Supersymmetry Enhancement

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DESY

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Based on...

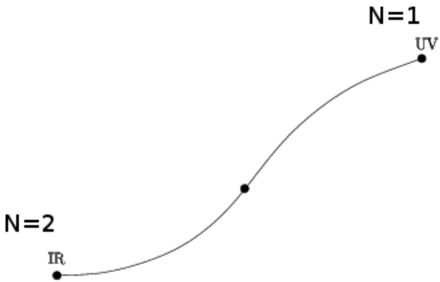
1. Supersymmetry enhancement from T-branes
   - F.C., S. Giacomelli, R.Savelli. 2018
2. Supersymmetry enhancement from Hitchin Systems
   - (Work in progress)
     F.C., A.Collinucci, S. Giacomelli, H. Hayashi, R.Savelli

For earlier related works in Susy Enhancement see

- K. Maruyoshi, J. Song, 2016
- P. Agarwal, K. Maruyoshi, J. Song, 2016
- P. Agarwal, A, Sciarappa, J. Song, 2017
- S. Giacomelli, 2018

Supersymmetry enhancement.

- A UV QFT follows an RG flow to a IR QFT with more explicit supersymmetry.
- Intrinsically interesting phenomenon in QFT.
- Can use the lagrangian to compute RG-protected quantities.



1. Maruyoshi-Song flows. $4d$ $\mathcal{N}=1 \to \mathcal{N}=2$
2. Benini-Giacomelli flows. $3d$ $\mathcal{N}=2 \to \mathcal{N}=4$
3. Ungauging quiver tails. (Gadde-Rastelli-Razamat).
4. $3d$ CS-Maxwell. (Yamazaki).

Maruyoshi-Song flows.

- Start in UV with a $4d\ \mathcal{N} = 2$ SCFT $\mathcal{T}$ with flavor symmetry F.
- Add by hand a $\mathcal{N} = 1$ chiral M.
- M is gauge singlet and in the adjoint of the F.
- Turn on a superpotential term $W_{def} = TrM\mu$ where μ is the moment map operator. ($\mu \simeq q\tilde{q}$ if $\mathcal{T}$ is lagrangian).
- Give a ***special kind of vev*** to M. This triggers a RG flow.
- Depending on the choice of $\mathcal{T}$ and $\langle M \rangle$ sometimes we find that $\mathcal{T}[\langle M \rangle]$ flows in the IR a *new* $\mathcal{N} = 2$ SCFT: call it $\mathcal{T}[\langle M \rangle]_{IR}$.
- “New" means $\mathcal{T}[\langle M \rangle]_{IR} \neq \mathcal{T}$

Nilpotent orbit vev.

- M is a matrix in the adjoint of F.
- We choose $\langle M \rangle$ to be along a nilpotent orbit of $\mathfrak{f}_{\mathbb{C}}$
- Nilpotent orbit is defined as ghg^{-1} for $g \in F$ and $h \in \mathfrak{f}_{\mathbb{C}}$ nilpotent.
- Nilpotent orbits are *completely classified*. Example:

$$\langle M \rangle = h = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (1)$$

is a nilpotent vev for M in the maximal nilpotent orbit of $\mathfrak{sl}_2$.

Remarkably, for ***some*** choices of $\langle M \rangle$ there will be Supersymmetry Enhancement.

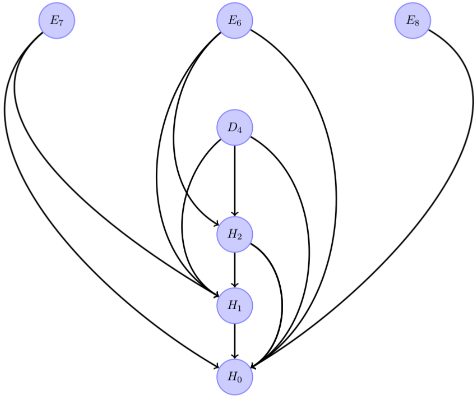
Example in SQCD

- Consider $SU(2)$ with $N_f = 4$. Flavory symmetry is $F = SO(8)$.
- Deform with $W_{def} = Mq\tilde{q}$
- Take $\langle M \rangle$ to be the maximal nilpotent orbit of $\mathfrak{so}(8)$.
- This theory flows to the H_0 Argyres-Douglas theory.
- Interesting because H_0 is the **minimal** $4d\ \mathcal{N} = 2$ SCFT. (Smallest central charges a and c)

$$a = \frac{43}{120} \quad c = \frac{11}{30} \tag{2}$$

  and longly believed to be non-lagrangian.
- H_0 has now been lagrangianized
  (K. Maruyoshi, J. Song, 2016)

Enhancing flows connecting rank 1 theories



A figure summarizing all the existent MS connecting rank one theories. Multiple flows happen for different nilpotent orbit deformations.

Evidence for the enhancement

So far I have just *said* that SUSY enhancement exists. Here is some evidence for it.

- a-maximization.
  1. $\mathcal{N}=2\; U(1)_r \times SU(2)_R$ R-symmetry is broken to $U(1)^{UV}_{\mathcal{N}=1} \times U(1)_{\mathcal{F}}$
  2. $U(1)_{\mathcal{N}=1} = U(1)^{UV}_{\mathcal{N}=1} + \epsilon\, U(1)_{\mathcal{F}}$ mixing of R-symmetry and flavor.
  3. Find the right mixing value $\epsilon^\star$ (Intriligator, Wecht)
  4. If central charges $a(\epsilon^\star)$ and $c(\epsilon^\star) \in \mathbb{Q}$, good evidence for enhancement.
- The full Superconformal Index.
  1. Use $\mathcal{L}_{UV}$ to compute the $\mathcal{N}=1$ Index by SUSY localization.
  2. Upon finding the mixing ϵ^*, shift the index fugacities
  3. Check limits of the index against previously computed cases. (Shur, McDonald, Coulomb, HL, HS of 3d mirror CB)

Open questions

1. Can one use this method to find more $\mathcal{N}=1$ lagrangians for $\mathcal{N}=2$ non-lagrangains SCTs?
2. Why only some specific nilpotent vevs for M give Susy enhancement?
3. Can one understand in a more direct way why the enhancement happens, from a physical point of view?
4. Is there a way to see this phenomenon from a String Theory engineering of the QFT?

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The big scan and a conjecture

1. MS flows starting form lagrangian SCFTs are **very** constrained.
   1. Have to tune matter content and gauge groups to have $\beta = 0$
   2. The number of nilpotent orbit is finite and $<$ rank^3
2. Write a program to run the a-max test for a very large class of $(\mathcal{T}, \langle M \rangle)$.
3. Find one new case in rank 1: E_7 MN with orbit with BC label E_6 flows to H_1.
4. $\sim$ 1000 cases checked among superconformal quivers of D and E shape, or also non-bifund matter.

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4. $\sim$ 1000 cases checked among superconformal quivers of D and E shape, or also non-bifund matter. ZERO enhancements. We conjecture that this class never give S.E.

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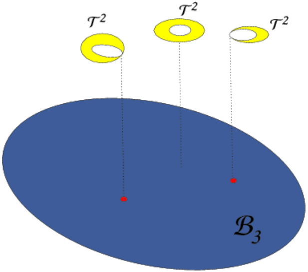
F-Theory introduction.

(Vafa '96)

- Consider IIB on $\mathbb{R}^4 \times Y_6$ with 7-branes. They source the axiodilaton and backreact.
- IIB Axiodilaton $\tau(z) = C_0(z) + ie^{-\phi(z)}$ is singular at the location of the 7-branes sources.
- Idea. Think of τ as the complex structure of an auxiliary elliptic curve, fibered over a Kahler base.



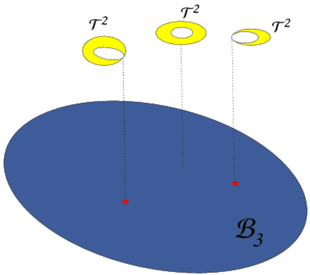
$\mathbb{T}^2$ pinches at the brane locations in the z-space.

Weierstrass Models

- Every elliptic curve can be put in Weierstrass form

$$y^2 = x^3 + fx + g \qquad (3)$$

- To fiber this over B take $f(z)$, $g(z)$ functions of the coordinates on the base.



Vanishing orders of f, g and Δ fix which is the gauge group G_{8d} on the 7-brane stack.

The geometrical picture. Part 1

- Consider a F-theory setup, on $\mathbb{R}^8 \times K3$. The $K3$ is non compact.
- Put a D3 on $\mathbb{R}^4 \subset \mathbb{R}^8$ probing the elliptic fibration.
- Write the Weierstrass model for the elliptic fibration. The theory on the D3 will be the theory $\mathcal{T}$ in the UV.
- From the D3 prespective $G_{8d} \simeq F$. In particular the Weiestrass model fixes the flavor group F.

Introduction to T-Branes

- Consider a stack of 2 D7-branes.
- On the worldvolume 8d SYM. There is a complex scalar field φ in the adjoint.
- If $[\varphi, \varphi^\dagger] = 0$ we can diagonalize it. We interpret the eigenvalues as the positions of the two D7s in transverse space.
- If $[\varphi, \varphi^\dagger] \neq 0$, then φ can be split in a diagonalizable and a nilpotent part. There is no geometrical interpretation of the nilpotent part. Yet 8d physics is different. We call this a T-brane.
- This can be generalized to N D7-branes, and to F-theory.

The geometrical picture. Part 2

- We know how to engineer the UV $\mathcal{N}=2$ starting point. Need to engineer now the chiral field M and its nilpotent vev.
- Idea. M is geometrized by a T-brane deformation of the 7-brane stack. $\implies$ Nilpotent orbit + fluctuation.
- Ex. For the case in which $\langle M \rangle$ is in the maximal orbit of $\mathfrak{sl}_2$, we take a T-brane profile given by:

$$\varphi = \langle \varphi \rangle + \delta\varphi = \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix} \tag{4}$$

How does such a vev for φ affect the geometry given by the Weierstrass model?

The geometrical picture. Part 3

Singularity	Curve	Flavor group
II^*	$u^2 = v^3 + v(M_2 z^3 + M_8 z^2 + M_{14} z + M_{20}) + (z^5 + M_{12} z^3 + M_{18} z^2 + M_{24} z + M_{30})$	E_8
III^*	$u^2 = v^3 + v(z^3 + M_8 z + M_{12}) + (M_2 z^4 + M_6 z^3 + M_{10} z^2 + M_{14} z + M_{18})$	E_7
IV^*	$u^2 = v^3 + v(M_2 z^2 + M_5 z + M_8) + (z^4 + M_6 z^2 + M_9 z + M_{12})$	E_6
I_0^*	$u^2 = v^3 + v(\tau z^2 + M_2 z + M_4) + (z^3 + \tilde{M}_4 z + M_6)$	$SO(8)$
IV	$u^2 = v^3 + v(M_{1/2} z + M_2) + (z^2 + M_3)$	$SU(3)$
III	$u^2 = v^3 + vz + (M_{2/3} v + M_2)$	$SU(2)$
II	$u^2 = v^3 + vM_{4/5} + z$	no

Table: Maximally deformed Weierstrass models

- The parameters M_i are the versal deformations of the model
- They correspond to casimir operators of the field φ in the F-theory picture, which we take to be $\langle \varphi \rangle = \langle M \rangle + \delta M$
- $\langle M \rangle$ is fixed by the chosen nilpotent orbit. δM will be the highest-spin singlet appearing in the decomposition of Adj.

The geometrical picture. Part 4

- When the D3 probes the deformed Weiestrass model, the theory is $\mathcal{N} = 1$, due to the T-brane presence. Original K3 $\to$ CY3.
- RG flow is a local zoom at the singularity. In the IR the probe does not have enough energy to resolve global aspects of the singularity.
- In the IR, some terms in the Weiestrass become subleading. We throw them away and we find a new elliptic curve.
- For all and only the choices of $(\mathcal{T}, \langle M \rangle)$ that give enhancement, we find in the IR a Weierstrass model for the correct $\mathcal{N} = 2$ theory.

Conclusions

1. We conjecture that no more $\mathcal{N} = 1$ lagrangians flowing to $\mathcal{N} = 2$ can be find by the MS method.
2. Can interpret geometrically the MS flows, for the rank 1 case.
   - We engineer the UV theory as a D3 probing the singularity locus of the elliptic fibration
   - We engeneer the nilpotent vev for M as a T-brane deformation of the 7-brane stack
   - We interpret the RG flow as a local zoom-in.
   - We recover the curve for the IR theory, in all cases which enhance.

Thanks for the attention