Curvature Perturbations From Stochastic Particle Production During Inflation

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Complexity in the early universe

**Inflation**
- Near scale invariant: $\Delta^2_\zeta \sim k^{n_s - 1}$
- Near Gaussian
- Weak self-interaction (slow roll)

**Particle theory**
- SM UV completions $N_F \gg 1$
- Coupling to $\phi$ weakly constrained
- Non-trivial field manifolds
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$$m_{\text{eff}}^2(t) = m_{\chi}^2 + g^2(\phi(t) - \phi_i) + \cdots$$

(trapped inflation)

(preheating)
Spectator field in dS

Spectator field in an expanding universe

\[
\left( \frac{d^2}{dt^2} - \frac{\nabla^2}{a^2} + 3H \frac{d}{dt} + M^2 + m^2(t) \right) \chi(t, x) = 0
\]

\[a = a_0 e^{H(t-t_0)} \]
(de Sitter)

\[M^2 = 2H^2 \]  
(conformal)

\[M^2 = 0 \]  
(massless)

\[m^2(t) = \sum_j m_j \delta(t - t_j) \]
(localized, non-adiabatic)
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\( k \frac{aH}{aH} = |k\tau| \leq 1 \) (horizon)

\[
X_k \equiv a \chi_k = \alpha_{k,j} f_k(\tau) + \beta_{k,j} f_k^*(\tau)
\]

\[
f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \times \left\{ \begin{array}{l}
1 \\
(1 - \frac{i}{k\tau})
\end{array} \right. 
\]

- \( m_j, t_j \) random
- \( \langle m_j \rangle = 0 \)
- \( \langle m_i m_j \rangle = \sigma^2 \delta_{ij} \)
- \( \langle N_s \rangle \frac{H(t_i - t_f)}{H(t_i - t_f)} = N_s \)
Fokker-Planck formalism

Numerical solution

\[ n = |\beta_j|^2 \ll 1 \quad \text{and} \quad n \gg 1 \]

\[ P(\ln|X_k|^2, t) \]
Fokker-Planck formalism

Numerical solution

$P(\ln|X_k|^2, t)$

$\ln|X_k|^2$

$m^2(t)$

$H(t - t_k)$

$M_j$

$(\beta_j \alpha_j) (\beta_{j+1} \alpha_{j+1})$
Fokker-Planck formalism

Numerical solution

\[ \mathbf{M}_j \mathbf{M}_{j-1} \cdots \mathbf{M}_1 = \mathbf{M}(j) \]

\[ \ln |X_k|^2 \]

\[ P(\ln |X_k|^2, t) \]

\[ m^2(t) \]

\[ H(t - t_k) \]
Fokker-Planck formalism

Numerical solution

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\[ m^2(t) \]

\[ \ln |X_k|^2 \]

\[ -20 -15 -10 -5 0 5 10 15 20 \]

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Numerical and analytical solution

\[ P(\mathbf{M}; t + \delta t) = \int d\mathbf{M}_j P(\mathbf{M}_j^{-1}\mathbf{M}; t) P(\mathbf{M}_j; \delta t) \]
1. Strength of non-adiabaticity is quantified by:
\[ N_s \frac{\text{Var}[m_j]}{H^2} \equiv N_s \left( \frac{\sigma}{H} \right)^2 \]

2. Distributions:

\[ |k\tau| \gg 1 \]

\[ |k\tau| \sim 1 \]

\[ |k\tau| \ll 1 \]
\( \text{Mean of } \ln |X_k|^2 \text{ grows linearly with time outside the horizon} \)
Mean of $\ln |X_k|^2$ grows linearly with time outside the horizon

$$\partial_{Ht} \langle \ln |X_k|^2 \rangle = \mu_1 \left( N_s \left( \frac{\sigma}{H} \right)^2 \right)$$
3 Mean of $\ln |X_k|^2$ grows linearly with time outside the horizon

$$\partial_{Ht} \langle \ln |X_k|^2 \rangle = \mu_1 \left( \mathcal{N}_s \left( \frac{\sigma}{H} \right)^2 \right)$$

4 The two-point function of $Z_k = \ln |X_k|^2 - \langle \ln |X_k|^2 \rangle$ is also linear
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4 The two-point function of $Z_k \equiv \ln |X_k|^2 - \langle \ln |X_k|^2 \rangle$ is also linear

$$\langle Z_k(t) Z_{k'}(t') \rangle = \mu_2 \left( \mathcal{N}_s \left( \frac{\sigma}{H} \right)^2 \right) H \min \left[ t - t_k, t - t_{k'}, t' - t_k, t' - t_{k'} \right]$$

$|X_k|^2$ performs a geometric (Brownian) random walk outside the horizon

$$\langle |X_{k_1}(t_1)|^2 \cdots |X_{k_n}(t_n)|^2 \rangle = \exp \left[ \sum_{i=1}^{n} \langle \ln |X_{k_i}(t_i)|^2 \rangle + \frac{1}{2} \sum_{i,j=1}^{n} \langle Z_{k_i}(t_i) Z_{k_j}(t_j) \rangle \right]$$
The quasi-de Sitter Goldstone $\pi$ couples to the spectator field $\chi$,

\[
S = \frac{1}{2} \int \sqrt{-g} \, d^4 x \left[ c(t + \pi) \partial_\mu \pi \partial^\mu \pi + \partial_\mu \chi \partial^\mu \chi - \left( M^2 + m^2 (t + \pi) \right) \chi^2 \right]
\]
The quasi-de Sitter Goldstone $\pi$ couples to the spectator field $\chi$,

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To lowest order in $\pi$, with $\zeta \simeq H\pi$

$$\delta \Delta^2_\zeta(k) = 4\pi^2(\Delta^2_\zeta)^2 \frac{k^3}{H^4} \int d\tau' d\tau'' \tau' \tau'' G_k(\tau, \tau') G_k(\tau, \tau'') \frac{dm^2(\tau')}{d\tau'} \frac{dm^2(\tau'')}{d\tau''} \times \int \frac{d^3p}{(2\pi)^3} \left[ X_p(\tau') X^*_p(\tau'') \right]_{AS} \left[ X_{|p-k|}(\tau') X^*_{|p-k|}(\tau'') \right]_{AS}$$
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$$\times \int \frac{d^3p}{(2\pi)^3} \left[ X_p(\tau') X_p^*(\tau'') \right]_{AS} \left[ X_{|p-k|}(\tau') X_{|p-k|}^*(\tau'') \right]_{AS}$$

$$\downarrow$$

$$\left\langle \delta \Delta_\zeta^2(k) \right\rangle = \left( \Delta_{\zeta,0}^2 \right)^2 N_s \left( \frac{\sigma}{H} \right)^2 e^\mathcal{F}(k, N_e, N_s(\sigma/H)^2)$$
Curvature power spectrum (conformal, \(N_e = 20\))

\[
\langle \delta \Delta^2 \rangle / \Delta^2 \approx N_s (\sigma/H)^2 = 3
\]

- Always super-horizon
- Cross horizon during scatterings
- Always sub-horizon
(conformal, \( N_e = 20, N_s (\sigma/H)^2 = 25 \))
Curvature power spectrum

(conformal, $N_e = 20, \mathcal{N}_s (\sigma / H)^2 = 25$)
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Curvature power spectrum

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Curvature power spectrum

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Conclusions

- Stochastically excited spectator fields undergo geometric random walks
- Lead to features in the curvature power spectrum $\rightarrow$ constraints
- Look for enhancement in the N-point function

$$\langle \zeta^n \rangle - \langle \zeta^n \rangle_{\chi=0} \sim \langle \zeta^2 \rangle_{\chi=0} \times \exp \left[ \frac{n^2}{2} F \left( N_s \frac{\sigma^2}{H^2} \right) \right]$$

- Higher spin spectators / higher spin observables
- Stochastic preheating
- Backreaction regime $\rightarrow$ dissipation

Thank You
Moment rates

\[ \partial_H \langle \ln |X|^2 \rangle + 2 \]

\[ \text{Var} [\ln |X|^2] \]

\[ N_s (\sigma/H)^2 \]

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Plenty of available parameter space

\[ N_{s}(\sigma/H)^2 \]

\[ <\rho_{\chi}> \]

\[ \rho_{\chi}^{\text{typ}} \]

\[ M^2 = 2H^2 \]