What type of exclusive measurements are missing?

Quarkonia As Tools
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Quarkonia advantages

- Strong coupling to gluons
- Heavy – extra “hard” scale
- Tool for study of gluonic structure of hadrons and hadronic/quark-gluon media
Main Topics

- Way to gluonic GPDs and (gravitational) formfactors
- Pressure of gluons in proton
- Transitions from exclusive to semi-exclusive:
- Probe of COME?!
- Elliptic Wigner Function as a precursor of elliptic flow?!
Recent development: pressure of QUARKS

- Published in Nature

- Based on previous work (Polyakov, OT, Anikin&OT, Pasquini, Vanderhaegen, Kumericky, Mueller, Goldstein&Liuti...)

- First time the nice picture presented (Link with stability of stars (Poincare, v. Laue))

- Link between hadronic and heavy ion physics
The pressure distribution inside the proton

V. D. Burkert, L. Elouadrhiri & F. X. Girod

\[ r^2 p(r) \times 10^{-2} \text{ GeV fm}^{-1} \]
Gravitational Formfactors

\[ \langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M \right] u(p) \]

- Conservation laws - zero Anomalous Gravitomagnetic Moment: \( \mu_G = J \) \( (g=2) \)
  - \( P_{q,g} = A_{q,g}(0) \) \( A_q(0) + A_g(0) = 1 \)
  - \( J_{q,g} = \frac{1}{2} \left[ A_{q,g}(0) + B_{q,g}(0) \right] \) \( A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1 \)

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons (Ji’s SRs)
- Describe interaction with both classical and TeV gravity
Generalized Parton Distributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin, OT ’09)

- Smaller mass square radius (attraction vs repulsion!?) – confirmed by analysis of pion time-like FFs (Kumano, Song, OT’17)

\[
\rho(b) = \sum_q e_q \int dxq(x, b) = \int d^2q F_1(Q^2 = q^2)e^{i\mathbf{q} \cdot \mathbf{b}}
\]

\[
= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1 + \tau}
\]

\[
\rho_0^{Gr}(b) = \frac{1}{2\pi} \int_0^\infty dq q J_0(qb) A(q^2)
\]

FIG. 17: Difference in the forms of charge density \(F_1^P\) and "matter" density \(A\).
Electromagnetism vs Gravity (OT’99)

- Interaction – field vs metric deviation

\[ M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \]

\[ M = \frac{1}{2} \sum_{q,G} \langle P' | T^\mu_{q,G} | P \rangle h_{\mu\nu}(q) \]

- Static limit

\[ \langle P | J_q^\mu | P \rangle = 2e_q P^\mu \]

\[ \sum_{q,G} \langle P | T^\mu_{i,G} | P \rangle = 2P^\mu P^\nu \]

\[ h_{00} = 2\phi(x) \]

\[ M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q) \]

\[ M_0 = \frac{1}{2} \sum_{q,G} \langle P | T^\mu_{i,G} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q) \]

- Mass as charge – equivalence principle
Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from
  \[ M = \frac{1}{2} \sum_{q,G} \langle P'|T^{\mu\nu}_{q,G}|P \rangle h_{\mu\nu}(q) \]
  \[ \vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \vec{g}_i \equiv g_{0i}. \]
  spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor 1/2 cancelled with 2 from Larmor frequency same as EM
  \[ h_{00} = 2\phi(x) \]

- Orbital and Spin momenta dragging – the same - Equivalence principle
  \[ \omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \vec{H}_L = \text{rot} \vec{g} \]
Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’; ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservation laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way
Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT’07) from Earth rotation – was considered as obvious (but it is just EP!) background

\[
\mathcal{H} = -g\mu_N \mathbf{B} \cdot \mathbf{S} - \zeta \hbar \omega \cdot \mathbf{S}, \quad \zeta = 1 + \chi \\
|\chi^{(\text{Hg})} + 0.369\chi^{(\text{Hg})}| < 0.042 \quad (95\% \text{C.L.})
\]
Indirect probe of spin-gravity coupling

- Matrix elements of energy-momentum tensors may be extracted from accurate high-energy experiments (“3D nucleon picture”)
- Allow to probe the couplings to quarks and gluons separately
Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics \( h_{zz} = h_{xx} = h_{yy} = h_{00} \)
- Matrix elements DIFFER

\[
\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \quad \mathcal{M}_a = \epsilon^2 h_{00}(q)
\]

- Ratio of accelerations: \( R = \frac{\epsilon^2 + p^2}{\epsilon^2} \) - confirmed by explicit solution of Dirac equation (Silenko, OT, ‘05)
Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim P^2$) is compensated by $1/P^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT’99) and
- Flip by “gravitoelectric” field: relic neutrino? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
  \[
  \langle P+q|T^{mn}|P-q\rangle = P^{m}\langle P+q|J^n|P-q\rangle/e \text{ up to the terms linear in } q
  \]
- Gravitomagnetic $g=2$ for any spin
- Special role of $g=2$ for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also $g=2$ for Black Holes. Indication of “quantum” nature?!
Cosmological implications of PNEP

- Necessary condition for Mach’s Principle (in the spirit of Weinberg’s textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat “Universe” - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!
Generalization of Equivalence principle

- Various arguments: AGM \approx 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)
Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs
Extended Equivalence Principle = Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smallness of quark and gluon “cosmological constants” (Polyakov, Schweitzer)
One more gravitational form factor

- Quadrupole
  \[ \langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2) (g^{\mu\nu} q^2 - q^\mu q^\nu) + ... \]

- Cf vacuum matrix element – cosmological constant (vacuum pressure)
  \[ \langle 0 | T^{\mu\nu} | 0 \rangle = \Lambda g^{\mu\nu} \]
  \[ \Lambda = C(q^2) q^2 \]

- Inflation ~ annihilation \((q^2 > 0)\) OT’15

- How to measure experimentally – Deeply Virtual Compton Scattering
QCD Factorization for DIS and DVCS (AND VM production)

- Manifestly spectral
- Extra dependence on $\xi$

\[
\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.
\]

\[
\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x, \xi)}{x - \xi + i\epsilon},
\]
Unphysical regions

- DIS: Analytical function – polynomial in $1/x_B$ if $1 \leq |X_B|$

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x, \xi)$

- Solved by using of Double Distributions Radon transform

$$H(z, \xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y)$$
Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

\[
f(x, y) = -\frac{1}{2\pi^2} \int_0^\infty \frac{dp}{p^2} \int_0^{2\pi} d\phi |\cos \phi| (H(p/\cos \phi + x + yt\phi, tg\phi) - H(x + yt\phi, tg\phi)) =
\]

\[
= -\frac{1}{2\pi^2} \int_{-\infty}^\infty \frac{dz}{z^2} \int_{-\infty}^{\infty} d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))
\]
Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS
  \[ H(x_B) = - \int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}} \]

- Non-positive powers of \( x_B \)

- DVCS
  \[ H(\xi) = - \int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x, \xi) \frac{x^n}{\xi^{n+1}} \]

- Polynomials (general property of Radon transforms): moments - integrals in \( x \) weighted with \( x^n \) - are polynomials in \( 1/\xi \) of power \( n+1 \)

- As a result, analyticity is preserved: only non-positive powers of \( \xi \) appear
Holographic property (OT’05)

Factorization Formula

\[ \mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x, \xi)}{x - \xi + i\epsilon} \]

Analyticity -> Imaginary part -> Dispersion relation:

\[ \mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x, x)}{x - \xi + i\epsilon} \]

\[ \Delta \mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x, x) - H(x, \xi)}{x - \xi + i\epsilon} \]

\[ = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x, \xi) dx (x - \xi)^{n-1} = \text{const} \]

“Holographic” equation (DVCS AND VM for both quarks and gluons)
Holographic property - II

1. Directly follows from double distributions

\[
H(z, \xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y)
\]

2. Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term \( G(x,y) \)

\[
\Delta H(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x, y)}{1 - y}
\]

\[
= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z - 1} = \text{const}
\]
Pressure in hadron pairs production (gravitational FFs from BELLE data – Kumano, Song, OT’17)

- Back to GDA region
- \[ \rightarrow \text{moments of } H(x,x) \text{ - define the coefficients of powers of cosine!} \]
- Higher powers of cosine in t-channel – threshold in s-channel
- Larger for pion than for nucleon pairs because of less fast decrease at \( x \rightarrow 1 \)
- Stability defines the sign of GDA: work in progress

\[
\mathcal{H}^c(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}
= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta \mathcal{H}.
\]
From D-term to pressure

- Inverse -> 1st moment (model)
- Kinematical factor – moment of pressure $C \sim <p r^4>$ ($<p r^2> = 0$) M.Polyakov’03

$$T^{Q}_{\mu\nu}(\vec{r}, \vec{s}) = \frac{1}{2E} \int \frac{d^3 \Delta}{(2\pi)^3} e^{i\vec{p}.\vec{\Delta}} \langle p', S' | \hat{T}_Q^{0}(0) | p, S \rangle$$

$$T_{ij}(\vec{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}$$

- Stable equilibrium (positive pressure in the core, negative at the edge): $C > 0$

- Justification of Polyakov’s definition: classical limit of the scattering with single graviton exchange
Gluonic GPDs – Quarkonia!? 

- Exclusive J/Ψ production – similar expression for very large $Q^2 >> m_c^2$
- $Q^2 \sim m_c^2$ – complicated analytic properties a’la DDVCS
- $Q^2 \rightarrow 0$: UPC, currently the most realistic (?!?) experiment
- Similar to TCS (work in progress)
- To measure: real and imaginary parts of J/Ψ exclusive photoproduction amplitude in wide kinematical region
From exclusive to semi-exclusive – probe of COME?

- NRQCD – tacitly assumes the emission of soft gluons
- cf pQCD/twist 3 phases
Phase: from 1-loop pQCD to twist 3

- Various options for factorisation – shift of SH separation

- New option for SSA: Instead of 1-loop twist 2 – Born twist 3 (quark-gluon correlator): Efremov, OT (85, Fermionic poles); Qiu, Sterman (91, GLUONIC poles)
Shift of SH separation for $J/\Psi$ production

- CSM $\rightarrow$ COM
Fate of soft gluon

- Must be combined with some hadronic remnants to form colour-singlet states (gluon-hadron duality)
- Typical for any QCD factorisation: “soft colour neutralization”
- Unitarity -> Duality:
  \[ I = \sum |q,G><q,G| = \sum |h><h| \]

- Remnants: absent in exclusive case
- May appear in semi-exclusive (GPD->TDA)
Semi-exclusive processes and COME

- Extra source of growth of cross-section with diffractive invariant mass $M$
- Normalization to vector $(V = \varrho,\phi)$ meson without COME

\[ R(M) = \frac{\sigma(M)_{J/\Psi}}{\sigma(M)_V} \]

- $R'(M) > 0$ – possible signal of COME
- To measure: diffractive invariant mass dependence of semi-exclusive $\varrho,\phi$ and $J/\Psi$ production
Ellipticity: from Wigner function to flow

- Wigner Function: most detailed description of hadronic structure (talk of Renaud Boussarie)
- Elliptic WF is related to elliptic flow in in pp and pA!
- Interference of L=0,2 quantum states
- How to explore this relation?

Elliptic Flow in Small Systems due to Elliptic Gluon Distributions?
Yoshikazu Hagiwara, Yoshitaka Hatta, Bo-Wen Xiao, and Feng Yuan

Elliptic flow from color-dipole orientation in pp and pA collisions
Edmond Iancu and Amir H. Rezaeian

Interference of L=0,2 quantum states
How to explore this relation?
Elliptic flow and exclusivity

- The azimuthal distribution may be studied at various invariant masses
- Suggestion: look for $\cos 2\phi$ distributions of dijets/quarkonia for various invariant masses of remnants: from inclusive via semi-exclusive ("T(transitional)WF") to exclusive
- To measure: $J/\Psi$ elliptic flow for various invariant masses of remnants
CONCLUSIONS

- Exclusive reactions with quarkonia may reveal various interfaces:
- “Macroscopical” aspects of GPDs: Pressure of gluons
- From exclusive to semi-exclusive: switching of COME
- From inclusive to semi-exclusive elliptic flows: quantum origins of collectivity
Main targets of “NICA Complex”:
- **study of hot and dense baryonic matter**
- investigation of nucleon spin structure,
  - polarization phenomena
- development of accelerator facility for HEP @ JINR providing
  - intensive beams of relativistic ions from $p$ to $Au$
  - polarized protons and deuterons
  with energy up to

$$\sqrt{S_{NN}} = 11 \text{ GeV} \ (Au^{79+}, L \sim 10^{32} \text{ cm}^{-2} \text{ c}^{-1})$$

$$\sqrt{S} = 27 \text{ GeV} \ (p, L \sim 10^{32} \text{ cm}^{-2} \text{ c}^{-1})$$
NICA: quarkonia important for both heavy ions and hadrons:
Many Thanks!

- To the Organizers for Excellent workshop!

- For your attention!
Is D-term independent?

- Fast enough decrease at large energy -

\[
\text{Re } \mathcal{A}(\nu) = \frac{P}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } \mathcal{A}(\nu')}{\nu'^2 - \nu^2} + C_0.
\]

\[
C_0 = \Delta - \frac{P}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } \mathcal{A}(\nu')}{\nu'^2}
= \Delta + P \int_{-1}^{1} dx \frac{H^+(x, x)}{x}.
\]

- FORWARD limit of Holographic equation

\[
\Delta = P \int_{-1}^{1} dx \frac{H^+(x, 0) - H^+(x, x)}{x}
= 2P \int_{-1}^{1} dx \frac{H(x, 0, t)}{x},
\]

\[
C_0(t) = 2P \int_{-1}^{1} dx \frac{H(x, 0, t)}{x}.
\]
“D – term” 30 years before…

- Cf Brodsky, Close, Gunion’72 (*seagull ~ pressure*) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?
Vector mesons and EEP

- $J=1/2 \rightarrow J=1$. QCD SR calculation of Rho’s AMM gives $g$ close to 2.

Maybe because of similarity of moments
- $g-2=<E(x)>; \ B=<xE(x)>$

Directly for charged Rho (combinations like $p+n$ for nucleons unnecessary!). Not reduced to non-extended EP:
Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)

- Provides $g=2$ identically!
- Experimental test at time –like region possible
EEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory - phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence - colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements
EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification: weighted TM moment of Sivers PROPORTIONAL to GPD $E$
  (OT’07, hep-ph/0612205):

\[ x \int_T f_T(x) : xE(x) \]

- Burkardt SR for Sivers functions is then related to Ji’s SR for $E$ and, in turn, to
  Equivalence Principle

\[ \sum_{q,G} \int dxx f_T(x) = \sum_{q,G} \int dxxE(x) = 0 \]
EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion of gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones – Brodsky, Gardner)
Another relation of Gravitational FF and NP QCD (first reported at 1992: hep-ph/9303228)

- **BELINFANTE** (relocalization) invariance:
  decreasing in coordinate – smoothness in momentum space

- Leads to absence of massless pole in singlet channel – $U_A(1)$

- Delicate effect of NP QCD

- **Equipartition** – deeply related to
  relocalization invariance by QCD evolution

\[
M^{\mu,\nu,\rho} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}
\]

\[
M^{\mu,\nu,\rho} = x^\nu T_B^{\mu\rho} - x^\rho T_B^{\mu\nu}
\]

\[
\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu,\rho} = 0.
\]

\[
(g_{\rho\sigma} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\sigma}) \partial^\rho (J^5_{S\alpha} x^\nu) = 0
\]

\[
q^2 \frac{\partial}{\partial q^\alpha} \langle P | J^5_{S\alpha} | P + q \rangle = (q^3 \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P | J^5_{S\gamma} | P + q \rangle
\]

\[
\langle P, S | J^5_{\mu}(0) | P + q, S \rangle = 2 M S_\mu G_1 + q_\mu (S q) G_2.
q^2 G_2 |_0 = 0
\]