TMDs in Associated Production : Experimental Review

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Quarkonia as Tools Workshop

Aussois

Introduction and Outline

Part One

What are the prospects of accessing gluon TMDs in associated production, mainly through double quarkonium and/or quarkonium + photon, via experimental methods, in the near or distant future.

Current

Non-prompt J/ψ + J/ψ production at the LHC :

LHCb at 7 and 13 TeV

Considered the associated production of J/ψ mesons originating from *b*-hadron decays at the LHC. Predictions for different gluon densities were then determined by theorists..

Non-prompt J/ψ + μ production

ATLAS experiment at 8 TeV.

Prompt J/ψ + J/ψ production at the LHCb :

First attempt to extract $\langle k_T^2 \rangle$ by theorists.

J/ψ + γ in ATLAS experiment

First experimental attempt to measure TMDs directly.

Observation of the associated production of a quarkonium with vector boson (e.g. J/ψ + Z) or heavy quark has seen nearly ~dozen experimental analyses with accompanying interpretations of results. Few have been interpreted in the context of TMDs.

.....Lots of theory papers, but an experimental desert for gluon TMDS. So far theorists have taken data and made predictions, but no published experimental search for TMDs.

Introduction and Outline

Part Two

Potential Future Analyses

Associated production of a dilepton and a Υ(J/ψ)

Several processes identified as being sensitive probes of TMDs. Theoretically cleanest is the production of a pair of (almost) back to back heavy quark and anti-quark or of a di-jet in lepton-nucleon collisions. A measurement of such processes could only be performed at a future Electron Ion Collider.

Other (non-discussed) include:

→ Single (pseudo) scalar quarkonium e.g. $\eta_{c,bB,}$ $\chi_{c,b}$, $\eta\eta_{c}$, J/ψ (or Υ) + Z_0 (or γ^*. Proposed at the LHC to test for gluon TMDs inside unpolarized protons. Impossible to measure with sufficient accuracy.

→ At the future (potential) FTP4LHC may be possible to measure TMD in gluons with single (pseudo)scalar quarkonium.

...Maybe the future will be less of a desert

Non-Prompt $J/\psi + J/\psi$ Production at the LHC

Non-Prompt J/ψ + J/ψ Production at LHCb

Part One

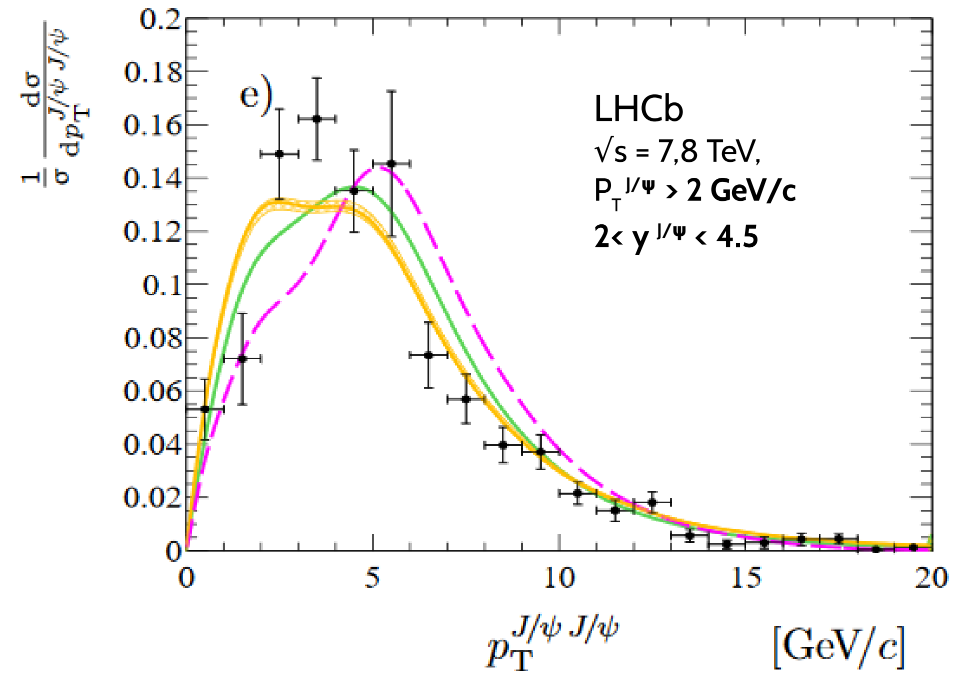
LHCb Experimental Data

Production of J/**ψ** mesons from *b*-hadron decays

LHCb Collaboration at √s = 7,8 TeV: Kinematic correlations for pairs of *b*-hadrons reconstructed via their inclusive decays into J/ψ mesons measured in forward rapidity region. **arXiv: 1708.05994** *LHCb Collaboration*

Several differential cross sections, including different angular correlations between the final decay muons, was measured for the first time. Hence corresponding calculations performed within framework of k_T factorization, and compared to LHCb data.

Theory paper then written to extract gluon TMD density information based on aforementioned data.
arXiv: 1808.06233 *S.P Baranov, A..V Lipatov, M.A, Malyshev.*



Powheg = orange, Pythia = green, Expectations for uncorrelated $b\bar{b}$ production = magenta, Uncertainties in Powheg + Pythia predictions due to choice of factorization and normalization scales = orange hatched.

TMD Cross Section

The cross sections of *b*-hadron production in *pp* collisions was calculated using the k_T factorization approach, based on the $\mathcal{O}(\alpha_s^2)$ off-shell gluon-gluon fusion subprocess:

$$g^*(k_1) + g^*(k_2) \to b(p_1) + \bar{b}(p_2)$$

Four-momenta of particles

K_T factorization approach uses two prescriptions for gluon dynamics at small *x*, that easily includes higher order radiative corrections, in the form of predictions for TMD parton distributions.

The *b*-flavour production cross section, obtained as a convolution of the off-shell partonic cross section $\hat{\sigma}^*_{gg}(x_1, x_2, \mathbf{k}^2_{1T}, \mathbf{k}^2_{2T}, \mu^2)$ and the TMD gluon distributions in a proton $f_g(x, \mathbf{k}^2_T, \mu^2)$

$$\sigma = \int dx_1 dx_2\, d\mathbf{k}^2_{1T} d\mathbf{k}^2_{2T}\, d\hat{\sigma}^*_{gg}(x_1, x_2, \mathbf{k}^2_{1T}, \mathbf{k}^2_{2T}, \mu^2) f_g(x_1, \mathbf{k}^2_{1T}, \mu^2) f_g(x_2, \mathbf{k}^2_{2T}, \mu^2),$$

$K_{iT,}$ component of the off-shell gluon momentum k_i perpendicular to beam axis

X_i, fraction of longitudinal momentum of the colliding proton

μ^2 , corresponding hard scale

Non-Prompt J/ψ + J/ψ Production at the LHC

Part Three

TMD gluon densities

Two approaches of the TMD gluon distribution functions in a proton were tested. For details see **ArXiv: 1808.0633** _*S.P Baranov, A..V Lipatov, M.A, Malyshev.*

.......Summarising

CCFM: Using a numerical solution of the equation (labelled *JH'2013*), which smoothly interpolates between small-*x* BFKL gluon dynmaics and high-*x* DGLAP dynamics. Two sets of predictions for TMD gluon densities have been determined from fits to high precision HERA data on the proton structure functions:

JH'2013 Set 1: Determined from fit to inclusive $F_2(x,Q^2)$ data only.

JH'2013 Set 2: Determined from fit to $F_2(x,Q^2)$ and $F_2^c(x,Q^2)$.

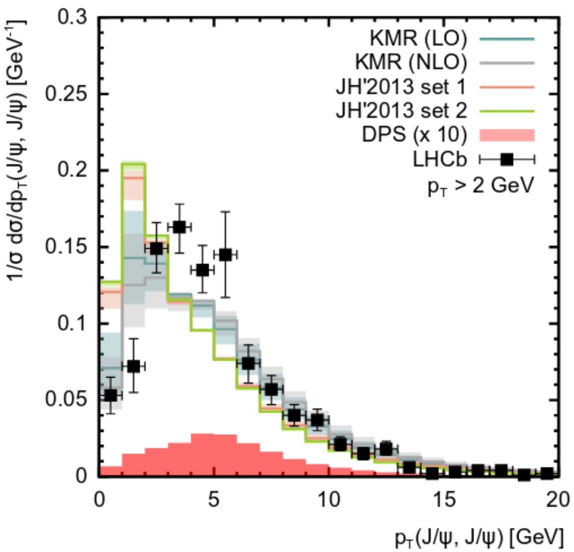
KMR: Construct TMD parton (quark and gluon) densities from well-known conventional ones, at LO and NLO. This approach assumes k_T dependence enters at the last evolution step, so DGLAP evolution can be used up to this step.

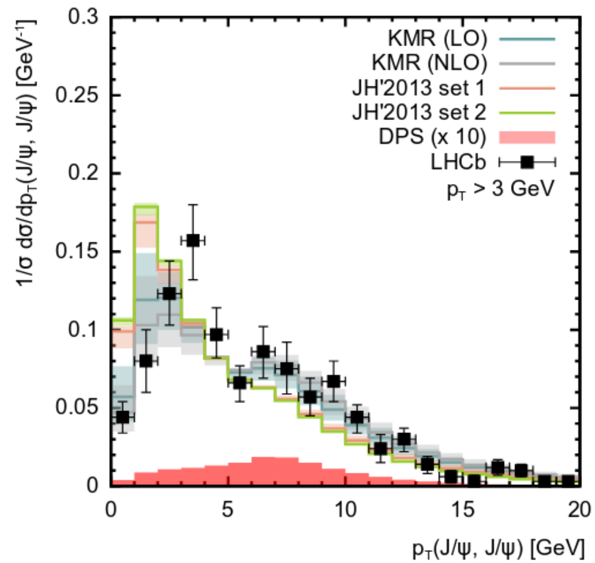
Non-Prompt J/ψ + J/ψ Production at the LHC

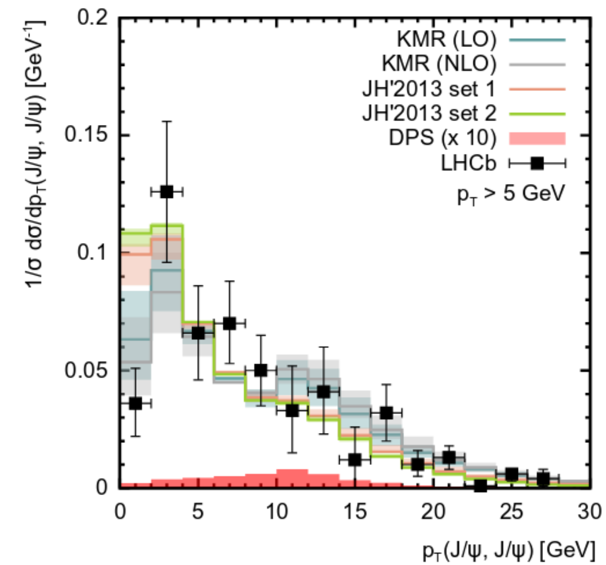
Part Four

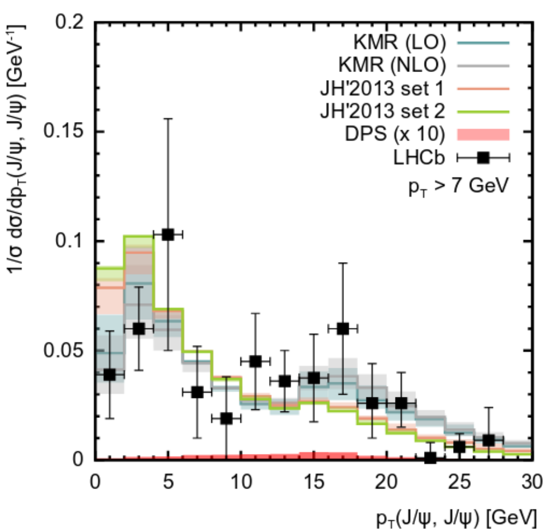
View in 2018 : Predictions for different gluon TMD densities

Results









Conclusions:

TMD gluon densities describe the data reasonably well for distributions within uncertainties.

CCFM evolved gluons tend to overestimate the measured transverse momentum spectra of the pair at low P_T.

KMR calculations agree well with LHCb data.

Figs: Normalized differential cross sections of non-prompt J/ψ + J/ψ production at √ s = 8 TeV as a function of the transverse momentum of the pair, using experimental data from LHCb. Predictions are made using the KMR (with NO and NLO accuracy) and the CCFM evolved TMD gluon distributions. Shaded bands represent the scale uncertainties of the calculation.

Non-Prompt $J/\psi + \mu$ Production at the LHC

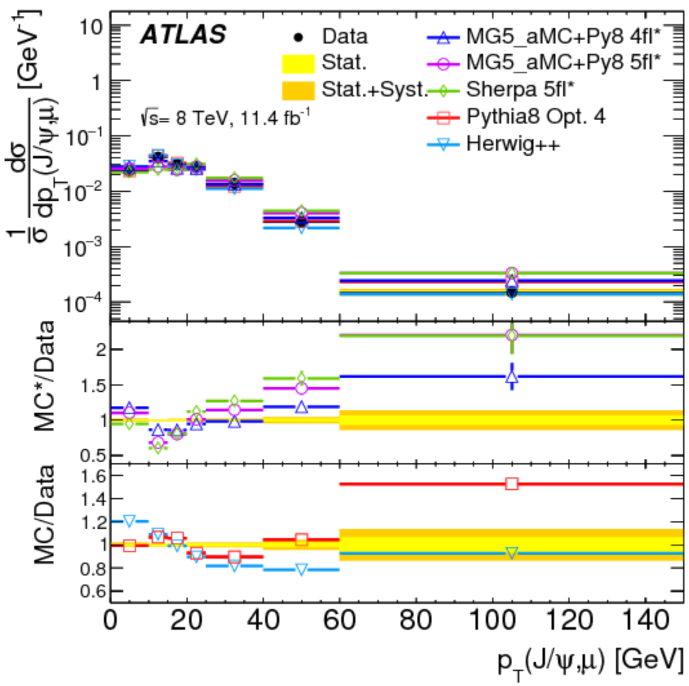
Non-Prompt J/ψ + μ Production at the LHC

Part One

...Similar story as for J/ψ + J/ψ production

ATLAS Collaboration at $\sqrt{s}$ = 8 TeV : Measurement of the production of two *b*-hadrons, where one *b*-hadron decays into a J/ψ meson, and the other decays into μ + *X* ***arXiv: 1705.03374***.

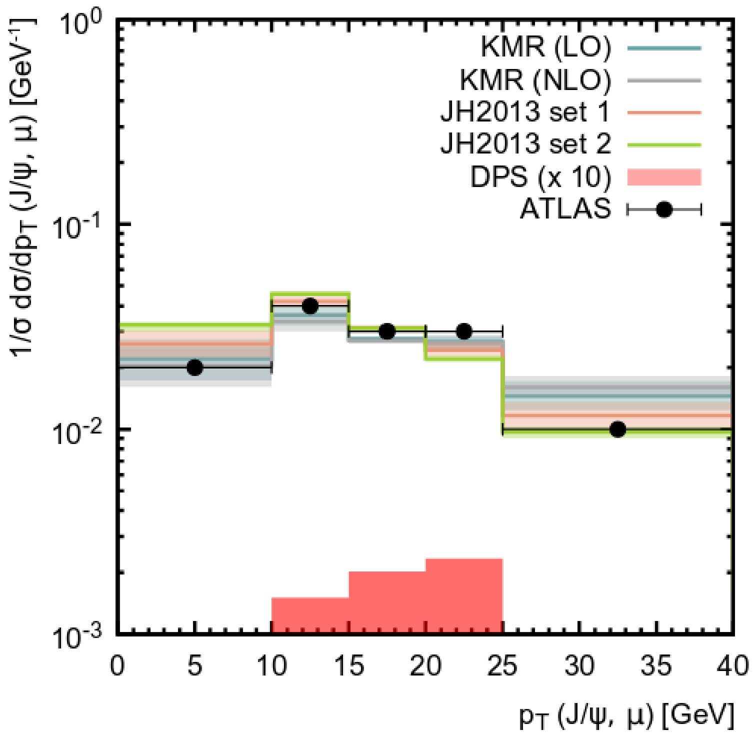
ATLAS collaboration measured the total and differential cross sections in a restricted part of the phase space (fiducial volume). Each muon was required to have transverse momentum $p_T > 6$ GeV, the two muons originating from J/ψ decay must have pseudorapidities $|\eta| < 2.3$ and the third muon must have $|\eta| < 2.5$.



The normalized cross section is presented time for the transverse momentum of the three muon system p_T(J/ψ, μ). Studying the normalized differential cross section could lead to more stringent comparison between data and theory due to the reduced experimental (systematic) uncertainties.

Non-Prompt J/ψ + μ Production at the LHC

Part Two



Normalized differential cross section of associated non prompt J/ψ + μ production at √s = 8 TeV with ATLAS experimental data.

Solid histogram represents central predictions calculated with fixed renormalization μ_R and factorization μ_F at default values (cite). Shaded regions correspond to scale uncertainties in predictions

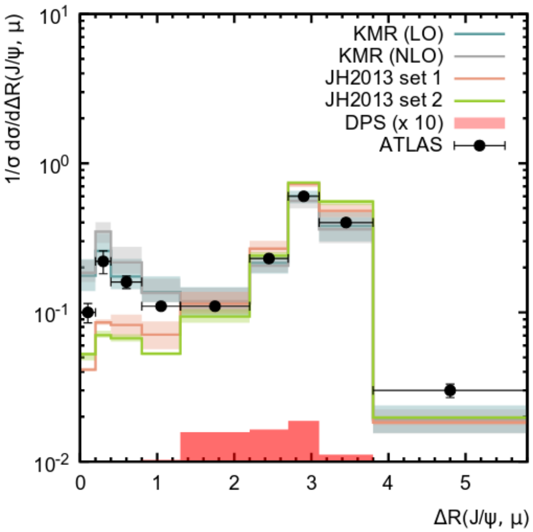
Predictions compared with available data.

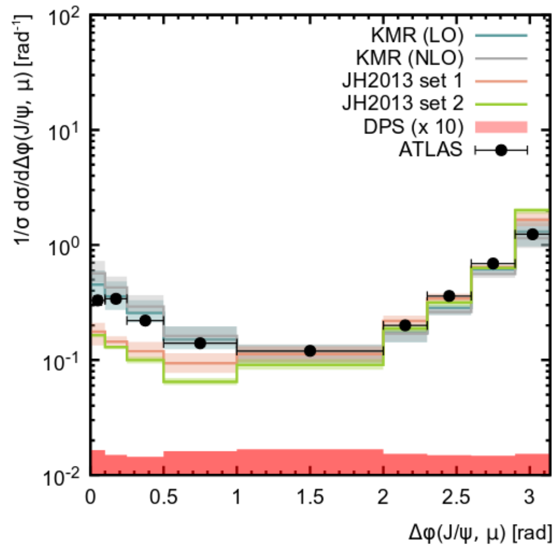
Calculated cross sections strongly depend on TMD gluon density used. Clear difference in shape between the *JH'2013* and KMR predictions is observed, where a better description is achieved with the KMR gluon distributions.

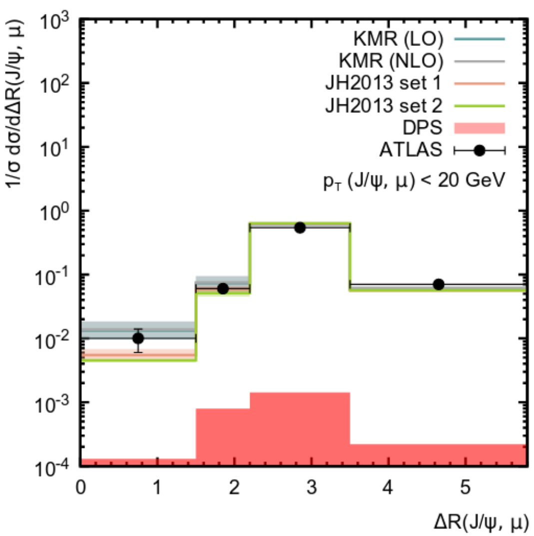
Small overestimation of the data in the last bins of p_T(J/ψ, μ), although data is close to estimated uncertainty bands.

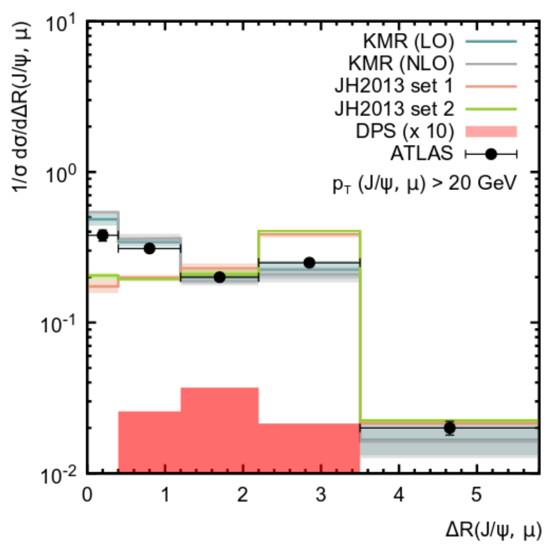
Non-Prompt J/ψ + μ Production at the LHC

Part Three









Clear difference in shape between *JH'2013* and KMR predictions is observed for Δ*R*(J/ψ, μ) and Δϕ(J/ψ, μ).

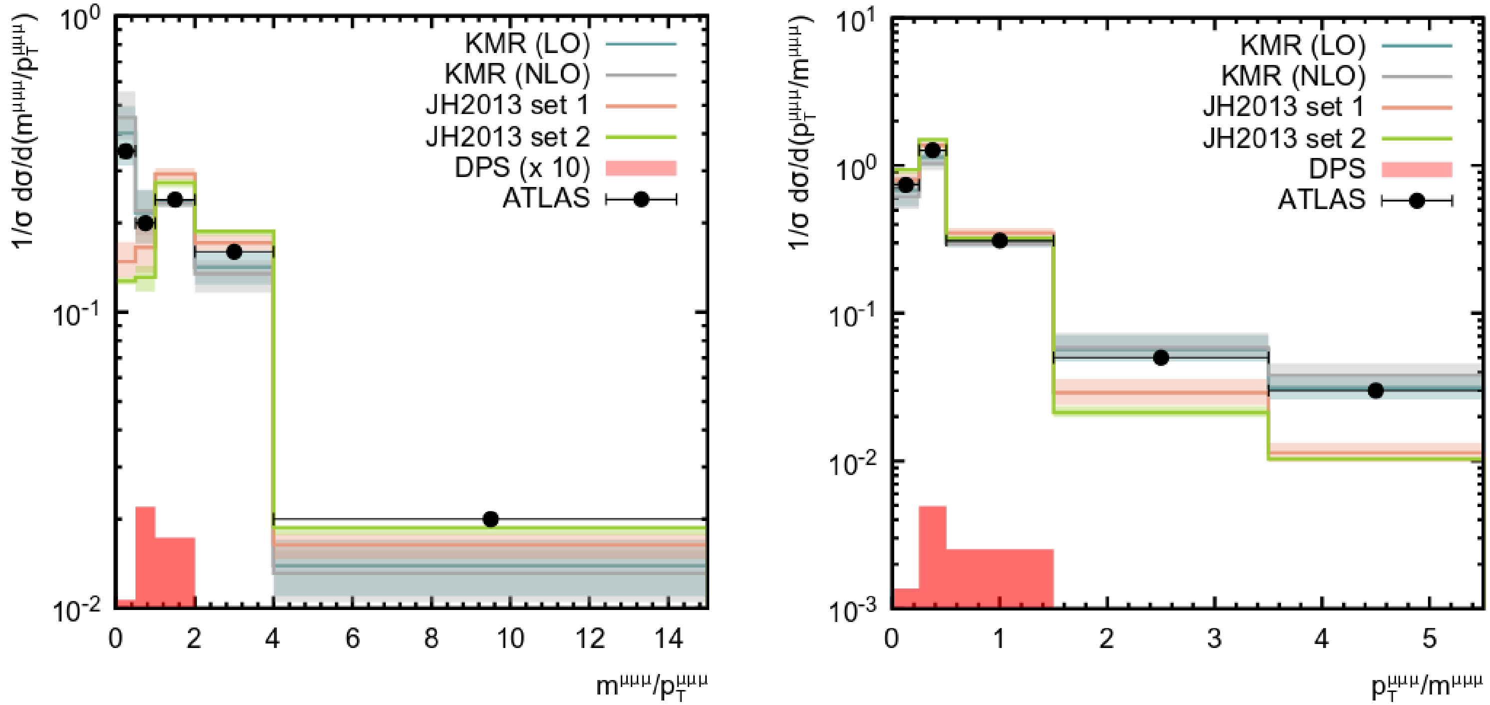
Small over estimation of the data at low ΔR(J/ψ, μ) and Δϕ(J/ψ, μ), but data close to estimated uncertainty bands.

Both *JH'2013* do not reproduce well the measured shape of angular correlations. Underestimate the data at low ΔR(J/ψ, μ) and Δϕ(J/ψ, μ), especially for p_T(J/ψ, μ) > 20 GeV. Overestimate the data at Δϕ(J/ψ, μ) ~ π.

Normalized differential cross sections of associated non-prompt J/ψ+ μ production at √s = 8 TeV as functions of Δ*R*(J/ψ, μ), Δϕ(J/ψ, μ), for low and high P_T ranges. Predictions made using KMR (LO and NLO accuracy) and CCFM evolved TMD gluon densities. Shaded bands represent the scale uncertainties of the calculations. Experimental data from ATLAS.

Non-Prompt J/ψ + μ Production at the LHC

Part Four



Observable differences between KMR and *JH'2013* predictions is very noticeable here.

Calculated that the ratio of KMR and *JH'2013* prediction reaches ~2.5-3 at $P_t^{\mu\mu\mu}/m^{\mu\mu\mu} > 2$. P_t/m is akin to k_T.

…. Observables are sensitive to non-collinear gluon dynamics. Well known properties of angular correlations between the momenta of produced particles could be very promising to constrain TMD gluon densities in a proton

Prompt J/ψ+ J/ψ production at the LHC

Prompt J/ψ + J/ψ Production at the LHC

Di-Q production and TMD factorization

Part One

arXiv:1710.01684v1 *J.P Lansberg, C.Pisano, F.Scarpa, M.Schlegel*

Argue that di-J/ψ production, amongst the several quarkonium associated production processes has seen numerous studies at the LHC and Tevatron, is the ideal process to perform the first measurement of linearly polarized gluons, $h_1^{\perp g}(x,\boldsymbol{k}_T^2)$. It exhibits the largest possible azimuthal asymmetries in regions already accessed by ATLAS and CMS experiments.

For TMD factorization to apply, di-Q production needs to satisfy:

- Should result from Single Parton Scattering
- Final State Interactions should be negligible, which is satisfied when quarkonia are produced via Colour-Singlet Transitions.

$$\frac{d\sigma}{dM_{QQ}dY_{QQ}d^2\boldsymbol{P}_{QQT}d\Omega} = \frac{\sqrt{Q^2-4M_Q^2}}{(2\pi)^2 8s\,Q^2}\Big\{F_1\,C\big[f_1^g f_1^g\big] + F_2\,C\big[w_2 h_1^{\perp g} h_1^{\perp g}\big] + \cos 2\phi_{CS}\Big(F_3 C\big[w_3 f_1^g h_1^{\perp g}\big] + F_3' C\big[w_3' h_1^{\perp g} f_1^g\big]\Big) + \cos 4\phi_{CS} F_4 C\big[w_4 h_1^{\perp g} h_1^{\perp g}\big]\Big\}, \qquad (3)$$

Prompt J/ψ + J/ψ Production at the LHC

Di-Q production and TMD factorization F factors

Part Two

As for the F_i we have

$$F_1 = \frac{\mathcal{N}}{\mathcal{D}M_Q^2}\Big[(6\alpha^8 - 38\alpha^6 + 83\alpha^4 + 480\alpha^2 + 256) + 2(1-\alpha^2)\left(6\alpha^8 + 159\alpha^6 - 2532\alpha^4 + 884\alpha^2 + 208\right) c_\theta^2$$
$$+ 2\left(1-\alpha^2\right)^2\left(3\alpha^8 + 19\alpha^6 + 7283\alpha^4 - 8448\alpha^2 - 168\right) c_\theta^4 - 2\left(1-\alpha^2\right)^3\left(159\alpha^6 + 6944\alpha^4 - 17064\alpha^2 + 3968\right) c_\theta^6$$
$$+ \left(1-\alpha^2\right)^4\left(4431\alpha^4 - 27040\alpha^2 + 17824\right) c_\theta^8 + 504\left(1-\alpha^2\right)^5\left(15\alpha^2 - 28\right) c_\theta^{10} + 3888\left(1-\alpha^2\right)^6 c_\theta^{12}\Big],$$

$$F_2 = \frac{2^4 3 M_Q^2 \mathcal{N}}{\mathcal{D}M_{QQ}^4}\Big[\alpha^4 - 2\left(\alpha^6 + 17\alpha^4 - 126\alpha^2 + 108\right) c_\theta^2 + \left(1-\alpha^2\right)^2\left(\alpha^4 + 756\right) c_\theta^4 - 36\left(1-\alpha^2\right)^3\left(\alpha^2 + 24\right) c_\theta^6 + 324\left(1-\alpha^2\right)^4 c_\theta^8\Big],$$

$$F_3' = F_3 = \frac{-2^3(1-\alpha^2)\mathcal{N}}{\mathcal{D}M_{QQ}^2} \times \Big[\alpha^2\left(16 - 3\alpha^2\right) + (6\alpha^6 + 159\alpha^4 - 1762\alpha^2 + 1584) c_\theta^2 + (1-\alpha^2)\left(3\alpha^6 + 19\alpha^4 + 5258\alpha^2 - 6696\right) c_\theta^4$$
$$- \left(1-\alpha^2\right)^2\left(159\alpha^4 + 5294\alpha^2 - 10584\right) c_\theta^6 + 18\left(1-\alpha^2\right)^3\left(99\alpha^2 - 412\right) c_\theta^8 + 1944\left(1-\alpha^2\right)^4 c_\theta^{10}\Big],$$

$$F_4 = \frac{(1-\alpha^2)^2\mathcal{N}}{\mathcal{D}M_Q^2} \times \Bigg[\left(3\alpha^4 - 32\alpha^2 + 256\right) - \left(6\left(\alpha^4 + 36\alpha^2 - 756\right)\alpha^2 + 4768\right) c_\theta^2 + \left(3\alpha^8 + 38\alpha^6 + 11994\alpha^4 - 32208\alpha^2 + 20400\right) c_\theta^4$$
$$- 2\left(1-\alpha^2\right)\left(105\alpha^6 + 5512\alpha^4 - 23120\alpha^2 + 19520\right) c_\theta^6 + \left(1-\alpha^2\right)^2\left(3459\alpha^4 - 30352\alpha^2 + 38560\right) c_\theta^8$$
$$+ 72\left(1-\alpha^2\right)^3\left(105\alpha^2 - 268\right) c_\theta^{10} + 3888\left(1-\alpha^2\right)^4 c_\theta^{12}\Bigg],$$

with $c_\theta = \cos\theta_{CS}$, $\alpha = 2M_Q/M_{QQ}$, $\mathcal{N} = 2^{11}3^{-4}\pi^2\alpha_s^4|R_Q(0)|^4$, $\mathcal{D} = M_{QQ}^4\left(1-(1-\alpha^2)\,c_\theta^2\right)^4$ and where $R_Q(0)$ is the Q radial wave function at the origin. Note that the expressions are symmetric about $\theta_{CS} = \pi/2$ since the process is forward-backward symmetric.

Prompt J/ψ + J/ψ Production at the LHC

Part Three

F-Factors

- $F^{(')}_{2,3,4} \leq F_1$,but for QQ production $F'_3 = F_3$

- When $M_{QQ} \gg M_Q$ then for $\cos\theta_{CS} = \cos\theta \to 0.$ The F-Factors become:

$$F_{1,4} \to \frac{256\mathcal{N}}{M^4_{QQ}M^2_Q}, \frac{F_2}{F_1} \to \frac{81 M^4_Q c^2_\theta}{2M^4_{QQ}}, \frac{F_3}{F_1} \to \frac{-24 M^2_Q c^2_\theta}{M^2_{QQ}}, \qquad \mathcal{N} = 2^{11}3^{-4}\pi^2\alpha_s^4|R_Q(0)|^4,$$

- At the threshold , $M_{QQ} \to 2M_Q$ and the F-factors are transformed to:

$$F_1 \to \frac{787\mathcal{N}}{16M^6_Q}, \quad F_2 \to \frac{3F_1}{787}, \quad F_{3,4} \to 0.$$

- $F_{2,3}$ are suppressed near $\Delta y \sim 0$, and $F_2(F_3)$ scales like $M^{-4}_{QQ}(M^{-2}_{QQ})$ relative to F_1 and F_4. Hence P_{QQT} dependence due to linearly polarized gluons encoded in F_2 vanishes at large scales, whereas the $\cos 4\phi$ modulation (double helicity flip) takes over the $\cos 2\phi$ one (single helicity flip).

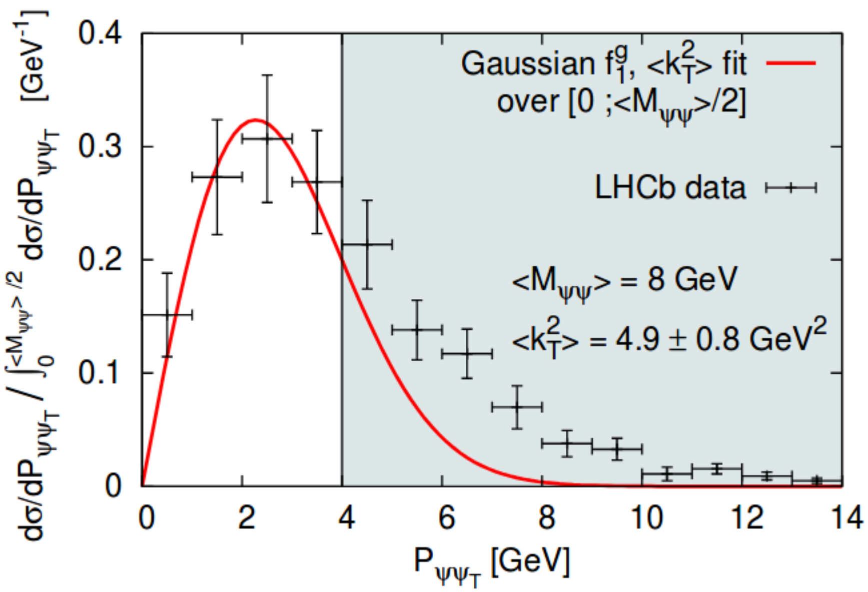
- $F_4 \to F_1$ for $\cos\theta \to 0$ away from the threshold (where $\cos\theta_{CS} \sim 0$ and $\Delta y \sim 0$). This is an important result, and a unique feature of di-J/psi and di- production. Hence di-J/ψ - is a good candidate to extract linearly polarized gluons from in light of recent experimental data.

- Knowing the F_i and an observed experimental yield one can extract the various TMD convolutions from their azimuthal (in)dependent parts. When the cross section is integrated over θ_{CS}, then the contribution from $F_{3,4}$ drops out of the TMD cross section.

Prompt J/ψ + J/ψ Production at the LHC

Part Four

Results



Normalized P_{QQT} dependence of the J/ψ yield obtained with a gaussian f_1^g with $\langle k_T^2\rangle$ fit to normalized LHCb data at 13 TeV. TMD factorization does not apply in the grey zone.

Have been able to use experimental data to fix $C\,[\,f_1 f_1]$ from P_{QQT} spectrum recently measured by the LHCb collaboration at 13TeV. Study was conducted without any TM cuts, so can neglect contributions from $h_1^{\perp g}$ given the size of F_2/F_1 near threshold.

For the TMD ansatze, with factorized dependencies on *x* and $\mathbf{k}^2_T$, the normalized P_{QQT} spectra does not depend on other kinematic variables. Fitting the P_{QQT} spectrum up to $M_{QQ}/2$ then obtain $\langle k_T^2\rangle$=4.9+/- 0.8 GeV^2. This is the first process indepdendent quantity to be experimentally determind in a pure gluon induced process with a colourless final state for which TMD factorization applies.

(Potential) future and current analyses

J/Psi + ɣ Production at the LHC

J/ψ + γ Production at the LHC

Part One

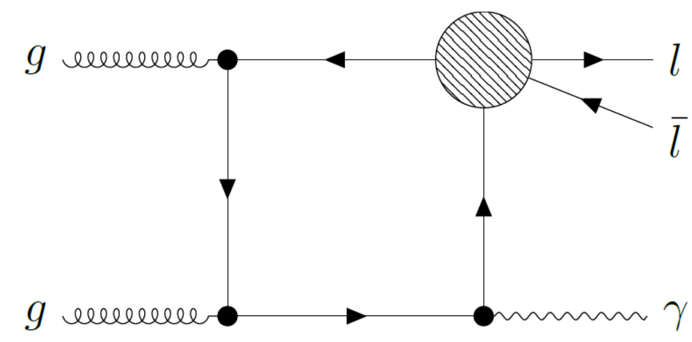
Cross Section

TMD cross section

Motivations:

The first measurement of gluon transverse momentum distributions ever, and/or

The first measurement of the polarization dependent part of gluon TMDs



$g + g \rightarrow J/\psi + \gamma$

$$\frac{d\sigma}{dQ\,dY\,d^2q_T\,d\Omega} = \frac{4\alpha_s^2\alpha e_Q^2|R_0(0)|^2}{3M_Q^3} \times \frac{Q^2 - M_Q^2}{sQ^3\{(\gamma^2+1)^2 - (\gamma^2-1)^2\cos^2\theta\}^2} \times$$

$$\times \left\{F_1 C_1\left[f_1^g f_1^g\right] + F_3 C_3\left[w_3 f_1^g h_1^{\perp g} + (x_a \leftrightarrow x_b)\right]\cos 2\phi + F_4 C_4\left[w_4 h_1^{\perp g} h_1^{\perp g}\right]\cos 4\phi\right\}$$

$$F_1 = 1 + 2\gamma^2 + 9\gamma^4 + (6\gamma^4 - 2)\cos^2\theta + (\gamma^2 - 1)^2\cos^4\theta$$

$$F_3 = 4\gamma^2\sin^2\theta$$

$$F_4 = (\gamma^2 - 1)^2\sin^4\theta$$

Unpolarized TMD parton dstribution function. $f_1(x,k_T,\xi,\mu)$. Transverse momentum distribution of an unpolarized gluon in an unpolarized proton.

Boer-Mulders function $h_1^{\perp}(x,k_T,\xi,\mu)$: Distribution of a linearly polarized gluon in an unpolarized proton.

Term responsible to for non trivial angular dependence in Collins-Sopher frame.

- Production of an isolated photon back to back with an isolated J/ψ in *pp* collisions is the ideal to observe transverse dynamics and polarization of gluons in the proton in terms of TMD factorization.

arXiv: 1401.7611v2 *W.J Den Dunnen, J.P Lansberg, C.Pisano, M.Schlegel*

J/ψ + χ Production at the LHC

Part Two

...After some manipulations

TMD cross section

$$\frac{d\sigma}{d\lambda dY\, d(q_T^2) d\phi} = \left[\mathbf{B}_0 \frac{1}{\lambda^2(\lambda-1)^3} \int \frac{1}{\left\{G^2 - z^2\right\}^2} \times \left\{F_1 T_1\left[q_T^2\right] + F_3 T_3\left[q_T^2\right]\cos 2\phi + F_4 T_4\left[q_T^2\right]\cos 4\phi\right\}\right] dz$$

$$d\Omega = d\cos\theta d\phi$$

$$z = \cos\theta$$

$$\mathbf{B}_0 = \frac{\pi\alpha_s^2\Gamma}{6M_Q s\alpha}$$

$$q_T^2 = [pT\ of\ J/\psi + \chi]^2$$

$$\lambda = [(Mass\ of\ J/\psi + \chi)/Mass\ of\ J/\psi]^2$$

$$G = \left(\frac{\lambda+1}{\lambda-1}\right)$$

$$F_1 = 1 + 2\lambda + 9\lambda^2 + (6\lambda^2 - 2)z^2 + (\lambda-1)^2 z^4$$

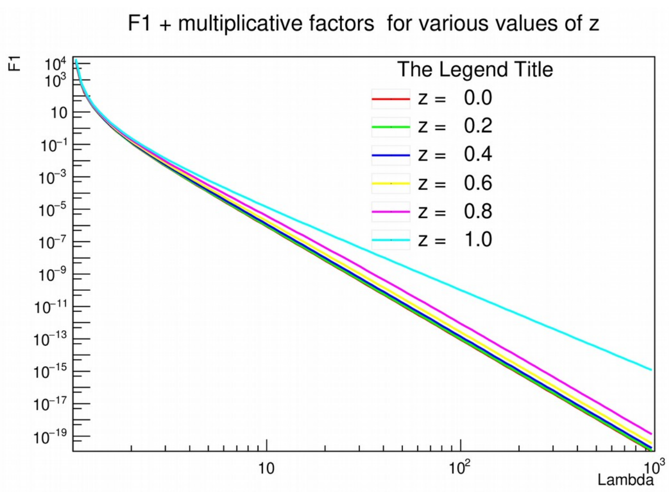
$$F_3 = 4\lambda(1 - z^2)$$

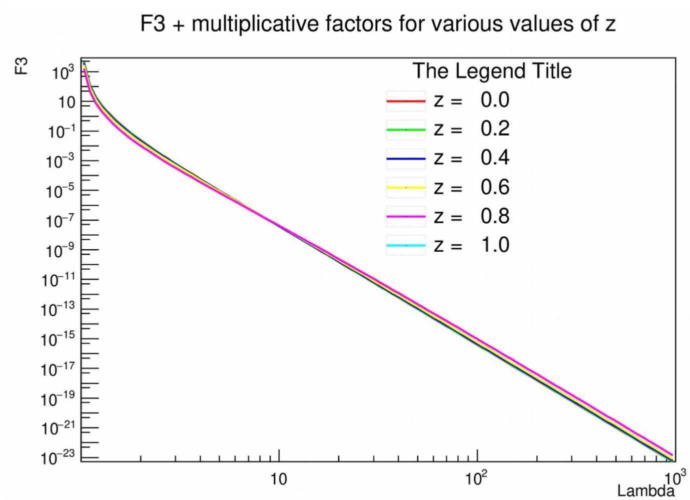
$$F_4 = (\lambda-1)^2\left(-2z^2 + z^4 + 1\right)$$

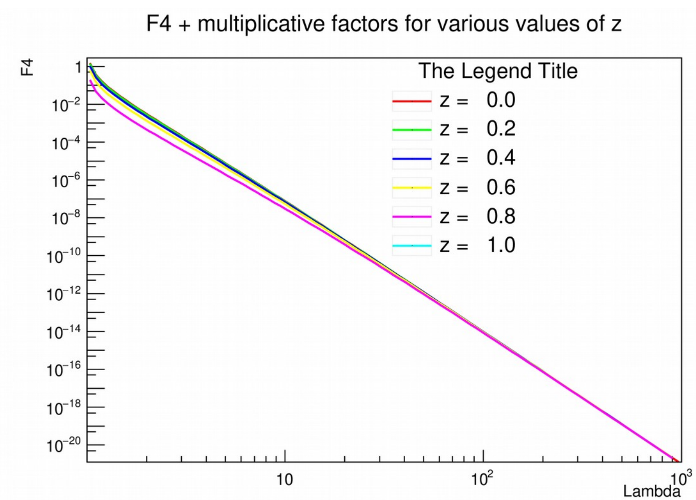
J/ψ + χ Production at the LHC

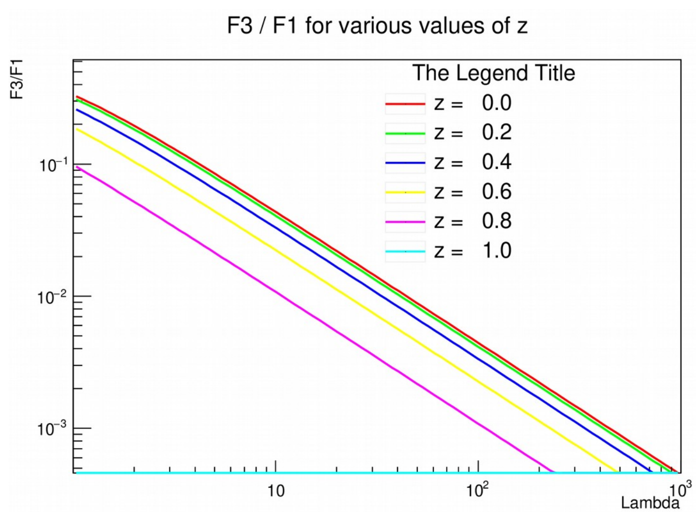
Part Three

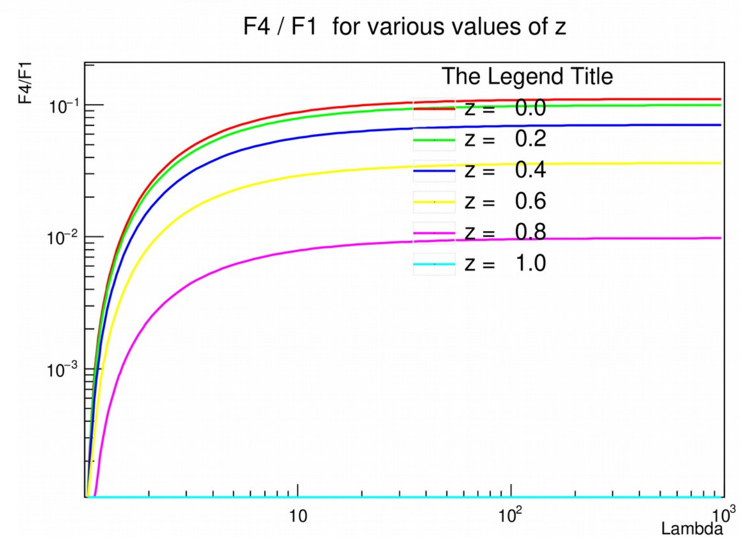
F factor plots and F factor ratios











$$F_1 = 1 + 2\lambda + 9\lambda^2 + (6\lambda^2 - 2)z^2 + (\lambda - 1)^2 z^4$$

$$F_3 = 4\lambda(1 - z^2) \qquad F_4 = (\lambda - 1)^2\left(-2z^2 + z^4 + 1\right)$$

$$\lambda = \left[\left(Mass\ of\ J/\psi + \chi\right) / Mass\ of\ J/\psi\right]^2$$

$J/\psi + \gamma$ Production at the LHC

Part Four

Monte Carlo

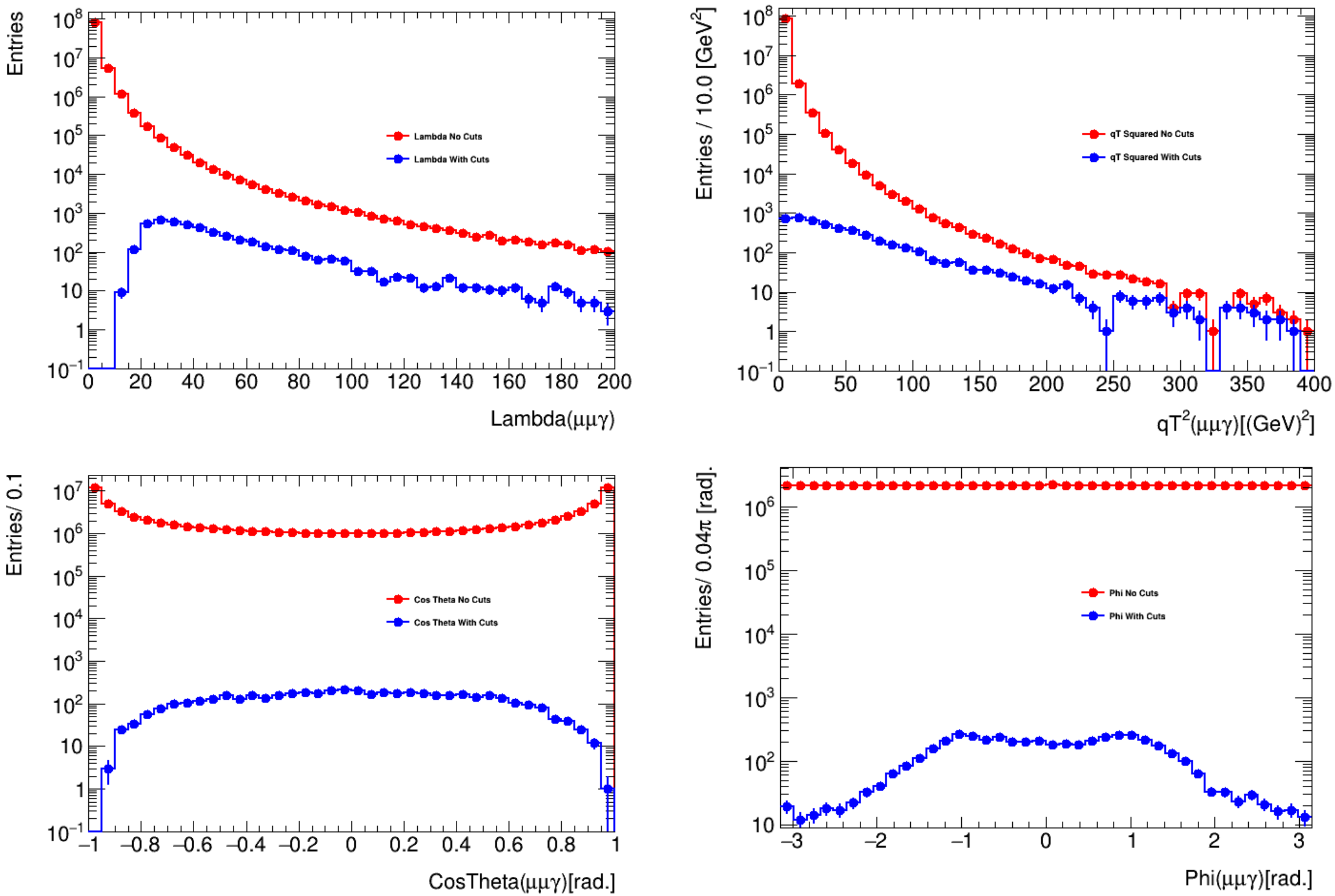
- Subprocess $g + g \longrightarrow J/\psi + \gamma$ was missing from Pythia8, but included at our request.
- Using a few Monte Carlo samples to see how cuts on the P_T of muons and photons affects angular distributions in the Collins-Sopher system.
- Using a 10M truth only sample with no generator level cuts on muons and photons.
- Will use Monte Carlo to correct for data.

J/ψ + χ Production at the LHC

Part Five

Monte Carlo Results

Plots of four of our variables



Red line: No P_T cuts. Blue line: P_T Cuts of 4 GeV on muons and 5 GeV on photons

J/ψ + χ Production at the LHC

Part Six

Monte Carlo Challenges

Challenges

- No access to low lambda values
- No cuts gives a nice flat phi distribution, but becomes modulated once cuts are applied
- Hope to see the small modulation of 2ϕ and 4ϕ on top of cuts.

Associated Production of a Dilepton and a Υ(J/ψ)

Associated Production of a Dilepton and a Υ(J/ψ)

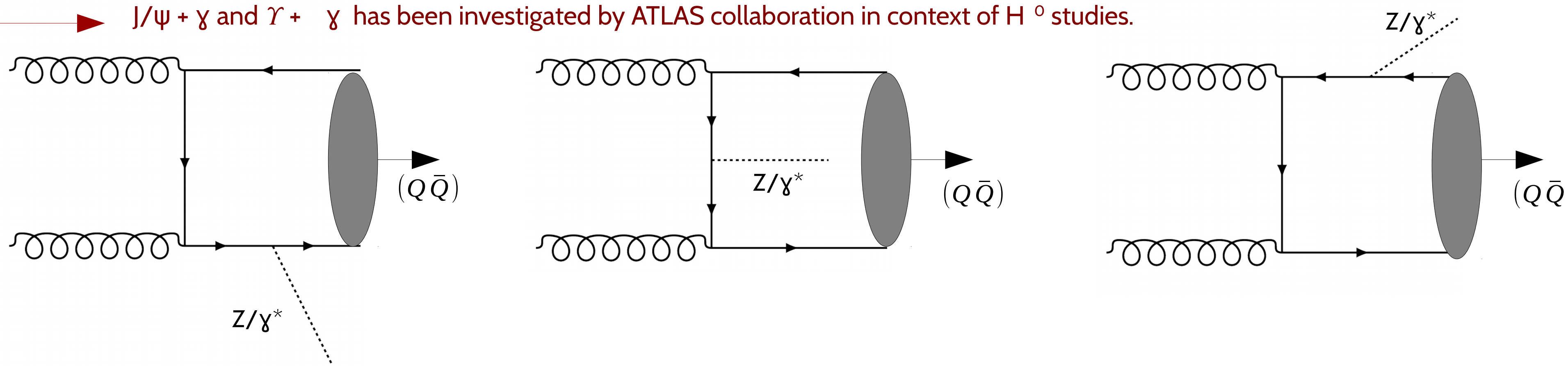
Part One

arXiv:1702.00305v1 *J.P Lansberg, C.Pisano, M. Schlegl*

Motivation

Motivation: Could extract information about linear polarization of gluons from a Υ(J/ψ) produced with a dilepton, be it from a Z boson or a virtual photon, in a kinematical configuration such that their transverse momentum are almost back to back. The detection of a dilepton may be experimentally easier or cleaner compared to a photon.

- Cross section for J/ψ production in association with a Z boson has been studied by the ATLAS collaboration, and compared with theoretical predictions..
- J/ψ + ɣ and Υ + ɣ has been investigated by ATLAS collaboration in context of H ^{0} studies.



Leading order diagrams from the subprocess gg ⟶ Q Z/ɣ*. Diagrams where the directions of quark lines are reversed also contribute.

$\alpha \equiv \frac{M_Q}{Q}$ $\beta \equiv \frac{M_B}{Q}$ Collins Sopher

$$\frac{d\sigma^{pp\to J/\psi[\Upsilon]\ell\bar{\ell}X}_{\text{TMD, LO}}}{d^4q\, dM_B^2\, d\Omega} = \hat{F}_1(Q,\alpha,\beta,\theta)\, C[f_1^g\, f_1^g] + \hat{F}_2(Q,\alpha,\beta,\theta)\, C[w_2\, h_1^{\perp g}\, h_1^{\perp g}]$$
$$+ \left\{\hat{F}_{3a}(Q,\alpha,\beta,\theta)\, C[w_{3a}\, h_1^{\perp g}\, f_1^g] + \hat{F}_{3b}(Q,\alpha,\beta,\theta)\, C[w_{3b}\, f_1^g\, h_1^{\perp g}]\right\} \cos 2\phi$$
$$+ \hat{F}_4(Q,\alpha,\beta,\theta)\, C[w_4\, h_1^{\perp g}\, h_1^{\perp g}] \cos 4\phi, \quad ($$

Associated Production of a Dilepton and a Υ(J/ψ)

Part Two

Cross Section

$$N^{(0)} \equiv \int d\Omega \frac{d\sigma^{pp\to J/\psi[\Upsilon]\ell\bar{\ell}X}_{\text{TMD, LO}}}{d^4q\, dM_B^2\, d\Omega} = \frac{d\sigma^{pp\to J/\psi[\Upsilon]\ell\bar{\ell}X}_{\text{TMD, LO}}}{d^4q\, dM_B^2} = \hat{F}_1(Q,\alpha,\beta)\, \mathcal{C}[f_1^g\, f_1^g] + \hat{F}_2(Q,\alpha,\beta)\, \mathcal{C}[w_2\, h_1^{\perp g}\, h_1^{\perp g}],$$

$$N^{(2)} \equiv \int d\Omega \cos 2\phi \frac{d\sigma^{pp\to J/\psi[\Upsilon]\ell\bar{\ell}X}_{\text{TMD, LO}}}{d^4q\, dM_B^2\, d\Omega} = \hat{F}_3(Q,\alpha,\beta) \left(\mathcal{C}[w_{3a}\, h_1^{\perp g}\, f_1^g] + \mathcal{C}[w_{3b}\, f_1^g\, h_1^{\perp g}]\right),$$

$$N^{(4)} \equiv \int d\Omega \cos 4\phi \frac{d\sigma^{pp\to J/\psi[\Upsilon]\ell\bar{\ell}X}_{\text{TMD, LO}}}{d^4q\, dM_B^2\, d\Omega} = \hat{F}_4(Q,\alpha,\beta)\, \mathcal{C}[w_4\, h_1^{\perp g}\, h_1^{\perp g}]. \quad (23)$$

- This is the form of the weighted cross section

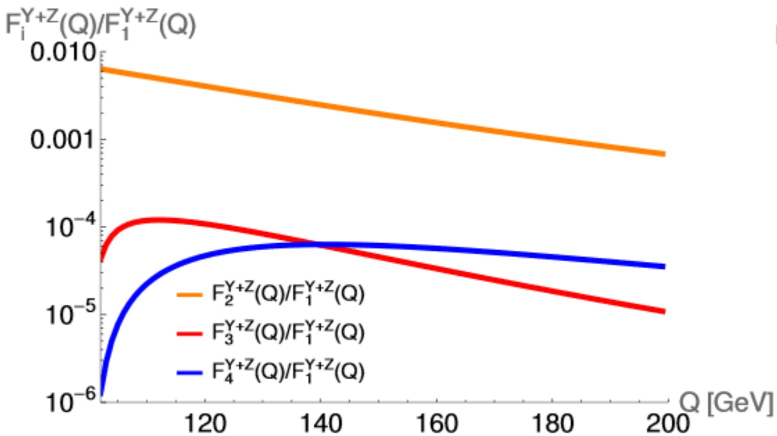
- It is more instructive to analyse the cross section that is integrated over Collins-Sopher angles, including a possible azimuthal weighting factor. These weighting factors could enable the theorists to disentangle the various azimuthal contributions in the weighted cross section.

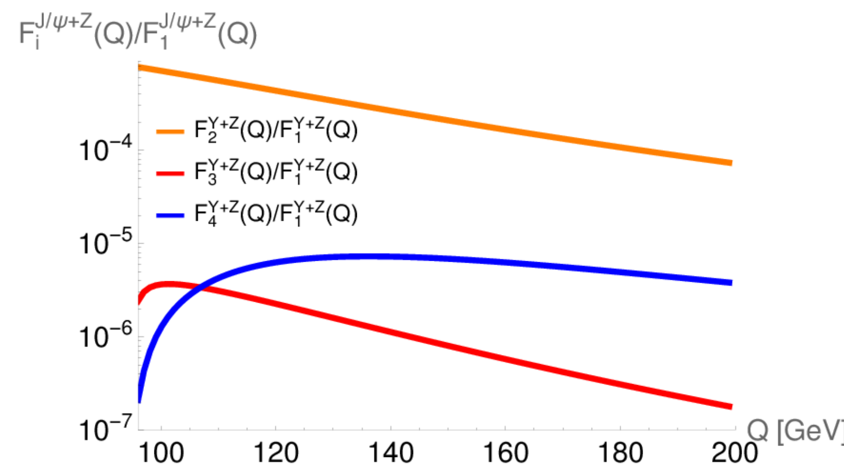
- The integrated cross section over θ can be found in the literature.

Associated Production of a Dilepton and a Υ(J/ψ)

Numerical Predictions for an associated quarkonium + Z

Part Three





- Numerically compare the relative size of both contributions from unpolarized and linearly polarized gluons to the angular integrated cross section, by considering the LO ratios $F_{2,3,4}(Q,, \beta)/F_1(Q,,\beta)$.
- Focus on the production of a real Z , I.e dileptons with an invariant mass around the Z-pole mass, $M_B \simeq m_Z$
- The ratios are rather small: ratio of F_2/F_1 ~ 0.5% at most for Υ production, and even smaller (< 10^{-3}) for J/ψ production.
- The convolution $C[w_2\, h_1^{\perp g}\, h_1^{\perp g}]$ factor from linearly polarized gluons does not exceed in size the convolution factor $C[f_1^g\, f_1^g]$
- Other literature suggest that the scale independent ratio $R = C[w_2\, h_1^{\perp g}\, h_1^{\perp g}]/C[f_1^g\, f_1^g]$ is at most 2/3 for a small scale Q ~ 3 GeV.
- Not reasonable to neglect contribution from linearly polarized gluons from the term N^(0) in the weighted cross section.

LHS: Result for an associated Υ state with mass m_Υ = 9.46 GeV.

RHS: Result for an associated J/ψ state with mass $m_{J/\psi}$ = 3.1 GeV.

Conclusions

- Lots of theoretical papers
- Little experimental papers
- First experimental attempts are being made to extract gluon TMDs.