Phenomenology of family-nonuniversal 3HDMs: an overview

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Motivation for extended Higgs sectors

Despite of remarkable consistency of the SM with to-date observations, it remains remarkably unsatisfactory.

We are looking for explanations why/where the SM features as they are/originate from such as neutrino sector features mass/mixing hierarchies origin of CPV Dark Matter baryon asymmetry etc

Non-minimal Higgs sectors can provide natural explanation to CPV and fermion puzzles, and more..

Active model-building activity with non-minimal scalar sectors (see e.g. Ivanov, 1702.03776)
“Bottom-up” approach

- **Assume a little extra**: take the plain SM and add a bit more on top to explain a particular peculiarity of the SM (very rarely most of them).

- **Postulate**: such an extension to be anomaly-free (just like the SM itself).

- **Fit**: parameters to the observables to see if it complies with the reality.

- **Extrapolate**: such an extension to much higher energies hoping the vacuum remains stable, Higgs mass does not receive GUT-scale corrections and the gauge couplings unify (e.g. MSSM).

**Know examples consider adding a SUSY, horizontal (family) symmetries, extra Higgs and vector-like fermions**
Example I. CP4 3HDM

- CP is not uniquely defined in QFT
  Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999

\[ J : \phi_i \xrightarrow{CP} X_{ij}\phi_j^*, \quad X \in U(N), \quad J^2 = XX^* \quad J^k = \mathbb{I} \]

- k=2 is usual CP, the first non-trivial case is CP4 with 3HDs
  Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016

\[ V_0 = -m_{11}^2(1\dagger1) - m_{22}^2(2\dagger2 + 3\dagger3) + \lambda_1(1\dagger1)^2 + \lambda_2 \left[(2\dagger2)^2 + (3\dagger3)^2\right] \]
\[ + \lambda_3(1\dagger1)(2\dagger2 + 3\dagger3) + \lambda'_3(2\dagger2)(3\dagger3) + \lambda_4 \left[(1\dagger2)(2\dagger1) + (1\dagger3)(3\dagger1)\right] + \lambda'_4(2\dagger3)(3\dagger2) \]

\[ V_1 = \lambda_5(3\dagger1)(2\dagger1) + \frac{\lambda_6}{2} \left[(2\dagger1)^2 - (3\dagger1)^2\right] + \lambda_8(2\dagger3)^2 + \lambda_9(2\dagger3) \left[(2\dagger2) - (3\dagger3)\right] + \text{h.c.} \]

\[ V = V_0 + V_1 \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad J^2 = \text{diag}(1, -1, -1), \quad J^4 = \mathbb{I} \]
Flavoured CP4 3HDM

CP4 can be extended to the Yukawa sector and must be spontaneously broken leading to particular patterns in the flavour sector

Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017

\[
\psi_i \rightarrow Y_{ij} \psi_j^{\text{CP}}, \quad \psi^{\text{CP}} = \gamma^0 C \psi^T \quad -L_Y = \bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \phi_a^* + \text{h.c.}
\]

CP4 invariance: \((Y^L)^\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^*, \quad (Y^L)^\dagger \Delta_a Y^u X_{ab} = \Delta_b^*\)

\[
Y = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & e^{i\alpha} \\
0 & e^{-i\alpha} & 0
\end{pmatrix}
\]

Example:

\[
\Gamma_1 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
g_{31} & g_{31}^* & g_{33}
\end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix}
g_{11} & g_{12} & g_{13} \\
g_{21} & g_{22} & g_{23} \\
0 & 0 & 0
\end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix}
-g_{22}^* & -g_{21}^* & -g_{23}^* \\
g_{12}^* & g_{11}^* & g_{13}^* \\
0 & 0 & 0
\end{pmatrix}
\]

Tree-level FCNCs

One avoids FCNCs from h125 by imposing the scalar alignment condition:

\[
m_{11}^2 = m_{22}^2
\]
Scalar sector scan

- Stick to the scalar alignment, take $h_{125}$ to be the lightest scalar, vary nine free parameters
- Checks for the boundedness from below and perturbativity
- Check that $S, T, U$ parameters are within $3\sigma$ of experimental values

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**Numerical scan procedure**

**Procedure:**

1. **Scalar sector scan:**
   - Stick to the scalar alignment, take $h_{125}$ to be the lightest scalar, vary 9 free parameters:
     - $v_3/v_2$, $u/v_1$, and 7 others.
   - Simplified checks of boundedness from below and perturbativity (all $|\ldots| < 5; \text{the exact conditions exist}$ [Bento, Haber, Romao, Silva, 2017]).
   - Check that $S, T, U$ parameters are within 3 of experimental values.

2. **Yukawa sector scan**
   - Fit all quark masses, mixing, and CPV phase (easy);
   - Add $K$ and $B$ oscillation parameters $|K|, m_K, m_B$ via expressions from [Buras et al, 2013] (tree-level contributions from neutral Higgses only).
Flavour CP4 3HDM: flavour observables

**Yukawa sector scan**

- Fit all quark masses, mixing, and CPV phase
- Add K and B oscillation parameters $|\epsilon_K|$, $\Delta m_K$, $\Delta m_{B_d}$, $\Delta m_{B_s}$ from Buras et al, 2013 (tree-level contributions from neutral Higgses only)

![Graph showing $\Delta m_K$ vs. $|\epsilon_K|$](image1)

![Graph showing $\Delta m_d$ vs. $\Delta m_s$](image2)

Typical points have light Higgses ($< 150$ GeV); a few points have moderately heavy Higgses.
Example II. \( U(1) \times U(1) \) 3HDM

- The most constraining realisable Abelian symmetry of 3HDM
  Keus, King, Moretti 2014; Ivanov, Keus, Vdovin, 2012

- Promote this symmetry to be a family symmetry of the fermion sector
  Camargo-Molina, Mandal, Pasechnik, Wessen, 2018

softly broken \( U(1)_x \times U(1)_z \)

\[
V_0 = -\sum_{i=1}^{3} \mu_i^2 |H_i|^2 + \sum_{i,j=1}^{3} \left( \frac{\lambda_{ij}}{2} |H_i|^2 |H_j|^2 + \frac{\lambda'_{ij}}{2} |H_i^\dagger H_j|^2 \right) , \quad V_{\text{soft}} = \sum_{i=1}^{3} \frac{1}{2} (m_{ij}^2 H_i^\dagger H_j + c.c)
\]

All the parameters can be taken real!

\[
\lambda_{ij} = \lambda_{ji} , \quad \lambda'_{ij} = \lambda'_{ji} , \quad m_{ij}^2 = m_{ji}^2 , \quad \lambda'_{11} = \lambda'_{22} = \lambda'_{33} = 0 , \quad m_{11}^2 = m_{22}^2 = m_{33}^2 = 0 .
\]
Flavoured U(1) x U(1) 3HDM: Yukawa sector

\[ \mathcal{L}_{\text{Yukawa}}^q = \sum_{i,j=1}^{2} \left\{ y_{ij}^d \bar{d}_R^i H_1^\dagger Q_L^j - y_{ij}^u \bar{u}_R^i \tilde{H}_2^\dagger Q_L^j \right\} + y_b \bar{b}_R H_3^\dagger Q_L^3 - y_t \bar{t}_R \tilde{H}_3^\dagger Q_L^3 + \text{c.c.} \]

| \begin{array}{ccc} 
\text{U(1)}_Y & \text{U(1)}_X & \text{U(1)}_Z \\
H_1 & \frac{1}{2} & -1 & -\frac{2}{3} \\
H_2 & \frac{1}{2} & 1 & \frac{1}{3} \\
H_3 & \frac{1}{2} & 0 & \frac{1}{3} \\
Q_{L}^{1,2} & \frac{1}{6} & \gamma & \delta \\
Q_{L}^{3} & \frac{1}{6} & \beta & \alpha \\
u_{R}^{1,2} & \frac{2}{3} & 1 + \gamma & \frac{1}{3} + \delta \\
t_{R} & \frac{2}{3} & \beta & \frac{1}{3} + \alpha \\
d_{R}^{1,2} & -\frac{1}{3} & 1 + \gamma & \frac{2}{3} + \delta \\
b_{R} & -\frac{1}{3} & \beta & -\frac{1}{3} + \alpha \\
\end{array} \]

\[ v_3 \gg v_{1,2} \quad \text{Heavy third generation!} \]

The model is treatable fully analytically in this limit!

**Dim-6 operators:**

\[ \bar{d}_R^{1,2} \left( H_i^\dagger Q_L^3 \right) \left( H_j^\dagger H_k \right) , \quad \bar{u}_R^{1,2} \left( \tilde{H}_i^\dagger Q_L^3 \right) \left( H_j^\dagger H_k \right) \]

\[ \bar{b}_R \left( H_i^\dagger Q_{L}^{1,2} \right) \left( H_j^\dagger H_k \right) , \quad \bar{t}_R \left( \tilde{H}_i^\dagger Q_{L}^{1,2} \right) \left( H_j^\dagger H_k \right) \]

**Full CKM?**

\((\beta - \gamma, \alpha - \delta) \notin \{(-1, -1), (-1, 0), (0, 0), (1, 0), (1, 1), (2, 1)\} \)
additional mass terms in the scalar potential. The scalar potential consistent with a $U(1)$ definition of a small parameter expand around the vacuum as

$$\mu \propto X_3$$

channel, when combined with the power of a multivariate analysis, leads to good signal-to-channel, which has not been explored before in the context of heavier charged Higgs we studied collider phenomenology of the lightest charged Higgs when its mass is in the

A generic prediction of the model is that the new scalars effectively probe realistic multi-Higgs theories with the current LHC data, and so we

On the other hand, the first and second family get their masses from

This is due to the fact that the parameters in

is generated by a combination of the $U(1)$

$|v_i| > v_i > |s_i|\,\, (i = 1, 2, 3)$

$|v_i| > v_i > |s_i|\,\, (i = 1, 2, 3)$

We will often focus on the case where

$\xi \ll 1$

$\xi \ll 1$

SM-like Higgs:  

$$\mathcal{L} \ni \sum_q \frac{m_q}{v_3} \bar{q}q h_{125} + \mathcal{O}(\xi)$$

Additional scalars’ Yukawa couplings:

$$\mathcal{L} \ni \cos \theta_C \frac{\sqrt{2m_s}}{v_1} \bar{s}_R c_L H^-_a - \cos \theta_C \frac{\sqrt{2m_c}}{v_2} \bar{c}_R s_L H^+_b + \text{c.c.} + \mathcal{O}(\xi)$$

$$+ \frac{m_s}{v_1} \bar{s}sh_a - \frac{m_c}{v_2} \bar{c}ch_b + i \frac{m_s}{v_1} \bar{s}\gamma^5 s A_a - i \frac{m_c}{v_2} \bar{c}\gamma^5 c A_b + \mathcal{O}(\xi)$$

Charged Higgses are mostly produced via $c\bar{s}$ fusion!

**Main focus:**  

$c\bar{s} \rightarrow H^+ \rightarrow W^+ h_{125}$

$m_{H^\pm} > 200$ GeV
Flavoured U(1) x U(1) 3HDM: results

\[ pp \rightarrow H^\pm \rightarrow W^\pm h_{125} \rightarrow \ell^\pm + E_T + b\bar{b} \]

Typical backgrounds:

<table>
<thead>
<tr>
<th>Process</th>
<th>W + n j</th>
<th>Wbj</th>
<th>Wb\bar{b}</th>
<th>( t\bar{t} + n j )</th>
<th>t( j )</th>
<th>tb</th>
<th>tW</th>
<th>WW</th>
<th>WZ</th>
<th>Wh( h_{125} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-sec (pb)</td>
<td>1.53 \times 10^5</td>
<td>308.9</td>
<td>41.7</td>
<td>431.3</td>
<td>174.6</td>
<td>2.6</td>
<td>54.0</td>
<td>67.8</td>
<td>25.4</td>
<td>1.1</td>
</tr>
</tbody>
</table>

- Lepton: \( p_T(\ell) > 25 \text{ GeV}, \ |\eta(\ell)| < 2.5 \)
- Jet: \( p_T(J) > 25 \text{ GeV}, \ |\eta(J)| < 4.5 \)
- Missing transverse energy: \( E_T > 25 \text{ GeV} \)
- \( \Delta R \) separation: \( \Delta R(J_1, J_2) > 0.4, \Delta R(\ell, J) > 0.4 \)
**Example III. U(1) x Z₂ 3HDM**

- **U(1)xU(1) symmetry of 3HDM does not lead to CKM (only Cabibbo!)**
  
- **We can relax the family symmetry and find an analog to Branco-Grimus-Lavoura (BGL) 2HDM where smallness of (tree-level) FCNCs relates to the smallness of off-diagonal CKM**

Dipankar, Morais, Padilla, Pasechnik, 2019 (in progress)

**Symmetry transformations:**

\[
\begin{align*}
U(1) & : & Q_{L,3} & \rightarrow e^{i\alpha}Q_{L,3} & Z_2 & : & Q_{L,3} & \rightarrow -Q_{L,3} \\
u_{R,3} & \rightarrow e^{2i\alpha}u_{R,3} & u_{R,3} & \rightarrow -u_{R,3} \\
H_1 & \rightarrow e^{i\alpha}H_1 & H_1 & \rightarrow H_1 \\
\Psi_{L,i} & \rightarrow e^{i\alpha}\Psi_{L,i} & \Psi_{L,i} & \rightarrow -\Psi_{L,i} \\
H_3 & \rightarrow e^{i\alpha}H_3 & d_{R,3} & \rightarrow -d_{R,3}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Allowed textures:</th>
<th>( Y_d^d )</th>
<th>( Y_u^u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1^d )</td>
<td>( \begin{pmatrix} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix} )</td>
</tr>
<tr>
<td>( Y_2^d )</td>
<td>( \begin{pmatrix} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{pmatrix} )</td>
</tr>
</tbody>
</table>

**Allowed textures:**

\[
\begin{align*}
Y_1^d & = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
Y_1^u & = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
Y_2^d, Y_2^u & = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
Y_3^d, Y_3^u & = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{align*}
\]

<table>
<thead>
<tr>
<th>BGL-Model</th>
<th>SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charged ( H_{1,2}^\pm )</td>
<td>Neutral ( H_{2,3} )</td>
</tr>
<tr>
<td>Tree</td>
<td>Loop</td>
</tr>
<tr>
<td>( M \rightarrow \ell \overline{\nu}, M' \ell \overline{\nu} )</td>
<td>√</td>
</tr>
<tr>
<td>( M^0 \rightarrow \ell_1 \ell_2 )</td>
<td>√</td>
</tr>
<tr>
<td>( M^0 - \overline{M}^0 ) oscillation</td>
<td>√</td>
</tr>
<tr>
<td>( \overline{B} \rightarrow X_s \gamma )</td>
<td>√</td>
</tr>
<tr>
<td>EW precision</td>
<td>√</td>
</tr>
</tbody>
</table>

- Dipankar, Morais, Padilla, Pasechnik, 2019 (in progress)
Flavoured U(1) x Z\(_2\) 3HDM: some first results

Figure 10: QFV observables normalized with respect to the SM prediction. In red are shown the points that passed Higgs data constraints and EW precision tests. Shown in blue are the points that, additionally, lay within the allowed uncertainty bounds for QFV observables, according to section (4.5).
“Top-down” approach

- **Postulate**: a consistent SUSY GUT valid at high energy scales

- **Explore**: the larger the symmetry, the fewer free parameters, the higher degree of unification

- **Study**: to what extent the SM emerges as an EFT at low energies. Anomalies of the visible sector with additional gauge groups in the EFT need not necessarily cancel

- **Hope**: to explain seemingly arbitrary features of the SM (number of families, mass/mixing hierarchies, neutrino status, cancellation of gauge anomalies in the observed sectors etc)

Any promising candidates?
Why Trinification?

**Positives:**

- The model accommodates any quark and lepton masses and mixings (Sayre et al 2016)
- Naturally light neutrinos via radiative see-saw with split-SUSY (Cauet et al 2011)
- Gauge symmetry preserve baryon number, i.e. no gauge-mediated proton decay (Achiman&Stech’78; Glashow&Kang'84)
- Motivated as a low-energy version of E8xE8 heterotic string theory (Gross et al 1985)
  - GUT scale fermion masses through $L \cdot L' \cdot L''$ type operators
    - Higher dimensional operators needed (Cauet et al. 2011)

**Negatives:**

- Considerable amount of particles and many couplings involved
  - Realistic calculations cumbersome
- Unmotivated Hierarchy between the trinification and the EW breaking scales (common to most GUTs)

Trinification-based models were left as the least developed GUT scenarios
Example I: SU(3)-flavoured T-GUT

Our proposal: Extend the SUSY trinification model (Georgi, Glashow and De Rujula 1984) with a local family \( \text{SU}(3)_F \) symmetry


\([\text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R] \rtimes \mathbb{Z}_3 \times \text{SU}(3)_F\]

We refer to the model as Supersymmetric Higgs-Unified Trinification

- Use the minimal field content:

\[
\begin{pmatrix}
H_u^0 & H_d^- & e_L \\
H_u^+ & H_d^0 & \nu_L \\
e_R & \nu_R & \phi
\end{pmatrix}^i,
\]

\[(1, 3, \overline{3}, 3) = (l^i)^l_r = (3, \overline{3}, 1, 3) = (Q_i^l)^x_l = \begin{pmatrix} u^x_L & d^x_L & D^x_L \end{pmatrix}^i,
\]

\[(\overline{3}, 1, 3, 3) = (Q_i^r)^x_r = \begin{pmatrix} u^c_R & d^c_R & D^c_R \end{pmatrix}^\dagger_i.\]

- Higgs and leptons unified in \( L \) due to SUSY 

- \( W_1 = \lambda_{27} \varepsilon_{ijk} (Q_L^i)^x_l (Q_R^i)^r_x (l^k)^l_r \)

  > One family of quarks and all leptons massless at tree-level \(\rightarrow\) radiatively generated,

  > Exact Yukawa unification for all three families.

- \( \text{SU}(3)_F \) also fits neatly into an \( E_8 \subset E_6 \times \text{SU}(3)_F \) embedding

\[
\text{SU}(3)_F \times E_6 \\
248 = (8, 1) \oplus (1, 78) \oplus (3, 27) \oplus (\overline{3}, 27) \Rightarrow \text{No gauge anomalies.}
\]
**On the top-bottom path down to a SM-like theory**

\[ \Delta_A = \begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \\ \Delta^4 \\ \Delta^5 \\ \Delta^6 \\ \Delta^7 \\ \Delta^8 \end{pmatrix}_{A=L,R,C,F} \]

**Q-GUT:**
\[ [\text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R] \times \text{SU}(3)_F \]

**Region - I**
\[ \langle \hat{\Delta}_{L,R,F}^8 \rangle \sim M_{\text{GUT}} \]

**SUSY-LRF:**
\[ \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(2)_F \times \text{U}(1)_L \times \text{U}(1)_R \times \text{U}(1)_F \]

**Region - II**
\[ \langle \phi^3 \rangle \sim M^{(1)}_{\text{soft}} \ll M_{\text{GUT}} \]
\[ T_{L+R} = T^8_L + T^8_R, \quad T_S = T^8_L - T^8_R - 2T_F \]

**non-SUSY-LRF:**
\[ \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(2)_F \times \text{U}(1)_{L+R} \times \text{U}(1)_S \]

**Region - III**
\[ \langle \phi^2 \rangle \sim M^{(2)}_{\text{soft}} \ll M^{(1)}_{\text{soft}} \]
\[ T_V = T^3_F - \frac{1}{2\sqrt{2}} T_S \]

**non-SUSY-LR:**
\[ \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{L+R} \times \text{U}(1)_V \]

**Region - IV**
\[ \langle \sqrt{3} R \rangle \sim M^{(3)}_{\text{soft}} \ll M^{(2)}_{\text{soft}} \]
\[ T_Y = \sqrt{3} T^3_R + T_{L+R}, \quad T_T = T_V - \frac{3}{2} T^3_R \]

**3HDM EFTs:**
\[ \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_T \]

**Region - V**
\[ \langle S \rangle \sim m_{\nu^c} \ll M^{(3)}_{\text{soft}} \]

**3HDM EFTs:**
\[ \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \]

---

**Natural hierarchy between GUT and EW scales!**

- U(1)_T provides family non-universal Z' boson.
- Mass scale in the fundamental sector emerges solely from soft-SUSY breaking interactions.
- SUSY stabilizes the \( M_{\text{GUT}} \gg M^{(1)}_{\text{soft}} \) hierarchy and "disappears" well above TeV-scale.

All symmetry breaking scales (including the electro-weak) except \( v_{\text{GUT}} \) are controlled by SSB parameters ⇒ No \( \mu \)-problem!
The simplest scenario that automatically provides CKM mixing with Cabibbo form.

\[
W = \varepsilon_{ijk} \left\{ y_{1-3} \Phi^i D_L^j D_R^k + y_{4-6} (H^i)^R_L (q_L^j) (q_R^k) + y_{7-9} (E_L^i)^R_L (q_L^j) + y_{10-12} (E_R^i)^R_L (q_R^k) \right\}.
\]

Introduce soft SUSY breaking sector!

The simplest scenario that automatically provides CKM mixing with Cabibbo form.

\[
\begin{align*}
\langle L^1 \rangle & \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \omega \end{pmatrix}, &
\langle L^2 \rangle & \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & v_3 & 0 \\ 0 & 0 & f \end{pmatrix}, &
\langle L^3 \rangle & \sim \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p \end{pmatrix}.
\end{align*}
\]

\[
M_{EW} \sim v_{1,2,3} \ll \omega \lesssim f \lesssim p \ll M_{GUT}, \quad (p, f, \omega) \sim M_{\text{soft}}^{(1,2,3)}
\]

Classical approach: \((y_{1-12} \text{ matched to } \lambda_{27})\)

\[
\begin{align*}
m_{c,t}^2 &= \frac{1}{2} \lambda_{27}^2 (v_1^2 + v_2^2), & m_b^2 &= \frac{1}{2} \lambda_{27}^2 v_3^2, & m_{u,d}^2 &= 0, & \tan \theta_C &= \frac{v_1}{v_2}, \\
m_B^2 &= \frac{1}{2} \lambda_{27}^2 (2p^2 + f^2 + \omega^2), & m_s^2 &= \frac{1}{2} \lambda_{27}^2 (p^2 + f^2), & m_D^2 &= \frac{1}{2} \lambda_{27}^2 \omega^2.
\end{align*}
\]

\[
V_{\text{CKM}} \sim \begin{pmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \text{Perturbations}
\]

BUT!

Unrealistic quark spectrum: RG+threshold corrections are needed!
The impact of quantum effects on fermion spectra

Top mass correction

\[ Z = \lambda_{162} \langle \tilde{\nu}_R^1 \rangle \]

\[ \tilde{u}_L^3 \xrightarrow{\lambda_{162}} \tilde{u}_R^3 \]

\[ \langle \tilde{E}_R^1 \rangle \xrightarrow{H_u^1} \tilde{D}_L^3 \]

\[ \{ m_1, m_2, m_3 \} = \{ m_{\tilde{u}_L^3}, m_{\tilde{u}_R^2 D_L^3}, m_{\tilde{g}} \} \]

One-loop top or charm mass correction:

\[ \Delta_3^1 \]

\[ \Delta_1^2 \]

Charm mass correction

\[ Z = \lambda_{70} \langle \tilde{\nu}_R^1 \rangle \]

\[ \tilde{u}_L^1 \xrightarrow{\lambda_{70}} \tilde{u}_R^3 \]

\[ \langle \tilde{E}_R^1 \rangle \xrightarrow{H_u^2} \tilde{D}_L^2 \]

\[ \{ m_1, m_2, m_3 \} = \{ m_{\tilde{u}_L^1}, m_{\tilde{u}_R^3 D_L^1}, m_{\tilde{g}} \} \]

\[ \Delta_3^1 \]

\[ \Delta_1^2 \]

A sufficient charm-top mass splitting at expense of a large hierarchy between squark masses!
Neutrino sector

\[ \mathcal{L}_N = \Psi_N \mathcal{M}^N \Psi_N^\top \]

\[ \Psi_N = \{ \phi^1 \phi^2 \phi^3 \nu^1_R \bar{\nu}^2_R \nu^3_R \nu^1_L \bar{\nu}^2_L \nu^3_L \tilde{H}_d^{10} \tilde{H}_d^{20} \tilde{H}_u^{10} \tilde{H}_u^{20} \tilde{H}_u^{30} \} \]

\[
\mathcal{M}^N = \left( \begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array} \right)
\]

\[ m_{N_{1,2}} \approx 11.80 \text{ TeV} \quad m_{N_3} \approx 4.654 \text{ GeV} \quad m_{N_{4,5}} \approx 840.0 \text{ GeV} \quad m_{N_6} \approx 683.9 \text{ GeV} \]

\[ m_{N_{7,8}} \approx 303.7 \text{ GeV} \quad m_{N_9} \approx 151.3 \text{ GeV} \quad m_{N_{10,11}} \approx 137.9 \text{ MeV} \quad m_{N_{12}} \approx 109.1 \text{ MeV} \]

\[ m_3 \approx 0.4260 \text{ eV} \quad m_2 \approx 0.07885 \text{ eV} \quad m_1 \approx 0.001302 \text{ eV} \quad \text{GREAT! PMNS??}
\]

Morais, Pasechnik, Porod, Varzielas 2019 (in progress)
Example II: SU(2)xU(1)-flavoured T-GUT

\[ E_8 \xrightarrow{M_8} E_6 \times SU(2)_F \times U(1)_F \]
\[ \xrightarrow{M_6} [SU(3)]^3 \times SU(2)_F \times U(1)_F \]

Morais, Pasechnik, Porod 2019 (in progress)

\[ 27 \supset (3, \overline{3}, 1) \oplus (\overline{3}, 1, 3) \oplus (1, 3, \overline{3}) \equiv L \oplus Q_L \oplus Q_R \]

\[ W_3 = \epsilon_{ij} \left[ \lambda_1 L^i \cdot Q_L^j \cdot Q_R^j - \lambda_2 \left( L^i \cdot Q_L^j \cdot Q_R^3 - L^3 \cdot Q_L^i \cdot Q_R^j \right) \right] \]

\[ i, j = 1, 2 \text{ are SU}(2)_F \text{ doublet indices} \]

\[ \langle \tilde{\Delta}_L^8 \rangle = \langle \tilde{\Delta}_R^8 \rangle \neq 0 \]
\[ SU(3)_L \times SU(3)_R \xrightarrow{u_L,R} SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \]

\[ SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \xrightarrow{f,p,\omega,s_{1,2,3}} SU(2)_L \times U(1)_Y \]

\[ \langle \tilde{L}^1 \rangle = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & d_1 & e_1 \\ 0 & \omega & s_1 \end{pmatrix}, \quad \langle \tilde{L}^2 \rangle = \begin{pmatrix} u_2 & 0 & 0 \\ 0 & d_2 & e_2 \\ 0 & s_2 & f \end{pmatrix}, \quad \langle \tilde{L}^3 \rangle = \begin{pmatrix} u_3 & 0 & 0 \\ 0 & d_3 & e_3 \\ 0 & s_3 & p \end{pmatrix} \]

\[ \begin{array}{c|c}
\text{VEVs} & \tan \theta_C \\
\hline
(u_1, u_2, d_1) & -u_2/u_1 \\
(u_1, u_2, d_2) & u_1/u_2 \\
(u_1, d_1, d_2) & d_2/d_1 \\
(u_2, d_1, d_2) & -d_1/d_2 \\
\end{array} \]

For more details, see presentation by Antonio Morais
Towards a consistent 3HDM: energy scales of the SHUT theory

low-scale GUT scenario

high-scale GUT scenario

vast pheno consequences!
## Classification of low-energy 3HDM EFTs

<table>
<thead>
<tr>
<th>3HDM</th>
<th>SU(3)$_C$</th>
<th>SU(2)$_L$</th>
<th>U(1)$_Y$</th>
<th>U(1)$_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{Li}$</td>
<td>3</td>
<td>2</td>
<td>1/3</td>
<td>(3, −1, −1)</td>
</tr>
<tr>
<td>$u_{Ri}$</td>
<td>3</td>
<td>1</td>
<td>4/3</td>
<td>(0, 4, 4)</td>
</tr>
<tr>
<td>$d_{Ri}$</td>
<td>3</td>
<td>1</td>
<td>−2/3</td>
<td>(−6, 2, −2)</td>
</tr>
<tr>
<td>$\ell_{Li}$</td>
<td>1</td>
<td>2</td>
<td>−1</td>
<td>(3, −1, −1)</td>
</tr>
<tr>
<td>$e_{Li}$</td>
<td>1</td>
<td>1</td>
<td>−2</td>
<td>(−6, −2, −2)</td>
</tr>
<tr>
<td>$H_i$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>($x_1, x_2, x_3$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>extra</th>
<th>SU(3)$_C$</th>
<th>SU(2)$_L$</th>
<th>U(1)$_Y$</th>
<th>U(1)$_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{L,R}$</td>
<td>3</td>
<td>1</td>
<td>−2/3</td>
<td>−2</td>
</tr>
<tr>
<td>$E_{Li}$</td>
<td>1</td>
<td>2</td>
<td>−1</td>
<td>(−1, −5, −5)</td>
</tr>
<tr>
<td>$E_{Ri}$</td>
<td>1</td>
<td>2</td>
<td>−1</td>
<td>(−5, −1, −1)</td>
</tr>
<tr>
<td>$\nu_{Rk}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>(0, ···, 4, ···, −4)</td>
</tr>
<tr>
<td>$T_i$</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>$S$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>−4</td>
</tr>
</tbody>
</table>

$(x_1, x_2, x_3) = \begin{cases} (5, 1, 5) & \text{Model I-A} \\ (1, 5, 1) & \text{Model I-B} \\ (5, 1, −3) & \text{Model II} \end{cases}$

- **$U(1)_T$** must be broken by $\langle S \rangle$ providing a TeV scale $Z'$ boson.
- **New exotic states offer rich phenomenology/cosmology:** EW-precision, flavour, collider, neutrino, dark-matter, baryogenesis, primordial GW.

**work in progress..**
Outlook

- 3HDMs are well-motivated in SM extensions via family symmetries and offer new opportunities for phenomenology.

- One of the most promising implications of 3HDMs is a capability to explain fermion mass/mixing hierarchies in quark, lepton and neutrino spectra.

- Several promising (but very specific!) 3HDMs have been found as low-energy (minimal) EFTs of the high-scale SHUT theory.

- Continuous exploration and verification of 3HDMs in the search for the most optimal candidate for BSM extension is ongoing.

Stay tuned!