Motivations for (weak scale) supersymmetry

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i.e., why are we here?

The Poincare group of spacetime transformations (boosts, rotations and translations) provides the basis for relativistic QFT as we know it

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0, \\
[M_{\mu\nu}, P_\lambda] &= i(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu), \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho})
\end{aligned}
$$

particle states labeled by mass & spin $|m, j>$

There is a general notion that everything is allowed in the laws of nature unless explicitly forbidden (Gell-Mann?)

We can extend the Poincare group to the superPoincare group by allowing as well anti-commutation relations

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0, \\
[M_{\mu\nu}, P_\lambda] &= i(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu), \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}) \\
[P_\mu, Q_a] &= 0, \\
[M_{\mu\nu}, Q_a] &= -(\sigma_{\mu\nu})_{ab}Q_b, \\
\{Q_a, \bar{Q}_b\} &= 2(\gamma^\mu)_{ab}P_\mu.
\end{aligned}
$$

There is no reason for nature *not* to use this structure

Gauge hierarchy

- In 4-d quantum field theory, scalar particle masses are typically unprotected by any symmetry and are quadratically divergent
- For spin-1 fields, they are introduced via gauge symmetry which also works to protect their masses
- For spin-1/2 objects, a left-right (chiral) symmetry acts to cancel linear divergences, leaving only log
- For this reason, it is thought by some that the only consistent QFTs which are valid at disparate energy scales are those with supersymmetry, wherein the scalars inherit the chiral symmetry of their fermionic superpartners.
- Since the Higgs has been discovered, and has all the properties of a fundamental scalar particle, it seems SUSY is necessary to tame its mass, cancel quadratic divergences which would otherwise lead to fine-tuning if the SM is assumed valid much beyond the weak scale
- This was noted already by Witten and Kaul in 1981- SUSY with weak scale superpartners can be a solution to the gauge hierarchy problem

- In fact, it seems to be a necessary ingredient in the only sensible quantum theory of gravity which we have: superstring theory where the super part refers to world sheet SUSY which is necessary to include fermions into the theory

- Then spacetime supersymmetry can easily emerge- in fact, one of the advantages of Calabi-Yau compactifications is that they lead to the presence of N=1 SUSY in the low energy theory

Thus, it has for long been expected that the
supersymmetric Standard Model is a reasonable target
for a low energy effective theory by string model builders.

The Minimal Supersymmetric Standard Model (MSSM) is what one gets by

1. starting with the SM gauge group
2. elevate SM fields to lie within superfields
3. add 2nd Higgs doublet Hd to maintain anomaly cancellation which
can be disrupted by the presence of higgsinos
4. break SUSY by adding in by hand all allowable soft SUSY breaking terms
5. invoke R-parity conservation to forbid Yukawa interactions leading to rapid p-decay

By gauging the SUSY transformations, making them local,
then one is forced to introduce supergravity theory which necessarily
includes a graviton/gravitino supermultiplet. The theory
obeys general co-ordinate invariance and thus includes
General Relativity. It is, however, a non-renormalizable theory.

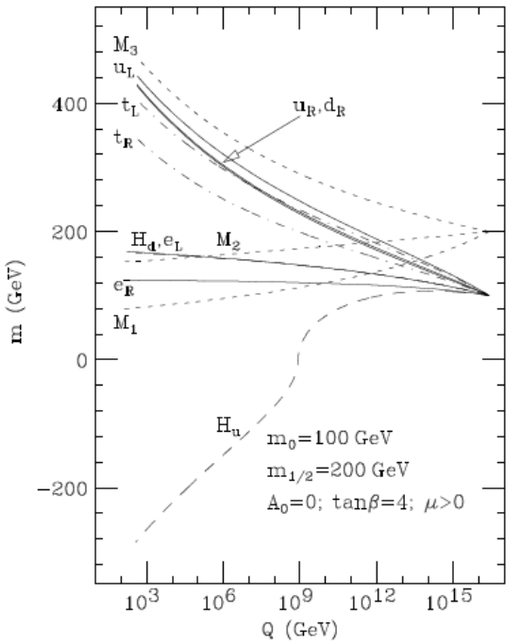
Supergravity breaking via the superHiggs mechanism
leads in the limit as mP-> infty keeping m32 fixed
to a theory with global SUSY and soft terms ~m(3/2)

For m(3/2)~TeV, then one gets weak scale SUSY as low energy effective theory

Radiative EWSB

One might expect that with all positive scalar masses, then EW symmetry might not break in SUGRA:

$$V_{\rm scalar} = (m_{H_u}^2 + \mu^2)|h_u^0|^2 \quad + \quad (m_{H_d}^2 + \mu^2)|h_d^0|^2$$
$$-B\mu(h_u^0 h_d^0 + \text{h.c.}) \quad + \quad \frac{1}{8}(g^2 + g'^2)\left(|h_u^0|^2 - |h_d^0|^2\right)^2$$



Need negative quadratic terms so that V develops non-zero minimum, breaking EW symmetry

This can happen radiatively if m(top) is big enough: drives mHu^2 negative

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right)$$

$$X_t = m_{Q_3}^2 + m_{\tilde{t}_R}^2 + m_{H_u}^2 + A_t^2$$

Occurs for m(top)~150-200 GeV, (which it is)

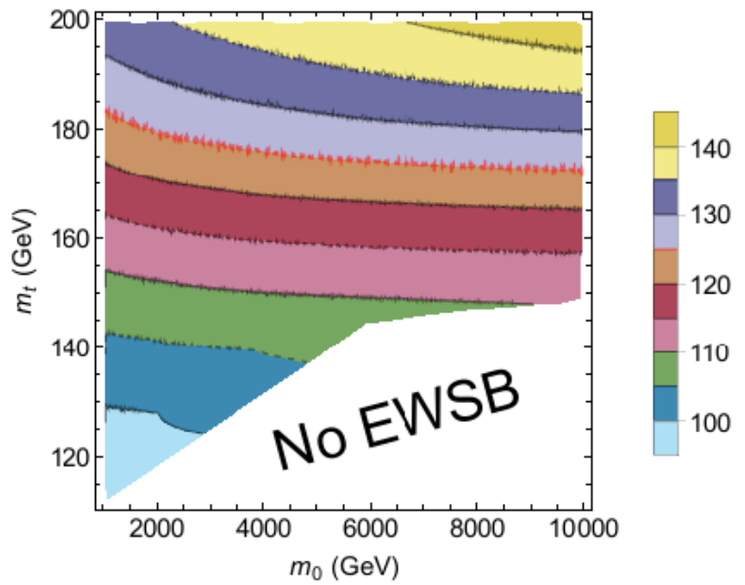
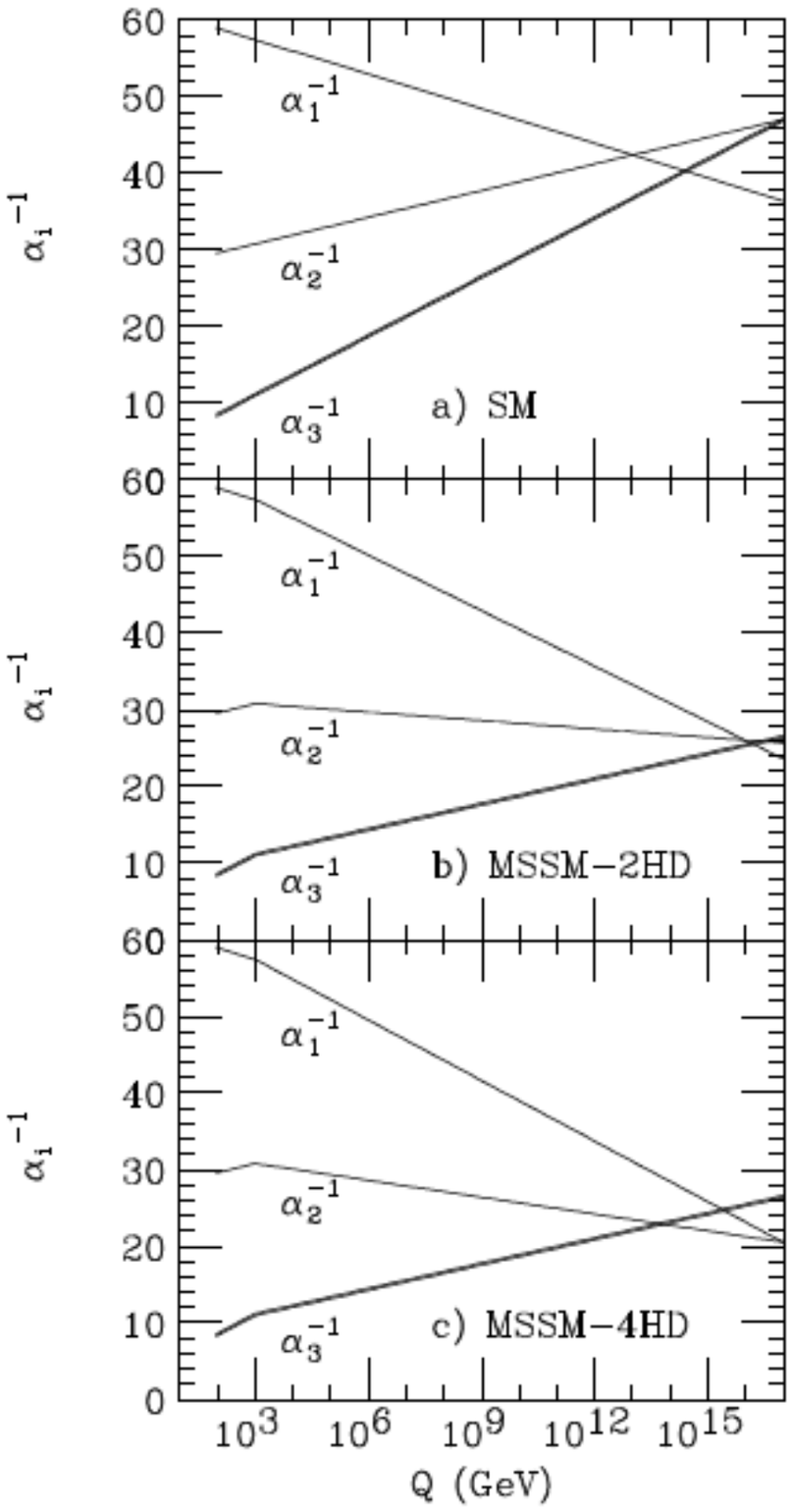


FIG. 1: Values of Higgs mass m_h which are generated for various values of m_t and m_0 in a SUSY model with $m_{1/2} = 700$ GeV and $A_0 = -1.6m_0$ and $\tan\beta = 10$.

Measured values of gauge couplings all unify under MSSM evolution; not so for SM



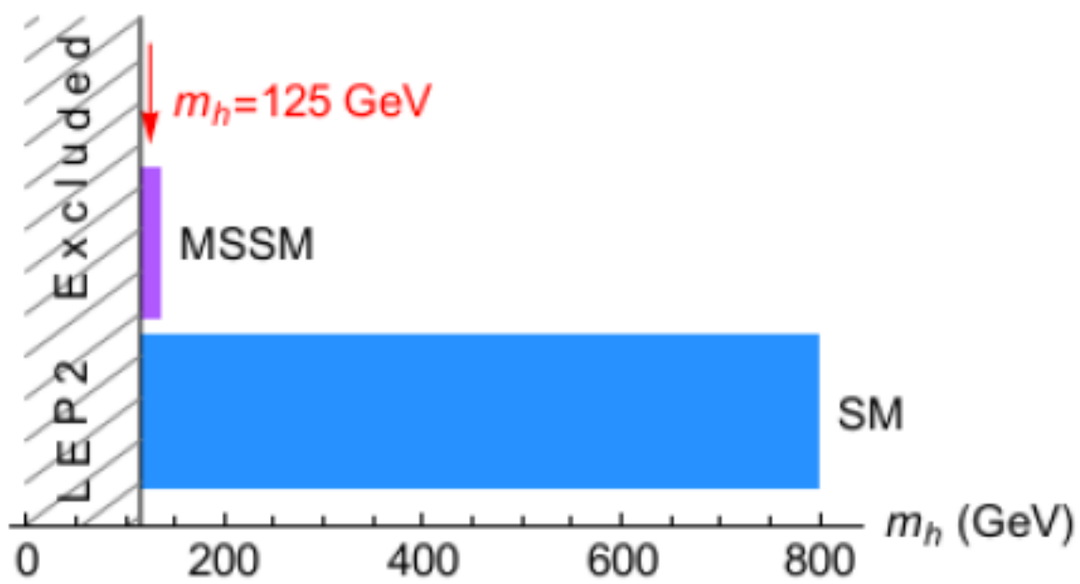
Suggests SUSY-GUT unification-
nowadays, ``local" GUTs
at compactified orbifold fixed points

Light Higgs mass highly constrained in SUSY but not in SM

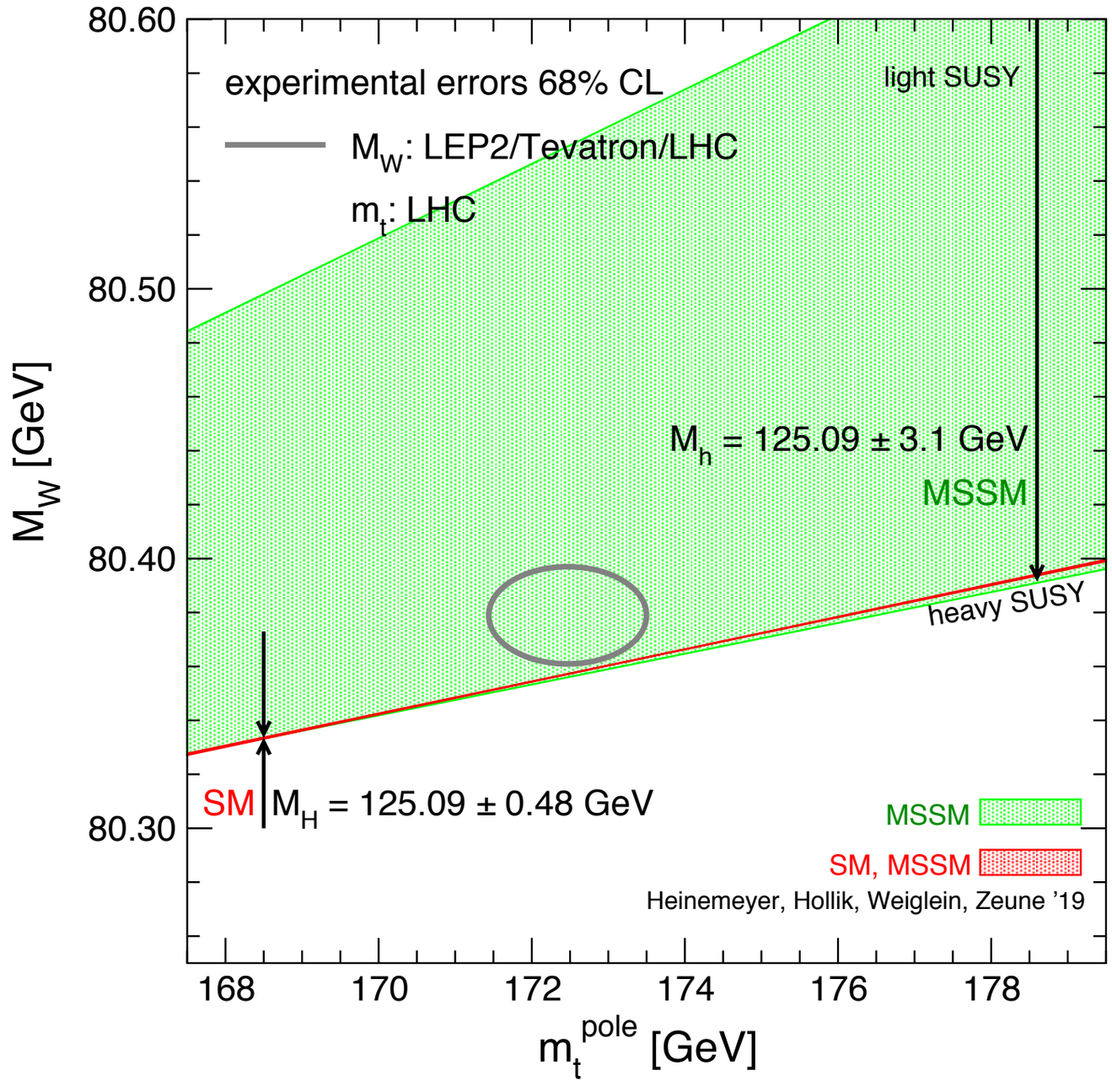
$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2}\frac{m_t^4}{m_W^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{X_t^2}{12m_{\tilde{t}}^2}\right)\right] \tag{1.7}$$

where $X_t = A_t - \mu \cot\beta$ and $m_{\tilde{t}}^2 \simeq m_{Q_3} m_{U_3}$. For a given $m_{\tilde{t}}^2$, this expression is maximal for large mixing in the top-squark sector with $X_t^{max} = \sqrt{6} m_{\tilde{t}}$. With top-squark masses below about 500 GeV, the radiative corrections to m_h are not large enough to yield $m_h \simeq 125$ GeV even with maximal mixing[13, 11]. This situation has been used to argue that additional

Need highly mixed (large A term) TeV-scale top squarks



Electroweak Precision Observables: mW, mt, mh



favors heavy, nearly decoupled SUSY

Baryogenesis

1. In SM, electroweak baryogenesis only works for mh<50 GeV

2. Leptogenesis is a possibility for mN~10^14 GeV- but then introducing an intermediate mass scale one expects mh->10^14 GeV

In SUSY, several mechanisms appear

0. SUSY EW baryogenesis: m(t1)<m(t)? also seems excluded

1. SUSY leptogenesis need TR>mN~10^9 GeV; but then one tends to overproduce gravitinos=> too much DM and BBN disruption

2. Non-thermal leptogenesis can occur where one augments N abundance via e.g inflaton decay: TR~10^6 can work, avoiding gravitino problem

3. Affleck-Dine leptogenesis via LHu flat direction works pretty much automatically down to TR~10^6 and avoids Q-ball formation which plagues older AD models

4. Oscillating sneutrino leptogenesis also a possibility

SUSY leptogenesis scenarios

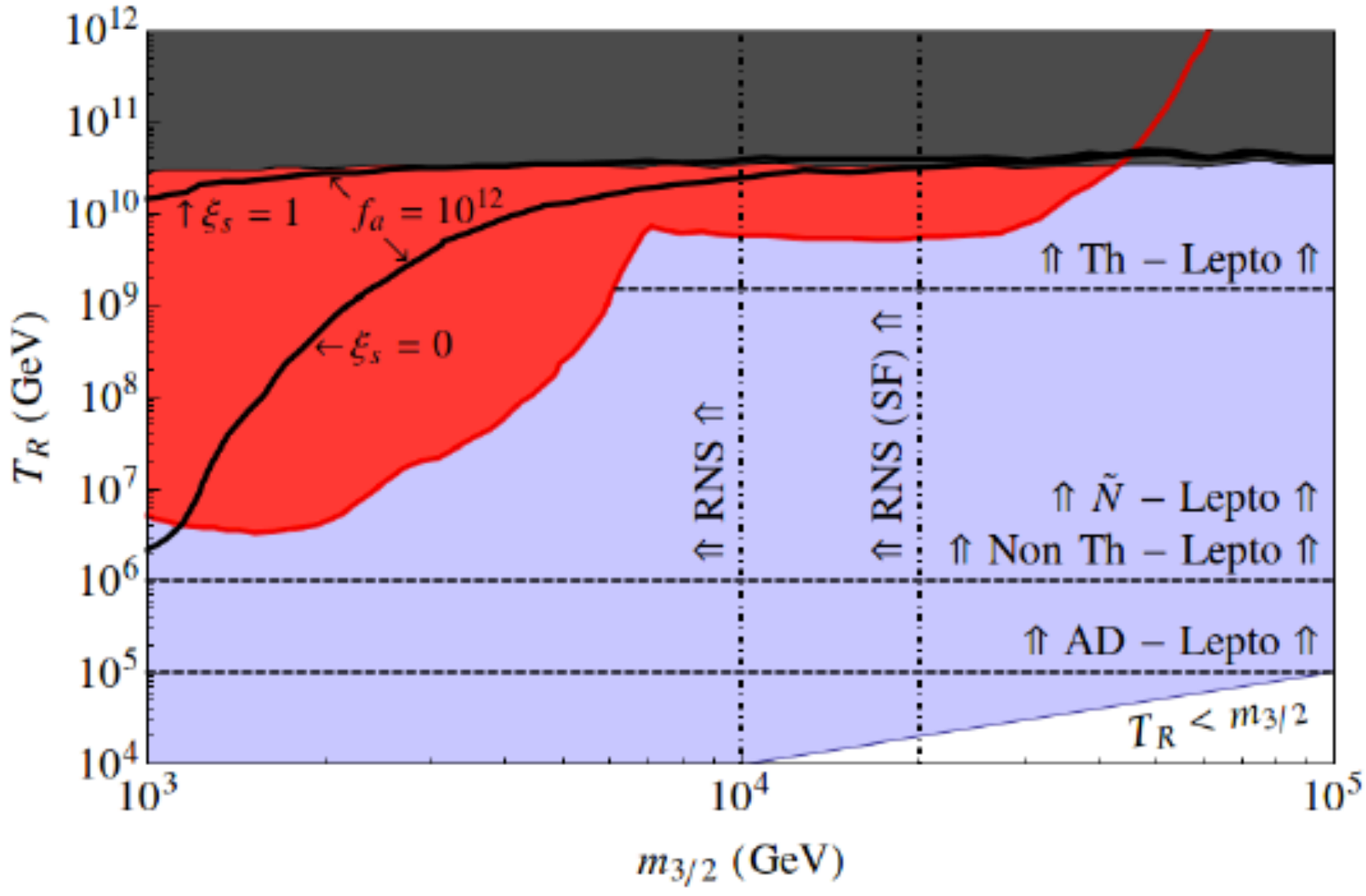


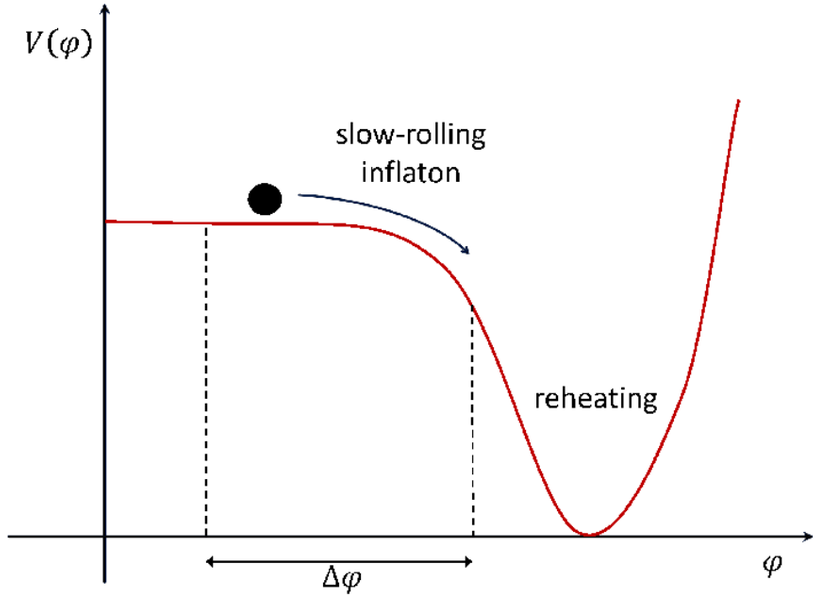
Figure 1: Plot of allowed regions in T_R vs. $m_{3/2}$ plane in the SUSY DFSZ axion model for *a*) $f_a = 10^{11}$ and 10^{12} GeV, *b*) $f_a = 10^{13}$ GeV, for $\xi_s = 0$ and 1 and *c*) $f_a = 10^{14}$ GeV for $\xi_s = 1$. For $f_a = 10^{11}$, $T_R > 10^{11}$ is forbidden to avoid PQ symmetry restoration. We take $m_s = m_{\tilde{a}} \equiv m_{3/2}$ in all plots.

Bae, HB, Serce,Zhang

Inflationary cosmology

We know the universe is spatially flat with uniform temperature
to 1 part in 10^5 even in causally disconnected regions

This can be explained in terms of an inflationary universe
where the scalar field inflaton lies in slow role
region of inflaton potential. At exit, inflaton
oscillates and then decays, reheating the universe to T_R
(which is the beginning of traditional Big Bang cosmology)



$$T_R = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4} \sqrt{m_P \Gamma_\phi} \qquad \Gamma_\phi \sim m_\phi^3/m_P^2$$

Without SUSY to stabilize m(inflaton), then
T_R would blow up to implausibly large values

Strong CP problem

Adding an axion sector directly to SM is not very plausible:

- global U(1)_PQ not compatible with gravity
- need some underlying gauge/discrete symmetry from which PQ emerges
- adding new PQ scale to theory: expect mh to blow up to fa
- expect fa -> mGUT-mP

SUSY cures ills associated with PQ

discrete R symmetries intrinsically supersymmetric: R(theta)=+1

discrete R symmetry can arise from compactification of extra dimensions in string theory

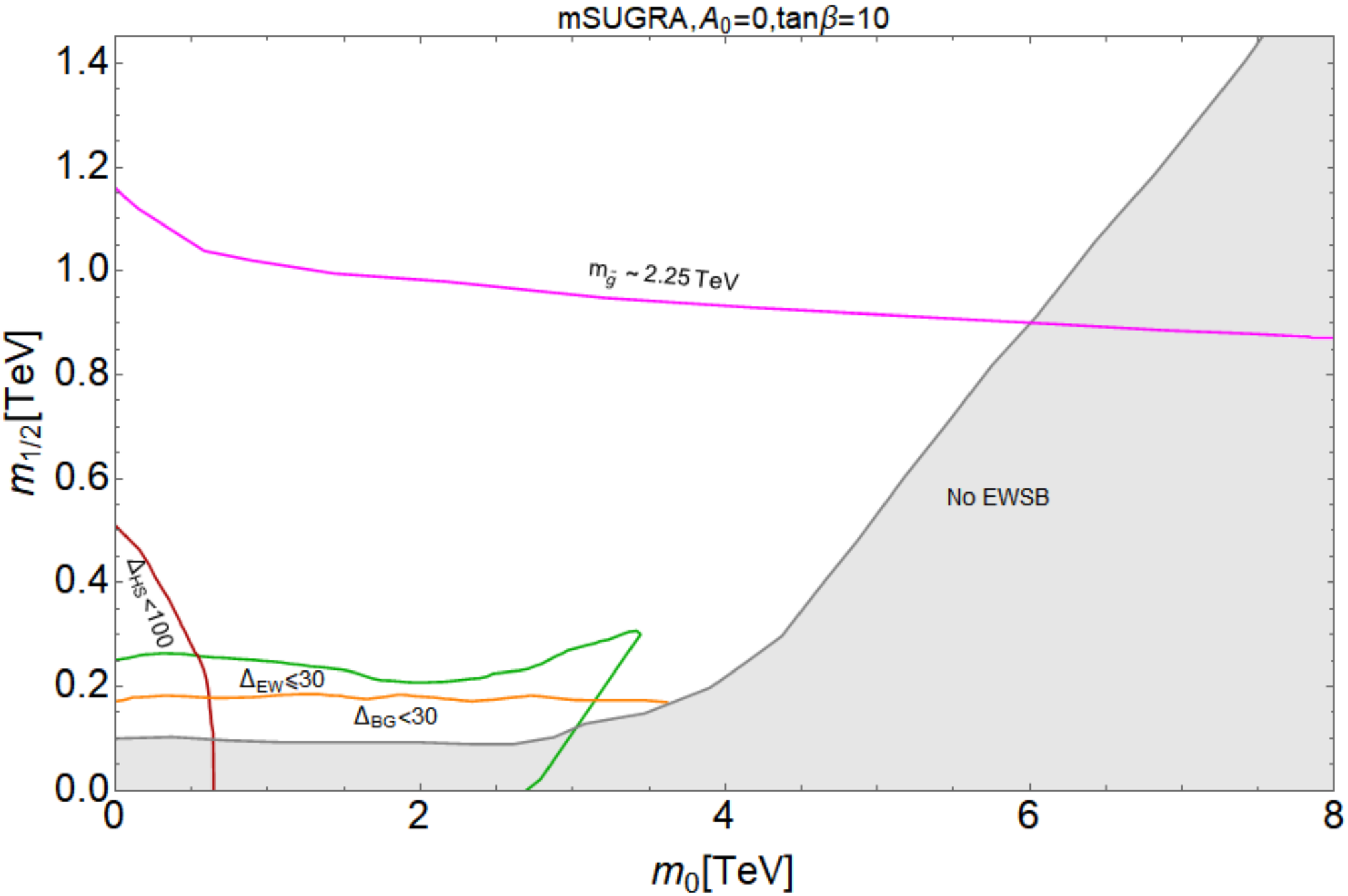
examples: hyCCK and hySPM models are gravity safe under Z(24)^R where R-parity and U(1)_PQ emerge as accidental approximate symmetries from underlying more fundamental Z(24)^R

HB, Barger, Sengupta

PQ scale set to cosmological sweet value fa~10^11 GeV via SUSY breaking:

$$f_a \sim \sqrt{m_{soft} m_P} \sim 10^{11}\ \mathrm{GeV}$$

But what about fact that LHC sees no sparticles?
Isn't there then a Little Hierarchy problem and hence SUSY is unnatural?



HB, Barger, Salam

Measures of fine-tuning:

#1: Simplest SUSY measure: Δ_{EW}

Working only at the weak scale, minimize scalar potential: calculate m(Z) or m(h)

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \quad \sim -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$ with $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ etc.

simple, direct, unambiguous interpretation:

- $|\mu| \sim m_Z \sim 100 - 200$ GeV
- $m_{H_u}^2$ should be driven to small negative values such that $-m_{H_u}^2 \sim 100 - 200$ GeV at the weak scale and
- that the radiative corrections are not too large: $\Sigma_u^u \lesssim 100 - 200$ GeV

CETUP*-12/002, FTPI-MINN-12/22, UMN-TH-3109/12, UH-511-1195-12

Radiative natural SUSY with a 125 GeV Higgs boson

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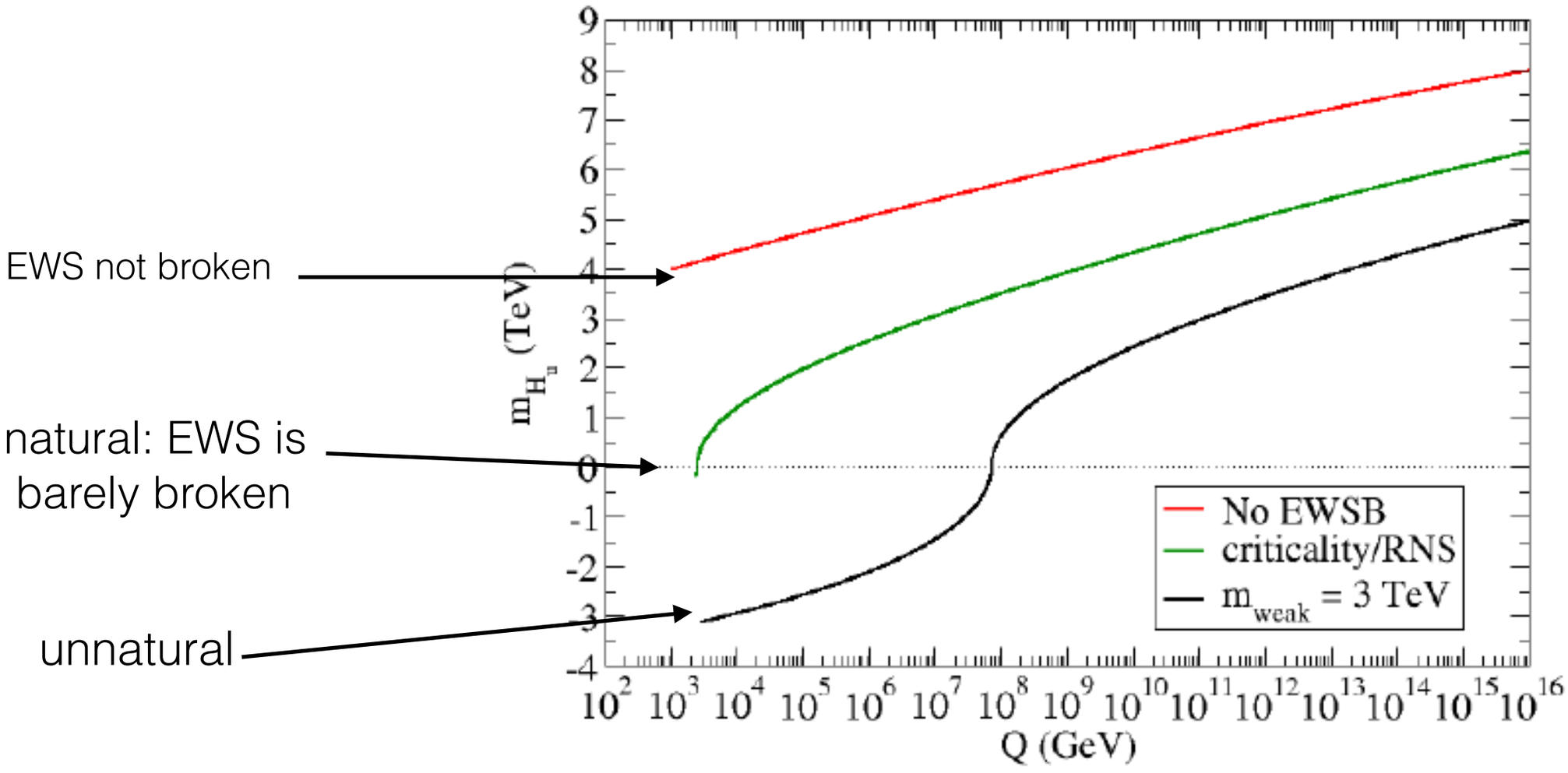
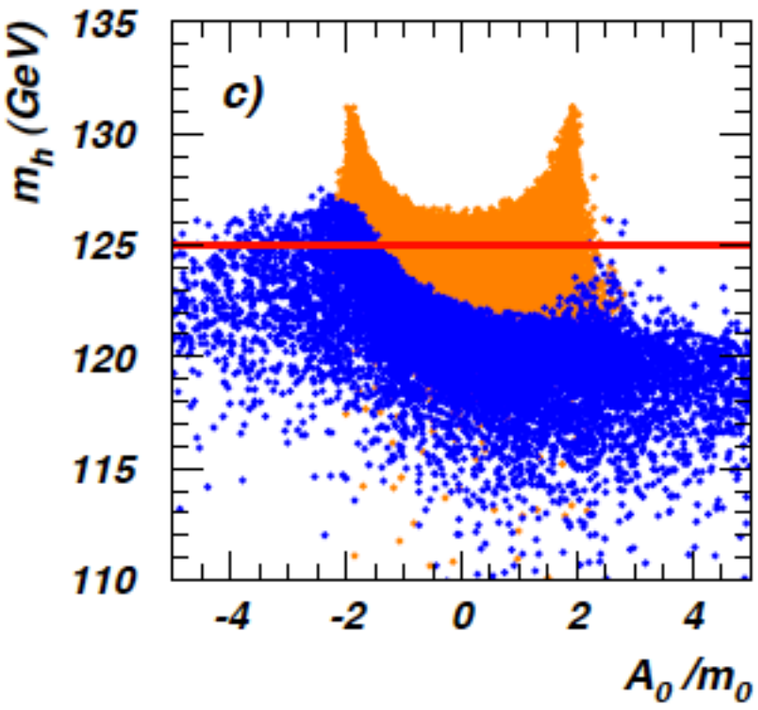
PRL109 (2012) 161802

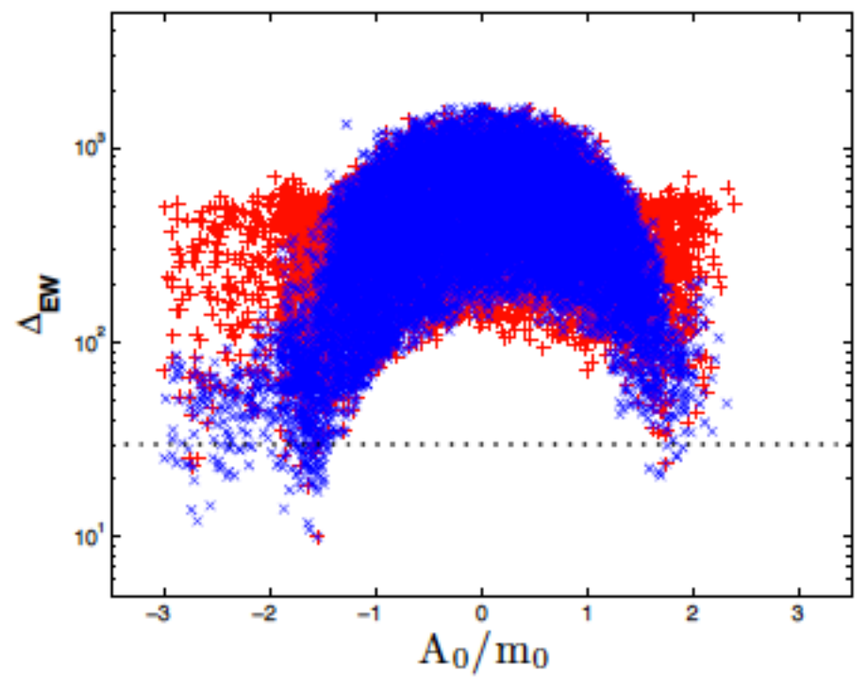
radiative corrections drive $m^2_{H_u}$ from unnatural GUT scale values to naturalness at weak scale: radiatively-driven naturalness



Evolution of the soft SUSY breaking mass squared term $sign(m^2_{H_u})\sqrt{|m^2_{H_u}|}$ vs. Q

Large value of A_t reduces $\Sigma_u^u(\tilde{t}_{1,2})$ contributions to Δ_{EW} while uplifting m_h to ~ 125 GeV





$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m^2_{\tilde{t}_{1,2}}) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8 g_Z^2 (\frac{1}{4} - \frac{2}{3} x_W) \Delta_t}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}} \right]$$

$$\Delta_t = (m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R})/2 + M_Z^2 \cos 2\beta (\frac{1}{4} - \frac{2}{3} x_W)$$

$$F(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right) \qquad Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$$

#2: Higgs mass or large-log fine-tuning Δ_{HS}

It is tempting to pick out one-by-one
quantum fluctuations but must combine log divergences
before taking any limit

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{rad}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from m(SUSY) to Lambda

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + A_t^2\right)\ln(\Lambda/m_{SUSY})$$

$$\Delta_{HS} \sim \delta m_h^2/(m_h^2/2) < 10$$

$$m_{\tilde{t}_{1,2},\tilde{b}_1} < 500 \text{ GeV}$$

$$m_{\tilde{g}} < 1.5 \text{ TeV}$$

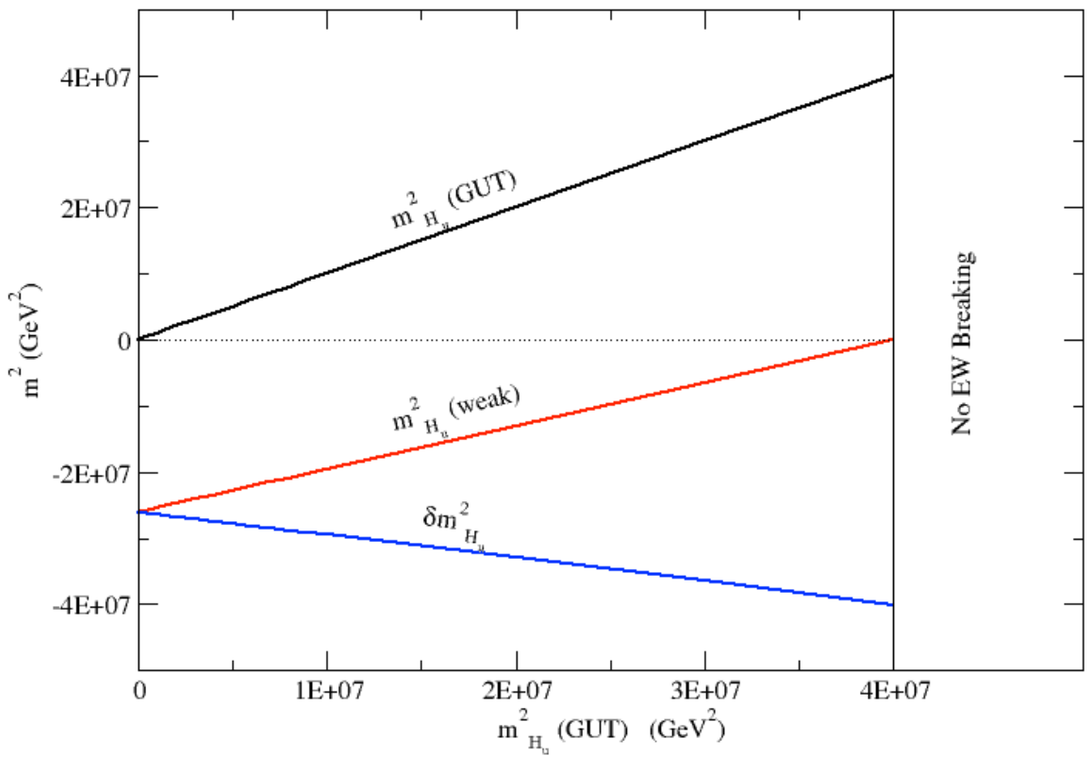
old natural SUSY then A_t can't be too big

What's wrong with this argument?

In zeal for simplicity, have made several simplifications: most egregious is that one sets m(Hu)^2=0 at beginning to simplify

$$m^2_{H_u}(\Lambda) \text{ and } \delta m^2_{H_u} \text{ are } not \text{ independent!}$$

violates prime directive!



The larger $m^2_{H_u}(\Lambda)$ becomes, then the larger becomes the cancelling correction!

HB, Barger, Savoy

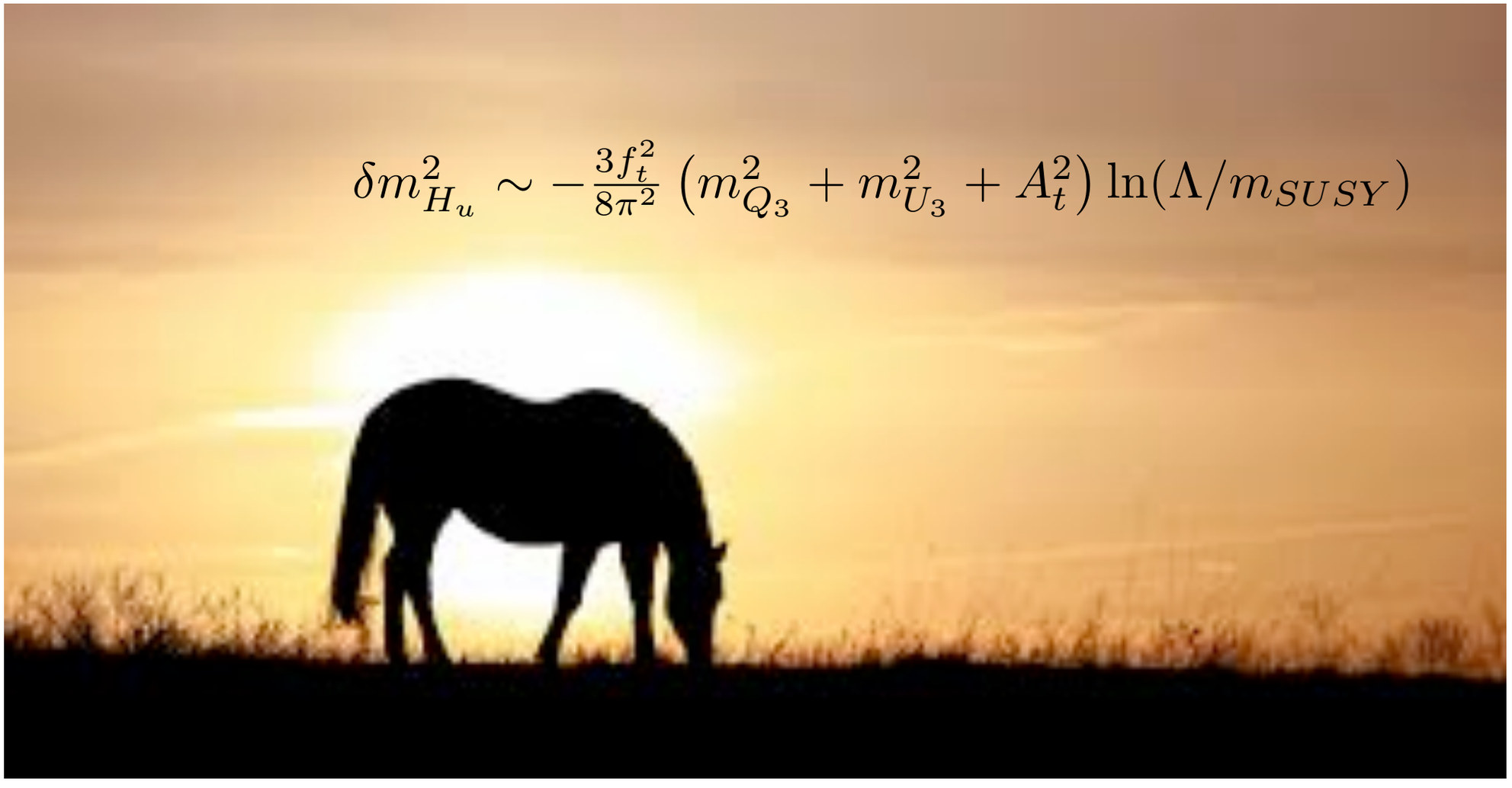
To fix: combine dependent terms:

$m_h^2 \simeq \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ where now both μ^2 and $\left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ are $\sim m_Z^2$

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

Instead of: the radiative correction $\delta m_{H_u}^2 \sim m_Z^2$
we now have: the radiatively-corrected $m_{H_u}^2 \sim m_Z^2$

Recommendation: put this horse out to pasture



R.I.P.

sub-TeV 3rd generation squarks not required for naturalness

#3. What about EENZ/BG measure?

$$\Delta_{BG} = max_i |\frac{\partial \log m_Z^2}{\partial \log p_i}| = max_i |\frac{p_i}{m_Z^2}\frac{\partial m_Z^2}{\partial p_i}|$$

$$p_i \text{ are the theory parameters}$$

applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

apply to high (e.g. GUT) scale parameters

$$\begin{aligned}
m_Z^2 \;\simeq\; & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\
& +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\
& -0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\
& -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\
& +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\
& +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\
& +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2,
\end{aligned}$$

applied to most parameters,

Δ_{BG} large, looks fine-tuned for *e.g.* $m_{\tilde{t}_1} \sim 1$ TeV

$\Delta_{BG}(Q_3) \simeq 0.73\frac{1000^2}{91.2^2} \sim 100$

#3. What about EENZ/BG measure?

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What if we apply to high (e.g. GUT) scale parameters ?

$$\begin{aligned}
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& +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\
& -0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\
& -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\
\hline
& +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\
\hline
& +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\
& +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2,
\end{aligned}$$

For correlated scalar masses $\equiv m_0$,
scalar contribution collapses:
what looks fine-tuned isn't: *focus point SUSY*
multi-TeV scalars are *natural*

Feng, Matchev, Moroi

Even with FP, still
fine-tuned on m(gluino) :(

But wait! in more complete models,
soft terms not independent

violates prime directive!

e.g. in SUGRA, for well-specified hidden sector,
each soft term calculated as multiple of m(3/2);
soft terms must be combined!

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3} m_{3/2}$

in general:

$$\begin{aligned} m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\ m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\ A_t &= a_{A_t} \cdot m_{3/2}, \\ M_i &= a_i \cdot m_{3/2}, \\ &\cdots. \end{aligned}$$

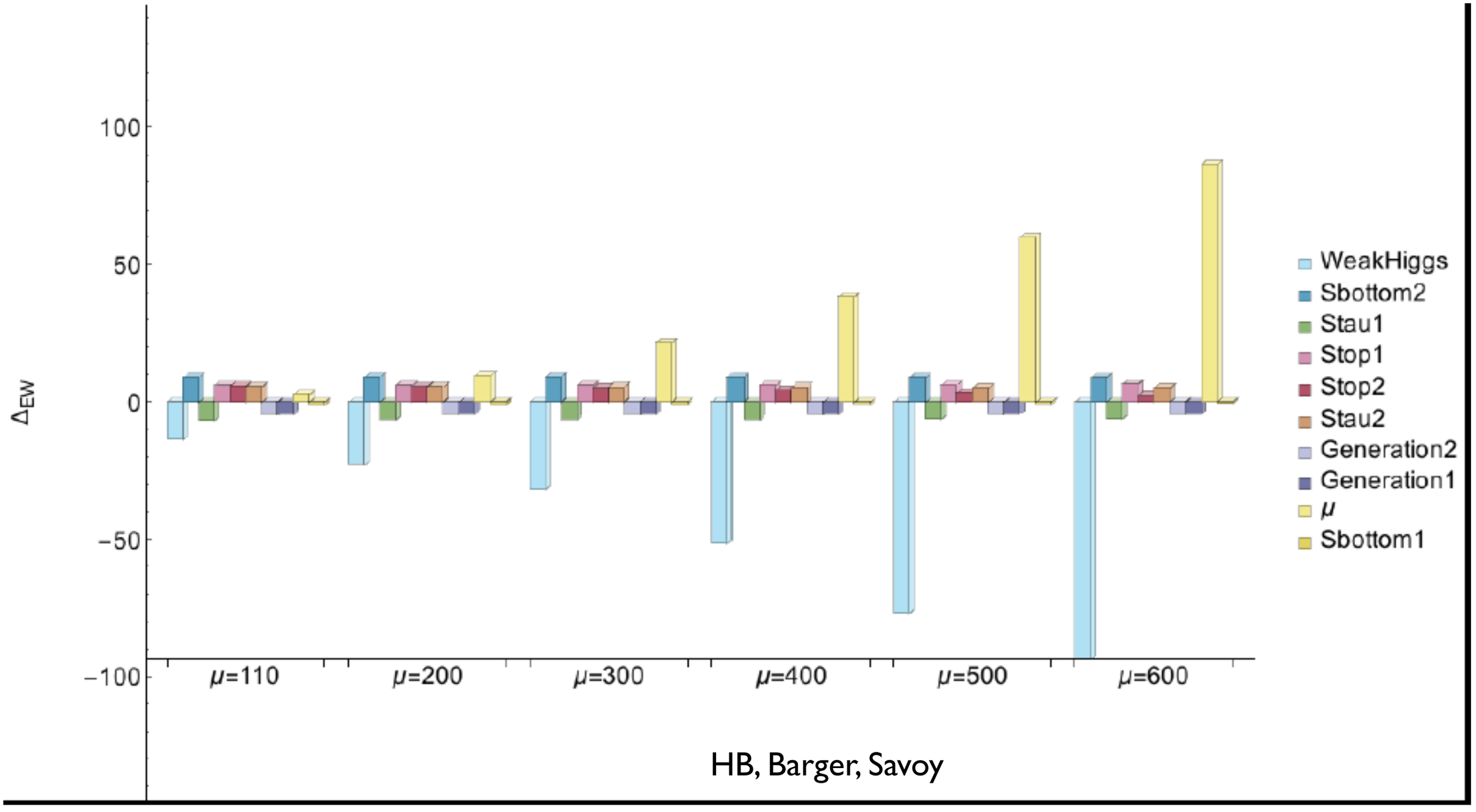
since μ hardly runs, then

$$\boxed{\begin{aligned} m_Z^2 &\simeq -2\mu^2 + a \cdot m_{3/2}^2 \\ &\simeq -2\mu^2 - 2m_{H_u}^2(weak) \end{aligned}}$$

$$m_{H_u}^2(weak) \sim -(100-200)^2 \text{ GeV}^2 \sim -a \cdot m_{3/2}^2/2$$

using μ^2 and $m_{3/2}^2$ as fundamental,
then $\Delta_{BG} \simeq \Delta_{EW}$ even using high scale parameters!

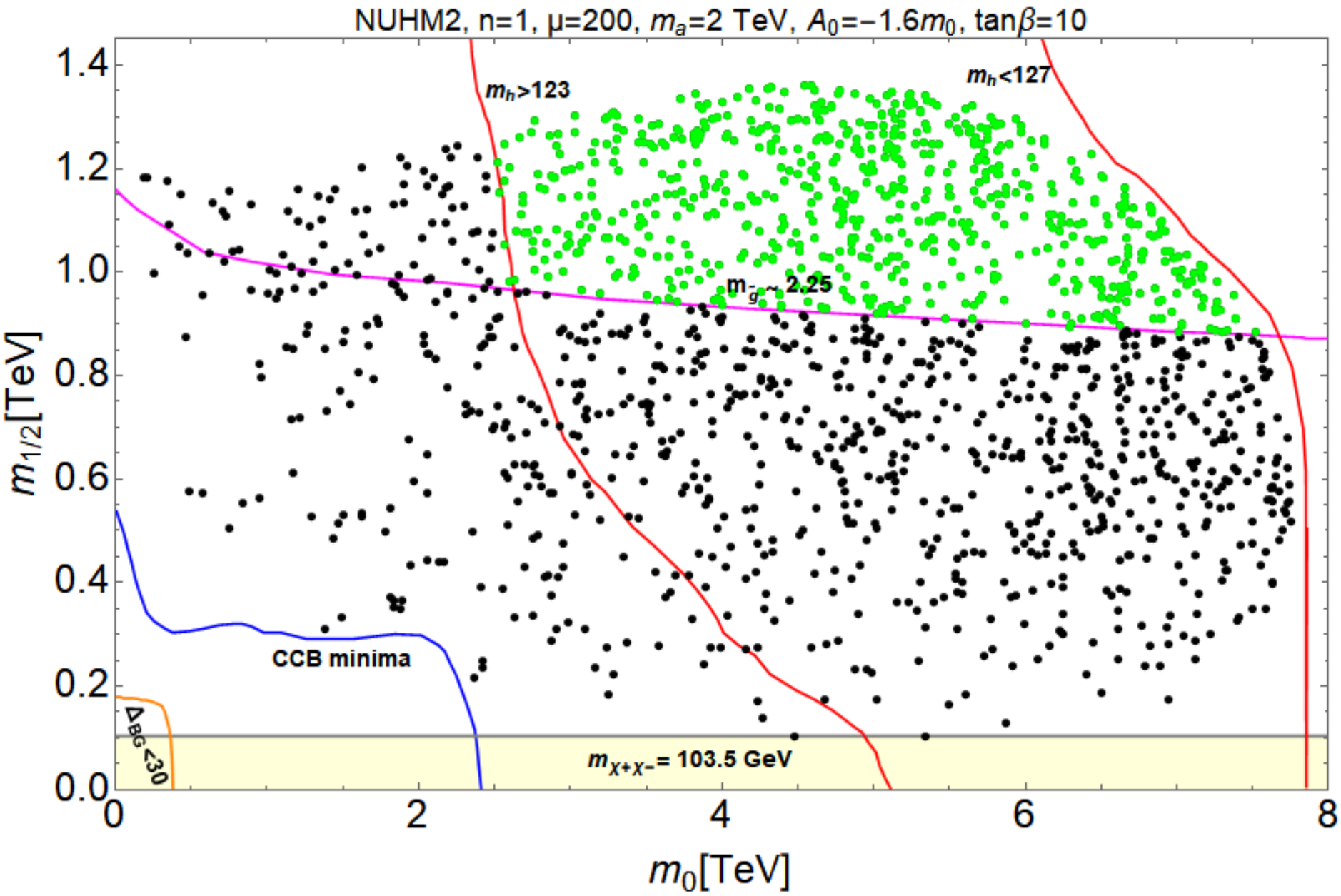
How much is too much fine-tuning?



Visually, large fine-tuning has already developed by $\mu \sim 350$ or $\Delta_{EW} \sim 30$

Stringy naturalness (Douglas):

landscape statistical pull to large soft terms balanced
by anthropic requirement of not too large a weak scale

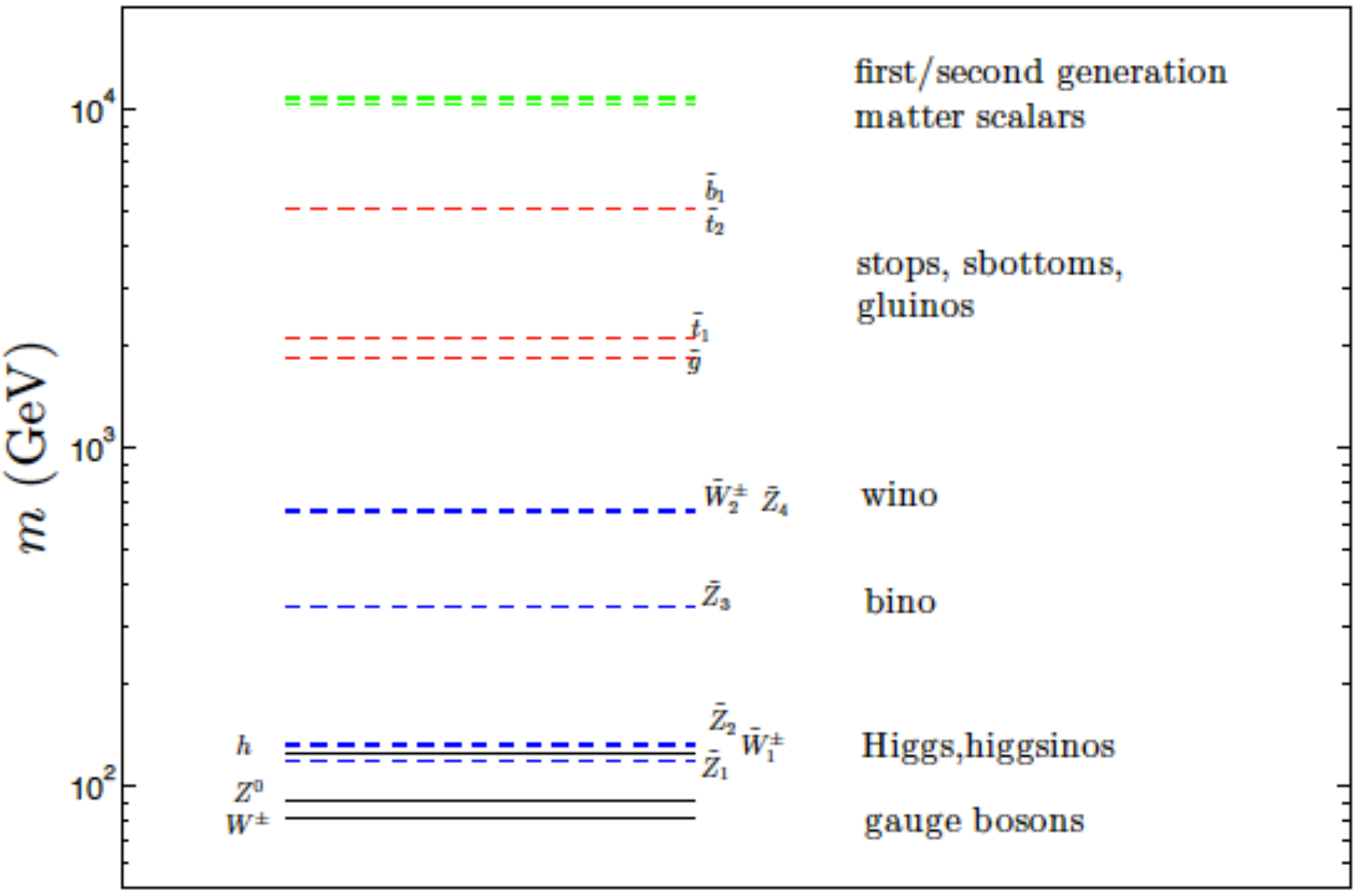


greater density of points is more stringy natural:
soft terms as big as possible such that EW symmetry
properly broken, not too big EW scale

bounds from naturalness (3%)	BG/DG	Delta_EW
mu	350 GeV	350 GeV
gluino	400-600 GeV	6 TeV
t1	450 GeV	3 TeV
sq/sl	550-700 GeV	10-30 TeV

h(125) and LHC limits are perfectly compatible with 3-10% naturalness: no crisis!

Typical spectrum for low Δ_{EW} models



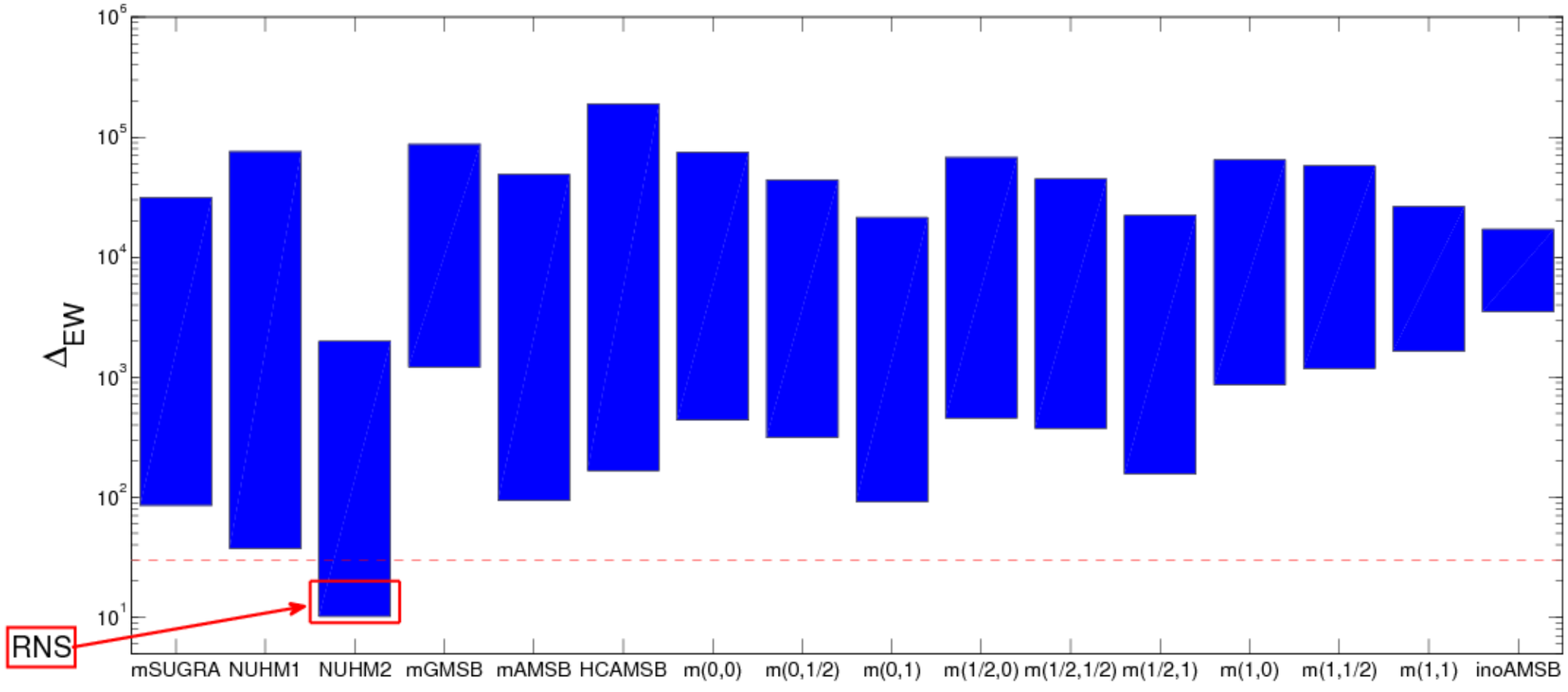
There is a Little Hierarchy, but it is no problem

$$\mu \ll m_{3/2}$$

Δ_{EW} is highly selective:
most constrained models are ruled out
except NUHM2 and its generalizations:

J. Ellis, K. Olive and Y. Santoso, *Phys. Lett.* **B 539** (2002) 107; J. Ellis, T. Falk, K. Olive and Y. Santoso, *Nucl. Phys.* **B 652** (2003) 259; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *J. High Energy Phys.* **0507** (2005) 065.

scan over p-space with m(h)=125.5+-2.5 GeV:



0.1%

1%

10%

HB, Barger, Mickelson,Padeffke-Kirkland, PRD89 (2014) 115019

What about WIMPs–
weren't they supposed to have been observed by now?

But so far we have addressed only Part 1
of fine-tuning problem:

In QCD sector, the term $\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu}\tilde{F}_A^{\mu\nu}$ must occur

But neutron EDM says it is not there: strong CP problem

(frequently ignored by SUSY types)

Best solution after 35 years:

PQWW/KSVZ/DFSZ invisible axion

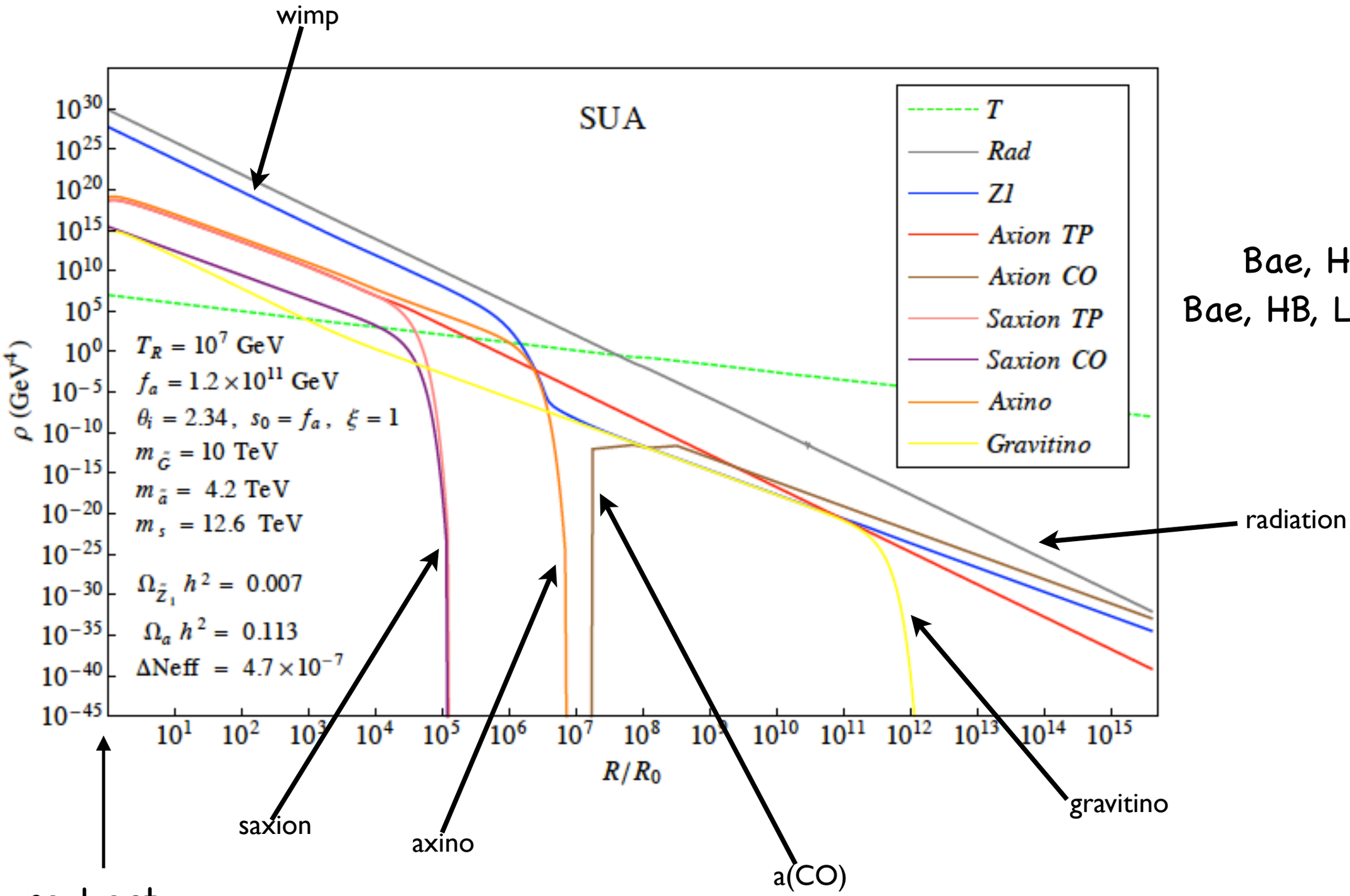
In SUSY, axion accompanied by axino and saxion

Changes DM calculus:
expect mixed WIMP/axion DM (2 particles)

mixed axion-neutralino production in early universe

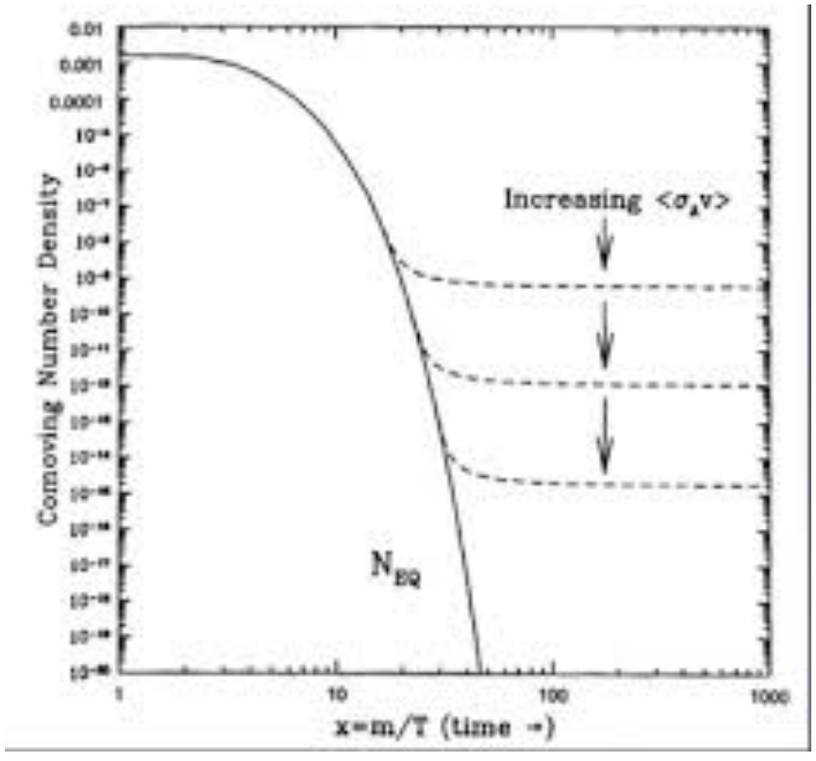
- neutralinos: thermally produced (TP) or NTP via $\tilde{a}$, s or $\tilde{G}$ decays
  - re-annihilation at $T_D^{s,\tilde{a}}$
- axions: TP, NTP via $s \to aa$, bose coherent motion (BCM)
- saxions: TP or via BCM
  - $s \to gg$: entropy dilution
  - $s \to SUSY$: augment neutralinos
  - $s \to aa$: dark radiation ($\Delta N_{eff} < 1.6$)
- axinos: TP
  - $\tilde{a} \to SUSY$ augments neutralinos
- gravitinos: TP, decay to SUSY

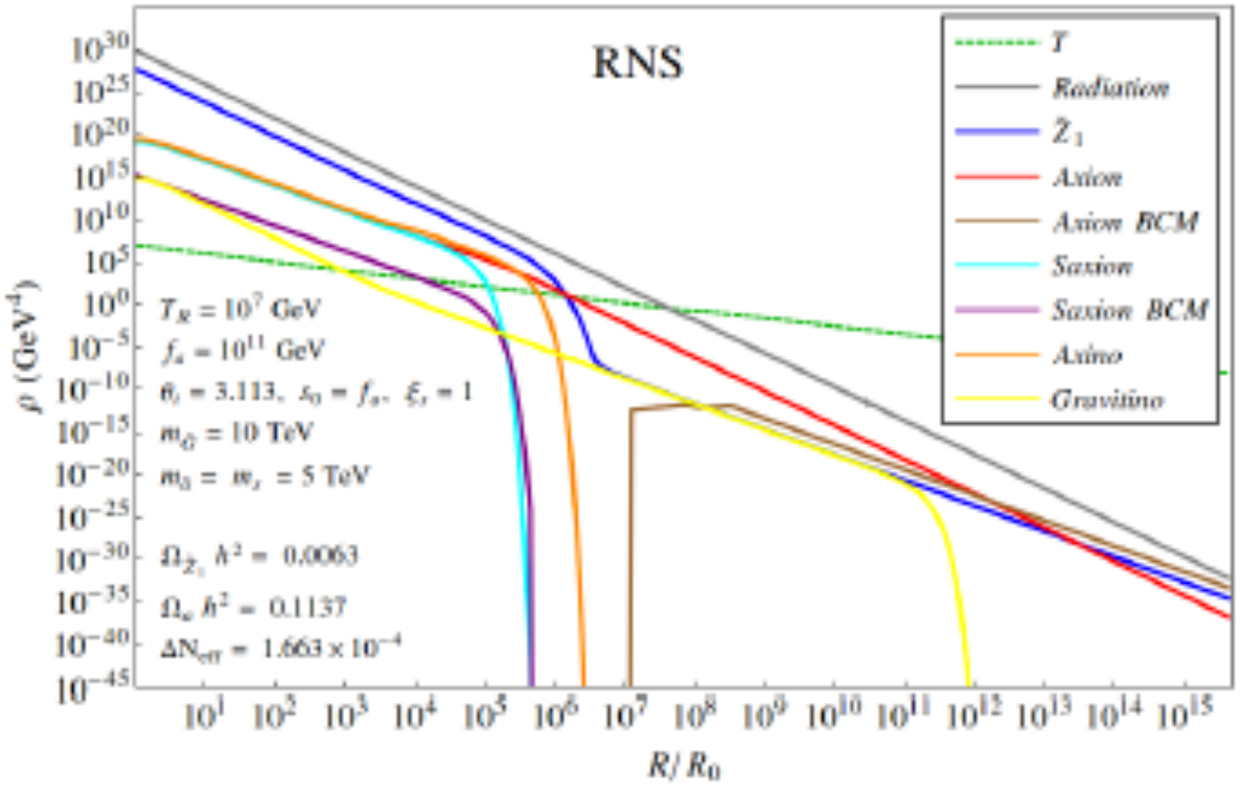
DM production in SUSY DFSZ: solve eight coupled Boltzmann equations



Bae, HB, Chun;
Bae, HB, Lessa, Serce

usual picture => mixed axion/WIMP

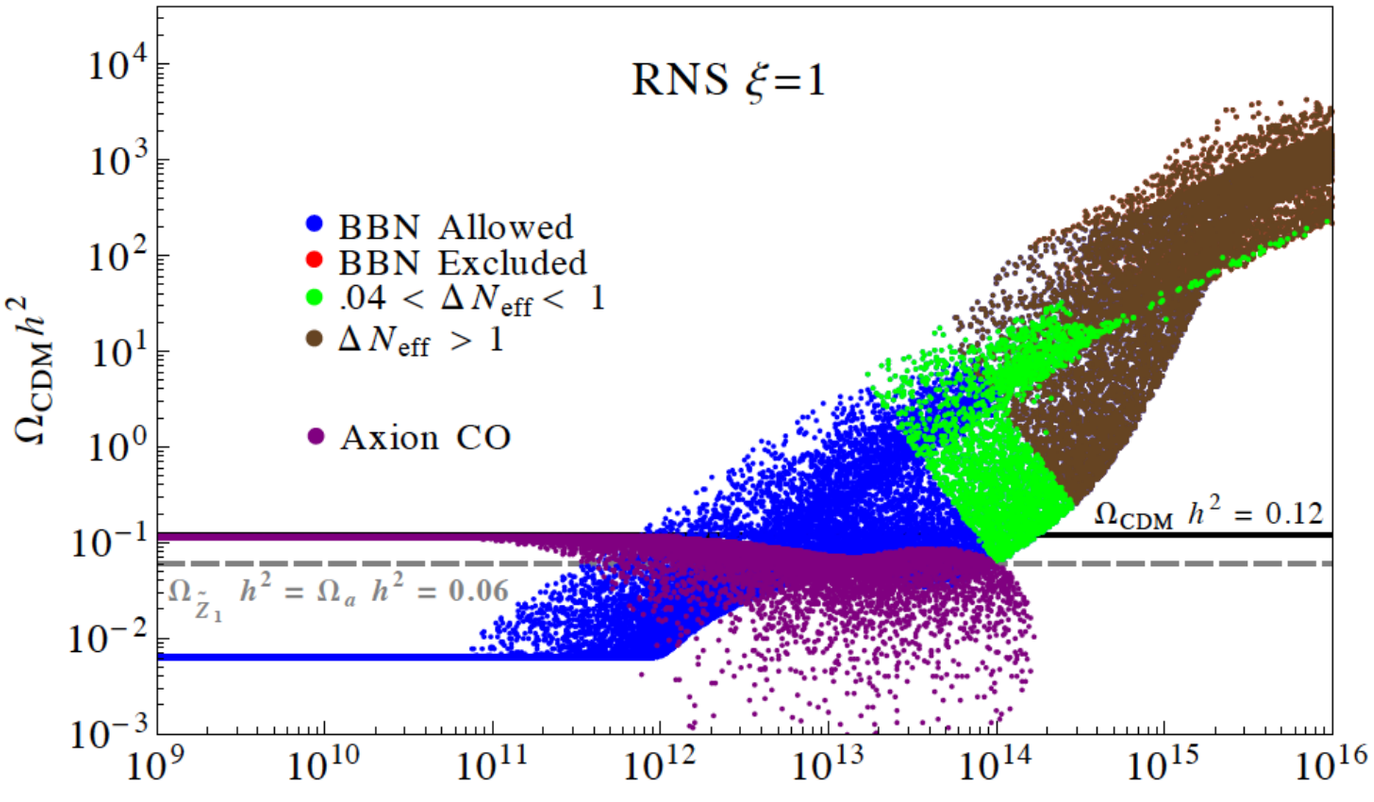




KJ Bae, HB, Lessa, Serce

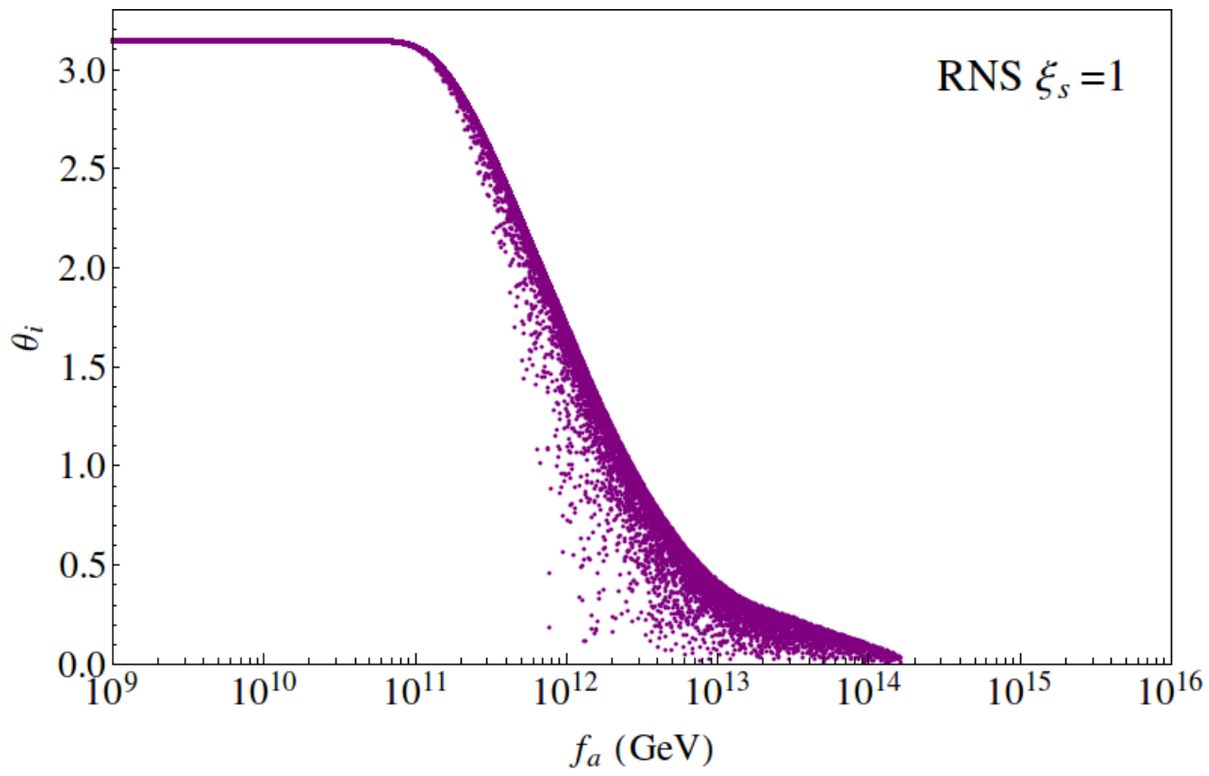
much of parameter space is axion-dominated
with 10-15% WIMPs

=>



higgsino abundance

axion abundance



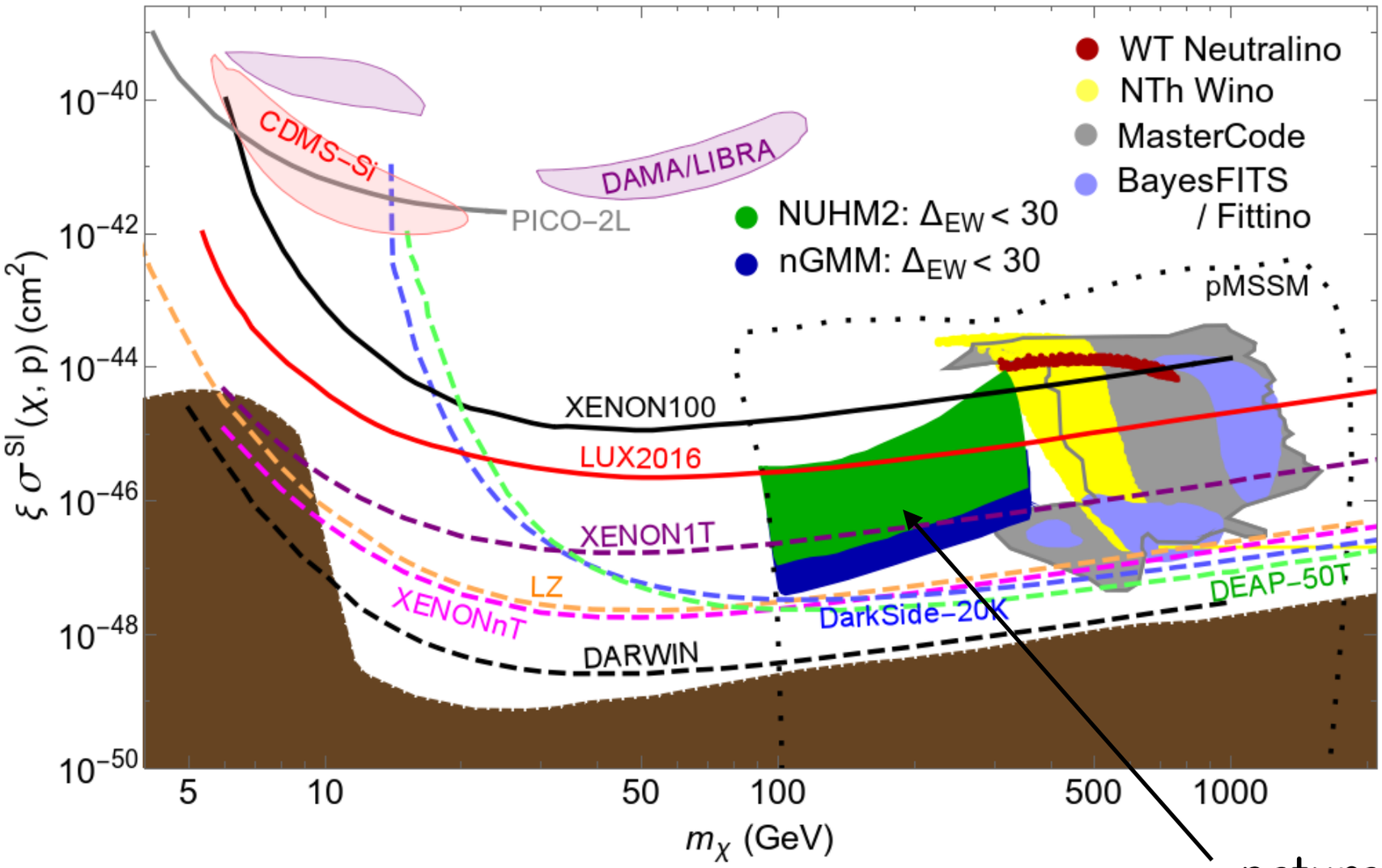
mainly axion CDM

for fa<~10^12 GeV;

for higher fa, then get increasing wimp abundance

Bae, HB,Lessa,Serce

Direct higgsino detection rescaled for minimal local abundance $\xi \equiv \Omega_\chi^{TP} h^2/0.12$



Bae, HB, Barger,Savoy,Serce

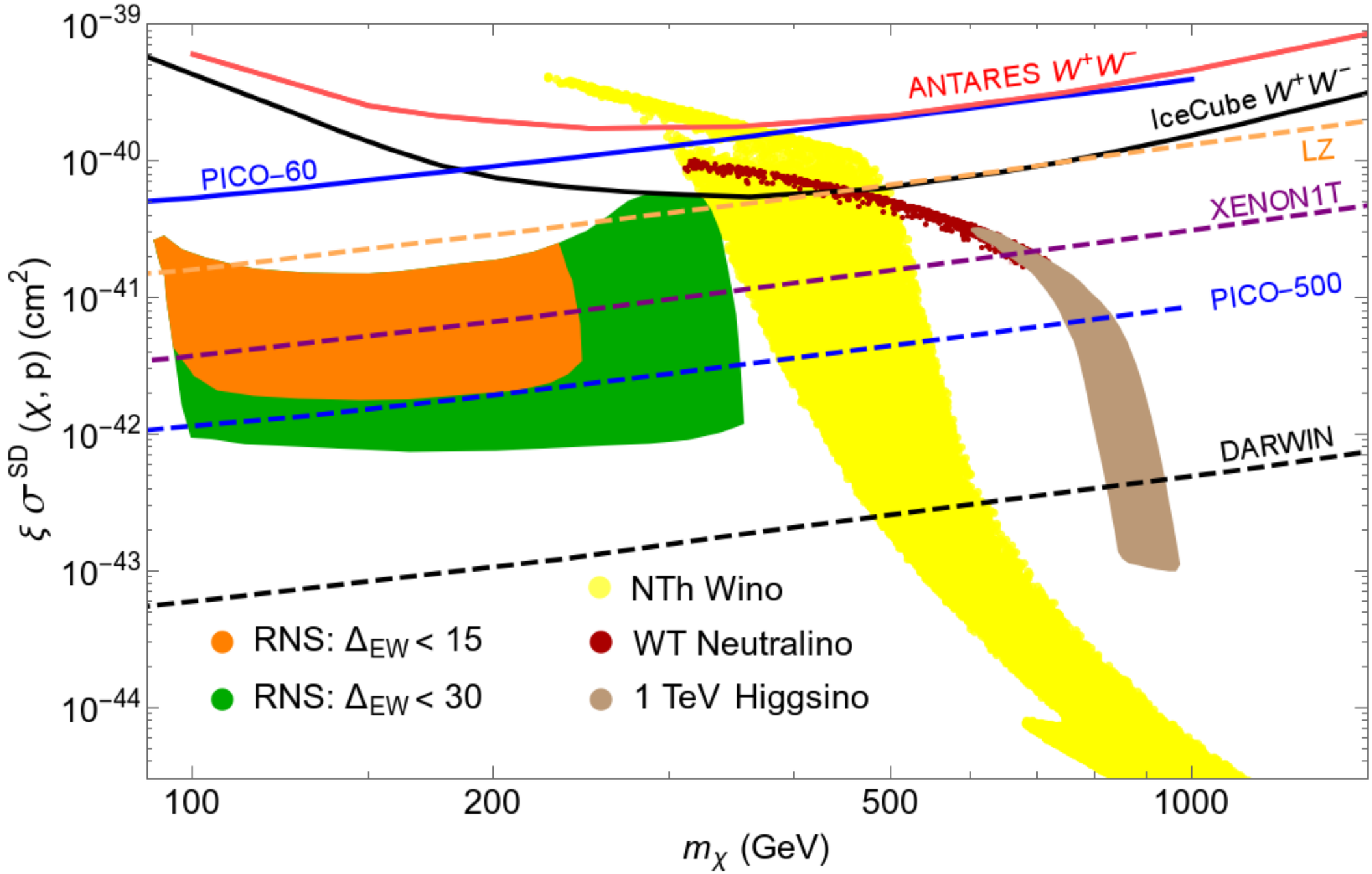
$$\mathcal{L} \ni -X_{11}^h \bar{\tilde{Z}}_1 \tilde{Z}_1 h$$

$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)} \sin\alpha - v_1^{(1)} \cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

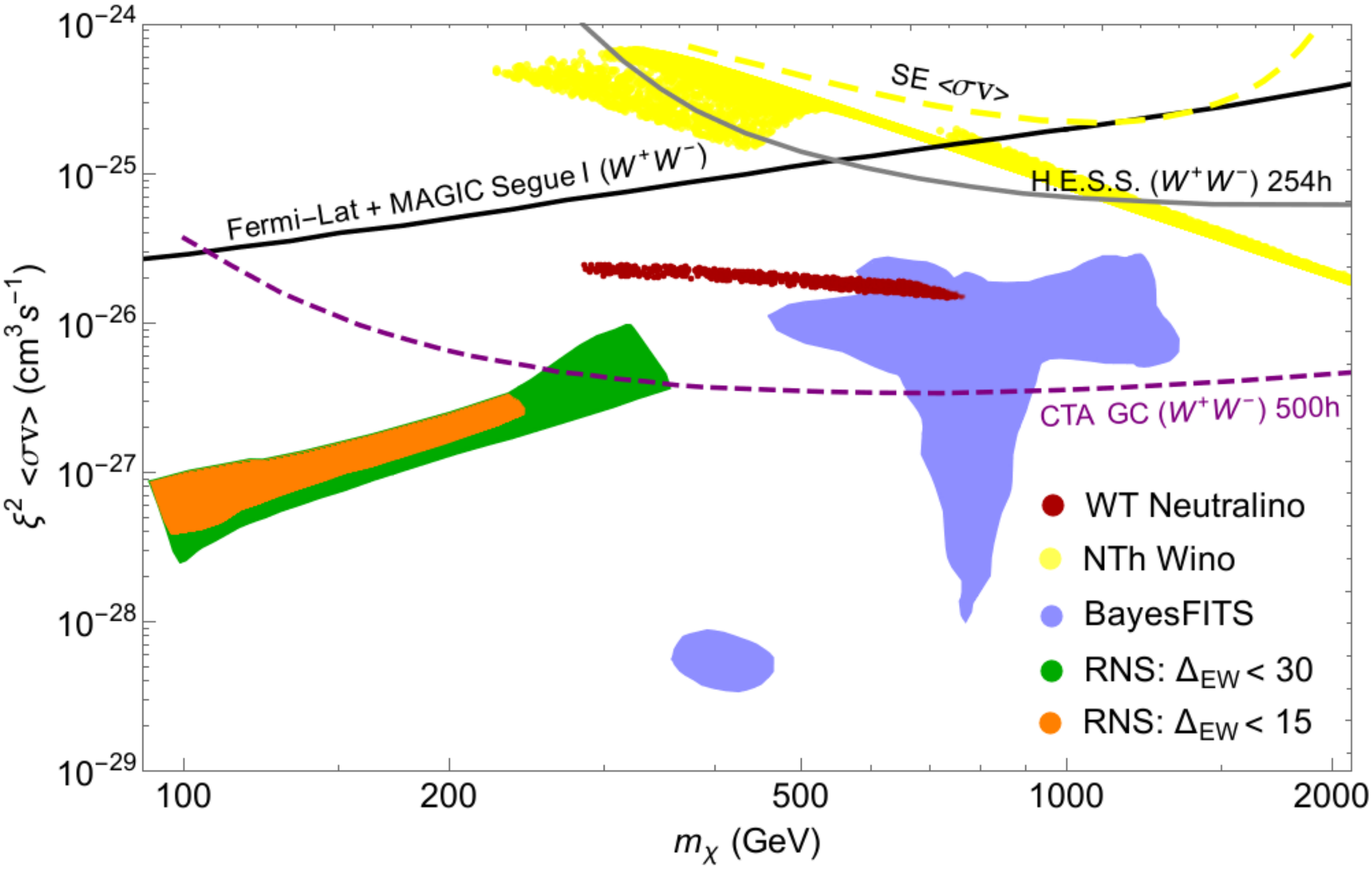
Xe-1-ton
now operating!

Can test completely with ton scale detector
or equivalent (subject to minor caveats)

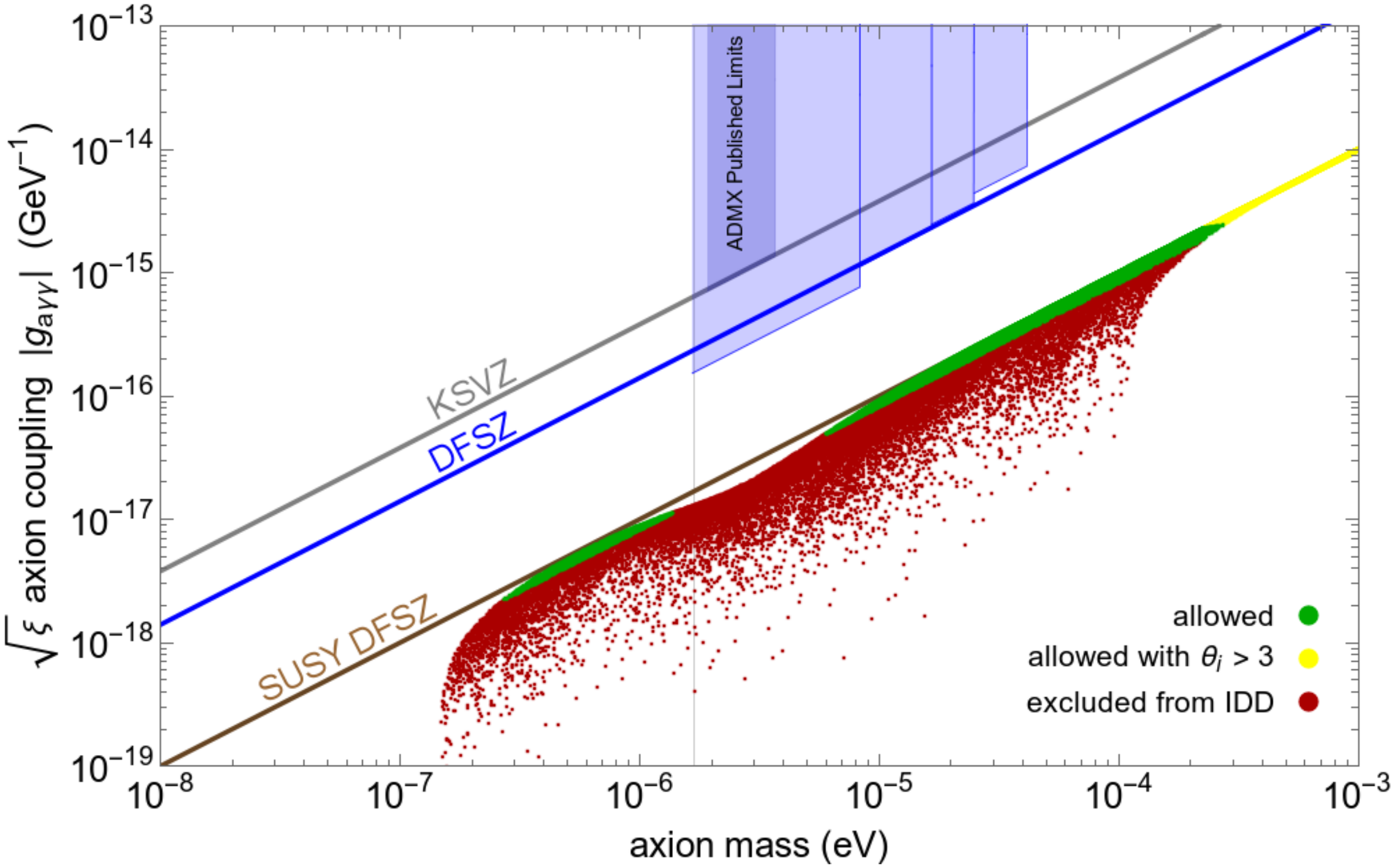
Prospects for SD WIMP searches:



Prospects for IDD WIMP searches:

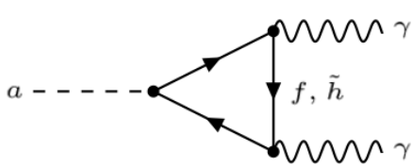


suppressed by square of diminished WIMP abundance



SUSY DFSZ axion: large range in m(a) but coupling reduced

may need to probe broader and deeper!



Bae, HB, Serce

Conclusions:

- Weak scale SUSY is more highly motivated than ever
- Gauge hierarchy, DM, strong CP, inflation,baryogenesis
- Supported by data via four virtual effects
- While mSUGRA/CMSSM,GMSB,AMSB likely ruled out, NUHM2,3, nAMSB, nMirageMediation still viable
- Axion may make bulk of dark matter, but WIMPs likely still around: should ultimately be detected: role of discrete R-symmetries
- HL-LHC may detect via SDLJ+MET signature
- Gluino pair and stop pair may have to await HE-LHC