

# HELAC(NLO) - STATUS REPORT

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## Contributors

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P. Draggiotis

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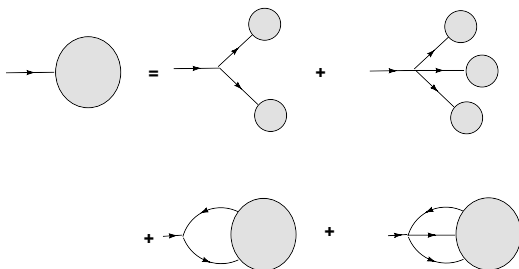
# HELAC TEAM



# DYSON-SCHWINGER RECURSIVE EQUATIONS

- **1999** HELAC: The first code to calculate recursively tree-order amplitudes for (practically) arbitrary number of particles

A. Kanaki and C. G. Papadopoulos, *Comput. Phys. Commun.* **132** (2000) 306 [arXiv:hep-ph/0002082].



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- **1999** HELAC: The first code to calculate recursively tree-order amplitudes for (practically) arbitrary number of particles
- **2000** PHEGAS: The first code to automatically produce phase-space mappings based on all FD

C. G. Papadopoulos, *Comput. Phys. Commun.* **137** (2001) 247 [[arXiv:hep-ph/0007335](https://arxiv.org/abs/hep-ph/0007335)].

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- 1999 HELAC: The first code to calculate recursively tree-order amplitudes for (practically) arbitrary number of particles
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- Including all SM, in both unitary and F-gauge, masses, CKM, unstable particle widths, complex mass scheme, etc.
- For QCD color connection representation: revival of the 't Hooft ideas ('71) in the modern era.

# COLOR REPRESENTATION

Color-assignment  $\rightarrow$  color connections

$$\mathcal{M}_{j_1 j_2, \dots, j_k}^{i_1, i_2, \dots, i_k} = \sum_{\sigma} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} A_{\sigma}$$

gluons  $\rightarrow (i, j)$ , quark  $\rightarrow (i, 0)$ , anti-quark  $\rightarrow (0, j)$ , other  $\rightarrow (0, 0)$



# COLOR REPRESENTATION

Color-assignment  $\rightarrow$  color connections

$$\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k} = \sum_{\sigma} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} A_{\sigma}$$

Color sum

$$\begin{aligned} & \sum_{\{i\}, \{j\}} |\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}|^2 \\ & \sum_{\sigma, \sigma'} A_{\sigma}^* C_{\sigma, \sigma'} A_{\sigma'} \\ C_{\sigma, \sigma'} & \equiv \sum_{\{i\}, \{j\}} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} \delta_{i_{\sigma'_1}, j_1} \delta_{i_{\sigma'_2}, j_2} \dots \delta_{i_{\sigma'_k}, j_k} \end{aligned}$$

# COLOR REPRESENTATION

Color-assignment  $\rightarrow$  color connections

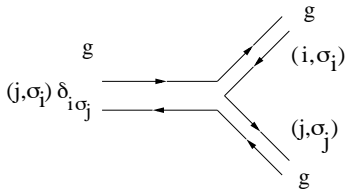
$$\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k} = \sum_{\sigma} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} A_{\sigma}$$

Color sum

$$\sum_{\{i\}, \{j\}} |\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}|^2$$
$$\sum_{\sigma, \sigma'} A_{\sigma}^* C_{\sigma, \sigma'} A_{\sigma'}$$

$$C_{\sigma, \sigma'} \equiv \sum_{\{i\}, \{j\}} \delta_{i_{\sigma_1}, j_1} \delta_{i_{\sigma_2}, j_2} \dots \delta_{i_{\sigma_k}, j_k} \delta_{i_{\sigma'_1}, j_1} \delta_{i_{\sigma'_2}, j_2} \dots \delta_{i_{\sigma'_k}, j_k}$$

Color-connection Feynman Rules



- 2007 HELAC: <http://helac-phegas.web.cern.ch/helac-phegas/>

A. Cafarella, C. G. Papadopoulos and M. Worek, *Comput. Phys. Commun.* **180** (2009) 1941 [arXiv:0710.2427 [hep-ph]].

- 2007 HELAC: <http://helac-phegas.web.cern.ch/helac-phegas/>
- Generate all subprocesses for  $pp$ ,  $p\bar{p}$  collisions, calculate cross sections, produce Les Houches accord file

# HELAC TREE ORDER CURRENT VERSION

- 2007 HELAC: <http://helac-phegas.web.cern.ch/helac-phegas/>
- Generate all subprocesses for  $pp$ ,  $p\bar{p}$  collisions, calculate cross sections, produce Les Houches accord file
- Very easy to use: just edit the `user.inp` file and then execute the command `./run.sh`

```
# Compulsory information
colpar 1      # colliding particles: 1=pp, 2=ppbar, 3=e+e-
inist 35 35  # initial state; enter 0 to sum over initial states
finst 35 35  # final state
energy 14000 # collision energy (GeV)

# For reference, here is the particle numbering:
# ve e u d vm mu c s vt ta t b a z w+ w- g h chi f+ f- jet
# 1 2 3 4 5 6 7 8 9 10 11 12 31 32 33 34 35 41 42 43 44 100
# The respective antiparticles have a minus sign (for example: positron is -2)
# A jet in the final state is denoted by the number 100

# Enter here your additional commands if you wish to alterate the default values
```

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- Generate all subprocesses for  $pp$ ,  $p\bar{p}$  collisions, calculate cross sections, produce Les Houches accord file
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- Including kt-reweight for jet matching

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- Generate all subprocesses for  $pp$ ,  $p\bar{p}$  collisions, calculate cross sections, produce Les Houches accord file
- Very easy to use: just edit the `user.inp` file and then execute the command `./run.sh`
- Including kt-reweight for jet matching
- Latest:  $W + 5$  jets at LHC

# THE HELAC-NLO ADVENTURE



- **2006** OPP: The method that enables us to think seriously about NLO calculations.

Based on previous work by Bern, Dixon, Kosower, Britto, Cachazo, Feng.

Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **425** (1994) 217 [arXiv:hep-ph/9403226].

R. Britto, F. Cachazo and B. Feng, Nucl. Phys. B **725** (2005) 275 [arXiv:hep-th/0412103].

**Complete framework: numerical (fast) & algebraic (stable)**

G. Ossola, C. G. Papadopoulos and R. Pittau, Nucl. Phys. B **763** (2007) 147 [arXiv:hep-ph/0609007].

- 2006 OPP: The method that enables us to think seriously about NLO calculations.

$$\begin{aligned}
 \int A &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) \\
 &+ \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) \\
 &+ \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) \\
 &+ \sum_{i_0}^{m-1} a(i_0) A_0(i_0) \\
 &+ \text{rational terms}
 \end{aligned}$$

Algebra & Integrals

- 2006 OPP: The method that enables us to think seriously about NLO calculations.

$$\begin{aligned}
 N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
 &+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
 &+ \sum_{i_0 < i_1}^{m-1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
 &+ \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
 \end{aligned}$$

Solving for known values of the loop momentum  $q$

- 2006 OPP: The method that enables us to think seriously about NLO calculations.
- 2007 CutTools: Reduction at the integrand level + rational terms  
Latest version to appear soon

G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0803** (2008) 042 [arXiv:0711.3596 [hep-ph]].

G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0805** (2008) 004 [arXiv:0802.1876 [hep-ph]].

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- 2008 HELAC1L: Based on HELAC to produce virtual one-loop amplitudes

A. van Hameren, C. G. Papadopoulos and R. Pittau, JHEP **0909** (2009) 106 [arXiv:0903.4665 [hep-ph]].

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Latest version to appear soon
- AVH\_0L0: One-loop scalar integrals including complex masses, IR regulators
- **2008** HELAC1L: Based on HELAC to produce virtual one-loop amplitudes
- **2009** HELAC-Dipoles: Based on HELAC to *automatically* produce Catani-Seymour dipoles, I-operator, KP-operator, arbitrary masses

M. Czakon, C. G. Papadopoulos and M. Worek, JHEP **0908** (2009) 085 [arXiv:0905.0883 [hep-ph]].

A. van Hameren, C. G. Papadopoulos and R. Pittau, JHEP **0909** (2009) 106 [arXiv:0903.4665 [hep-ph]].

Process		$6 D_i$	$5 D_i$	$4 D_i$	$3 D_i$	$2 D_i$	$R_2$	CT	Total
$gg \rightarrow t\bar{t}b\bar{b}$	non-planar	18	120	268	220	112	51	6	795
	planar	13	32	35	40	48	25	2	195
$gg \rightarrow t\bar{t}gg$	non-planar	168	576	480	224	56	50	14	1568
	planar	2	14	40	60	60	34	3	213



$pp \rightarrow t\bar{t}b\bar{b}$			
$u\bar{u} \rightarrow t\bar{t}b\bar{b}$			
	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
HELAC-1L	-2.347908989000179E-07	-2.082520105681483E-07	3.909384299635230E-07
$I(\epsilon)$	-2.347908989000243E-07	-2.082520105665445E-07	
$gg \rightarrow t\bar{t}b\bar{b}$			
HELAC-1L	-1.435108168334016E-06	-2.085070773763073E-06	3.616343483497464E-06
$I(\epsilon)$	-1.435108168334035E-06	-2.085070773651439E-06	

	$p_x$	$p_y$	$p_z$	$E$
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
$t$	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
$\bar{t}$	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
$b$	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
$\bar{b}$	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

$pp \rightarrow VVb\bar{b}$ and $pp \rightarrow VV + 2$ jets			
$u\bar{u} \rightarrow W^+W^-b\bar{b}$			
	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
HELAC-1L	-2.493916939359002E-07	-4.885901774740355E-07	1.592538533368835E-07
$I(\epsilon)$	-2.493916939359001E-07	-4.885901774752593E-07	
$gg \rightarrow W^+W^-b\bar{b}$			
HELAC-1L	-2.686310592221201E-07	-6.078682316434646E-07	-2.431624440346638E-07
$I(\epsilon)$	-2.686310592221206E-07	-6.078682340168020E-07	

	$p_x$	$p_y$	$p_z$	$E$
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
$W^+$	22.40377113462118	-16.53704884550758	129.4056091248114	154.8819879118765
$W^-$	92.64238702192333	-0.4920930146078141	30.48443210132545	126.4095336206695
$b$	-71.68369328357026	6.716416578342183	-158.5329205583824	174.1159068988160
$\bar{b}$	-43.36246487297426	10.31272528177322	-1.357120667754454	44.59257156863792

$pp \rightarrow V + 3 \text{ jets}$			
$ud \rightarrow W^+ ggg$			
	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
HELAC-1L	-1.995636628164684E-05	-5.935610843551600E-05	-5.323285370666314E-05
$I(\epsilon)$	-1.995636628164686E-05	-5.935610843566534E-05	
$u\bar{u} \rightarrow Zggg$			
HELAC-1L	-7.148261887172997E-06	-2.142170009323704E-05	-1.906378375774021E-05
$I(\epsilon)$	-7.148261887172976E-06	-2.142170009540120E-05	

	$p_x$	$p_y$	$p_z$	$E$
$u$	0	0	250	250
$\bar{d}$	0	0	-250	250
$W^+$	23.90724239064912	-17.64681636854432	138.0897548661186	162.5391101447744
$g$	98.85942812363483	-0.5251163702879512	32.53017998659339	104.0753327455388
$g$	-76.49423931754684	7.167141557113385	-169.1717405928078	185.8004692730082
$g$	-46.27243119673712	11.00479118171890	-1.448194259904179	47.58508783667868

$pp \rightarrow t\bar{t} + 2 \text{ jets}$			
$u\bar{u} \rightarrow t\bar{t}gg$			
	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
HELAC-1L	-6.127108113312741E-05	-1.874963444741646E-04	-3.305349683690902E-04
$I(\epsilon)$	-6.127108113312702E-05	-1.874963445081074E-04	
$gg \rightarrow t\bar{t}gg$			
HELAC-1L	-3.838786514961561E-04	-9.761168899507888E-04	-5.225385984750410E-04
$I(\epsilon)$	-3.838786514961539E-04	-9.761168898436521E-04	

	$p_x$	$p_y$	$p_z$	$E$
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
$t$	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
$\bar{t}$	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
$g$	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
$g$	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

$pp \rightarrow b\bar{b}b\bar{b}$			
$u\bar{u} \rightarrow b\bar{b}b\bar{b}$			
	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
HELAC-1L	-9.205269484951069E-08	-2.404679886692200E-07	-2.553568662778129E-07
$I(\epsilon)$	-9.205269484951025E-08	-2.404679886707971E-07	
$gg \rightarrow b\bar{b}b\bar{b}$			
HELAC-1L	-2.318436429821683E-05	-6.958360737366907E-05	-7.564212339279291E-05
$I(\epsilon)$	-2.318436429821662E-05	-6.958360737341511E-05	

	$p_x$	$p_y$	$p_z$	$E$
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
$b$	24.97040523056789	-18.43157602837212	144.2306511496888	147.5321146846735
$\bar{b}$	103.2557390255471	-0.5484684659584054	33.97680766420219	108.7035966213640
$b$	-79.89596300367462	7.485866671764871	-176.6948628845280	194.0630765341365
$\bar{b}$	-48.33018125244035	11.49417782256567	-1.512595929362970	49.70121215982584

# HELAC-DIPOLES

$\mathcal{E}_0$  - massless emitter,  $\mathcal{S}_0$  - massless spectator,  $\mathcal{E}_M$  - massive emitter,  $\mathcal{S}_M$  - massive spectator,  $\mathcal{E}_I$  - initial state emitter,  $\mathcal{E}_F$  - final state emitter,  $\mathcal{S}_I$  - initial state spectator,  $\mathcal{S}_F$  - final state spectator, ✓ - check, ■ - does not occur.

	$\mathcal{E}_0/\mathcal{S}_0$	$\mathcal{E}_0/\mathcal{S}_M$	$\mathcal{E}_M/\mathcal{S}_0$	$\mathcal{E}_M/\mathcal{S}_M$		$\mathcal{E}_0/\mathcal{S}_0$	$\mathcal{E}_0/\mathcal{S}_M$	$\mathcal{E}_M/\mathcal{S}_0$	$\mathcal{E}_M/\mathcal{S}_M$
$\mathcal{E}_I/\mathcal{S}_I$					$\mathcal{E}_F/\mathcal{S}_I$				
$g \rightarrow gg$	✓	■	■	■	$g \rightarrow gg$	✓	■	■	■
$g \rightarrow qq$	✓	■	■	■	$g \rightarrow qq$	✓	■	✓	■
$q \rightarrow qq$	✓	■	■	■	$q \rightarrow qq$	✓	■	✓	■
$q \rightarrow gq$	✓	■	■	■	$q \rightarrow gq$	✓	■	✓	■
$\mathcal{E}_I/\mathcal{S}_F$					$\mathcal{E}_F/\mathcal{S}_F$				
$g \rightarrow gg$	✓	✓	■	■	$g \rightarrow gg$	✓	✓	■	■
$g \rightarrow qq$	✓	✓	■	■	$g \rightarrow qq$	✓	✓	✓	✓
$q \rightarrow qq$	✓	✓	■	■	$q \rightarrow qq$	✓	✓	✓	✓
$q \rightarrow gq$	✓	✓	■	■	$q \rightarrow gq$	✓	✓	✓	✓

**Table 1:** Independent dipole splitting formulae, which need to be tested in order to ensure the correctness of the code. In the splitting description, e.g.  $g \rightarrow gg$ , the left hand side particle always denotes the virtual state.

# HELAC-DIPOLES

PROCESS	REAL EMISSION + DIPOLES [msec]	REAL EMISSION [msec]	NR OF DIPOLES
$gg \rightarrow ggg$	3.8	1.0	27
$gg \rightarrow gggg$	8.5	2.6	56
$gg \rightarrow ggggg$	300	42	100
$u\bar{d} \rightarrow W^+ gggg$	9.3	2.4	56
$gg \rightarrow t\bar{t}b\bar{b}g$	12	2.9	55

**Table 2:** *The CPU time needed to evaluate the real emission matrix element together with all of the dipole subtraction terms per phase-space point (this corresponds to  $\alpha_{max} = 1$ ). All numbers have been obtained on an Intel 2.53 GHz Core 2 Duo processor with the Intel Fortran compiler using the -fast option.*

- Arbitrary processes QCD+EW



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- Massive and massless external states

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# HELAC-DIPOLES

- Arbitrary processes QCD+EW
- Massive and massless external states
- Helicity (& color) sampling for partons
- Random helicities for non-partons
- Restrictions on PS  $\alpha_{min}$  and  $\alpha_{max}$

```
-----  
Dipole Subtraction Configuration  
-----  
only real emission:           F  
only last particle soft/collinear: F  
only divergent dipoles:      T  
random polarization for non-partons: T  
sign mode (0-both,1-positive,2-negative): 0  
helicity sum (0-fast,1-slow,2-flat MC):    1  
events with polarization sum=             10000  
events for sampling optimization=         20000  
event increment for sampling update=      10000  
alphaMinCut=                            1.0000000000000000E-006  
alphaMaxI=                               1.0000000000000000  
alphaMaxIF=                              1.0000000000000000  
alphaMaxFI=                              1.0000000000000000  
alphaMaxFF=                              1.0000000000000000  
kappa=                                   0.0000000000000000E+000  
jet veto included:                     F  
pt of vetoing jet=                     50.00000000000000  
color sampling: F  
-----  
Number of Dipoles:                    55  
Number of Processes:                   7  
-----
```

# HOW HELAC-NLO WORKS-VIRTUAL

Generate  $w = 1$  events (Les Houches format) using HELAC at tree order.  
Information included: LH + color assignment, helicity. Optimization!

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Generate  $w = 1$  events (Les Houches format) using HELAC at tree order.  
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```
<event>
6 81 1.000000E+00 1.726000E+02 7.546772E-03 1.180000E-01
21 -1 0 0 103 101 0.0000000000000000E+00 0.0000000000000000E+00 4.885658920243087E+02 4.885658920243087E+02 0.0000000000000000E+00 0.000000E+00 9.0000E+00
21 -1 0 0 104 102 0.0000000000000000E+00 0.0000000000000000E+00 -4.885658920243087E+02 4.885658920243087E+02 0.0000000000000000E+00 0.000000E+00 9.0000E+00
6 1 1 2 103 0 1.648551153938704E+02 -2.128833463956879E+01 1.563411288268662E+01 2.401366022681282E+02 1.7260000000000000E+02 0.000000E+00 9.0000E+00
-6 1 1 2 0 102 -6.677109609683933E+01 6.109017946596872E+01 -3.256227583127494E+02 3.794882475549945E+02 1.7260000000000000E+02 0.000000E+00 9.0000E+00
5 1 1 2 104 0 4.725480269309031E+00 2.281431584259000E+01 1.753945210216305E+02 1.769351891952198E+02 0.0000000000000000E+00 0.000000E+00 9.0000E+00
-5 1 1 2 0 101 -1.028094995663402E+02 -6.261616066898994E+01 1.345941244084323E+02 1.805717450302747E+02 0.0000000000000000E+00 0.000000E+00 9.0000E+00
# 9.193930413382987E-08 4 3 4 14 13
# 0.0000000000000000E+00 0.0000000000000000E+00 0.0000000000000000E+00 0.0000000000000000E+00 7.869627745360847E-01 -1.175027485420859E+00 -7.869627745360847E-01
-1.175027485420859E+00
# 0.0000000000000000E+00 0.0000000000000000E+00 0.0000000000000000E+00 0.0000000000000000E+00 1.376454726499085E+00 -3.246111302748730E-01 -1.376454726499085E+00
-3.246111302748730E-01
# 1.863667555432868E+01 -3.562491121497572E+00 -9.077135267012881E+00 6.153194387677511E+00 -1.970622777463714E+01 -1.717507312227297E+00 -6.433090024792207E+00
-6.899515402964241E+00
# -6.580432295368123E+00 -2.321633716694498E-01 2.264652765353805E+01 1.423921666814779E+01 -2.316151832172334E+01 1.257559440674843E+01 4.439749203374159E+00
6.883084353093276E+00
# -5.059138841333641E-01 -1.133454593457765E+00 1.833599061114253E+01 -4.015116252979888E+00 1.833599061114253E+01 4.015116252979888E+00 5.059138841333641E-01
-1.133454593457765E+00
# 2.672004287479594E+00 1.755067698199695E+01 2.615285048432793E+00 -6.256029470621641E+00 -2.615285048432793E+00 -6.256029470621641E+00 2.672004287479594E+00
-1.755067698199695E+01
# pdf 3.605966723564206E-02 1.350916463377768E-01
</event>
```

# HOW HELAC-NLO WORKS-VIRTUAL

Do this sum by MC (sample a configuration  $\{i\} = 1, 2, 3$   $\{j\} = 1, 2, 3$ )

$$\sum_{\{i\}, \{j\}} |\mathcal{M}_{j_1 j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}|^2$$

Express in terms of color connections  $A_\sigma$

$$\mathcal{M}_{j_1 j_2, \dots, j_k}^{i_1, i_2, \dots, i_k} = \sum_{\sigma} \delta_{i_{\sigma_1} j_1} \delta_{i_{\sigma_2} j_2} \dots \delta_{i_{\sigma_k} j_k} A_\sigma$$

Very significant reduction in CPU-time

Process	$n_{conn}$	$\langle n_{conn} \rangle_{MC}$	Ratio
$gg \rightarrow b\bar{b} W^+ W^-$	6	1.74	3.5
$gg \rightarrow t\bar{t} b\bar{b}$	24	3.04	7.9
$gg \rightarrow t\bar{t} gg$	120	6.27	19.1



# HOW HELAC-NLO WORKS-VIRTUAL

Generate  $w = 1$  events (Les Houches format) using HELAC at tree order. Information included: LH + color assignment, helicity. Optimization!

Calculate using HELAC-1L virtual part for each  $w = 1$  event. Produce a new LH file including virtual corrections. Includes UV renormlization

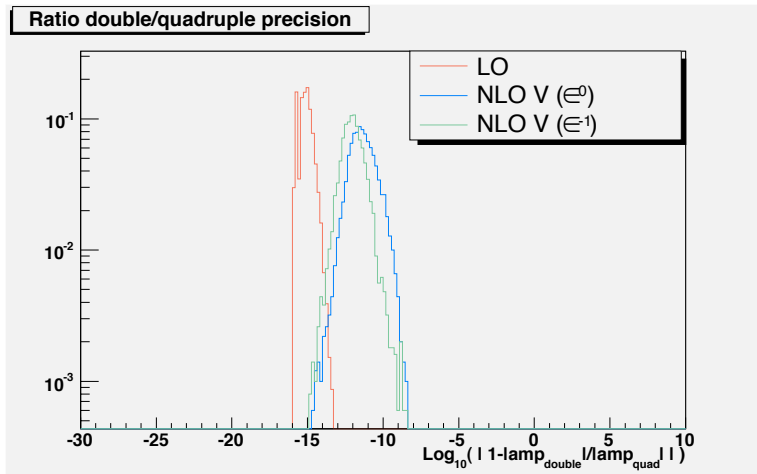
# HOW HELAC-NLO WORKS-VIRTUAL

Generate  $w = 1$  events (Les Houches format) using HELAC at tree order. Information included: LH + color assignment, helicity. Optimization!

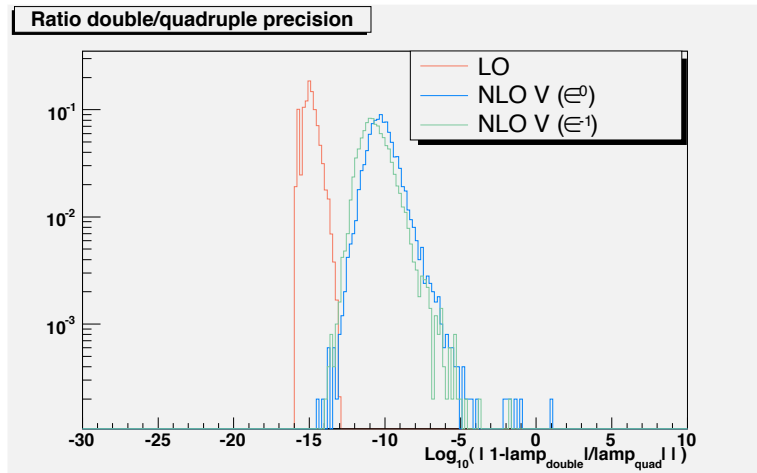
Calculate using HELAC-1L virtual part for each  $w = 1$  event. Produce a new LH file including virtual corrections. Includes UV renormlization

The final LH file can now be used to produce any kinematical distribution !

# HOW HELAC-NLO WORKS - STABILITY



# HOW HELAC-NLO WORKS - STABILITY



# HOW HELAC-NLO WORKS-REAL

## HELAC-DIPOLES

Generate CS Dipoles and calculate  $R - D$ , jet-algorithm, histograms

# HOW HELAC-NLO WORKS-REAL

## HELAC-DIPOLES

Generate CS Dipoles and calculate  $R - D$ , jet-algorithm, histograms

Calculate  $I$  operator contributions, histograms

# HOW HELAC-NLO WORKS-REAL

## HELAC-DIPOLES

Generate CS Dipoles and calculate  $R - D$ , jet-algorithm, histograms

Calculate  $I$  operator contributions, histograms

Calculate  $KP$  operator contributions, histograms

T. Binoth, G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0806** (2008) 082 [arXiv:0804.0350 [hep-ph]].

Process	scale $\mu$	Born cross section [fb]	NLO cross section [fb]
ZZZ	$3M_Z$	9.7(1)	15.3(1)
WZZ	$2M_Z + M_W$	20.2(1)	40.4(2)
WWZ	$M_Z + 2M_W$	96.8(6)	181.7(8)
WWW	$3M_W$	82.5(5)	146.2(6)

**Table 1:** Cross section for the four processes, corresponding to the distributions in Fig 4. Different values of the factorization(renormalization) scale are used for the different processes.



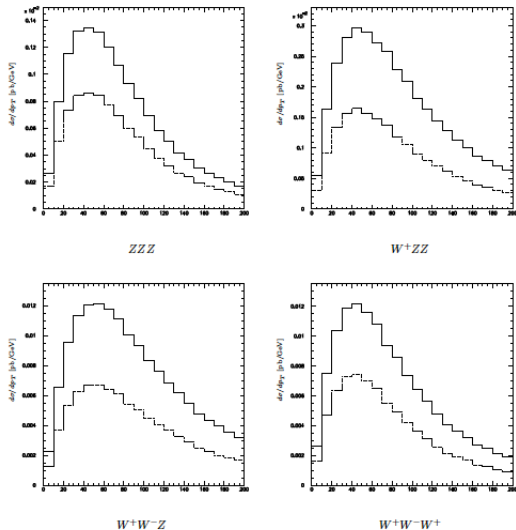


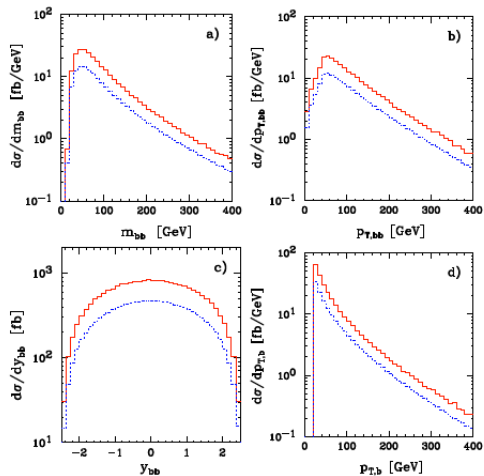
Figure 4: Transverse momentum distribution, as defined in the text, for the four processes  $pp \rightarrow VVV$ : NLO (solid line) compared with the LO contribution (dashed line).

Setup:  $\sqrt{s} = 14$  TeV,  $p_{T,b} > 20$  GeV,  $\Delta R > 0.8$ ,  $|y_b| < 2.5$  $\mu_R = \mu_F = m_t$ 

Process	$\sigma_{[23,24]}^{\text{LO}}$ [fb]	$\sigma^{\text{LO}}$ [fb]	$\sigma_{[23,24]}^{\text{NLO}}$ [fb]	$\sigma_{\alpha_{\text{max}}=1}^{\text{NLO}}$ [fb]	$\sigma_{\alpha_{\text{max}}=0.01}^{\text{NLO}}$ [fb]
$q\bar{q} \rightarrow t\bar{t}b\bar{b}$	85.522(26)	85.489(46)	87.698(56)	87.545(91)	87.581(134)
$pp \rightarrow t\bar{t}b\bar{b}$	1488.8(1.2)	1489.2(0.9)	2638(6)	2642(3)	2636(3)

**Table 1:** Cross sections for  $pp \rightarrow t\bar{t}b\bar{b} + X$  at the LHC at LO and NLO for the scale choice  $\mu_F = \mu_R = m_t$ , in comparison with the results of Refs. [23, 24]. The statistical errors are quoted in parentheses.

$$K = 1.77$$



**Figure 3:** Distribution of the invariant mass  $m_{bb}$  of the bottom-anti-bottom pair (a), distribution in the transverse momentum  $p_{T,bb}$  of the bottom-anti-bottom pair (b), distribution in the rapidity  $y_{bb}$  of the bottom-anti-bottom pair (c) and distribution in the transverse momentum  $p_{T,b}$  of the bottom quark (d) for  $pp \rightarrow t\bar{t}b\bar{b} + X$  at the LHC at LO (blue dashed line) and NLO (red solid line). All distributions have been obtained with  $\alpha_{max} = 0.01$ .

$$pp \rightarrow t\bar{t}(H \rightarrow)b\bar{b}$$

"The SM and NLO multileg working group: Summary report," arXiv:1003.1241 [hep-ph].

With realistic cuts on  $b\bar{b}$

$$\mu_R = \mu_F = m_t + m_H/2$$

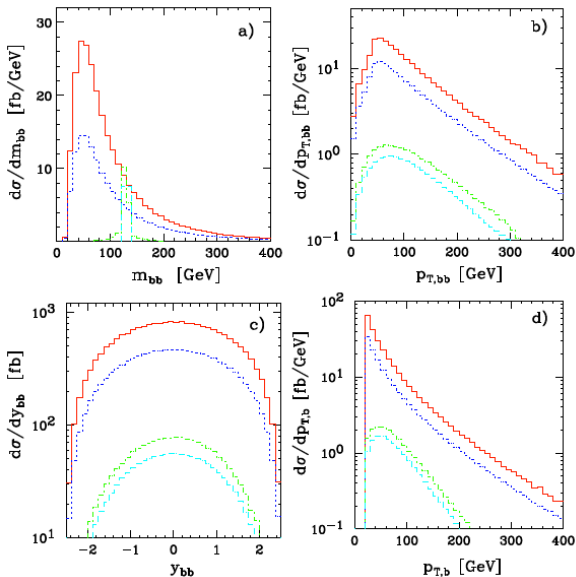
$$\sigma_{\text{LO}}^{\text{S}} = (150.375 \pm 0.077) \text{ fb.}$$

At the NLO we obtain

$$\sigma_{\text{NLO}}^{\text{S}} = (207.473 \pm 0.232) \text{ fb} \quad \text{for } \alpha_{\text{max}} = 0.01,$$

$$\sigma_{\text{NLO}}^{\text{S}} = (207.268 \pm 0.150) \text{ fb} \quad \text{for } \alpha_{\text{max}} = 1$$

$$K = 1.38$$

$pp \rightarrow t\bar{t}(H \rightarrow) b\bar{b}$ 

G. Bevilacqua, M. Czakon, C. G. Papadopoulos and M. Worek, arXiv:1002.4009 [hep-ph].

Setup:  $\sqrt{s} = 14 \text{ TeV}$ ,  $p_{T,j} > 20 \text{ GeV}$ ,  $\Delta R > 1$ ,  $|y_j| < 4.5$

$$\sigma_{t\bar{t}jj}^{NLO} = (106.94 \pm 0.17) \text{ pb} \quad K = 0.89$$

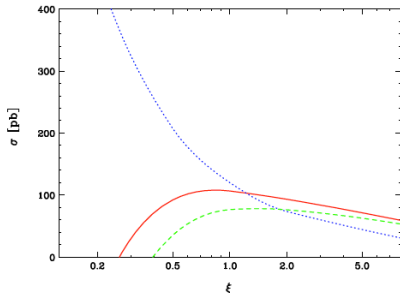
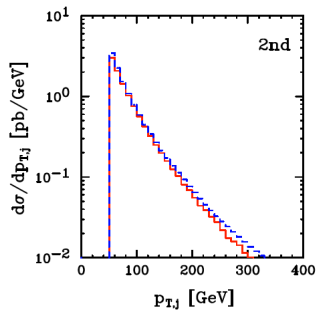
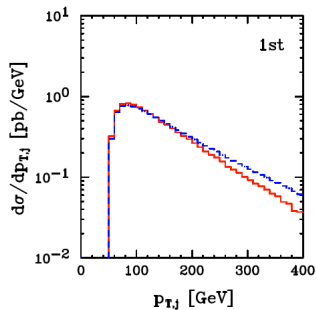


FIG. 2: Scale dependence of the total cross section for  $pp \rightarrow t\bar{t}jj + X$  at the LHC with  $\mu_R = \mu_F = \xi \cdot \mu_0$  where  $\mu_0 = m_t$ . The blue dotted curve corresponds to the LO, the red solid to the NLO result whereas the green dashed to the NLO result with a jet veto of 50 GeV.

# $pp \rightarrow t\bar{t} + 2 \text{ JETS}$



- Complete software for NLO-QCD at LHC:  
LO (lhc) + Virtual(lhc) + Real(lhc ?)  
lhc=Les Houches Compatible

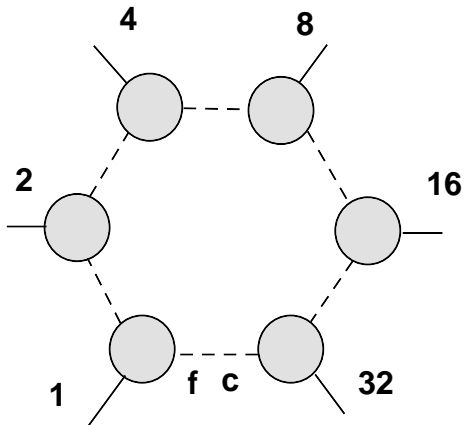


- Complete software for NLO-QCD at LHC:  
LO (lhc)+ Virtual(lhc) + Real(lhc ?)  
lhc=Les Houches Compatible
- Speed, stability, efficiency issues under control  
Improvements in PS for real corrections

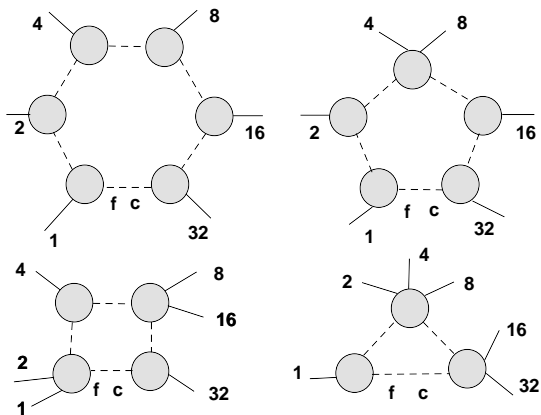
- Complete software for NLO-QCD at LHC:  
LO (lhc)+ Virtual(lhc) + Real(lhc ?)  
lhc=Les Houches Compatible
- Speed, stability, efficiency issues under control  
Improvements in PS for real corrections
- Provide NLO calculator for all processes  $2 \rightarrow n$ , where 4(5) out of  $n$ ,  
i.e. 6(7) particles attached to the loop  
MCFM-type approach ?

For those who are interested in more details

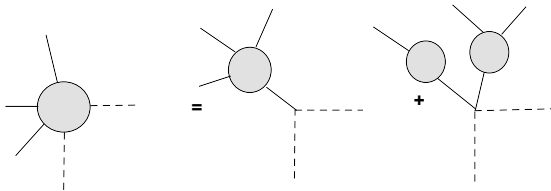
# HELAC 1-LOOP



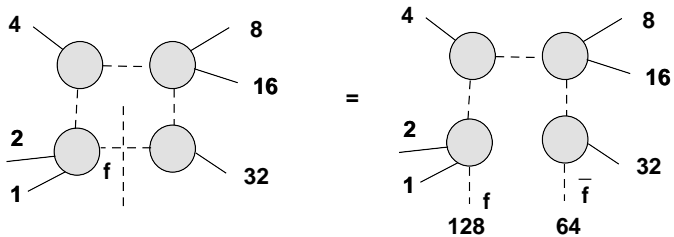
# HELAC 1-LOOP



# HELAC 1-LOOP



# HELAC 1-LOOP



$$\mathcal{M}_{j_2, \dots, j_k}^{a_1, i_2, \dots, i_k} t_{i_1 j_1}^{a_1} \rightarrow \mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}$$

$$\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k} = \sum_{\sigma} \delta_{i_{\sigma_1} j_1} \delta_{i_{\sigma_2} j_2} \dots \delta_{i_{\sigma_k} j_k} A_{\sigma}$$

$$\sum_{\{i\}, \{j\}} |\mathcal{M}_{j_1, j_2, \dots, j_k}^{i_1, i_2, \dots, i_k}|^2$$

$$\sum_{\sigma, \sigma'} A_{\sigma}^* C_{\sigma, \sigma'} A_{\sigma'}$$

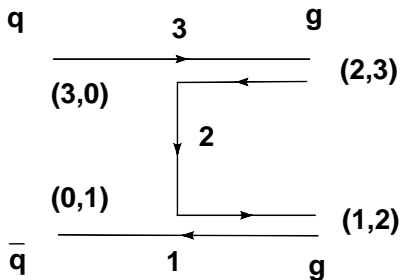
$$C_{\sigma, \sigma'} \equiv \sum_{\{i\}, \{j\}} \delta_{i_{\sigma_1} j_1} \delta_{i_{\sigma_2} j_2} \dots \delta_{i_{\sigma_k} j_k} \delta_{i_{\sigma'_1} j_1} \delta_{i_{\sigma'_2} j_2} \dots \delta_{i_{\sigma'_k} j_k}$$



# HELAC COLOR TREATMENT

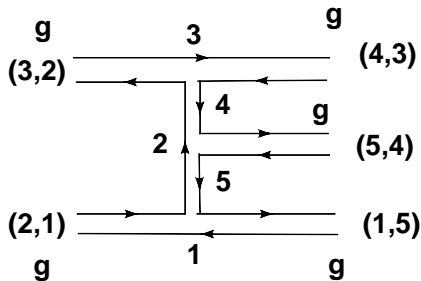
$$(x_1, y_1) \dots (x_n, y_n)$$

where  $y_i$  take the values  $\{1, 2, \dots, n_l\}$  if  $i$  is a gluon or an outgoing quark (incoming anti-quark) otherwise  $y_i = 0$ , whereas  $x_i$  take the values  $\{\sigma_1, \sigma_2, \dots, \sigma_{n_l}\}$  if  $i$  is a gluon or an incoming quark (outgoing anti-quark) otherwise  $x_i = 0$ . So for instance for a  $q\bar{q} \rightarrow gg$  process,  $n_l = 3$  and a possible color connection is given by  $(3, 0)(0, 1)(1, 2)(2, 3)$



# HELAC COLOR TREATMENT

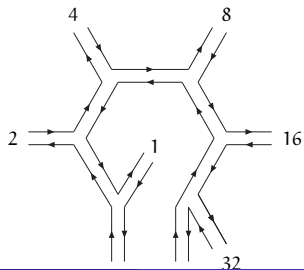
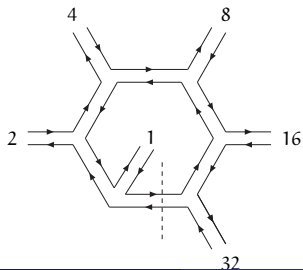
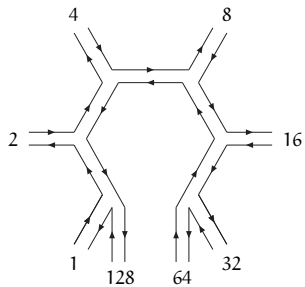
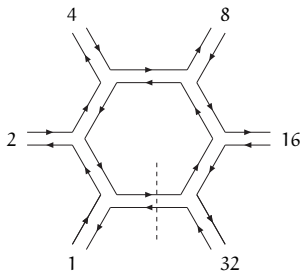
whereas for  $gg \rightarrow ggg$ ,  $n_l = 5$  and a possible color connection is given by  $(2, 1)(3, 2)(4, 3)(5, 4)(1, 5)$



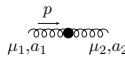
$$C_{\sigma, \sigma'} = N_c^{m(\sigma, \sigma')}$$

where  $m(\sigma, \sigma')$  count the number of common cycles of the two permutations.

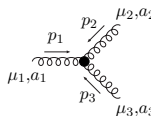
# HELAC COLOR TREATMENT - 1 LOOP



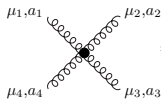
# HELAC R2 TERMS



$$= \frac{ig^2 N_{col}}{48\pi^2} \delta_{a_1 a_2} \left[ \frac{p^2}{2} g_{\mu_1 \mu_2} + \lambda_{HV} \left( g_{\mu_1 \mu_2} p^2 - p_{\mu_1} p_{\mu_2} \right) + \frac{N_f}{N_{col}} (p^2 - 6 m_q^2) g_{\mu_1 \mu_2} \right]$$

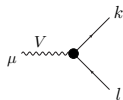


$$= -\frac{g^3 N_{col}}{48\pi^2} \left( \frac{7}{4} + \lambda_{HV} + 2 \frac{N_f}{N_{col}} \right) f^{a_1 a_2 a_3} V_{\mu_1 \mu_2 \mu_3} (p_1, p_2, p_3)$$

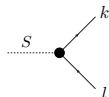


$$= -\frac{ig^4 N_{col}}{96\pi^2} \sum_{P(234)} \left\{ \left[ \frac{\delta_{a_1 a_2} \delta_{a_3 a_4} + \delta_{a_1 a_3} \delta_{a_4 a_2} + \delta_{a_1 a_4} \delta_{a_2 a_3}}{N_{col}} + 4 Tr(t^{a_1} t^{a_3} t^{a_2} t^{a_4} + t^{a_1} t^{a_4} t^{a_2} t^{a_3}) (3 + \lambda_{HV}) - Tr(\{t^{a_1} t^{a_2}\} \{t^{a_3} t^{a_4}\}) (5 + 2\lambda_{HV}) \right] g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} + 12 \frac{N_f}{N_{col}} Tr(t^{a_1} t^{a_2} t^{a_3} t^{a_4}) \left( \frac{5}{3} g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} - g_{\mu_2 \mu_3} g_{\mu_1 \mu_4} \right) \right\}$$

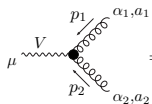
# HELAC R2 TERMS



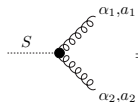
$$= -\frac{g^2}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl} \gamma_\mu (v + a\gamma_5) (1 + \lambda_{HV})$$



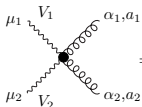
$$= -\frac{g^2}{8\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl} (c + d\gamma_5) (1 + \lambda_{HV})$$



$$= a \frac{ig^2}{12\pi^2} \delta_{a_1 a_2} \epsilon_{\mu\alpha_1\alpha_2\beta} (p_1 - p_2)^\beta$$



$$= c \frac{g^2}{8\pi^2} \delta_{a_1 a_2} g_{\alpha_1 \alpha_2} m_q$$



$$= -\frac{ig^2}{24\pi^2} \delta_{a_1 a_2} (v_1 v_2 + a_1 a_2) (g_{\mu_1 \mu_2} g_{\alpha_1 \alpha_2} + g_{\mu_1 \alpha_1} g_{\mu_2 \alpha_2} + g_{\mu_1 \alpha_2} g_{\mu_2 \alpha_1})$$

# HELAC 1-LOOP

```

=====
INFO
INFO COLOR 1 out of 6
INFO 1 12 35 7 1 1 4 -4 3 8 4 4 0 0 0 0 1 2
INFO 1 48 35 8 1 1 16 -8 5 32 8 6 0 0 0 0 1 2
INFO 2 14 -3 9 1 1 12 35 7 2 -3 2 0 0 0 0 1 2
INFO 2 14 -3 9 0 1 12 35 7 2 -3 2 0 0 0 0 2 1 2
INFO 2 28 -8 10 1 1 12 35 7 16 -8 5 0 0 0 0 1 1 2
INFO 2 28 -8 10 0 1 12 35 7 16 -8 5 0 0 0 0 2 1 2
INFO 3 44 8 11 1 1 12 35 7 32 8 6 0 0 0 0 1 1 2
INFO 3 44 8 11 0 1 12 35 7 32 8 6 0 0 0 0 2 1 2
INFO 2 50 -3 12 1 1 48 35 8 2 -3 2 0 0 0 0 1 1 2
INFO 2 50 -3 12 0 1 48 35 8 2 -3 2 0 0 0 0 2 1 2
INFO 2 52 -4 13 1 1 48 35 8 4 -4 3 0 0 0 0 1 1 2
INFO 2 52 -4 13 0 1 48 35 8 4 -4 3 0 0 0 0 2 1 2
INFO 3 56 4 14 1 1 48 35 8 8 4 4 0 0 0 0 1 1 2
INFO 3 56 4 14 0 1 48 35 8 8 4 4 0 0 0 0 2 1 2
INFO 1 60 35 15 1 4 4 -4 3 56 4 14 0 0 0 0 0 1 2
INFO 1 60 35 15 2 4 16 -8 5 44 8 11 0 0 0 0 0 1 2
INFO 1 60 35 15 3 4 28 -8 10 32 8 6 0 0 0 0 0 1 2
INFO 1 60 35 15 4 4 52 -4 13 8 4 4 0 0 0 0 0 1 2
INFO 2 62 -3 16 1 3 12 35 7 50 -3 12 0 0 0 0 1 1 2
INFO 2 62 -3 16 0 3 12 35 7 50 -3 12 0 0 0 0 2 1 2
INFO 2 62 -3 16 2 3 48 35 8 14 -3 9 0 0 0 0 1 1 2
INFO 2 62 -3 16 0 3 48 35 8 14 -3 9 0 0 0 0 2 1 2
INFO 2 62 -3 16 3 3 60 35 15 2 -3 2 0 0 0 0 1 1 2
INFO 2 62 -3 16 0 3 60 35 15 2 -3 2 0 0 0 0 2 1 2
=====
INFO
INFO COLOR 2 out of 6
INFO 1 12 35 7 1 1 4 -4 3 8 4 4 0 0 0 0 1 1
INFO 1 48 35 8 1 1 16 -8 5 32 8 6 0 0 0 0 1 1

```

# HELAC 1-LOOP

```
papadopo@aiolos:/tmp - Shell - Konsole
INFO =====
INFO COLOR 4 out of 6
INFO number of nums 143
INFO NUM 1 of 143 10
INFO 3 96 8 9 1 1 64 35 7 32 8 6 0 0 0 1 1 2
INFO 3 96 8 9 0 1 64 35 7 32 8 6 0 0 0 2 1 2
INFO 1 112 35 10 1 1 16 -8 5 96 8 9 0 0 0 0 1 1
INFO 3 120 4 11 1 1 112 35 10 8 4 4 0 0 0 1 1 1
INFO 3 120 4 11 0 1 112 35 10 8 4 4 0 0 0 2 1 1
INFO 1 124 35 12 1 1 4 -4 3 120 4 11 0 0 0 0 1 1
INFO 2 126 -3 13 1 1 124 35 12 2 -3 2 0 0 0 1 1 1
INFO 2 126 -3 13 0 1 124 35 12 2 -3 2 0 0 0 2 1 1
INFO 2 254 -3 14 1 1 128 35 8 126 -3 13 0 0 0 1 1 2
INFO 2 254 -3 14 0 1 128 35 8 126 -3 13 0 0 0 2 1 2
INFO 6 32 16 8 4 2 1 35 8 35 4 35 -3 0 0 0 3 1
INFOFY 1
INFO NUM 2 of 143 10
INFO 3 96 8 9 1 1 64 35 7 32 8 6 0 0 0 1 1 1
INFO 3 96 8 9 0 1 64 35 7 32 8 6 0 0 0 2 1 1
INFO 1 112 35 10 1 1 16 -8 5 96 8 9 0 0 0 0 1 1
INFO 3 120 4 11 1 1 112 35 10 8 4 4 0 0 0 1 1 1
INFO 3 120 4 11 0 1 112 35 10 8 4 4 0 0 0 2 1 1
INFO 1 124 35 12 1 1 4 -4 3 120 4 11 0 0 0 0 1 2
INFO 2 126 -3 13 1 1 124 35 12 2 -3 2 0 0 0 1 1 2
INFO 2 126 -3 13 0 1 124 35 12 2 -3 2 0 0 0 2 1 2
INFO 2 254 -3 14 1 1 128 35 8 126 -3 13 0 0 0 1 1 1
INFO 2 254 -3 14 0 1 128 35 8 126 -3 13 0 0 0 2 1 1
INFO 6 32 16 8 4 2 1 35 8 35 4 35 -3 0 0 0 3 1
INFOFY 1
INFO NUM 3 of 143 10
INFO 2 80 -8 9 1 1 64 35 7 16 -8 5 0 0 0 1 1 2
```

# HELAC 1-LOOP

```
papadopo@aiolos:/tmp - Shell - Konsole
INFO NUM 127 of 143 15
INFO 1 48 35 9 1 1 16 -8 5 32 8 6 0 0 0 0 1 1
INFO 3 112 3 10 1 1 48 35 9 64 3 7 0 0 0 0 1 1 1
INFO 3 112 3 10 0 1 48 35 9 64 3 7 0 0 0 0 2 1 1
INFO 1 12 35 11 1 1 4 -4 3 8 4 4 0 0 0 0 0 1 1
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