

# Minimal $SO(10)$ -based GUT

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Based on:  
Boucenna, Ohlsson, MP  
1812.10548  
Ohlsson, MP 1903.08241



## Open questions in the Standard Model

- Neutrino masses
- Dark matter
- Baryon asymmetry of the universe
- Higgs vacuum stability





## Open questions in the Standard Model

- Neutrino masses
- Dark matter
- Baryon asymmetry of the universe
- Higgs vacuum stability
- Why three gauge groups?
- Near unification of gauge couplings
- Fermion charge assignment
- Anomaly cancellation





$$\text{SO}(10) \supset \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$$

- Three gauge groups unified
- Gauge coupling unification
- Anomalies vanish identically

## Fermionic representation

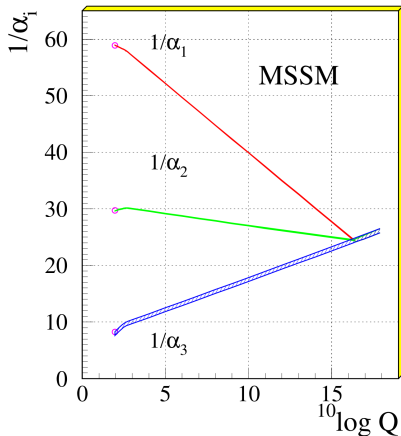
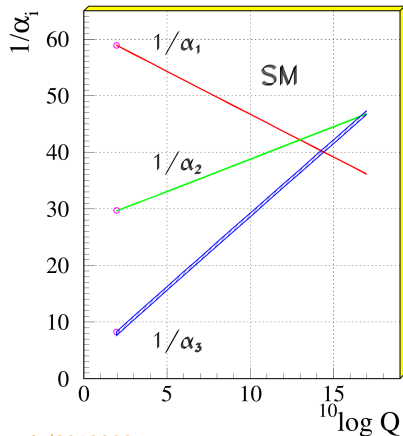
$$\mathbf{16}_F \rightarrow (\mathbf{3}, \mathbf{2})_{1/6} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{1}, \mathbf{1})_1 \oplus (\mathbf{1}, \mathbf{1})_0$$
$$Q_L + u_R^c + d_R^c + \ell_L + e_R^c + N_R^c$$

- One fermionic representation
- Right-handed neutrinos
- Relation between hypercharges

# Gauge Coupling Unification



How do we unify the coupling constants without SUSY...?



hep-ph/0012288

# Gauge Coupling Unification

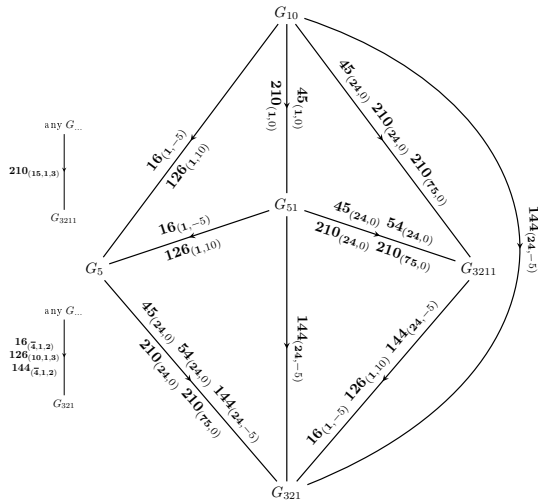
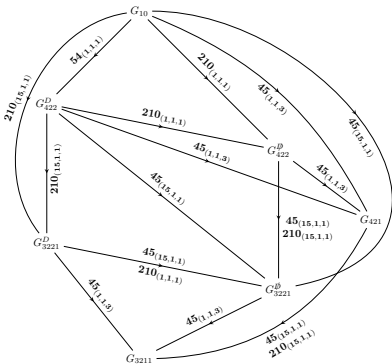


... And without introducing multiple steps?

Ferrari et al. 1811.07910

Chang et al. 1985

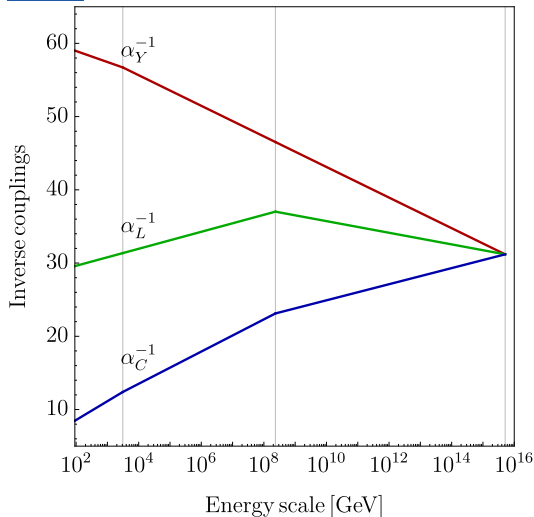
Deshpande et al. 1993



# Gauge Coupling Unification in One Step



$$\text{Split } \mathbf{210} \supset S_1(\mathbf{8}, \mathbf{1})_1 \oplus S_2(\mathbf{8}, \mathbf{3})_0$$



$$M_1 = 3.1 \text{ TeV}$$

$$M_2 = 2.34 \times 10^8 \text{ GeV}$$

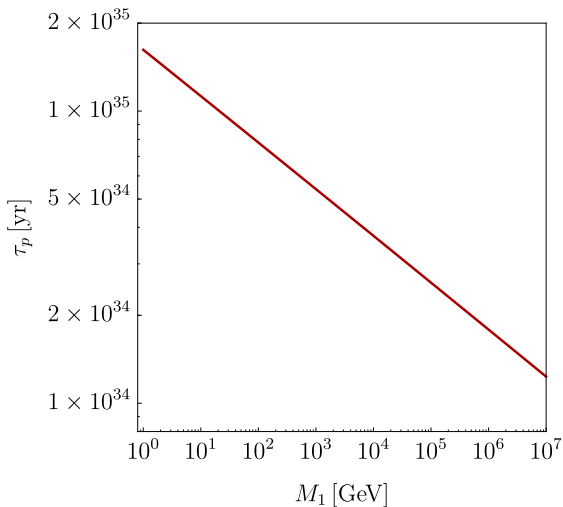
$$M_{\text{GUT}} = 4.51 \times 10^{15} \text{ GeV}$$

$M_2$  and  $M_{\text{GUT}}$  determined uniquely by  $M_1$  and requirement of gauge coupling unification

Similar constructions in:  
Frigerio, Hambye 0912.1545  
Parida et al. 1608.03956

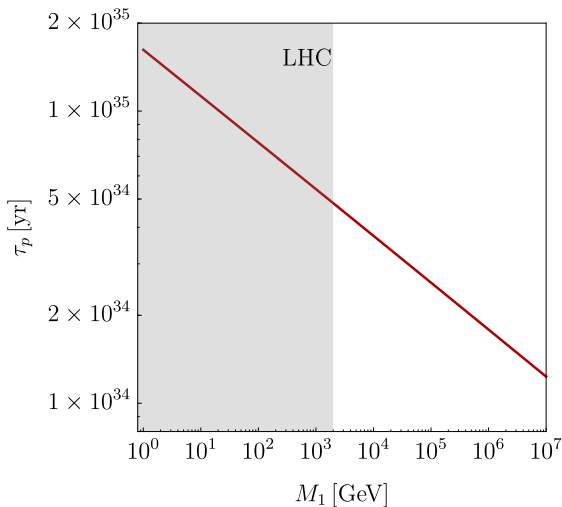
Babu et al. hep-ph/0506312

# Proton Lifetime and LHC



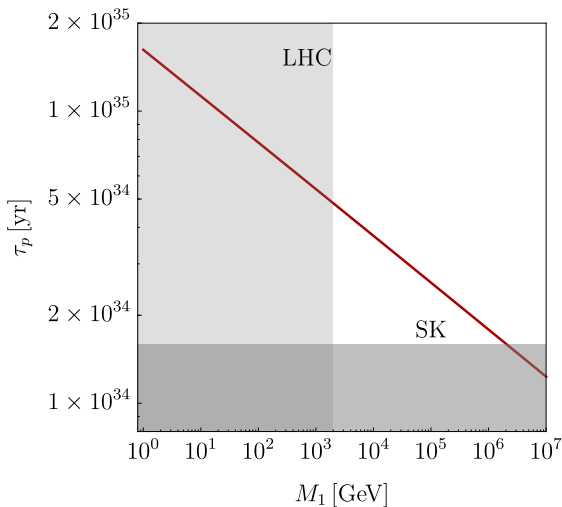


# Proton Lifetime and LHC



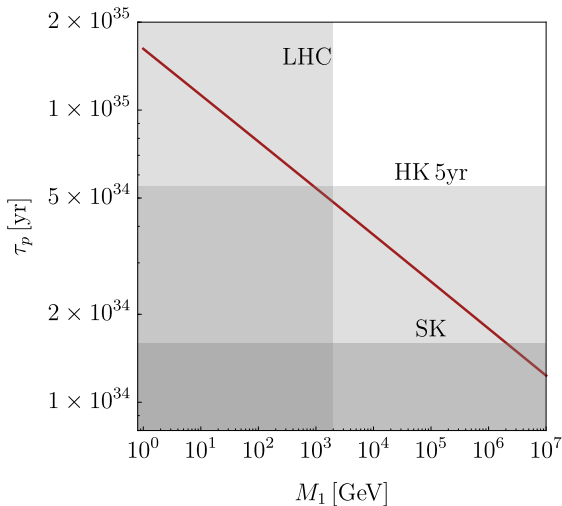
- $M_1 > 3.1 \text{ TeV}$   
CMS 1512.01224

# Proton Lifetime and LHC



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- $\tau_p > 1.6 \times 10^{34} \text{ yr}$   
( $p \rightarrow e^+ \pi^0$ )  
SK 1610.03597

# Proton Lifetime and LHC



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- $\tau_p > 1.6 \times 10^{34} \text{ yr}$   
( $p \rightarrow e^+ \pi^0$ )  
SK 1610.03597
- $\tau_p > 5.5 \times 10^{34} \text{ yr}$   
( $p \rightarrow e^+ \pi^0$ )  
HK 1805.04163



## Higgs

- Higgs  $H_u = (\mathbf{1}, \mathbf{2})_{1/2}$  and  $H_d = (\mathbf{1}, \mathbf{2})_{-1/2}$  reside in  $\mathbf{10}_H$  and  $\overline{\mathbf{126}}_H$
- SM Higgs doublet is a combination of these
- Yukawa Lagrangian  $\mathbf{16}_F(Y_{10}\mathbf{10}_H + Y_{126}\overline{\mathbf{126}}_H)\mathbf{16}_F$



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## Neutrino Masses

- Neutrino Dirac mass through  $\overline{\ell}_L H_u N_R$
- $\sigma \equiv (\mathbf{1}, \mathbf{1})_0 \subset \overline{\mathbf{126}}_H$  gives Majorana mass through  $\sigma \overline{N}_R^c N_R$
- Type-I seesaw
- (Type-II seesaw also possible using  $\Delta_L \equiv (\mathbf{1}, \mathbf{3})_{-1} \subset \overline{\mathbf{126}}_H$ )



Bajc et al. hep-ph/0510139

Altarelli, Meloni 1305.1001

Babu, Khan 1507.06712

Babu et al. 1612.04329

- $\mathbf{10}_H$  is a real representation  $\implies v_{10}^u = v_{10}^d$ .

Not enough freedom to fit

- Solution: Complexify it as  $\mathbf{10}_H \equiv \mathbf{10}_{H,1} + i\mathbf{10}_{H,2}$
- But then have extra Yukawa couplings

$$Y_{10} \mathbf{16}_F \mathbf{10}_H \mathbf{16}_F + \tilde{Y}_{10} \mathbf{16}_F \mathbf{10}_H^* \mathbf{16}_F$$



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$$Y_{10} \mathbf{16}_F \mathbf{10}_H \mathbf{16}_F + \tilde{Y}_{10} \mathbf{16}_F \mathbf{10}_H^* \mathbf{16}_F$$

- Introduce  $U(1)_{PQ}$  symmetry with charges

$$\mathbf{16}_F \rightarrow e^{i\alpha} \mathbf{16}_F, \mathbf{10}_H \rightarrow e^{-2i\alpha} \mathbf{10}_H, \overline{\mathbf{126}}_H \rightarrow e^{-2i\alpha} \overline{\mathbf{126}}_H$$

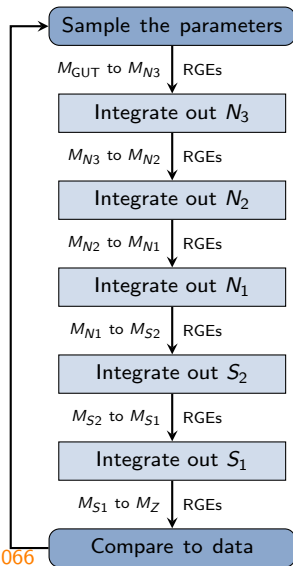
- Axions: Strong CP problem and DM.  $f_a = M_{GUT} = 4.51 \times 10^{15}$  GeV

# Fitting to the Standard Model



$$\begin{aligned}v_{\text{SM}} Y_u &= v_{10}^u Y_{10} + v_{126}^u Y_{126}, \\v_{\text{SM}} Y_d &= v_{10}^d Y_{10} + v_{126}^d Y_{126}, \\v_{\text{SM}} Y_\nu &= v_{10}^u Y_{10} - 3v_{126}^u Y_{126}, \\v_{\text{SM}} Y_\ell &= v_{10}^d Y_{10} - 3v_{126}^d Y_{126}, \\M_R &= v^\sigma Y_{126}.\end{aligned}$$

- 19 free parameters and 19 data
- Integrate out RH neutrinos:  
 $\kappa \rightarrow \kappa + \frac{2}{M_{N_i}} (Y_\nu^i)^T (Y_\nu^i)$  at each RH neutrino threshold
- Find acceptable fit ( $\chi^2 \simeq 21$ )
- **Simple leptogenesis not successfully fit**



Babu, Mohapatra hep-ph/9209215, Fukuyama, Okada hep-ph/0205066  
Bertolini et al. hep-ph/0605006, Joshipura, Patel 1102.5148

Dueck, Rodejohann 1306.4468, Antusch et al. hep-ph/0203233, arXiv:hep-ph/0501272

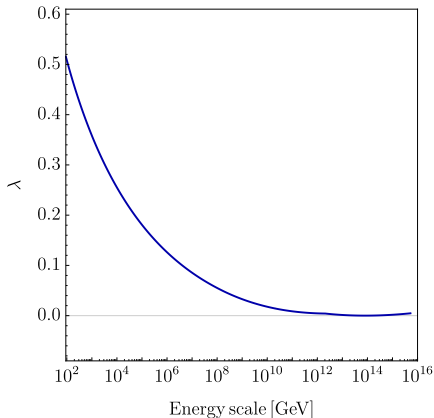
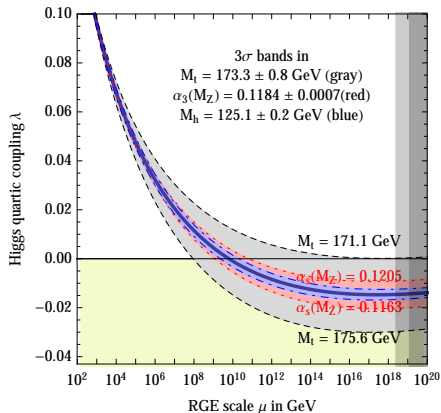


# Vacuum Stability



$$16\pi^2 \frac{d\lambda}{d \ln \mu} = \dots - 3\lambda (3g_2^2 + \frac{3}{5}g_1^2) + 3g_2^4 + \frac{3}{2} (\frac{3}{5}g_1^2 + g_2^2)^2 + 4\lambda \text{Tr} [Y_\nu^\dagger Y_\nu] - 8\text{Tr} [Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + \dots$$

Machacek, Vaughn 1985



Buttazzo et al. 1307.3536



Let us relax some model specifications:

- Do not specify the extra scalars
- Do not know exact  $M_{\text{GUT}}$
- Both Type-I and Type-II seesaw



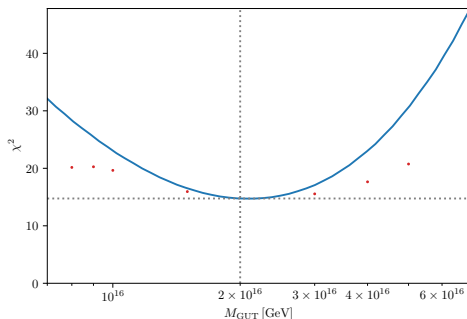
Let us relax some model specifications:

- Do not specify the extra scalars
- Do not know exact  $M_{\text{GUT}}$
- Both Type-I and Type-II seesaw
  
- Remove the colored octet scalars
- Set  $M_{\text{GUT}} = 2 \times 10^{16}$  GeV, but check sensitivity to  $M_{\text{GUT}}$
- Introduce another scale: Mass of  $\Delta_L \equiv (\mathbf{1}, \mathbf{3})_{-1}$

Neglects some model details, but indicative



- Acceptable fits to normal neutrino mass ordering ( $\chi^2 \simeq 14.7$ ), but not inverted
- Type-II seesaw is sub-dominant:  $M_\Delta \sim M_{\text{GUT}}$ ,  $v_\Delta \sim 10^{-6}$  GeV
- Largest contribution to  $\chi^2$  is  $\sin^2 \theta_{23}^\ell$ : fit favours value in first octant (0.474), but actual value is in second octant (0.547)





## Minimal $SO(10)$ model with $U(1)_{PQ}$ symmetry

- Neutrino masses ✓
- Dark matter ✓
- Baryon asymmetry of the universe (**More detailed analysis needed**)
- Higgs vacuum stability ✓
- Fit the Yukawa sector to SM ✓

# BACKUP SLIDES

$$M_1 \lesssim 5.92 \times 10^{10} \text{ GeV},$$

$$M_2 \approx \left( \frac{M_1}{\text{GeV}} \right)^{0.330} \times 1.65 \times 10^7 \text{ GeV},$$

$$M_{\text{GUT}} \approx \left( \frac{M_1}{\text{GeV}} \right)^{-0.0447} \times 7.34 \times 10^{15} \text{ GeV}.$$

$$\tau_p \equiv \tau(p \rightarrow e^+ \pi^0) \simeq \frac{4}{\pi} \frac{f_\pi^2}{m_p} \frac{1}{\alpha_H^2 A_R^2} \frac{1}{F_q} \frac{M_{\text{GUT}}^4}{\alpha (M_{\text{GUT}})^2},$$

where  $f_\pi \approx 139 \text{ MeV}$  is the pion decay constant,  $m_p \approx 938.3 \text{ MeV}$  is the proton mass,  $\alpha_H \approx 0.012 \text{ GeV}^3$  is the hadronic matrix element,  $A_R \approx 2.726$  is a renormalisation factor, and  $F_q \approx 7.6$  is a quark-mixing factor.

$$\tau_p \approx 3.22 \times \frac{M_{\text{GUT}}^4}{\alpha (M_{\text{GUT}})^2}.$$

$$H \equiv \frac{v_{10}^d}{v_{SM}} Y_{10}, \quad F \equiv \frac{v_{126}^d}{v_{SM}} Y_{126}, \quad r \equiv \frac{v_{10}^u}{v_{10}^d},$$

$$s \equiv \frac{1}{r} \frac{v_{126}^u}{v_{126}^d} = \frac{v_{10}^d}{v_{10}^u} \frac{v_{126}^u}{v_{126}^d}, \quad r_R \equiv v_{126}^R \frac{v_{SM}}{v_{126}^d}, \quad r_L \equiv \frac{v_{SM}}{v_{126}^d}.$$

$$Y_u = r(H + sF), \quad Y_d = H + F,$$

$$Y_\nu = r(H - 3sF), \quad Y_\ell = H - 3F,$$

$$M_R = r_R F, \quad Y_\Delta = r_L F.$$



Observable	Value	Error
$m_u$ (MeV)	1.36	0.15
$m_c$ (MeV)	635	32
$m_t$ (GeV)	172	8.7
$m_d$ (MeV)	2.90	0.15
$m_s$ (MeV)	54.1	2.8
$m_b$ (GeV)	2.87	0.15
$m_e$ (MeV)	0.487	0.025
$m_\mu$ (MeV)	103	5.2
$m_\tau$ (GeV)	1.75	0.088
$\Delta m_{21}^2$ ( $10^{-5} \text{eV}^2$ )	7.55	0.38
$\Delta m_{31}^2$ ( $10^{-3} \text{eV}^2$ ) (NO)	2.50	0.13
$\Delta m_{32}^2$ ( $10^{-3} \text{eV}^2$ ) (IO)	-2.42	0.13
$\sin \theta_{12}^q$	0.225	0.012
$\sin \theta_{13}^q$	0.00372	0.00019
$\sin \theta_{23}^q$	0.0418	0.0021
$\delta_{\text{CKM}}$	1.14	0.058
$\sin^2 \theta_{12}^\ell$	0.320	0.020
$\sin^2 \theta_{13}^\ell$ (NO)	0.0216	0.0011
$\sin^2 \theta_{13}^\ell$ (IO)	0.0222	0.0012
$\sin^2 \theta_{23}^\ell$ (NO)	0.547	0.030
$\sin^2 \theta_{23}^\ell$ (IO)	0.551	0.030
$\lambda$	0.516	0.026

Parameter	Value
$m_1$	$3.70 \times 10^{-3} \text{ eV}$
$m_2$	$9.55 \times 10^{-3} \text{ eV}$
$m_3$	$4.93 \times 10^{-2} \text{ eV}$
$M_1$	$1.87 \times 10^{10} \text{ GeV}$
$M_2$	$4.46 \times 10^{11} \text{ GeV}$
$M_3$	$2.34 \times 10^{12} \text{ GeV}$
$m_{ee}$	$1.56 \times 10^{-3} \text{ eV}$
$\delta_{\text{CP}}$	0.441