SUSY 2019,
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String Models

[Landscape of three-family supersymmetric Standard Models in F-theory]

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Outline

I. String Theory compactification:

Heterotic and String theory w/ D-branes

II. F-theory: key ingredients

gauge symmetries, matter & Yukawa couplings

[Recent development: global constraints on gauge symmetry

→ implications for F-theory ``swampland”] No time

III. Particle physics model building in F-theory:

Building blocks; First globally consistent

three-family supersymmetirc Standard Models

IV. Landscape of three family SUSY Standard Models:

Scan of globally consistent models Highlight

V. Summary/Outlook: work in progress & open issues

Apologies, Upenn-centric

I. String Theory compactification

w/ D-branes

Perturbative String Theories → consistent theory of quantum gravity

Green, Schwarz'84

Phenomenologically most promising

SM x $U(1)_{B-L}$ w/ two Higgs doublets: He, Ovrut, Pantev '04...

MSSM (w/ one Higgs doublets): Bouchard, M.C., Donagi '05

Orbifold constructions: Lebedev,Nilles,Raby,Ramos,Ratz,Vaudrevange,Wingerter'07-'10

Landscape analysis (Complete intersection Calabi-Yau's): Anderson, Gray, Lukas`09-'18...

(10^{10} solutions w/ 3-family SM's; phenomenological issues)

Heterotic E_8xE_8 string

Type IIA superstring (closed)

Type IIB superstring (closed)

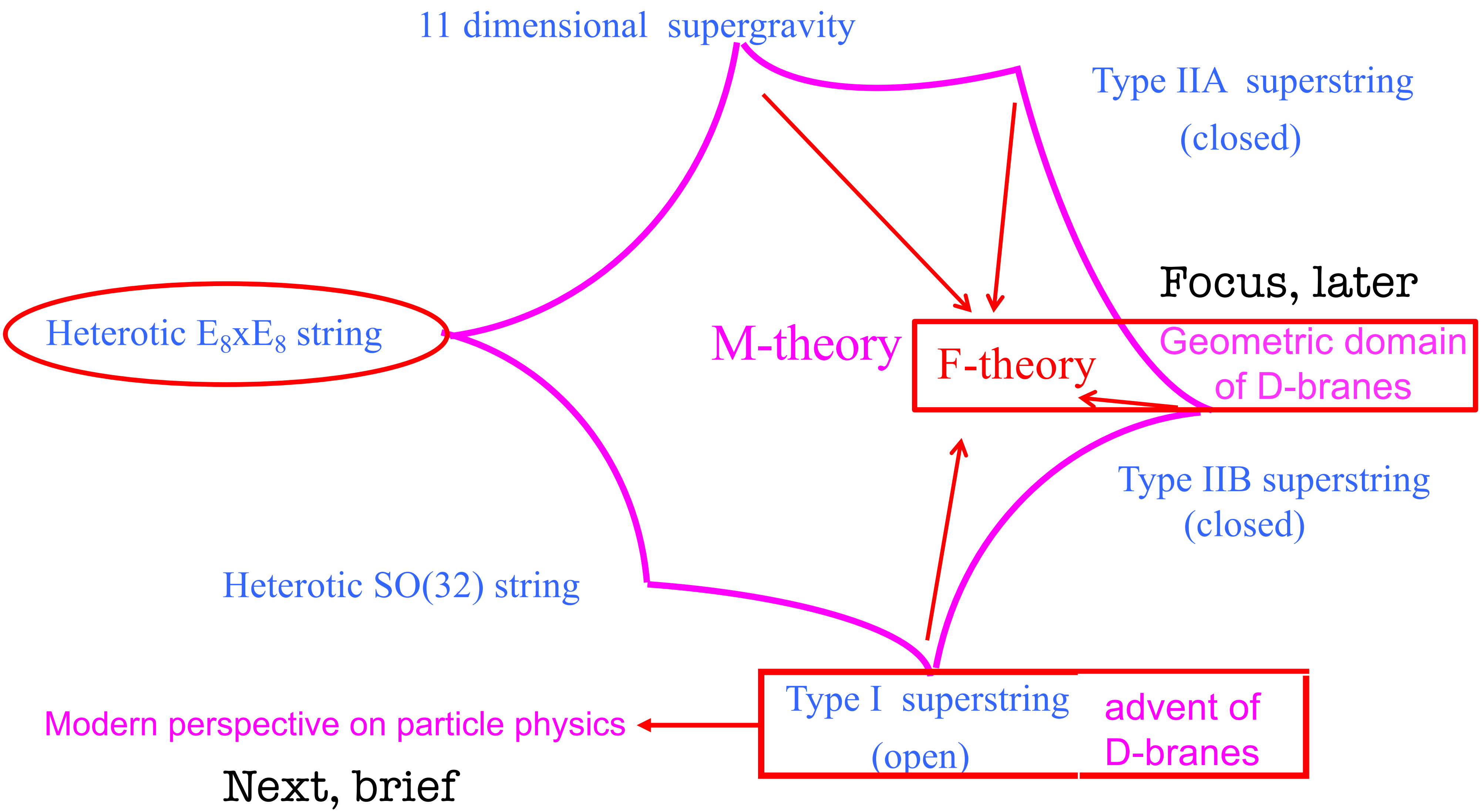
Heterotic SO(32) string

Type I superstring (open)

Perturbative String Theories → consistent theory of quantum gravity

Hull, Townsend'94
Witten'95

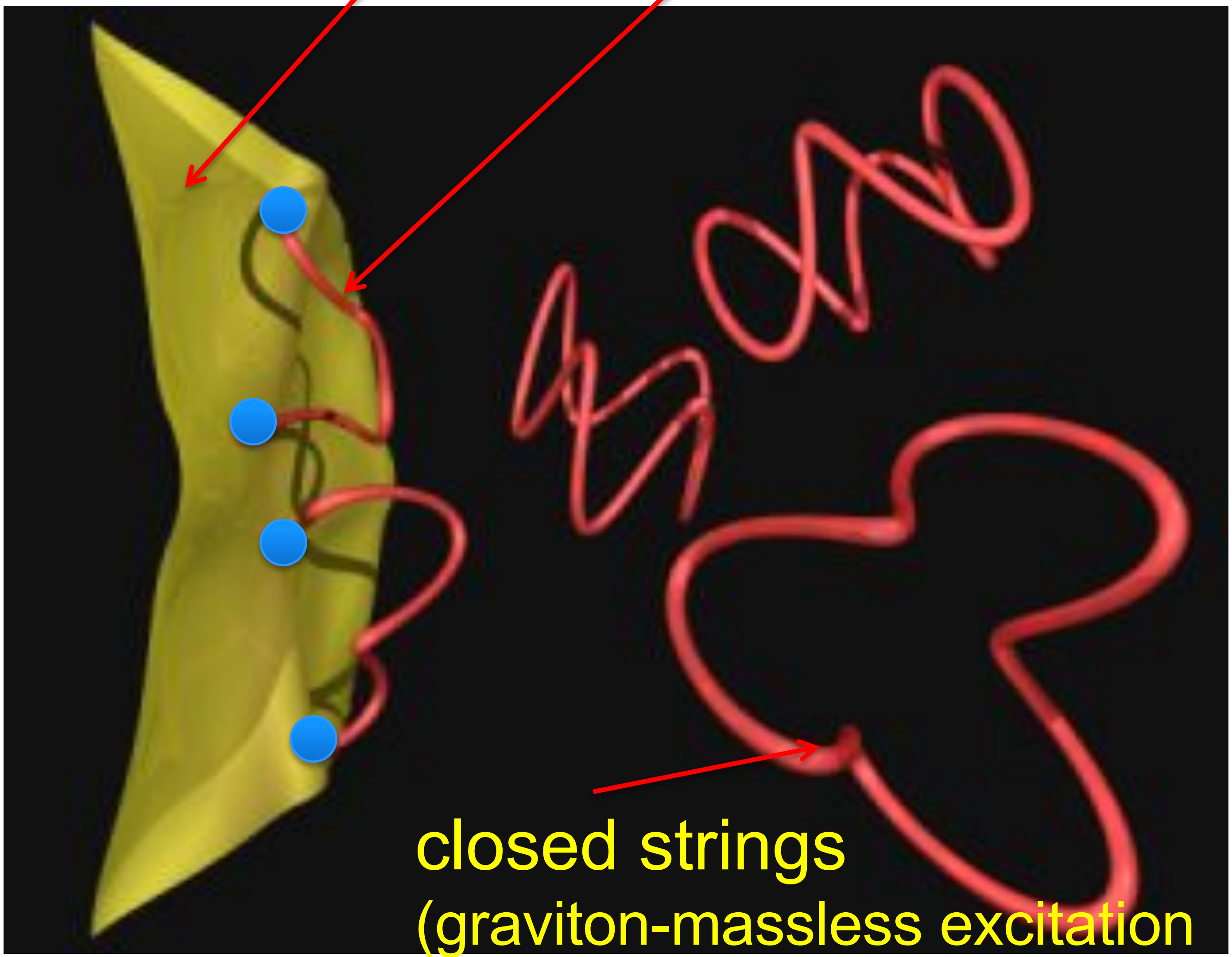
Non-perturbative Unification



Different String theories related to each other by weak-strong coupling duality

String Theory with D(irichlet)-branes

boundaries of open strings with charges at their ends

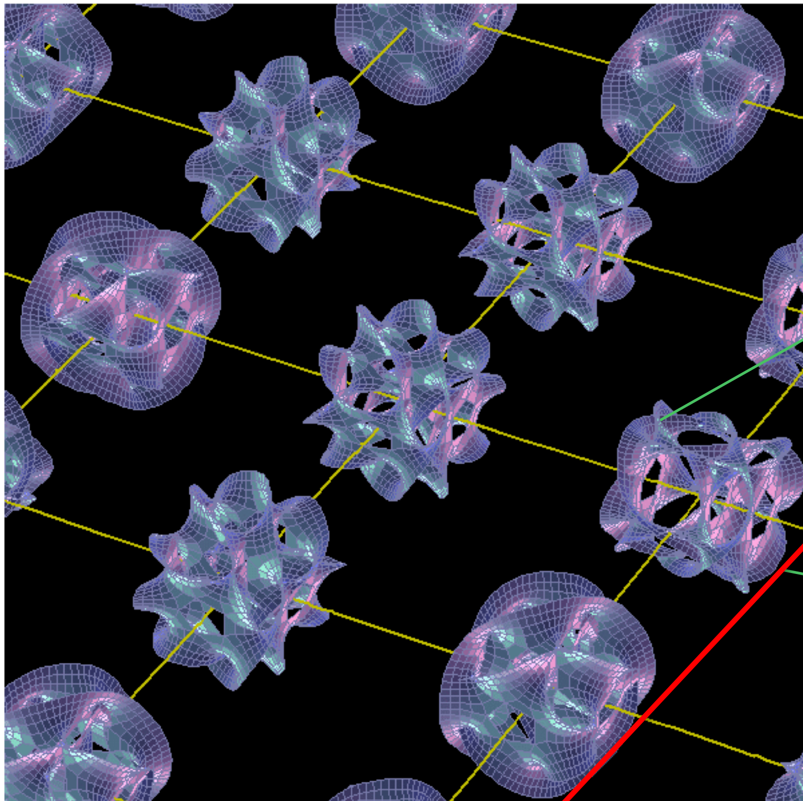


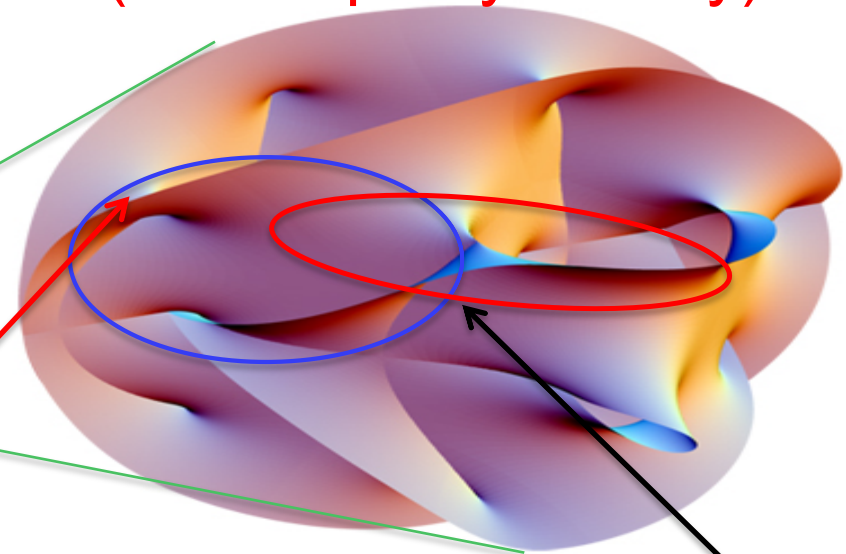
Polchinski'96

D-branes → field theory of charged excitations

String Compactification with D-branes

on compact 6 dim Calabi-Yau space
(N=1 supersymmetry)





In compact space D-branes wrap different cycles (divisors) which intersect
(gauge boson) (matter)

Implications for particle physics (charged excitations)

→ intersecting D - brane solutions of particle physics

Standard Models with intersecting D-branes

Aldazabal, Franco, Ibanez, Rabadan, Uranga 0011132;
Blumenagen, Kors, Lust 0012156...

First three-family supersymmetric Standard Model

M.C., Shiu, Uranga 0107166; 0107143 ...

Status review: Blumenhagen, M.C., Langacker , Shiu 0502005
Ann. Rev. Nucl. & Particle Sci., 55, 71 (2005)

Extensive follow-up: O(100) realistic Standard Models

M.C., Richter, Halverson `09-`10...
M.C., Halverson ,Langacker '11,'12...,'16

→ typically w/ chiral & non-chiral exotics

[Further developments: D-instantons → neutrino masses]

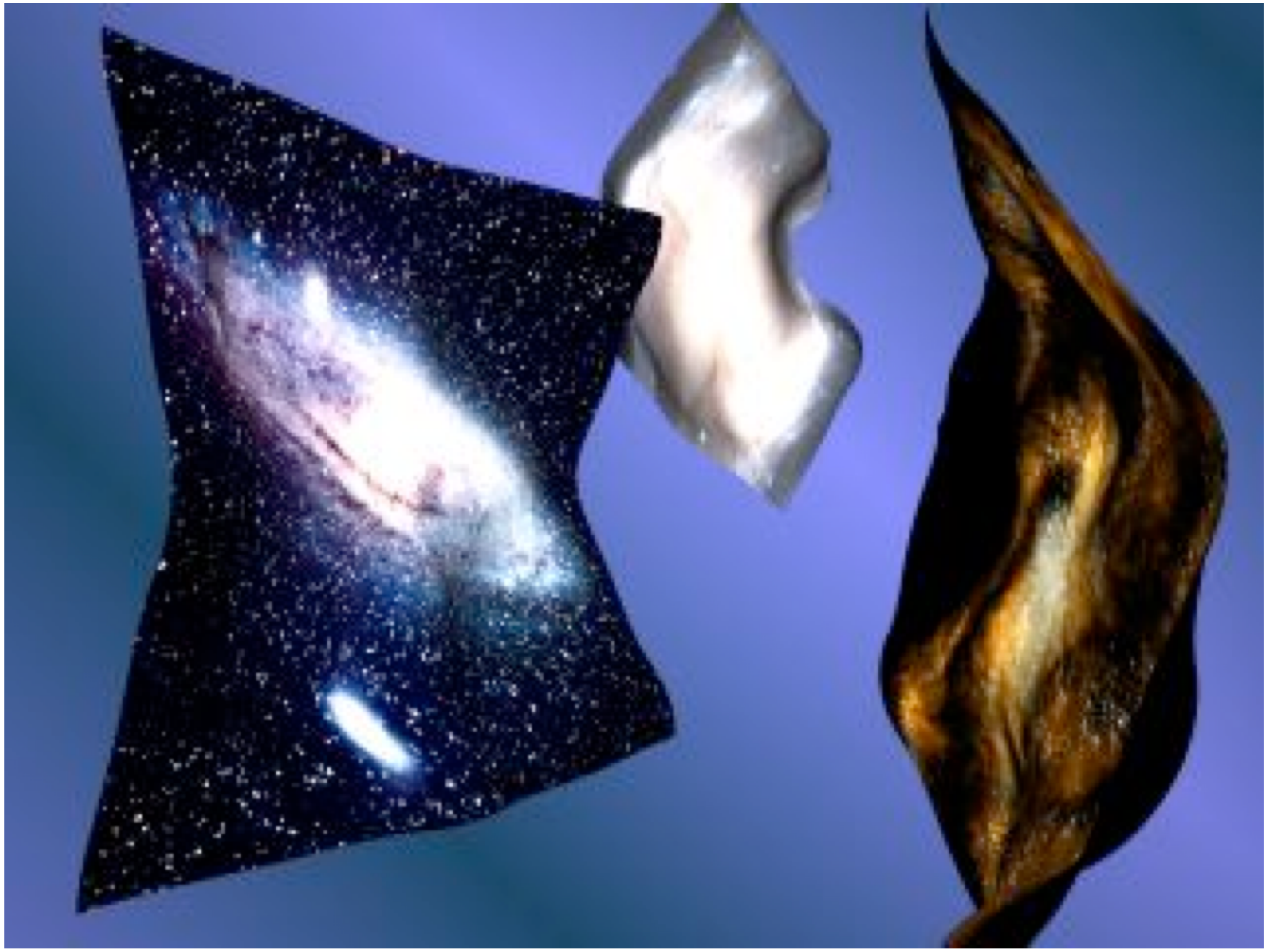
M.C.,Blumenhagen , Weigand 0609191; Ibanez & Uranga 0609213...

No time

Upshot: landscape analysis limited – on orbifolds, only
(due limitations of conformal field theory techniques)

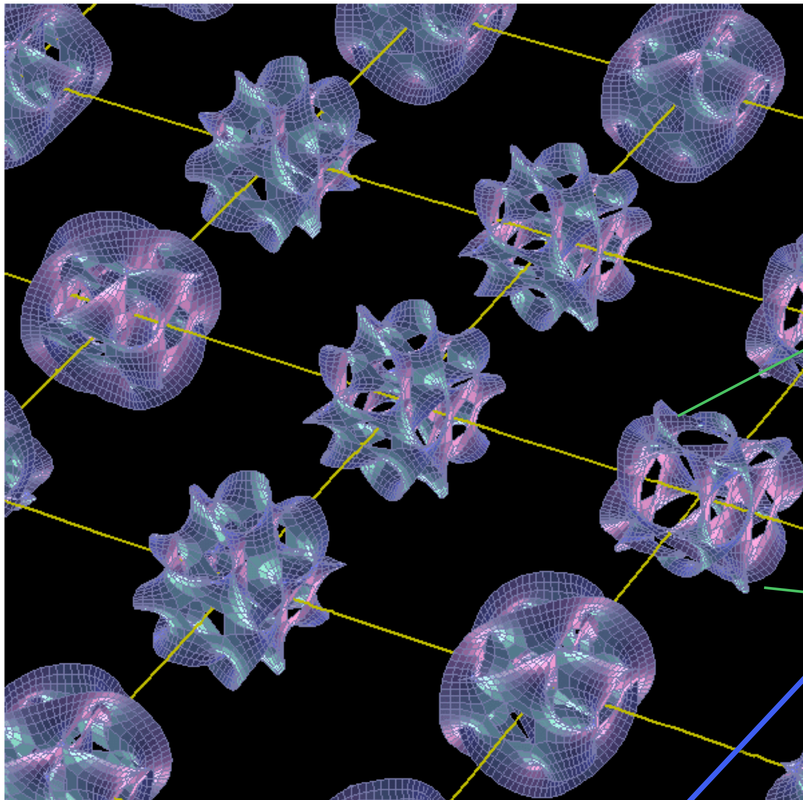
Dual D-brane interpretation:

extended massive sources, curve space-time
("back-reacted"objects at finite - large string coupling g_s)

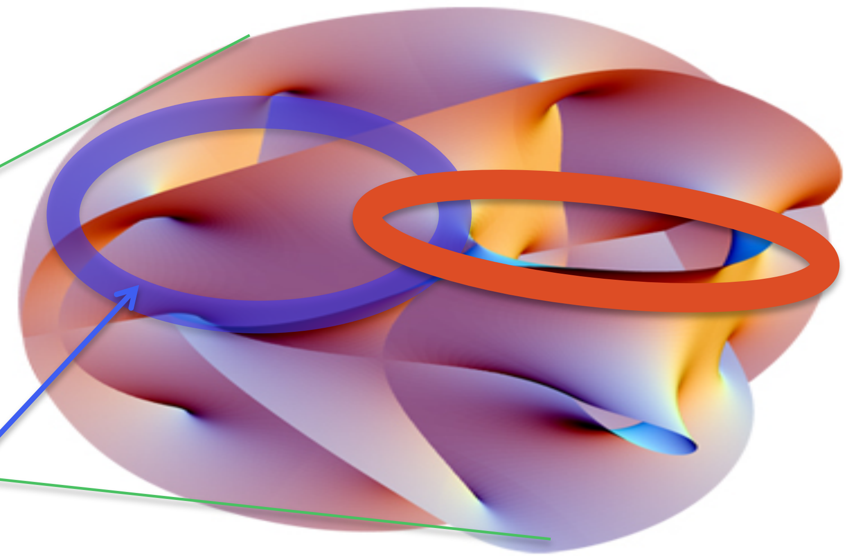


→ Implication for particle physics at finite g_s

D-branes as gravitational object



on compact 6-dim Calabi-Yau space



In compact space D-branes, wrapping divisors ``back-react'' → cause highly curved - singular space along divisors ($g_s \to \infty$)

Calabi-Yau space with backreacted D-branes:
new six dimensional space B

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F-theory

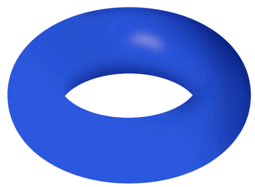
II. F-theory basic ingredients

Type IIB string perspective

F-theory compactification

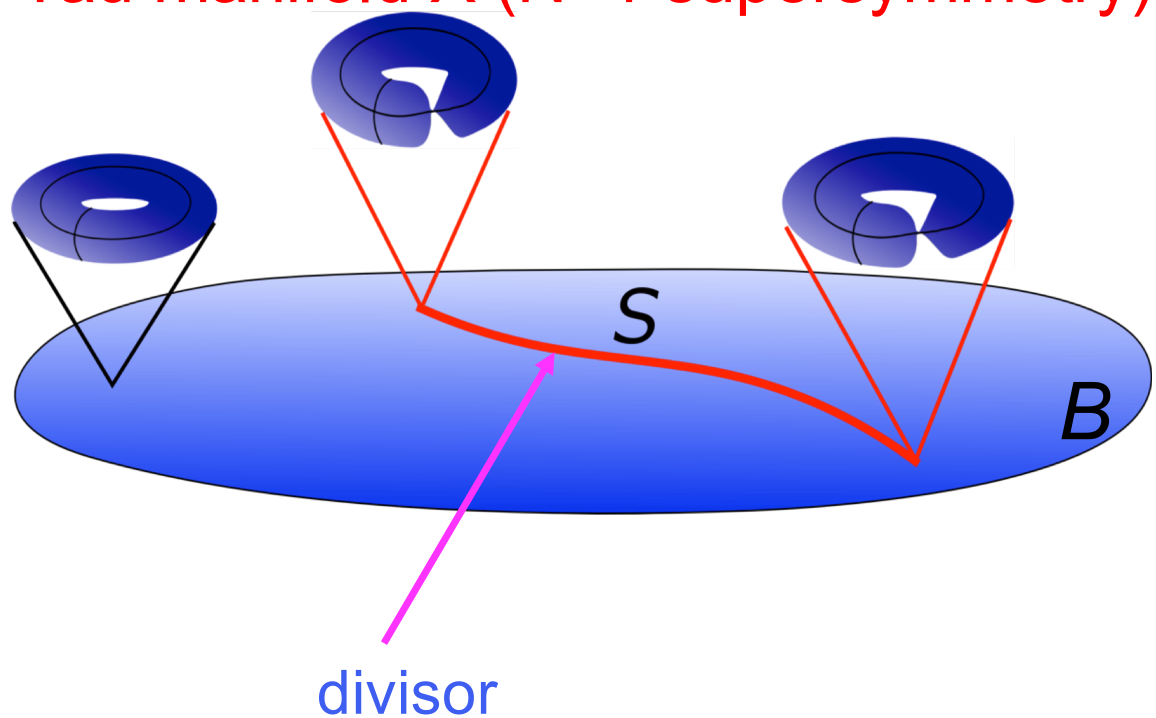
[Vafa'96], [Morrison,Vafa'96],...
c.f., review [Weigand 1806.01854]

Singular torus fibered Calabi-Yau manifold X (N=1 supersymmetry)

To B add torus: 
Modular parameter of torus
(elliptic curve)

$$\tau \equiv C_0 + i g_s^{-1}$$

(SL(2,Z) of Type IIB string)



Weierstrass normal form for torus (elliptic) fibration of *X*

$$y^2 = x^3 + f x z^4 + g z^6$$

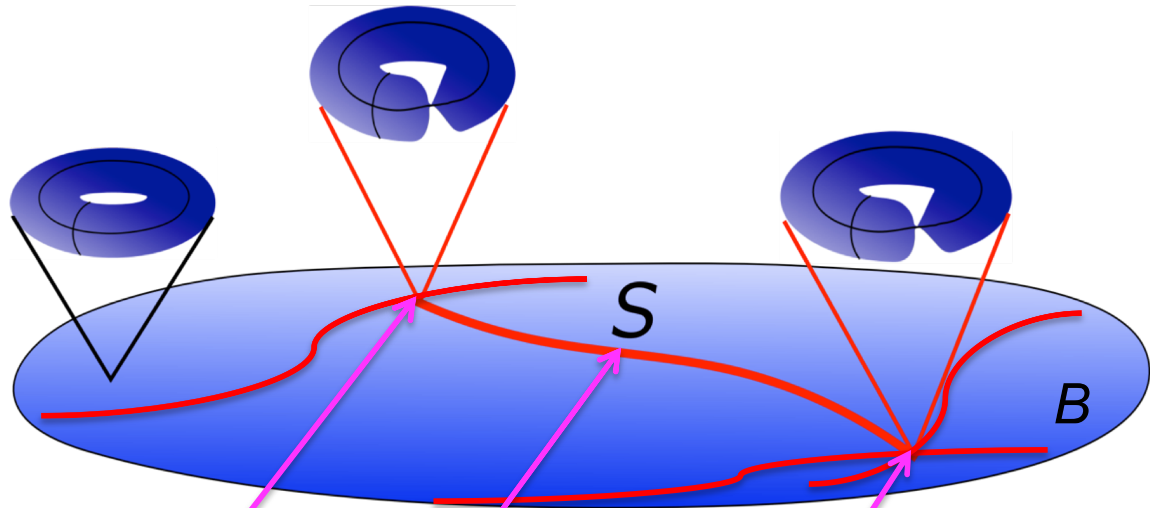
[z:x:y] - homogeneous coordinates on **P**²(1,2,3) $(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$
weighted projective space
f, g – sections on (holomorphic functions of) B

F-theory compactification

Singular torus fibered Calabi-Yau manifold X

Modular parameter of two-torus (elliptic curve)

$$\tau \equiv C_0 + i g_s^{-1}$$



Weierstrass normal form for elliptic fibration of X

$$y^2 = x^3 + fxz^4 + gz^6$$

Matter
(co-dim 2; chirality- G_4-flux)

Yukawa couplings
(co-dim 3)

divisor- singular elliptic-fibration, $g_s \rightarrow \infty$
location of (p,q) 7-branes

non-Abelian gauge symmetry
(co-dim 1) – ADE singularities

III. Particle physics in F-theory

Globally consistent models

Initial focus: F-theory with SU(5) grand unification

[10 10 5 coupling,...] [Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

Local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

Global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

Also SO(10) ...[Buchmüller, Dierigl,Oehlmann, Rühle'17]

Other particle physics models:

Standard Model building blocks (via toric techniques)

[Lin,Weigand'14] SM x U(1) [1604.04292]

First global 3-family Standard, Pati-Salam, Trinification models
[M.C., Klevers, Peña, Oehlmann, Reuter, 1503.02068]

Global 3-family Standard Model with Z_2 matter parity
[M.C., Lin, Liu, Oehlmann, 1807.01320]

Construction of elliptically fibered Calabi-Yau manifold

i. Elliptic curve E

Examples of constructions via toric techniques:

E_{F_i} as a hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$
(generalized weighted projective spaces, associated with16 reflexive polytopes F_i):

c.f., [Klevers, Pena, Oehlmann, Piragua, Reuter '14]

$$E_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau space: X_{F_i}

Impose Calabi-Yau condition:
coordinates in $\mathbb{P}_{F_i}$ and coeffs. of E_{F_i} lifted to
sections on (specific polynomial functions of) B

$$E_{F_i} \subset \mathbb{P}_{F_i} \longrightarrow X_{F_i}$$
$$\downarrow$$
$$B$$

Fibration depends only on anti-canonical divisor $\overline{\mathcal{K}}$
& two additional S_7 and S_9 divisor classes

iii. Chiral index for D=4 matter:

$$c_{mat} \longrightarrow \boxed{\mathcal{C}_{\mathbf{R}}} \longrightarrow \Sigma_{\mathbf{R}}$$

$$\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} G_4$$

a) construct G_4 flux by computing $H_V^{(2,2)}(\hat{X})$

b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

iv. Global consistency – D3 tadpole cancellation:

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

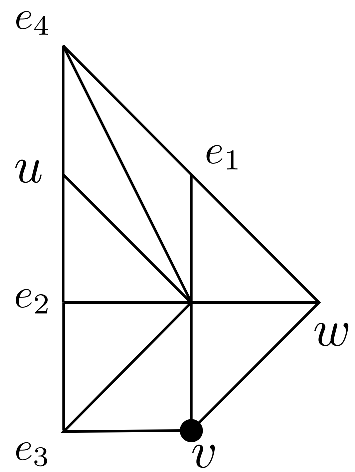
a) satisfied for integer and positive n_{D3}

b) constraint on integer valued flux G_4 $\quad G_4 + \frac{1}{2}c_2(X) \in H^4(\mathbb{Z}, \hat{X})$

Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter,1503.02068]

F$_{11}$ polytope



Elliptic curve:

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

(hypersurface constraint in $\mathbb{P}^2$ [u:v:w] with four blow-ups [e_1:e_2:e_3:e_4])

Global [geometric origin of U(1)] [M.C., Lin, 1706.08521]

Gauge Symmetry: $[\mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)}] \,/\, \mathbb{Z}_6$

Matter: $[s_3]=0$ $[s_9]=0$ $[e_4]=0$ rational section

Representation	$(\mathbf{3},\mathbf{2})_{1/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{-1}$

Compatibility with global constraint

Construct G_4 for chiral index & D3-tadpole constraint

Standard Model:

Hyperplane divisor class

$H = 4\overline{\mathcal{K}}$

Base B = $\mathbb{P}^3$ Divisors in the base:

$$\mathcal{S}_7 = n_7 H$$
$$\mathcal{S}_9 = n_9 H$$
$$n_7, n_9 \in \mathbb{Z}$$

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	(27; 16)	–	–			
6	–	(12; 81)	(21; 42)	–	–		
5	–	–	(12; 57)	(30; 8)	–	(3; 46)	
4	(42; 4)	–	(30; 32)	–	–	–	–
3	–	(21; 72)	–	–	–	(15; 30)	
2	(45; 16)	(24; 79)	(21; 66)	(24; 44)	(3; 64)		
1	–	–	–	–			
0	–	–	(12; 112)				
-1	(36; 91)	(33; 74)					
-2	–						

Tip of the Iceberg?

IV. Landscape of Standard Models

Toric analysis

[M.C., J. Halverson, L. Lin, M. Liu and J. Tian, arXiv:1903.0009]

a) Take the same toric elliptic fibration as before:

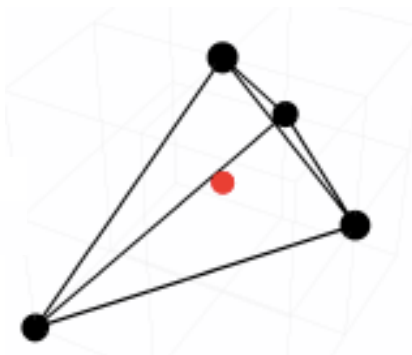
hyperplane constraint in 2D reflexive polytope F_{11}

Gauge symmetry: $$\frac{\mathbf{SU(3)\times SU(2)\times U(1)}}{\mathbb{Z}_6}$$ Global Gauge Symmetry

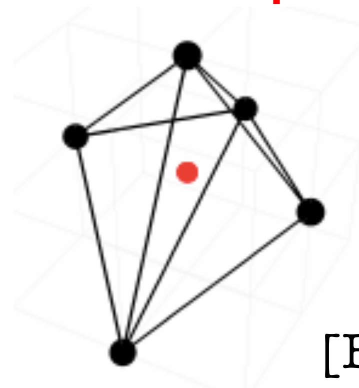
b) Take bases B, associated with 3D reflexive polytopes.

E.g.,

$\mathbb{P}^3$



$\mathbb{P}^2 \times \mathbb{P}^1$



[Batyrev; Kreuzer-Skarke]

For each reflexive polytope, different bases B are associated with different (fine-star-regular) triangulations of a chosen polytope.

[Triangulations determine intersections of divisors.]

Triangulations grow exponentially with the complexity of a polytope.

c) Specific choice of divisors: $S_{7,9} = \overline{\mathcal{K}}$

[anti-canonical divisor of the base B – fixed by the polytope]

SU (3) and SU (2) divisors S_9 and S_3 with class $\overline{\mathcal{K}} \rightarrow$

$$g_{3,2}^2 = 2/\mathrm{vol}(\overline{\mathcal{K}})$$

U(1) - (height-pairing) divisor class $5\overline{\mathcal{K}}/6 \rightarrow$

(accounting for a factor of 2 mismatch w/ Cartan generators)

$$\frac{5}{3} g_Y^2 = \frac{2}{\mathrm{vol}(\overline{\mathcal{K}})}$$

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SUSY Standard Model with gauge coupling unification

$$g_3^2 = g_2^2 = 5/3 g_Y^2$$

Geometrically connected to Pati-Salam SU(4) x SU(2) x SU(2)

[but did not find manifest SO(10) GUT].

c.f., [M.C., Klevers, Peña, Oehlmann, Reuter,1503.02068]

d) Remaining conditions:

iii. 3-families of quarks and leptons (chiral index)

iv. D3-tadpole constraints

- Construct G_4 flux (in terms of base divisors)
- Chirality, D3 tadpole and G_4 integrality expressed in terms of intersection numbers of divisors in the base B → Geometric conditions!
- In the case $S_{7,9} = \overline{\mathcal{K}}$ and 3-families it reduces to:

$$n_{\mathrm{D3}} = 12 + \frac{5}{8}\overline{\mathcal{K}}^3 - \frac{45}{2\overline{\mathcal{K}}^3} \in \mathbb{Z}_{\geq 0}$$

Depends only on a polytope and not on triangulation!

Landscape count:

$$12 + \frac{5}{8}\overline{\mathcal{K}}^3 - \frac{45}{2\overline{\mathcal{K}}^3} \in \mathbb{Z}_{\geq 0}$$

- Out of 4319 3D reflective polytopes → 708 satisfy the constraint. (many of them with large number of lattice points)

- Triangulation of polytopes can be handled combinatorially. (each corresponds to a different base B)
  It can be implemented on computer, e.g., in SageMath:
  i) for 237 polytopes w/ $<$ 15 lattice points →414310 MSSM models
  ii) for 471 polytopes w/ ≥ 15 lattice points →
  exponentially growing computation time

c.f., [Halverson, Tian, 1610.08864]

- Provide bound: counting via fine-regular triangulations of each facets → estimate on regular fine-star triangulations:

$$7.667 \times 10^{13} \lesssim N_{\rm SM}^{\rm toric} \lesssim 1.622 \times 10^{16}$$

(dominated by $\mathcal{P}_8$ polytope)

V. Summary

String theory compactification with focus on recent
F- theory advances

Particle physics models:
first global three family Standard Models

Anticipated: tip of the iceberg

Indeed, geometric advances

Landscape of globally consistent Standard Models with the exact chiral spectrum of three-families of quarks & leptons
via toric techniques > quadrillion models!

Outlook

Work in progress:

Number of massless Higgs doublets & vector exotics:

- technically difficult
- at generic points in moduli space number small/zero (MSSM excepted to be abundant)

work in progress, M.C., Bies, Lin, Liu

Yukawa Couplings

Expected to have R-parity suppressed couplings as it is geometrically connected Pati-Salam model

work in progress, M.C., Lin, Liu, Zoccarato, Zhang

Outstanding issues:

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D3-brane gauge dynamics,…moduli stabilization,… supersymmetry breaking,…

Further studies

Thank you!