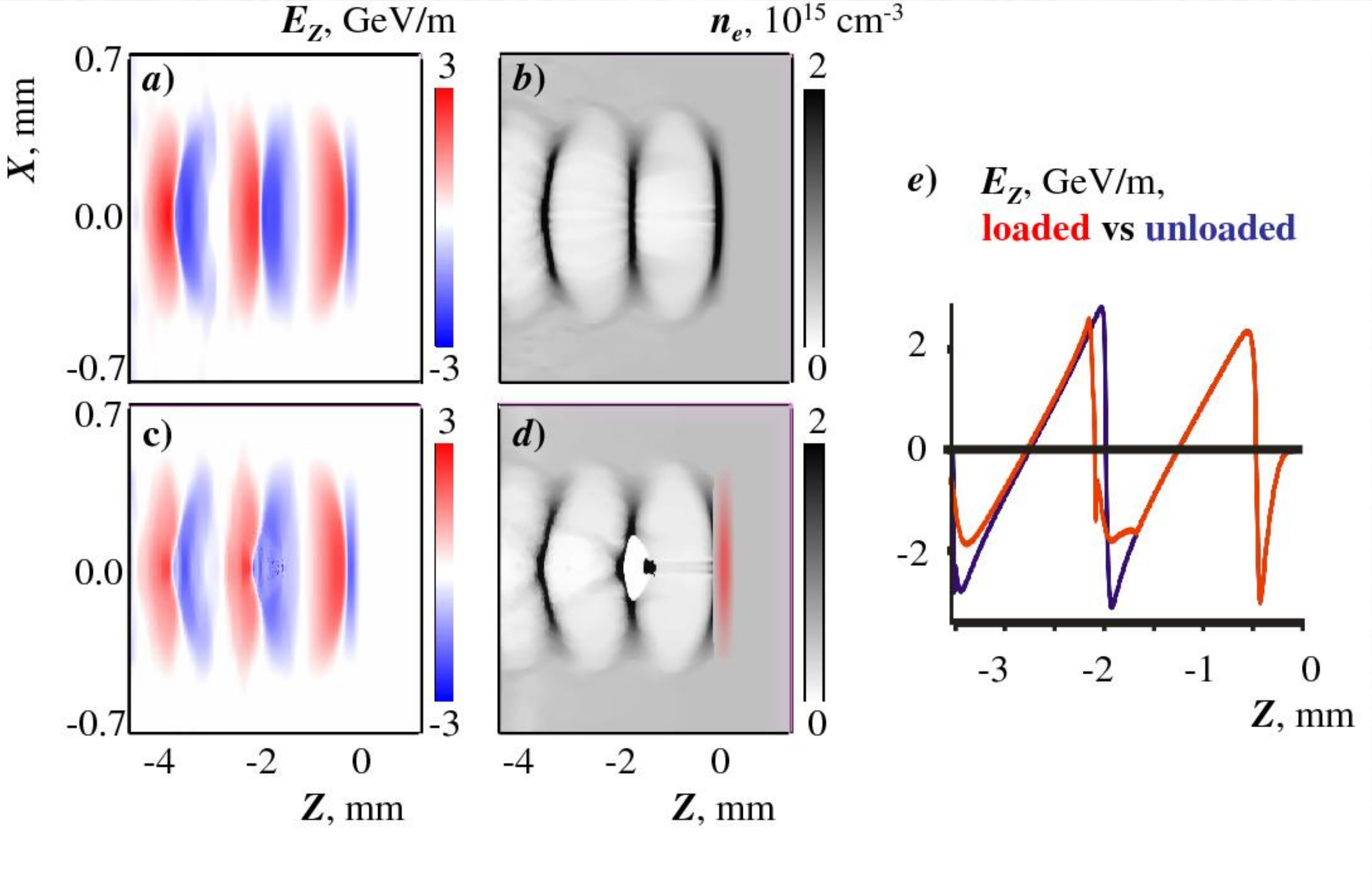
Plasma wave excitation by long proton beams in plasmas

Plasma Wake Field Acceleration

VLPL

- Analytical theory of SM-PDPWA
- VLPL3D simulations for PS-beam
- VLPL3D simulations for SPS beam
- Dependence on plasma density

Short proton driver, $\tau\omega_p$~1: Excellent wakefield generation



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Wake field excitation

If we have a beam with the density $\rho(r,\xi) = \rho_0 g(r) f(\xi)$

$\xi = \beta_0 ct - z$

$$E_z(r,\xi) = 4\pi k_p^2 \int_0^{\xi} \int_0^{\infty} r' dr' \rho(r', \xi')\, I_0(k_p r_<) K_0(k_p r_>)\, d\xi' f(\xi') \cos k_p(\xi - \xi'),$$

$$(E_r - B_\theta)(r,\xi) = 4\pi k_p \int_0^{\xi} \int_0^{\infty} r' dr'\, \partial_{r'} \rho(r', \xi')\, I_1(k_p r_<) K_1(k_p r_>)\, d\xi' f(\xi') \sin k_p(\xi - \xi')$$

Only short beams, $\tau\omega_p \sim 1$, or beams structured at $k_p = \omega_p/c$ generate wakes

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Long proton beam

For a long proton beam with the step-like radial profile

$$g(r) = H(r_b(\xi) - r)$$

$$E_z(r,\xi) = 4\pi\rho_0 \int_0^{\xi} \left\{1 - k_p r_b I_0(k_p r) K_0(k_p r_b)\right\} f(\xi') \sin k_p(\xi - \xi') d\xi',$$

$$W_\perp(r,\xi) = (E_r - B_\theta)(r,\xi) = -4\pi\rho_0 k_p \int_0^{\xi} r_b(\xi') I_1(k_p r) K_1(k_p r_b) f(\xi') \ \sin k_p(\xi - \xi') d\xi'$$

$r_b(\xi)$ is the beam radius changing due to pinching

$\rho_0 = e n_b$ is the beam charge density

Self-modulation instability

Equation for the beam envelope radius r_b

$$\frac{\partial^2 r_b}{\partial \tau^2} - \frac{L^2}{r_b^3} = -\frac{\omega_b^2}{\gamma_0}\int_0^{\xi} r_b(\xi')I_1(k_p r_b)K_1(k_p r_b)f(\xi')\,k_p \sin k_p(\xi-\xi')d\xi'$$

Positive feedback

$\omega_b^2 = 4\pi e\rho_0/m_i$ is the beam plasma frequency

$L = r_{b0}^2\omega_{\beta 0}^2$ is the beam angular momentum

Linear analysis

For simplicity, we assume a step-like density profile $en(r,\xi)= \theta(r_b-r)\,\theta(\xi)$ and the beam is thin, $k_p r_b << 1$. Then, for the normalized radius $r_b = r_b / r_0$ we obtain

$$\frac{\partial^2 r_b(\xi)}{\partial\tau^2} - \frac{\omega_{\beta 0}^2}{r_b^3(\xi)} = -\omega_{\beta 0}^2 \int_0^\xi r_b(\xi')\, k_p \sin k_p(\xi - \xi')d\xi',$$

$$\omega_{\beta 0}^2 = \omega_b^2 / 2\gamma_0$$

We perturb this eq. $r_b = 1 + \delta r_b$ where $\delta r_b = \delta\hat{r}_b \exp(ik_p\xi)$

Dispersion relation

$$\left(\frac{\partial^2}{\partial\xi^2}+k_p^2\right)\left(\frac{\partial^2}{\partial\tau^2}+\Delta\right)\delta\hat{r}_b=-\omega_{\beta 0}^2\,k_p^2\delta\hat{r}_b,$$

where $\Delta = 3\omega_{\beta 0}^2$.

we assume $\delta\hat{r}_b \sim \exp(ik\xi - i\delta\omega\tau)$

and obtain the dispersion relation

$$D = (k^2-k_p^2)(\delta\omega^2-\Delta)+\omega_{\beta 0}^2 k_p^2.$$

Instability growth rate

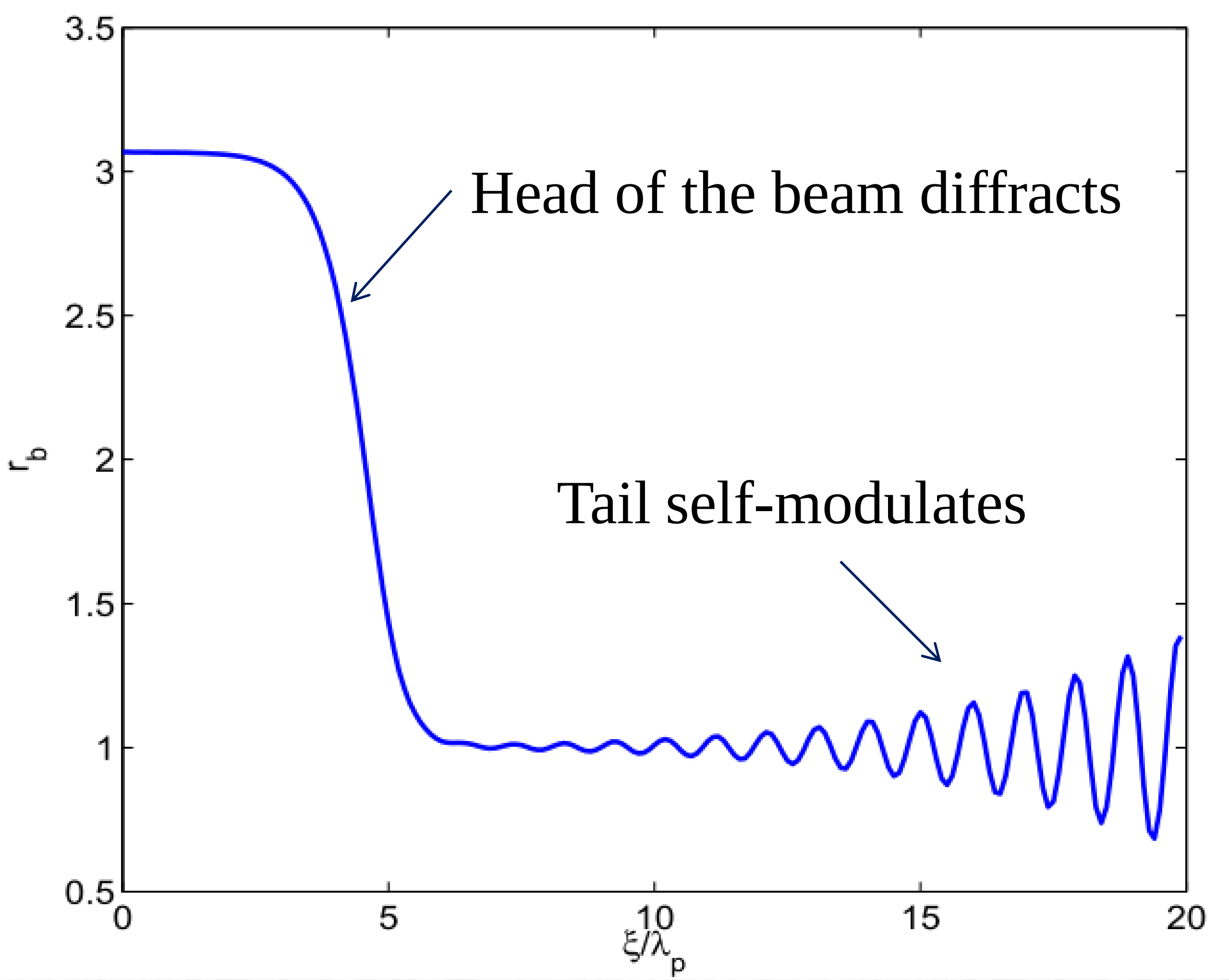
We express the instability growth as the number of exponentiations at time τ and distance ξ from the beam head

$$\Gamma\tau = (\omega_{\beta 0} k_{\mathrm{p}} \xi \tau / 2)^{1/2}.$$

where

$$\omega_{\beta 0}{}^2 = 4\pi n_{\mathrm{b}} e^2 / m_{\mathrm{i}} \gamma_0.$$

“Analytical” simulation



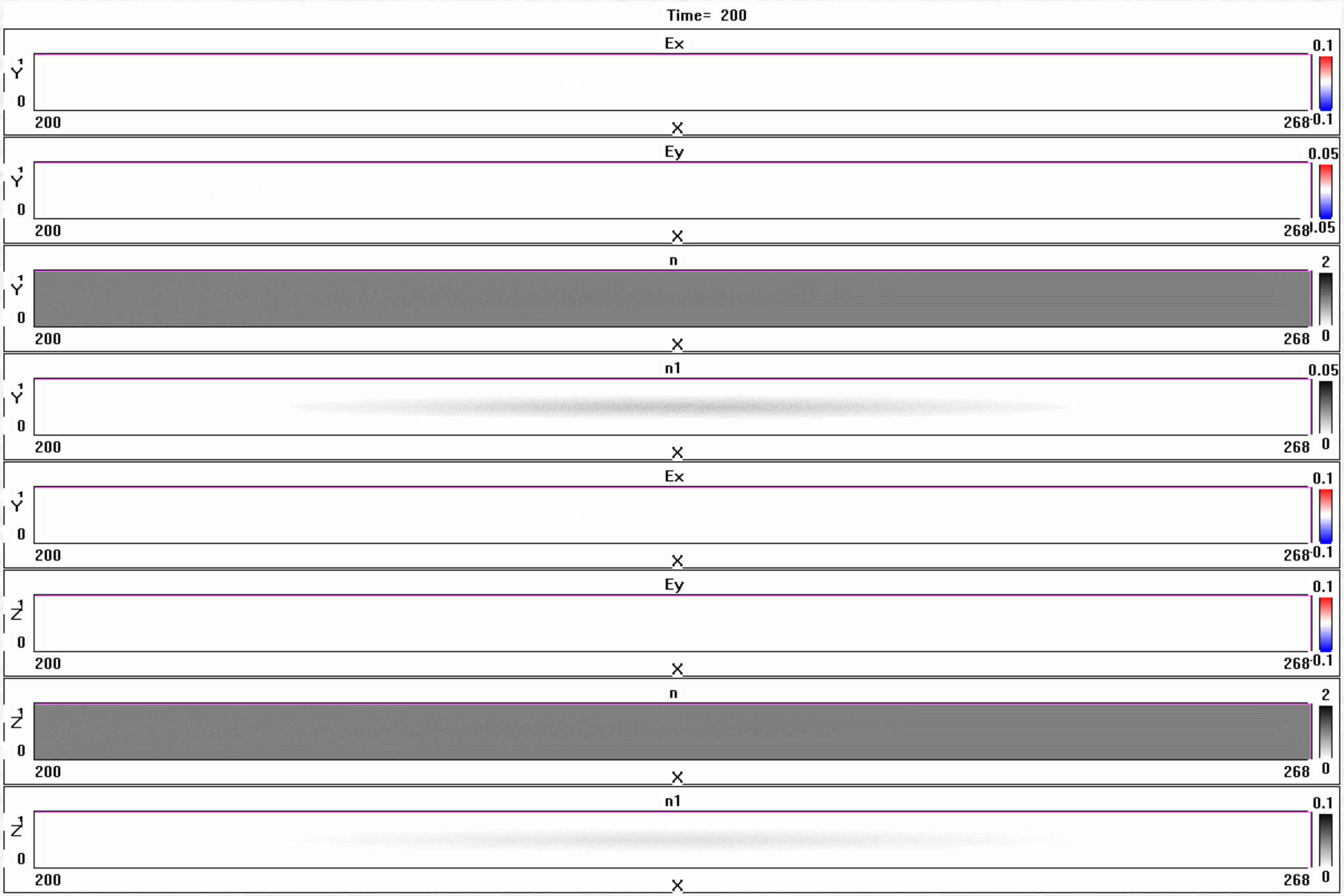
3D PIC simulations for PS-beam

Simulation parameters:

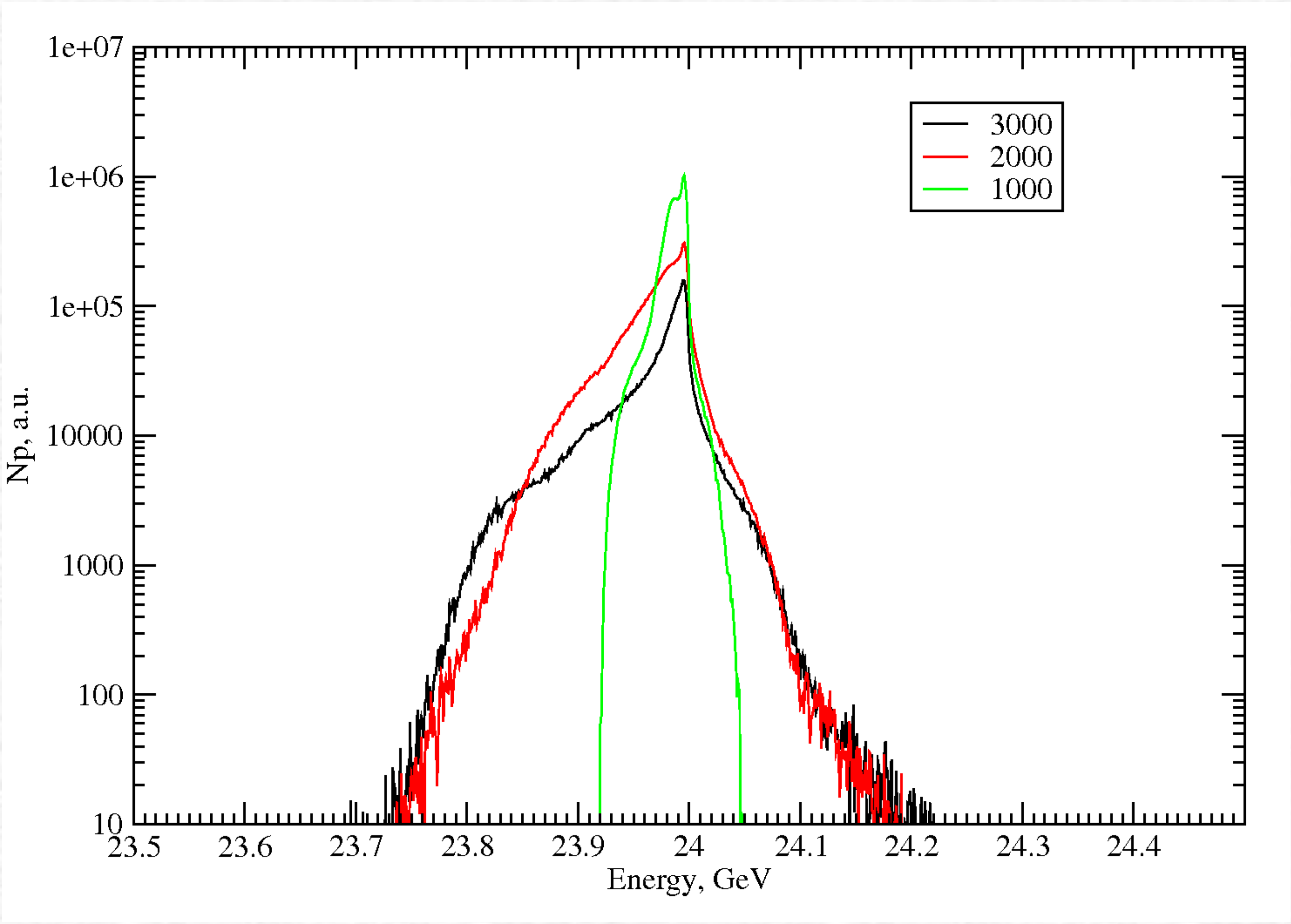
Proton energy 24 GeV
Plasma density $n_p = 100\ n_{b0}$.
Beam density profile $n_p = n_{b0} \exp(-z^2/\sigma_z^2) \exp(-r^2/\sigma_r^2)$
The beam length $k_p\sigma_z = 90$
The beam radius $k_p\sigma_r = 2$

The simulation time is measured in $T_p = 2\pi/\omega_p$
The simulation distances are measured in $\lambda_p = 2\pi c/\omega_p$

3D PIC simulations for PS-beam



3D PIC simulations for PS-beam



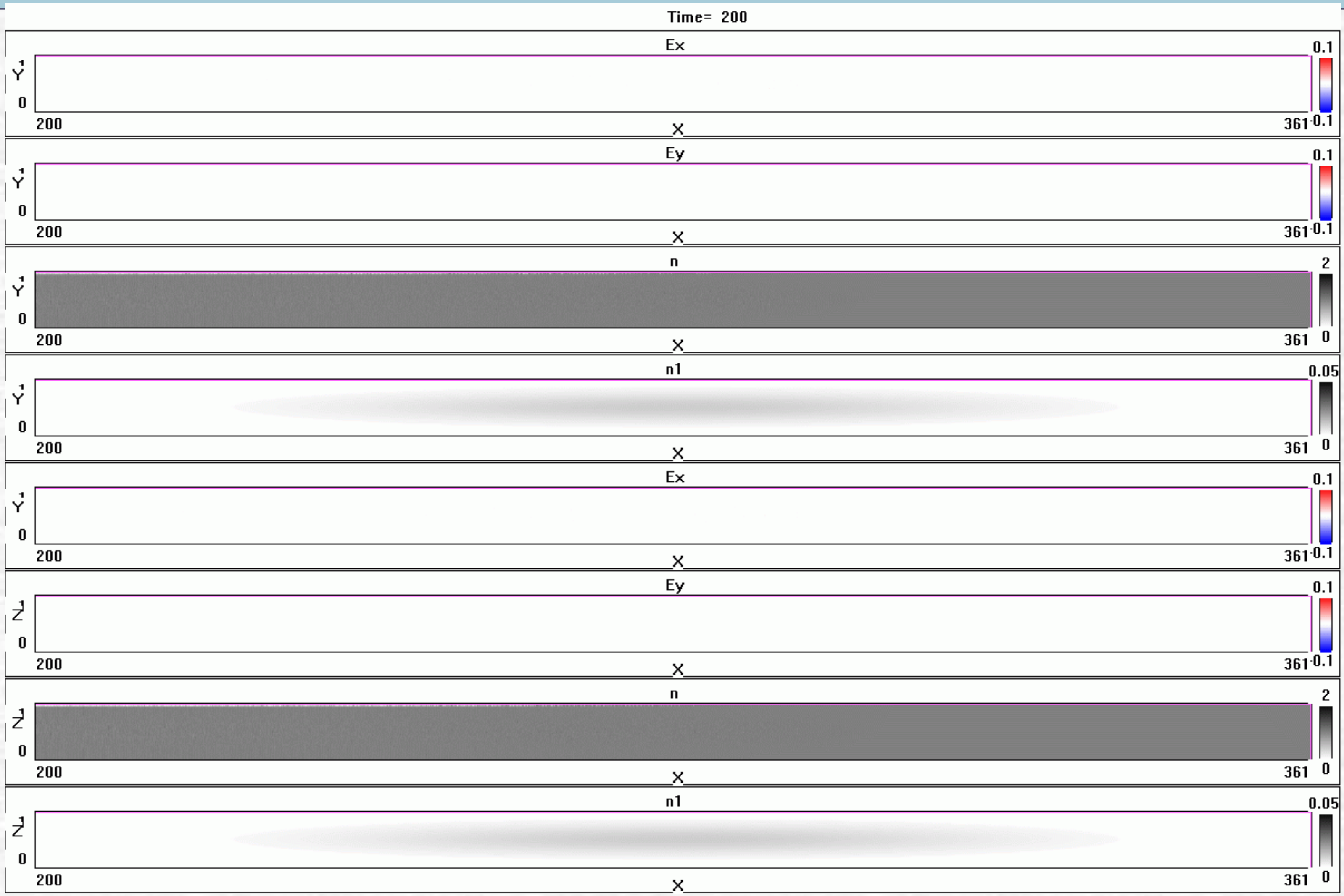
3D PIC simulations for SPS-beam

Simulation parameters:

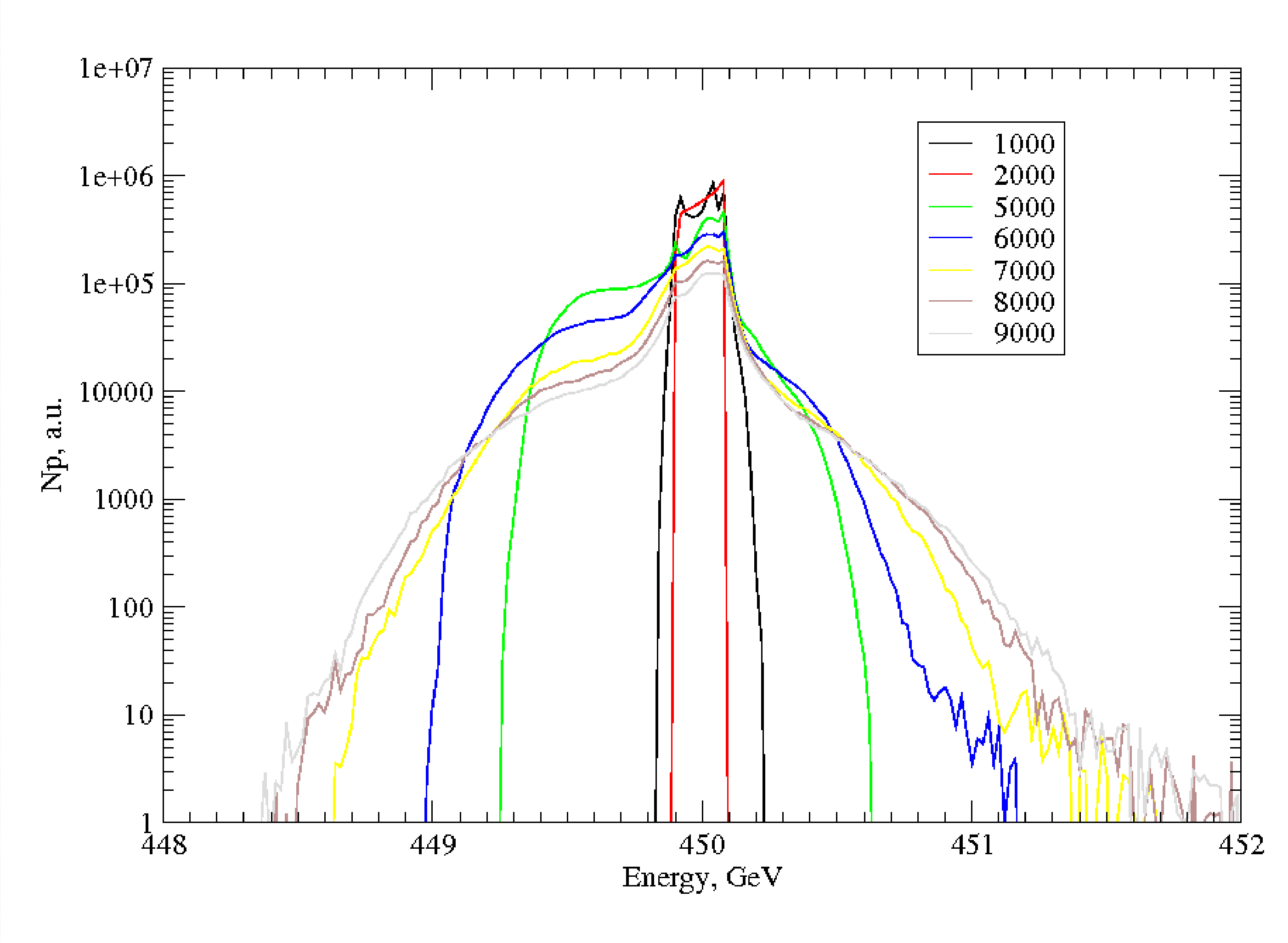
Proton energy 450 GeV
Plasma density $n_p = 100\, n_{b0}$.
Beam density profile $n_p = n_{b0} \exp(-z^2/\sigma_z^2) \exp(-r^2/\sigma_r^2)$
The beam length $k_p\sigma_z = 240$
The beam radius $k_p\sigma_r = 2$

The simulation time is measured in $T_p = 2\pi/\omega_p$
The simulation distances are measured in $\lambda_p = 2\pi c/\omega_p$

3D PIC simulations for SPS-beam



3D PIC simulations for SPS-beam



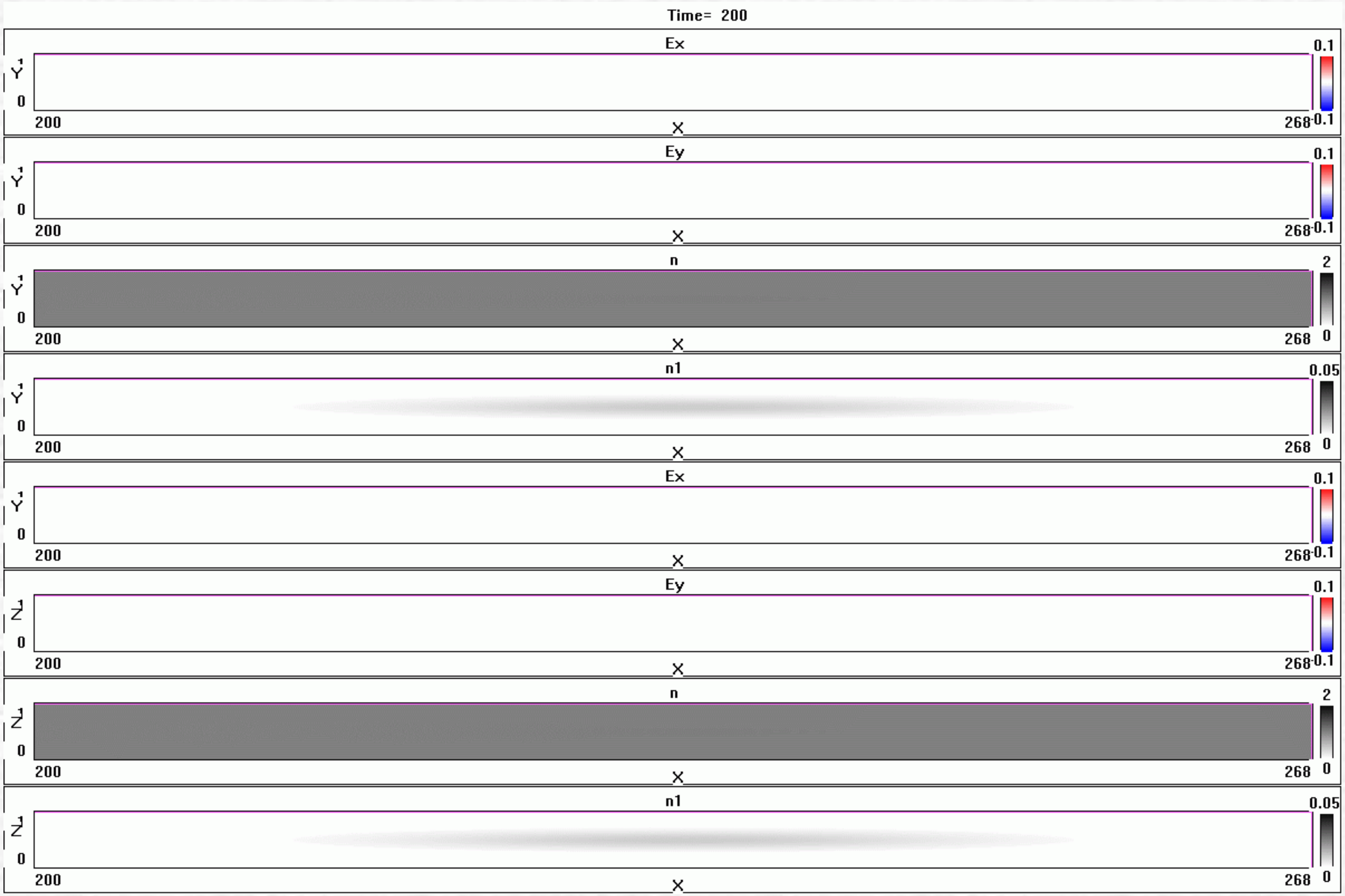
SPS beam in a lower plasma density

Simulation parameters:

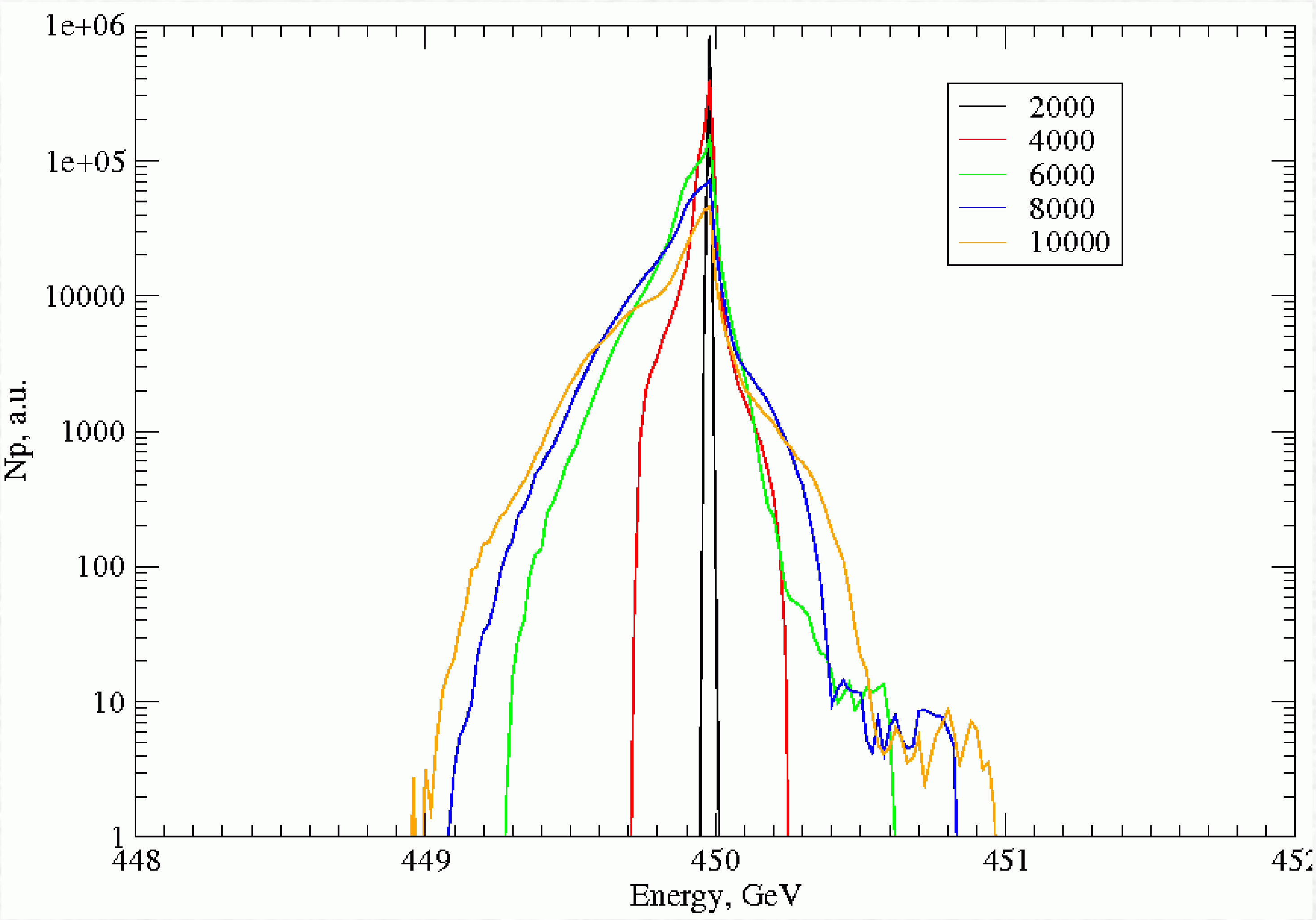
Proton energy 450 GeV
Plasma density n_p =100 n_{b0}.
Beam density profile $n_p = n_{b0} \exp(-z^2/\sigma_z^2) \exp(-r^2/\sigma_r^2)$
The beam length $k_p\sigma_z$=90
The beam radius $k_p\sigma_r$=2

The simulation time is measured in $T_p = 2\pi/\omega_p$
The simulation distances are measured in $\lambda_p = 2\pi c/\omega_p$

SPS beam in a lower plasma density



SPS beam in a lower plasma density



Discussion

- Proton beams are subject to self-modulation when propagating through plasmas
- The instability growth rate depends weakly on the particle mass and γ-factor
- The plasma wave is generated, although poorly controlled and is transient
- Beam split in beamlets via the “mask” might be useful and must be studied in detail