The GOLEM Project

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in collaboration with
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MC4LHC readiness, 29 March – 01 April 2010

Overview

The GOLEM Method

golem95 One-Loop Integral Library

Golem-2.0 Virtual Matrix Element Generator

Results

GOLEM Readiness

The GOLEM Method: Overview

GOLEM: General One-Loop Evaluator for Matrix Elements

- GOLEM = a method for evaluating one-loop Feynman diagrams
- GOLEM = a library for one-loop integrals (golem95)
- GOLEM = a matrix element generator at the one-loop level

Why Feynman Diagrams?

- No distinction between cut-constructible and rational part
  $\Rightarrow$ conceptually simple
- Gram determinant problem avoidable by dedicated tensor reduction ($\Rightarrow$ golem95)
- Combinatorial complexity of Feynman diagrams
  $\Rightarrow$ problematic only beyond $2 \to 4$

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The GOLEM Method: Overview

The GOLEM method uses

- Feynman diagrams
- Helicity projections
- Improved tensor reduction

The GOLEM method is designed for

- any number of ext. particles ($\lesssim 6$ feasable)
- massless and massive particles
- QCD and EW corrections
- physics within and beyond the Standard Model

The GOLEM method is aiming at

- NLO "Plug In" for MC generators
  $\rightarrow$ see also Rikkert's talk

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The GOLEM Method: Implementation

$$\mathcal{A}^{\{\lambda\}}(\{p_j\};\{m_j\}) = \sum_{\{c_i\},\alpha} f^{\{c_i\}}\mathcal{G}_\alpha^{\{\lambda\}}(\{p_j\};\{m_j\})$$

$$\mathcal{G}_\alpha^{\{\lambda\}}(\{p_j\};\{m_j\}) = \int \frac{\mathrm{d}^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}(k)}{D_1\cdots D_N}$$
$$= \sum_r \mathcal{N}^{\{\lambda\}}_{\mu_1\ldots\mu_r}(\{p_j\};\{m_j\}) \cdot I_N^{n\,\mu_1\ldots\mu_r}(\{p_j\};\{m_j\})$$

Idea: Split implementation into two steps

1. Numerically stable reduction of tensor integrals
2. Matrix element generator for one-loop amplitudes

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$$I_N^{n\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S)=\int\frac{\mathrm{d}^nk}{i\pi^{n/2}}\frac{q_{a_1}^{\mu_1}\cdots q_{a_r}^{\mu_r}}{\prod_{j\in S}(q_j^2-m_j^2+i\delta)}$$

$$S_{ij}=(q_i-q_j)^2-m_i^2-m_j^2,\quad q_j=k+r_j$$

Decomposition into Lorentz invariant integrals:

$$I_N^{n\,\mu_1\ldots\mu_r}=(-1)^N\Gamma(N-n/2)\sum_{p,j_1,\ldots,j_p}T^{\mu_1\cdots\mu_r}(\{r_j\},g^{\cdot\cdot})\times$$
$$\int\mathrm{d}z_1\cdots\mathrm{d}z_N\delta(1-z_1-\ldots z_N)\frac{z_{l_1}\ldots z_{l_p}}{(-1/2z^TSz-i\delta)^{N-n/2}}$$

- Can be reduced further $\to I_{N-1}^n+I_N^{n+2}$
- Can be evaluated numerically (degenerated kinematics)

$\Rightarrow$ Gram determinants can be avoided

golem95 One-Loop Integral Library

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Reminder: PV reduction and Gram determinants

$$\int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{p \cdot k}{(k+r_1)^2(k+r_2)^2(k+r_3)^2 k^2}$$

- If $\{r_1, r_2, r_3\}$ linearly independent:

$$p^\mu = \alpha_1 r_1^\mu + \alpha_2 r_2^\mu + \alpha_3 r_3^\mu + \alpha_\perp \epsilon^{\mu\nu\rho\sigma} r_{1\nu} r_{2\rho} r_{3\sigma}$$

- Since $2r_i \cdot k = (k+r_i)^2 - k^2 - r_i^2$
  $\Rightarrow$ decomposition into scalar integrals
- Need to solve: (which introduces Gram determinant)

$$\left(\begin{array}{ccc|c} r_1 \cdot r_1 & r_1 \cdot r_2 & r_1 \cdot r_3 & 0 \\ r_2 \cdot r_1 & r_2 \cdot r_2 & r_2 \cdot r_3 & 0 \\ r_3 \cdot r_1 & r_3 \cdot r_2 & r_3 \cdot r_3 & 0 \\ \hline 0 & 0 & 0 & \det G \end{array}\right) \cdot \left(\begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \hline \alpha_\perp \end{array}\right) = \left(\begin{array}{c} p \cdot r_1 \\ p \cdot r_2 \\ p \cdot r_3 \\ \hline \epsilon^{p r_1 r_2 r_3} \end{array}\right)$$

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golem95 One-Loop Integral Library

Current version of golem95

- http://lappweb.in2p3.fr/lapth/Golem/golem95.html
- algebraic separation of IR poles
- cache avoiding multiple evaluation
- all required integrals for $N \leq 6$, massless
- documentation, examples available

Under development

- version with propagator masses
- currently: finite box (D_0) by call to LoopTools [T. Hahn]

Early stage of development

- Complex propagator masses

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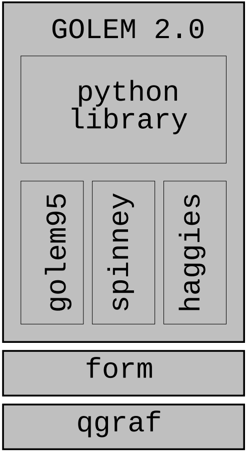
GOLEM-2.0: One-Loop Matrix Element Generator

Overview

- implementation of the Golem method
- very modular
  - python library (command line tools)
  - spinney: helicity spinors in Form
  - haggies: optimizing code generator
  - golem95: integral library
- based on Form and QGraf [Vermaseren;Nogueira]
- Fortran 95 matrix element code



- create configuration file
- enter process, here: $gg \longrightarrow s\bar{s}b\bar{b}$ @ NLO in QCD
- set up process directory
- generate code and draw diagrams

shell

```
$ golem-main.py --template process.in
$
```

GOLEM-2.0: Matrix Elements Made Easy

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- enter process, here: $gg \longrightarrow s\bar{s}b\bar{b}$ @ NLO in QCD
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editor: process.in

```
process_path=<a directory>
in=g,g
out=s,s~,b,b~
order=gs,4,6
model=sm

# more settings optional
...
```

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$ make dist # -> matrix.tar.gz
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```

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$ golem-main.py process.in
$ make dist # -> matrix.tar.gz
$ make doc # -> process.ps
$
```

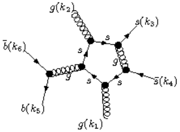


Diagram 1109

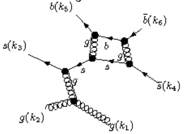


Diagram 1111

golem-2.0: Work in Progress

Features not fully implemented (but planned/in progress):

- Les Houches interface
- FeynRules import [C. Duhr]
- Renormalisation of massive theories

Implemented but not fully tested:

- Majorana fermions and higher spins
- massive processes

+ improvements considering size, speed and interface

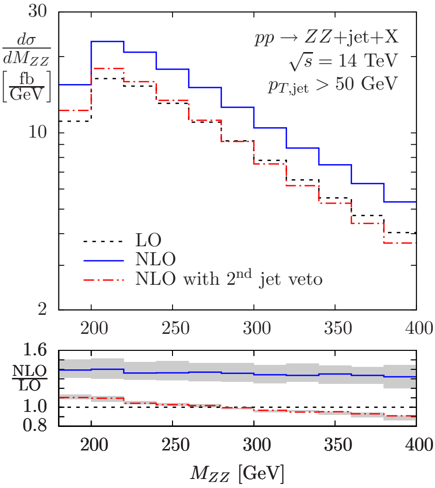
Some recent results

GOLEM method has been used for

- $gg \to W^* W^* \to l\nu l'\nu'$ [Binoth,Ciccolini,Kauer,Krämer]
- $gg \to HH, HHH$ [Binoth,Karg,Kauer,Rückl]
- $pp \to Hjj$ (VBF/GF) [Andersen,Binoth,Heinrich,Smillie]
- $pp \to VVj$ [Binoth,Gleisberg,Karg,Kauer,Sanguinetti]
- $q\bar{q} \to b\bar{b}b\bar{b}$ [Binoth,Greiner,Guffanti,Guillet,TR,Reuter]
- $pp \to \text{Graviton} + j$ [Karg et al.]
- $gg \to b\bar{b}b\bar{b}$ (in progress)
- ...

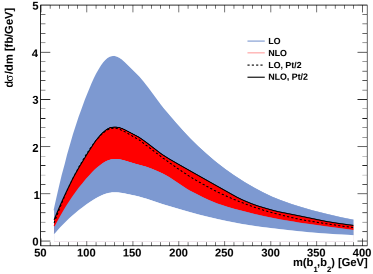
- high priority "wishlist" process
- algebraic reduction of tensor integrals
- at most pentagon diagrams
- successful comparison with

[Dittmaier,Kallweit,Uwer]



$q\bar{q} \rightarrow b\bar{b}b\bar{b}$ [Binoth,Greiner,Guffanti,Guillet,TR,Reuter]

- added to "wishlist" 2007
- background to BSM Higgs search
- calculation using `golem-2.0` and `golem95`
- $gg \rightarrow b\bar{b}b\bar{b}$ missing
  $\Rightarrow$ to be completed $\approx$ June



GOLEM Readiness

golem95

massless version

real propagator masses (due: May)

complex propagator masses

golem-2.0

massless processes

massive processes

LanHEP interface

Majorana fermions

FeynRules interface

Les Houches interface

Automatic renormalisation

- To appear in first release this summer

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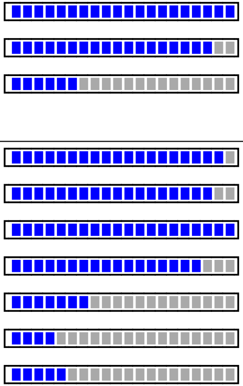

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