ALPs
and
X-ray astronomy

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Introduction

ALPs: strong CP problem, string axiverse, etc.

Detection typically relies on ALP couplings:

\[ \mathcal{L} \supset -\frac{g}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} + g_{aN} (\partial_{\mu} a) \bar{N} \gamma^{\mu} \gamma_{5} N. \]

Strong magnetic fields are especially useful

Neutron stars have the strongest magnetic fields

Also lots of data
Helioscope

\[
\mathcal{L} \supset -\frac{g}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}
\]

CAST (2017)
Main Idea

\[ B_0 = 20 \times 10^{14} \text{ G} \]

\[ B = B_0 \left( \frac{r_0}{r} \right)^3 \]

Hard X-rays

Heyl/Lai (2006)

Perna et. al. (2012)

Soft X-rays

Radio Waves

Pshirkov/Popov (2007)

Hook/Kahn/Safdi (2018)
Probability of Conversion
ALP Wave Equation

\[ E, a \propto e^{i\omega t} \]

photon wavelength much smaller than magnetar radius

linearized wave equation

\[
\begin{pmatrix} a \\ E_\parallel \end{pmatrix} = \begin{pmatrix} \omega + \Delta_a & \Delta_M \\ \Delta_M & \omega + \Delta_\parallel \end{pmatrix} \begin{pmatrix} a \\ E_\parallel \end{pmatrix}
\]

axion mass term \( \Delta_a = -\frac{m_a^2}{2\omega} \)

mixing term \( \Delta_M = \frac{1}{2}gB\sin\theta \)

Euler-Heisenberg term \( \Delta_\parallel = \frac{1}{2}q\omega\sin^2\theta \)

\[ q = \frac{7\alpha}{45\pi}b^2\hat{q} \]

\[ \hat{q} = \frac{1 + 1.2b}{1 + 1.33b + 0.56b^2} \]

Raffelt/Stodolsky (1988)

magnetic field \( b = B/B_Q \)

\[ B_Q = m_e^2c^3/(e\hbar) = 4.414 \times 10^{13} \text{ G} \]

Schwinger (1951), Adler (1971), Heyl/Hernquist (2010)
Mixing Angle

\[ \frac{i}{\gamma} \frac{d}{dx} \begin{pmatrix} a \\ E_{||} \end{pmatrix} = \begin{pmatrix} \omega r_0 + \Delta_a r_0 & \Delta_M r_0 \\ \Delta_M r_0 & \omega r_0 + \Delta_{||} r_0 \end{pmatrix} \begin{pmatrix} a \\ E_{||} \end{pmatrix} = \begin{pmatrix} A(x) \\ D(x) \end{pmatrix} \begin{pmatrix} a \\ E_{||} \end{pmatrix} \]

Mixing angle: \[ \frac{1}{2} \tan 2\theta = \frac{D(x)}{B(x) - A(x)} \]

\[ \omega = 100 \text{ keV}, \quad m_a = 10^{-8} \text{ keV}, \]
\[ r_0 = 10 \text{ km}, \quad B_0 = 20 \times 10^{14} \text{ G} \]
\[ g = 10^{-15} \text{ keV}^{-1} \]

\[ D(x) = \Delta_M r_0 = \frac{\Delta_M r_0}{x^3} \approx \frac{3.0 \times 10^5}{x^3} \]

\[ B(x) = \frac{\Delta_{||} q(x)}{x^6} r_0 \approx \frac{8.6 \times 10^{13}}{x^6} \]

\[ A(x) \approx \Delta_a r_0 \approx -2.5 \times 10^{-5} \]
nitude. Although in a detailed calculation of the axion-photon conversion rates the inhomogeneity of the field must be properly included as in our perturbative solution Eq. (33), it is clear that the conversion is now dramatically suppressed due to the magnetically induced vacuum index of refraction. Given this suppression it is difficult to imagine the occurrence of observable effects.\(^{36}\)
Solution: n-body

general n-body oscillation problem

\[ i \frac{d a_i(x)}{dx} = \sum_{j=1}^{n} A_{ij}(x) a_j(x) \]

\[ A^*_j(x) = A_{ij}(x) \quad \frac{d}{dx} \sum_{i=1}^{n} |a_i(x)|^2 = 0 \]

\[ a_i(x) = \left\{ \prod_{j=i}^{n-1} \sin[\chi_j(x)] \right\} \cos[\chi_{i-1}(x)] e^{-i\phi_i(x)} \]

\[ \frac{d \chi_{i-1}(x)}{dx} = \sum_{j=1}^{n} A_{ij}(x) S_{ij}(x) + \cot[\chi_{i-1}(x)] \sum_{j=i+1}^{n} \left\{ \sum_{\ell=1}^{n} A_{j\ell}(x) S_{j\ell}(x) \right\} \cot[\chi_{j-1}(x)] \left\{ \prod_{k=i}^{j-2} \csc^2[\chi_k(x)] \right\} \]

\[ \frac{d \phi_i(x)}{dx} = \sum_{j=1}^{n} A_{ij}(x) C_{ij}(x) \]

coefficients

\[ S_{ij}(x) = \left\{ \prod_{k=j}^{n-1} \sin[\chi_k(x)] \right\} \cos[\chi_{j-1}(x)] \frac{\cos[\phi_j(x) - \phi_i(x)]}{\sin[\phi_j(x) - \phi_i(x)]} \]

\[ C_{ij}(x) = \left\{ \prod_{k=i}^{n-1} \sin[\chi_k(x)] \right\} \sin[\chi_{i-1}(x)] \frac{\cos[\phi_j(x) - \phi_i(x)]}{\cos[\phi_j(x) - \phi_i(x)]}. \]
Solution: 2-body

\[ i \frac{d}{dx} \begin{pmatrix} a \\ E_\parallel \end{pmatrix} = \begin{pmatrix} \omega r_0 + \Delta_a r_0 & \Delta_M r_0 \\ \Delta_M r_0 & \omega r_0 + \Delta_\parallel r_0 \end{pmatrix} \begin{pmatrix} a \\ E_\parallel \end{pmatrix} = \begin{pmatrix} A(x) & D(x) \\ D(x) & B(x) \end{pmatrix} \begin{pmatrix} a \\ E_\parallel \end{pmatrix} \]

\[ a(x) = \cos[\chi(x)] e^{-i\phi_a(x)} \]

\[ E_\parallel(x) = i \sin[\chi(x)] e^{-i\phi_E(x)} \]

\[ \frac{d\chi(x)}{dx} + i \cot[\chi(x)] \left[ \frac{d\phi_a(x)}{dx} - A(x) \right] = -D(x)e^{i[\phi_a(x) - \phi_E(x)]}, \]

\[ \frac{d\chi(x)}{dx} - i \tan[\chi(x)] \left[ \frac{d\phi_E(x)}{dx} - B(x) \right] = -D(x)e^{-i[\phi_a(x) - \phi_E(x)]} \]

\[ \frac{d\chi(x)}{dx} = -D(x) \cos[\Delta \phi(x)], \]

\[ \frac{d\Delta \phi(x)}{dx} = A(x) - B(x) + 2D(x) \cot[2\chi(x)] \sin[\Delta \phi(x)]. \]
Conversion Probability

\[ P_{a \rightarrow \gamma}(x) = \sin^2[\chi(x)] \]

\[ \omega = 100 \text{ keV}, \quad m_a = 10^{-8} \text{ keV}, \quad g = 10^{-15} \text{ keV}^{-1}, \quad r_0 = 10 \text{ km}, \quad B_0 = 20 \times 10^{14} \text{ G} \]
Probability Dependences

\[ \omega = 100 \text{ keV}, \ m_\alpha = 10^{-8} \text{ keV}, \ g = 10^{-15} \text{ keV}^{-1}, \ r_0 = 10 \text{ km}, \ B_0 = 20 \times 10^{14} \text{ G} \]
Parameter Scan
Semi-analytic Treatment

\[ P_{a \rightarrow \gamma}(x) = \left| \int_1^x dx' \Delta_M(x') r_0 \exp \left\{ i \int_1^{x'} dx'' \left[ \Delta_a - \Delta_\| (x'') \right] r_0 \right\} \right|^2 \]

\[ = (\Delta_{M0} r_0)^2 \left| \int_1^x dx' \frac{1}{x'^3} \exp \left[ i \Delta_a r_0 \left( x' - \frac{x_{a \rightarrow \gamma}^6}{5x'5} \right) \right] \right|^2 . \]

\[ = \left( \frac{\Delta_{M0} r_0^3}{r_{a \rightarrow \gamma}^2} \right)^2 \cos(2\chi_0) \times \left\{ \begin{array}{ll}
\frac{\pi}{3|\Delta_a r_{a \rightarrow \gamma}|} e^{\frac{6|\Delta_a r_{a \rightarrow \gamma}|}{5}} & |\Delta_a r_{a \rightarrow \gamma}| \geq 0.45 \\
\frac{\Gamma\left(\frac{2}{5}\right)^2}{\frac{6}{5} |\Delta_a r_{a \rightarrow \gamma}|^{\frac{4}{5}}} & |\Delta_a r_{a \rightarrow \gamma}| \lesssim 0.45
\end{array} \right. \]
Production in Core
Photons Luminosity

\[
L_{a\rightarrow\gamma} = \frac{N_a}{2\pi} \int_0^{2\pi} d\theta \int_{\omega_i}^{\omega_f} d\omega \omega \frac{dN_a}{d\omega} P_{a\rightarrow\gamma}(\omega, \theta)
\]

\[
\frac{1}{2\pi} \int_0^{2\pi} d\theta P_{a\rightarrow\gamma}(\omega, \theta) \rightarrow R_\theta P_{a\rightarrow\gamma}(\omega, \pi/2)
\]

\[R_\theta = 0.6\]

total axion number: just subdominant to neutrino cooling

\[
L_a = N_a \int_0^{\infty} d\omega \omega \frac{dN_a}{d\omega} \leq L_\nu = 4\pi \int_0^{r_0} dr r^2 \dot{q}_\nu
\]

Sedrakian (2017), Balantekin et. al. (2017),...
ALP Production

\[ N_a \leq \frac{4\pi r_0^3 \dot{q}_\nu}{3 \int_0^\infty d\omega \omega \frac{dN_a}{d\omega}} \]

\[ L_{a\rightarrow\gamma} = \frac{4\pi r_0^3 \dot{q}_\nu R_\theta}{3 \int_0^\infty d\omega \omega \frac{dN_a}{d\omega}} \int_{\omega_i}^{\omega_f} d\omega \omega \frac{dN_a}{d\omega} P_{a\rightarrow\gamma}(\omega, \pi/2) \]

\[ \frac{dN_a}{d\omega} = \frac{x^2(x^2 + 4\pi^2)e^{-x}}{8(\pi^2 \zeta_3 + 3\zeta_5)(1 - e^{-x})} \]

\[ \int_0^\infty d\omega \frac{dN_a}{d\omega} = 1 \]
\[ x = \omega/k_B T \]

modified Urca production

\[ \dot{q}_\nu = \left( 7 \times 10^{20} \text{ erg s}^{-1} \text{ cm}^{-3} \right) \left( \frac{\rho}{\rho_0} \right)^{2/3} R_M \left( \frac{T}{10^9 \text{ K}} \right)^8 \]

\[ \rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3} \] is the nuclear saturation density
Neutrino Cooling

direct Urca

\[ \dot{q}_\nu^D \sim 10^{27} T_9^6 \mathcal{R}_D \text{ erg s}^{-1} \text{ cm}^{-3} (\rho \gtrsim 10^{15} \text{ g cm}^{-3}) . \]

modified Urca

\[ \dot{q}_\nu^M \sim 7 \times 10^{20} T_9^8 \left( \frac{\rho}{\rho_{\text{nuc}}} \right)^{2/3} \mathcal{R}_M \text{ erg s}^{-1} \text{ cm}^{-3} . \]

Cooper pair cooling

\[ \dot{q}_\nu^{CP} \sim 10^{21} \left( \frac{\rho}{\rho_{\text{nuc}}} \right)^{1/3} T_9^6 f \left( \frac{T_{\text{core}}}{T_{\text{crit}}} \right) \text{ erg s}^{-1} \text{ cm}^{-3} . \]

Beloborodov (2017)

Figure 2. Neutrino cooling rate as a function of temperature in the core at density \( \rho_{\text{nuc}} = 2.8 \times 10^{14} \text{ g cm}^{-3} \). The black curve shows Murca cooling assuming no superfluidity (\( T_{\text{crit}} < 10^8 \text{ K} \)). The colored curves show the cooling of matter with non-superfluid protons and superfluid neutrons, for two cases: \( T_{\text{crit}} = 10^9 \text{ K} \) (blue curves) and \( T_{\text{crit}} = 3 \times 10^9 \text{ K} \) (red curves). The dashed curve shows the Murca contribution, and the dashed–dotted curve shows the Cooper pair contribution; the net cooling rate is shown by the solid curve. A triplet-state neutron pairing is assumed (model B in Yakovlev et al. 2001).
Core Temperature

\[ q_\nu = (7 \times 10^{20} \text{ erg} \cdot \text{s}^{-1} \cdot \text{cm}^{-3}) \left( \frac{\rho}{\rho_0} \right)^{2/3} R_M \left( \frac{T}{10^9 \text{ K}} \right)^8 \]

\[ q_a = (1.3 \times 10^{25} \text{ erg} \cdot \text{s}^{-1} \cdot \text{cm}^{-3}) \left( \frac{g_{aN}}{10^{-10} \text{ GeV}^{-1}} \right) \left( \frac{\rho}{\rho_0} \right)^{1/3} \left( \frac{T}{10^9 \text{ K}} \right)^6 \]

\[ \left( \frac{g_{aN}^*}{10^{-10} \text{ GeV}^{-1}} \right) = 7.3 \times 10^{-3} \left( \frac{\rho}{\rho_0} \right)^{1/6} \sqrt{R_M} \left( \frac{T}{10^9 \text{ K}} \right) \]

observed at infinity. Each symbol shows a calculated model of steady heat transfer from the core to the stellar surface. The star is assumed to have a dipole magnetic field near the surface, in the heat-blanketing envelope. Two cases are considered: the iron envelope and the maximal light-element envelope, which is called “fully accreted” in Potekhin et al. (2003). The luminosity is shown for two values of the polar magnetic field: \( B_p = 3 \times 10^{13} \text{ G} \) and a more typical one for magnetars \( B_p = 10^{15} \text{ G} \). As \( T_{\text{core}} \) approaches \( 10^9 \text{ K} \), \( L_s^\infty \) approaches the ceiling imposed by neutrino cooling (Potekhin et al. 2007); heating the core to higher temperatures would not significantly increase the surface luminosity.
Total Photon Luminosity

\[ L_{a \rightarrow \gamma} = \frac{4\pi r_0^3 q_{\nu} R_{\theta}}{3} \int_0^{\omega_f} d\omega \omega \frac{dN_a}{d\omega} \int_{\omega_i}^{\omega_f} d\omega \frac{dN_a}{d\omega} P_{a \rightarrow \gamma}(\omega, \pi/2) \]

**Figure 4.** Photon luminosity from ALP-photon oscillations in the broad band from 1 keV to 200 keV in the \((m_a, g)\) plane. The computations are done for SGR 1806-20 assuming \(r_0 = 10\) km and \(B_0 = 20 \times 10^{14}\) G. The magnetar core temperature is assumed to be \(T = 10^9\) K. The two panels show the same conversion probability from different points of view.
Comparison with Observations
Non-thermal Emission

Besides spectacular outbursts, magnetars produce persistent or decaying X-ray emission with luminosity $L \sim 10^{34} - 10^{36}$ erg s$^{-1}$. Two peaks are observed in their X-ray spectra, with comparable luminosities. The first peak is near 1 keV; it is associated with thermal emission from the neutron star surface. The second peak is above 100 keV. Its low-energy slope (between 10 and 100 keV) was observed in 7 magnetars$^1$ (Kuiper et al. 2008; Enoto et al. 2010), with a typical photon index $\Gamma \sim 1 - 1.5$. 

Beloborodov (2012)
In this letter we present the broad band spectrum (1-200 keV) of the persistent emission of SGR 1806-20 obtained in autumn 2003. The source flux was \(1.3 \times 10^{-10}\) erg s\(^{-1}\) cm\(^{-2}\), which corresponds to a source luminosity \(3.6 \times 10^{36}\) erg s\(^{-1}\) in this energy band (assuming a source...
Results

\[ L_{\gamma}^{\text{obs}} = 1.2 \times 10^{36} \text{ erg} \cdot \text{s}^{-1} \]
Polarization
X-mode Domination

Electron cyclotron frequency: \( \omega_{ce} = m_e (B/B_c) \sim 50 \text{ MeV} \)

\[ \omega \ll \omega_{ce} \]

Description in terms of X-mode and O-mode

\[ \frac{I_{\parallel}(\chi_0, x = 1)}{I_{\perp}(x = 1)} \sim \left(\frac{\omega}{\omega_{ce}}\right)^2 \sim 10^{-6} \]

Most of the thermal radiation is X-mode

Lai/Ho (2003), Heyl/Lai (2007)
QED Effect

\[ i \frac{d}{dr} \begin{pmatrix} E_\parallel \\ E_\perp \end{pmatrix} = \frac{\omega}{2} \begin{pmatrix} 2 + \sigma_{11} & \sigma_{12} \\ \sigma_{21} & 2 + \sigma_{22} \end{pmatrix} \begin{pmatrix} E_\parallel \\ E_\perp \end{pmatrix} \]

\[ \sigma_{12} = -\sigma_{21} \sim 0 \quad \sigma_{11} \sim \left( q_\parallel - \frac{\omega_{pl}^2}{\omega^2} \right) \sin^2 \theta \quad \sigma_{22} \sim -q_\perp \sin^2 \theta \]

QED contribution

\[ q_\parallel = \frac{7 \alpha}{45 \pi} b^2 \hat{q}_\parallel \]

\[ q_\perp = \frac{4 \alpha}{45 \pi} b^2 \hat{q}_\perp \]

Plasma contribution

\[ \omega_{pl}^2 = 4\pi e^2 n_e/m_e \]

\[ n_e = Y_e \rho/m_p \]
Vacuum Resonance

\[ i \frac{d}{dr} \left( \begin{array}{c} E_\parallel \\ E_\perp \end{array} \right) = \frac{\omega}{2} \left( \begin{array}{cc} 2 + \sigma_{11} & \sigma_{12} \\ \sigma_{21} & 2 + \sigma_{22} \end{array} \right) \left( \begin{array}{c} E_\parallel \\ E_\perp \end{array} \right) \]

\[ \sigma_{12} = -\sigma_{21} \sim 0 \quad \sigma_{11} \sim (q_\parallel - \frac{\omega_{pl}^2}{\omega^2}) \sin^2 \theta \quad \sigma_{22} \sim -q_\perp \sin^2 \theta \]

\[ \sigma_{11} = \sigma_{22} \]

\[ \frac{\omega_{pe}^2}{\omega^2} = q_\parallel + q_\perp \]

\[ \rho \sim 0.964 Y_e^{-1} \left( \frac{\omega}{1 \text{ keV}} \frac{B}{10^{14} \text{ G}} \right)^2 \]
X-modes in Hard X-rays

Beloborodov (2012)
Baring (2017)

Net 75% polarization in X-mode
X-modes in Hard X-rays

- Relativistic plasma ejected from magnetar; Large Lorentz factors.
  \[ \Phi \sim 10^9 \text{ V} \]
  \[ \gamma \sim e\Phi/m_e c^2 \sim 10^3 \]

- Resonantly scatters with thermal keV photons
  Photon energy in e rest frame = Landau energy
  \[ \tilde{E} = \hbar \omega_B = b m_e c^2 \]

- Photons get tremendous energy boost. Parallel modes convert into e+e- pairs via pair production; perp modes undergo photon splitting for energies above 1 MeV. The two produced photons have parallel polarization and split into e+e-. Both modes thus reprocessed into e+e-; trapped.

- When \( B < 0.01B_{\text{crit}} \), photons escape. Pink region. Mostly perpendicular modes.

- Photon of energy \( E \) is emitted where \( b \approx 0.1(E/m_e c^2)^{1/2} \). For our case, \( x \sim 8 \)
Polarization Radius

\[ r_{PL} = \left( \frac{\alpha \nu}{45 c} \right)^{1/5} \left( \frac{B_0}{B_c} \frac{r_0^3}{\sin \beta} \right)^{2/5} \approx 923.4 \left( \frac{\omega}{1 \text{keV}} \right)^{1/5} \left( \frac{B_0}{10^{14} \text{G}} \right)^{2/5} \left( \frac{r_0}{10 \text{km}} \right)^{6/5} \text{ km}, \]

\[ r_{PL} \sim 300 r_0 \]
O-Modes from ALPs

\[ i \frac{d}{dx} \begin{pmatrix} a \\ E_{\parallel} \\ E_{\perp} \end{pmatrix} = \begin{pmatrix} \omega r_0 + \Delta_a r_0 & \Delta_M r_0 & 0 \\ \Delta_M r_0 & \omega r_0 + \Delta_\parallel r_0 & 0 \\ 0 & 0 & \omega r_0 + \Delta_\perp r_0 \end{pmatrix} \begin{pmatrix} a \\ E_{\parallel} \\ E_{\perp} \end{pmatrix} \]

\[ \Delta_a = -\frac{m_a^2}{2\omega}, \quad \Delta_\parallel = \frac{1}{2} q_\parallel \omega \sin^2 \theta, \quad \Delta_\perp = \frac{1}{2} q_\perp \omega \sin^2 \theta, \quad \Delta_M = \frac{1}{2} gB \sin \theta. \]

\[ a(x) = A \cos[\chi(x)] e^{-i\phi_a(x)}, \quad E_{\parallel}(x) = iA \sin[\chi(x)] e^{-i\phi_\parallel(x)} \]

Axion \quad O-mode

\[ E_{\perp}(x) = A_\perp e^{-i\phi_\perp(x)} \]

X-mode
**ALP-O-mode Evolution**

\[ \frac{d\chi(x)}{dx} = -\Delta_M r_0 \cos[\Delta \phi(x)], \]
\[ \frac{d\Delta \phi(x)}{dx} = (\Delta_a - \Delta_{||}) r_0 + 2 \Delta_M r_0 \cot[2\chi(x)] \sin[\Delta \phi(x)] \]

**Sum of phases:**

\[ \frac{d\Sigma \phi(x)}{dx} = (2\omega + \Delta_a + \Delta_{||}) r_0 - 2 \Delta_M r_0 \csc[2\chi(x)] \sin[\Delta \phi(x)]. \]

**X-mode evolution**

\[ \frac{d\phi_{\perp}(x)}{dx} = (\omega + \Delta_{\perp}) r_0. \]
Initial Conditions

Intensities:

\[ I_a(x) = A^2 \cos^2[\chi(x)] \quad I_\parallel(x) = A^2 \sin^2[\chi(x)] \quad I_\perp(x) = A^2_\perp \]

axion \quad O-mode \quad X-mode

Free initial conditions:

\[ \chi(1) = \chi_0 \quad I_\parallel(1) \quad I_\perp(1) \]

Uncorrelated ALP+O-mode states

integrate over \[ \Delta \phi(1) = \Delta \phi_0 \]
Averaged Stokes Parameters

Phase-averaged intensities at position $x$:

\[
\bar{I}_a(\chi_0, x) = \int_0^{2\pi} \frac{d\Delta \phi_0}{2\pi} I_a(\chi_0, \Delta \phi_0, x) \quad \bar{I}_\parallel(\chi_0, x) = \int_0^{2\pi} \frac{d\Delta \phi_0}{2\pi} I_\parallel(\chi_0, \Delta \phi_0, x)
\]

Stokes parameters

\[
I(\chi_0, x) = \bar{I}_\perp(x) + \bar{I}_\parallel(\chi_0, x) \quad Q(\chi_0, x) = \bar{I}_\perp(x) - \bar{I}_\parallel(\chi_0, x)
\]

Closed analytic form?
Q Behavior

\[ Q(\chi_0, x) = A_\perp^2 - \frac{A^2}{2} \{ 1 + [2P_{a\rightarrow\gamma}(x) - 1] \cos(2\chi_0) \} \]

- **X-mode**
- **O-mode**
- **Conversion probability**
- **Initial condition**

\[ \chi_0 = 0 \quad \text{(if there are no O-modes at the surface)} \]

\[ \frac{I_\parallel}{I_\perp} \sim 0 \quad \text{assume only X-modes produced astrophysically} \]

\[ \frac{I_a}{I_\perp} \sim 0 - \mathcal{O}(10^4) \quad \text{no ALPs to limit from luminosity} \]
Fig. 4: (Normalized) Stokes parameter $Q/\bar{I}_\perp$ at infinity for the benchmark point $m_a = 10^{-8}$ keV, $g = 5 \times 10^{-17} \text{keV}^{-1}$, $r_0 = 10 \text{km}$, $B_0 = 20 \times 10^{14} \text{G}$ and $\theta = \pi/2$ in function of $\bar{I}_a/\bar{I}_\perp$ at the surface assuming $\bar{I}_\parallel/\bar{I}_\perp \sim 0$ at the surface. The left panel corresponds to ALP energy $\omega = 1$ keV ($P_{a\rightarrow\gamma} \approx 1.2 \times 10^{-3}$) while the right panel corresponds to $\omega = 100$ keV ($P_{a\rightarrow\gamma} \approx 9.6 \times 10^{-5}$).
**Q Behavior**

![Graphs showing Q/I_\perp for ω = 1 keV and ω = 100 keV](image)

**Fig. 3:** (Normalized) Stokes parameter \( Q/I_\perp \) at infinity for the benchmark point \( m_a = 10^{-8} \) keV, \( g = 5 \times 10^{-17} \) keV\(^{-1} \), \( r_0 = 10 \) km, \( B_0 = 20 \times 10^{14} \) G and \( \theta = \pi/2 \) in the plane \((I_\parallel/I_\perp, I_a/I_\perp)\) at the surface. The left panel corresponds to ALP energy \( \omega = 1 \) keV \( (P_{a\rightarrow\gamma} \approx 1.2 \times 10^{-3}) \) while the right panel corresponds to \( \omega = 100 \) keV \( (P_{a\rightarrow\gamma} \approx 9.6 \times 10^{-5}) \).
Radius of Conversion

$$r_{a \rightarrow \gamma} = \left( \frac{7\alpha}{45\pi} \right)^{1/6} \left( \frac{\omega B_0}{m_a B_c} |\sin \theta| \right)^{1/3} r_0$$

$$\sim 1626.9 \left( \frac{\omega}{1\,\text{keV}} \right)^{1/3} \left( \frac{B_0}{10^{14}\,\text{G}} \right)^{1/3} \left( \frac{10^{-8}\,\text{keV}}{m_a} \right)^{1/3} \left( \frac{r_0}{10\,\text{km}} \right) \text{ km.}$$

$$\frac{r_{PL}}{r_{a \rightarrow \gamma}} \sim 0.57 \left( \frac{1\,\text{keV}}{\omega} \right)^{2/15} \left( \frac{B_0}{10^{14}\,\text{G}} \right)^{1/15} \left( \frac{m_a}{10^{-8}\,\text{keV}} \right)^{1/3} \left( \frac{r_0}{10\,\text{km}} \right)^{1/5}$$
Future

Exciting time for ALP searches!

generally, I expect the interface between X-ray astronomy and axion physics to be a fruitful area in the future
Future

polarization signals in hard X-rays

generalized initial states

spectral analysis

populations of magnetars?

other signals?


generally, I expect the interface between X-ray astronomy and axion physics to be a fruitful area in the future
Plasma Effects

\[ i \frac{d}{dz} \Phi = \begin{pmatrix} \omega + \Delta_a & \Delta_M & 0 \\ \Delta_M & \omega + \Delta_{||} + \Delta_p & \sigma_{12} \omega / 2 \\ 0 & \sigma_{21} \omega / 2 & \omega + \sigma_{22} \omega / 2 \end{pmatrix} \Phi, \quad \Phi = \begin{pmatrix} a \\ \frac{E_{||}}{E_{\perp}} \end{pmatrix} \]

\[ \Delta_p \approx -\omega^2_{pl} / 2 \omega \approx -2 \times 10^{-14} \text{ eV} \]

\[ \omega^2_{pl} = 4 \pi \alpha N_e / m_e \]

\[ N_e = (7 \times 10^{-2} \text{ cm}^{-3}) [B_z / (1 \text{ G})] [(1 \text{ sec}) / P] \]

\[ \Delta_p r_0 \approx -10 \]

\[ \Delta_{||0} r_0 \approx 8.6 \times 10^{13} \]

plasma important at \( \Delta_p \approx \Delta_{||} \)

\[ \gamma_{\text{res}} = \frac{4 \Delta_M^2 H}{\Delta_{||}} = 1.915 \times 10^{-8} \frac{g_2^2 H_1}{\omega_1 \hat{q}} \]
The Hard X-ray Modulation Telescope (HXMT), named "Insight", is China’s first X-ray astronomy satellite. There are three main payloads onboard Insight-HXMT, the high energy X-ray telescope (20-250 keV, 5100 cm$^2$), the medium energy X-ray telescope (5-30 keV, 952 cm$^2$), and the low energy X-ray telescope (1-15 keV, 384 cm$^2$). The main scientific objectives of Insight-HXMT are: (1) to scan the Galactic Plane to find new transient sources and to monitor the known variable sources, (2) to observe X-ray binaries to study the dynamics and emission mechanism in strong gravitational or magnetic fields, and (3) to find and study gamma-ray bursts with its anti-coincidence CsI detectors.
Urca Processes

Direct URCA

Modified URCA
Neutrino Production

\[ \dot{q}_v^D \sim 10^{27} \, T_9^6 \, R_D \, \text{erg s}^{-1} \, \text{cm}^{-3} \, (\rho \gtrsim 10^{15} \, \text{g cm}^{-3}) \]

\[ \dot{q}_v^M \sim 7 \times 10^{20} \, T_9^8 \left( \frac{\rho}{\rho_{\text{nuc}}} \right)^{2/3} \, R_M \, \text{erg s}^{-1} \, \text{cm}^{-3} \]

\[ \dot{q}_v^{CP} \sim 10^{21} \left( \frac{\rho}{\rho_{\text{nuc}}} \right)^{1/3} \, T_9^7 \, f \left( \frac{T_{\text{core}}}{T_{\text{crit}}} \right) \, \text{erg s}^{-1} \, \text{cm}^{-3} \]
Initial Conditions

Since the focus is on the conversion probability and only the relative phase $\Delta \phi(x) = \phi_a(x) - \phi_E(x)$ appears in the equations above, one gets to

$$\frac{d\chi(x)}{dx} = -D(x) \cos[\Delta \phi(x)],$$

$$\frac{d\Delta \phi(x)}{dx} = A(x) - B(x) + 2D(x) \cot[2\chi(x)] \sin[\Delta \phi(x)],$$

(2.9)

where $\chi(1)$ determines the initial state at the surface of the magnetar. To avoid singularities for $\chi(1) = n\pi/2$ with $n \in \mathbb{Z}$, i.e. for pure initial states, the initial condition for $\Delta \phi(1)$ must satisfy $\Delta \phi(1) = m\pi$ with $m \in \mathbb{Z}$. It is therefore possible to set $\Delta \phi(1) = 0$ for a pure ALP initial state and the ALP-photon conversion probability is simply $P_{a \rightarrow \gamma}(x) = \sin^2[\chi(x)]$. 
Besides spectacular outbursts, magnetars produce persistent or decaying X-ray emission with luminosity $L \sim 10^{34} - 10^{36}$ erg s$^{-1}$. Two peaks are observed in their X-ray spectra, with comparable luminosities. The first peak is near 1 keV; it is associated with thermal emission from the neutron star surface. The second peak is above 100 keV. Its low-energy slope (between 10 and 100 keV) was observed in 7 magnetars (Kuiper et al. 2008; Enoto et al. 2010), with a typical photon index $\Gamma \sim 1 - 1.5$. 

![Graph of gamma-ray spectral representation from 1E 1841-045](image1.png)

**Kuiper et. al. (2004)**

Fig. 5. – A $\nu F_{\nu}$ spectral representation of the total pulsed high-energy emission from 1E 1841-045 is shown in red (RXTE PCA; open circles, RXTE HEXT; open squares). The total spectrum from the Kes73/1E 1841-045 complex is represented in blue (triangles; XMM EPIC PN, filled squares; RXTE HEXT). The total (pulsed plus DC) 1-7 keV X-ray spectrum from 1E 1841-045 (Mori et al. 2003) is plotted as a solid dark orange line. The two magenta flux points are INTEGRAL IBIS ISGRI measurements given in Molkov et al. (2004). Fits (> 10 keV) to the total complex (blue) and total pulsed (red) spectra of 1E 1841-045 are drawn as dashed lines.
Axion Emissivity (pair breaking)

\[ \mathcal{L}_{\text{int}} = \frac{1}{f_a} B^\mu L_\mu \]

\[ B^\mu = C_a \bar{\psi} \gamma^\mu \gamma_5 \psi, \quad L_\mu = \partial_\mu \phi. \]

\[ |\mathcal{M}_a|^2 = \frac{1}{2} f_a^{-2} (B^\mu B^{\nu \dagger})(L_\mu L^{\nu \dagger}) \]

The energy radiated per unit time in axions (axion emissivity) is given by the phase-space integral over the probability of the process of emission

\[ \epsilon_a = -f_a^{-2} \int \frac{d^3 q}{(2\pi)^3 2\omega} \omega g(\omega) q_\mu q_\nu \text{Im} \Pi_{a}^{\mu \nu}(q), \quad (7) \]

where \( q \) and \( \omega \) are the axion momentum and energy. Here we defined the polarization tensor of baryonic matter

\[ \text{Im} \Pi_{a}^{\mu \nu}(\omega, \vec{q}) = \frac{1}{2} \sum_n (B_\mu B^{\nu \dagger}_n) \delta^4(q - \sum_i p_i), \quad (8) \]

where the \( i \) sum is over the four-momenta of the baryons. Upon carrying out the angular integral in Eq. (7) we write the emissivity in terms of a one-dimensional integral

\[ \epsilon_a = -\frac{f_a^{-2}}{4\pi^2} \int_0^\infty d|\vec{q}| \bar{\varphi} g(\omega) \kappa_a(q), \quad (9) \]

where the contraction of the axion currents with the baryonic polarization tensor is given by

\[ \kappa_a(q) = q_\mu q_\nu \text{Im} \Pi_{a}^{\mu \nu}(q). \quad (10) \]
Axion Emissivity (pair breaking)

\[ \epsilon_a = \frac{8}{3\pi} f_a^{-2} \nu(0) v_F^2 T^5 I_a, \]

\[ I_a = z^5 \int_1^\infty dy \frac{y^3}{\sqrt{y^2 - 1}} f_F(zy)^2, \]

where \( z = \Delta(T)/T \) and \( f_F(x) = [1 + \exp(x)]^{-1} \) is the Fermi distribution function. The \( T^5 \) scaling of the emissivity is understood as follows. The integration over the phase space of neutrons carries a power of \( T \), since for degenerate neutrons the phase-space integrals are confined to a narrow strip around the Fermi surface of thickness \( T \). The axion is emitted thermally and being relativistic contributes a factor \( T^3 \) to the emissivity. The one power of \( T \) from the energy of the axion and the inverse one power of \( T \) from the energy conserving delta function cancel. The transition matrix element is proportional to the combinations of \( u_p \) and \( v_p \) amplitudes, which are dimensionless, but contain implicit temperature dependence due to the temperature dependence of the gap function. This dependence is not manifest in Eq. (22), i.e., was absorbed in the definition of the integral \( I_a \). Thus, the explicit temperature dependence of the axion emission rate Eq. (22) is \( T^5 \). In the cgs units the axion emissivity Eq. (22) is

\[ \epsilon_a = 1.06 \times 10^{21} \left( \frac{10^{10} \text{GeV}}{f_a} \right)^2 \left( \frac{m^*}{m} \right)^2 \left( \frac{v_F}{c} \right)^3 \left( \frac{T}{10^9 \text{K}} \right)^5 I_a \text{ erg cm}^{-3} \text{ s}^{-1}, \]
Comparing Neutrino and Axion Emissivity

Keller/Sedrakian (2012)

where two powers of $v_F/c$ arise from the small momentum transfer expansion and one power - from the density of states. At temperatures of order the critical temperature $T_c \simeq 10^9$ K the superfluid cools primarily by emission of neutrinos via the pair-breaking processes driven by the axial-vector currents (we continue to assume that potential fast cooling via direct Urca processes is prohibited). The emissivity of this processes in the case of $^1S_0$-wave superfluid is given by [19, 20, 23]

$$\epsilon_\nu = \frac{4 G_F^2 g_A^2}{15 \pi^3} \zeta_\nu \nu(0) v_F^2 T^7 I_\nu,$$

(25)

where $G_F$ is the weak Fermi coupling constant, $g_A = 1.25$ is the axial-vector current coupling constant, $\zeta_\nu = 6/7$ and

$$I_\nu = z^7 \int_1^\infty dy \frac{y^5}{\sqrt{y^2 - 1}} f_F (zy)^2.$$

(26)

We now require that the axion luminosity does not exceed the neutrino luminosity, i.e.,

$$\frac{\epsilon_a}{\epsilon_\nu} = \frac{10 \pi^2}{f_a G_F^2 g_A^2 \zeta_\nu I_\nu} \frac{I_a}{I_\nu} < 1.$$

(27)
Comparing Neutrino and Axion Emissivity

Keller/Sedrakian (2012)

Substituting the free-space value of the axial vector coupling $g_A = 1.25$ and introducing $r(z) \equiv z^2 (I_a/I_\nu)$ we transform Eq. (27)

$$\frac{\epsilon_a}{\epsilon_\nu} = \frac{59.2}{f_a^2 G_F^2 \Delta(T)^2} r(z).$$  \hspace{1cm} (28)

Not far from the critical temperature $\Delta(T) \simeq 3.06 T_c \sqrt{1 - T/T_c}$, which translates into $z = 3.06 t^{-1} \sqrt{1 - t}$, where $t = T/T_c$. Numerical evaluations of the integrals provides the following values $r(0.5) = 0.07$, $r(1) = 0.26$, $r(2) = 0.6$ and asymptotically $r(z) \to 1$ for $z \gg 1$. Substituting the value of the Fermi coupling constant $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ in Eq. (28) and noting that $r(z) \leq 1$, we finally obtain

$$f_a > 5.92 \times 10^9 \text{GeV} \left[ \frac{0.1 \text{ MeV}}{\Delta(T)} \right]$$ \hspace{1cm} (29)
Light Species

There’s a lot of interest in light BSM particles these days
Light Species

UV models: ALPs/dark photons in string theory

Probes at the LHC/HL-LHC etc.

Probes using atomic/molecular physics

Direct detection techniques: interferometers, light shining through wall, haloscope, etc.

Cosmological/astrophysical probes

Lots of new experimental ideas/proposals!
Why Axions?
Neutron EDM

Historically, the strong CP problem motivated axions

The classical version is visually intuitive:

$$d_n = qx$$

electric dipole moment of the neutron measured to be tiny!

$$d_n < 2.9 \times 10^{-26} \text{ e cm}$$

How about the theory calculation?
An Unnaturally Small Angle

\[ |d_n| \sim e x \sqrt{1 - \cos \theta} \]
\[ \sim 10^{-14} e \sqrt{1 - \cos \theta} \text{ cm} \]
\[ \Rightarrow \theta < 10^{-12} !! \]

As far as we know, no “anthropic” reason

Kaloper et. al. (2017)

An example of fine-tuning
A Dynamical Angle

In chemistry, a similar question is resolved by making the angle itself dynamical.

Axions are the dynamical degree of freedom.
A Dynamical Angle

\[ \theta \, G_{\mu\nu} \tilde{G}^{\mu\nu} \Rightarrow \frac{a}{f} \, G_{\mu\nu} \tilde{G}^{\mu\nu} \]

Axions are the dynamical degree of freedom
The Axiverse
String Theory

The most important prediction of string theory is the existence of extra dimensions.

These extra dimensions are compactified.

What are the generic features?

- Extra scalar fields: Moduli (gravitationally coupled)

- Hundreds of pseudoscalar fields: Axion-like Particles (ALPs)
String Axiverse

Although it has been argued that there is no exact global symmetry in string theory, there can be a bunch of well-controlled approximate shift symmetries for light axions.

\[ \mathcal{L}_a \sim \frac{1}{2} (\partial_\mu a)^2 + \frac{c_1}{f} \partial_\mu a J^\mu - \frac{c_2}{f} aG_{\mu\nu} \tilde{G}^{\mu\nu} - \frac{c_3}{f} aF_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{2} m_a^2 a^2 + \ldots \]

\[ m_a \quad \{ \quad \text{Free, independent parameters for us} \quad \}
\]

\[ g \sim 1/f \quad \}

axion decay constant: depends on internal CY geometry

axion mass: mechanism of PQ breaking
ALP Detection
WIMP Detection Strategies

thermal freeze-out (early Univ.)
indirect detection (now)

production at colliders
ALP Detection Strategies

emission/conversion

indirect detection (now)

direct detection

production at colliders

LHC/lepton collider studies
Collider Prospects

\[ \mathcal{L}_a \sim \frac{1}{2} (\partial^\mu a)^2 - \frac{1}{2} m_a^2 a^2 + \frac{c_1}{f} \partial_\mu a \bar{f} \gamma_\mu \gamma_5 f \]

- \frac{c_2}{f} a G_{\mu\nu} \tilde{G}^{\mu\nu}
- \frac{c_3}{f} a F_{\mu\nu} \tilde{F}^{\mu\nu}
- \frac{c_4}{f} a F_{\mu\nu} \tilde{Z}^{\mu\nu}
- \frac{c_5}{f} a Z_{\mu\nu} \tilde{Z}^{\mu\nu}

+ \frac{c_6}{f^2} (\partial^\mu a)(\partial^\nu a) \phi^\dagger \phi
+ \frac{c_7}{f^3} (\partial^\mu a)(\phi^\dagger i D_\mu \phi + h.c.) \phi^\dagger \phi + \ldots

\begin{align*}
\ell^+ \ell^- & \quad a \to \ell^+ \ell^- \\
Z & \quad Z \to \gamma a \\
gg & \quad a \to gg \\
\gamma \gamma & \quad a \to \gamma \gamma \\
aa & \quad h \to aa \\
Za & \quad h \to Z a
\end{align*}

Bauer/Neubert (2018)
Lian-Tao Wang et. al. (2017)
A. Alves, KS (2017)
Collider Prospects

**Figure 4:** Left: Summary plot of constraints on the parameter space spanned by the ALP mass and ALP-photon coupling. Right: Enlarged display of the constraints from collider searches: LEP (light blue and blue), CDF (purple), LHC from associated production and Z decays (orange), LHC from photon fusion (light orange), and from heavy-ion collisions at the LHC (green).
Conversion as ALP Detection Tool

Strategy:

1. Photons/ALPs travel from A to B
2. There’s a magnetic field between A and B
3. Photon-ALP interconversion happens
4. Photon spectrum at B shows changes
Astrophysical Probes of Conversion

Supernova dimming with ALPs

ALPs from AGNs giving X-rays

Conversion Probability

\[ p(a \rightarrow \gamma) = \sin^2(2\theta) \sin^2(\pi z / L_{\text{osc}}) \]

mixing strength

follows directly from Maxwell’s equations

Constant B field, simple analytic solution

Fig.: John Terning

Csaki et. al. (2001)

Conlon et. al. (2014)
Strong QED

Strong
\[ B_k = \frac{m^2 c^3}{e \hbar} \approx 4.4 \times 10^{13} \text{ G} \]

Euler-Heisenberg Lagrangian:
\[ \omega < m \sim 500 \text{ keV} \]

Weak field limit:

Heyl/Hernquist (2012)

\[ I = F_{\mu \nu} F^{\mu \nu} = 2 \left( |B|^2 - |E|^2 \right) \]
\[ K = \left( \frac{1}{2} \epsilon^{\lambda \rho \mu \nu} F_{\lambda \rho} F_{\mu \nu} \right)^2 = -(4E \cdot B)^2 \]

\[ \mathcal{L} \approx -\frac{1}{4} I + \frac{E_k^2 e^2}{\hbar c} \left[ \frac{1}{E_k^4} \left( \frac{1}{180} I^2 - \frac{7}{720} K \right) + \frac{1}{E_k^6} \left( \frac{13}{5040} K I - \frac{1}{630} I^3 \right) \right] \]
Why Neutron Stars?

Strong magnetic fields are especially useful

Magnetars have the strongest magnetic fields

Also lots of data

Formal side:

- Non-constant magnetic field, interesting equations
- Strong magnetic field, so non-linear QED effects
Full QED Lagrangian

$$\mathcal{L}_0 = -\frac{1}{4}I,$$

$$\mathcal{L}_1 = \frac{e^2}{\hbar c} \int_0^{\infty} e^{-\zeta} \frac{d\zeta}{\zeta^3} \left( i\zeta^2 \frac{\sqrt{-K}}{4} \times \right)$$

$$\cos \left( \frac{\zeta}{B_k} \sqrt{-\frac{I}{2} + i\frac{\sqrt{-K}}{2}} \right) + \cos \left( \frac{\zeta}{B_k} \sqrt{-\frac{I}{2} - i\frac{\sqrt{-K}}{2}} \right) + |B_k|^2 + \frac{\zeta^2}{6}I \right)$$

QED effects can change the spectrum from magnetars

Studied extensively by astrophysics community

Heyl, Ho, Perna, Taverna, Turolla, etc.

$$n_\perp = 1 + \epsilon_\perp b^2$$

$$n_\parallel = 1 + \epsilon_\parallel b^2$$
Refractive Indices

Perpendicular (X-mode)

\[ n_\perp \sim 1 + \frac{\Delta_\perp}{\omega} \]

\[ q_\perp = \frac{4\alpha}{45\pi} b^2 \hat{q}_\perp \]

\[ \Delta_\perp = \frac{1}{2} q_\perp \omega \sin^2 \theta \]

\[ \hat{q}_\perp = \frac{1}{1 + 0.72 b^{5/4} + (4/15)b^2} \]

Parallel (O-mode)

\[ n_\parallel \sim 1 + \frac{\Delta_\parallel}{\omega} \]

\[ q_\parallel = \frac{7\alpha}{45\pi} b^2 \hat{q}_\parallel \]

\[ \Delta_\parallel = \frac{1}{2} q_\parallel \omega \sin^2 \theta \]

\[ \hat{q}_\parallel = \frac{1 + 1.2b}{1 + 1.33b + 0.56b^2} \]

Heyl/Hernquist (2012)