Precision Standard Model parameters for matching to SUSY and other ultraviolet completions

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SUSY 2019

Corpus Christi

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Based on work with David G. Robertson, `arXiv:hep-ph/1906.?????`, and new computer software library soon to be released: `SMDR`

The LHC has now:

- Discovered the Higgs boson with $M_h = 125$ GeV and interactions consistent with the minimal Standard Model
- Set strong bounds on new physics beyond the Standard Model

The Standard Model, defined by its Lagrangian, is mathematically and technically complete, valid up to 1-2 TeV.

One might perhaps expect to find in the Review of Particle Properties a list of the best fit values of the Lagrangian parameters of the Standard Model, but it isn't there...

Instead, the RPP provides only derived, or on-shell, quantities that are closer to experiment but not directly related to the fundamental definition of the theory.

Goal: to determine the fundamental Lagrangian parameters of the theory as precisely as possible.

Standard Model $\overline{\text{MS}}$ parameters (fundamental inputs):

- Renormalization scale Q
- Higgs VEV, self-coupling $v, \ \lambda$
- gauge couplings $g_3, \ g, \ g'$
- Yukawa couplings $y_t, \ y_b, \ y_c, \ y_s, \ y_d, \ y_u, \ y_\tau, \ y_\mu, \ y_e$
- CKM mixing angles and CP phase, neutrino masses and PMNS angles (neglected here; very tiny effect on numerical results below)
- Light-quark contribution to fine-structure constant $\Delta\alpha^{(5)}_{\rm had}(M_Z)$

All are running parameters evaluated at Q, except the last. In principle, $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ is not independent, but in practice its determination relies on non-perturbative physics, so its effectively an independent parameter.

Can trade VEV for negative Higgs squared mass parameter m^2:

$$V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4$$

Two possible definitions of the VEV

- Tree-level VEV: $v_{\rm tree} = \sqrt{-m^2/\lambda}$
  - Advantage: manifestly gauge-invariant
  - Disadvantage: must include tadpole graphs, perturbation theory includes factors $1/\lambda^n$ at loop order n
- Loop-corrected VEV: $v =$ minimum of the full effective potential
  - Advantage: tadpole graphs vanish, need not be included. Sum of all Higgs tadpoles $\propto \partial V_{\rm eff}/\partial\phi$.
  - Disadvantage: depends on gauge choice; Landau gauge is sensible

Relationship between the two definitions:

$$v_{\rm tree}^2 = v^2 + \frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{1}{(16\pi^2)^n}\Delta_n,$$

where Δ_n are related to derivatives of the n-loop effective potential.

Many other works choose the tree-level VEV scheme.

Perturbation theory converges somewhat slower because of $1/\lambda^n$ factors.

Intuitively: expanding around the “wrong” = tree-level vacuum.

We choose to use the loop-corrected VEV scheme = **tadpole-free pure $\overline{\textbf{MS}}$ scheme**. All calculations use Landau gauge, so can’t vary gauge-fixing parameter for checks. However, one does gain checks from cancellation of Goldstone boson contributions.

Gauge dependence of VEV shouldn’t bother you: it isn’t an observable anyway (depends on RG scale).

The current state-of-the-art for the effective potential V_{eff} and Δ_n:

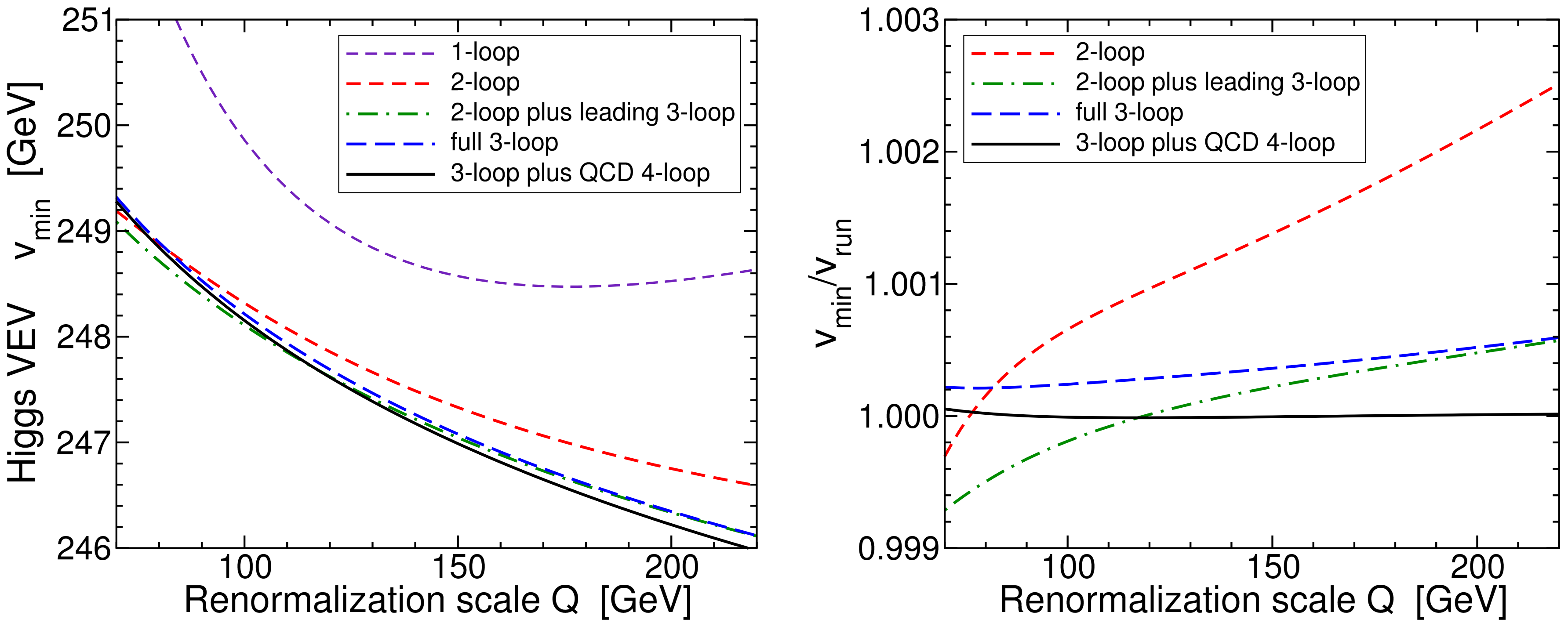
- Full 2-loop (Ford, Jack, Jones hep-ph/0111190)
- Full 3-loop (SPM 1310.7553, 1709.02397)
- 4-loop leading order in QCD (SPM 1508.00912)

Also include Goldstone boson resummation (SPM 1406.2355, Elias-Miro, Espinosa, Konstandin 1406.2652) to fix infrared problems and spurious imaginary parts associated with vanishing or negative field-dependent Goldstone boson squared masses.

In practice, evaluate the necessary 3-loop vacuum integral functions using code:
`3VIL = 3-loop Vacuum Integral Library`
(SPM and Robertson, 1610.07720)

This is incorporated in our new code to appear very soon:
`SMDR = Standard Model in Dimensional Regularization`

Scale-dependence of VEV v, for a typical set of input parameters specified at $Q = M_{\rm top}$:

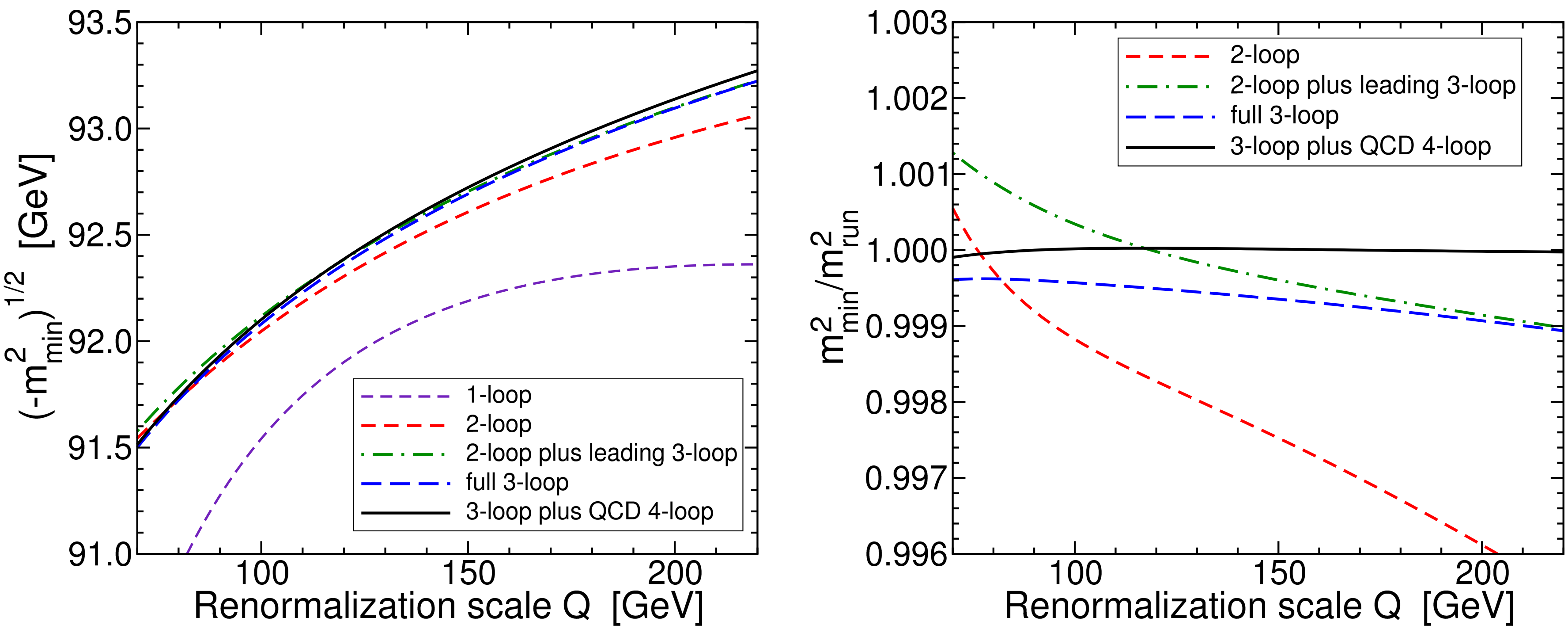


Right graph shows ratio of v to value $v_{\rm run}$ obtained by RG running from the input scale.

For 100 GeV $< Q <$ 200 GeV, RG scale dependence $< 1 \times 10^{-5}$, so about $\Delta v = \pm 2$ MeV.

Conservatively, purely theoretical error might be an order of magnitude larger, $\sim$20 MeV.

Converse of previous slide: take v as given at $Q = M_{\rm top}$, require $V_{\rm eff}$ to be minimized to obtain m^2:



Note the small RG scale dependence shown above involves only purely theoretical sources of error. Will return to this issue. . .

Tadpole-free pure $\overline{\text{MS}}$ inputs:

$$Q, v, \lambda, g_3, g, g', y_t, y_b, y_c, y_s, y_d, y_u, y_\tau, y_\mu, y_e, \Delta\alpha^{(5)}_{\text{had}}(M_Z), \ldots$$

$\updownarrow$ **SMDR**

On-shell observable outputs:

heavy particle pole masses:	$M_t,\ M_h,\ M_Z,\ M_W,$
running light quark masses:	$m_b(m_b),\ m_c(m_c),\ m_s(2\text{ GeV}),\ m_d(2\text{ GeV}),\ m_u(2\text{ GeV}),$
lepton pole masses:	$M_\tau,\ M_\mu,\ M_e,$
5-quark QCD coupling:	$\alpha_S^{(5)}(M_Z),$
Fermi constant:	$G_F = 1.1663787\ldots \times 10^{-5}\text{ GeV}^{-2},$
fine structure constant:	$\alpha = 1/137.035999139\ldots$ and $\Delta\alpha^{(5)}_{\text{had}}(M_Z)$

SMDR provides a complete map between $\overline{\text{MS}}$ inputs and observable outputs.

A benchmark model point in parameter space, defined by a fit to central values from the Particle Data Group's 2018 Review of Particle Properties:

$$M_t \;=\; 173.1 \text{ GeV}, \qquad M_h \;=\; 125.18 \text{ GeV},$$

$$M_{Z,\,\text{Breit-Wigner}} \;=\; 91.1876 \text{ GeV},$$

$$G_F \;=\; 1.1663787 \times 10^{-5}\,\text{GeV}^2,$$

$$\alpha \;=\; 1/137.035999139,$$

$$\alpha_S^{(5)}(M_Z) = 0.1181,$$

$$m_b(m_b) \;=\; 4.18 \text{ GeV}, \qquad m_c(m_c) \;=\; 1.275 \text{ GeV},$$

$$m_s(2 \text{ GeV}) \;=\; 0.095 \text{ GeV} \quad m_d(2 \text{ GeV}) \;=\; 0.0047 \text{ GeV}, \quad m_u(2 \text{ GeV}) \;=\; 0.0022 \text{ GeV},$$

$$M_\tau \;=\; 1.77686 \text{ GeV}, \qquad M_\mu \;=\; 0.1056583745 \text{ GeV}, \qquad M_e \;=\; 0.000510998946 \text{ GeV},$$

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) \;=\; 0.02764$$

The corresponding benchmark values for the $\overline{\text{MS}}$ parameters are found to be:

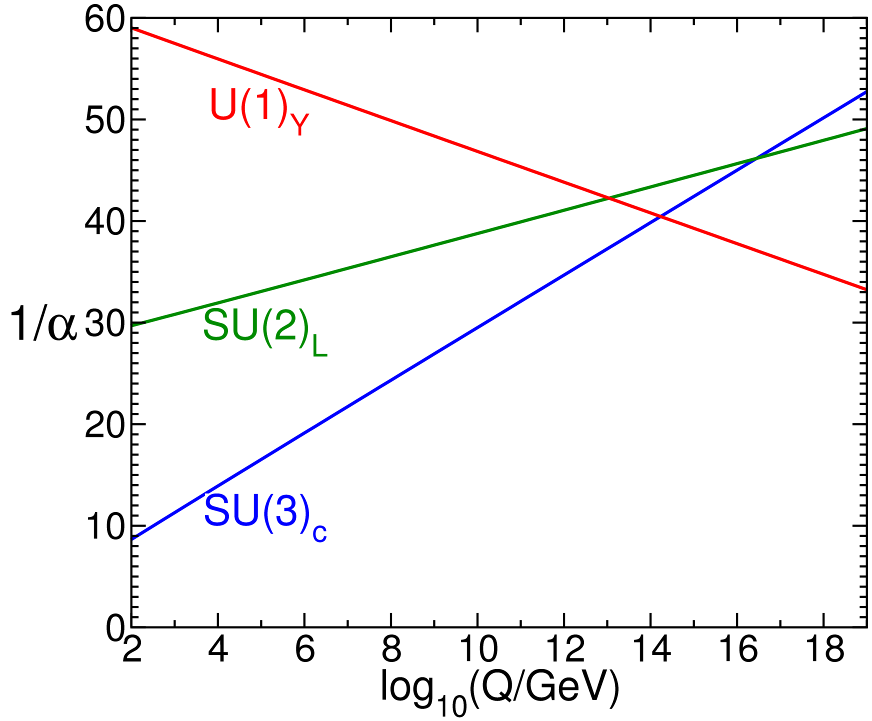
Benchmark input $\overline{\text{MS}}$ parameters:

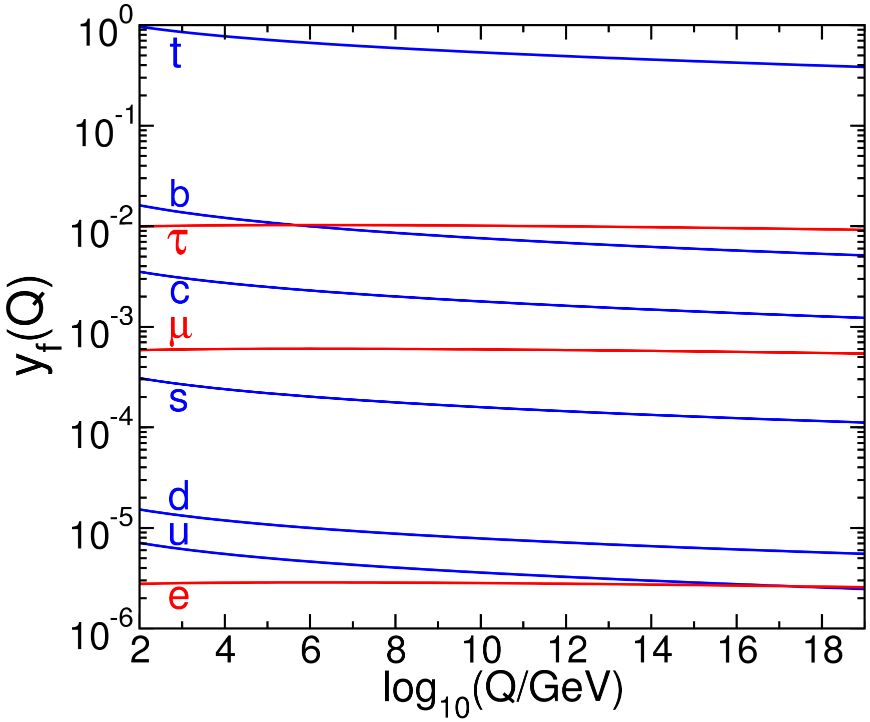
$$
\begin{aligned}
Q_0 &= 173.1\,\text{GeV}, \\
v(Q_0) &= 246.601\,\text{GeV}, \qquad \lambda(Q_0) \,=\, 0.126203, \\
g_3(Q_0) &= 1.163624, \qquad g_2(Q_0) \,=\, 0.647659, \qquad g'(Q_0) \,=\, 0.358539, \\
y_t(Q_0) &= 0.934799, \qquad y_b(Q_0) \,=\, 0.0154801, \qquad y_\tau(Q_0) \,=\, 0.00999446, \\
y_c(Q_0) &= 0.0034009, \qquad y_s(Q_0) \,=\, 0.000297202, \qquad y_\mu(Q_0) \,=\, 0.000588381, \\
y_d(Q_0) &= 1.47037 \times 10^{-5}, \qquad y_u(Q_0) \,=\, 6.84728 \times 10^{-6}, \\
y_e(Q_0) &= 2.792985 \times 10^{-6}.
\end{aligned}
$$

SMDR (incorporating state-of-the-art theory) allows you to:

- change the RPP observables, and find the corresponding best-fit $\overline{\text{MS}}$ parameters, or vice versa.
- compute individual observables in terms of the $\overline{\text{MS}}$ inputs
- do renormalization group running and decoupling of $\overline{\text{MS}}$ parameters

Run gauge couplings and Yukawa couplings from the input scale up to very high scales:





Uses 5-loop QCD and 3-loop for everything else.

Note: the Review of Particle Properties definitions of $\overline{\text{MS}}$ electroweak couplings $\widehat{\alpha}(M_Z)$ and $\widehat{s}^2_W(M_W)$ decouple the top quark, but not the W boson.

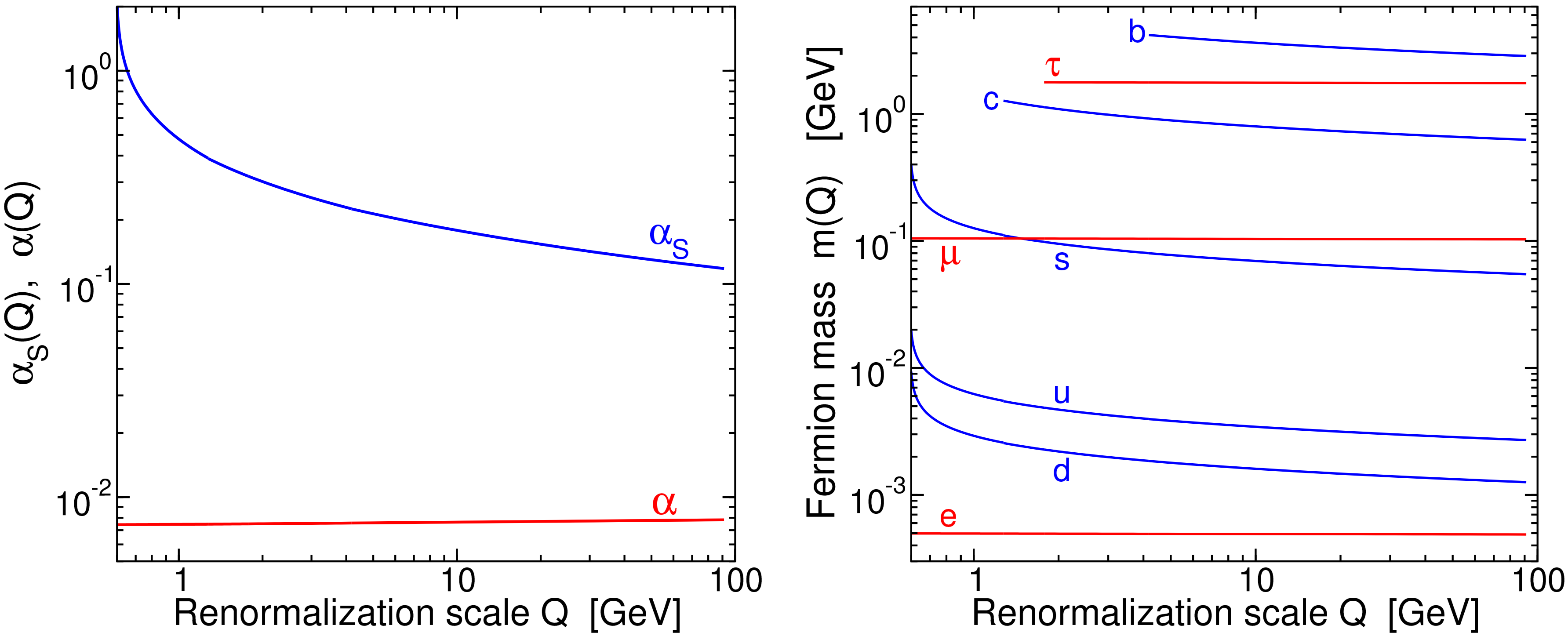
Not actually $\overline{\text{MS}}$ couplings in the usual sense, since the effective theory with a massive W boson is not renormalizable.

We prefer to decouple t, h, Z, W **simultaneously**, at a common matching scale.

The high-energy ("non-decoupled") theory has gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

The low-energy theory has gauge group $SU(3)_c \times U(1)_{\rm EM}$, with 5 massive quarks and 3 massive leptons.

Decouple top, Higgs, Z, W simultaneously at a scale $Q_{\rm dec}$ of your choice, and run down to lower Q, decoupling bottom, tau, charm at scales of your choice:



5-loop RG running and 4-loop decoupling for QCD, 3-loop running and 2-loop decoupling for other parameters. Complete 2-loop decoupling from SPM 1812.04100.

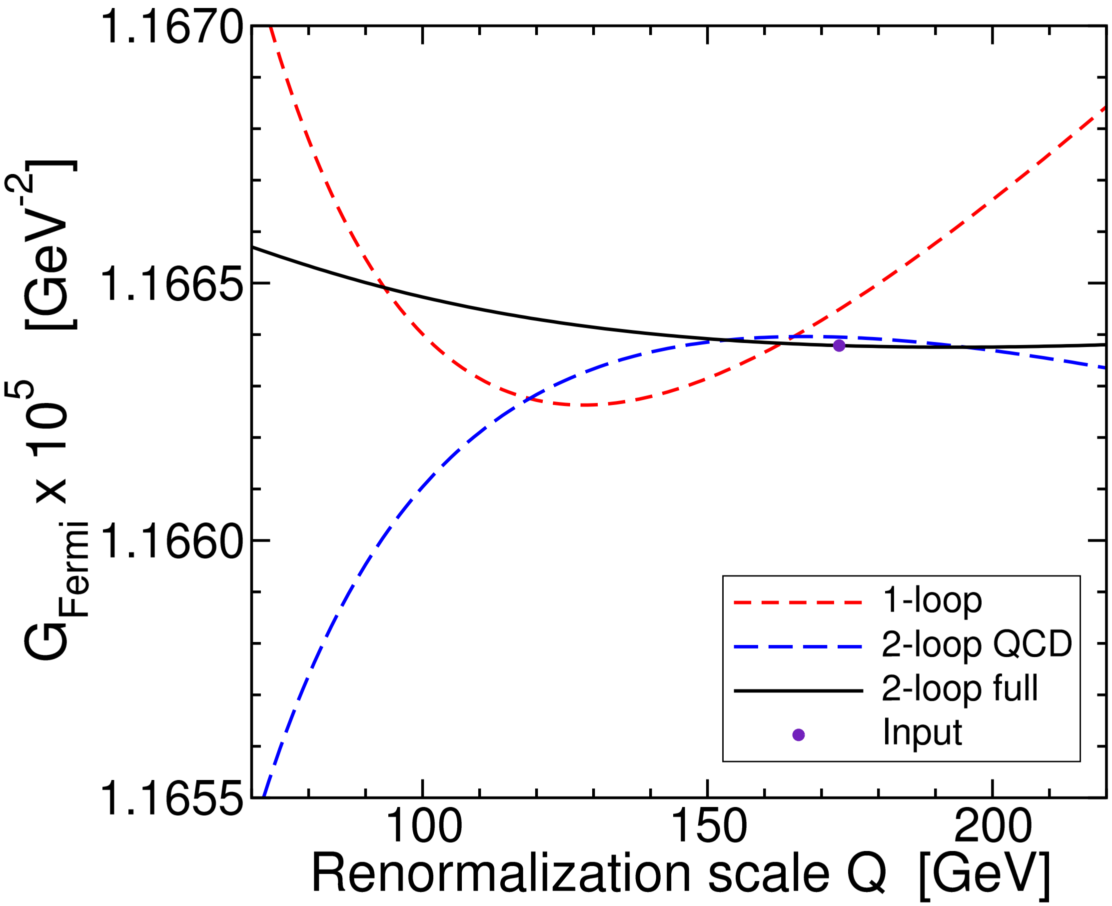
Determination of Fermi decay constant in terms of $\overline{\text{MS}}$ parameters:

$$G_F = \frac{1 + \Delta\overline{r}}{\sqrt{2} v_{\text{tree}}^2} = \frac{1 + \Delta\widetilde{r}}{\sqrt{2} v^2}.$$

In the tree-level VEV scheme, $\Delta\overline{r}$ has been at full 2-loop order given by Kniehl, Pikelner and Veretin 1401.1844, 1503.02138, 1601.08143 in their computer program `mr`.

We have obtained $\Delta\widetilde{r}$ at full 2-loop order in the tadpole-free VEV scheme, and checked that $1/\lambda$ and $1/\lambda^2$ terms are absent.

RG scale dependence of $G_{\rm Fermi}$, as a function of the Q where it is computed:



Scale dependence is less than 1 part in 10^{-4}, for 100 GeV $< Q <$ 220 GeV.

Compares well to an interpolating formula given by Degrassi, Gambino, Giardino 1411.7040 using another scheme; difference corresponds to $\Delta M_W \sim 5$ MeV.

Relate Sommerfeld fine structure constant $\alpha = 1/137.035999139\ldots$ to $\overline{\text{MS}}$ parameters in non-decoupled theory at $Q = M_Z$:

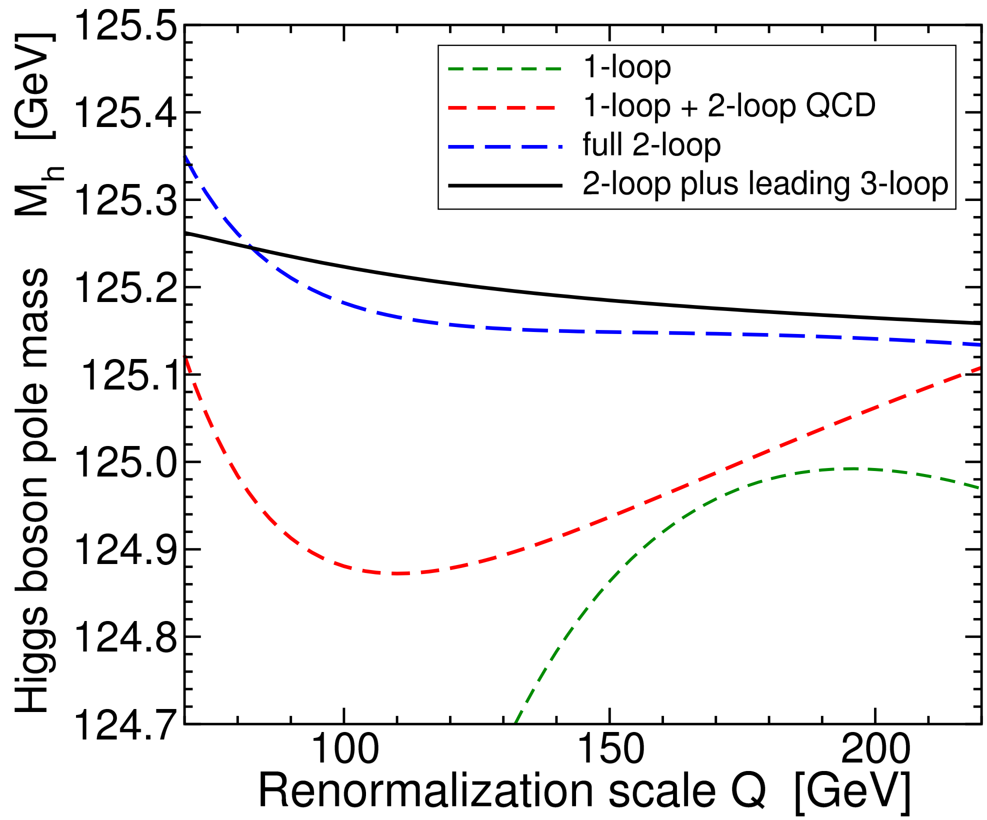
$$\alpha = \frac{g^2(M_Z) g'^2(M_Z)}{4\pi \left[g^2(M_Z) + g'^2(M_Z)\right]} \left[1 - \Delta\alpha^{(5)}_{\text{had}}(M_Z) - \Delta\alpha^{\text{LO}}_{\text{pert}} - \Delta\alpha^{\text{HO}}_{\text{pert}}\right],$$

where $\Delta\alpha^{(5)}_{\text{had}}(M_Z)$ contains contributions from b, c, s, d, u including non-perturbative effects, and the sum of 1-loop contributions from t, W, τ, μ, e are:

$$\begin{aligned}\Delta\alpha^{\text{LO}}_{\text{pert}} \quad = \quad & \frac{\alpha}{4\pi}\left[\frac{202}{27} + 14\ln(M_W/M_Z) - \frac{32}{9}\ln(M_t/M_Z) - \frac{8}{3}\ln(M_\tau/M_Z)\right. \\ & \left. -\frac{8}{3}\ln(M_\mu/M_Z) - \frac{8}{3}\ln(M_e/M_Z)\right]\end{aligned}$$

The higher-order contributions $\Delta\alpha^{\text{HO}}_{\text{pert}}$ were given in an interpolating formula by Degrassi Gambino Giardino 1411.7040.

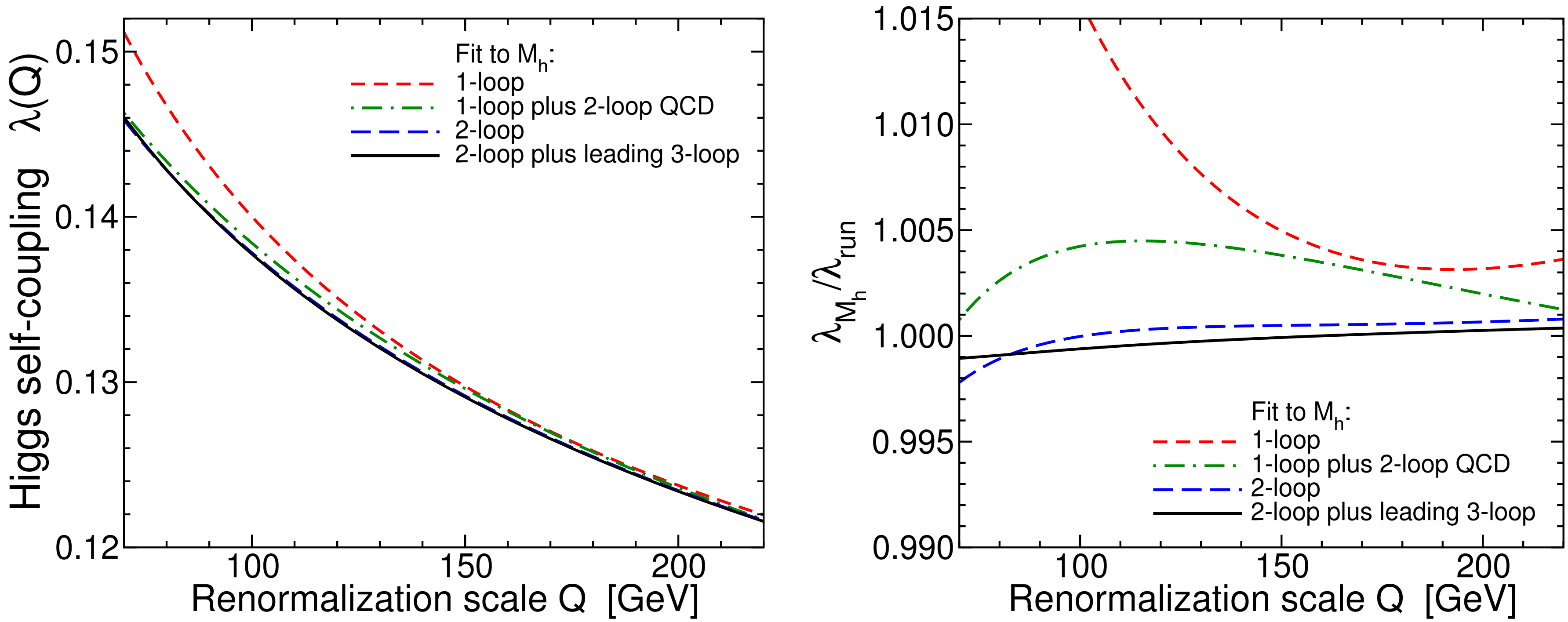
Higgs boson pole mass state-of-the-art: full 2-loop plus leading 3-loop approximation $g_3^2, y_t^2 \gg g^2, g'^2, \lambda$. (SPM and D.G. Robertson, 1407.4336)



`SMDR` subsumes and replaces our previous program `SMH`.

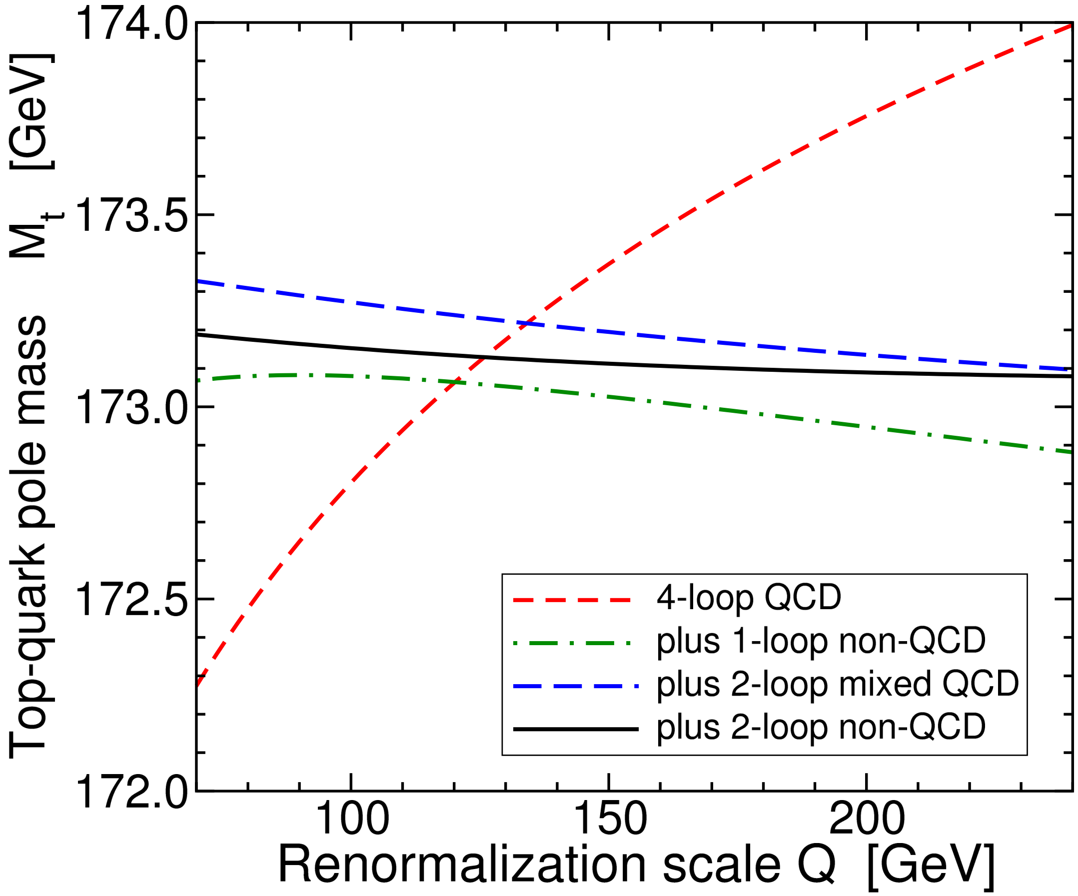
Scale dependence is tens of MeV, but we showed that $Q \sim 160$ GeV is preferred; higher-order effects are minimized with that choice.

Conversely, given the pole mass M_h as an input, can determine the Higgs self-coupling parameter λ as an output:



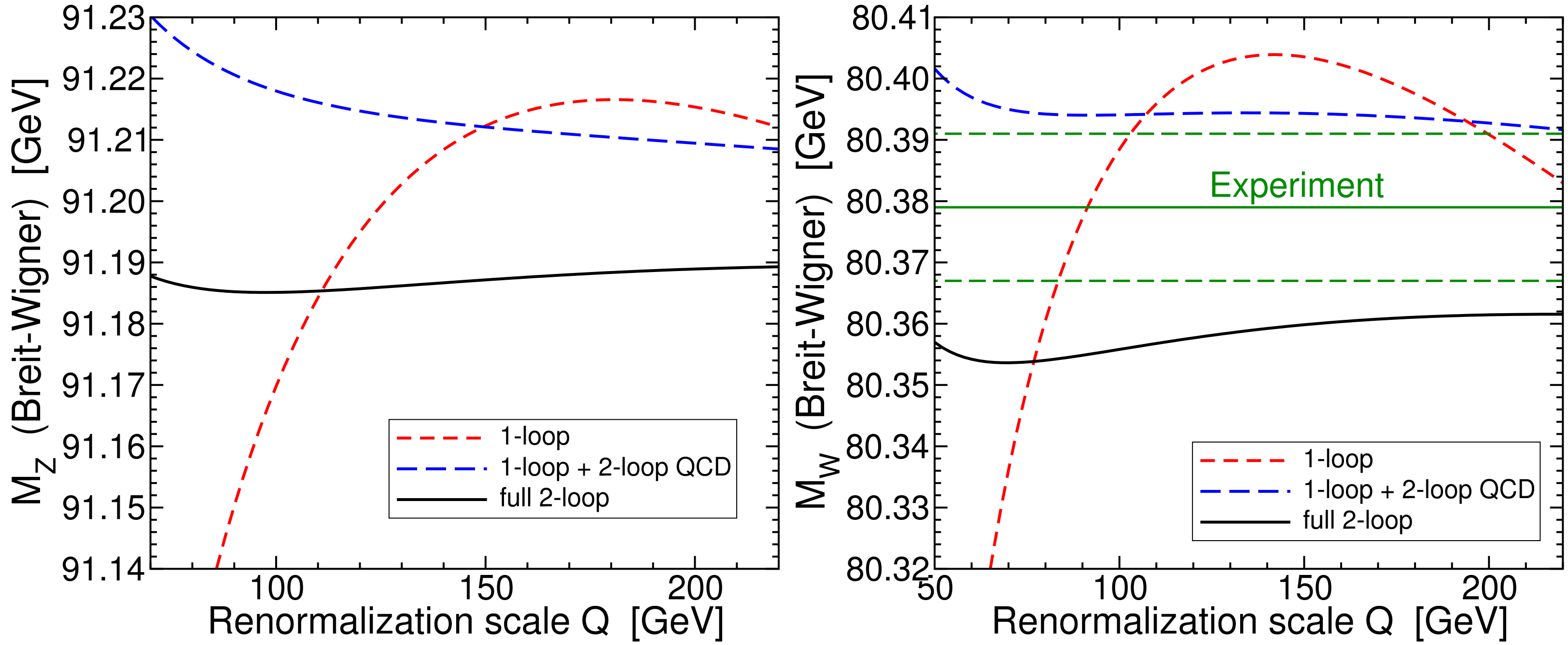
Again, the effects of higher-order contributions are minimized by choosing $Q \approx 160$ GeV, so that is the default used by SMDR when fitting λ.

Top-quark pole mass: state-of-the-art is 4-loop order in QCD plus full 2-loop order.



Note that neglecting electroweak and y_t effects is not justified. (Bad scale dependence is hidden if one also neglects electroweak and y_t contributions to beta functions.)

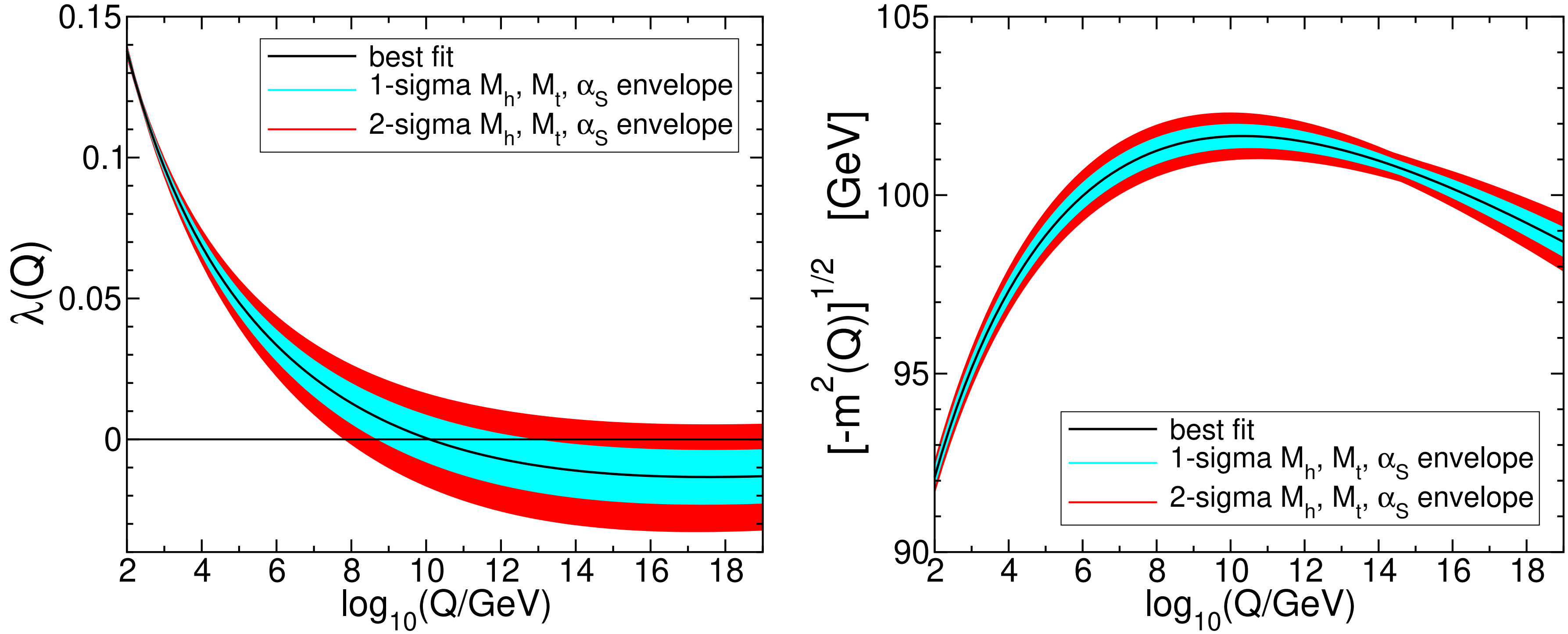
Z and W boson pole masses at full 2-loop order in tadpole-free pure $\overline{\text{MS}}$ scheme, from SPM 1505.04833 and 1503.03782 respectively:



Note: at this level of accuracy, need to convert pole masses (theoretical calculation, gauge invariant) to Breit-Wigner variable-width masses (used by experimentalists and PDG), using $M^2_{\rm BW} = M^2_{\rm pole} + \Gamma^2$.

$M_{Z,{\rm BW}} = M_{Z,{\rm pole}} +$ 34.1 MeV, $M_{W,{\rm BW}} = M_{W,{\rm pole}} +$ 27.1 MeV.

Taking into account the largest parametric uncertainties, can form 1-sigma and 2-sigma envelopes for the Higgs potential parameters λ and m^2:



Can match onto your favorite theory at an appropriate decoupling scale Q.

Left panel illustrates well-known quasi-instability of Standard Model.

Right panel illustrates the hierarchy problem: $m^2 \sim -(100\ \text{GeV})^2$ has trouble matching onto high-scale physics characterized by much larger masses.

SMDR uses many multi-loop calculations by other authors, including:

- Standard Model 3-loop beta functions: Tarasov 1982, Mihaila, Salomon, Steinhauser 1201.5868, Chetyrkin, Zoller 1205.2892 and 1303.2890, Bednyakov, Pikelner, Velizhanin 1210.6873, 1212.6829, 1303.4364, 1310.3806, and 1406.7171
- QCD 5-loop beta functions van Ritbergen, Vermaseren, Larin hep-ph/9701390, Czakon hep-ph/0411261, Baikov, Chetyrkin, Kuhn 1606.08659, Herzog, Ruijl, Ueda, Vermaseren, Vogt 1701.01404, Chetyrkin hep-ph/9703278, Vermaseren, Larin, van Ritbergen hep-ph/9703284, Baikov, Chetyrkin, Kühn 1402.6611
- QCD contributions to quark pole masses at 4-loop order: Melnikov, van Ritbergen hep-ph/9912391, Marquard, A. Smirnov, V. Smirnov, M. Steinhauser 1502.01030
- Matching relations for decoupling in QCD at 4-loop order: Chetyrkin, Kniehl, Steinhauser 9708255, Grozin, Hoeschele, Hoff, Steinhauser 1107.5970, Schroder and Steinhauser 0512058, Bednyakov 1410.7603, Liu and Steinhauser 1502.04719
- Matching relations for α and y_f at full 2-loop order: SPM 1812.04100

Some other public code with overlapping aims:

- **RunDec, CRunDec** by Chetyrkin, Kuhn, Herren, Schmidt, Steinhauser hep-ph/0004189, 1201.6149, 1703.03751

  QCD 5-loop running, 4-loop decoupling and pole masses

- **mr** by Kniehl, Pikelner, Veretin 1601.08143

  Standard Model, uses tree-level VEV scheme

Outlook

SMDR software library provides precise map between $\overline{\text{MS}}$ parameters in tadpole-free scheme and on-shell observables.

- written in C, interfaces to C++, FORTRAN, Python
- user library functions easy to incorporate in your own programs
- “calculator mode” illustrates all capabilities, serves as example code
- all figures shown were produced by simple programs provided with the SMDR distribution as further examples
- modular structure, so it will be straightforward to update in future as more loops and/or BSM physics effects are added to calculations
- Lots of work to do: full 3-loop effects will be needed. For example, $\Delta M_W \lesssim 1$ MeV at an e^+e^- collider.

Since the Standard Model is now complete, perhaps it is time for the Particle Data Group to provide results for fundamental Lagrangian parameters in the $\overline{\text{MS}}$ scheme.

BACKUP

Sample run in calculator mode:

```
[smdr]# ./calc_all ReferenceModel.dat
INPUT PARAMETERS read from "ReferenceModel.dat":
Q = 173.100000;
Higgs vev = 246.600746;
Higgs mass^2 parameter = -8636.365174;
Higgs self-coupling lambda = 0.126203;
gauge  couplings: g3 = 1.163624;      g = 0.647659;      gp = 0.358539;
Yukawa couplings: yt = 0.934799;     yb = 0.015480;    ytau = 0.00999446;
                  yc = 0.0034009;    ys = 0.00029720;   ymu = 0.000588381;
                  yu = 0.0000068473; yd = 0.000014704;   ye = 0.00000279299;
Delta_hadronic^(5) alpha(MZ) = 0.027640

OUTPUT QUANTITIES:

Mt = 173.100000; Gammat = 1.372897;   (* complex pole *)

Mh = 125.180000; Gammah = 0.003409;   (* complex pole *)

MZ = 91.153552;  GammaZ = 2.491674;   (* complex pole *)
MZ = 91.187600;  GammaZ = 2.490744;   (* Breit-Wigner, compare to PDG *)

MW = 80.333307;  GammaW = 2.084131;   (* complex pole *)
MW = 80.360337;  GammaW = 2.083430;   (* Breit-Wigner, compare to PDG *)
```

```
MSbar quantities at Q = MZ, full Standard Model, nothing decoupled:
  alphaS = 0.117053;   alpha = 1/128.114214;   sin^2_thetaW = 0.231417;

MSbar quantities at Q = MZ, only top quark decoupled (PDG convention):
  alphaS = 0.118100;   alpha = 1/127.945062;   sin^2_thetaW = 0.231228;

MSbar bottom and charm masses:
  mb(mb) = 4.180000;   (MSbar mass in 5-quark + 3-lepton QCD+QED theory)
  mc(mc) = 1.275000;   (MSbar mass in 4-quark + 2-lepton QCD+QED theory)

Light quark MSbar masses (at Q = 2 GeV, in 4-quark + 3-lepton QCD+QED theory):
  ms = 0.095000;     mu = 0.0022000;     md = 0.0047000;

Lepton pole masses:
  Mtau = 1.776860;      Mmuon = 0.105658375;      Melectron = 0.0005109989;

Sommerfeld fine structure and Fermi constants:
  alpha = 1/137.03599914;     GFermi = 1.16637870 10^-5;

Total calculation time: 2.68 seconds
```