Clockwork Neutrinos

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Based on: S. Hong, G. Kurup, M. Perelstein [arXiv:1903.06191]

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Outline of this talk

- Motivation
- Overview of the clockwork mechanism for fermions
- Analytical results: Spectrum and couplings
- Constraints on the model
- Generalized Clockwork
- Collider Signatures
- Conclusion

Motivation: A Dirac Mass for the Neutrino

- Neutrino masses are very small compared to other fermion masses in the SM
- Most popular model for small neutrino masses so far: the **see-saw** mechanism, which has Majorana mass terms
- Can we design a model with *only* **Dirac** masses, and naturally give very small masses for the SM neutrino?
- The Clockwork mechanism provides one such solution

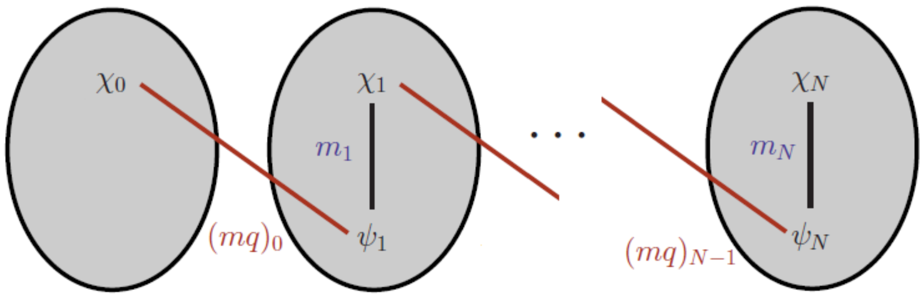
Clockwork Theory

G. Giudice, M. McCullough [1610.07962]

Possible to have hierarchically small neutrino masses from $O(1)$ Lagrangian parameters.

Theory with N+1 right-chiral & N left-chiral fermions + nearest-'neighbour' interactions:

$$\mathcal{L}_{\text{cw}} = \mathcal{L}_{\text{kin}} - m \sum_{i=1}^{N} \left(\psi_i^\dagger \chi_i - q\, \psi_i^\dagger \chi_{(i-1)} + \text{h.c.} \right) = \mathcal{L}_{\text{kin}} - \Psi^\dagger \tilde{M} X$$



$$\tilde{M}^\dagger \tilde{M} = m^2 \begin{pmatrix} q^2 & -q & 0 & \cdots & 0 & 0 \\ -q & 1+q^2 & -q & \cdots & 0 & 0 \\ 0 & -q & 1+q^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & 1 \end{pmatrix}$$

Clockwork Theory

Eigenvalues: $\boxed{m_0^2 = 0}, \quad m_i^2 = m^2\lambda_i = m^2\left(1 + q^2 - 2q\cos\frac{i\pi}{N+1}\right)$

One massless mode from remaining chiral symmetry: $U(1)_L^N \times U(1)_R^{N+1} \to U(1)$

Clockwork Theory

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One massless mode from remaining chiral symmetry: $U(1)_L^N \times U(1)_R^{N+1} \to U(1)$

Eigenvectors: Oscillatory, except for **exponentially decreasing** zero mode:

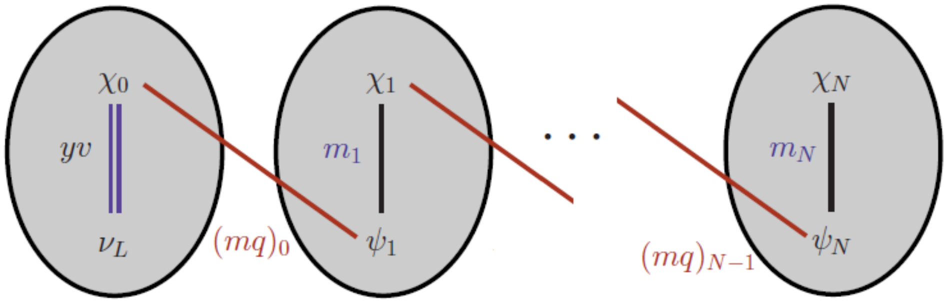
$$\boxed{(U_R)_{j0} = \frac{\mathcal{N}_0}{q^{N-j}}}, \; (U_R)_{jk} = \mathcal{N}_k\left[q\sin\frac{(N-j)k\pi}{N+1} - \sin\frac{(N-j+1)k\pi}{N+1}\right]$$

$$\mathcal{N}_0 \equiv \sqrt{\frac{q^2-1}{q^2-q^{-2N}}}, \; \mathcal{N}_k \equiv \sqrt{\frac{2}{(N+1)\lambda_k}}$$

Clockwork Theory + SM

Exponentially small coupling of zero mode to 0^{th} site $\propto \dfrac{1}{q^N} \Rightarrow$

Yukawa coupling to SM Higgs at 0^{th} site to generate small mass $\sim \dfrac{yv}{q^N}$



This gives very small neutrino masses! e.g. q = 3, N = 20, y = 0.01 $\Rightarrow m_\nu \sim 0.5\,\text{eV}$

$$M^\dagger M = m^2 \begin{pmatrix} q^2+p^2 & -q & 0 & \cdots & 0 & 0 \\ -q & 1+q^2 & -q & \cdots & 0 & 0 \\ 0 & -q & 1+q^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & 1 \end{pmatrix}, \quad p = \frac{y\ v}{\sqrt{2}m}$$

$\Rightarrow$ No known analytical solution to diagonalization problem

Large-N Expansion

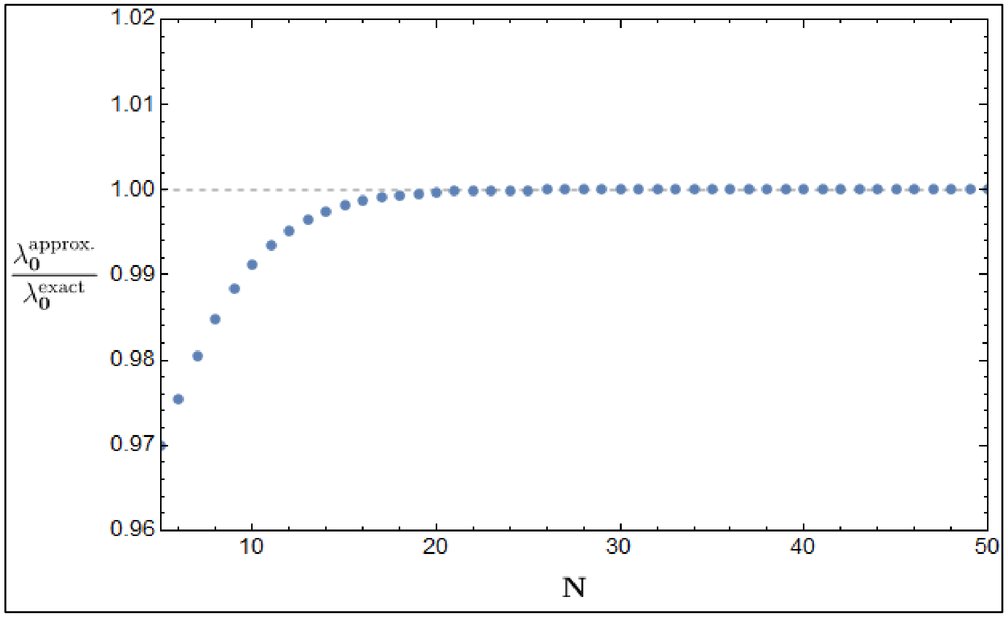
Expand characteristic equation in $\frac{p\,q}{N} = \frac{y\,v}{\sqrt{2}\,m}\frac{q}{N}$

Mass eigenstates: (α is the flavour index)

$$m_{0,\alpha} = m\frac{p_\alpha}{q^N}\left(\frac{q^2-1}{q^2-q^{-2N}}\right)^{1/2} \times \left(1 - \frac{p_\alpha^2}{2(N+1)}\sum_{k=1}^{N}\frac{C_k}{\lambda_k}\right),$$

$$m_{k,\alpha} = m\lambda_k^{1/2}\left(1 + \frac{p_\alpha^2}{2(N+1)}\frac{C_k}{\lambda_k}\right),$$

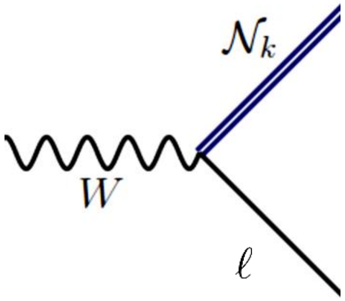
$$C_k = \frac{2q^2}{\lambda_k}\sin^2\frac{Nk\pi}{N+1}, \quad k = 1\ldots N$$



Large-N Expansion

Charged current:

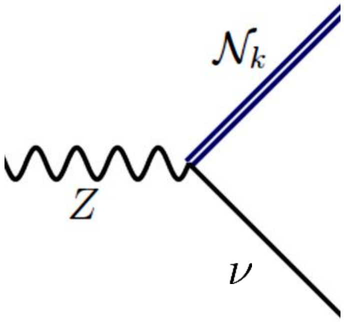
$$J_W^{\mu +} = \frac{V_{\alpha\beta}}{\sqrt{2}} \bar{e}_{L\alpha} \gamma^\mu P_L \left(\kappa_{0\beta} \mathcal{N}_{0\beta} + \sum_{j=1}^{N} \kappa_{j\beta} \mathcal{N}_{j\beta} \right) + \mathcal{O}(p^3)\,,$$



where ,

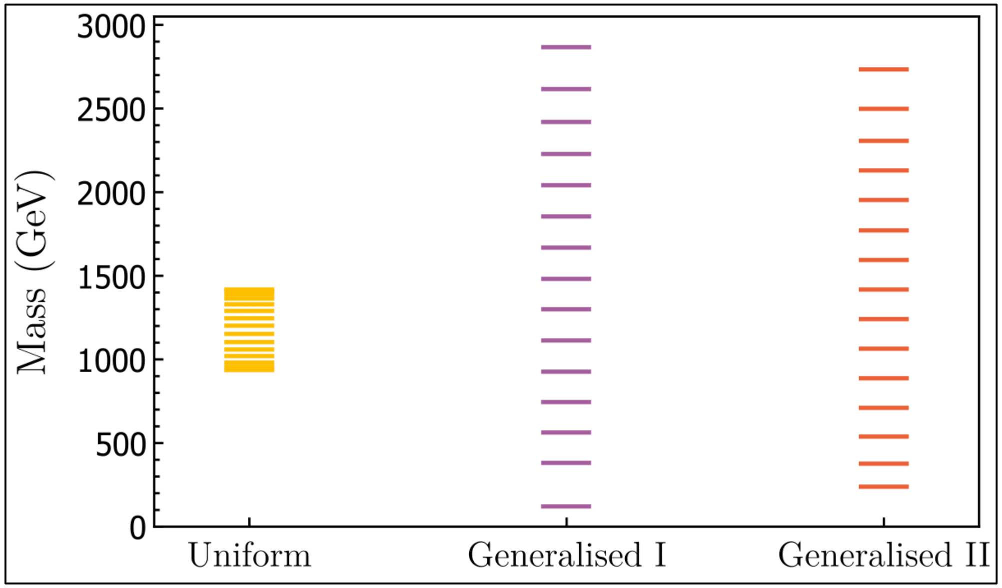
$$\kappa_{0\beta} = 1 - \frac{p_\beta^2}{N+1} \sum_{k=1}^{N} \frac{C_k}{2\lambda_k}$$

$$\kappa_{j\beta} = -p_\beta \sqrt{\frac{1}{N+1} \frac{C_j}{\lambda_j}}\,, j = 1 \ldots N$$



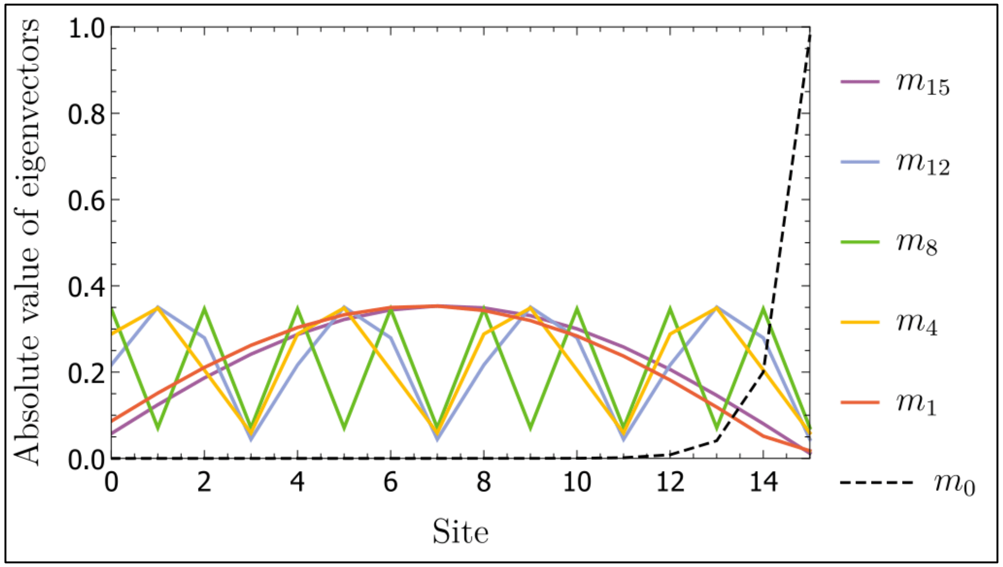
Similar corrections to neutral current.

Eigenmodes and Mass Spectrum



Mass spectrum of N=15 Clockwork theories which give appropriate neutrino masses

Eigenmodes of Uniform Clockwork theories are oscillatory!



Constraints on the model

i. Masses of the SM neutrinos

Constraints on the model

i. Masses of the SM neutrinos:

- Sum of masses from CMB data, $\sum_{i=1,2,3} m_{\nu_i} \lesssim 0.20 \text{ eV}$
- Mass squared differences from oscillation data
- For this talk, normal hierarchy with a nearly degenerate spectrum is chosen
  - (Constraints are similar for other spectra)

Constraints on the model

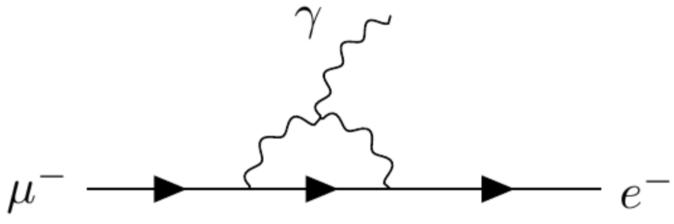
i. Masses of the SM neutrinos

ii. Lepton Flavour Violation

Constraints on the model

i. Masses of the SM neutrinos

ii. Lepton Flavour Violation: Strongest bound from $\mu \to e\gamma$ process:



$\Rightarrow$ Branching ratio constrained to be $< 4.2 \times 10^{-13}$

Minimal Flavour Violation }

$$BR(\mu \to e\gamma) = \frac{3\alpha_{em}}{8\pi} \left| \sum_{\alpha=1}^{3} \sum_{j=0}^{N} V_{\mu\alpha} V_{e\alpha}^{*} |(U_{L\alpha})^{0j}|^2 F\left(\frac{m_{j,\alpha}^2}{m_W^2}\right) \right|^2 \sim \frac{y^4 v^4}{m^4}$$

T. P. Cheng, L.-F. Li - Phys. Rev. Lett. 45 (1980)

Constraints on the model

i. Masses of the SM neutrinos

ii. Lepton Flavour Violation

iii. Precision Electroweak Constraints

Constraints on the model

i. Masses of the SM neutrinos

ii. Lepton Flavour Violation

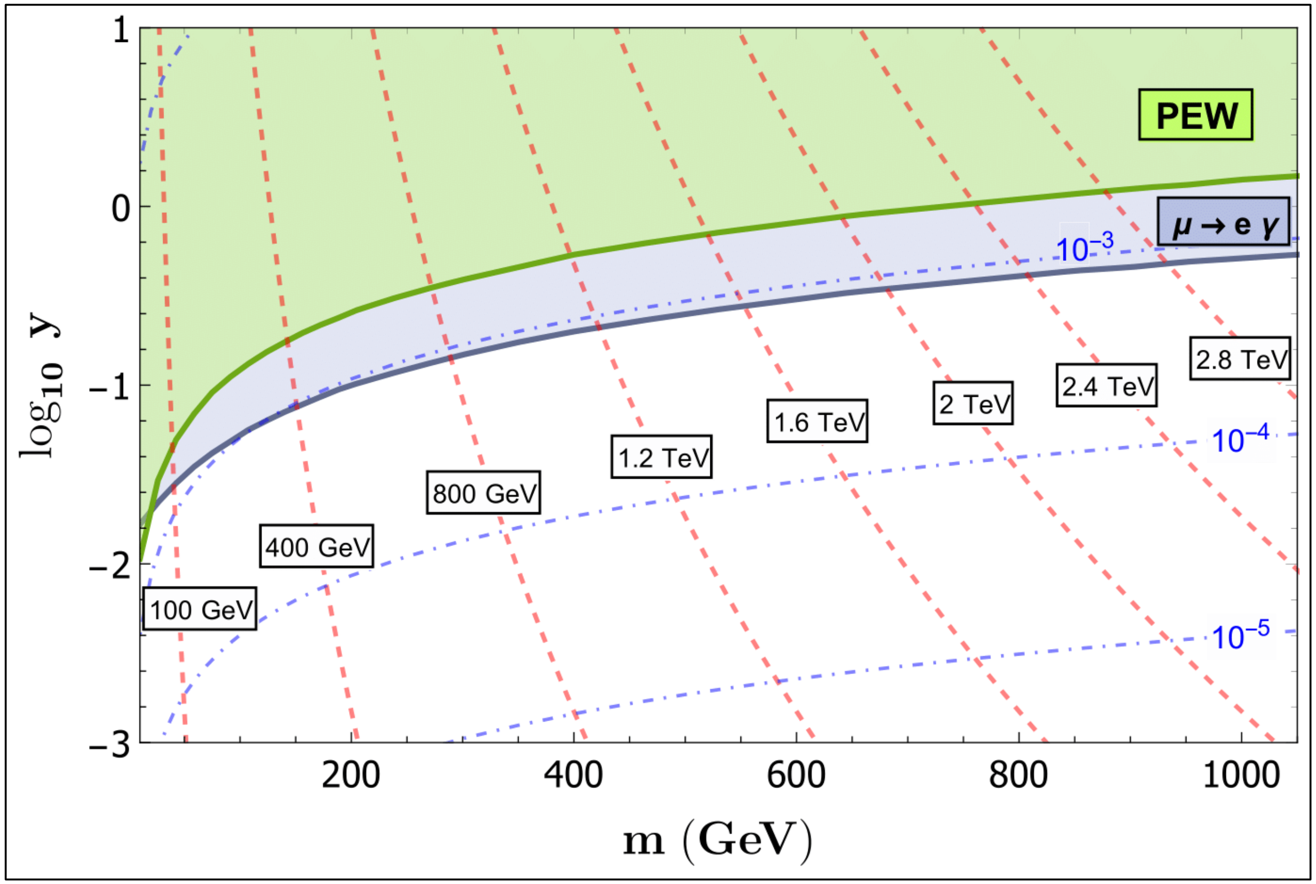
iii. Precision Electroweak Constraints:

- Use α, m_Z, and Γ_μ as fixed inputs to determine g, g' and v, including shifts due to clockwork
- Do χ^2 fit on other PEW observables
- $\Rightarrow$ Limit on fractional changes in SM neutrino gauge couplings

Recall,

$$(J_W^{\mu+})_0 = \frac{V_{\alpha\beta}}{\sqrt{2}} \bar{e}_{L\alpha} \gamma^\mu P_L \kappa_{0\beta} \mathcal{N}_{0\beta} + \mathcal{O}(p^3)\,, \ \kappa_{0\beta} = 1 - \frac{p_\beta^2}{N+1} \sum_{k=1}^{N} \frac{C_k}{2\lambda_k}$$

Constraints on the model



N = 20

$q \; \epsilon \; [2.32, 4.64]$

Red contours:
Mass of first excited state

Blue contours:
Coupling of W-boson to first excited state

Generalized Clockwork Theory

q need not be a constant: $\mathcal{L} = \mathcal{L}_{kin} - m \sum_{j=0}^{N-1} (\psi_i^\dagger \chi_i - q(j)\, \psi_i^\dagger \chi_{(i-1)}) + \text{h.c.}$

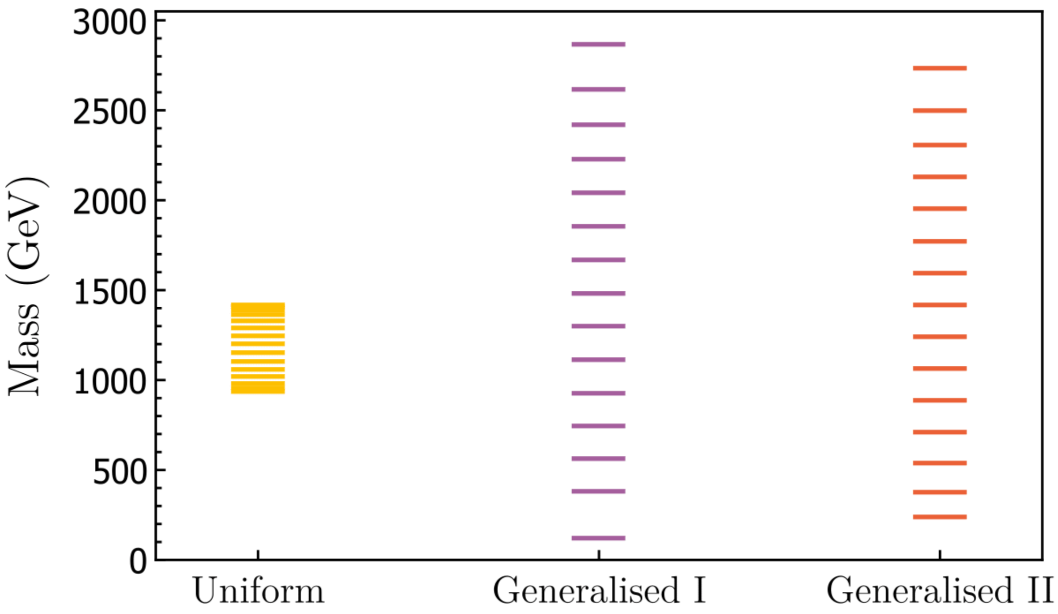
(Symmetry breaking occurs for any $q \Rightarrow$ generically leads to exponentially small mass)

Generalized CW models can have more efficient suppression of zero mode + localization

Generalized Clockwork Theory

Generalized CW models can have more efficient suppression of zero mode + localization

e.g.- For linearly increasing q, $(U_L)_{N0} \sim \dfrac{1}{N!\, q^N}$ (similar behaviour for linear decrease)

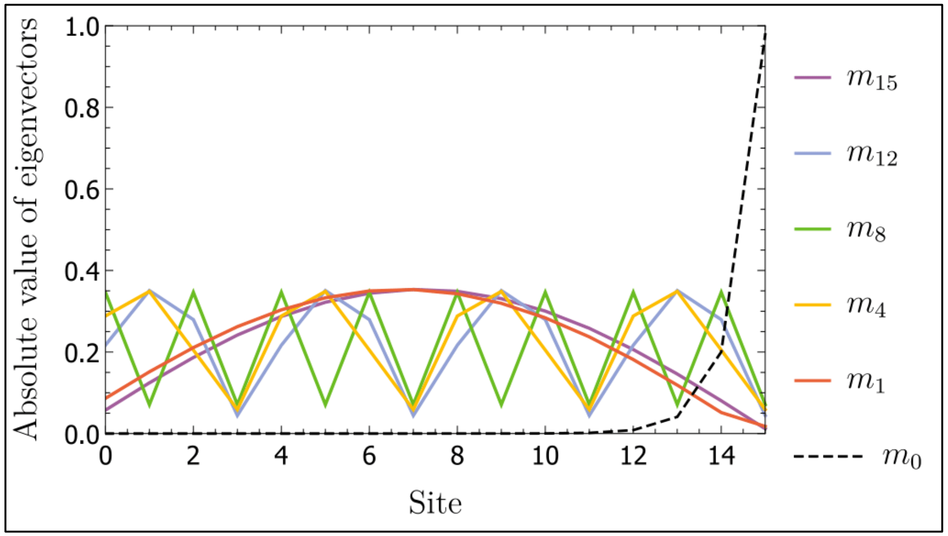


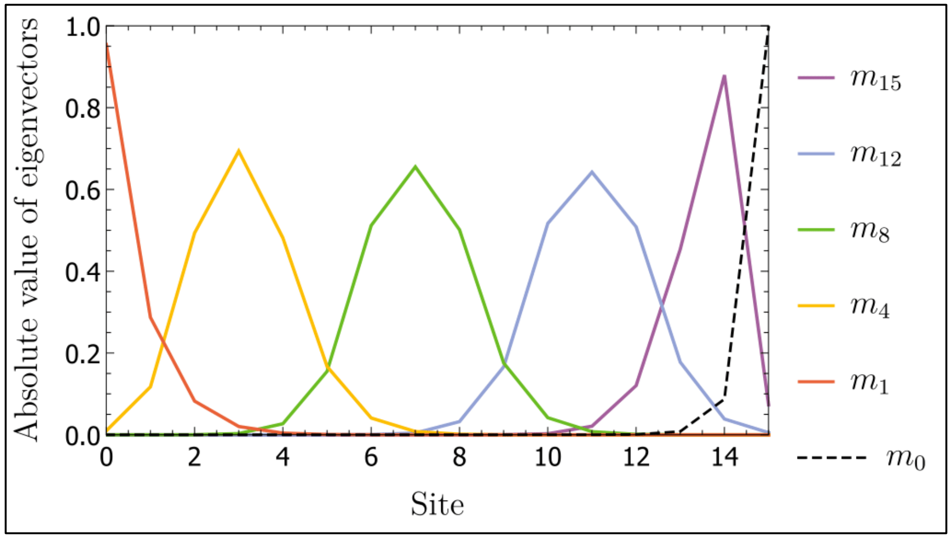
Masses in the generalized model examples we considered are generically more spread out, due to higher q values in the mass matrix.

Generalized Clockwork Theory

Generalized CW models can have more efficient suppression of zero mode + localization

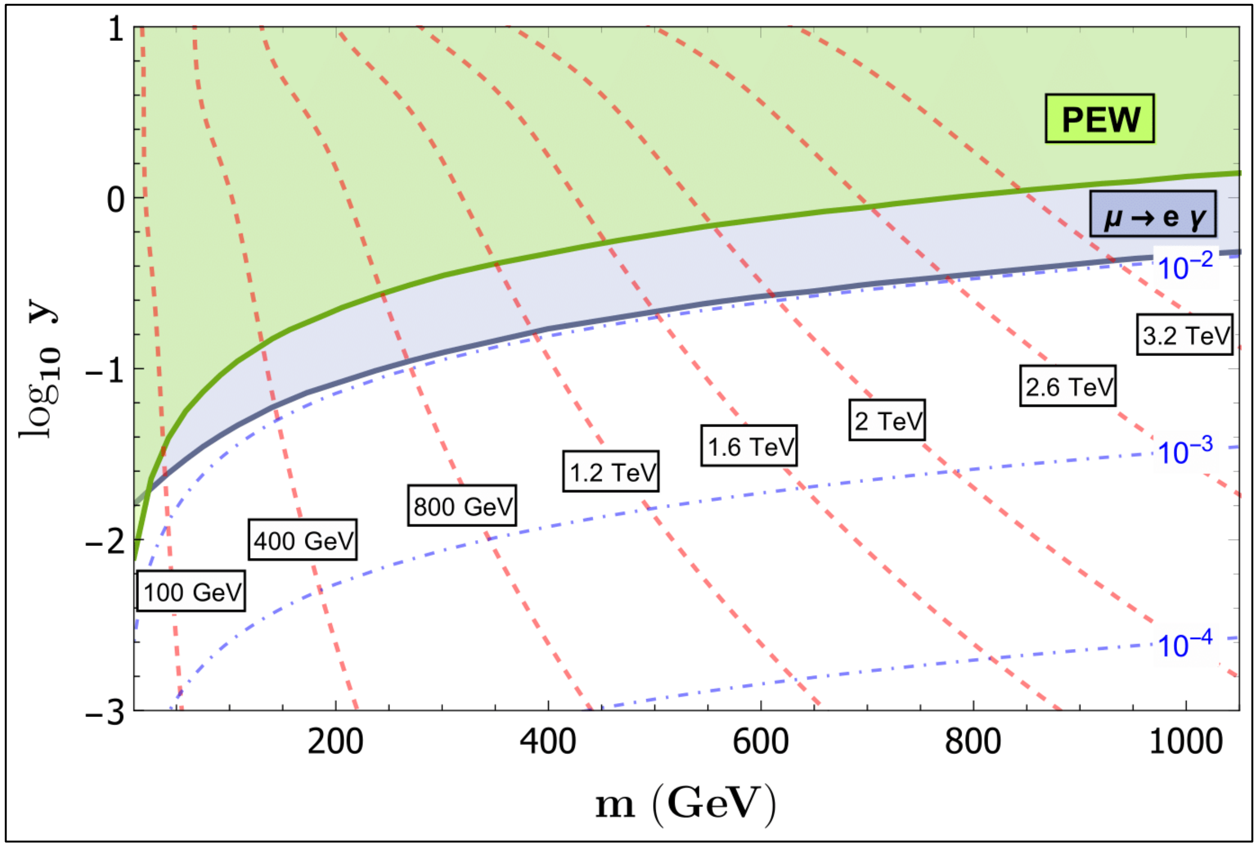
e.g.- For linearly increasing q, $(U_L)_{N0} \sim \dfrac{1}{N!\, q^N}$ (similar behaviour for linear decrease)





Uniform CW eigenmodes are oscillatory and spread out; generalized CW eigenmodes are localised.

Constraints for generalized clockwork



N = 10

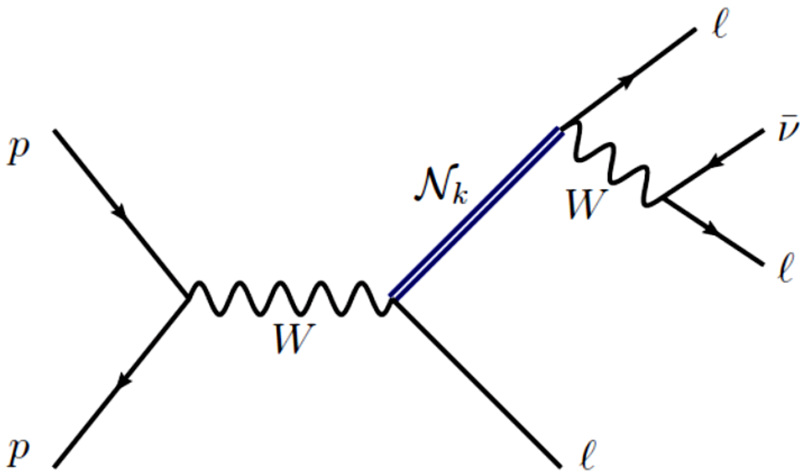
$q(i) \;=\; q_0 \times i$

$q_0 \;\epsilon\; [0.97, 4.85]$

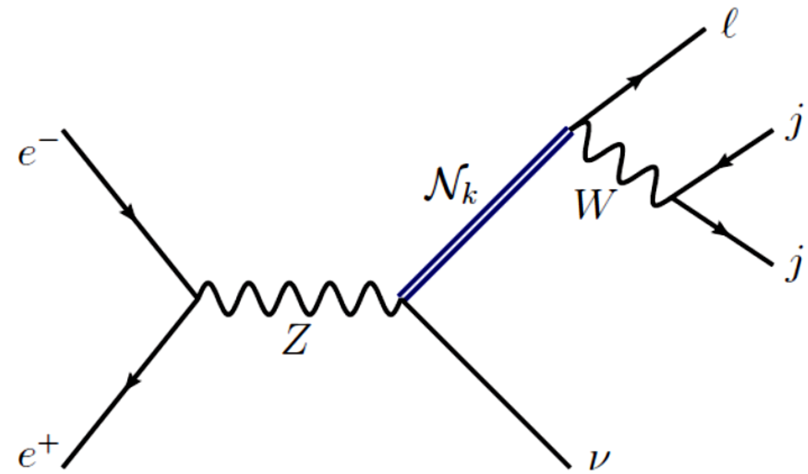
Red contours:
Mass of first excited state

Blue contours:
Coupling of W-boson to first excited state

Collider Signatures



Hadron Colliders: 3l + MET



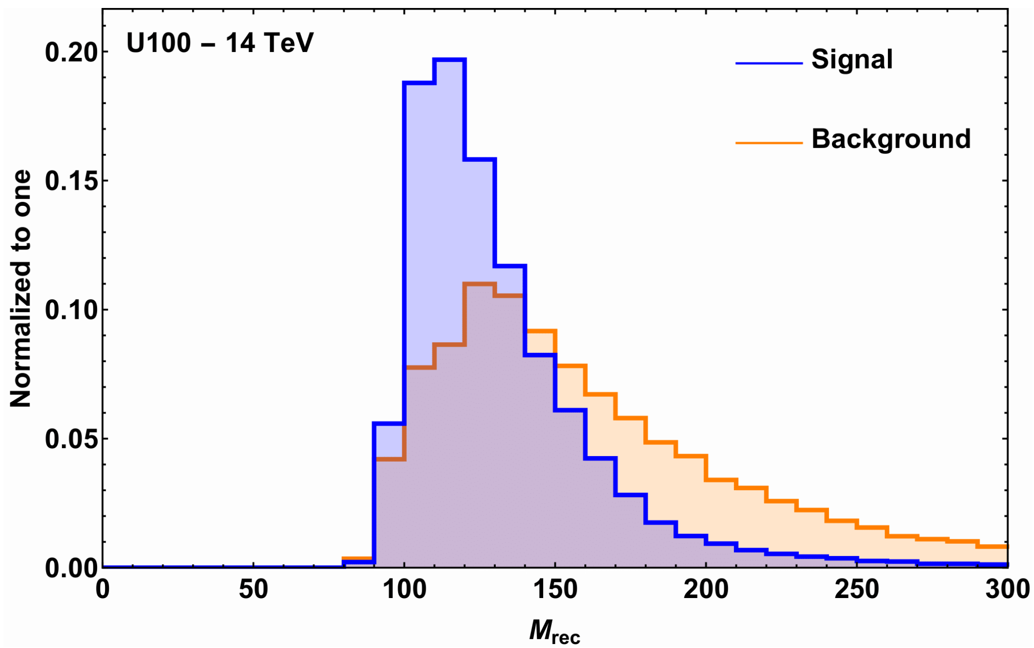
Lepton Colliders: ljj + MET

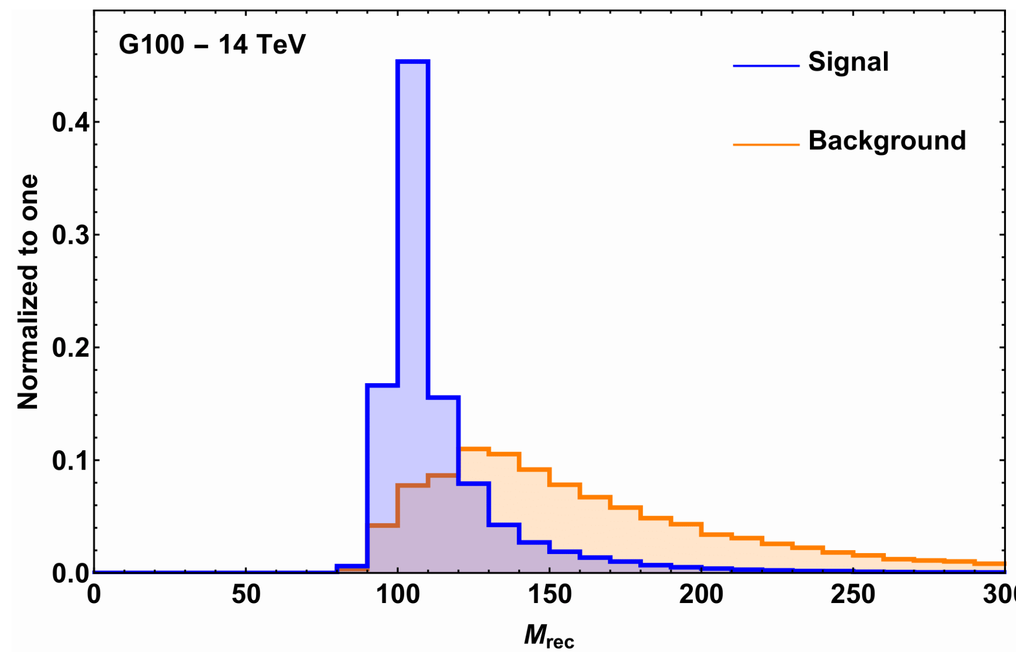
Higher mass of sterile neutrinos ⇒ high invariant mass and p_T for final products

Lepton colliders: Can detect any reasonable CW model within kinematic range

LHC: Low mass ($\sim$100 GeV) models are detectable with projected future luminosities

Collider Signatures





Universal Clockwork
- Smeared out bump;
Couplings to excited states are roughly similar;
Resonance includes multiple states ⇒
shape analysis?

Generalized Clockwork
- Sharper bump;
Higher coupling to lowest excited state;
Resonances are separated, with much smaller
significance for heavier neutrinos

Collider Analysis Example

LHC - 14 TeV ($\mathcal{L} = 3000\ fb^{-1}$)	**U100**		**G100**	
σ (fb) with parton-level cuts	0.66		1.39	
# of signal events	1965		4180	
Cuts:	S (fb)	BG (fb)	S (fb)	BG (fb)
Pre-selection cuts	0.29	93	0.62	93
$s_{12} < 80$ GeV	0.13	13.5	0.38	13.5
$s_{23} < 80$ GeV	0.12	8.7	0.37	8.7
$M_{rec} < 120$ GeV	–	–	0.29	3.8
S/B	0.01		0.077	
$S/\sqrt{B}$	2.3		8.2	
$S/\sqrt{S+B}$	2.3		7.9	

ℓ_1, ℓ_2 : Same Flavor Opposite Charge Leptons

ℓ_3 : Same charge as ℓ_1

M_{rec} : Mass reconstructed using W boson mass

Summary

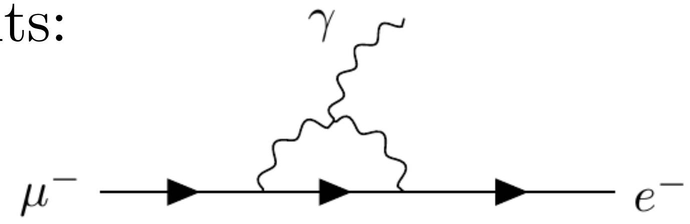
- The Clockwork framework offers a way to get small Dirac neutrino masses from O(1) Lagrangian parameters
- Approximate analytical solutions to the eigenvalue problem have been obtained through large-N expansions/perturbation theory
- Large sections of the parameter space satisfy flavour and EW constraints
- Low mass CW models may be observable at HL-LHC, and most kinematically accessible models can be discovered at future lepton colliders
- Can be generalized to have interesting phenomenology, more efficient 'clockworking' and sharper collider signatures

Thank you!

See arXiv:1903.06191 for more details.

Backup: Lepton Flavour Violation Details

LFV branching ratio in terms of analytical results:



$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{8\pi} |\mathcal{A}|^2$$

$$\text{where,} \quad \mathcal{A} = \sum_{\alpha=1}^{3} \sum_{j=0}^{N} V_{\mu\alpha} V_{e\alpha}^{*} |(U_{L\alpha})^{0j}|^2 F\left(\frac{m_{j,\alpha}^2}{m_W^2}\right)$$

$$F(x) = \frac{1}{6(1-x)^4}\left(10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \log x\right)$$

$$\mathcal{A} = \left(\frac{y_3^2 v^2}{2m^2}\right) \cdot \left[\frac{V_{e3}^{*} V_{\mu 3} \Delta m_{32}^2 - V_{e1}^{*} V_{\mu 1} \Delta m_{21}^2}{m_{0,3}^2}\right] \cdot \mathcal{F}(m, q, N)$$

$$\mathcal{F}(m, q, N) = \frac{1}{N+1} \sum_{k=1}^{N} \frac{C_k}{\lambda_k}, \left(F\left(\frac{m^2 \lambda_k}{m_W^2}\right) - F(0)\right)$$

Backup: Chosen Benchmark Points

Name	N	q	m (GeV)	y	m_1 (GeV)	g_1	m_N (GeV)
U100	20	3.45	40	[0.0207, 0.0209, 0.0269]	98.6	0.0014	177.7
U400	16	5.00	100	[0.0541, 0.0546, 0.0704]	402.1	0.0011	598.6
U750	17	4.70	200	[0.0947, 0.0956, 0.1232]	743.9	0.0010	1137.5
U1000	18	4.40	310	[0.1362, 0.1376, 0.1773]	1059.5	0.0009	1670.6
G100	10	2.60	40	[0.0178, 0.0180, 0.0232]	101.6	0.0182	1044.7
G300	11	2.05	160	[0.0373, 0.0377, 0.0485]	315.6	0.0122	3631.2
G750	11	2.20	360	[0.0811, 0.0819, 0.1055]	765.9	0.0109	8760.9

Backup: Cross-sections at colliders

BP	$pp \to 3\ell + MET$		$e^+e^- \to \ell jj + MET$		
	14 TeV	100 TeV	250 GeV	500 GeV	3 TeV
U100	0.66	4.2	7.4	12.8	3.9
G100	1.40	8.6	4.3	7.4	1.34
U400	3.0×10^{-3}	0.032	–	0.81	7.6
G300	0.014	0.12	–	5.8	6.1
U750	2.6×10^{-4}	5.0×10^{-3}	–	–	5.9
G750	5.0×10^{-4}	8.0×10^{-3}	–	–	7.9
U1000	5.0×10^{-5}	1.7×10^{-3}	–	–	2.3

Backup: Lepton colliders have clear signals

BP	U100	G100	U400	G300	U750	G750	U1000
$\sqrt{s}$, GeV	250	250	500	500	3000	3000	3000
$\mathcal{L}_{3\sigma}$, fb^{-1}	220	50	4300	20	55	25	720

Table 1: Center-of-mass energy and integrated luminosity required for a 3-sigma observation of the CW neutrino signal in electron-positron collisions.