Anomalous Gauge Couplings from Diboson Production

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SUSY
Useful to have a “model independent” formulation of deviations from the Standard Model.

Philosophy:
- We know the Standard Model is there at the 100 GeV-1 TeV scale with a very Standard Model-like Higgs boson.
- Treat $SU(2) \times U(1)_Y$ as a good symmetry.
Effective Field Theory

- Useful to have a “model independent” formulation of deviations from the Standard Model.

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- Standard Model effective field theory (EFT)  

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_k \frac{c_{1,k}}{\Lambda} O_{1,k} + \sum_k \frac{c_{2,k}}{\Lambda^2} O_{2,k} + \cdots \]

- $O_{n,k}$: $SU(3) \times SU(2)_L \times U(1)_Y$ gauge invariant $4 + n$ dimensional higher order operators.
- $\Lambda$: scale of new physics.
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- $O_{n,k}$: $SU(3) \times SU(2)_L \times U(1)_Y$ gauge invariant $4+n$ dimensional higher order operators.
- $\Lambda$: scale of new physics.
- Allows for a systematic parameterization of deviations from Standard Model predictions without doing too much damage to lower energy measurements.
Informative to focus on one process.

- Global fits take known measurements and fit to them.
- Focusing on a single process allows us to learn the most about that process.
- Of particular interest is the electroweak sector.
- Focus on $W^+W^-$ production at the LHC. [Baglio, Dawson, I. Lewis PRD96 (2017) 073003; Baglio, Dawson I. Lewis, PRD99 (2019) 035029]
- Sensitive to anomalous trilinear gauge boson couplings (ATGCs)
$W^+W^-$ production

- Anomalous coupling language *Hagiwara, Peccei, Zeppenfeld, Hikasa NPB482 (1987)*:

$$\delta \mathcal{L} = -ig_{WWV} \left( g_1^V (W^+ W^\mu V^\nu - W^- W^\mu V^\nu) + \kappa^V W^+ W^- V^{\mu \nu} + \frac{\lambda^V}{M_W^2} W^+ W^- V^{\nu \rho} \right)$$

- $V = Z, \gamma$
- $gwWZ = g \cos \theta_w$, $gWW\gamma = e$
Anomalous coupling language Hagiwara, Peccei, Zeppenfeld, Hikasa NPB482 (1987):

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- $V = Z, \gamma$
- $g_{WWZ} = g \cos \theta_w, \quad g_{WW\gamma} = e$

Parameterize deviations from Standard Model:

$$g_1^Z = 1 + \delta g_1^Z \quad g_1^\gamma = 1 + \delta g_1^\gamma \quad \kappa^Z = 1 + \delta \kappa^Z \quad \kappa^\gamma = 1 + \delta \kappa^\gamma$$

$\lambda^Z = 0$ and $\lambda^\gamma = 0$ in Standard Model.
$W^+W^-$ production

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- Parameterize deviations from Standard Model:

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- $\lambda^Z = 0$ and $\lambda^\gamma = 0$ in Standard Model.
- $SU(2)_L$ invariance implies:

$$\delta g_1^\gamma = 0, \quad \lambda^\gamma = \lambda^Z, \quad \delta \kappa^\gamma = \frac{\cos^2 \theta_W}{\sin^2 \theta_W} (\delta g_1^Z - \delta \kappa^Z)$$

- Three independent parameters: $\lambda^Z, \delta g_1^Z, \delta \kappa^Z$
Have not included anomalous quark gauge boson couplings:

\[
\mathcal{L} = g_V V^\mu \left( (g_L V^q + \delta g_L V^q) \bar{q}_L \gamma_\mu q_L + (g_R V^q + \delta g_R V^q) \bar{q}_R \gamma_\mu q_R \right)
\]

\[
\delta g_L^W = \delta g_L^{Zu} - \delta g_L^{Zd}
\]

- Highly constrained by LEP.
- Each diagram individually violates unitarity and grow uncontrollably with energy.
  - Will eventually get probabilities greater than one.
  - Standard Model contains cancellations to unitarize amplitudes and growth with energy cancels.
- Anomalous quark couplings can spoil cancellation and have growth with energy.
- This was recently pointed out Zhang PRL118 (2017) 011803
Differential Distributions

13 TeV, pp→W⁺W⁻→e⁺ νν, \( \delta g_L^{Zu} = 0.0163, \delta g_R^{Zu} = 0.00452, \delta g_L^{Zd} = 0.0239 \)

13 TeV, pp→W⁺W⁻→e⁺ νν, \( \delta g_L^{Zu} = -0.00239, \delta g_R^{Zu} = -0.0069, \delta g_L^{Zd} = 0.00271, \delta g_R^{Zd} = 0.0212 \)

- 1/Λ⁴ terms dominate in tails and the bounds on anomalous couplings. [Falkowski, Gonzalez-Alonso, Greijo, Marzocca, Son JHEP 1702 (2017) 115]
- Ferm: ATGCs set to zero.
- 3GB: Anomalous fermion couplings set to zero.
- Assuming \( C_i \lesssim 1 \), anomalous couplings correspond to \( \Lambda \gtrsim 2.8 \) TeV.
7, 8, 13 TeV diboson data.

Yellow: Includes all operators.

Red: Only anomalous trilinear gauge boson couplings.

Pink: Only anomalous quark-gauge boson couplings.

Anomalous quark couplings make impact on anomalous trilinear gauge boson coupling.

Bounds on down-type quark coupling comparable to LEP (LEP bounds on up-type quarks still more stringent.)

See also Baglio, Lewis, Dawson PRD96 (2017) 073003; Alves, Rosa-Agostinho, Éboli, Gonzalez-Garcia, PRD98 (2018) 013006
### Negligible interference between SM and ATGCs for fully transversely polarized $W$s.

Hagiwara, Peccei, Zeppenfeld, Hikasa NPB282 (1987); Azatov, Contino, Machado, Riva, PRD95 (2017)

### Effects of EFT depend on polarization of gauge bosons.

- $\Lambda^{-2}$: SM amplitude squared+interference with EFT.
- $|A|^2$: full amplitude squared.
Importance of Decays

- Have clear indication that different vector boson polarizations depend on anomalous couplings differently.
- Observables sensitive to the polarizations can be more sensitive to the EFT, and maybe “resurrect” the interference between the SM and EFT.
- Additionally, once the vector bosons are not in the final state, different polarizations of the internal vector bosons can interfere with each other.
  - In particular, the angles between the two bosons decay planes or decay and production planes can be sensitive the interference between the SM and EFT. 


Panico, Riva, Wulzer PLB776 (2018) 473
How important are quark couplings at HL-LHC and HE-LHC?

- Only consider the last bin.
  - Define last bin where $\delta_{\text{statistical}} \sim \delta_{\text{systematic}} \sim 16\%$ ATLAS, JHEP 1609 (2016) 029
  - 14 TeV HL-LHC (3 ab$^{-1}$):
    \[ p_{T,\text{lead}}^\ell > 750 \text{ GeV}. \]
  - 27 TeV HE-LHC (15 ab$^{-1}$):
    \[ p_{T,\text{lead}}^\ell > 1350 \text{ GeV}. \]

- Scan over the allowed LEP ranges Falkowski, Riva JHEP 1502:
  \[
  \begin{align*}
  \delta g_{Zd}^L &= (2.3 \pm 1) \times 10^{-3} \\
  \delta g_{Zu}^L &= (-2.6 \pm 1.6) \times 10^{-3} \\
  \delta g_{Zd}^R &= (16.0 \pm 5.2) \times 10^{-3} \\
  \delta g_{Zu}^R &= (-3.6 \pm 3.5) \times 10^{-3}
  \end{align*}
  \]

- Note: $\delta g_{Zd}^R$ is not centered around zero.
  - Need non-zero anomalous trilinear gauge boson couplings to compensate for non-zero $\delta g_{Zd}^R$ and get expected rate from the SM.
14 TeV HL-LHC, 3 ab\(^{-1}\)

- Black, red: ATGCs only
- Blue, green: ATGCs+anomalous quark couplings.
- Black, blue: \(\delta_{\text{sys}} = 16\%\)
- Red, green: \(\delta_{\text{sys}} = 4\%\)
- Areas inside contours allowed.
- Non-overlapping contours.

HL-LHC 14 TeV NLO projections with \(pp \rightarrow e^\pm \mu^\mp \nu \nu\) | \(\mathcal{L} = 3\) ab\(^{-1}\)

- 3GB, \(\delta_{\text{sys}} = 4\%\)
- 3GB, \(\delta_{\text{sys}} = 16\%\)
- 3GB+Ferm, \(\delta_{\text{sys}} = 4\%\)
- 3GB+Ferm, \(\delta_{\text{sys}} = 16\%\)

![Graph showing HL-LHC 14 TeV NLO projections](image)
27 TeV HE-LHC, 15 ab$^{-1}$

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“Primitive cross sections” and NLO:

\[
\frac{d\sigma^2}{d\vec{C}} = d\sigma_{SM}(1 - \sum_{i=1}^{m} C_i) + \sum_{i=1}^{m} C_i d\sigma(1; \vec{R}_i) + \sum_{i=1}^{m} C_i^2 \left( d\sigma(2; \vec{R}_i) - d\sigma(1; \vec{R}_i) \right) \\
+ \sum_{i>j=1}^{m} C_i C_j \left[ d\sigma(2; \vec{M}_{ij}) - d\sigma(2; \vec{R}_i) - d\sigma(2; \vec{R}_j) + d\sigma_{SM} \right]
\]

- $C_i$ are Wilson coefficients.
- Primitive cross sections $d\sigma(n, \vec{R}_i)$ and $d\sigma(n, \vec{M}_{ij})$ defined by setting one or two Wilson coefficients to one and all others to zero.
- Primitive cross sections in Warsaw and HISZ bases are included in supplemental data in arXiv submission 1812.00214.
  - Realistic collider cuts.
  - Several distributions available.
  - We provide the method to take our primitive cross sections and recast into your own favorite basis.
- Can download yourself and test operator-by-operator or perform your own fits.
Dipole Operators

- Dipole operators of the form

\[ O_{fW} = i F_L \sigma^\mu_\nu f_R \sigma^a \Phi W_{\mu\nu}, \quad O_{fB} = i F_L \sigma^\mu_\nu f_R \Phi B_{\mu\nu} \]

- \( F_L \) is an \( SU(2)_L \) doublet and \( f_R \) is an \( SU(2)_L \) singlet.
- \( \Phi \) is SM Higgs doublet.

- Have new three-point and four-point quark-gauge boson couplings.
  - 3-point vertex is momentum dependent, important at high energies.
  - 4-point vertex will also grow quickly with energy.

- Note: dipoles couple left-chiral to right-chiral.
  - SM and other anomalous coupling contributions to \( WW/WZ \) considered so far couple left-left or right-right.
  - Since initial state fermions massless, there is no interference between dipole operators and other contributions.
  - Hence, only contribute at \((dimension - 6)^2\).
Dipole Operators

- **Red:** With Dipoles, **Blue:** Without Dipoles
- No interference ⇒ Dipoles only add ⇒ no flat directions/cancellations with dipoles.

Almeida, Rosa-Agostinho, Êboli, Gonzalez-Garcia arXiv:1905.05187
**Dipole Operators**

- **LHS**: Electroweak diboson production
  - Red: electroweak diboson production, Black: electroweak precision data.
- **RHS**: Drell Yan
  - Red: ATLAS, Blue: CMS, Solid: Run 1, Dashed: Run 2, Black: All combined

Almeida, Rosa-Agostinho, Éboli, Gonzalez-Garcia arXiv:1905.05187
Comment on Calculating Cross Sections

- Amplitude has terms up to $\Lambda^{-2}$.
- Amplitude squared includes terms that go as $\Lambda^{-4}$.

$$|A|^2 \sim |g_{SM} + \frac{c_{\text{dim}-6}}{\Lambda^2}|^2 \sim g_{SM}^2 + g_{SM} \times \frac{c_{\text{dim}-6}}{\Lambda^2} + \frac{c_{\text{dim}-6}^2}{\Lambda^4}$$

- $g_{SM}$ is a generic Standard Model coupling.
Comment on Calculating Cross Sections

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- $g_{SM}$ is a generic Standard Model coupling.
- Same order as dimension-8 contributions:

$$|A|^2 \sim |g_{SM} + \frac{c_{\text{dim-6}}}{\Lambda^2} + \frac{c_{\text{dim-8}}}{\Lambda^4}|^2$$

$$\sim g_{SM}^2 + g_{SM} \times \frac{c_{\text{dim-6}}}{\Lambda^2} + \frac{c_{\text{dim-6}}^2}{\Lambda^4} + g_{SM} \times \frac{c_{\text{dim-8}}}{\Lambda^4} + O(\Lambda^{-6})$$

- Validity of keeping dimension-6 squared without dimension-8:
  - Strongly interacting theory: $c \gg g_{SM}$ so that $c_{\text{dim-6}}^2 \gg c_{\text{dim-8}} \times g_{SM}$.
  - Or the UV completion suppresses the dimension-8 terms.
Linear vs. Quadratic terms

Red: including quadratic contributions, Blue: Linear terms only.

Quadratic terms dominate for operators with non-interference.

Little difference between fits with quadratic or linear when operator limited by precision Higgs or EW data, not di-boson distributions.

The LHC has completed two very successful runs and the data analysis is under way.
By the end, expect 20 times more data to be accumulated.
Still may expect to see new physics.
Effective field theories have the benefit of searching for new physics and parameterizing how well we understand the Standard Model at high energies.
We showed a case study of $W^+W^-$ production.
  - Important effects from anomalous quark couplings have been neglected thus far.
  - LHC constraints on anomalous quark couplings can be comparable to LEP already.
  - Can be very significant at 3 ab$^{-1}$ and a proposed 27 TeV machine.
  - We have incorporated anomalous trilinear gauge boson couplings and quark couplings into POWHEG at NLO (publicly available)
  - Have provided sufficient supplemental material so you can perform your own scans using your favorite operator basis.
LHC can constrain dipole operators with electroweak di-boson and Drell Yan data.
Need to be careful with interference and counting:
  - With fully transverse gauge boson production, the dimension-6 EFT has negligible interference with the SM.
  - Observables sensitive to gauge boson polarization are essential to resurrecting this interference.
Thank You
$W^+W^-$ production

- Operators affecting ATGCs:
  \[ O_{3W} = \varepsilon^{abc} W_\mu^{av} W_\nu^{bp} W_\rho^{c\mu} \quad O_{HD} = |\Phi^\dagger D_\mu \Phi|^2 \quad O_{HWB} = \Phi^\dagger \sigma^a \Phi W^a_{\mu \nu} B^{\mu \nu} \]
  \[ O^{(3)}_{H\ell} = i \left( \Phi^\dagger \overleftrightarrow{D}_\mu \sigma^a \Phi \right) \overline{\ell}_L \gamma^\mu \sigma^a \ell_L \quad O_{ll} = (\overline{\ell}_L \gamma^\mu \ell_L)(\overline{\ell}_L \gamma_\mu \ell_L) \]

- In the EW sector have to choose input parameters: $G_F, M_W, M_Z$
- EFT alters relationships between other parameters and input parameters:
  \[ g_Z \to g_Z + \delta g_Z \quad v \to v(1 + \delta v) \quad s_W^2 \to s_W^2 + \delta s_W^2, \]
  where $s_W = \sin \theta_W$, $c_W = \cos \theta_W$ and
  \[ g_Z = \frac{g}{\cos \theta_W} \quad s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \quad G_F = \frac{1}{\sqrt{2}v^2} \]
  \[ \delta v = C_{H\ell}^{(3)} - \frac{1}{2} C_{\ell \ell} \quad \delta \sin^2 W = -\frac{v^2}{\Lambda^2} \frac{s_W c_W}{c_W^2 - s_W^2} \left[ 2s_W c_W \left( \delta v + \frac{1}{4} C_{HD} \right) + C_{HWB} \right] \]
  \[ \delta g_Z = -\frac{v^2}{\Lambda^2} \left( \delta v + \frac{1}{4} C_{HD} \right) \]
Matching ATGCs

- Operators affecting ATGCs:
  \[
  O_{3W} = \varepsilon^{abc} W^a_\mu W^b_\nu W^c_\rho \quad O_{HD} = |D_\mu \Phi|^2 \quad O_{HWB} = \Phi^+ \sigma^a W^a_{\mu\nu} B^{\mu\nu}
  \]
  \[
  O^{(3)}_{H\ell} = i \left( \Phi^+ \gamma^\mu \sigma^a \Phi \right) \bar{\ell} \gamma^\mu \sigma^a \ell 
  \]
  \[
  O_{ll} = (\bar{\ell} \gamma^\mu \ell_L)(\bar{\ell} \gamma^\mu \ell_L)
  \]

- Anomalous Couplings Framework:
  \[
  \delta L = -i g_{WWV} \left( g_1 W^{+\mu}_\nu W^{-\mu}_\nu + \frac{\lambda^V}{M_W^2} W^{+\mu}_\nu W^{-\mu}_\nu V^{\nu\rho} \right)
  \]

- Had 5 dimension-6 operators, only three independent combinations.
Matching ATGCs

- Operators affecting ATGCs:

\[ O_{3W} = \epsilon^{abc} W^a_{\mu} W^b_{\nu} W^c_{\rho} \]
\[ O_{HD} = |\Phi^\dagger D_\mu \Phi|^2 \]
\[ O_{HWB} = \Phi^\dagger \sigma^a \Phi W^{a}_{\mu} B^{\mu \nu} \]

\[ O^{(3)}_{H_L} = i \left( \Phi^\dagger \bar{D}_\mu \sigma^a \Phi \right) \bar{\ell}_L \gamma^\mu \sigma^a \ell_L \]
\[ O_{ll} = \left( \bar{\ell}_L \gamma^\mu \ell_L \right) \left( \bar{\ell}_L \gamma^\mu \ell_L \right) \]

- Anomalous Couplings Framework:

\[ \delta L = -ig_{WWV} \left( g^V_1 (W^+_{\mu\nu} W^{-\mu} V^\nu - W^{-\mu\nu} W^+ V^\nu) + \kappa^V W^+_{\mu} W^{-\mu} V^\nu + \frac{\lambda^V}{M_W^2} W^{+\mu} W^{-\mu\nu} V^{\nu\rho} \right) \]

- Had 5 dimension-6 operators, only three independent combinations.
- In Warsaw basis:

\[ \delta g^Z_1 = \frac{\nu^2}{\Lambda^2} \frac{1}{\cos^2 \theta_W - \sin^2 \theta_W} \left( \frac{\sin \theta_W}{\cos \theta_W} C_{HWB} + \frac{1}{4} C_{HD} + \delta \nu \right) \]
\[ \delta \kappa^Z = \frac{\nu^2}{\Lambda^2} \frac{1}{\cos^2 \theta_W - \sin^2 \theta_W} \left( 2 \sin \theta_W \cos \theta_W C_{HWB} + \frac{1}{4} C_{HD} + \delta \nu \right) \]
\[ \delta \lambda^Z = \frac{\nu}{\Lambda^2} 3 M_W C_{3W} \]

- In the electroweak sector have to choose input parameters: \( G_F, M_W, M_Z \)
- Effective field theory alters relationships between other parameters and input parameters:
Experimental results

- ATGCs actively being searched for in $W^+W^-$ production by both ATLAS \cite{JHEP1609} and CMS \cite{Phys.Lett. B772 (2017)}.

\begin{itemize}
  \item [\textbf{ATLAS}]
    \begin{itemize}
      \item $\sqrt{s} = 8 \text{ TeV}, 20.3 \text{ fb}^{-1}$
      \item 95\% C.L. with LEP Scenario
      \item $\Lambda_{\infty}$
    \end{itemize}
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  \end{itemize}
Anomalous Quark-Gauge Boson Couplings

Anomalous quark-gauge boson couplings occur from the operators

\[
O^{(3)}_{HF,ij} = i \left( \Phi^\dagger \sigma^a D_\mu \Phi - (D_\mu \Phi)^\dagger \sigma^a \Phi \right) \bar{Q}_L i \gamma^\mu \sigma^a Q_L j
\]

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O^{(1)}_{HF,ij} = i \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) \bar{Q}_L i \gamma^\mu Q_L j
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O_{Hf,ij} = i \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) \bar{q}_R i \gamma^\mu q_R j
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Anomalous Quark-Gauge Boson Couplings

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\[ O_{Hf,ij} = i \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) \bar{q}_R i \gamma^\mu q_{Rj} \]

They alter the amplitudes:

\[ \tilde{A}_{\pm \lambda \lambda'} = g_Z^2 \cos^2 \theta_W \left( g_R^{Zq} + \delta g_R^{Zq} \right) \beta_W \frac{E_{CM}^2}{E_{CM}^2 - M_Z^2} A_{\lambda \lambda'}^Z + e^2 Q_q \beta_W A_{\lambda \lambda'}^Y + e^2 Q_q \beta_W A_{\lambda \lambda'}^Y + 2 T_3 \frac{g^2}{\beta_W} (1 + \delta g_W)^2 A_{\lambda \lambda'}^W, \]

We assume flavor diagonal (\( i = j \)) and universal.

From SU(2)_L invariance we have

\[ \delta g_W = \delta g_{Zu} - \delta g_{Zd} \]

4 free anomalous quark couplings.
Anomalous quark-gauge boson couplings occur from the operators

\[ O^{(3)}_{HF,ij} = i \left( \Phi^\dagger \sigma^a D_\mu \Phi - (D_\mu \Phi)^\dagger \sigma^a \Phi \right) \bar{Q}_L \gamma^\mu \sigma^a Q_L j \]

\[ O^{(1)}_{HF,ij} = i \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) \bar{Q}_L i \gamma^\mu Q_L j \]

\[ O_{Hf,ij} = i \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) \bar{q}_R i \gamma^\mu q_R j \]
Anomalous Quark-Gauge Boson Couplings

Anomalous quark-gauge boson couplings occur from the operators

\[ O_{HF,ij}^{(3)} = i \left( \Phi^\dagger \sigma^a D_\mu \Phi - (D_\mu \Phi)^\dagger \sigma^a \Phi \right) \bar{Q}_L \gamma^\mu \sigma^a Q_L j \]

\[ O_{HF,ij}^{(1)} = i \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) \bar{Q}_L \gamma^\mu Q_L j \]

They alter the amplitudes:

\[ \tilde{A}_{-\lambda\lambda'}^{+\lambda\lambda'} = \ g_Z^2 \cos^2 \theta_W \left( g_{R}^{Zq} + \delta g_{R}^{Zq} \right) \beta_W \frac{s}{s - M_Z^2} A_{\lambda\lambda'}^{Z} + e^2 Q_{q} \beta_W A_{\lambda\lambda'}^{Y} + 2 T_3 \ g_Z^2 \beta_W \left( 1 + \delta g_W \right)^2 A_{\lambda\lambda'}^{W} \]

where, assuming flavor diagonal \((i = j)\) and universal,

\[ \delta g_{L}^{Zu} = - \frac{v^2}{2\Lambda^2} \left( C_{HF}^{(1)} - C_{HF}^{(3)} \right) \quad \delta g_{L}^{Zd} = - \frac{v^2}{2\Lambda^2} \left( C_{HF}^{(1)} + C_{HF}^{(3)} \right) \]

\[ \delta g_{R}^{Zu} = - \frac{v^2}{2\Lambda^2} C_{Hu} \quad \delta g_{R}^{Zd} = - \frac{v^2}{2\Lambda^2} C_{Hd} \]

\[ \delta g_W = \delta g_{L}^{Zu} - \delta g_{L}^{Zd} \]
Higher Order QCD Corrections

Known up to NNLO in QCD and NLO in electroweak

Frixione NPB410; Ohnemus PRD44; Dixon, Kunszt, Signer NPB531; Dicus, Kao, Repko PRD36; Glover, van der Bij PLB219; Binoth, Ciccolini, Kauer, Kramer JHEP 0612, JHEP 0503; Baglio, Ninh, Weber PRD94; Bierweiler, Kasprzik, Kuhn, Uccirati JHEP 1211; Bierweiler, Kasprzik, Kuhn JHEP 1312; Billoni, Dittmaier, Jager, Speckner JHEP 1312; Biedermann, Billoni, Denner, Dittmaier, Hofer, Jager, Salfelder JHEP 1606; Gehrmann et al. PRL113; Grazzini et al. JHEP 1608;

Biedermann et al. JHEP 1606
Differential Distributions at NLO by Helicity up to $1/\Lambda^2$

Baglio, Dawson, I. Lewis PRD96 (2017) 073003
Baglio, Dawson, I. Lewis PRD96 (2017) 073003
Choice of Basis

- Have worked in the anomalous coupling framework, however as seen can match the EFT onto these.

- The operators listed before are in a certain basis, the “Warsaw Basis” Grzadkowski, Iskrzynski, Misiak, Rosiek JHEP 10 (2010) 085

\[
O_{3W} = \varepsilon^{abc} W_{\mu}^{a} W_{\nu}^{b} W_{\rho}^{c\mu}, \quad O_{HD} = |\Phi^\dagger D_\mu \Phi|^2, \quad O_{HWB} = \Phi^\dagger \sigma^a W_{\mu\nu}^a B^{\mu\nu}
\]

\[
O_{H\ell}^{(3)} = i \left( \Phi^\dagger \vec{D}_\mu \sigma^a \Phi \right) \bar{\ell}_L \gamma^\mu \sigma^a \ell_L \quad \text{and} \quad O_{\ell\ell} = (\bar{\ell}_L \gamma^\mu \ell_L)(\bar{\ell}_L \gamma^\mu \ell_L)
\]

- There is another set of operators in the so-called HISZ basis Hagiwara, Ishihara, Szalapski, Zeppenfeld PRD48 (1993) 2182:

\[
O_{3W} = \varepsilon^{abc} W_{\mu}^{a} W_{\nu}^{b} W_{\rho}^{c\mu}, \quad O_{DW} = i \frac{g}{2} (D_\mu \Phi)^\dagger \sigma^a W_{\mu\nu}^a D_\nu \Phi
\]

\[
O_{DB} = i \frac{g'}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} D_\nu \Phi
\]

- Many other bases, such as SILH (Strongly Interacting Light Higgs).

- One set of operators can be related to another via the SM equations of motion.
  - Hence, to dimension-6, all these complete sets of operators are equivalent.
  - It’s a choice which basis we work in.
Primitive Cross Sections

- Specialize to a basis with a set of Wilson coefficients

\[ \vec{C} = (C_1, C_2, \ldots, C_m), \]

where \( C_i \sim \Lambda^{-2} \)

- Typically, the amplitude will be linear in \( C_i \) and hence the cross section will be quadratic in \( C_i \).

- Keeping only linear terms

\[
d\sigma^1(\vec{C}) = d\sigma_{SM}(1 - \sum_{i=1}^{m} C_i) + \sum_{i=1}^{m} C_i d\sigma(1; \vec{R}_i)
\]

- \( \vec{R}_i = (0, 0, \ldots, 1, \ldots, 0) \) the Wilson coefficient vector with the i’th Wilson coefficient set to one.

- Hence, \( d\sigma(1; \vec{R}_i) \) is the cross section linear in Wilson coefficients with the i’th Wilson coefficient set to one.

- \( d\sigma(1; \vec{R}_i) \) is a “primitive cross section” and is just a number at this point and independent of the Wilson coefficients.
**Primitive Cross Sections**

- Similar composition at quadratic order:

\[
d\sigma^2(\vec{C}) = d\sigma_{SM}(1 - \sum_{i=1}^{m} C_i) + \sum_{i=1}^{m} C_i d\sigma(1; \vec{R}_i) \\
\quad + \sum_{i=1}^{m} C_i^2 \left(d\sigma(2; \vec{R}_i) - d\sigma(1; \vec{R}_i)\right) \\
\quad + \sum_{i>j=1}^{m} C_i C_j \left[d\sigma(2; \vec{M}_{ij}) - d\sigma(2; \vec{R}_i) - d\sigma(2; \vec{R}_j) + d\sigma_{SM}\right]
\]

- \(\vec{M}_i = (0,0,\ldots,1,\ldots,1,\ldots,0)\) the Wilson coefficient vector with *both* the i’th and j’th Wilson coefficients set to one.
- \(d\sigma(1; \vec{R}_i)\) is the primitive cross linear in Wilson coefficients with the i’th Wilson coefficient set to one.
- \(d\sigma(2; \vec{R}_i)\) is the primitive cross quadratic in Wilson coefficients with the i’th Wilson coefficient set to one.
- \(d\sigma(2; \vec{M}_i)\) is the primitive cross quadratic in Wilson coefficients with the i’th and j’th Wilson coefficient set to one.
- All primitive cross sections are numbers independent of the Wilson coefficients.
Different operator basis, get different primitive cross sections (although total cross section does not change)

\[
d\sigma^2(\vec{C}') = d\sigma_{SM} (1 - \sum_{i=1}^{m} C'_i) + \sum_{i=1}^{m} C'_i d\sigma'(1; \vec{R}_i) \\
+ \sum_{i=1}^{m} C'_i^2 \left( d\sigma'(2; \vec{R}_i) - d\sigma'(1; \vec{R}_i) \right) \\
+ \sum_{i>j=1}^{m} C'_i C'_j \left[ d\sigma'(2; \vec{M}_{ij}) - d\sigma'(2; \vec{R}_i) - d\sigma'(2; \vec{R}_j) + d\sigma_{SM} \right]
\]

If you know the relationship between $\vec{C}$ and $\vec{C}'$, can find relationship between the primitive cross sections $d\sigma$ and $d\sigma'$.

Master formulas can be found in Baglio, Dawson, I. Lewis, PRD99 (2019) 035029

In supplemental material of Baglio, Dawson, I. Lewis, PRD99 (2019) 035029 can find primitive cross sections for Warsaw and HISZ basis at LO and NLO for a variety of distributions.

Using master formula, can take these primitive cross sections and change to your favorite set of operators.
LEP bounds on anomalous quark-gauge boson couplings are not centered about zero \cite{Falkowski1502}:

\[
\begin{align*}
\delta g_{LZd}^L &= (2.3 \pm 1) \times 10^{-3} \\
\delta g_{LZu}^L &= (-2.6 \pm 1.6) \times 10^{-3} \\
\delta g_{Zd}^R &= (16.0 \pm 5.2) \times 10^{-3} \\
\delta g_{Zu}^R &= (-3.6 \pm 3.5) \times 10^{-3}
\end{align*}
\]

The “last bin” was determined using the Standard Model cross section.

- At high luminosity, to obtain Standard Model cross section need non-zero anomalous trilinear gauge boson couplings to cancel non-zero anomalous quark-gauge boson couplings.
- Hence, the contours including anomalous quark-gauge boson couplings are centered off zero and do not overlap with those including only anomalous trilinear gauge boson couplings.
- Now, center anomalous quark gauge boson couplings at zero and keep same uncertainties.
**14 TeV HL-LHC, 3 ab$^{-1}$, LEP bounds centered at zero**

- **Black, red:** ATGCs only
- **Blue, green:** ATGCs+anomalous quark couplings.
- **Black, blue:** $\delta_{sys} = 16\%$
- **Red, green:** $\delta_{sys} = 4\%$
- Areas inside contours allowed.

HL-LHC 14 TeV NLO projections with $pp \to e^\pm \mu^\mp \nu \nu | \mathcal{L} = 3$ ab$^{-1}$

- 3GB, $\delta_{sys} = 4\%$
- 3GB, $\delta_{sys} = 16\%$
- 3GB+Ferm$'$, $\delta_{sys} = 4\%$
- 3GB+Ferm$'$, $\delta_{sys} = 16\%$
27 TeV HE-LHC, 15 ab$^{-1}$, LEP bounds centered at zero

- Black, red: ATGCs only
- Blue, green: ATGCs+anomalous quark couplings.
- Black, blue: $\delta_{\text{sys}} = 16\%$
- Red, green: $\delta_{\text{sys}} = 4\%$
- Areas inside contours allowed.