ATLAS Measurements of Rare Decays and CP Violation in Beauty Mesons

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On Behalf of the ATLAS Collaboration

SUSY2019
May 20, 2019
Introduction

• Precision measurement of ultra-rare decays $B^0 \rightarrow \mu^+\mu^-$ and $B_s^0 \rightarrow \mu^+\mu^-$
  
  • *JHEP 04 (2019) 098*
  
  • High Luminosity LHC sensitivity projections: ATL-PHYS-PUB-2018-005

• CP-violation in the $B_s^0 \rightarrow J/\psi\phi$ channel
  
  • ATLAS-CONF-2019-009
  
  • High Luminosity LHC sensitivity projections: ATL-PHYS-PUB-2018-041
$B^0_{(s)} \rightarrow \mu^+ \mu^-$

Motivation for the Measurement

- The smallness and precision of the predicted branching fractions provides a favorable environment for observing contributions from new physics.
- Significant deviations could arise in models involving non-SM heavy particles such as those predicted in:
  - Minimal Supersymmetric Standard Model*
  - Minimal Flavor Violation**
  - Two Higgs-Doublet Models†
  - And others‡

“New Physics”

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* Huang, Chao-Shang and Liao, Wei and Yan, Qi-Shu, Promising process to distinguish supersymmetric models with large tan $\beta$ from the standard model: $B \rightarrow X_s \mu^+ \mu^-$, Phys. Rev. D 59 (1998) 011701, arXiv: hep-ph/9803460 [hep-ph].
$B^0_{(s)} \rightarrow \mu^+ \mu^-$ Analysis Background

- The SM precisely predicts* the branching ratios of the two decays:
  - $\mathcal{B}(B^0_{s} \rightarrow \mu^+ \mu^-) = (3.65 \pm 0.23) \times 10^{-9}$
  - $\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) = (1.06 \pm 0.09) \times 10^{-10}$

- Run 1 results reported by ATLAS** have produced the following branching ratios for the $B^0_{(s)} \rightarrow \mu^+ \mu^-$ decays
  - $\mathcal{B}(B^0_{s} \rightarrow \mu^+ \mu^-) = (0.9^{+1.1}_{-0.8}) \times 10^{-9}$
  - $\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) < 4.2 \times 10^{-10}$ at 95% confidence level

- These results are consistent with the Standard Model with a $p$ value of 4.8%, corresponding to 2.0 standard deviations

- LHCb has reported their results from 2015 and 2016 data†:
  - $\mathcal{B}(B^0_{s} \rightarrow \mu^+ \mu^-) = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$ with 7.8 $\sigma$ significance
  - $\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) < 3.4 \times 10^{-10}$ at 95% confidence level

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Performing the Measurement

• The aim of the measurement is to obtain the branching fraction of the $B_{(s)}^0 \rightarrow \mu^+ \mu^-$ channels

• The branching ratios are made relative to the normalization decay $B^+ \rightarrow J/\psi K^+$ which is abundant and has a well measured branching fraction

\[
\mathcal{B}(B_{(s)}^0 \rightarrow \mu^+ \mu^-) = \frac{N_{d(s)}}{\varepsilon_{\mu^+ \mu^-}} \times [\mathcal{B}(B^+ \rightarrow J/\psi K^+) \times \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-)] \frac{\varepsilon_{J/\psi K^+}}{N_{J/\psi K^+}} \times \frac{f_u}{f_{d(s)}}
\]

• Here $N_{d(s)}$ is the signal yield, $N_{J/\psi K^+}$ is the reference yield, $\varepsilon_{d(s)}$ and $\varepsilon_{J/\psi K^+}$ are the acceptance times the efficiency and $f_u/f_{d(s)}$ is the ratio of the hadronization probabilities of a b-quark into $B^+$ and $B_{(s)}^0$. 
Preliminary Steps of the Analysis

• Collect the data with the ATLAS detector for analysis
  • Inner Detector
  • Muon Spectrometer

• Perform a **blind analysis**
  • The signal region of the dimuon invariant mass (5166 to 5526 MeV) is concealed while procedures of the event selection and details of the signal extraction are defined

• Simulate the data
  • Dimuon events – for signal and background regions
  • $B^+ \rightarrow J/\psi K^+$ candidates (reference channel)
Sources of Background

- Continuum background: the dominant combinatorial component
  - Consists of muons from uncorrelated hadron decays
  - Characterized by a weak dependence on the dimuon invariant mass

- Partially reconstructed decays: one or more of the final-state particles (X) in a b hadron decay is not reconstructed
  - These candidates accumulate in the low dimuon invariant mass sideband

- Peaking background: $B_{(s)}^0 \rightarrow hh'$ decays with both hadrons misidentified as muons
Suppressing the Continuum Background: Multivariate Analysis

- A multivariate approach, implemented as a Boosted Decision Tree (BDT), is used to enhance the signal relative to the continuum background.

- Here is the final BDT output for various datasets used in the analysis.

- A larger BDT output corresponds to more suppression of the continuum background.

![BDT Output Diagram](attachment:image.png)
Data-Simulation Comparisons

• The BDT is optimized when trained with 15 selected input variables - used to characterize a B meson event and the produced muons

• A grid search is performed to optimize the other BDT parameters

• Shown here are two of the input variables used in the training

• Care is taken to ensure that BDT output is not correlated with the invariant mass of the muons
Yield Extraction
The Normalization Channel

- The $B^\pm$ yield for the normalization channel is extracted with an unbinned extended maximum-likelihood fit to the $J/\psi K^+$ invariant mass distribution.

- The fit includes 4 components:
  - $B^+ \rightarrow J/\psi K^+$ decays
  - Cabibbo-suppressed $B^+ \rightarrow J/\psi \pi^+$ decays
  - Partially reconstructed $B$ decays
  - Continuum background (composed mostly of $b\bar{b} \rightarrow J/\psi X$ decays)
Efficiency Ratio

- The efficiency ratio is required for the calculation of the signal branching fraction:

\[ R_\varepsilon = \frac{\varepsilon(B^+ \rightarrow J/\psi K^+)}{\varepsilon(B^{0(s)} \rightarrow \mu^+ \mu^-)} \]

- Both channels are measured in the fiducial acceptance for the B meson:
  - \( p_T^B > 8 \text{ GeV} \) and \( |\eta_B| < 2.5 \)
- The total efficiencies include acceptance and trigger, reconstruction and selection efficiencies.
  - Muon acceptance: \( p_T^{\mu_1} > 6.0 \text{ GeV} \), \( p_T^{\mu_2} > 4.0 \text{ GeV} \) and \( |\eta_{\mu_{1,2}}| < 2.5 \)
  - Kaon acceptance: \( p_T^K > 1.0 \text{ GeV} \) and \( |\eta_K| < 2.5 \)
  - The signal reference BDT selection: BDT > 0.2455
Extraction of the Signal Yield

• The data in the dimuon signal region is **unblinded**

• The dimuon candidates are classified according to **four intervals** in the BDT output

• Each interval is chosen to give an equal efficiency of 18% for signal MC events

• The figure here shows the BDT interval with the lowest signal to background ratio

• Continuum and partially reconstructed B decays are the main backgrounds
Signal Yield

- Increasing the restrictions of the BDT improve suppression of the continuum background
- Superimposed on each figure is the maximum likelihood fit
- SM Expected:
  \( N_s = 91 \) and \( N_d = 10 \)
- Determined from the fit of highest bin:
  \( N_s = 80 \pm 22 \) and \( N_d = -12 \pm 20 \)
Comparison to the Standard Model

- Simultaneous maximum likelihood fits of the branching ratios of the $B^0$ and $B_S^0$.
  - The blue point and contours is the result from 2015 and 2016 data,
  - the green point and contours is the Run 1 result,
  - and the black point and contours is the combination of the two
- Also shown is the Standard Model prediction; the data and the model differ by 2.4 standard deviations
- The significance of $B_S^0 \rightarrow \mu^+ \mu^-$ combined result is estimated to be $4.6\sigma$
Comparison to the LHCb Result

- Comparison of the branching ratios of the $B^0$ and $B_s^0$ for the latest ATLAS and LHCb results
  - The black point and contours are the ATLAS combined Run 1 and 2015 and 2016 data result
  - The red point and contours are the LHCb combined Run 1 and partial Run 2 data result
  - Also shown is the Standard Model prediction in purple
$B^0_{(s)} \rightarrow \mu^+ \mu^-$ Projections

- Full Run 2 projections are shown on the right
- HL-LHC projected results shown below

Conservative yield:

High-yield:
CP Violation in $B_s^0 \rightarrow J/\psi\phi$
CP Violation in $B_s^0 \rightarrow J/\psi \phi$

- In the $B_s^0 \rightarrow J/\psi \phi$ decay, CP violation occurs due to interference between a direct decay and a decay with $B_s^0 - \bar{B}_s^0$ mixing

- Shown on the left are the lowest order mixing diagrams that show the mixing oscillations
- On the right is the decay channel of interest
CP Violation Parameters

- The CP violating phase $\phi_s$ is defined as the weak phase difference between the $B_s^0 - \bar{B}_s^0$ mixing amplitude and the $b \rightarrow c\bar{c}s$ decay amplitude.
- In the SM the phase $\phi_s$ is small and is related to Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix elements via the relation $\phi_s \approx -2\beta_s$.
- In terms of the CKM quark mixing matrix elements, $\beta_s = \text{arg}(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*})$.
- In the Standard Model (without New Physics contributions): $\ -2\beta_s = -0.0363^{+0.0016}_{-0.0015}$
- Other physical quantities involved in the $B_s^0 - \bar{B}_s^0$ mixing are:
  - The decay width: $\Gamma_s = (\Gamma_L + \Gamma_H)/2$
  - And the width difference: $\Delta\Gamma_s = \Gamma_L - \Gamma_H$
  - Here, $\Gamma_L$ and $\Gamma_H$ are the decay widths of the light and heavy mass eigenstates.
Recap of the ATLAS LHC Run 1 Results

- In LHC Run 1 ATLAS experimentally determined $\phi_s$ and $\Delta\Gamma_s$ to be
  - $\phi_s = -0.090 \pm 0.078$ (stat.) $\pm 0.041$ (syst.) rad
  - $\Delta\Gamma_s = 0.085 \pm 0.011$ (stat.) $\pm 0.007$ (syst.) ps$^{-1}$

- These results are compatible with the Standard Model and with other experiments
- The results also leave room for new physics contributions to the CP Violation phase
Run 2 Analysis Strategy

• Select events with the use of flavor tagging to identify the flavor of the signal B Meson

• Determine $B_s^0$ properties:
  • Mass ($m_i$), mass error ($\sigma_{m_i}$), proper decay time ($t_i$), decay time error ($\sigma_{t_i}$), transverse momentum ($p_{T_i}$), and the tag probability of a B meson flavor: $P_i(\text{B}|Q_x)$

• Determine transversity angles:
  • $\theta_T$, $\phi_T$, and $\psi_T$

• Perform an unbinned maximum likelihood fit to extract nine parameters simultaneously:
  • The CPV phase and decay widths: $\phi_s$, $\Delta \Gamma_s$, $\Gamma_s$
  • The amplitudes and strong phases: $|A_0(0)|^2$, $|A_{||}(0)|^2$, $\delta_{||}$, $\delta_{\perp}$, $|A_s(0)|^2$, $\delta_{\perp} - \delta_s$
Run 2: $B_s^0$ Flavor Tagging (I)

- The measured charge of a lepton (electron or muon) from the semileptonic decay of a B meson provides strong discrimination for determining the $B_s^0$ flavor.

- Input for $B_s^0$ flavor tagging:
  - Electron tagging
  - Muon tagging
  - $b$-hadron jet tagging (in the case of no lepton)
Run 2: $B_s^0$ Flavor Tagging (II)

- $b \rightarrow l$ transitions are diluted through processes that can change the charge of the observed lepton, such as through neutral B meson oscillations, or through cascade decays $b \rightarrow c \rightarrow l$

- The separation power of lepton tagging is enhanced by considering $p_T$ weighted charge of the tracks in a cone around the electron or muon or charged tracks in the $b$-jet:

$$Q_x = \frac{\sum_{i}^{N \text{ tracks}} q_i \cdot (p_{Ti})^\kappa}{\sum_{i}^{N \text{ tracks}} (p_{Ti})^\kappa}$$

- $B^\pm \rightarrow J/\psi K^\pm$ is used to calibrate the flavor tagging
Measured $B_s^0$ Properties

- The $B_s^0$ mass fit and the proper decay time fit are shown here

$$t = \frac{L_{xy} m_B}{p_{T_B}}$$
• The measured transversity angles, $\theta_T$, $\phi_T$, and $\psi_T$ are shown here.
Run 2 Results

- The Run 2 results with 80.5 fb\(^{-1}\) of data are shown in the table on the left.
- The combined Run 1 and Run 2 results are shown in the red circle in the figure on the right.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Statistical uncertainty</th>
<th>Systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_s) [rad]</td>
<td>-0.068</td>
<td>0.038</td>
<td>0.018</td>
</tr>
<tr>
<td>(\Delta \Gamma_s) [ps(^{-1})]</td>
<td>0.067</td>
<td>0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>(\Gamma_s) [ps(^{-1})]</td>
<td>0.669</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>(</td>
<td>A_\parallel(0)</td>
<td>^2)</td>
<td>0.219</td>
</tr>
<tr>
<td>(</td>
<td>A_0(0)</td>
<td>^2)</td>
<td>0.517</td>
</tr>
<tr>
<td>(</td>
<td>A_S(0)</td>
<td>^2)</td>
<td>0.046</td>
</tr>
<tr>
<td>(\delta_\perp) [rad]</td>
<td>2.946</td>
<td>0.101</td>
<td>0.097</td>
</tr>
<tr>
<td>(\delta_\parallel) [rad]</td>
<td>3.267</td>
<td>0.082</td>
<td>0.201</td>
</tr>
<tr>
<td>(\delta_\perp - \delta_S) [rad]</td>
<td>-0.220</td>
<td>0.037</td>
<td>0.010</td>
</tr>
</tbody>
</table>
Combination Results

- The combined Run 1 and Run 2 results are shown in the table to the left.
- The preliminary HFLAV average combination of the best experimental results is shown in the figure on the right.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Statistical uncertainty</th>
<th>Systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s [\text{rad}]$</td>
<td>$-0.076$</td>
<td>0.034</td>
<td>0.019</td>
</tr>
<tr>
<td>$\Delta \Gamma_s [\text{ps}^{-1}]$</td>
<td>0.068</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>$\Gamma_s [\text{ps}^{-1}]$</td>
<td>0.669</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$</td>
<td>A_\parallel(0)</td>
<td>^2$</td>
<td>0.220</td>
</tr>
<tr>
<td>$</td>
<td>A_0(0)</td>
<td>^2$</td>
<td>0.517</td>
</tr>
<tr>
<td>$</td>
<td>A_\perp</td>
<td>^2$</td>
<td>0.043</td>
</tr>
<tr>
<td>$\delta_{\perp} [\text{rad}]$</td>
<td>3.075</td>
<td>0.096</td>
<td>0.091</td>
</tr>
<tr>
<td>$\delta_{\parallel} [\text{rad}]$</td>
<td>3.295</td>
<td>0.079</td>
<td>0.202</td>
</tr>
<tr>
<td>$\delta_{\perp} - \delta_\perp S [\text{rad}]$</td>
<td>$-0.216$</td>
<td>0.037</td>
<td>0.010</td>
</tr>
</tbody>
</table>
$B_s^0 \rightarrow J/\psi \phi$ CP Violation Projections

- HL-LHC projections of the CP violation phase and decay width difference are shown here.
- With higher statistics due to trigger selection the precision of the result will improve.
Final Remarks

• The combined ATLAS results from Run 1 and the 2015 and 2016 results of the $B_{(s)}^0 \rightarrow \mu^+\mu^-$ analysis were presented
  - The results are compatible with the Standard Model within 2.4σ

• The combined ATLAS results from Run 1 and the 2015, 2016, and 2017 results of the CP Violation in $B_s^0 \rightarrow J/\psi\phi$ were presented
  - The result is the most stringent measurement on parameters $\phi_s, \Delta\Gamma_s, \Gamma_s$ and the helicity functions parameters of the $B_s^0 \rightarrow J/\psi\phi$ decay from a single measurement

• With the HL-LHC statistics, more precise measurements will be possible
Additional Slides

- The following slides provide supplementary and follow up material to the presentation
Description of BDT Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T^B$</td>
<td>Magnitude of the B candidate transverse momentum $\vec{p}_T^B$.</td>
</tr>
<tr>
<td>$\chi^2_{PV,DV,xy}$</td>
<td>Compatibility of the separation $\Delta x$ between production (i.e. associated PV) and decay (DV) vertices in the transverse projection: $\Delta x_T \cdot \Sigma^{-1} \Delta x_T$, where $\Sigma^{-1} \Delta x_T$ is the covariance matrix.</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>Three-dimensional opening between $\vec{p}_T^B$ and $\Delta x$: $\sqrt{\alpha^{2D} + \Delta \eta^2}$</td>
</tr>
<tr>
<td>$</td>
<td>\alpha_{2D}</td>
</tr>
<tr>
<td>$L_{xy}$</td>
<td>Projection of $\Delta x_T$ along the direction of $\vec{p}_T^B$: $(\Delta x_T \cdot \vec{p}_T^B)/</td>
</tr>
<tr>
<td>$IP_{3D}$</td>
<td>Three-dimensional impact parameter of the B candidate to the associated PV.</td>
</tr>
<tr>
<td>$DOCA_{\mu\mu}$</td>
<td>Distance of closest approach (DOCA) of the two tracks forming the B candidate (three-dimensional).</td>
</tr>
<tr>
<td>$\Delta \phi_{\mu\mu}$</td>
<td>Difference in azimuthal angle between the momenta of the two tracks forming the B candidate.</td>
</tr>
<tr>
<td>$</td>
<td>d_0</td>
</tr>
<tr>
<td>$</td>
<td>d_0</td>
</tr>
<tr>
<td>$P_{\text{min}}^L$</td>
<td>Value of the smaller projection of the momenta of the muon candidates along $\vec{p}_T^B$.</td>
</tr>
<tr>
<td>$I_{0.7}$</td>
<td>Isolation variable defined as ratio of $</td>
</tr>
<tr>
<td>$DOCA_{\text{str}}$</td>
<td>DOCA of the closest additional track to the decay vertex of the B candidate. Tracks matched to a PV different from the B candidate are excluded.</td>
</tr>
<tr>
<td>$N_{\text{close}}$</td>
<td>Number of additional tracks compatible with the decay vertex (DV) of the B candidate with $\ln(\chi^2_{\text{str},DV}) &lt; 1$. The tracks matched to a PV different from the B candidate are excluded.</td>
</tr>
<tr>
<td>$\chi^2_{\mu,SPV}$</td>
<td>Minimum $\chi^2$ for the compatibility of a muon in the B candidate with any PV reconstructed in the event.</td>
</tr>
</tbody>
</table>
Calculation of the Branching Ratio

• Each piece of the branching ratio is now obtained:
  • \( N_s = 80 \pm 22 \) and \( N_d = -12 \pm 20 \)
  • \( R_{\epsilon} = \frac{\epsilon(B^+ \rightarrow J/\psi K^+)}{\epsilon(B^{0(s)} \rightarrow \mu^+ \mu^-)} = 0.1176 \pm 0.0009 \) (stat.) \( \pm 0.0047 \) (syst.)
  • \( N_{J/\psi K^+} = 334351 \) with a statistical uncertainty of 0.3%
  • \( \mathcal{B}(B^+ \rightarrow J/\psi K^+) = (1.010 \pm 0.029) \times 10^{-3} \)
  • \( \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-) = (5.961 \pm 0.033)\% \)
  • \( f_s/f_d = 0.256 \pm 0.013 \) (HFLAV average *)

\[
\mathcal{B}(B^{0(s)} \rightarrow \mu^+ \mu^-) = \frac{N_{d(s)}}{\epsilon_{\mu^+ \mu^-}} \times [\mathcal{B}(B^+ \rightarrow J/\psi K^+) \times \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-)] \frac{\epsilon_{J/\psi K^+}}{N_{J/\psi K^+}} \times \frac{f_u}{f_d(s)}
\]

Decision Trees

- A machine learning algorithm can be taught to make predictions based on experience with example datasets.
- This machine learning approach is called classification.
- Obtaining a probability that an event is either background or signal.
• The sidebands and the signal Monte Carlo are used for the BDT training
• The sample is subdivided into three randomly selected separate and equally populated sub-samples
• These samples are used in rotation to train, verify, and evaluate the selection efficiency of 3 individual BDTs

• The 3 BDTs produce statistically compatible performances
• They are then combined into one single classifier in such a way that each BDT is applied only to the part of the data sample not involved in the BDT training.
Comparison of Sidebands and Simulation

ATLAS Preliminary

$\sqrt{s} = 13$ TeV, $26.3$ fb$^{-1}$

\[ \chi^2_{PV,DV,xy} \]

\[ \text{DOCA}_{\mu\mu} \text{ [mm]} \]
Correlation of Variables Before the BDT Selection

Slide 36
Correlation of Variables After the BDT Selection

**ATLAS Simulation Preliminary**

- $B_s^0 \rightarrow \mu^+ \mu^-$ MC
- $B_s^0 \rightarrow \mu^+ \mu^-$ sidebands

**ATLAS** Preliminary $\sqrt{s} = 13$ TeV, 26.3 fb$^{-1}$ After BDT selection

- $m_2$ = 13 TeV, 26.3 fb$^{-1}$ After BDT selection

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20 May 2019

Slide 37
Comment on Negative Likelihood Values

- The central value of the maximum likelihood fit is allowed to be negative as a consequence of this particular statistical observable. We know that a negative value is not physical for the number of events $N_d$ in the branching fraction and also the branching fraction of $B_d$. Because we know that a negative value is not physical we can use a robust statistical tool called the Neyman construction to find an upper bound on $N_d$. The Neyman construction takes as input the measured branching ratios for $B_s$ and $B_d$ from the maximum likelihood and returns the possible number of events $N_d$ and $N_s$. A feature of the Neyman construction is that it cannot return negative values for $N_d$ or $N_s$.

- In the end we quote a measurement for $BR(B_s)$ and an upper limit for $BR(B_d)$. These are calculated from the output of the $N_d$ and $N_s$ belts in the Neyman construction.
\[ B_S^0 \to J/\psi\phi \]

- Differential decay rate:

\[
\frac{d^4\Gamma}{dt \ d\Omega} = \sum_{k=1}^{10} O^{(k)}(t) g^{(k)}(\theta_T, \psi_T, \phi_T)
\]

Table 4: The ten time-dependent functions, \( O^{(k)}(t) \) and the functions of the transversity angles \( g^{(k)}(\theta_T, \psi_T, \phi_T) \). The amplitudes \( |A_0(0)|^2 \) and \( |A_\perp(0)|^2 \) are for the CP-even components of the \( B_S^0 \to J/\psi\phi \) decay, \( |A_\perp(0)|^2 \) is the CP-odd amplitude; they have corresponding strong phases \( \delta_0, \delta_\parallel \) and \( \delta_\perp \). By convention \( \delta_0 \) is set to be zero. The \( S \)-wave amplitude \( |A_S(0)|^2 \) gives the fraction of \( B_s^0 \to J/\psi K^+ K^- (f_0) \) and has a related strong phase \( \delta_S \). The factor \( \alpha \) is described in the text of Section 5.1. The \( \pm \) and \( \mp \) terms denote two cases: the upper sign describes the decay of a meson that was initially a \( B_s^0 \) meson, while the lower sign describes the decays of a meson that was initially \( \bar{B}_s^0 \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( O^{(k)}(t) )</th>
<th>( g^{(k)}(\theta_T, \psi_T, \phi_T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{2}</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{2}</td>
<td>A_\parallel(0)</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{2}</td>
<td>A_\perp(0)</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{2}</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>5</td>
<td>(</td>
<td>A_\parallel(0)</td>
</tr>
<tr>
<td>6</td>
<td>(</td>
<td>A_0(0)</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{1}{2}</td>
<td>A_S(0)</td>
</tr>
<tr>
<td>8</td>
<td>( \alpha</td>
<td>A_S(0)</td>
</tr>
<tr>
<td>9</td>
<td>( \frac{1}{2} \alpha</td>
<td>A_S(0)</td>
</tr>
<tr>
<td>10</td>
<td>( \alpha</td>
<td>A_0(0)</td>
</tr>
</tbody>
</table>
$B_d^0 \rightarrow K^{*0}(892)\mu^+\mu^-$
\[ B_d^0 \rightarrow K^{*0}(892)\mu^+\mu^- \]

- differential decay rate as a function of the angular parameters

\[
\frac{1}{d\Gamma/dq^2} \frac{d^4\Gamma}{d\cos\theta_L d\cos\theta_K d\phi dq^2} = \frac{9}{32\pi} \left[ \frac{3(1-F_L)}{4} \sin^2\theta_K + F_L \cos^2\theta_K + \frac{1-F_L}{4} \sin^2\theta_K \cos 2\theta_L \right. \\
- F_L \cos^2\theta_K \cos 2\theta_L + S_3 \sin^2\theta_K \sin^2\theta_L \cos 2\phi \\
+ S_4 \sin 2\theta_K \sin 2\theta_L \cos \phi + S_5 \sin 2\theta_K \sin \theta_L \cos \phi \\
+ S_6 \sin^2\theta_K \cos \theta_L + S_7 \sin 2\theta_K \sin \theta_L \sin \phi \\
+ S_8 \sin 2\theta_K \sin 2\theta_L \sin \phi + S_9 \sin^2\theta_K \sin^2\theta_L \sin 2\phi \left. \right] . \quad (2.1)

\[
P_1 = \frac{2S_3}{1-F_L} \\
P_2 = \frac{2}{3} \frac{A_{FB}}{1-F_L} \\
P_3 = -\frac{S_9}{1-F_L} \\
P'_{j=4,5,6,8} = \frac{S_{i=4,5,7,8}}{\sqrt{F_L(1-F_L)}} .
\]
Angular Analysis $B^0_d \rightarrow K^* \mu^+ \mu^-$

- Search for heavy new particles that may contribute to FCNC decay amplitudes
- The lowest order Feynman diagrams for $B^0_d \rightarrow K^* \mu^+ \mu^-$ are the box diagram shown on the upper left and the two penguin diagrams
- The angular parameters of the measurement are shown in the upper right diagram in the rest frame of the $K^*$
Analysis Scheme

- The differential decay amplitude of $B^0_d \rightarrow K^* \mu^+ \mu^-$ can be written in terms of:
  - $q^2 = 4m^2_\mu$, $\cos(\theta_K)$, $\cos(\theta_L)$ and $\phi$
  - The fraction of longitudinally polarized $K^*$ mesons ($F_L$)
  - And 7 angular coefficients, $S_i$ where $i = 3, 4, 5, 6, 7, 8, 9$
  - Theoretical uncertainties can be reduced in the decay amplitude using ratios of $F_L$ and $S_i$ to form $P_1, P_2, P_3$ and $P'_j$ where $j = 4, 5, 6, 8$
- The equation can be simplified using trigonometric transformations to “fold” certain angular distributions so that only 3 coefficients remain in the decay amplitude

### Folding Schemes:

<table>
<thead>
<tr>
<th>$F_L, S_3, S_4, P'_4$</th>
<th>$F_L, S_3, S_5, P'_5$</th>
<th>$F_L, S_3, S_7, P'_6$</th>
<th>$F_L, S_3, S_8, P'_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \rightarrow -\phi$ for $\phi &lt; 0$</td>
<td>$\phi \rightarrow -\phi$ for $\phi &lt; 0$</td>
<td>$\phi \rightarrow -\pi - \phi$ for $\phi &lt; -\frac{\pi}{2}$</td>
<td>$\phi \rightarrow -\pi - \phi$ for $\phi &lt; -\frac{\pi}{2}$</td>
</tr>
<tr>
<td>$\theta_L \rightarrow \pi - \theta_L$ for $\theta_L &gt; \frac{\pi}{2}$</td>
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<td>$\theta_L \rightarrow \pi - \theta_L$ for $\theta_L &gt; \frac{\pi}{2}$</td>
</tr>
</tbody>
</table>

### Resulting angular variable ranges:

- $\cos \theta_L \in [0, 1]$, $\cos \theta_K \in [-1, 1]$ and $\phi \in [0, \pi]$,
- $\cos \theta_L \in [0, 1]$, $\cos \theta_K \in [-1, 1]$ and $\phi \in [0, \pi]$,
- $\cos \theta_L \in [0, 1]$, $\cos \theta_K \in [-1, 1]$ and $\phi \in [-\pi/2, \pi/2]$,
- $\cos \theta_L \in [0, 1]$, $\cos \theta_K \in [-1, 1]$ and $\phi \in [-\pi/2, \pi/2]$. 
An extended maximum likelihood fit process is used to extract the 3 coefficients of interest in a particular folding scheme:

- \( F_L, S_3 \) and \( S_j \) where \( j = 4,5,7,8 \) (or \( F_L, S_3 \) and \( P'_j \) where \( j = 4,5,6,8 \))
- Nuisance parameters (mass of \( K\pi\mu\mu \) and mass width coefficient) are also extracted

The fitting procedure is performed in 6 bins of \( q^2 \) between 0.04 and 6 GeV for each folding scheme in order to probe the dependence on \( q^2 \)
Control Regions

- Two $K^* c \bar{c}$ decay control sample fits, $K^* J/\psi$ and $K^* \psi(2S)$, are shown in $q^2 \in [8, 11]$ and $[12, 15]$ GeV$^2$ regions, respectively
- Control samples are used to extract values for nuisance parameters describing the signal
- The fit to data includes a combinatorial background component that does not peak in the $m_{K\pi\mu\mu}$ distribution
Subset of Results

- Fit to the mass $K\pi\mu\mu$ and angle $\phi$ in the dilepton mass region $q^2 \in [2.0, 4.0]$ GeV$^2$
- The fit is performed using the $F_L$, $S_3$ and $S_5$ folding scheme
• Fitted signal and background yields are shown in the table for the various bins of $q^2$

<table>
<thead>
<tr>
<th>$q^2$ [GeV$^2$]</th>
<th>$n_{\text{signal}}$</th>
<th>$n_{\text{background}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.04, 2.0]</td>
<td>128 ± 22</td>
<td>122 ± 22</td>
</tr>
<tr>
<td>[2.0, 4.0]</td>
<td>106 ± 23</td>
<td>113 ± 23</td>
</tr>
<tr>
<td>[4.0, 6.0]</td>
<td>114 ± 24</td>
<td>204 ± 26</td>
</tr>
<tr>
<td>[0.04, 4.0]</td>
<td>236 ± 31</td>
<td>233 ± 32</td>
</tr>
<tr>
<td>[1.1, 6.0]</td>
<td>275 ± 35</td>
<td>363 ± 36</td>
</tr>
<tr>
<td>[0.04, 6.0]</td>
<td>342 ± 39</td>
<td>445 ± 40</td>
</tr>
</tbody>
</table>

• The fits to $\cos(\theta_K)$ and $\cos(\theta_L)$ in the $q^2 \in [2.0, 4.0]$ GeV$^2$ in the $F_L$, $S_3$ and $S_5$ folding scheme are shown here
Results

- There is good agreement between theory and measurement for all regions with the exception of:
  - the $P'_4$ and $P'_5$ parameters in $q^2 \in [4.0, 6.0] \text{ GeV}^2$
  - $P'_8$ parameter in $q^2 \in [2.0, 4.0] \text{ GeV}^2$