

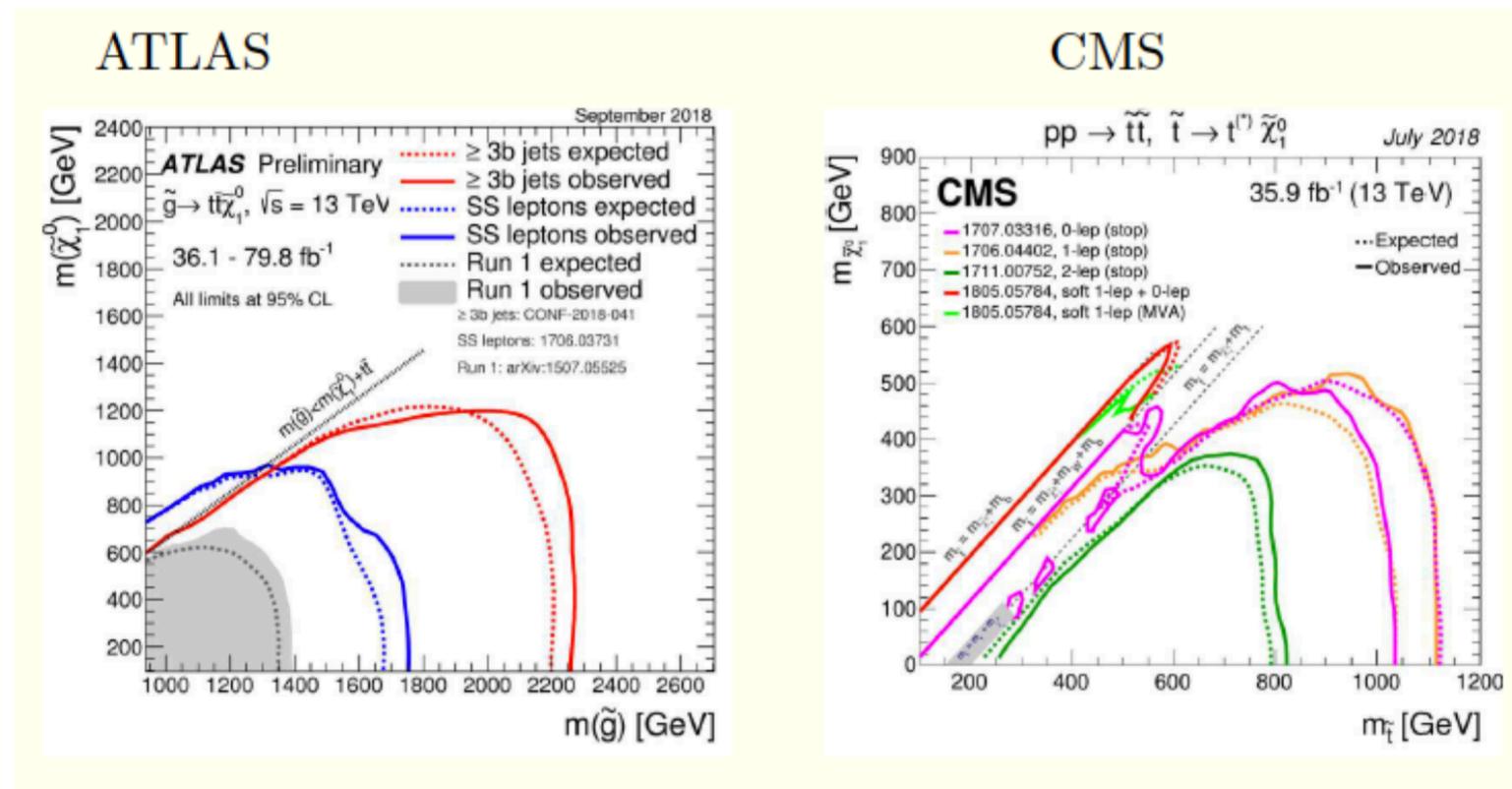
The Landscape: Suggestive Trends in Low-energy Phenomenology

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Based on arXiv:1712.01399, arXiv:1502.07746
with Howie Baer, Gordy Kane, ...

Where we are

No superpartners at LHC



$$m_{\tilde{g}} > 2.25 \text{ TeV}$$

$$m_{\tilde{t}_1} > 1.1 \text{ TeV}$$

No thermal WIMP

Summary

Simple statistical arguments + weak-scale anthropic requirements explain lack of super partners

The ubiquity of moduli fields explains why the thermal WIMP miracle hasn't materialized

A Prediction from 2012

Mike Douglas

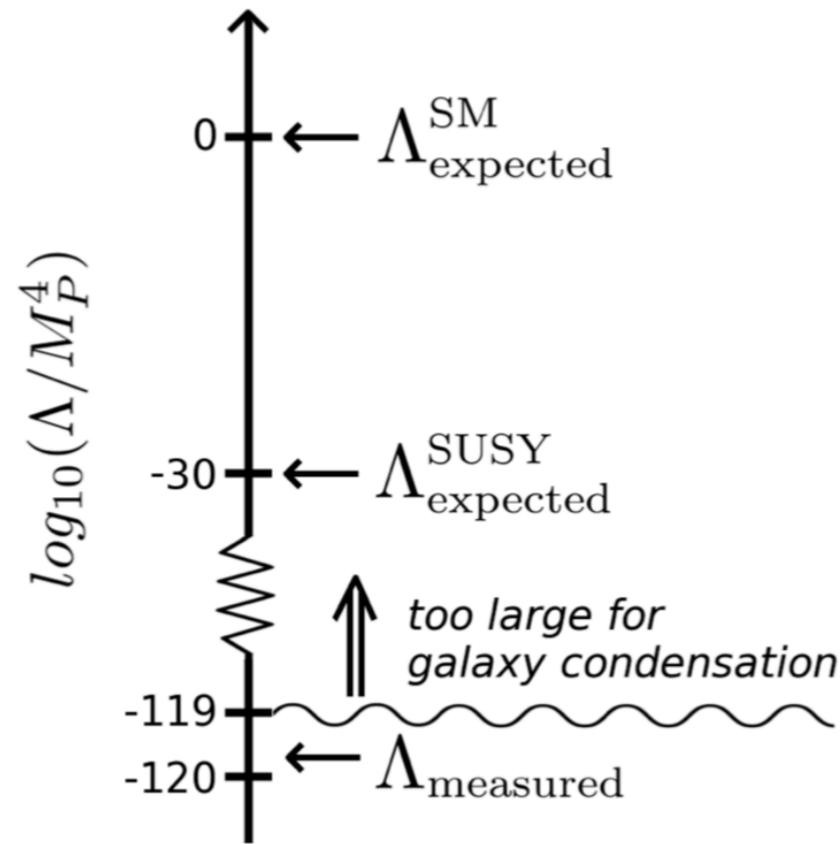
still try to see as far as he or she can, I am going to go out on a limb and argue that

String/M theory will predict that our universe has supersymmetry, broken at the 30 – 100 TeV scale. If at the lower values, we may see gluinos at LHC, while if at the higher values, it will be very hard to see any evidence for supersymmetry.

- String theory and low-energy supersymmetry ([arXiv:1204.6626](https://arxiv.org/abs/1204.6626))

The arguments relied on statistics of flux vacua and the string landscape

A Prediction from 1988



Steven Weinberg

the great predictions of the latter 20th century [4]: namely that given a multiverse which includes a vast assortment of pocket-universes with varying cosmological constants, then it may not be surprising to find $\Lambda \sim 10^{-120} m_P^4$ since if it was much bigger, then galaxy condensation would not occur and we would not even be here to discuss the issue. The situation is portrayed in Fig. 1 which depicts the fact that the cosmological constant ought to be at its most natural value *subject to the constraint that we can exist to observe it*. Such anthropic reasoning relies on the existence of a vast landscape of possibilities that is provided for by the discretuum of flux vacua from string theory [1–3, 5].

The Setting

- String/M theory has many consistent vacuum states which at least roughly match the Standard Model, and might be candidates to describe our world.
- The number of vacua is so large that the problems of reproducing the Standard Model in detail, and the classic problems of “beyond the Standard Model physics” such as the hierarchy problem and cosmological constant problem, might admit statistical solutions. The basic example is that in an ensemble of N vacua which differ only in a parameter Λ (say the c.c. as in [6]), and in which Λ is uniformly distributed, it is likely that a quantity which appears fine tuned by an amount $\epsilon > 1/N$ (for the c.c., 10^{-120} in a generic nonsupersymmetric theory) will be realized by at least one vacuum, just on statistical grounds.
- No single vacuum is favored by the theory. Although selection principles might be found, they will not determine a unique vacuum *a priori*, but rather cut down the possibilities in a way which is useful only when combined with other information.

Douglas, 2004: Statistical analysis of the SUSY breaking scale, hep-th/0405279

Ensembles of Flux Vacua

d=4, N=1 effective supergravity, type IIb

$$V = e^{K/m_P^2} \left(g^{i\bar{j}} D_i W D_{\bar{j}} W^* - \frac{3}{m_P^2} |W|^2 \right) + \frac{1}{2} \sum_{\alpha} D_{\alpha}^2$$

$$= e^{K/m_P^2} \left(\sum_i |F_i|^2 - 3 \frac{|W|^2}{m_P^2} \right) + \frac{1}{2} \sum_{\alpha} D_{\alpha}^2$$

$$\Lambda_{cc} = m_{hidden}^4 - 3e^{K/m_P^2} |W|^2 / m_P^2$$

$$\partial V / \partial z^i = \partial V / \partial \bar{z}^{\bar{i}} = 0.$$

Ensembles of Flux Vacua

$$dN_{F,metastable}(z) = \sum_i \delta_z(V'(z)) \theta(V''(z))$$

where $\theta(V'')$ is 1 when the $2n \times 2n$ real matrix of squared bosonic masses $M = V''$ is positive definite. The derivatives of V appearing here are

$$\begin{aligned}\partial_a V &= e^{\mathcal{K}} (D_a D_b W \bar{D}^b \bar{W} - 2D_a W \bar{W}) \\ D_a \partial_b V &= e^{\mathcal{K}} (D_a D_b D_c W \bar{D}^c \bar{W} - D_a D_b W \bar{W}) \\ \bar{D}_{\bar{a}} \partial_b V &= e^{\mathcal{K}} (R^d{}_{c\bar{a}b} D_d W \bar{D}^c \bar{W} + g_{b\bar{a}} D_c W \bar{D}^c \bar{W} - D_b W D_{\bar{a}} \bar{W} \\ &\quad - 2g_{b\bar{a}} W \bar{W} + D_b D_c W \bar{D}_{\bar{a}} \bar{D}^c \bar{W}),\end{aligned}$$

where R is the curvature of the cotangent bundle, i.e. $R^d{}_{c\bar{a}b} X_d \equiv [\nabla_a, \bar{\nabla}_{\bar{b}}] X_c = \bar{\partial}_{\bar{b}}(g^{\bar{e}d} \partial_a g_{c\bar{e}}) X_d$.

$$W(z), \mathcal{K}(z, \bar{z})$$

Ensembles of Flux Vacua

$$W = \int_X G_4 \wedge \Omega = N^\alpha \Pi_\alpha$$

X : elliptically fibered CY 4-fold. F-theory compactification

$\Pi_\alpha = \int \Sigma_\alpha \wedge \Omega$ are the periods of some basis $\{\Sigma_\alpha\}$ of $H^4(X, \mathbb{Z})$

Weil Petersson metric on complex structure moduli space

$$\mathcal{K} = -\ln \hat{\Pi}_\alpha (\eta^{-1})^{\alpha\beta} \hat{\Pi}_\beta$$

Can replace sum over flux quanta by integral

Useful to change variables using the fact that Ω and its derivatives supply a Hodge decomposition basis of $H^4(X, \mathbb{Z})$

Douglas and Denef, 2004: Distributions of Flux Vacua [hep-th/0404116](https://arxiv.org/abs/hep-th/0404116)

Ensembles of Flux Vacua

Finally left with density per unit volume in moduli space

$$\int_R d^{2n} z \rho(z)$$

A simplified picture of the results is that one can define an “average density of vacua” in the moduli space, which can be integrated over a region of interest and then multiplied by a “total number of allowed choices of flux,” to estimate the total number of vacua which stabilize moduli in that region. This estimate becomes exact in the limit of large flux,

Computed explicitly for simple compactifications: T6, conifold, mirror quintic. Based on results, general arguments for landscape were advanced

Douglas and Denef, 2004: Distributions of Flux Vacua [hep-th/0404116](https://arxiv.org/abs/hep-th/0404116)

SUSY Breaking Vacua

$$dN_{vac}[F_i, D_\alpha, \hat{\Lambda}] = \prod dF dD d\hat{\Lambda} \rho(F_i, D_\alpha, \hat{\Lambda}).$$

$$dN_{\Lambda=0}[F_i, D_\alpha] = \prod d^2 F_i dD_\alpha d\hat{\Lambda} \rho(F_i, D_\alpha, \hat{\Lambda}) \delta\left(\sum_i |F_i|^2 + \sum_\alpha D_\alpha^2 - 3e^K |W|^2\right)$$

$$\Lambda = \sum_i |F_i|^2 + \sum_\alpha D_\alpha^2 - 3e^K |W|^2$$

distribution for SUSY breaking vacua decouples from the cosmological constant problem

$$\underline{dN_{\Lambda=0}[F_i, D_\alpha]} = \prod d^2 F_i dD_\alpha \rho(F_i, D_\alpha)$$

CC Distribution

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

$$m_{hidden}^2 = \sum_i |F_i|^2 + \frac{1}{2} \sum_\alpha D_\alpha^2$$

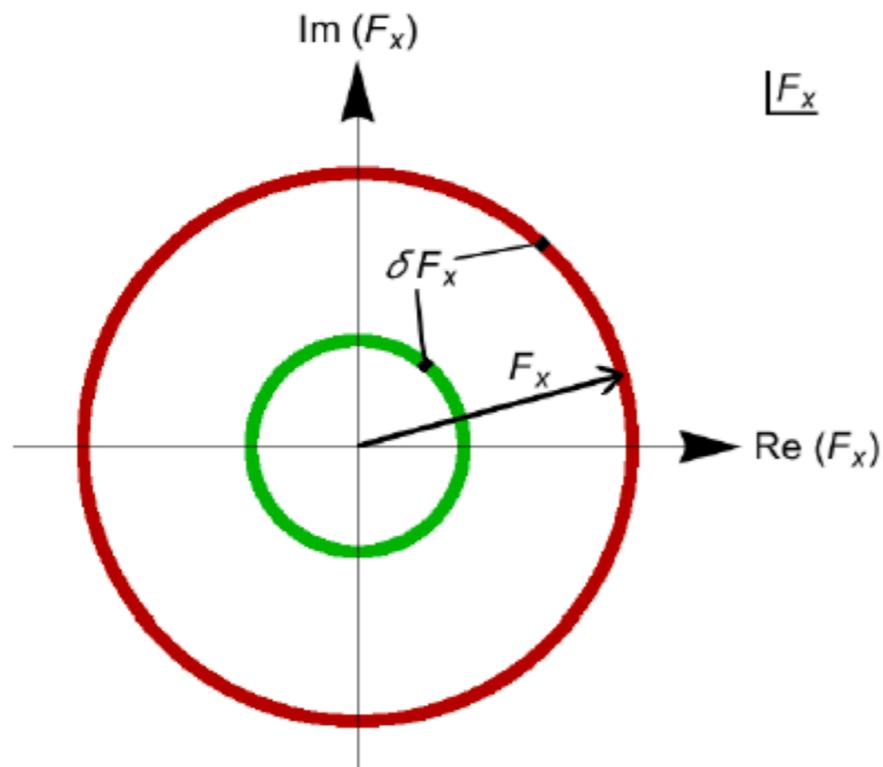
$$m_{weak} \simeq m_{W,Z,h} \simeq 100 \text{ GeV}$$

f_{cc} decouples

Regarding the role of the cosmological constant in determining the SUSY breaking scale, a key observation of Denef and Douglas [10, 11] and Susskind [9] was that W at the minima is distributed uniformly as a complex variable, and the distribution of $e^{K/m_P^2} |W|^2 / m_P^2$ is not correlated with the distributions of F_i and D_α . Setting the cosmological constant to nearly zero, then, has no effect on the distribution of supersymmetry breaking scales. Physically, this can be understood by the fact that the superpotential receives contributions from many sectors of the theory, supersymmetric as well as non-supersymmetric. The cosmological fine-tuning penalty is $f_{cc} \sim \Lambda / m^4$ where the above discussion leads to $m^4 \sim m_{string}^4$ rather than $m^4 \sim m_{hidden}^4$, rendering this term inconsequential for determining the number of vacua with a given SUSY breaking scale.

Distribution of Vacua

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) f_{EFT} \cdot f_{cc} \cdot dm_{hidden}^2$$



$$\underline{dN_{\Lambda=0}[F_i, D_\alpha]} = \prod d^2 F_i dD_\alpha \rho(F_i, D_\alpha)$$

$$f_{SUSY}(m_{hidden}^2) \sim (m_{hidden}^2)^{2n_F + n_D - 1}$$

In words, the total supersymmetry breaking scale is the distance from the origin in the space of supersymmetry breaking parameters, and in a high dimensional space most of the volume is near the boundary. Essentially the same observation

- more complicated regimes or multi-modulus compactifications?

Caveats

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) f_{EFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

$$f_{SUSY}(m_{hidden}^2) \sim (m_{hidden}^2)^{2n_F + n_D - 1}$$

- enhancement of number of vacua near conifold points: should skew vacua, but still vast majority of vacua do not produce exponentially small scales
- multiple F and D terms will skew distribution to high scales
- partially supersymmetric sectors may skew the distribution to low scales

Summary

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

- $f_{cc} \sim \Lambda/m^4$ where DD maintain $m \sim m_{string}$ and not m_{hidden}
- $f_{SUSY}(m_{hidden}^2) \sim (m_{hidden}^2)^{2n_F+n_D-1}$ for uniformly distributed values of F and D breaking fields
- $f_{EWFT} \sim m_{weak}^2/m_{soft}^2$ (?) where $m_{soft} \sim m_{3/2} \sim m_{hidden}^2/m_P$

$$n = 2n_F + n_D - 1$$

$$f_{SUSY} \sim m_{soft}^n$$

n_F	n_D	n
0	1	0
1	0	1
0	2	1
1	1	2
0	3	2
2	0	3
2	1	4

landscape favors high-scale SUSY breaking tempered by electroweak fine-tuning penalty

EWFT Penalty

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

$$f_{EWFT} = ?$$

An effective field theory (or specific coupling or observable) T_1 is more natural in string theory than T_2 if the number of phenomenologically acceptable vacua leading to T_1 is larger than the number leading to T_2 .

Douglas, 2012

anthropics lies here

Question: can we quantify these ideas into what the LHC is seeing?

EWFT Penalty

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

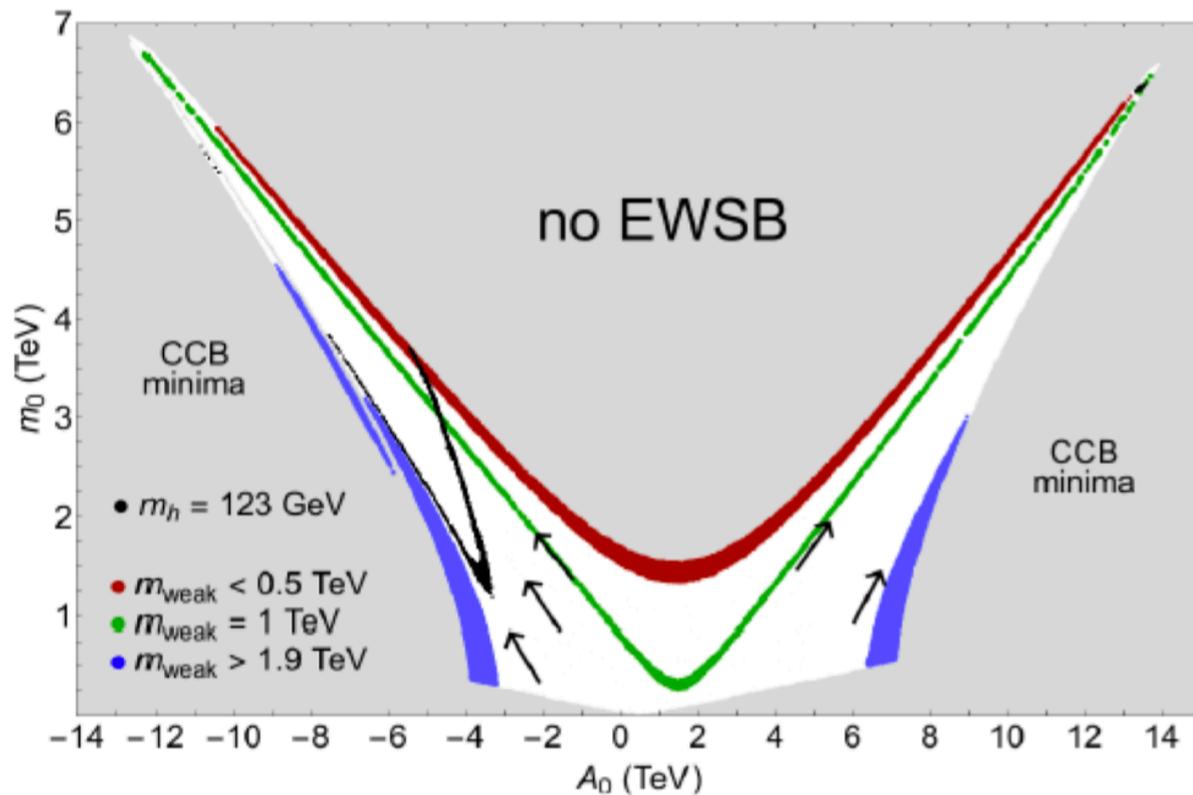
The final term f_{EWFT} merits some discussion. Following Ref. [17], an initial guess [9, 11, 13] for f_{EWFT} was that $f_{EWFT} \sim m_{weak}^2/m_{soft}^2$ which may be interpreted as conventional naturalness in that the larger the Little Hierarchy between m_{weak} and m_{soft} , then the greater is the fine-tuning penalty. As pointed out in Ref. [18], there are several problems with this ansatz.

$$f_{EWFT} \sim m_{weak}^2/m_{soft}^2 \quad ?$$

This particular term needs to be treated with care

EWFT Penalty

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{hidden}^2$$



$$f_{EWFT} \sim m_{weak}^2 / m_{soft}^2 \quad ?$$

As soft terms such as the trilinear A_t terms increase, one is ultimately forced into charge-or-color-breaking vacua of the MSSM [19, 20]. These sorts of vacua must be entirely vetoed on anthropic grounds.

As high-scale soft terms such as $m_{H_u}^2$ increase too much, then they are no longer driven to negative values and electroweak symmetry isn't even broken. These non-EWSB solutions also should be vetoed on anthropic grounds.

Fine-tuning Penalty

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

$$f_{EWFT} \sim m_{weak}^2 / m_{soft}^2$$

Even in the event of appropriate EWSB, the factor $f_{EWFT} \sim m_{weak}^2 / m_{soft}^2$ penalizes but does not forbid vacua with a weak scale far larger than its measured value. In contrast, Agrawal *et al.* [7] have shown that a weak scale larger than ~ 3 times its measured value would lead to much weaker weak interactions and a disruption in nuclear synthesis reactions, and likely an unlivable universe as we know it. In addition, Susskind posits that an increased weak scale would lead to larger SM particle masses and consequent disruptions in both atomic and nuclear physics. From these calculations, it seems reasonable to *veto* SM-like vacua which lead to a weak scale more than (conservatively) four times its measured value.

Agrawal, Barr, Donoghue, Seckel (1998)

Donoghue Penalty

Minimization of MSSM Higgs potential

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2.$$

Radiative corrections from (s)particles with Yukawa/gauge coupling to Higgs

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2 (\frac{1}{4} - \frac{2}{3}x_W) \Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

$$\Delta_{EW} \equiv \max_i (C_i) / (M_Z^2/2)$$

where $C_{H_u} = | -m_{H_u}^2 \tan^2 \beta / (\tan^2 \beta - 1) |$, $C_{H_d} = | m_{H_d}^2 / (\tan^2 \beta - 1) |$ and $C_\mu = | -\mu^2 |$, along with analogous definitions for $C_{\Sigma_u^u(k)}$ and $C_{\Sigma_d^d(k)}$. Low Δ_{EW} means less fine-tuning.

Donoghue Penalty

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} \cdot dm_{hidden}^2$$

$$case \mathbf{A} : f_{EWFT} \rightarrow \Theta(30 - \Delta_{EW}),$$

Σ_u^u generate large values of the weak scale $m_{weak} \gg 100$ GeV. The value of $\Delta_{EW} < 30$ then corresponds to calculated anthropic requirements from Agrawal *et al.* that the weak scale not deviate by more than a factor of several from its measured value [33]. In this case, $\Delta_{EW} = 30$ corresponds to a Z mass nearly four times its measured value.

$$case \mathbf{B} : f_{EWFT} \rightarrow \Delta_{EW}^{-1}$$

$$\Delta_{EW}^{-1} \sim (m_Z^2/2)/\max [|m_{H_u}^2(weak)| \text{ or } \mu^2 \text{ or } |\Sigma_u^u(i)|]$$

Instead of placing a generic m_{soft}^2 in the denominator of Eq. 11, we place the maximal weak scale contribution to the magnitude of the weak scale. Rather than placing a sharp cutoff on

m_{soft}^n Scan Results

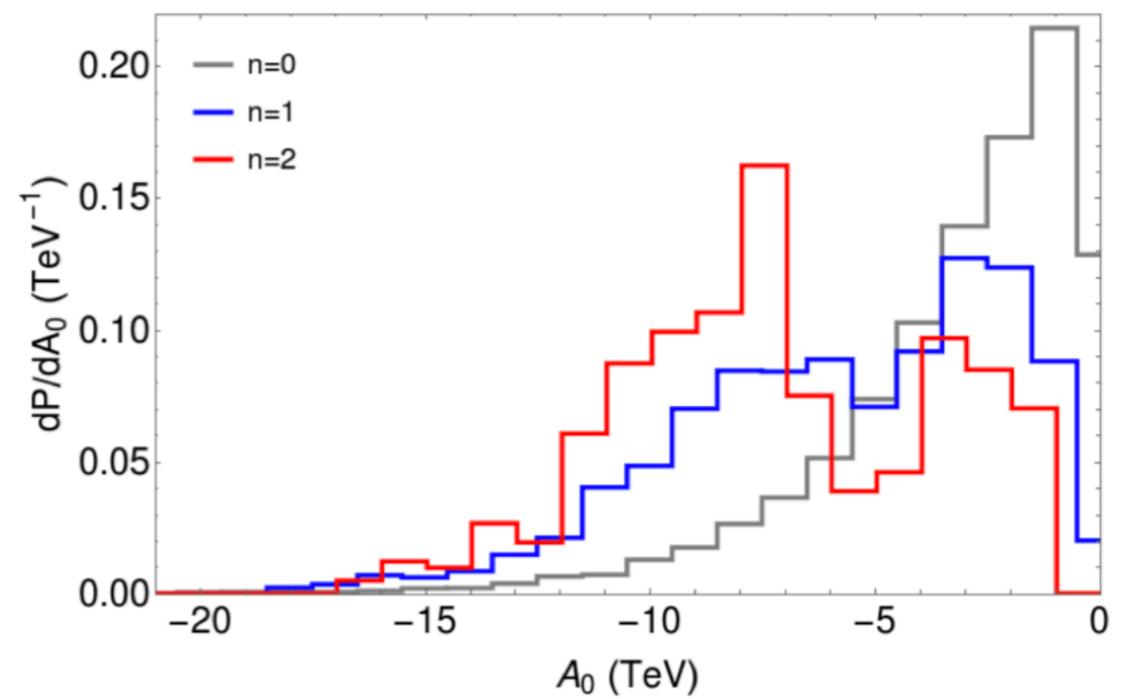
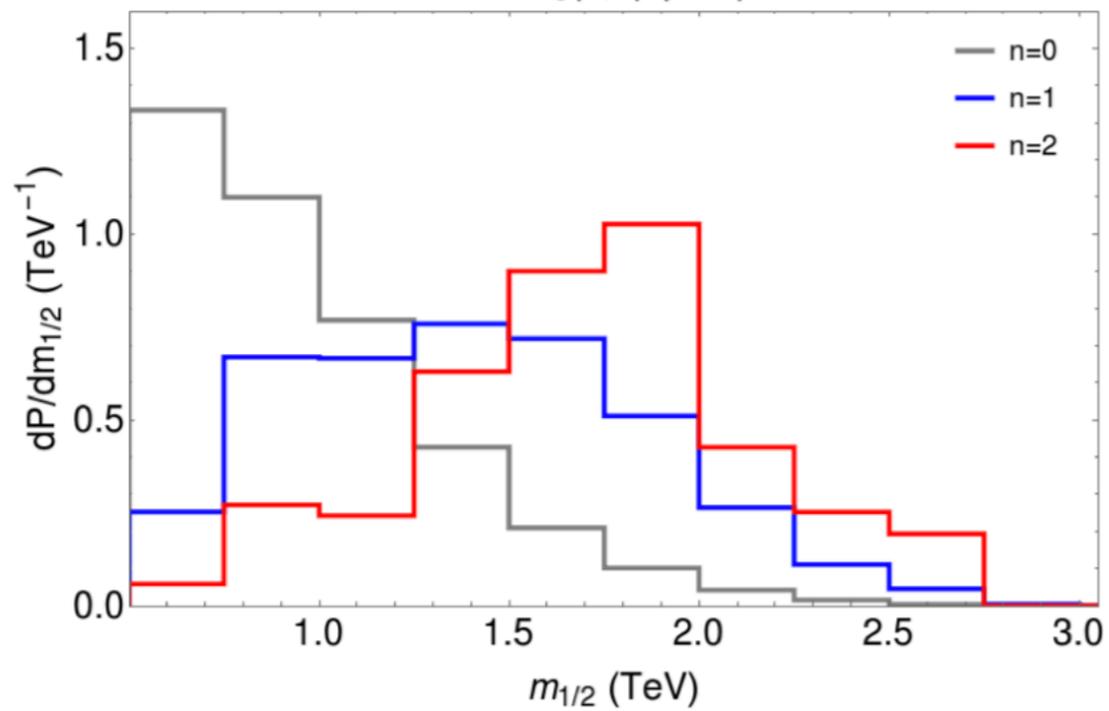
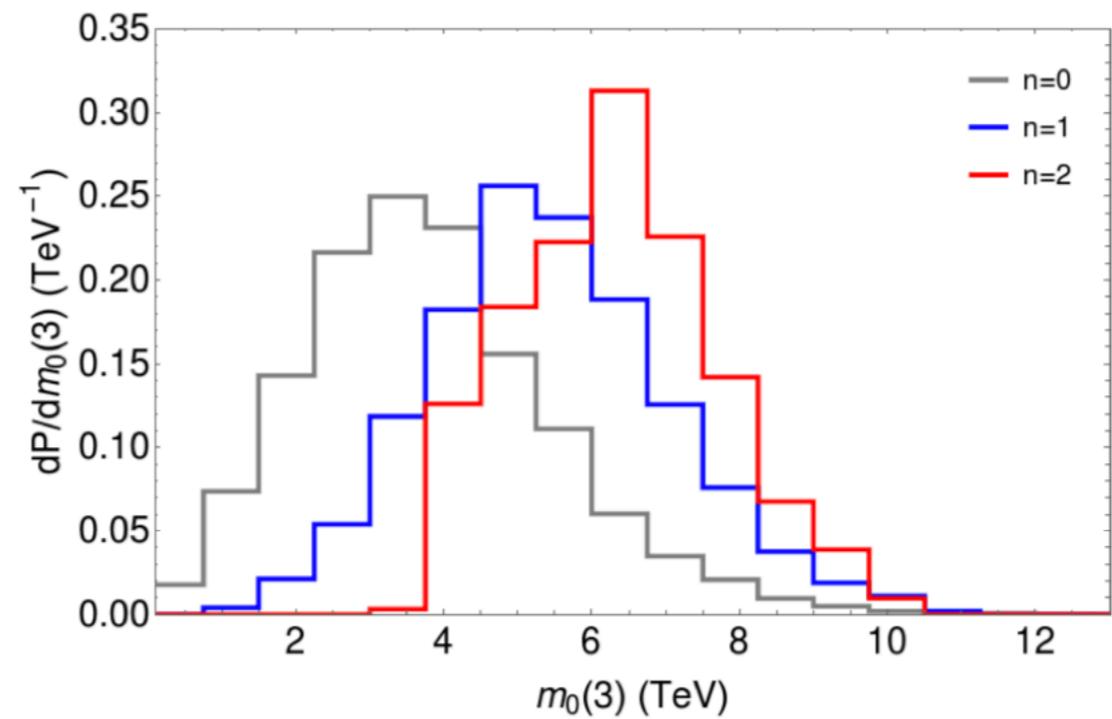
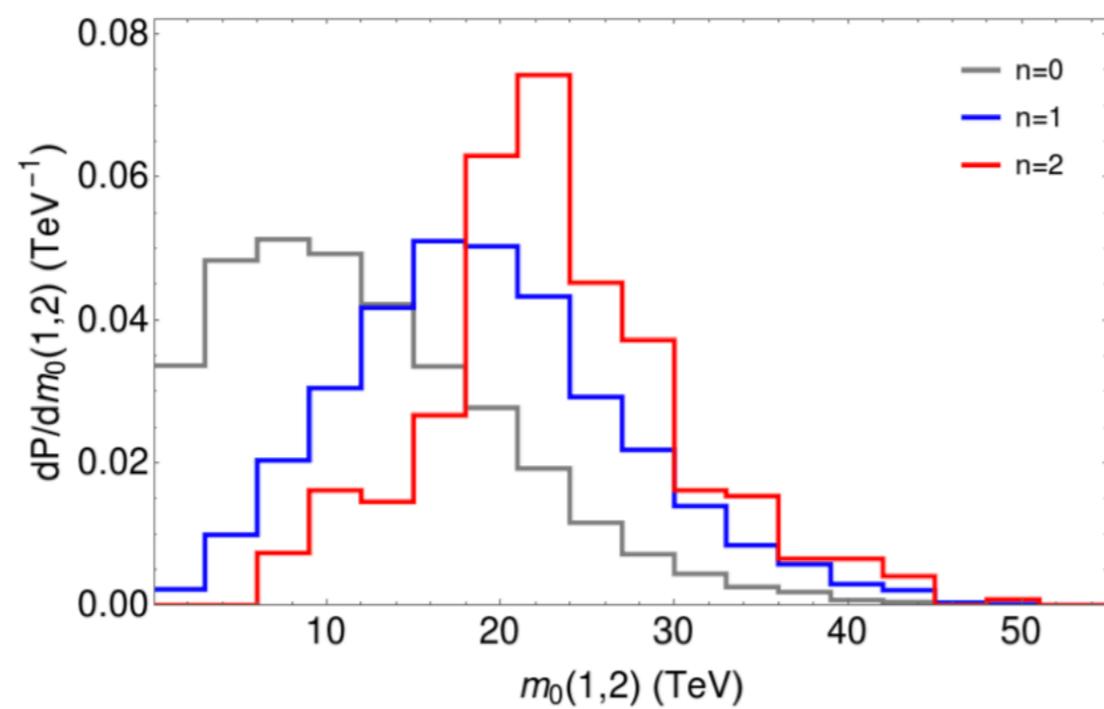
$m_0(1, 2), m_0(3), m_{1/2}, A_0, \tan \beta, \mu, m_A$ (NUHM3)

- $m_0(1, 2) : 0.1 - 60$ TeV,
 - $m_0(3) : 0.1 - 20$ TeV,
 - $m_{1/2} : 0.5 - 10$ TeV,
 - $A_0 : -50 - 0$ TeV,
 - $m_A : 0.3 - 10$ TeV,
- ‡ $\mu = 150$ GeV

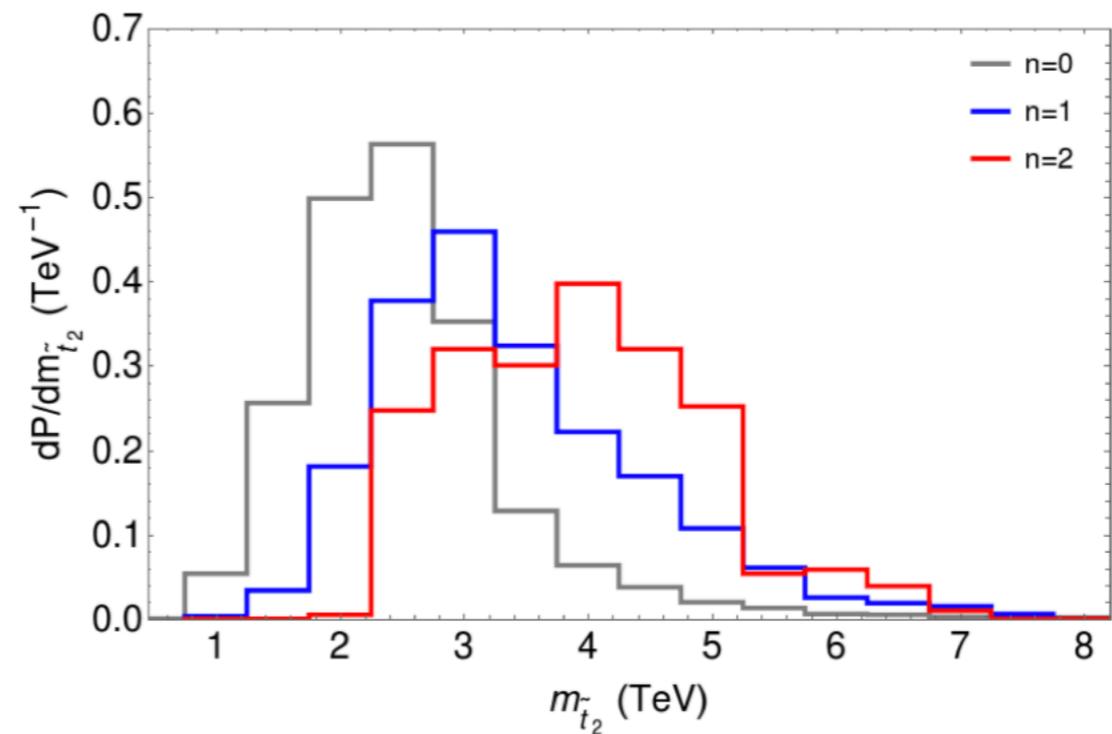
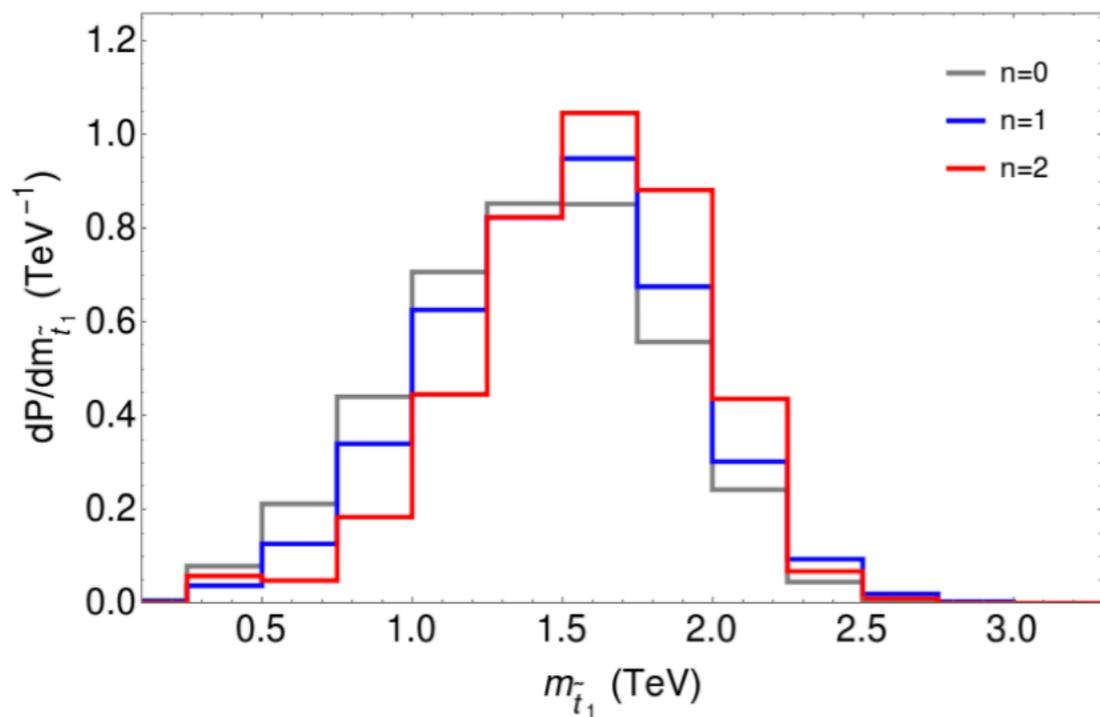
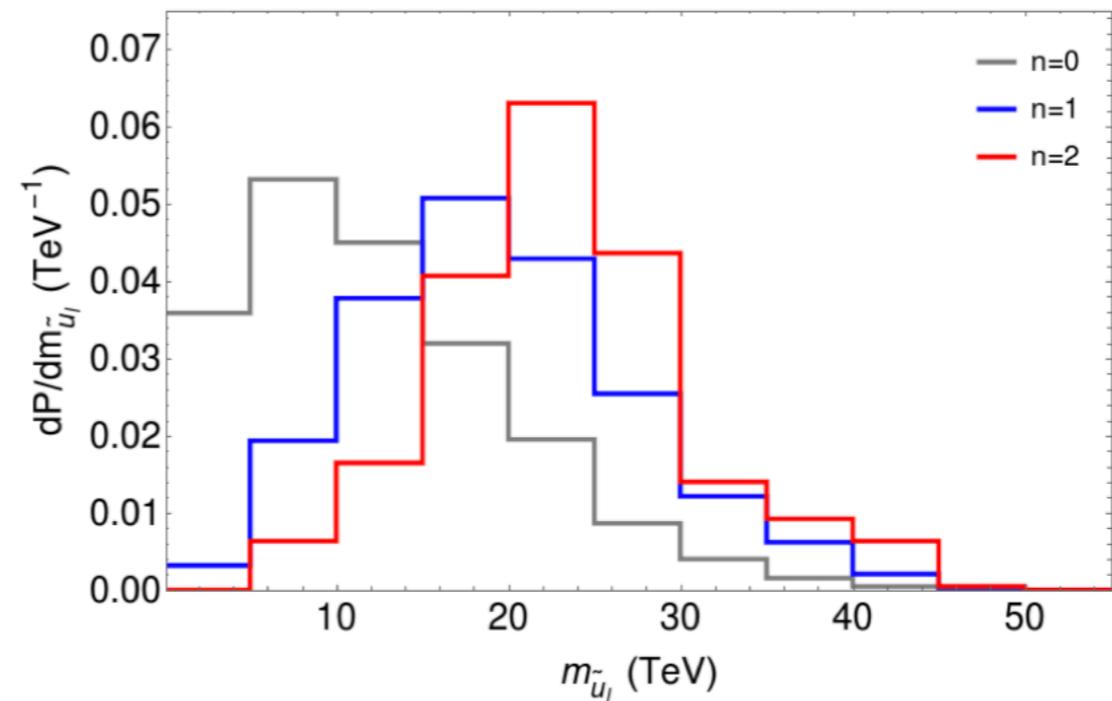
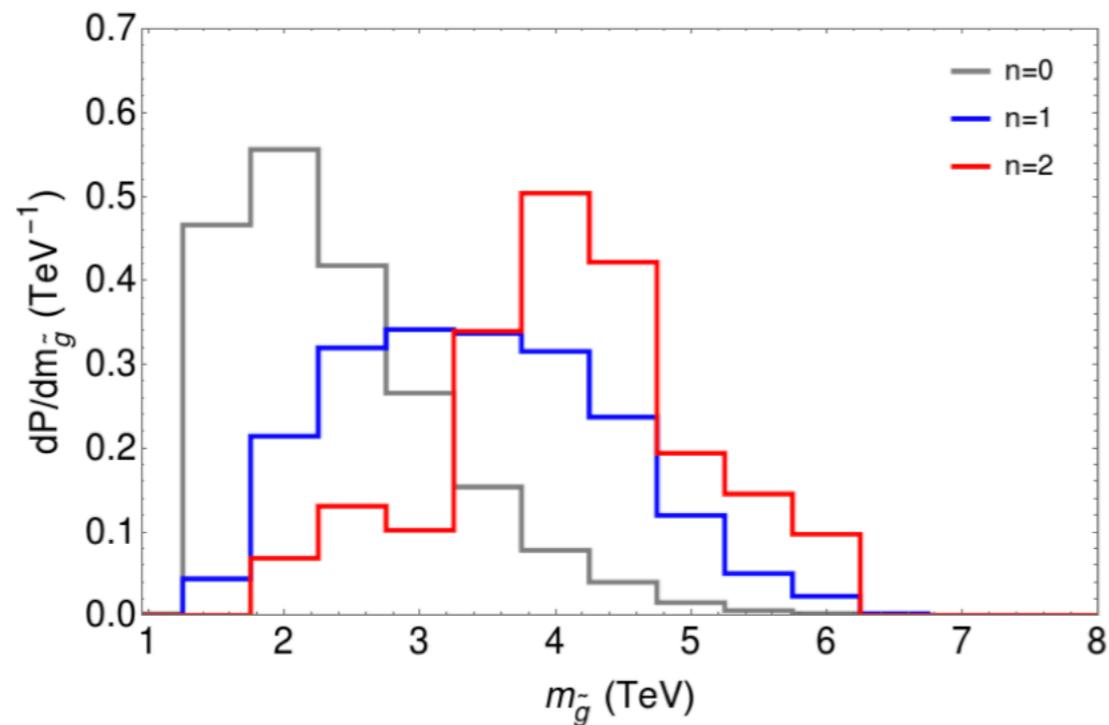
$\tan \beta : 3 - 60$ scanned uniformly.

neutral (n=0), linear (n=1), quadratic (n=2) scan

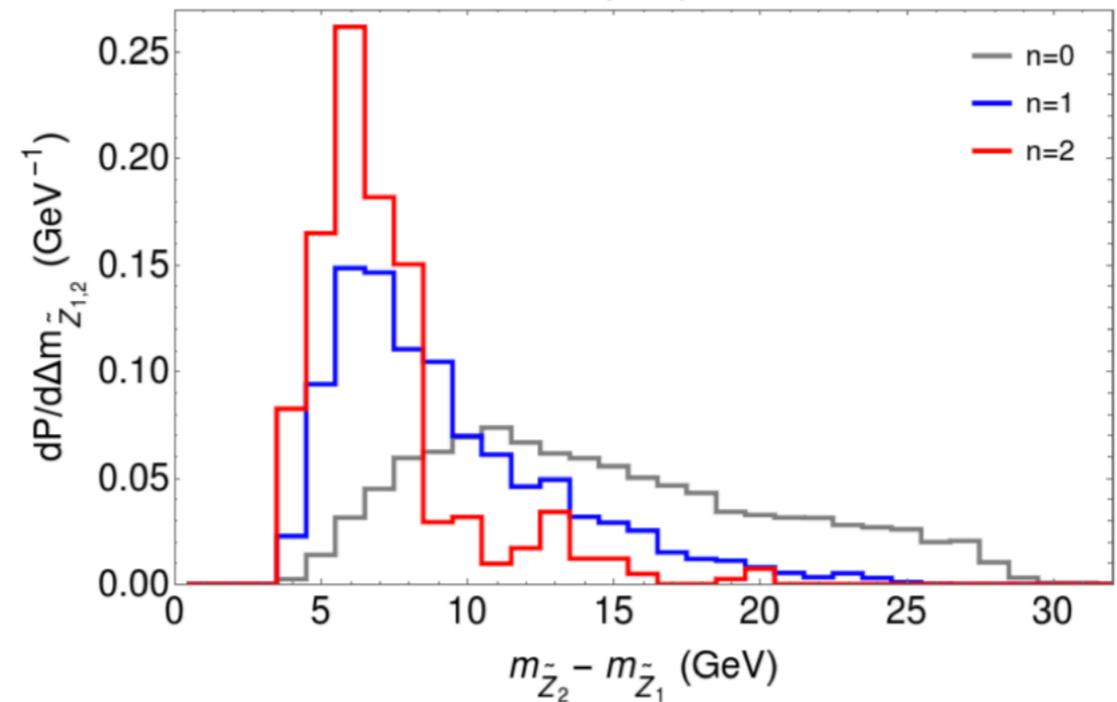
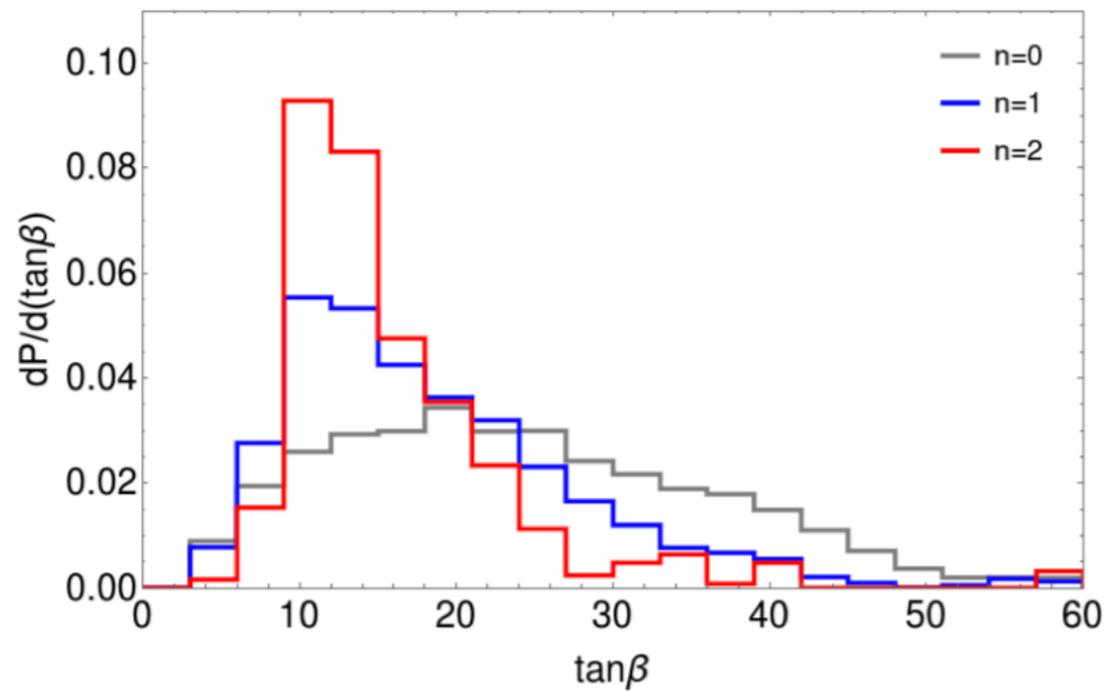
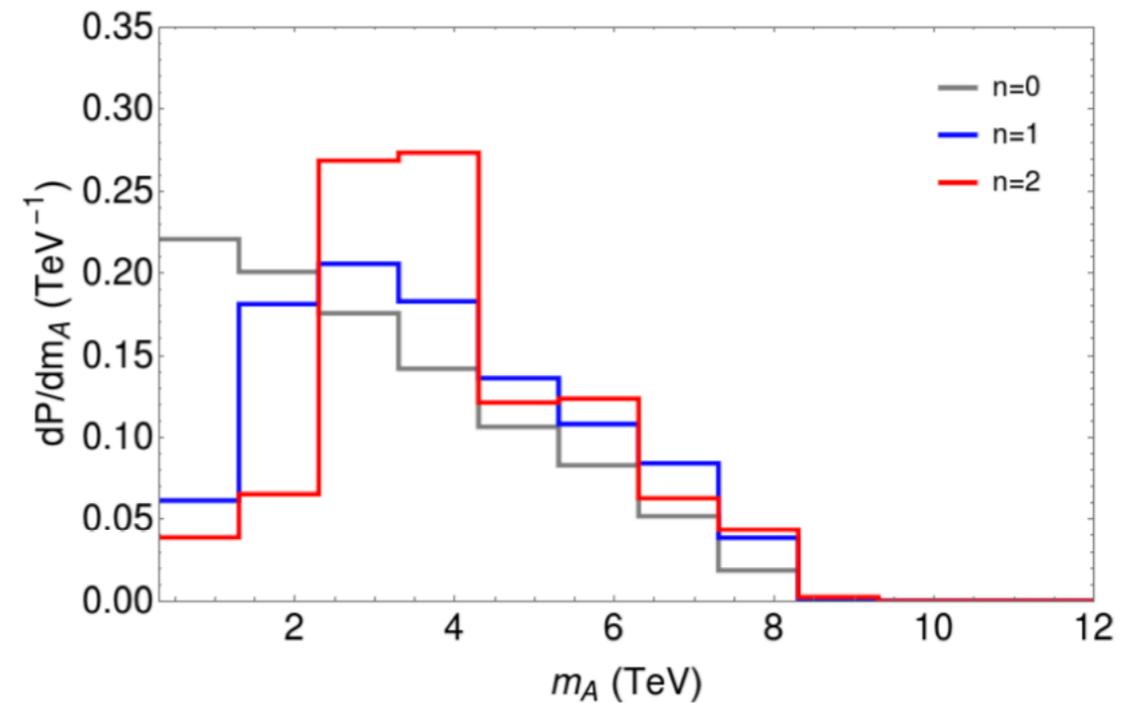
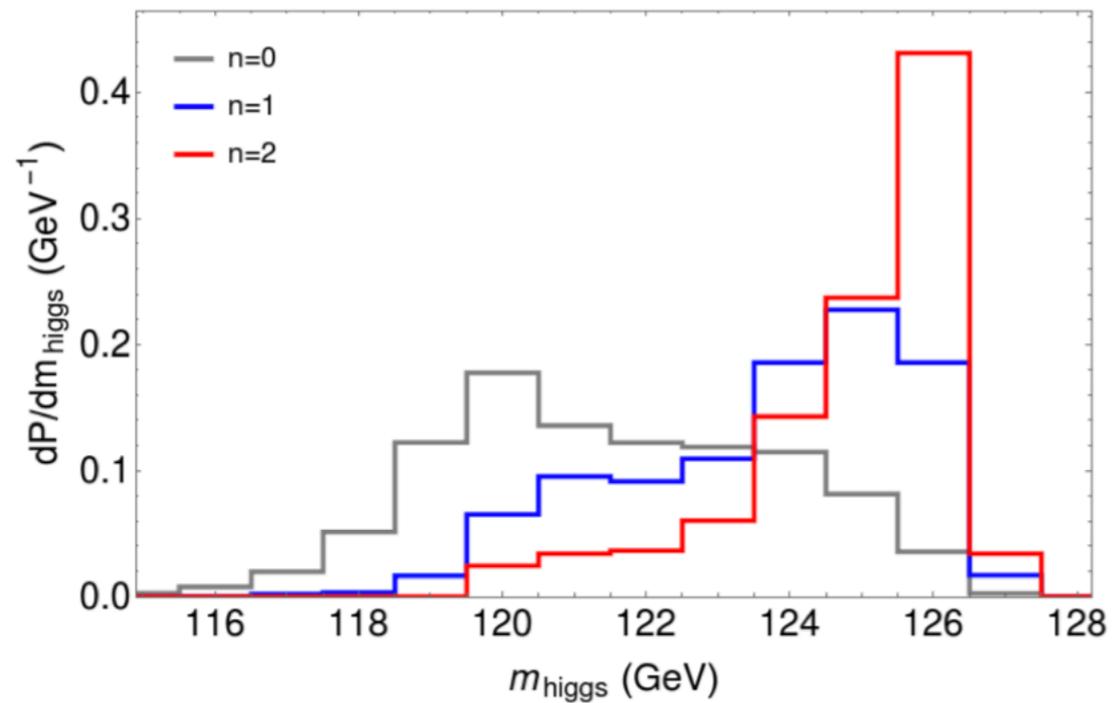
m_{soft}^n Scan Results



m_{soft}^n Scan Results



m_{soft}^n Scan Results



m_{soft}^n Scan Results

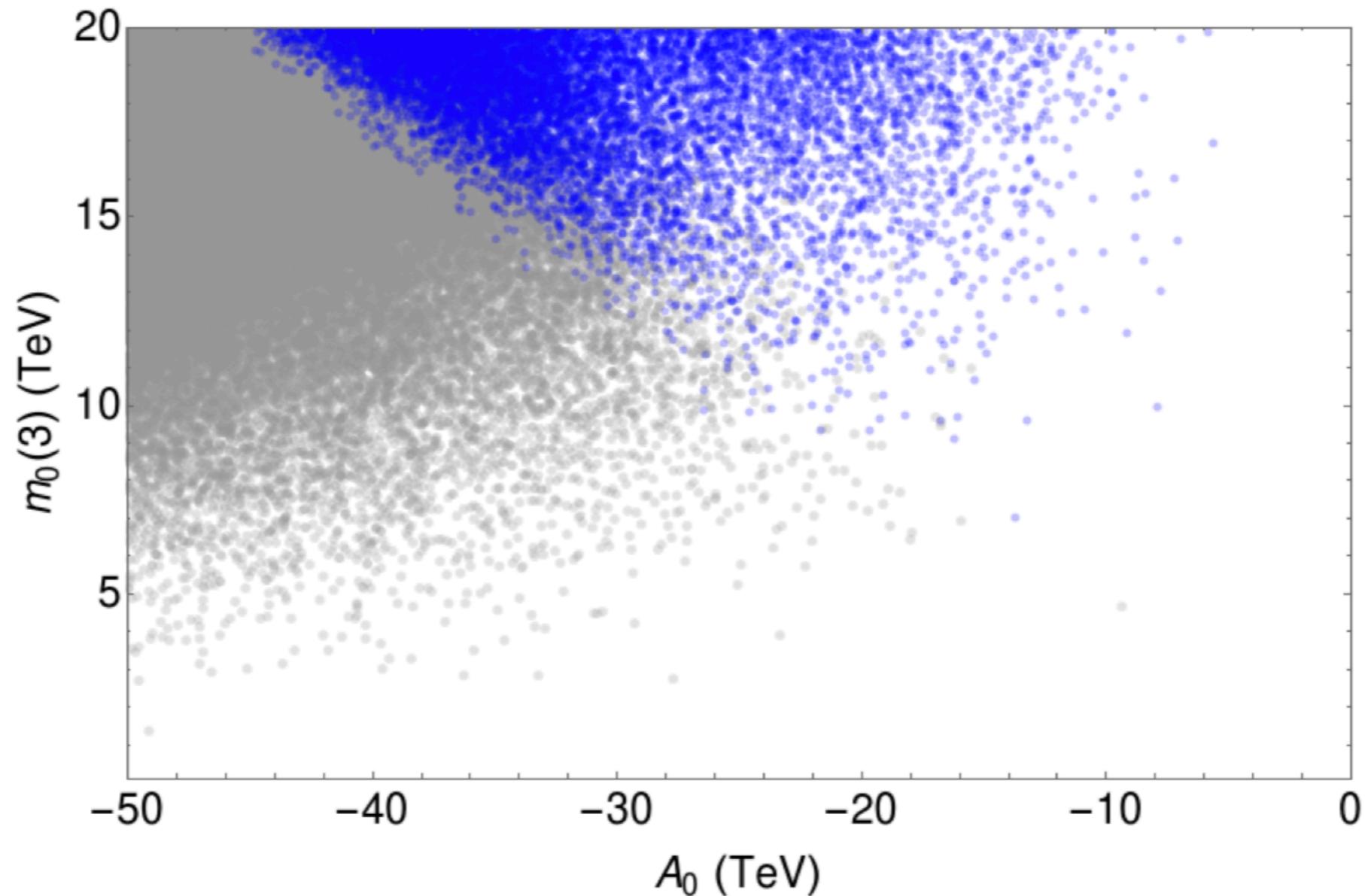


Figure 5: Locus of 100K scan points from a scan with $n = 4$ and scan range as below Eq. [15]. The gray points have either CCB scalar potential minima or no EWSB. The blue points admit EWSB but all have $\Delta_{EW} > 240$ corresponding to a weak scale greater than ~ 1 TeV.

m_{soft}^n Scan Results

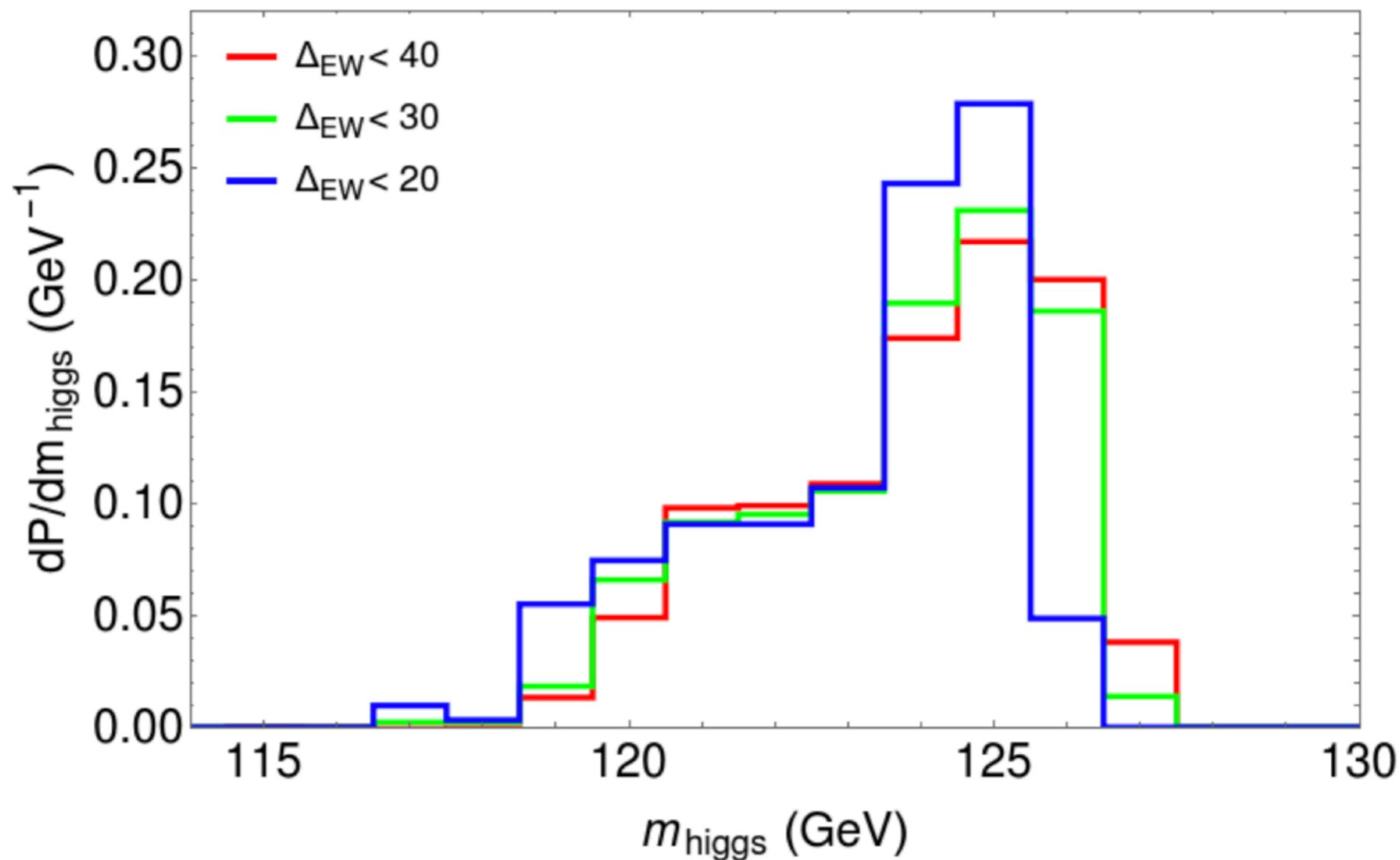


Figure 6: Probability distribution for Higgs mass m_h for the case of $n = 1$ but with varying cutoff $\Delta_{EW} < 20, 30$ and 40 .

m_{soft}^n Scan Results

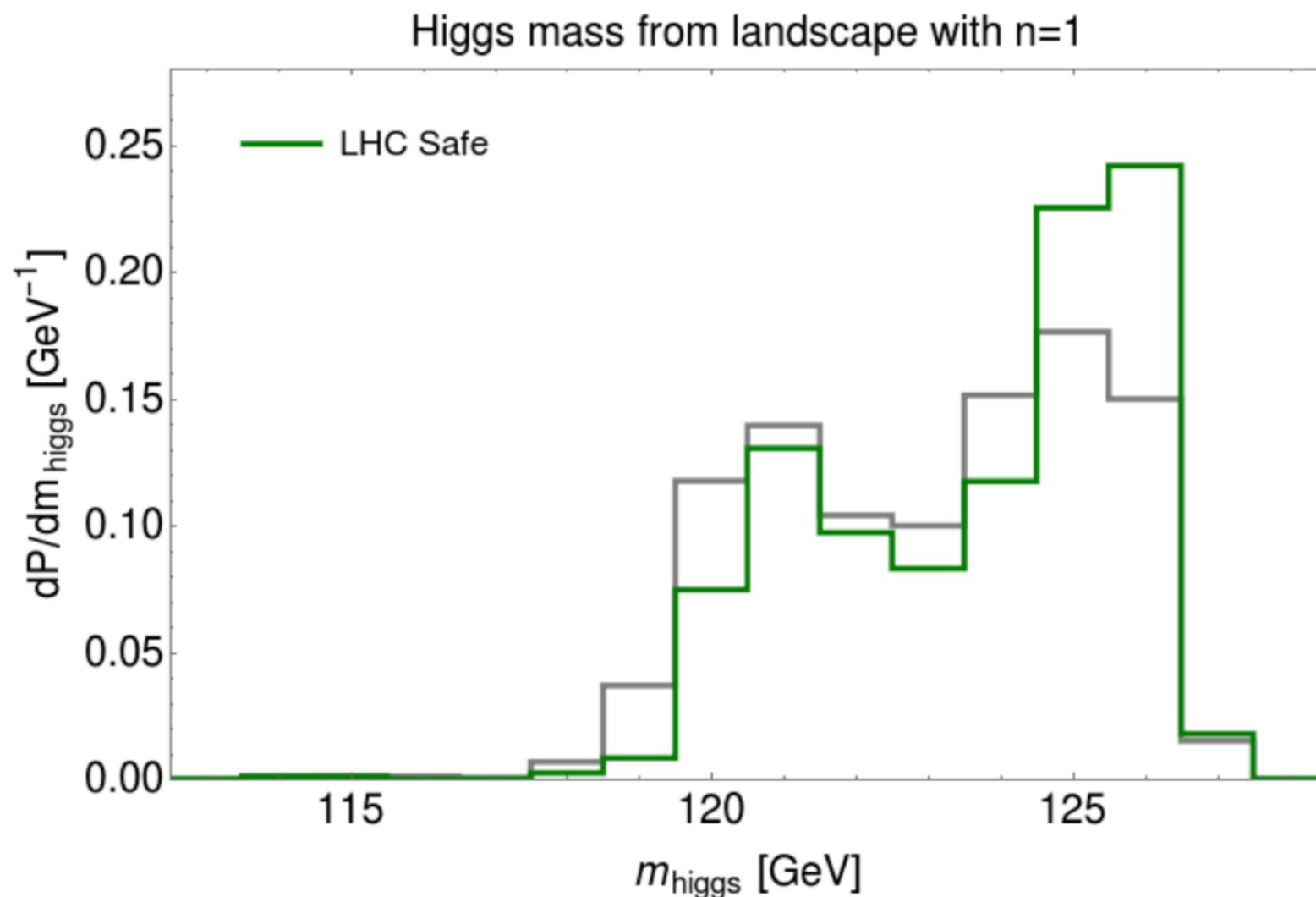


Figure 2: Statistical expectation for the mass of the Higgs boson from the string theory landscape which scans over single F -term SUSY breaking. The green histogram includes only LHC Run 2 safe points.

More Penalties?

- inflation and SUSY breaking scales correlated in string theory
Kallosch/Linde (2004)
- can lower inflationary scale, and sequester
Cicoli (2016)
- still difficult to obtain TeV scale SUSY and be OK with density perturbations
- perhaps penalty biasing high-scale SUSY?

Moduli and Non-thermal Dark Matter

$$m_{\text{modulus}} \sim m_{3/2}$$

moduli decay around $t \sim M_P^2/m_{\text{modulus}}^3$

$$\sim 10^3 \text{ s for } m_{\text{modulus}} \sim 1 \text{ TeV}$$

$$T_r = c^{1/2} \left(\frac{10.75}{g_*} \right)^{1/4} \left(\frac{m_{\text{modulus}}}{50 \text{ TeV}} \right)^{3/2} T_{BBN} \quad \Gamma = \frac{c}{2\pi} \frac{m_{\text{modulus}}^3}{M_P^2}$$

$$dN_{\text{vac}} \sim \Theta(m_{\text{modulus}} - 50 \text{ TeV}) \times f_{EWFT} \times (m_{\text{hidden}}^2)^n d(m_{\text{hidden}}^2)$$

Summary

LHC data suggests we should revisit these old arguments

- penalty from cosmological constant tuning decouples: is this generically true?
- is the democracy of F-terms suggested by Douglas/Denef generically correct?
- how about alternatives to weak scale anthropic penalty?

Backup

Practical Naturalness

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

$$\Delta_{EW} \equiv \max_i |C_i| / (m_Z^2/2)$$

The main requirements for low fine-tuning ($\Delta_{EW} \lesssim 30^1$) are the following.

- $|\mu| \sim 100 - 350$ GeV [23–27] (where $\mu \gtrsim 100$ GeV is required to accommodate LEP2 limits from chargino pair production searches).
- $m_{H_u}^2$ is driven radiatively to small, and not large, negative values at the weak scale [21, 28].
- The top squark contributions to the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized for TeV-scale highly mixed top squarks [28]. This latter condition also lifts the Higgs mass to $m_h \sim 125$ GeV. For $\Delta_{EW} \lesssim 30$, the lighter top squarks are bounded by $m_{\tilde{t}_1} \lesssim 3$ TeV.
- The gluino mass, which feeds into the $\Sigma_u^u(\tilde{t}_{1,2})$ via renormalization group contributions to the stop masses [27], is required to be $m_{\tilde{g}} \lesssim 6$ TeV, possibly beyond the reach of the $\sqrt{s} = 13 - 14$ TeV LHC.²
- First and second generation squark and slepton masses may range as high as 5–30 TeV with little cost to naturalness [21, 22, 29, 30].

HB, Barger, Huang, Mustafayev, Tata

Predicted Spectrum

- $m_{\tilde{g}} \sim 4 \pm 2$ TeV,
- $m_{\tilde{t}_1} \sim 1.5 \pm 0.5$ TeV,
- $m_A \sim 3 \pm 2$ TeV,
- $\tan \beta \sim 13 \pm 7$,
- $m_{\tilde{W}_1, \tilde{Z}_{1,2}} \sim 200 \pm 100$ GeV and
- $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 7 \pm 3$ GeV with
- $m(\tilde{q}, \tilde{\ell}) \sim 20 \pm 10$ TeV (for first/second generation matter scalars).

Ensembles of Flux Vacua

$$W = \int_M G \wedge \Omega(z)$$

- Type IIB string theory (Gukov Vafa Witten superpotential)
- Omega: holomorphic three form on CY
- G: sum of NS and RR three-form gauge field strengths

$$G = F^{RR} - \tau H^{NS}$$

- flux quanta: start with a general F-theory compactification on an elliptically fibered CY 4-fold:

$$W = \int_X G_4 \wedge \Omega = N^\alpha \Pi_\alpha$$

$\Pi_\alpha = \int \Sigma_\alpha \wedge \Omega$ are the periods in some basis $\{\Sigma_\alpha\}$ of $H^4(X, \mathbb{Z})$

Douglas and Denef, 2004: Distributions of Flux Vacua [hep-th/0404116](https://arxiv.org/abs/hep-th/0404116)

NUHM3

boundary conditions for the soft terms. The NUHM3 model is convenient in that it allows for μ as an independent input parameter, and since we require μ not too far from $m_{W,Z,h} \sim 100$ GeV. The NUHM3 model is inspired by previous work on mini-landscape investigations of heterotic string theory compactified on a $Z_6 - II$ orbifold [49]. In these models, sparticle masses are dictated by the geography of their wavefunctions within the compactified manifold. These models exhibit *localized* grand unification [50] wherein the first/second generation matter superfields lie near fixed points (the twisted sector) and thus lie in **16**-dimensional spinor reps of SO(10). Meanwhile, third generations fields and Higgs and vector boson multiplets lie more in the bulk and thus occur in split multiplets (solving the doublet-triplet splitting problem) and receive smaller soft masses [51]. Such a set-up motivates the NUHM3 model with the following parameters $m_0(1,2)$, $m_0(3)$, $m_{1/2}$, A_0 , $\tan \beta$, m_{H_u} , m_{H_d} where all mass parameters are taken

m_0 Distribution

the vicinity of $m_0(1, 2) \sim 20$ TeV with tails extending out to 30 TeV. Such large scalar masses occur because of the linear ($n = 1$) and quadratic ($n = 2$) pull on these soft terms with only minimal suppression which sets in at $m_0(1, 2) \gtrsim 20$ TeV. One avenue for suppression arises from electroweak D -term contributions to the $\Sigma_{u,d}^{u,d}$ terms which depend on weak isospin and electric charge assignments. For nearly degenerate scalars of each generation, these nearly cancel out [52]. Another avenue for suppression comes from two loop terms in the MSSM RGEs [53]: if scalar masses enter the multi-TeV range, then these terms can become large and help drive third generation scalar masses tachyonic leading to CCB minima in the scalar potential [54]. Both these rather mild suppressions are insufficient to prevent first/second generation scalar masses from rising to the 20-30 TeV range. Such heavy scalars go a long way to suppressing possible FCNC and CP violating SUSY processes [21]. For the $n = 0$ case, $dP/dm_0(1, 2)$ peaks around 5-10 TeV before suffering a drop-off.

3rd Generation

around 10 TeV for $n = 1$ and 12 TeV for $n = 2$. Large values of $m_0(3)$ generate large stop masses which result in $\Sigma_u^u(\tilde{t}_{1,2})$ exceeding ~ 30 *i.e.* generating a weak scale typically in excess of $m(\text{weak}) \sim 400$ GeV. For $n = 0$, the distribution peaks around 3 TeV.

$m_{1/2}$ Distribution

In frame *c*), we plot the distribution in $m_{1/2}$. In this case, the $n = 1$ distribution peaks around 1.5 TeV whilst $n = 2$ peaks slightly higher. If the (unified) gaugino masses become too big, then the large gluino mass also lifts the top squarks to higher masses thus causing the $\Sigma_u^u(\tilde{t}_{1,2})$ to again become too large. The distributions fall to near zero by $m_{1/2} \sim 3$ TeV leading to upper limits on gaugino masses. The $n = 0$ distribution actually peaks at its lowest allowed

A_0 distribution

In frame d), we show the distribution versus A_0 . Here we only show the more lucrative negative A_0 case which leads to higher Higgs masses m_h [46]. The $n = 0$ distribution peaks at $A_0 \sim 0$ with a steady fall-off at large negative A_0 values. In this case, the typically small mixing in the stop sector leads to values of m_h below its measured result. In contrast, for $n = 1, 2$ the distributions increase (according to the statistical pull) to peak values around $A_0 \sim -(5 - 10)$ TeV. Such large A_0 values lead to large mixing in the top-squark sector which can enhance m_h whilst decreasing the $\Sigma_u^u(\tilde{t}_{1,2})$ values [14]. The $n = 1$ curve actually features a double bump structure: we have traced the lower peak to the presence of large $m_A \sim m_{H_d} \sim 5 - 10$ TeV values which increase the S term in the third generation matter scalar RGEs. This term (along with large two-loop effects from first/second generation matter scalars) acts to suppress $m_{U_3}^2$ leading to lighter \tilde{t}_1 states even without large mixing. For even larger negative A_0 values, the distributions rapidly fall to zero since they start generating CCB minima in the MSSM scalar potential.

Higgs Distribution

to little mixing in the stop sector and hence too light values of m_h . Taking $n = 1$, instead we now see that the distribution in m_h peaks at ~ 125 GeV with the bulk of probability between $123 \text{ GeV} < m_h < 127 \text{ GeV}$ – in solid agreement with the measured value of $m_h = 125.09 \pm 0.24$ GeV [55].⁵ This may not be surprising since the landscape is pulling the various soft terms towards large values including large mixing in the Higgs sector which lifts up m_h into the 125 GeV range. By requiring the $\Sigma_u^u(\tilde{t}_{1,2}) \lesssim 30$ (which would otherwise yield a weak scale in excess of 350 GeV) then too large of Higgs masses are vetoed. For the $n = 2$ case with a stronger draw towards large soft terms, the m_h distribution hardens with a peak at $m_h \sim 126$ GeV.

m_A , $\tan\beta$ Distribution

In Fig. 3b), we show the distribution in pseudoscalar mass m_A . Here, for $m_A \gg m_h$, then $m_A \sim m_{H_d}$ (at the weak scale) and we have a statistical draw to large m_A values which is tempered by the presence of $m_{H_d}/\tan\beta$ in Eq. 1. While the $n = 0$ uniform draw peaks at the lowest m_A values, the $n = 1$ and 2 cases yield a broad distribution peaking around $m_A \sim 3$ TeV which drops thereafter. In frame c), we show the distribution in $\tan\beta$. Here, the $n = 0$ case has a broad distribution with a peak around $\tan\beta \sim 20$ while the $n = 1$ and 2 cases have sharper distributions peaking around $\tan\beta \sim 10 - 15$. The suppression of $\tan\beta$ for large values can be understood due to the draw towards large soft terms in the sbottom sector. As $\tan\beta$ increases, the b (and τ) Yukawa couplings increase so that the $\Sigma_u^u(\tilde{b}_{1,2})$ terms become large. Then the anthropic cutoff on $\Delta_{EW} < 30$ disfavors the large $\tan\beta$ regime. In frame d), we show the $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ mass splitting. For our case with $\mu = 150$ GeV, the light higgsinos $\tilde{W}_1^\pm, \tilde{Z}_{1,2}$ all have masses around 150 GeV. The phenomenologically important mass gap $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ becomes smaller the more gauginos are decoupled from the higgsinos. The landscape draw towards large gaugino masses thus suppressed $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ for the $n = 1$ and 2 cases so that the mass gap peaks at around 5 – 8 GeV. For the uniform scan with $n = 0$, then the gap is larger— typically 10 – 20 GeV.

Ensembles of Flux Vacua

$$W = \int_M G \wedge \Omega(z)$$

- Omega: holomorphic three form on CY
- G: sum of NS and RR three-form gauge field strengths

$$G = F^{RR} - \tau H^{NS}$$

- Weil-Petersson metric on complex str. moduli space
- an explicit statistical study with a single complex structure modulus