Measurement of the phase space density increase of a muon beam through ionization cooling

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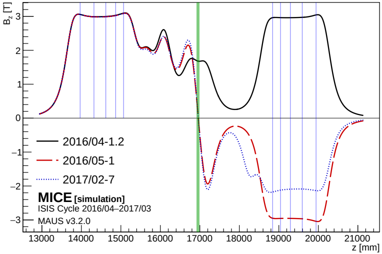
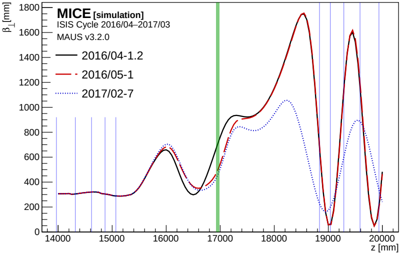
Run settings under consideration

Preliminary dissertation uploaded at indico.cern.ch/event/739039

Three main cooling settings, will **only present 2017/02-7 here**

	2016/04-1.2	2016/05-1	2017/02-2
Mode	Solenoid	Flip	Flip
Momentum [MeV/c]	140	140	140
Emittances [mm]	3, 6, 10	3, 6, 10	3, 6, 10
$\beta_\perp^*$	$\sim 800\,\mathrm{mm}$	$\sim 550\,\mathrm{mm}$	$\sim \mathbf{520\,mm}$

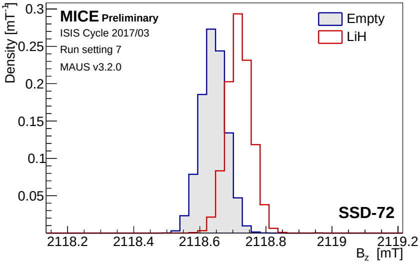


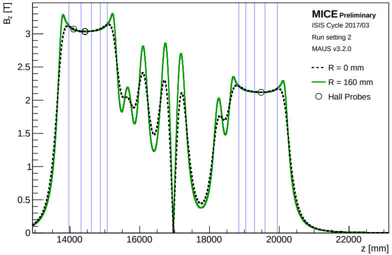


Simulations

Using the MAUS 3.2.0 simulations (latest, thanks DR, DM and PF)

- ~ 100k particles per setting, D2 tuned up to match observed;
- High ρ glue in trackers, matched within the tracker reconstruction;
- Fields scaled up by $1.8\,\%$ in SSU and $1.55\,\%$ in SSD to match the hall probe measurements;
- Hall probe readings stable at the 10^{-4} level between runs.





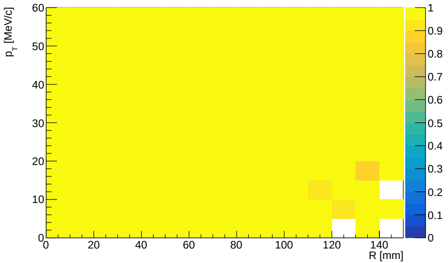
Tracker p_T hole issue

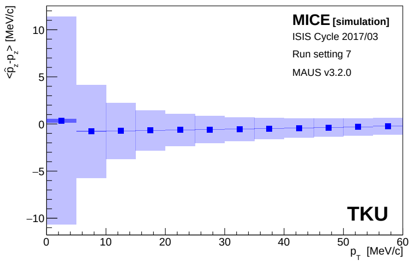
The trackers are highly efficient. If a muon goes through their fiducial volume, they almost certainty record a track (*bottom left*).

At low p_T, however, the p_z resolution is extremely poor (*bottom right*)

→ If one applies a momentum cut to preserve monochromaticity, one will inevitably **loose more low p_T tracks**;

→ It creates a **hole** at the centre of the phase space which hinders any hope of recovering an unbiased phase space density.





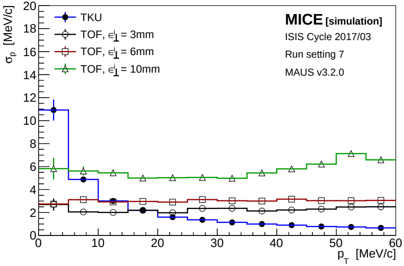
Tracker p_T hole fix

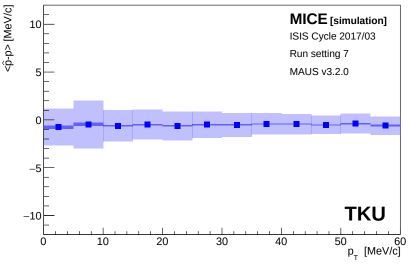
The low p_z resolution is a **inherent limitation** of the tracker technology. If the radius is too small, the uncertainty diverges.

The **ToF** of $\sim 140\,\text{MeV}/c$ particles offers an exquisite estimate of p:

- $\rightarrow$ Use it to recover the phase space of low p_T tracks;
- $\rightarrow$ Only real uncertainty is energy straggling, worse for large $\epsilon_\perp$ beams;
- $\rightarrow$ Use the weighted momentum estimate, scale accordingly.

For the downstream section, use the TKU best estimate and propagate it downstream. Once again, use the weighted average.

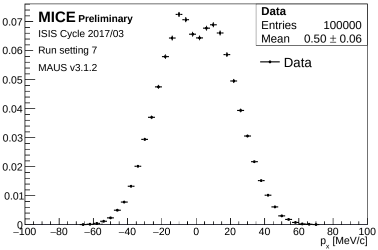


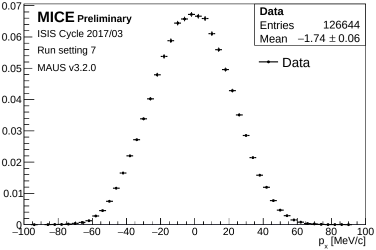


Comparison of the 6 mm beam, TKU S1 (!)

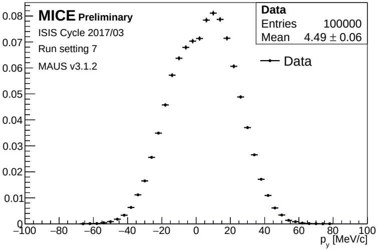
Before fix | After fix

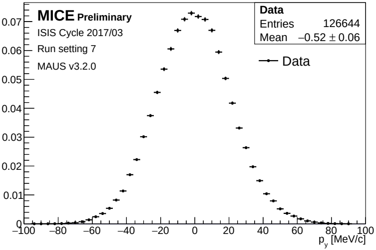
p_x





p_y

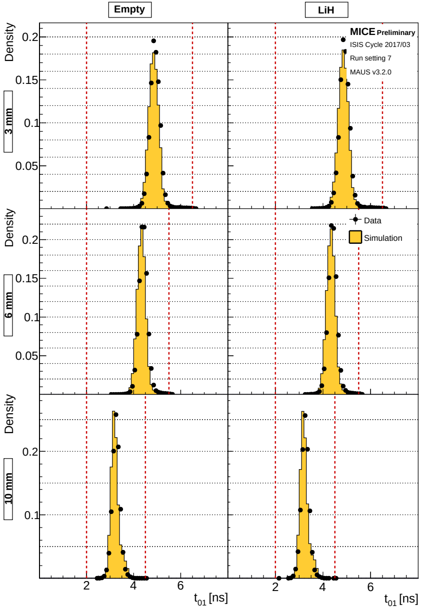




Sample selection

Series of selection criteria applied to the data and the simulation:

- 1 SP in TOF0
- 1 SP in TOF1
- ToF01 compatible with μ
- 1 tracker track (**TKU+TKD**)
- $\chi^2/\text{ndf} < 10$ (**TKU+TKD**)
- Fiducial radius $< 150\,\text{mm}$ (**TKU+TKD**)
- TKU total momentum $\in [135, 145]\,\text{MeV}/c$
- Energy loss between TOF1 and TKU compatible with μ and true diffuser thickness

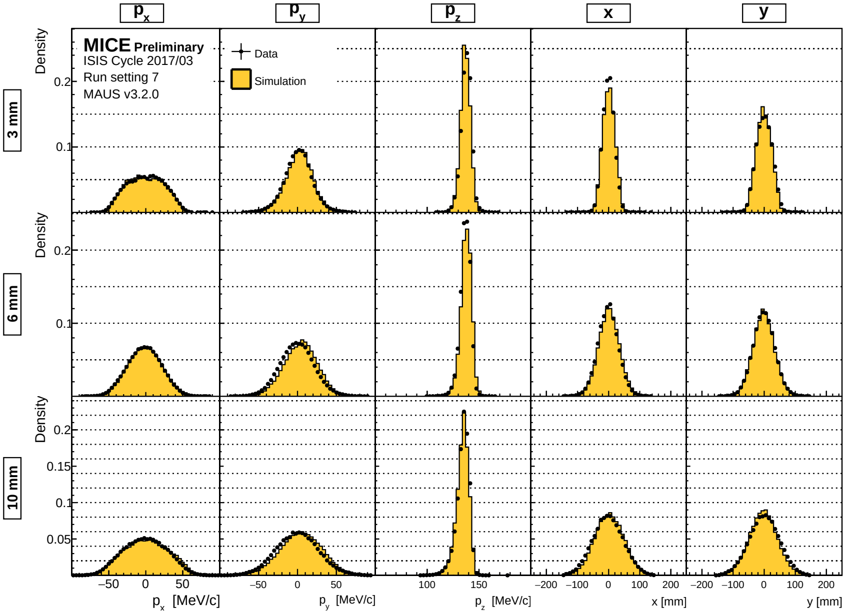


2017/02-7 data samples

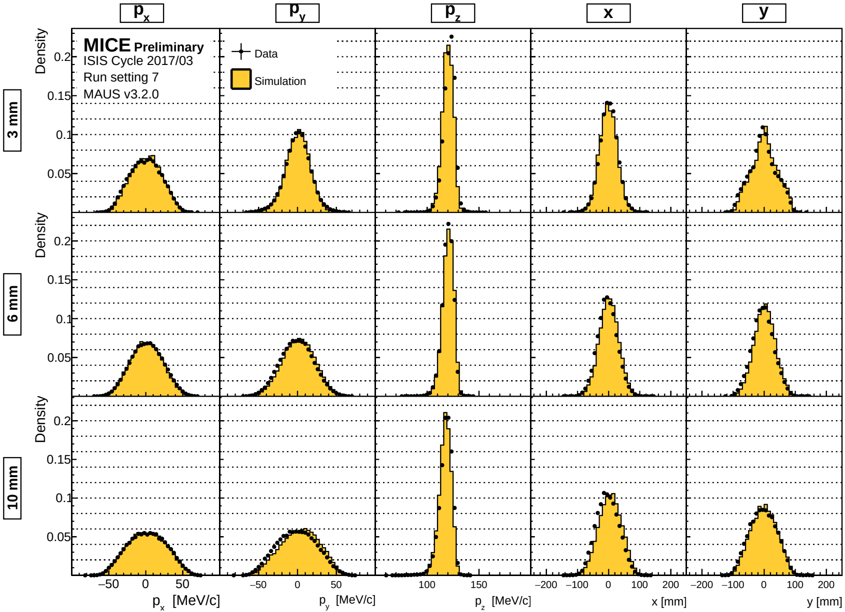
Each number in the table below represents the amount of tracks that pass the row's cut and that cut alone. **All the data available is included**.

Cuts	No absorber			LiH absorber		
Input $\epsilon_\perp$	3 mm	6 mm	10 mm	3 mm	6 mm	10 mm
None	719334	1458158	1212980	677811	857507	1024734
TOF0 SP	613401	1216732	963969	569998	708432	802106
TOF1 SP	687660	1396488	1126896	646037	820382	946978
Time-of-flight	293958	563262	429728	272118	327895	355393
TKU track	271421	969672	623020	257795	576315	529450
TKU χ^2/ndf	252299	891481	574656	239333	527072	487189
TKU fiducial	268867	962979	595927	255364	572451	506443
TKU Momentum	81103	288399	132168	76067	168826	111180
Energy loss	119744	463164	266932	111700	269547	219638
All US	**54884**	**219146**	**87197**	**50602**	**126644**	**71213**
TKD track	53656	204415	60326	49056	118088	48193
TKD χ^2/ndf	52760	201679	59464	48361	116683	47676
TKD fiducial	53227	195329	51847	47928	114054	42774
All DS	**52368**	**192864**	**51203**	**47271**	**112777**	**42375**

2017/02-7 upstream profiles



2017/02-7 downstream profiles

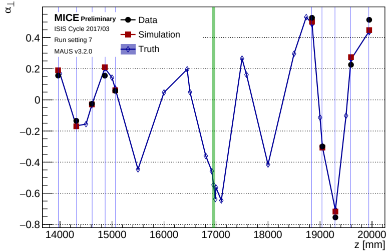


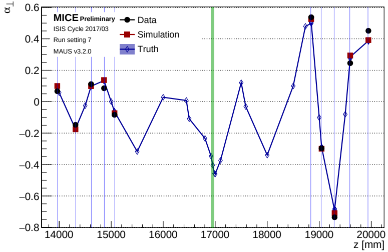
Optical functions

6 mm beam

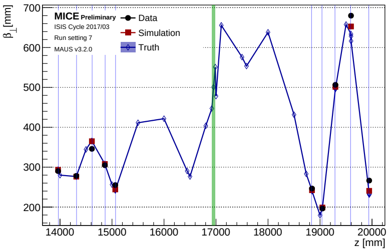
10 mm beam

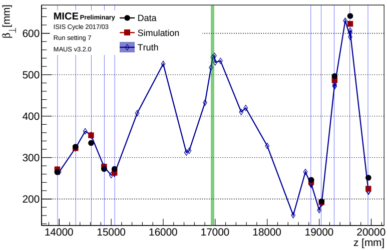
$\alpha_\perp$





$\beta_\perp$



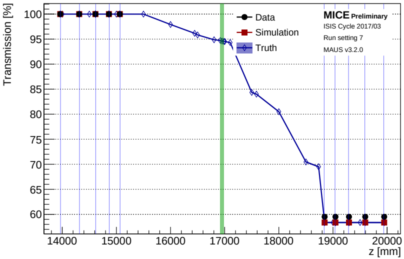


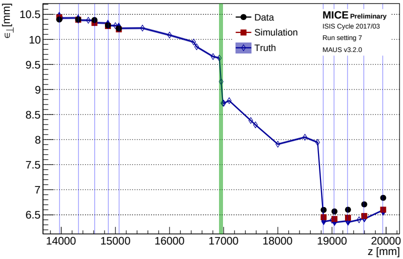
Transverse normalised RMS emittance

Transverse normalised RMS emittance defined as $\epsilon_\perp = \frac{1}{m}|\mathbf{\Sigma}_\perp|^{\frac{1}{4}}$, with $|\mathbf{\Sigma}_\perp|$ the determinant of the covariance matrix defined as

The RMS emittance is directly related to the **volume of the RMS ellipsoid** through $\epsilon_\perp = \sqrt{2V_{\text{RMS}}}/(m\pi)$ and as such is the most common probe of average phase-space density.

→ Poor estimate in the case of low transmission or non-linear transport





Transmission and emittance evolution in the 10 mm beam, with LiH

Transverse single-particle amplitude

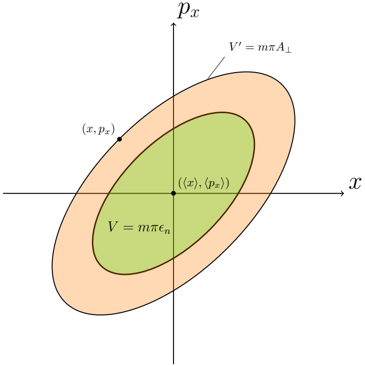
Single particle amplitude is defined as

$$A_\perp = \epsilon_\perp \mathbf{u}^T \mathbf{\Sigma}_\perp^{-1} \mathbf{u}, \qquad (1)$$

with $\mathbf{u} = \mathbf{v} - \boldsymbol{\mu}$, the centered phase-space vector of the particle.

It is related to the **volume** of an ellipse similar to the RMS ellipse, going through $\mathbf{v}$.

High amplitudes **iteratively removed** from ensemble to prevent bias

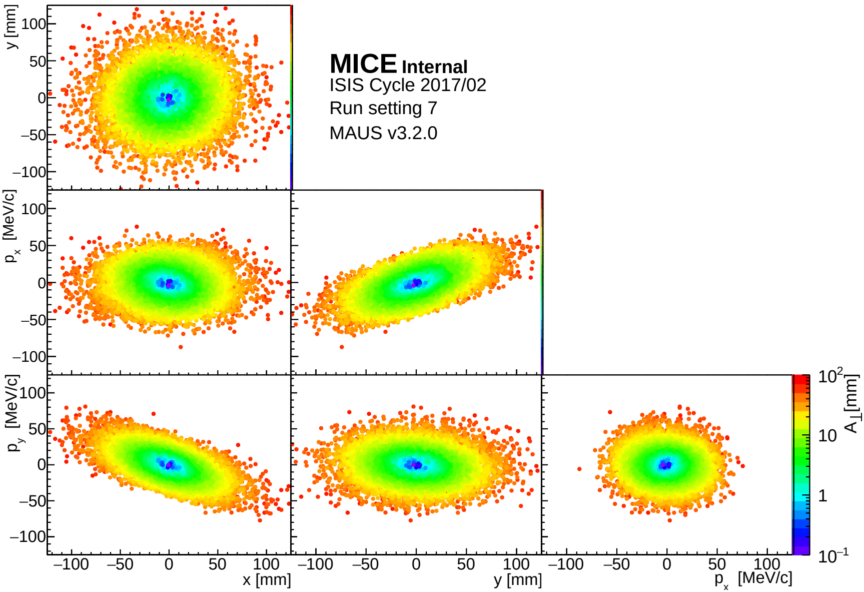


Particle amplitude provides an density estimate in every sample point

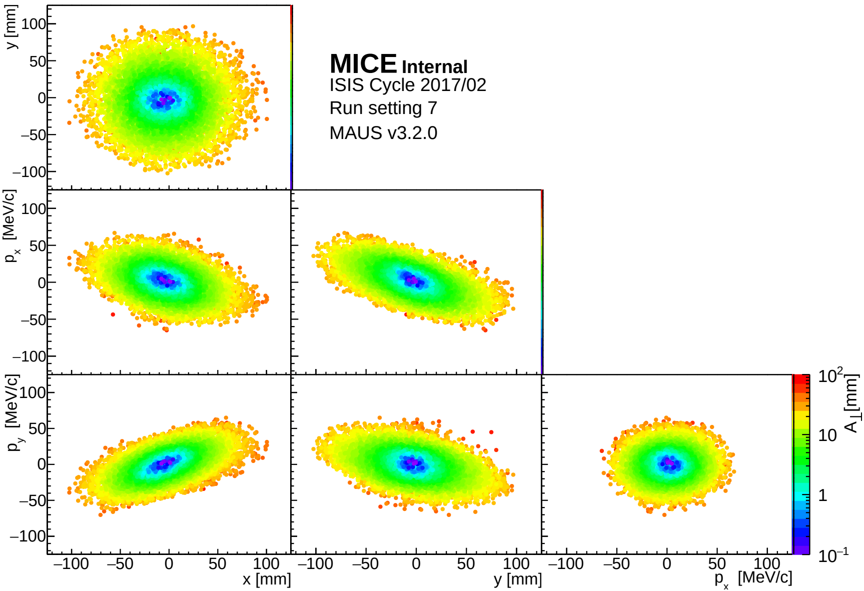
$$\rho(\boldsymbol{v}_i) = \rho_{\mathsf{max}} \exp\left[-\mathbf{u}^T \mathbf{\Sigma}_\perp^{-1} \mathbf{u}/2\right] = \boxed{\rho_{\mathsf{max}} \exp\left[-\frac{A_\perp}{2\epsilon_\perp}\right]}. \qquad (2)$$

$\rightarrow$ Allows for the selection of a **high density core** !

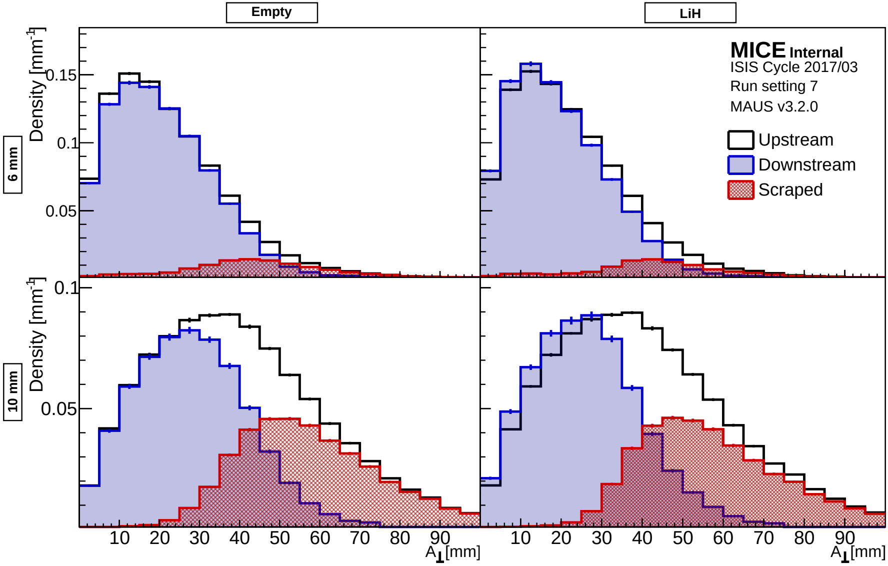
Poincaré sections upstream (6 mm, LiH)



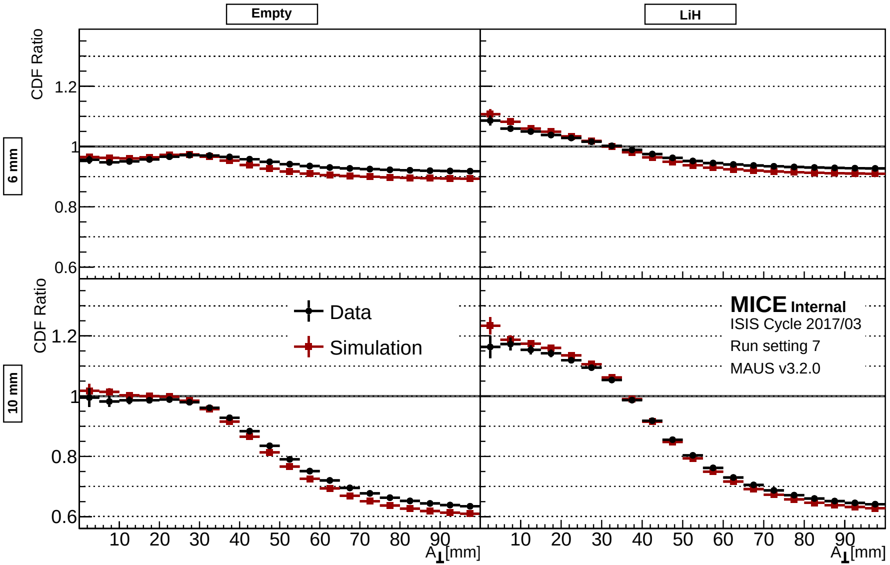
Poincaré sections downstream (6 mm, LiH)



Amplitude distributions in data



CDF ratios



Empty
LiH
6 mm
10 mm
CDF Ratio
CDF Ratio
1.2
1
0.8
0.6
Data
Simulation
MICE Internal
ISIS Cycle 2017/03
Run setting 7
MAUS v3.2.0
10 20 30 40 50 60 70 80 90
$A_\perp$[mm]

Summary statistics

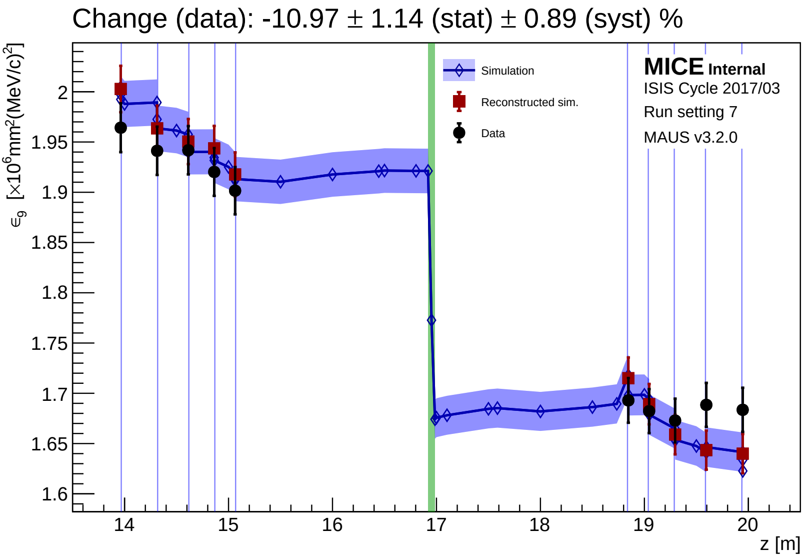
Several summary statistics studied and viable, as summarized in here:

	A_α	e_α	ϵ_α
Name	α-amplitude	α-subemittance	α-emittance
Value	$\boldsymbol{\epsilon}_\perp \chi^2_4(\alpha)$	$\boldsymbol{\epsilon}_\perp P(3, \chi^2_4(\alpha)/2)/\alpha$	$\frac{1}{2}m^2\pi^2\boldsymbol{\epsilon}^{\mathbf{2}}_\perp \chi^2_4(\alpha)$
$\alpha = 9\,\%$	$\boldsymbol{\epsilon}_\perp$	$\sim 0.16\boldsymbol{\epsilon}_\perp$	V_{RMS}
σ_x/x	$g(\alpha)\sqrt{\alpha(1-\alpha)/n}$	$1/\sqrt{2\alpha n}$	$2g(\alpha)\sqrt{\alpha(1-\alpha)/n}$
$\alpha = 9\,\%$	$\sim 1.9/\sqrt{n}$	$\sim 2.4/\sqrt{n}$	$\sim 3.8/\sqrt{n}$
$\Delta x/x$	δ	δ	$(\delta+1)^2 - 1$

- $P(n, x)$ the regularized Gamma function;
- $g(\alpha)$ a complicated function of α (see dissertation);
- δ is the relative normalised emittance change.

A fraction of $\boldsymbol{\alpha = 9\,\%}$ **is chosen** as it represents the RMS ellipse in 4D.

Fractional emittance evolution (6 mm, LiH)



Non-parametric density estimation

Amplitude methods work well for beams with a **Gaussian core**, so they are restricted to a small fraction of a non-linear beam (statistical limitation)

Three classes of estimators considered for this measurement

	Histograms	k-Nearest Neighbour	PBATDE
$\rho(\boldsymbol{x})$	$n_i(\boldsymbol{x})/(n\Delta_i)$	$k/(n\kappa_d R_k^d)$	$\frac{1}{M}\sum_{i=1}^{M} v_i^{-1}$
MISE	$\mathcal{O}(\Delta^{2/d} + 1/(n\Delta))$	$\mathcal{O}((k/n)^{4/d} + 1/k)$	$\mathcal{O}(J^{-4/d} + J/n)$
$d = 4$	$\mathcal{O}(\Delta^{1/2} + 1/(n\Delta))$	$\mathcal{O}(k/n + 1/k)$	$\mathcal{O}(1/J + J/n)$
Convergence	$n^{-2/(2+d)}$	$n^{-4/(4+d)}$	$n^{-4/(4+d)}$
$d = 4$	$n^{-1/3}$	$\boldsymbol{n^{-1/2}}$	$\boldsymbol{n^{-1/2}}$
Speed	**Fast**	**Fast**	Slow

***k*-Nearest Neighbour** (kNN) estimator has highest rate of convergence (on par with the PBATDE) and is the least computationally intensive.

Systematic studies on a broad array of distribution classes showed robustness of the kNN algorithm (see dissertation).

k-Nearest Neighbor

To find the density at a point $\boldsymbol{x}$ in phase space, find the k closest data points, with the distance defined as

$$D_i^2 = (\boldsymbol{x} - \boldsymbol{x}_i)^T(\boldsymbol{x} - \boldsymbol{x}_i), \qquad (3)$$

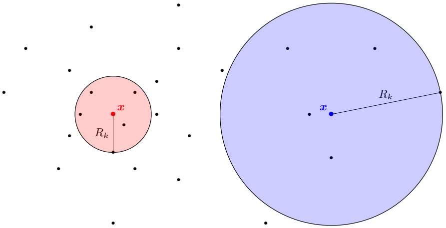
the Euclidean distance.

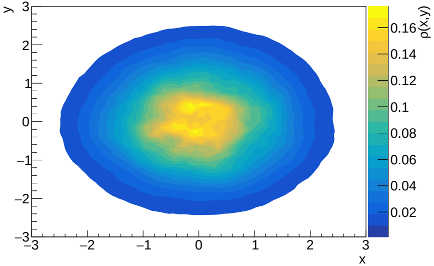
If R_k is the distance to the k^{th} point, the 4D density then reads

$$\boxed{\rho(\boldsymbol{x}) = \frac{k}{n}\frac{1}{\kappa_4 R_k^4},} \qquad (4)$$

with $\kappa_4 = \pi^2/2$, the volume of the Euclidean unit 4-ball.

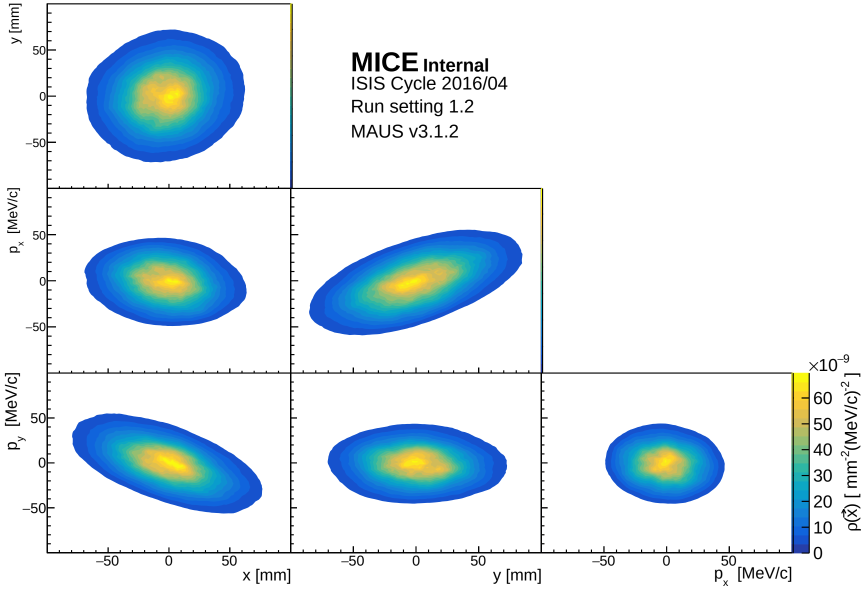
The optimal k in 4D follows $k \sim \sqrt{n}$.



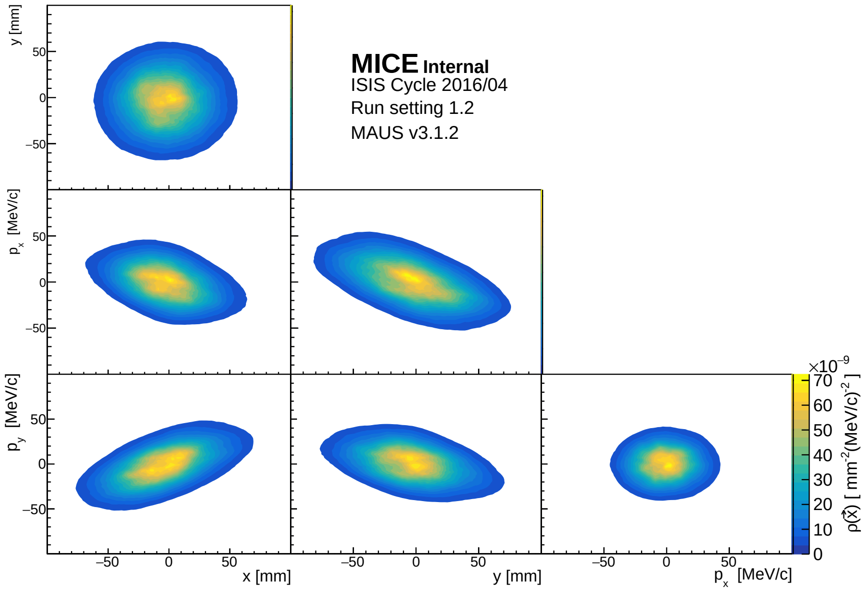


kNN density of 10^3 Gaussian points

Poincaré sections upstream (6 mm, LiH)



Poincaré sections downstream (6 mm, LiH)



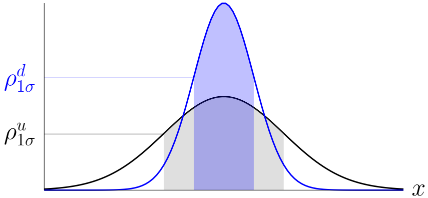
Contour levels

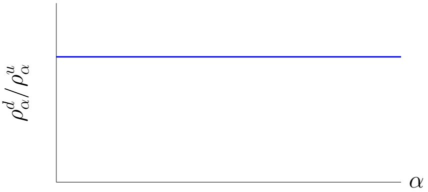
Goal: Need an equivalent to $A_\perp$, to represent at which density the points lie upstream and downstream of the absorber in a 1D plot.

Challenge: The probability density function is unknown; the concept of radius is meaningless. Must find a different quantity to $A_\perp$.

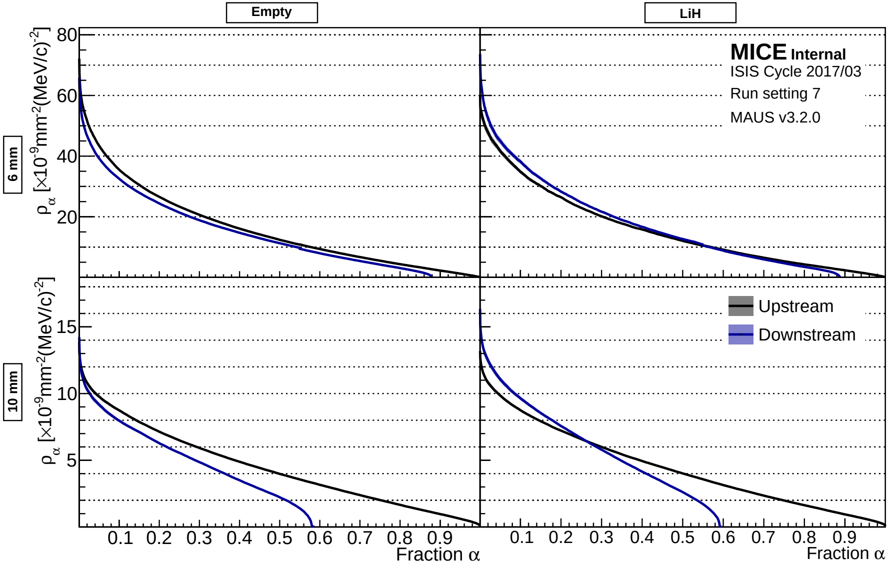
Solution: **Contour levels**. For a core fraction of the beam α, find the level of the corresponding contour. If the level has raised at a given α, the density of particles has increased. With F^{-1} the inverse CDF,

$$\boxed{\rho_\alpha = \rho(F^{-1}(\alpha)).} \quad (5)$$





Density profiles in data



Density summary statistics

As a summary statistic, one can represent the evolution of an arbitrary contour level: **9 % as it corresponds to the RMS ellipse contour**, ρ_9.

If the beam core is Gaussian, the relative change in density corresponds to

$$\frac{\rho_\alpha^d - \rho_\alpha^u}{\rho_\alpha^u} = \frac{1}{(\delta+1)^2} - 1 \simeq -2\delta \tag{6}$$

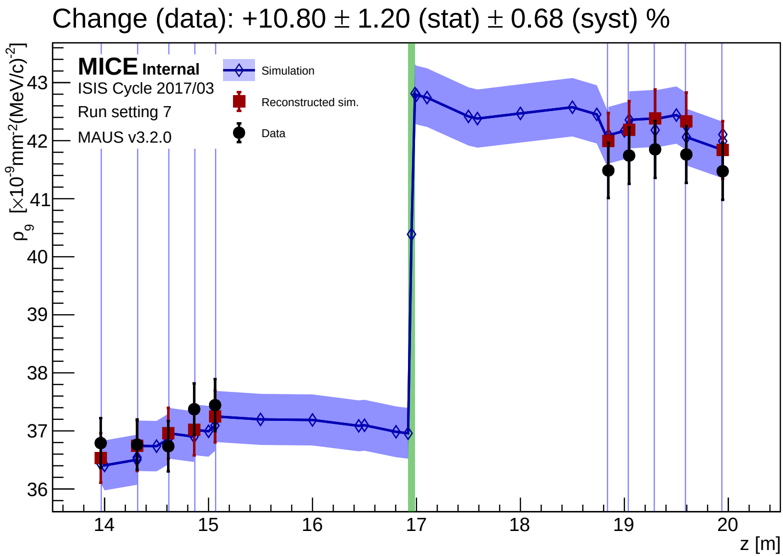
with δ the relative RMS emittance change. If the emittance decreases by δ, the density approximatively increases twice as much.

An alternative is to compute the volume of phase space that contains the particles above ρ_9, V_9. This yields the generalised fractional emittance. If the beam core is Gaussian, the relative change in density corresponds to

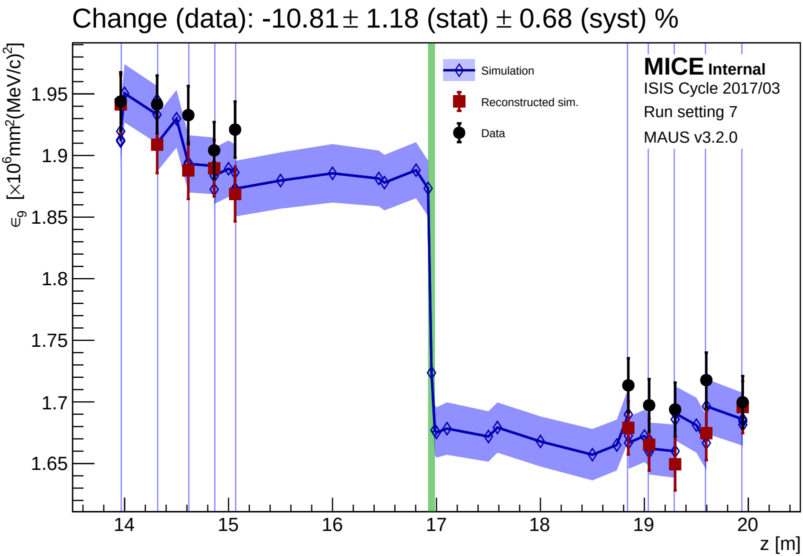
$$\frac{V_\alpha^d - V_\alpha^u}{V_\alpha^u} = (\delta+1)^2 - 1 \simeq 2\delta, \tag{7}$$

i.e. twice the emittance change, same as before.

Density evolution (6mm, LiH)



Contour volume evolution (6mm, LiH)



Conclusions

Amplitude-based analysis:

- Selecting the core amplitude-wise **gets rid of artificial cooling** due to scraping and **artificial growth** due to non-linearities
- Systematics study complete
- Method shows a **clear cooling signal** in data, agrees with MC.

Nonparametric density estimation analysis:

- Thorough studies finished, **kNN most convergent, robust and fast in 4D**. Allows for the reconstruction of the probability density function in phase space upstream and downstream of absorber.
- Systematics study complete
- Method shows a **clear cooling signal** in data, agrees with MC.

Could show all or a subset of the plots presented here at NuFact.

Find more details at indico.cern.ch/event/739039

Systematics

Same systematics sources as CR:

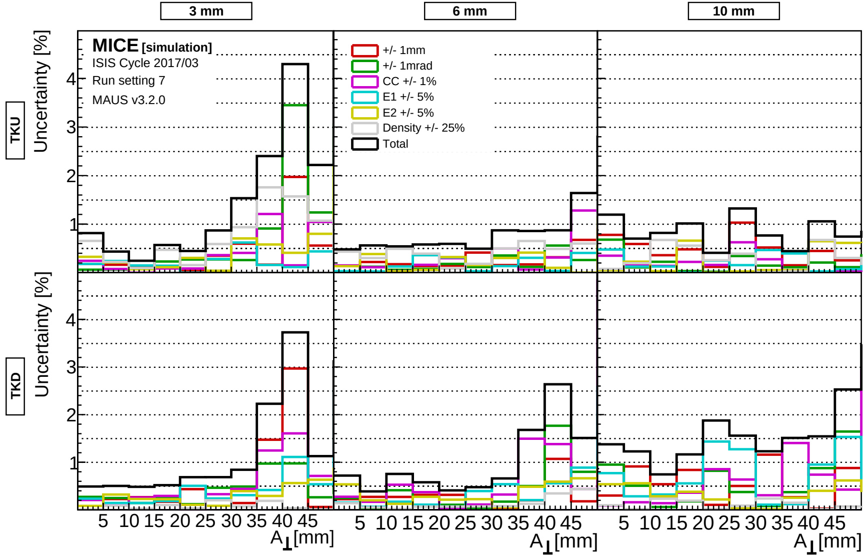
- $+1$ mm position
- $+1$ mrad rotation
- $+1$ % CC current
- $+5$ % E1/E2 current
- $+25$ % glue density

For each source, 10^6 muons resampled from the measured distribution. Same cuts applied to the so-produced simulation.

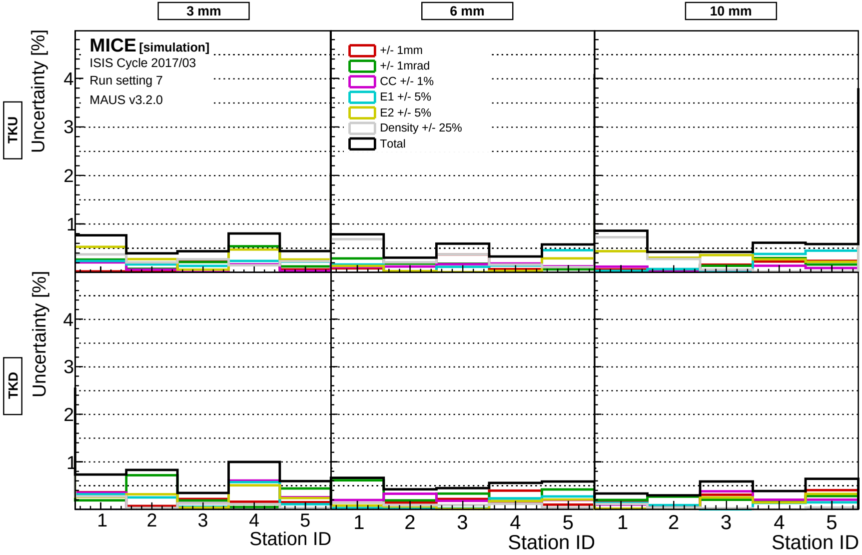
Each quantity represented is computed in each case and a systematic study of the uncertainty is produced.

Next slides show the effect on the amplitude distributions, the fractional emittance and the density estimation for 2017/02-7.

Amplitude distribution systematics



9 % fractional emittance systematics



Density levels systematics