Phenomenology of family-nonuniversal 3HDMs:
an overview

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Motivation for extended Higgs sectors

Despite of remarkable consistency of the SM with to-date observations, it remains remarkably unsatisfactory.

We are looking for

explanations why/where the SM features as they are/originate from

such as

neutrino sector features **mass/mixing hierarchies** **origin of CPV**

Dark Matter **baryon asymmetry etc**

Non-minimal Higgs sectors can provide natural explanation to CPV and fermion puzzles, and more..

Active model-building activity with non-minimal scalar sectors (see e.g. Ivanov, 1702.03776)

"Bottom-up" approach

- **Assume a little extra**: take the plain SM and add a bit more on top to explain a particular peculiarity of the SM (very rarely most them)

- **Postulate**: such an extension to be anomaly-free (just like the SM itself)

- **Fit**: parameters to the observables to see if it complies with the reality

- **Extrapolate**: such an extension to much higher energies hoping the vacuum remains stable, Higgs mass does not receive GUT-scale corrections and the gauge couplings unify (e.g. MSSM)

Know examples consider adding a SUSY, horizontal (family) symmetries, extra Higgs and vector-like fermions

Example I. CP4 3HDM

- **CP is not uniquely defined in QFT**
  Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^* \,, \quad X \in U(N)\,, \quad J^2 = XX^* \qquad J^k = \mathbb{I}$$

- **k=2 is usual CP, the first non-trivial case is CP4 with 3HDs**
  Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)$$

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

$$V = V_0 + V_1 \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad J^2 = \mathrm{diag}(1, -1, -1)\,, \quad J^4 = \mathbb{I}$$

Flavoured CP4 3HDM

- **CP4 can be extended to the Yukawa sector and must be spontaneously broken leading to particular patterns in the flavour sector**

Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017

$$\psi_i \to Y_{ij}\psi_j^{CP}, \quad \psi^{CP} = \gamma^0 C\bar{\psi}^T \qquad -\mathcal{L}_Y = \bar{q}_L\Gamma_a d_R\phi_a + \bar{q}_L\Delta_a u_R\phi_a^* + h.c.$$

CP4 invariance: $(Y^L)^\dagger\Gamma_a Y^d X_{ab} = \Gamma_b^*, \quad (Y^L)^\dagger\Delta_a Y^u X_{ab}^* = \Delta_b^*$ $\qquad Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} \\ 0 & e^{-i\alpha} & 0 \end{pmatrix}$

<u>Example:</u>

case B_1

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}$$

- **Tree-level FCNCs**

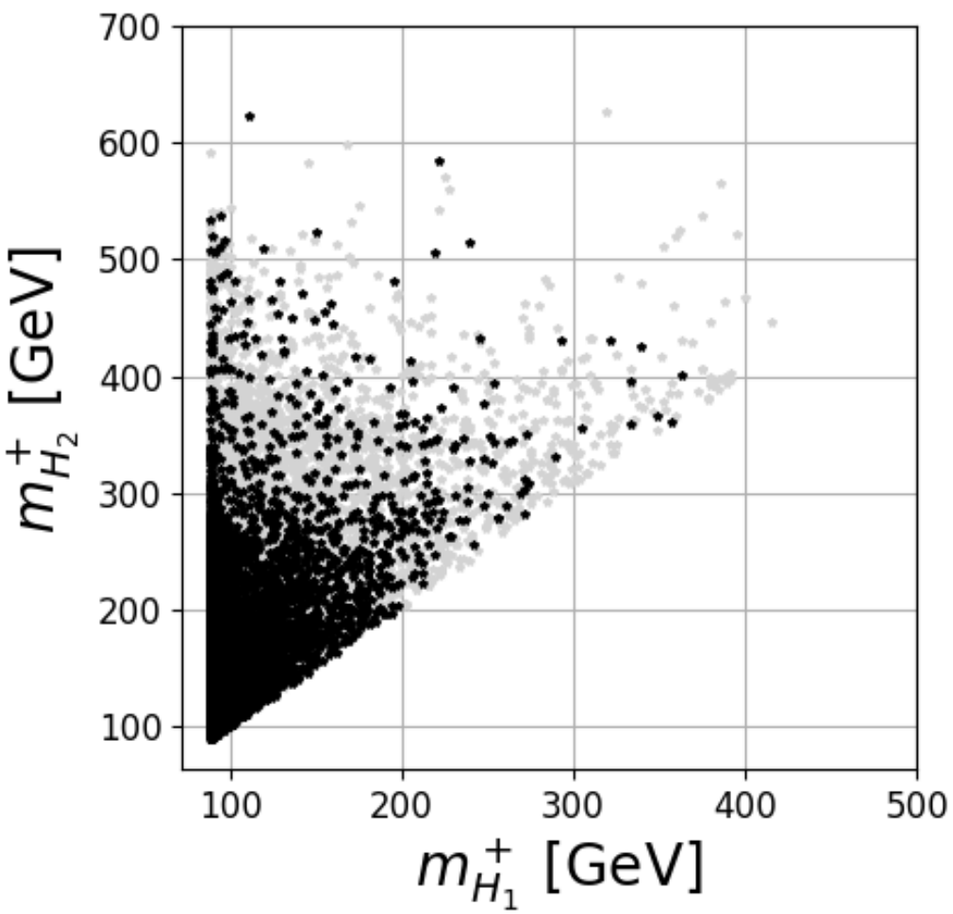
One avoids FCNCs from h125 by imposing the scalar alignment condition:

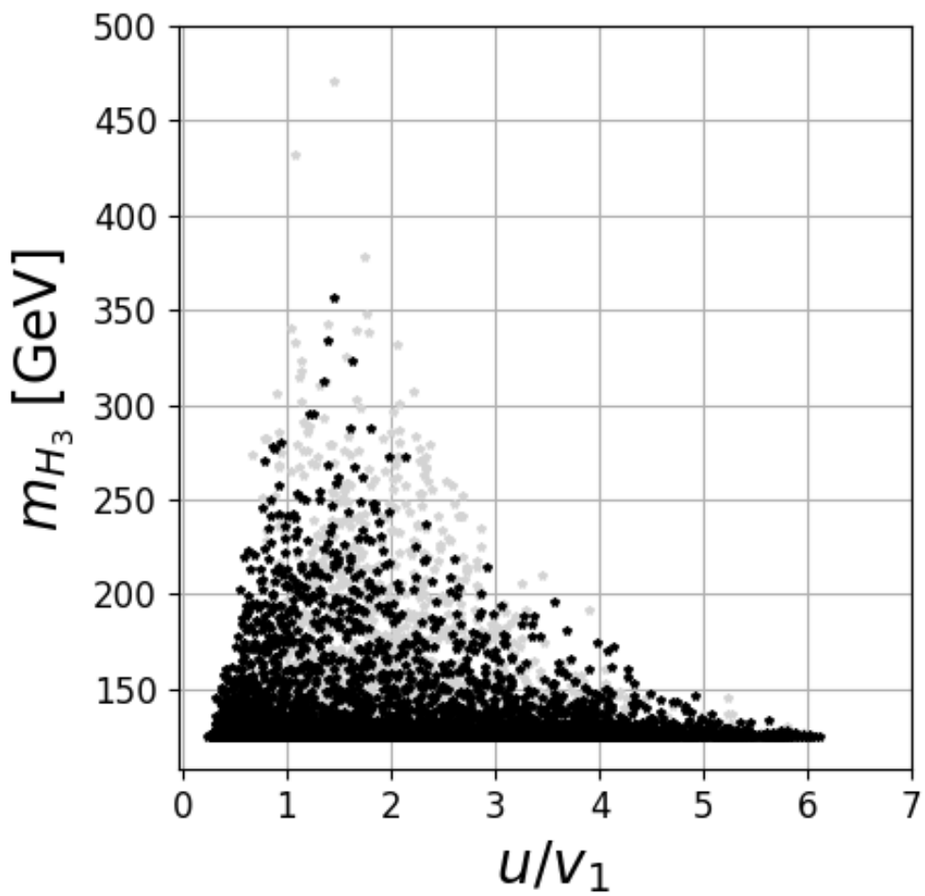
$$m_{11}^2 = m_{22}^2$$

Flavour CP4 3HDM: extra Higgses

Scalar sector scan

- Stick to the scalar alignment, take h125 to be the lightest scalar, vary nine free parameters
- Checks for the boundedness from below and perturbativity
- Check that S,T,U parameters are within 3σ of experimental values

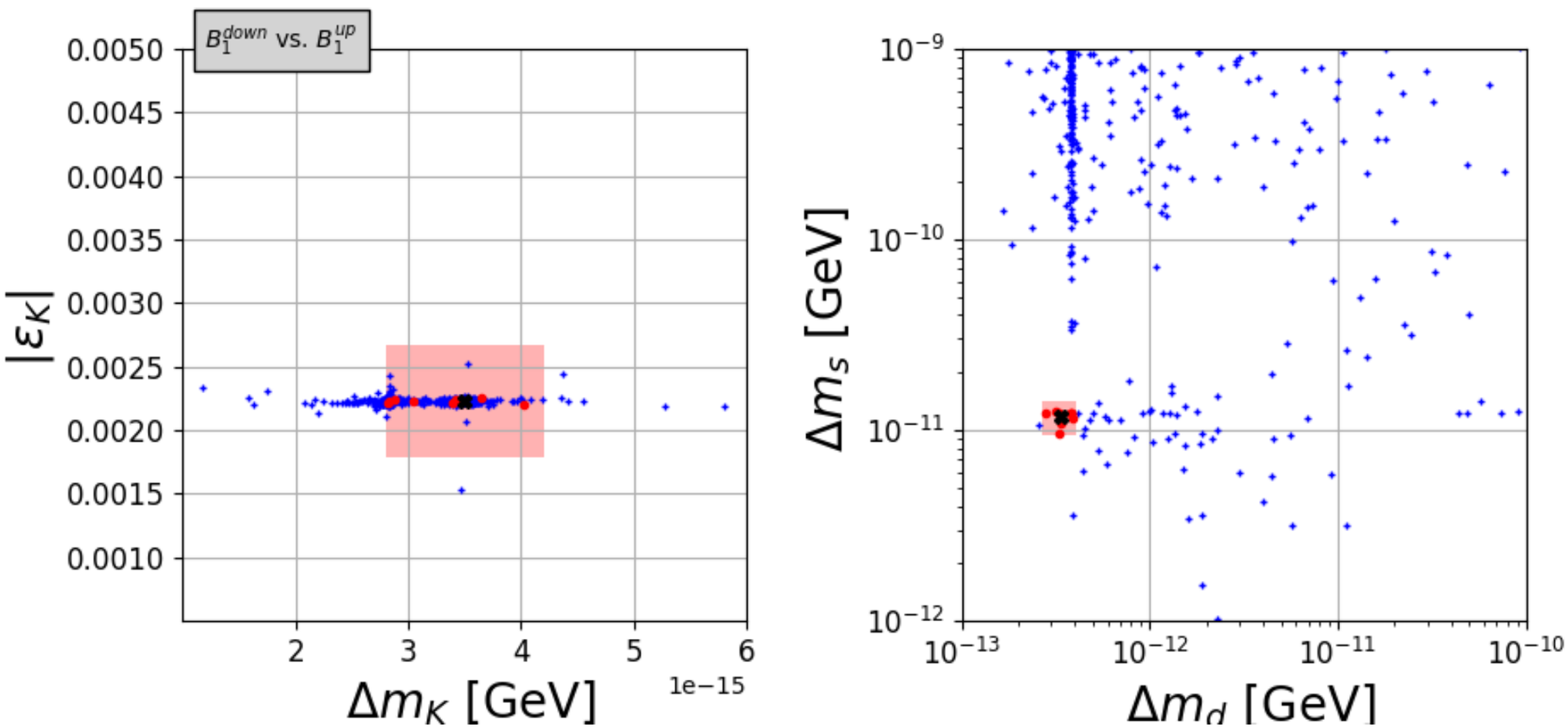




Flavour CP4 3HDM: flavour observables

Yukawa sector scan

- Fit all quark masses, mixing, and CPV phase
- Add K and B oscillation parameters $|\epsilon_K|$, Δm_K, Δm_{B_d}, Δm_{B_s} from Buras et al, 2013 (tree-level contributions from neutral Higges only)



Typical points have light Higgses (< 150 GeV); a few points have moderately heavy Higgses.

Example II. U(1) x U(1) 3HDM

- **The most constraining realisable Abelian symmetry of 3HDM**
  Keus, King, Moretti 2014; Ivanov, Keus, Vdovin, 2012

- **Promote this symmetry to be a family symmetry of the fermion sector**
  Camargo-Molina, Mandal, Pasechnik, Wessen, 2018

softly broken

$U(1)_X \times U(1)_Z$

- No tree-level FCNCs
- Cabbibo-like mixing at tree-level
- Fermion mass hierarchies are related to hierarchy of VEVs
- New scalar states couple dominantly to the second quark family (exotic collider signatures)

$$V_0 = -\sum_{i=1}^{3} \mu_i^2 |H_i|^2 + \sum_{i,j=1}^{3} \left(\frac{\lambda_{ij}}{2} |H_i|^2 |H_j|^2 + \frac{\lambda'_{ij}}{2} |H_i^\dagger H_j|^2 \right) , \quad V_{\text{soft}} = \sum_{i=1}^{3} \frac{1}{2} (m_{ij}^2 H_i^\dagger H_j + \text{c.c})$$

All the parameters can be taken real!

$$\lambda_{ij} = \lambda_{ji} , \quad \lambda'_{ij} = \lambda'_{ji} , \quad m_{ij}^2 = m_{ji}^2 ,$$

$$\lambda'_{11} = \lambda'_{22} = \lambda'_{33} = 0 , \quad m_{11}^2 = m_{22}^2 = m_{33}^2 = 0 .$$

Flavoured U(1) x U(1) 3HDM: Yukawa sector

$$\mathcal{L}^{\mathrm{q}}_{\mathrm{Yukawa}} = \sum_{i,j=1}^{2} \left\{ y^{\mathrm{d}}_{ij} \bar{d}^{i}_{\mathrm{R}} H^{\dagger}_{1} Q^{j}_{\mathrm{L}} - y^{\mathrm{u}}_{ij} \bar{u}^{i}_{\mathrm{R}} \tilde{H}^{\dagger}_{2} Q^{j}_{\mathrm{L}} \right\} + y_{\mathrm{b}} \bar{b}_{\mathrm{R}} H^{\dagger}_{3} Q^{3}_{\mathrm{L}} - y_{\mathrm{t}} \bar{t}_{\mathrm{R}} \tilde{H}^{\dagger}_{3} Q^{3}_{\mathrm{L}} + \mathrm{c.c.}$$

	$\mathrm{U(1)_Y}$	$\mathrm{U(1)_X}$	$\mathrm{U(1)_Z}$
H_1	$\frac{1}{2}$	-1	$-\frac{2}{3}$
H_2	$\frac{1}{2}$	1	$\frac{1}{3}$
H_3	$\frac{1}{2}$	0	$\frac{1}{3}$
$Q^{1,2}_{\mathrm{L}}$	$\frac{1}{6}$	γ	δ
Q^{3}_{L}	$\frac{1}{6}$	β	α
$u^{1,2}_{\mathrm{R}}$	$\frac{2}{3}$	$1+\gamma$	$\frac{1}{3}+\delta$
t_{R}	$\frac{2}{3}$	β	$\frac{1}{3}+\alpha$
$d^{1,2}_{\mathrm{R}}$	$-\frac{1}{3}$	$1+\gamma$	$\frac{2}{3}+\delta$
b_{R}	$-\frac{1}{3}$	β	$-\frac{1}{3}+\alpha$

$v_3 \gg v_{1,2}$ ⇒ **Heavy third generation!**

The model is treatable fully analytically in this limit!

Dim-6 operators:

$$\bar{d}^{1,2}_{\mathrm{R}} \left(H^{\dagger}_{i} Q^{3}_{\mathrm{L}} \right) \left(H^{\dagger}_{j} H_{k} \right) , \quad \bar{u}^{1,2}_{\mathrm{R}} \left(\tilde{H}^{\dagger}_{i} Q^{3}_{\mathrm{L}} \right) \left(H^{\dagger}_{j} H_{k} \right)$$

$$\bar{b}_{\mathrm{R}} \left(H^{\dagger}_{i} Q^{1,2}_{\mathrm{L}} \right) \left(H^{\dagger}_{j} H_{k} \right) , \quad \bar{t}_{\mathrm{R}} \left(\tilde{H}^{\dagger}_{i} Q^{1,2}_{\mathrm{L}} \right) \left(H^{\dagger}_{j} H_{k} \right)$$

⇓

Full CKM?

$$(\beta - \gamma, \alpha - \delta) \notin \{(-1,-1), (-1,0), (0,0), (1,0), (1,1), (2,1)\}$$

Flavoured U(1) x U(1) 3HDM: exotic channels

$$\xi \equiv \frac{\sqrt{v_1^2 + v_2^2}}{v_3}$$

$$\xi \ll 1$$

SM-like Higgs: $$\mathcal{L} \supset \sum_q \frac{m_q}{v_3} \bar{q} q \, h_{125} + \mathcal{O}(\xi)$$

Additional scalars' Yukawa couplings:

$$\mathcal{L} \supset \cos\theta_{\rm C} \frac{\sqrt{2} m_{\rm s}}{v_1} \bar{s}_{\rm R} c_{\rm L} H_{\rm a}^- - \cos\theta_{\rm C} \frac{\sqrt{2} m_{\rm c}}{v_2} \bar{c}_{\rm R} s_{\rm L} H_{\rm b}^+ + {\rm c.c.} + \mathcal{O}(\xi)$$
$$+ \frac{m_{\rm s}}{v_1} \bar{s} s h_{\rm a} - \frac{m_{\rm c}}{v_2} \bar{c} c h_{\rm b} + {\rm i} \frac{m_{\rm s}}{v_1} \bar{s} \gamma^5 s A_{\rm a} - {\rm i} \frac{m_{\rm c}}{v_2} \bar{c} \gamma^5 c A_{\rm b} + \mathcal{O}(\xi) \, ,$$

Charged Higgses are mostly produced via $c\bar{s}$ fusion!

Main focus: $c\bar{s} \to H^+ \to W^+ h_{125}$ $\quad m_{H^\pm} > 200$ GeV

Flavoured U(1) x U(1) 3HDM: results

$$pp \to H^{\pm} \to W^{\pm} h_{125} \to \ell^{\pm} + \not{E}_T + b\bar{b}$$

Typical backgrounds:

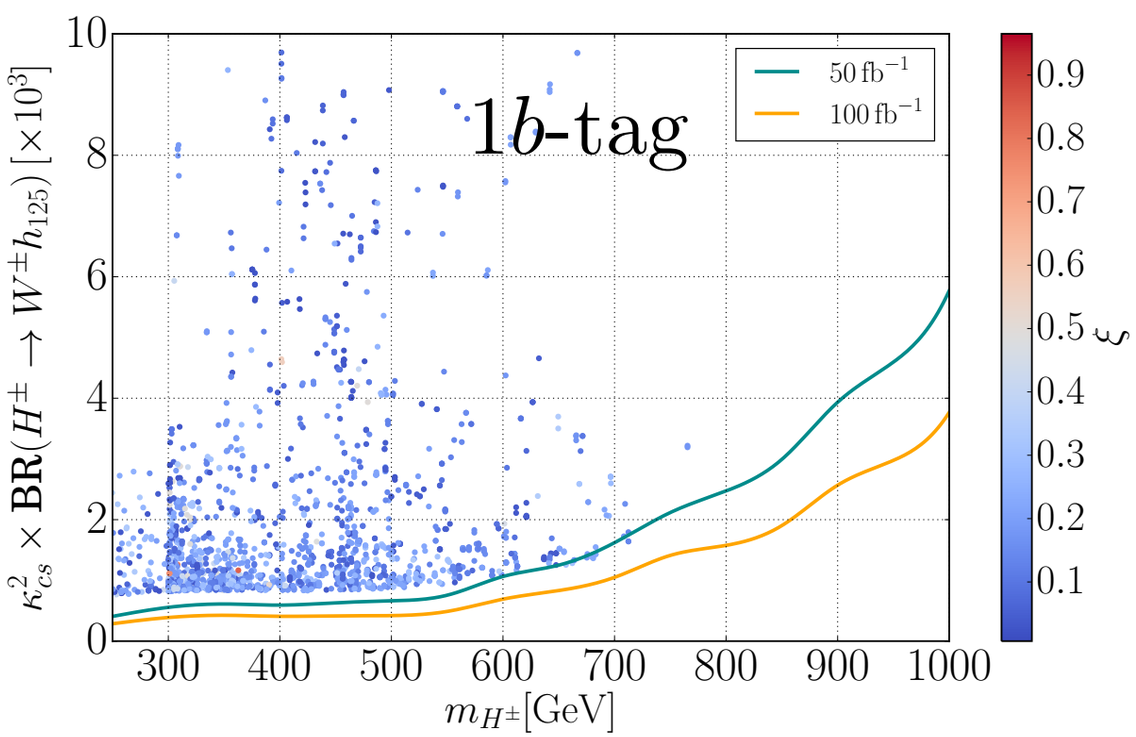
LHC ($\sqrt{s} = 13$ TeV)

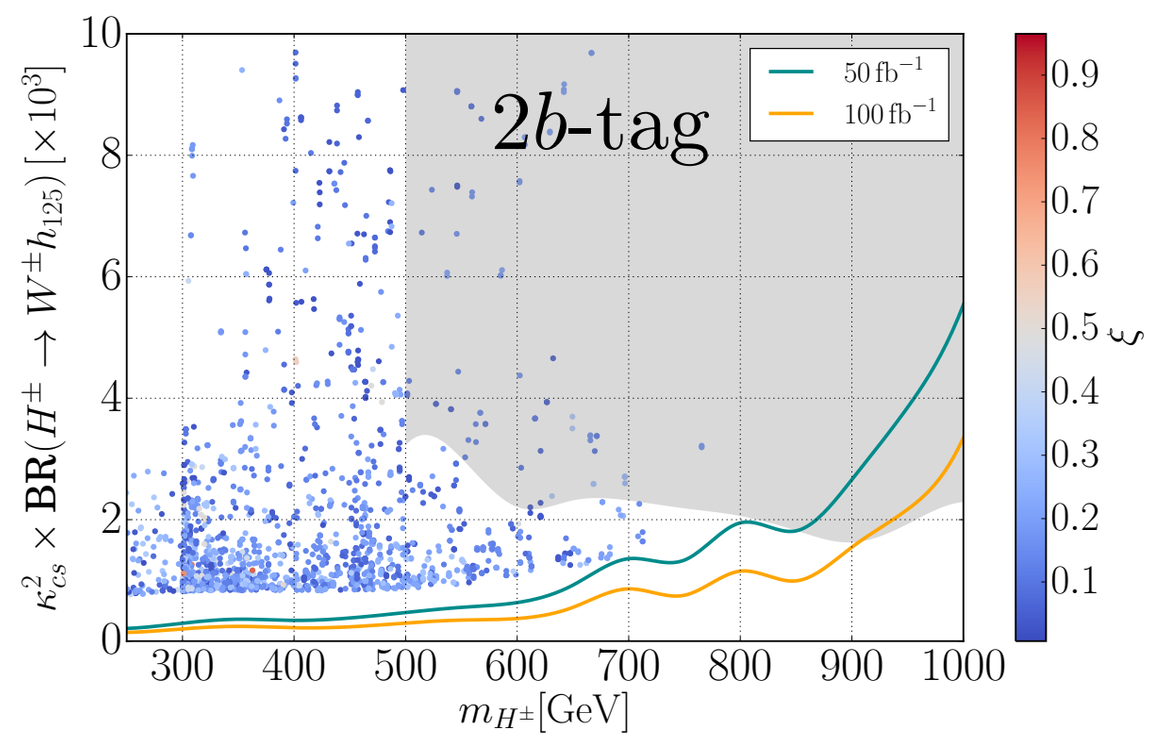
Process	$W + n\ j$	Wbj	$Wb\bar{b}$	$t\bar{t} + n\ j$	tj	tb	tW	WW	WZ	Wh_{125}
x-sec (pb)	1.53×10^5	308.9	41.7	431.3	174.6	2.6	54.0	67.8	25.4	1.1

Selection cuts:

- Lepton: $p_T(\ell) > 25$ GeV, $|\eta(\ell)| < 2.5$
- Jet: $p_T(J) > 25$ GeV, $|\eta(J)| < 4.5$
- Missing transverse energy: $\not{E}_T > 25$ GeV
- ΔR separation: $\Delta R(J_1, J_2) > 0.4$, $\Delta R(\ell, J) > 0.4$

5σ discovery contours





Example III. U(1) x Z_2 3HDM

- **U(1)xU(1) symmetry of 3HDM does not lead to CKM (only Cabibbo!)**

- **We can relax the family symmetry and find an analog to Branco-Grimus-Lavoura (BGL) 2HDM where smallness of (tree-level) FCNCs relates to the smallness of off-diagonal CKM**
  Dipankar, Morais, Padilla, Pasechnik, 2019 (in progress)

Symmetry transformations:

$$U(1): \quad Q_{L,3} \to e^{i\alpha} Q_{L,3}, \quad u_{R,3} \to e^{2i\alpha} u_{R,3}, \quad H_1 \to e^{i\alpha} H_1, \quad \Psi_{L,i} \to e^{i\alpha} \Psi_{L,i}, \quad H_3 \to e^{i\alpha} H_3$$

$$Z_2: \quad Q_{L,3} \to -Q_{L,3}, \quad u_{R,3} \to -u_{R,3}, \quad H_1 \to -H_1, \quad \Psi_{L,i} \to -\Psi_{L,i}, \quad d_{R,3} \to -d_{R,3}$$

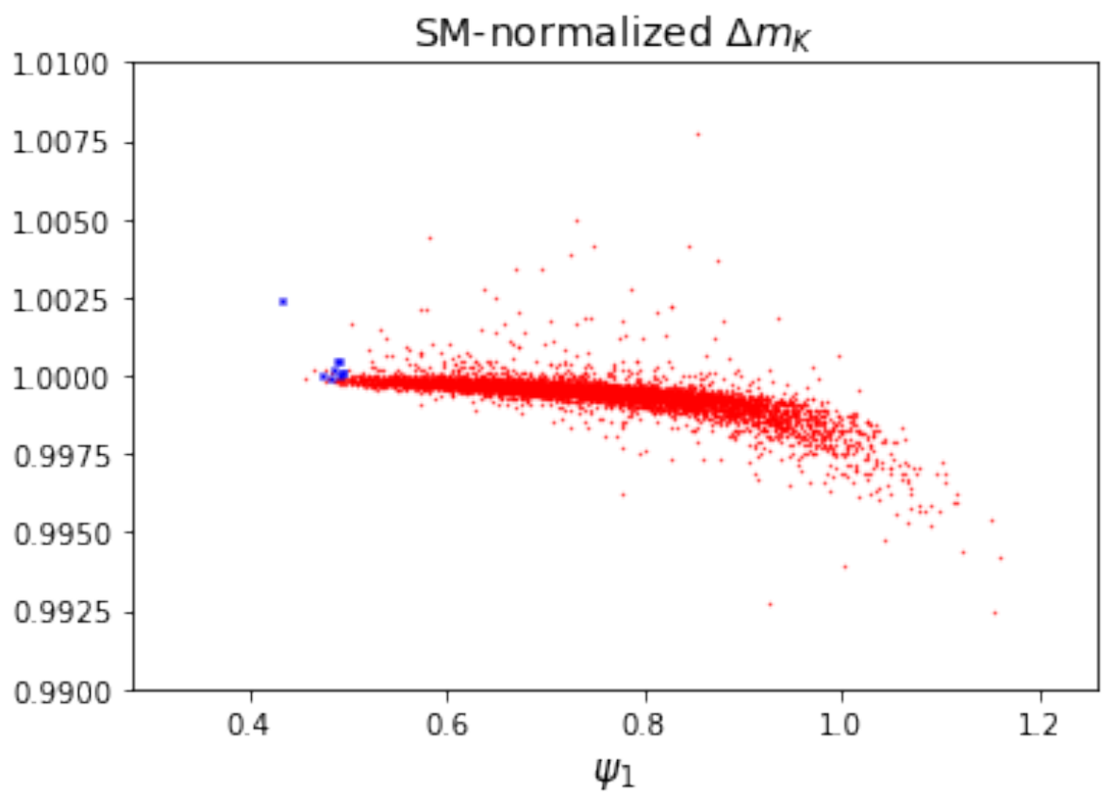
Allowed textures:

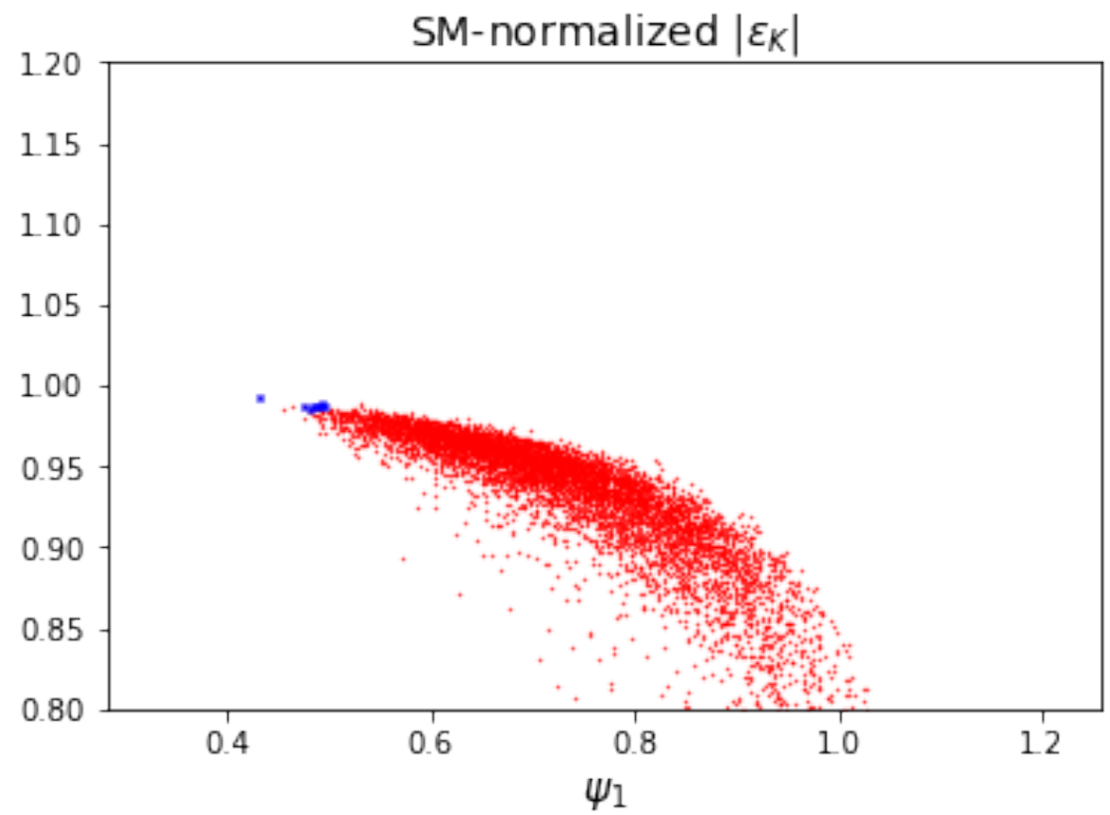
$$Y_1^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & 0 \end{pmatrix}; \quad Y_1^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

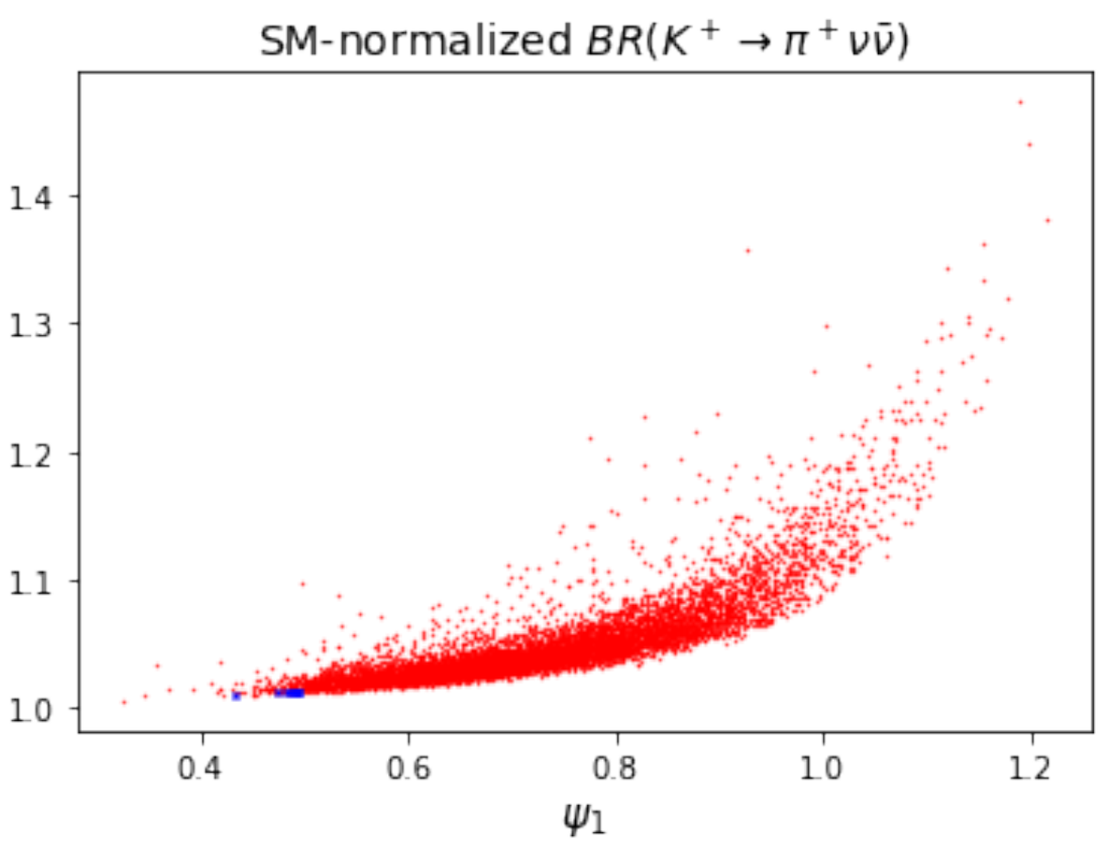
$$Y_2^d, Y_2^u = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad Y_3^d, Y_3^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

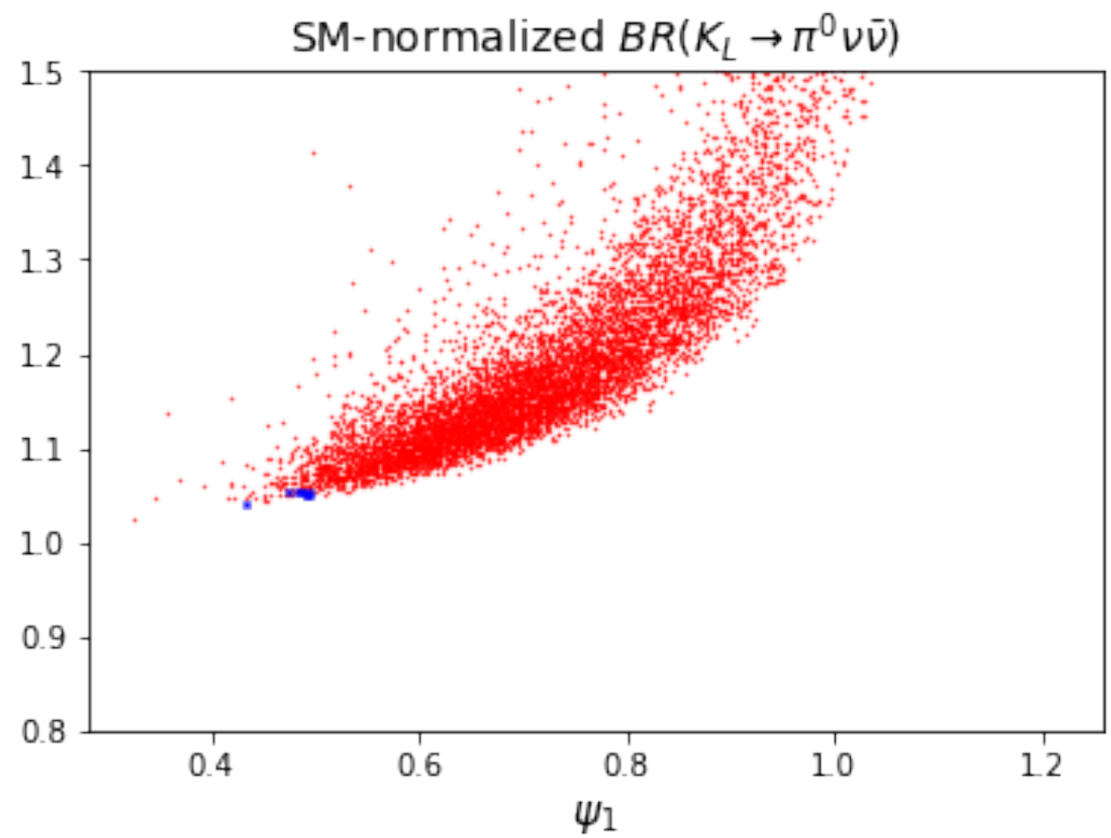
	BGL-Model				SM	
	Charged $H_{1,2}^{\pm}$		Neutral $H_{2,3}$		Tree	Loop
	Tree	Loop	Tree	Loop		
$M \to \ell\overline{\nu}, M'\ell\overline{\nu}$	✓	✓		✓	✓	✓
$M^0 \to \ell_1\overline{\ell}_2$		✓	✓	✓		✓
$M^0 - \overline{M}^0$ oscillation		✓	✓	✓		✓
$\overline{B} \to X_s\gamma$		✓		✓		✓
EW precision		✓		✓		✓

Flavoured U(1) x Z_2 3HDM: some first results









"Top-down" approach

- **Postulate**: a consistent SUSY GUT valid at high energy scales

- **Explore**: the larger the symmetry, the fewer free parameters, the higher degree of unification

- **Study**: to what extent the SM emerges as an EFT at low energies. Anomalies of the visible sector with additional gauge groups in the EFT need not necessarily cancel

- **Hope**: to explain seemingly arbitrary features of the SM (number of families, mass/mixing hierarchies, neutrino status, cancellation of gauge anomalies in the observed sectors etc)

Any promising candidates?

Why Trinification?

Positives:

- The model accommodates any quark and lepton masses and mixings (Sayre et al 2016)
- Naturally light neutrinos via radiative see-saw with split-SUSY (Cauet et al 2011)
- Gauge symmetry preserve baryon number, i.e. no gauge-mediated proton decay (Achiman&Stech'78; Glashow&Kang'84)
- Motivated as a low-energy version of E8xE8 heterotic string theory (Gross et al 1985)

Negatives:

- GUT scale fermion masses through $L \cdot L' \cdot L''$ type operators
  - Higher dimensional operators needed **(Cauet et al. 2011)**
- Considerable amount of particles and many couplings involved
  - Realistic calculations cumbersome
- Unmotivated Hierarchy between the trinification and the EW breaking scales (common to most GUTs)

Trinification-based models were left as the least developed GUT scenarios

Example I: SU(3)-flavoured T-GUT

Our proposal: Extend the SUSY trinification model (Georgi, Glashow and De Rujula 1984) with a local family $\mathrm{SU}(3)_\mathrm{F}$ symmetry

A. Morais, A. Ordell, J.-E. Camargo-Molina, RP, J. Wessen, PRD95 (2017) 075031

$$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \rtimes \mathbb{Z}_3 \times \mathrm{SU}(3)_\mathrm{F}$$

We refer to the model as **Supersymmetric Higgs-Unified Trinification**

- Use the minimal field content:

$$(1,\mathbf{3},\overline{\mathbf{3}},\mathbf{3}) = \left(L^i\right)^l{}_r = \begin{pmatrix} H_\mathrm{u}^0 & H_\mathrm{d}^- & e_\mathrm{L} \\ H_\mathrm{u}^+ & H_\mathrm{d}^0 & \nu_\mathrm{L} \\ e_\mathrm{R} & \nu_\mathrm{R} & \phi \end{pmatrix}^i, \quad \begin{aligned} &(\mathbf{3},\overline{\mathbf{3}},1,\mathbf{3}) = \left(Q_\mathrm{L}^i\right)^x{}_l = \begin{pmatrix} u_\mathrm{L}^x & d_\mathrm{L}^x & D_\mathrm{L}^x \end{pmatrix}^i, \\ &(\overline{\mathbf{3}},1,\mathbf{3},\mathbf{3}) = \left(Q_\mathrm{R}^i\right)^r{}_x = \begin{pmatrix} u_{\mathrm{R}x}^c & d_{\mathrm{R}x}^c & D_{\mathrm{R}x}^c \end{pmatrix}^{\top\, i}. \end{aligned}$$

- Higgs and leptons unified in L due to SUSY

full unification of Yukawa couplings

- $W_1 = \lambda_{27}\,\varepsilon_{ijk}\left(Q_\mathrm{L}^i\right)^x{}_l \left(Q_\mathrm{R}^j\right)^r{}_x \left(L^k\right)^l{}_r$
  - > **One** family of quarks and **all** leptons massless at tree-level $\rightarrow$ **radiatively generated**,
  - > Exact Yukawa unification for all three families.

- $\mathrm{SU}(3)_\mathrm{F}$ also fits neatly into an $\mathrm{E}_8 \subset \mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F}$ embedding

$\mathbf{SU(3)_F} \times \mathbf{E_6}$ $\qquad \mathbf{248} = (\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{78}) \oplus (\mathbf{3},\mathbf{27}) \oplus (\overline{\mathbf{3}},\overline{\mathbf{27}})$ $\qquad \Rightarrow$ **No gauge anomalies**.

On the top-bottom path down to a SM-like theory

$$\Delta_A = \begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \\ \hline \Delta^4 \\ \Delta^5 \\ \Delta^6 \\ \Delta^7 \\ \hline \Delta^8 \end{pmatrix}_{A=L,R,C,F}$$

Q-GUT:
$[SU(3)_C \times SU(3)_L \times SU(3)_R] \times SU(3)_F$

Region - I $\left\langle \widetilde{\Delta}^8_{L,R,F} \right\rangle \sim M_{GUT}$

SUSY-LRF:
$SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(2)_F \times U(1)_L \times U(1)_R \times U(1)_F$

Region - II $\left\langle \widetilde{\phi}^3 \right\rangle \sim M^{(1)}_{soft} \ll M_{GUT}$

$$T_{L+R} = T^8_L + T^8_R\,, \quad T_S = T^8_L - T^8_R - 2T_F$$

non-SUSY-LRF:
$SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(2)_F \times U(1)_{L+R} \times U(1)_S$

Region - III $\left\langle \widetilde{\phi}^2 \right\rangle \sim M^{(2)}_{soft} \lesssim M^{(1)}_{soft}$

$$T_V = T^3_F - \tfrac{1}{2\sqrt{2}} T_S$$

non-SUSY-LR:
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times U(1)_V$

Region - IV $\langle \widetilde{\nu}^1_R \rangle \sim M^{(3)}_{soft} \lesssim M^{(2)}_{soft}$

$$T_Y = \sqrt{3} T^3_R + T_{L+R}\,, \quad T_T = T_V - \tfrac{3}{2} T^3_R$$

3HDM EFTs:
$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_T$

Region - V $\langle S \rangle \sim m_{z'} \lesssim M^{(3)}_{soft}$

3HDM EFTs:
$SU(3)_C \times SU(2)_L \times U(1)_Y$

Natural hierarchy between GUT and EW scales!

- $U(1)_T$ provides family non-universal Z' boson.
- Mass scale in the fundamental sector emerges solely from soft-SUSY breaking interactions.
- SUSY stabilizes the $M_{GUT} \gg M^{(1)}_{soft}$ hierarchy and "disappears" well above TeV-scale.

All symmetry breaking scales (including the elecro-weak) except v_{GUT} are controlled by SSB parameters $\Rightarrow$ No μ-problem!

From low-scale T-GUT to low-scale SMEFT: Why 3HDM?

After Q-GUT breaking the SUSY-LRF theory reads

Introduce soft SUSY breaking sector!

$$W = \varepsilon_{ijk}\left\{y_{1-3}\boldsymbol{\Phi}^i\boldsymbol{D}_{\mathrm{L}}{}^j\boldsymbol{D}_{\mathrm{R}}{}^k + y_{4-6}(\boldsymbol{H}^i)^L{}_R(\boldsymbol{q}_{\mathrm{L}}{}^j)_L(\boldsymbol{q}_{\mathrm{R}}{}^k)^R \right.$$
$$\left. + y_{7-9}(\boldsymbol{E}_{\mathrm{L}}{}^i)^L(\boldsymbol{q}_{\mathrm{L}}{}^j)_L\boldsymbol{D}_{\mathrm{R}}{}^k + y_{10-12}(\boldsymbol{E}_{\mathrm{R}}{}^i)_R\boldsymbol{D}_{\mathrm{L}}{}^j(\boldsymbol{q}_{\mathrm{R}}{}^k)^R\right\}\ .$$

The simplest scenario that automatically provides CKM mixing with Cabibbo form.

$$\langle L^1\rangle \sim \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \qquad \langle L^2\rangle \sim \frac{1}{\sqrt{2}}\begin{pmatrix} v_2 & 0 & 0 \\ 0 & v_3 & 0 \\ 0 & 0 & f \end{pmatrix} \qquad \langle L^3\rangle \sim \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p \end{pmatrix}.$$

$$M_{\mathrm{EW}} \sim v_{1,2,3} \ll \omega \lesssim f \lesssim p \lll M_{\mathrm{GUT}}, \qquad (p, f, \omega) \sim M_{\mathrm{soft}}^{(1,2,3)}$$

Classical approach: (y_{1-12} matched to λ_{27})

Cabibbo form readily at tree-level

$$m^2_{\mathrm{c,t}} = \frac{1}{2}\lambda_{27}^2({v_1}^2 + {v_2}^2), \quad m^2_{\mathrm{b}} = 3m^2_{\mathrm{s}} = \frac{1}{2}\lambda_{27}^2 v_3^2, \quad m^2_{\mathrm{u,d}} = 0, \quad \tan\theta_{\mathrm{C}} = \frac{v_1}{v_2}$$

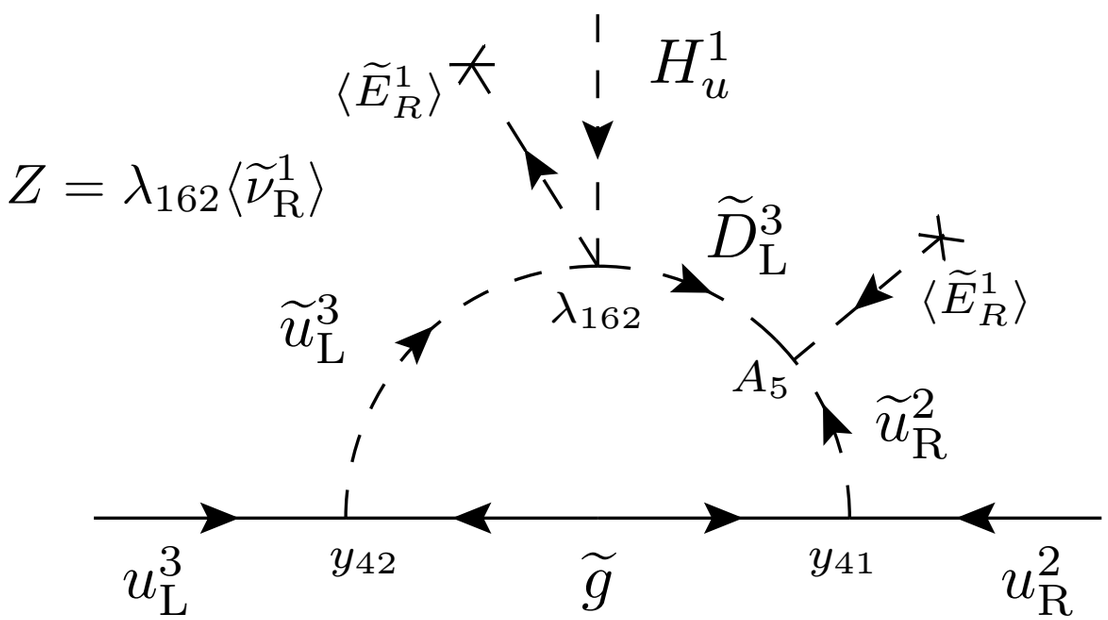
$$m^2_{\mathrm{B}} = \frac{1}{2}\lambda_{27}^2(2p^2 + f^2 + \omega^2), \quad m^2_{\mathrm{S}} = \frac{1}{2}\lambda_{27}^2(p^2 + f^2), \quad m^2_{\mathrm{D}} = \frac{1}{2}\lambda_{27}^2\omega^2.$$

$$V_{\mathrm{CKM}} \sim \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \text{Perturbations}$$

BUT!
Unrealistic quark spectrum: RG+threshold corrections are needed!

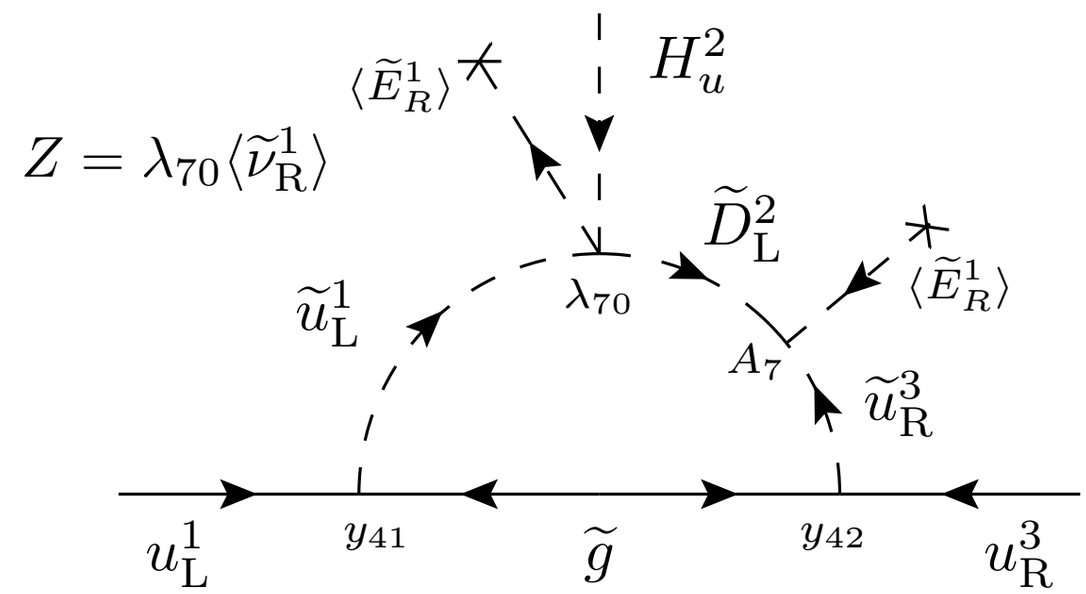
The impact of quantum effects on fermion spectra

Top mass correction



$$\{m_1, m_2, m_3\} = \{m_{\widetilde{u}^3_{\rm L}}, m_{\widetilde{u}^2_{\rm R}\widetilde{D}^3_{\rm L}}, m_{\widetilde{g}}\}$$

Charm mass correction



$$\{m_1, m_2, m_3\} = \{m_{\widetilde{u}^1_{\rm L}}, m_{\widetilde{u}^3_{\rm R}\widetilde{D}^1_{\rm L}}, m_{\widetilde{g}}\}$$

One-loop top or charm mass correction:

$$\frac{2i\alpha_{\rm S} Z m_3}{3\pi\left(m_2^2 - m_3^2\right)}\left(\frac{m_3^2\log\left(\frac{m_1^2}{m_3^2}\right)}{m_3^2 - m_1^2} - \frac{m_2^2\log\left(\frac{m_1^2}{m_2^2}\right)}{m_2^2 - m_1^2}\right)$$

Δ^1_{32}	λ_{162}	$m_{\tilde{g}}$	$m_{\tilde{u}^3_{\rm L}}$	$m_{\tilde{u}^2_{\rm R},\, D^3_{\rm L}}$
1	10^{-2}	10^8	10^3	10^3
1	10^{-2}	10^6	10^6	10^6

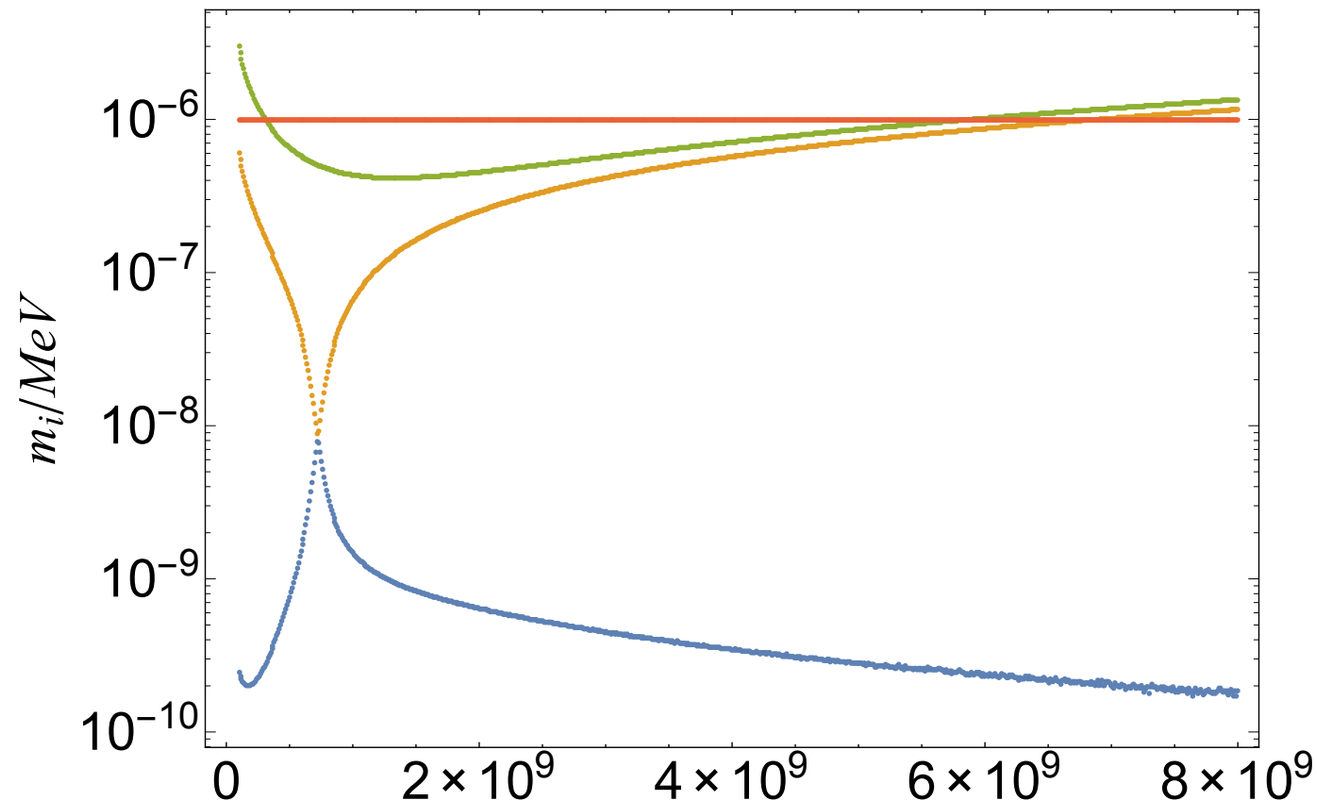
Δ^2_{13}	λ_{70}	$m_{\tilde{g}}$	$m_{\tilde{u}^1_{\rm L}}$	$m_{\tilde{u}^3_{\rm R},\, D^2_{\rm L}}$
10^{-5}	10^{-2}	10^8	10^{10}	10^3
10^{-6}	10^{-2}	10^6	10^{10}	10^6

A sufficient charm-top mass splitting at expense of a large hierarchy between squark masses!

Neutrino sector

$$\mathcal{L}_{\mathrm{N}} = \Psi_N \mathcal{M}^N \Psi_N^\top \qquad \Psi_N = \{\phi^1\, \phi^2\, \phi^3\, \nu_{\mathrm{R}}^1\, \nu_{\mathrm{R}}^2\, \nu_{\mathrm{R}}^3\, \nu_{\mathrm{L}}^1\, \nu_{\mathrm{L}}^2\, \nu_{\mathrm{L}}^3\, \widetilde{H}_{\mathrm{d}}^{1\,0}\, \widetilde{H}_{\mathrm{d}}^{2\,0}\, \widetilde{H}_{\mathrm{d}}^{3\,0}\, \widetilde{H}_{\mathrm{u}}^{1\,0}\, \widetilde{H}_{\mathrm{u}}^{2\,0}\, \widetilde{H}_{\mathrm{u}}^{3\,0}\}$$

$$\mathcal{M}^N = \begin{pmatrix}
0 & 0 & 0 & 0 & \otimes & \otimes & 0 & 0 & 0 & 0 & \vee & \vee & 0 & \vee & \vee \\
0 & \times & \times & \times & 0 & 0 & 0 & \vee & \vee & \vee & \vee & \vee & \vee & \vee & \vee \\
0 & \times & \cup & \times & 0 & 0 & 0 & \vee & \vee & \vee & \vee & \vee & \vee & \vee & \vee \\
0 & \times & \times & \times & 0 & 0 & 0 & \vee & \vee & \vee & \vee & \vee & \vee & \vee & \vee \\
\otimes & 0 & 0 & 0 & 0 & 0 & \vee & \vee & \vee & \vee & 0 & 0 & \vee & 0 & 0 \\
\otimes & 0 & 0 & 0 & 0 & 0 & \vee & \vee & \vee & \vee & 0 & 0 & \vee & 0 & 0 \\
0 & 0 & 0 & 0 & \vee & \vee & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \vee & \vee & \vee & \vee & \vee & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \otimes & \otimes \\
0 & \vee & \vee & \vee & \vee & \vee & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \otimes & \otimes \\
0 & \vee & \vee & \vee & \vee & \vee & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \otimes & \otimes \\
\vee & \vee & \vee & \vee & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \otimes & 0 & 0 \\
\vee & \vee & \vee & \vee & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \otimes & 0 & 0 \\
0 & \vee & \vee & \vee & \vee & \vee & 0 & 0 & 0 & 0 & \otimes & \otimes & 0 & 0 & 0 \\
\vee & \vee & \vee & \vee & 0 & 0 & 0 & \otimes & \otimes & \otimes & 0 & 0 & 0 & 0 & 0 \\
\vee & \vee & \vee & \vee & 0 & 0 & 0 & \otimes & \otimes & \otimes & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$



$m_{N_{1,2}} \simeq 11.80$ TeV $\quad m_{N_3} \simeq 4.654$ GeV $\quad m_{N_{4,5}} \simeq 840.0$ GeV $\quad m_{N_6} \simeq 683.9$ GeV

$m_{N_{7,8}} \simeq 303.7$ GeV $\quad m_{N_9} \simeq 151.3$ GeV $\quad m_{N_{10,11}} \simeq 137.9$ MeV $\quad m_{N_{12}} \simeq 109.1$ MeV

$m_3 \simeq 0.4260$ eV $\quad m_2 \simeq 0.07885$ eV $\quad m_1 \simeq 0.001302$ eV GREAT! **PMNS???**

Morais, Pasechnik, Porod, Varzielas 2019 (in progress)

Example II: SU(2)xU(1)-flavoured T-GUT

$$E_8 \xrightarrow{M_8} E_6 \times SU(2)_F \times U(1)_F$$
$$\xrightarrow{M_6} [SU(3)]^3 \times SU(2)_F \times U(1)_F$$

Morais, Pasechnik, Porod 2019 (in progress)

$$\mathbf{27} \supset (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) \equiv \boldsymbol{L} \oplus \boldsymbol{Q}_L \oplus \boldsymbol{Q}_R$$

$$W_3 = \varepsilon_{ij} \left[\lambda_1 \boldsymbol{L}^i \cdot \boldsymbol{Q}_L^3 \cdot \boldsymbol{Q}_R^j - \lambda_2 \left(\boldsymbol{L}^i \cdot \boldsymbol{Q}_L^j \cdot \boldsymbol{Q}_R^3 - \boldsymbol{L}^3 \cdot \boldsymbol{Q}_L^i \cdot \boldsymbol{Q}_R^j \right) \right]$$

$i, j = 1, 2$ are $SU(2)_F$ doublet indices

$$\langle \tilde{\Delta}_L^8 \rangle = \langle \tilde{\Delta}_R^8 \rangle \neq 0$$

$$SU(3)_L \times SU(3)_R \xrightarrow{v_{L,R}} SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$$

$$SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \xrightarrow{f, p, \omega, s_{1,2,3}} SU(2)_L \times U(1)_Y$$

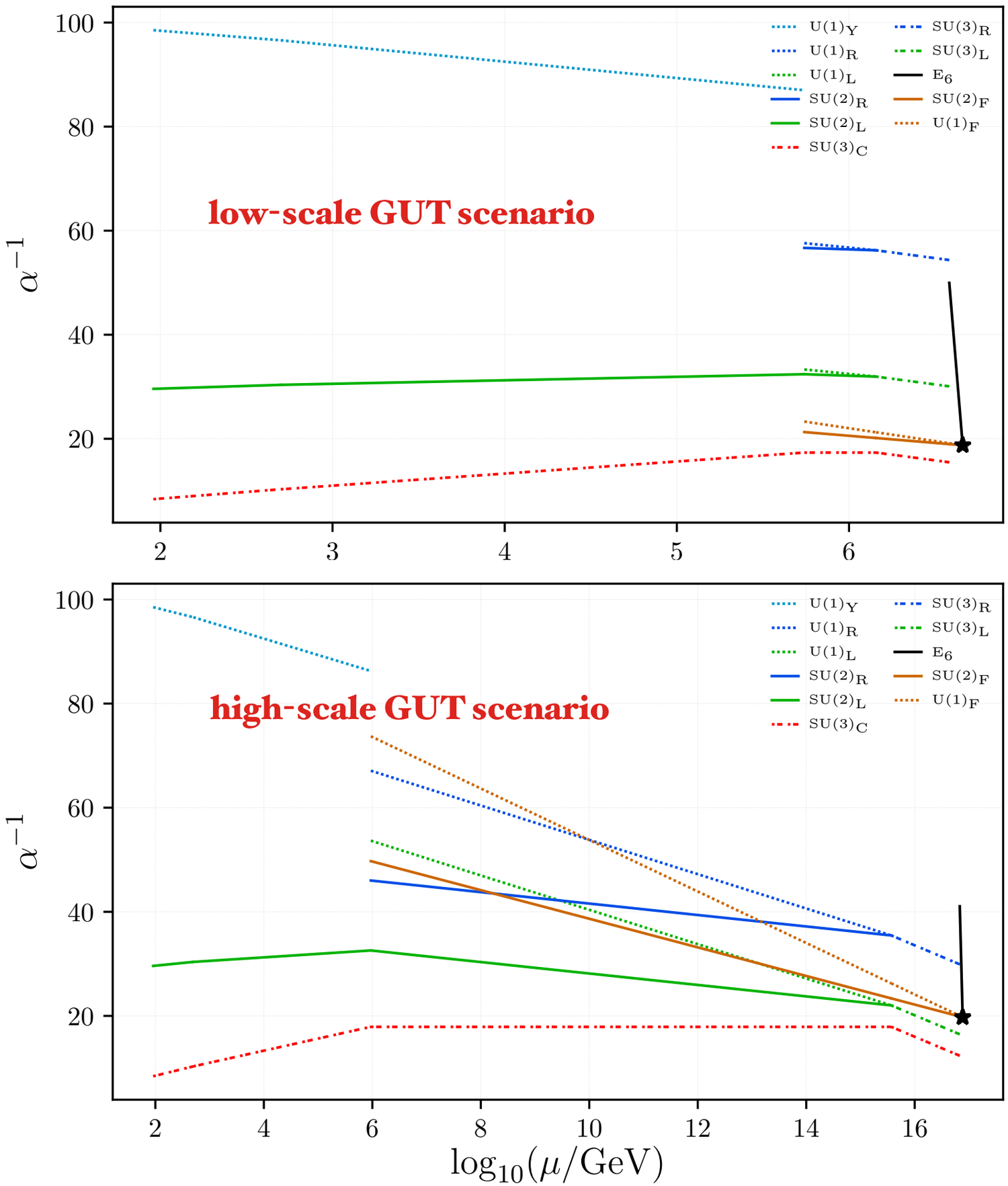
$$\langle \tilde{L}^1 \rangle = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & d_1 & e_1 \\ 0 & \omega & s_1 \end{pmatrix}, \quad \langle \tilde{L}^2 \rangle = \begin{pmatrix} u_2 & 0 & 0 \\ 0 & d_2 & e_2 \\ 0 & s_2 & f \end{pmatrix}, \quad \langle \tilde{L}^3 \rangle = \begin{pmatrix} u_3 & 0 & 0 \\ 0 & d_3 & e_3 \\ 0 & s_3 & p \end{pmatrix}$$

$$\frac{m_t}{m_c} \sim \mathcal{O}(100) \qquad \frac{m_b}{m_s} \sim \mathcal{O}(50) \qquad \frac{\lambda_1}{\lambda_2} \sim 10^2$$

VEVs	$\tan\theta_C$
(u_1, u_2, d_1)	$-u_2/u_1$
(u_1, u_2, d_2)	u_1/u_2
(u_1, d_1, d_2)	d_2/d_1
(u_2, d_1, d_2)	$-d_1/d_2$

For more details, see presentation by Antonio Morais

Towards a consistent 3HDM: energy scales of the SHUT theory



vast pheno consequences!

Classification of low-energy 3HDM EFTs

3HDM	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_T$
q_{Li}	**3**	**2**	$1/3$	$(3,-1,-1)$
u_{Ri}	**3**	**1**	$4/3$	$(0,4,4)$
d_{Ri}	**3**	**1**	$-2/3$	$(-6,2,-2)$
ℓ_{Li}	**1**	**2**	-1	$(3,-1,-1)$
e_{Li}	**1**	**1**	-2	$(-6,-2,-2)$
H_i	**1**	**2**	1	(x_1,x_2,x_3)

extra	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_T$
$D_{L,R}$	**3**	**1**	$-2/3$	-2
E_{Li}	**1**	**2**	-1	$(-1,-5,-5)$
E_{Ri}	**1**	**2**	-1	$(-5,-1,-1)$
ν_{Rk}	**1**	**1**	0	$(\underbrace{0,\cdots,0}_{17\text{ entries}}\vert\underbrace{4,\cdots,4}_{6\text{ entries}}\vert\underbrace{-4,\cdots,-4}_{9\text{ entries}})$
T_l	**1**	**3**	0	$(0,0)$
S	**1**	**1**	0	-4

$$(x_1,x_2,x_3)=\begin{cases}(5,1,5) & \text{Model I-A}\\ (1,5,1) & \text{Model I-B}\\ (5,1,-3) & \text{Model II}\end{cases}$$

- $U(1)_T$ must be broken by $\langle S\rangle$ providing a TeV scale Z' boson.
- **New exotic states offer rich phenomenology/cosmology:** EW-precision, flavour, collider, neutrino, dark-matter, baryogenesis, primordial GW.

work in progress..

Outlook

- 3HDMs are well-motivated in SM extensions via family symmetries and offer new opportunities for phenomenology

- One of the most promising implications of 3HDMs is a capability to explain fermion mass/mixing hierarchies in quark, lepton and neutrino spectra

- Several promising (but very specific!) 3HDMs have been found as low-energy (minimal) EFTs of the high-scale SHUT theory

- Continuous exploration and verification of 3HDMs in the search for the most optimal candidate for BSM extension is ongoing

Stay tuned!