SUSY phenomenology

circa 2019

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SUSY has for long been the dominant paradigm in BSM physics-and for good reason!

- Stabilizes weak scale, tames mHiggs quadratic divergences: all of them
- Generalized extension of Poincare group of spacetime symmetries
- Critical ingredient in string theory: world sheet/spacetime SUSY
- Likely low energy effective theory of Calabi-Yau compactifications
SUSY is supported by data via virtual effects

- measured gauge couplings unify under MSSM RGEs
- $m(\text{top})$ just what is needed to drive EWSB in SUSY
- $m(h)$ lies within narrow window of MSSM prediction
- $m(W)$ vs. $m(\text{top})$: favors heavy SUSY over SM

Hard to believe this is all coincidence
$m_{W}$: LEP2/Tevatron/LHC
$m_{t}$: Tevatron/LHC

$m_{h}=125$ GeV

$M_{W}$: MSSM

$M_{h}$: $125.09 \pm 0.48$ GeV

$M_{h}$: $125.09 \pm 3.1$ GeV

$M_{h}$: SM, MSSM

Heinemeyer, Hollik, Stockinger, Weiglein, Zeune '18

Experimental errors 68% CL
But where are the sparticles? And where are the WIMPs?
Where are the sparticles?

BG naturalness: \( m(\text{gluino,stop}) < \sim 400 \text{ GeV} \)

\[
\begin{align*}
m_{\tilde{g}} &> 2.25 \text{ TeV} \\
m_{\tilde{t}_1} &> 1.1 \text{ TeV}
\end{align*}
\]
Where are WIMPs?

pre-LHC prime WIMP real estate now excluded
Is SUSY a failed enterprise (as is often claimed in popular press)?
“The appearance of fine-tuning in a scientific theory is like a cry of distress from nature, complaining that something needs to be better explained”

S. Weinberg
“...settling the ultimate fate of naturalness is perhaps the most profound theoretical question of our time”

Arkani-Hamed et al., arXiv:1511.06495

“Given the magnitude of the stakes involved, it is vital to get a clear verdict on naturalness from experiment”

This should be matched by theoretical scrutiny of what we mean by naturalness
Putting Dirac and 't Hooft naturalness aside, what we usually refer to as natural is **practical naturalness**

An observable $\mathcal{O} \equiv o_1 + \cdots + o_n$ is natural if all *independent* contributions $o_i$ to $\mathcal{O}$ are comparable to or less than $\mathcal{O}$.

$$ \frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 $$

$$ \Delta_{EW} \equiv \max_i |C_i|/(m_Z^2/2) $$

The main requirements for low fine-tuning ($\Delta_{EW} \lesssim 30$) are the following.

- $|\mu| \sim 100 - 350$ GeV[23–27] (where $\mu \gtrsim 100$ GeV is required to accommodate LEP2 limits from chargino pair production searches).
- $m_{H_u}^2$ is driven radiatively to small, and not large, negative values at the weak scale [21, 28].
- The top squark contributions to the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized for TeV-scale highly mixed top squarks[28]. This latter condition also lifts the Higgs mass to $m_h \sim 125$ GeV. For $\Delta_{EW} \lesssim 30$, the lighter top squarks are bounded by $m_{\tilde{t}_1} \lesssim 3$ TeV.
- The gluino mass, which feeds into the $\Sigma_u^u(\tilde{t}_{1,2})$ via renormalization group contributions to the stop masses[27], is required to be $m_{\tilde{g}} \lesssim 6$ TeV, possibly beyond the reach of the $\sqrt{s} = 13 - 14$ TeV LHC.
- First and second generation squark and slepton masses may range as high as 5-30 TeV with little cost to naturalness[21, 22, 29, 30].

HB, Barger, Huang, Mustafayev, Tata
The bigger the soft term, the more natural is its weak scale value, until EW symmetry no longer broken: living dangerously!

Radiative corrections drive $m^2_{H_u}$ from unnatural GUT scale values to naturalness at weak scale: radiatively-driven naturalness.

Evolution of the soft SUSY breaking mass squared term $\text{sign}(m^2_{H_u})\sqrt{|m^2_{H_u}|}$ vs. $Q$.
How much is too much fine-tuning?

Visually, large fine-tuning has already developed by $\mu \sim 350$ or $\Delta_{EW} \sim 30$
a natural sparticle spectrum for SUSY
NUHM2: \( m_0, m_{1/2}, A_0, \tan \beta, \mu, m_A \)
High scale (HS, stop mass) measure

\[ m_h^2 \simeq \mu^2 + m_{H_u}^2(\Lambda) + \delta m_{H_u}^2. \]

\[ \delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda^2/m_{SUSY}^2) \]

Implies 3 3rd generation squarks < 500 GeV:
SUSY ruled out under $\Delta_{HS}$

BUT! too many terms ignored! NOT VALID!

The bigger $m_{H_u}^2(\Lambda)$ is, the bigger is the cancelling correction-
these terms are not independent.

For big enough $m_{H_u}^2(\Lambda)$, then $m_{H_u}^2$ driven to natural value at weak scale:
radiatively driven naturalness (RNS)

HB, Barger, Mickelson, Padeffke, Savoy
EENZ/BG naturalness

\[ \Delta_{EENZ/BG} \equiv \max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right| \]

- depends on input parameters of model
- different answers for same inputs assuming different models

<table>
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<th>model</th>
<th>(c_{m_0})</th>
<th>(c_{m_{1/2}})</th>
<th>(c_{A_0})</th>
<th>(c_\mu)</th>
<th>(c_{H_u})</th>
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parameters introduced to parametrize our ignorance of SUSY breaking; not expected to be fundamental

e.g. SUSY with dilaton-dominated breaking: \(m_0^2 = m_{3/2}^2\) with \(m_{1/2} = -A_0 = \sqrt{3}m_{3/2}\)

(doesn’t make sense to use independent m0, mhf, A0)

while \(\Delta_{BG}\) tells us about fine-tuning in our computer codes, what we really want to know is: is nature fine-tuned or natural?
Prospects for gluino pair and top-squark pair searches at LHC

An energy upgrade of LHC to ~27 TeV will be needed to test natural SUSY in gluino pair or stop pair search channels.

(nAMSB can allow m(glino) up to 9 TeV)

HB, Barger, Gainer, Serce, Sengupta, Tata
Natural SUSY: only higgsinos need lie close to weak scale
Soft dilepton+jet+MET signature from higgsino pair production

It appears that HL-LHC can see entire natural SUSY p-space; signal in this channel should emerge slowly as more integrated luminosity accrues
It is sometimes invoked that maybe we should abandon naturalness: after all, isn’t the cosmological constant (CC) fine-tuned?

In a multiverse with $10^{500}$ vacua with different CCs, then the value of the CC may not be surprising since larger values would lead to runaway pocket universes where galaxies wouldn’t condense—anthropics: no observers in such universes (Weinberg)

The CC is as natural as possible subject to the condition that it leads to galaxy condensation
To handle string theory and concomitant multiverse, Douglas introduced concept of \textit{stringy naturalness}.

An effective field theory (or specific coupling or observable) $T_1$ is more natural in string theory than $T_2$ if the number of \textit{phenomenologically acceptable} vacua leading to $T_1$ is larger than the number leading to $T_2$.

(anthropics hides here)

This embodies Weinberg’s prediction of CC

Can we apply similar reasoning to magnitude of weak scale?
$$m(\text{weak}) \sim m(W,Z,h) \sim 100 \text{ GeV}$$
\[ \frac{dP}{d\mathcal{O}} \sim f_{\text{prior}} \cdot f_{\text{selection}} \]

What is \( f(\text{prior}) \) for SUSY breaking scale?

In string theory, usually complicated hidden sector containing a variety of F- and D- breaking fields

For comparable \( \langle F_i \rangle \) and \( \langle D_j \rangle \) values, then expect

\[ f_{\text{prior}} \sim m_{\text{soft}}^{2n_F + n_D - 1} \]

Under single F-term SUSY breaking, expect linear increasing statistical selection of soft terms

Figure 1: Annuli of the complex \( F_X \) plane giving rise to linearly increasing selection of soft SUSY breaking terms.
What about \( f(\text{selection}) \)?

Originally, people adopted \( f_{\text{EWFT}} \sim \frac{m_{\text{weak}}^2}{m_{\text{soft}}^2} \)

to penalize soft terms straying too far from weak scale

This doesn’t work for variety of cases

- Too big soft terms can lead to CCB minima: must veto such vacua
- Bigger \( m(\text{Hu})^2 \) leads to more natural value at weak scale
- Bigger \( A(t) \) trilinear suppresses \( t_1, t_2 \) contribution to weak scale

Adopt \( \mu \) solution which leads to natural, weak scale value of \( \mu \sim 100-350 \text{ GeV} \)

Then for statistically selected soft terms, \( m(\text{weak}) \) is output, not input

Must veto too large \( m(\text{weak}) \) values: nuclear physics screwed up
(\text{Agrawal, Barr, Donoghue, Seckel, 1998})

Factor four deviation of weak scale from measured value \( \Rightarrow \Delta_{EW} < 30 \)
Agrawal, Barr, Donoghue, Seckel result:
weak scale cannot deviate by more than
factor 2-5 from its measured value
lest disasters occur in nuclear physics
Veto pocket universes with CCB minima or minima leading to weak scale a (conservative) factor four greater than our value $m(W,Z,h) \sim 100$ GeV
Living dangerously with stringy naturalness

Must avoid CCB/noEWSB minima; soft terms drawn to red, where pocket universe weak scale close to our value

EW symmetry barely broken

HB, Barger, Savoy, Serce
Putting it all together: $m_0$ vs. $m_{hf}$ plane

greater density of points is more stringy natural:
soft terms as big as possible such that EW symmetry
properly broken, not too big EW scale

HB, Barger, Salam
What does stringy naturalness imply for m(h)?

For n=1 or 2, expect m(h)~125 GeV!

HB, Barger, Serce, Sinha
From our $n = 1, 2$ results which favor a value $m_h \sim 125$ GeV, then we also expect

- $m_{\tilde{g}} \sim 4 \pm 2$ TeV,
- $m_{\tilde{t}_1} \sim 1.5 \pm 0.5$ TeV,
- $m_A \sim 3 \pm 2$ TeV,
- $\tan \beta \sim 13 \pm 7$,
- $m_{\tilde{W}_1, \tilde{Z}_{1,2}} \sim 200 \pm 100$ GeV and
- $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 7 \pm 3$ GeV with
- $m(\tilde{q}, \tilde{\ell}) \sim 20 \pm 10$ TeV (for first/second generation matter scalars).

From stringy naturalness, expect LHC to see Higgs with $m(h) \sim 125$ GeV but as yet no sign of sparticles!
To test SUSY in gluino-pair or stop-pair channels, may need upgrade to HE-LHC
Only higgsinos required to lie near weak scale

Signal in soft-dilepton+jet+MET channel should gradually emerge at LHC14 as more and more integrated luminosity accrues!
Another fine-tuning problem in SM: strong CP in QCD sector

\[ \mathcal{L}_{\text{QCD}} \ni \frac{\alpha_s}{8\pi} G_{\mu\nu} A \tilde{G}^\mu\nu \quad \tilde{\theta} = \theta_{\text{QCD}} + \text{Arg}[\text{det}(m_q)] \]

Why is \( \tilde{\theta} < 10^{-10} \)?

Only plausible solution: global U(1)$_{\text{PQ}}$ symmetry with concomitant axion

But problems occur with PQ solution

- Global U(1)s not consistent with gravity completion/string theory
- Expect axion scale \( f_a \sim m_{\text{GUT}} - m_P \) but cosmology favors \( f_a \sim 10^{11} \) GeV

SUSY can help solve both these

In SUSY, axion superfield also contains spin-1/2 axino and spin-0 saxion

SUSY DFSZ axion model: solution to SUSY mu problem via Kim-Nilles operator

\[ W_{KN} \ni \frac{\lambda_\mu}{m_P} S^2 H_u H_d \]

\( \mu \sim \lambda_\mu f_a^2 / m_P \sim 100 \) GeV for \( f_a \sim 10^{11} \) GeV!
Discrete R-symmetries: expected to arise from compactification of extra space dimensions in string theory

\( R \)-symmetries: superspace co-ordinates \( \theta \) carry \( R \)-charge +1;

superpotential \( W \) carries \( R \)-charge +2

Anomaly-free discrete \( R \)-symmetries with charges consistent with GUTs

- forbid \( \mu \) term
- forbid \( RPV \) operators
- forbid dim-5 \( p \)-decay op’s
- allow usual Yukawas
- allow see-saw term
Kamionkowski&March-Russell constraint for gravity safety in axion model: 
PQ violating terms in axion potential must be suppressed 
by at least 1/m_P^8 else \( \theta_{\text{bar}} > 10^{-10} \) !

Almost all 2-PQ field SUSY DFSZ models *not* gravity safe
One that works: hybrid between Choi-Chun-Kim (CCK) and 
Martin/Babu-Gogoladze-Yang (MBGY) models

\[
W \ni f_u Q H_u U^c + f_d Q H_d D^c + f_L L H_d E^c + f_\nu L H_u N^c \\
+ f X^3 Y / m_P + \lambda_\mu X^2 H_u H_d / m_P + M N^c N^c / 2 \tag{9}
\]

\[
V_{\text{soft}} \ni m_X^2 |\phi_X|^2 + m_Y^2 |\phi_Y|^2 + (f A_f \phi_X^3 \phi_Y / m_P + h.c.)
\]

HB, V. Barger, D. Sengupta, 1810.03713

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<th>( H_d )</th>
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The lowest order PQ violating terms in the superpotential are \( X^8 Y^2 / m_P^7 \), \( X^4 Y^6 / m_P^7 \) and 
\( Y^{10} / m_P^7 \) which implies that the lowest order PQ breaking term in the scalar potential is sup-
pressed by \( 1/m_P^8 \). Therefore, this model satisfies the KMR condition for being gravity-safe.
SUSY with $Z(24)^R$ discrete symmetry saves the day for PQ solution to strong CP!

$$f_a \sim \sqrt{m_{\text{soft}} m_P} \sim 10^{11} \text{ GeV}$$  sweet spot!
What about WIMPs?

To be natural in QCD sector, must solve strong CP via axion:
DM=axion+higgsino-like WIMP admixture

Bae, HB, Serce
Typically, DM is axion dominated unless non-thermal processes (axino, saxion decay) augment WIMP abundance.

WIMPs haven’t been detected so far because there are likely a lot less of them floating around than in WIMP-only DM scenario.

K.J. Bae, HB, H. Serce
Even tho WIMPs make up typically 10-15% DM,

Ultimately expect WIMP signal from multi-ton noble liquid detectors!

Figure 11: Locus of $n = 1$ landscape scan points for the NUHM3 model with $\mu = 100 - 360$ GeV in the (a) $\xi \sigma^{SD}(\tilde{\chi}_1^0, p)$ vs. $m_{\tilde{\chi}_1^0}$ and (b) $\xi^2(\sigma v)$ vs. $m_{\tilde{\chi}_1^0}$ planes versus recent WIMP search constraints.

HB, Barger, Salam, Serce, Sinha
Conclusions

- SUSY still highly motivated
- Natural regions of p-space with light higgsinos exists
- Higgsinos pairs => Soft Dilepton+Jet+MET signature at HL-LHC
- Gluinos, stops might have to wait for HE-LHC
- Stringy naturalness: LHC should see $m_h \sim 125$ GeV plus no sparticles so far
- Discrete R-symmetries solve SUSY mu, RPV, p-decay
- $Z(24)^R$ yields gravity safe axion model with $f_a \sim 10^{11}$ GeV
- Amusingly, both R-parity and $U(1)_{PQ}$ arise as accidental, approximate symmetries from underlying $Z(24)^R$
- WIMPs not seen because subdominant component of DM compared to axions
- But should see WIMPs at multi-ton noble liquid detectors
- Axion coupling suppressed by presence of higgsinos- likely invisible with present tech.
Gauge hierarchy problem (SM):

\[ V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \]

\[ m_h^2 \approx 2\mu^2 + \delta m_h \]

\[ \delta m_h^2 \approx \frac{3}{4\pi^2} \left( -\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2 \theta_W} + \lambda \right) \Lambda^2 \]

Figure 2: Value of \( m_h(SM) \) versus SM \( \mu \) parameter for theory cut-off values \( \Lambda_{SM} = 10^3, 10^{11} \) and \( 10^{16} \) GeV.

Hardly plausible that SM is valid much beyond the TeV scale