

# Clockwork mechanisms

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**SUSY2019**

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Fundamental Interactions**

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Hosted by Texas A&M University – Corpus Christi

# CW mechanism

[Choi, Kim, Yun 14], [Choi, Im 15], [Kaplan, Rattazzi 15], [Giudice, McCullough 16]

A mechanism generating **exponentially small couplings/ large interaction scales**  
from  $O(1)$  couplings

# CW models

A new useful tool to build a ‘natural’ model

- **CW Dimensions** [Ahmed, Dillon 16], [Giudice, Kats, McCullough, Torre, Urbano 17], [Choi, Im, Shin 17], [Craig, Garcia, Sutherland 17]...
- **CW Axion** [Choi, Kim, Yun 14], [Farina, et.al. 16],[Coy, Frigerio, Ibe 17],[Agrawal, Fan, Reece 18], [Long 18], [Bonnefoy, Dudas, Pokorski 19], [Bae, Kost, Shin 19]...
- **CW Inflation** [Kehagias, Riotto 17], [SCP, Shin 18]  
...
- **CW Dark matter** [Hambey, Teresi, Tytgat 17], [Marzora, Raidal, Urban 18], [Kim, McDonald 18], [Goudelis, Mohan, Sengupta 18]...
- **CW Neutrino** [SCP, Shin 17], [Ibarra, Kushwaha, Vempati 17], [Patel 17], [Banerjee, Ghosh, Ray 18], [Alonso, et.al. 18], [Hong, Kurup, Perelstein 19], [Kitabayashi 19]...
- **CW gravity** [Ibanez, Montero 17], [Saraswat17], [Hong, Kim, Shin 18], [Terresi 18], [Im, Nilles, Olechowski 19], [Sannino, Smirnov, Wang 19]...

# Contents

- Basic ideas of CW mechanism
- CW models
- Summary & Discussion

# Basic ideas

[Giudice, McCullough 16] [Choi, Im, Shin 17]

**We can easily make an exponentially small number by multiplying small numbers multiple times:**

$$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \cdots \times \frac{1}{3} = \frac{1}{3^N} = e^{-N \ln 3}$$

$$N = 15 : \text{book} \rightarrow \text{atom}$$

A **chain** of N-fields can generate small effective coupling:

$$\phi_0 \xrightarrow{1/q < 1} \phi_1 \xrightarrow{1/q} \phi_2 \xrightarrow{1/q} \dots \xrightarrow{1/q} \phi_N \xrightarrow{g} \mathcal{O}_{\text{SM}}$$

$$\phi_{\text{light}} \approx \phi_0 \longrightarrow \phi_{\text{light}} - \mathcal{O}_{\text{SM}} \sim \frac{g}{q^N}$$

light field by  
symmetry

# General CW mechanism

[Giudice, McCullough 16]

- 1D lattice in theory space with  $(N+1)$  sites
- Each site has a symmetry  $S$ ,  $G=S^{\{N+1\}}$ : symmetry of full theory space
- $(N+1)$  massless particles  $P=(P_0, P_1, \dots, P_{\{N+1\}})$
- At a scale  $m$ ,  $G$  is broken due to mass mixings between site(i)-site(i+1);  $N$ -links ==> remaining one symmetry  $S_0$ , one massless particle  $P_0$  (a linear combination of  $P$ 's)
- **CW assumption: Symmetry breaking is asymmetrically with  $q (>1$  or  $<1$ ) for site(i) and site(i+1),  $i=0,1,2,\dots,N-1$ .**
- **$P_0$  is exponentially localized towards one of the boundaries. ( $P_0$  is exponentially small ( $\sim 1/q^N$ ) at the opposite site ==> leads to suppressed coupling with the localized operator)**

# CW theory space

**Basic ingredients:**



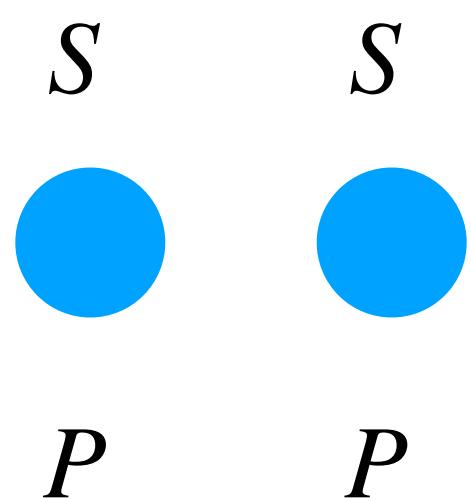
$S$  : symmetry group

$P$  : massless particle

“One-site” model

# CW theory space

We can extend to 2  
on 1D lattice

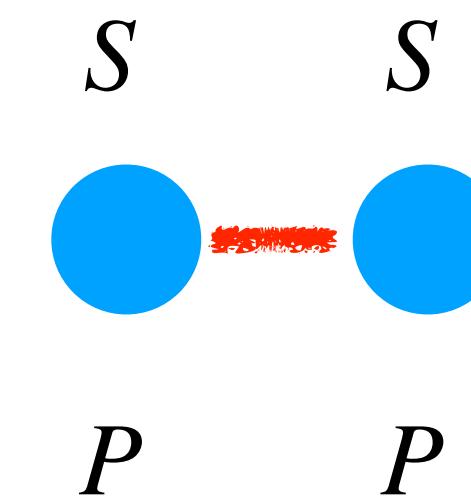


Full symmetry:  $\mathcal{G} \supset S^2$

2 massless particles:  $P = (P_0, P_1)$

# CW theory space

**asymmetric soft breaking  
parametrized by  $q > 1$**



**Full symmetry:**  $\mathcal{G} \supset S^2 \xrightarrow{\hspace{1cm}} S_0$

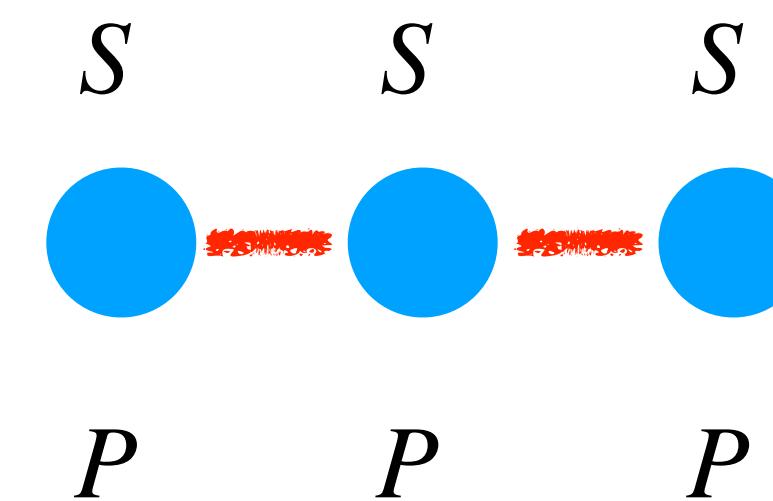
**1 massless particle  
remains**

$$P = (P_1, P_2) \xrightarrow{\hspace{1cm}} P_0 = P_1 + \frac{1}{q} P_2$$

**(semi-localization)**

# CW theory space

**“3-sites model”**



Symmetry breaking by 2 links:

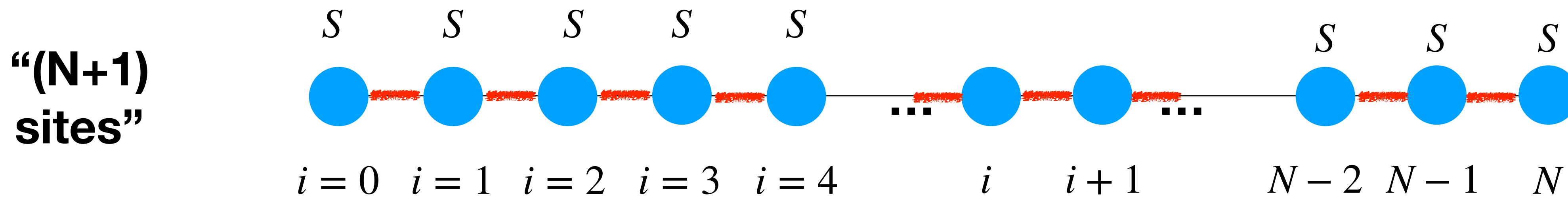
$$\mathcal{G} \supset S^3 \xrightarrow{\quad\quad\quad} S_0$$

1 massless particle  
remains:

$$P = (P_0, P_1, P_2) \xrightarrow{\quad\quad\quad} P_0 = P_1 + \frac{1}{q} P_2 + \frac{1}{q^3} P_3$$

(semi-localization)

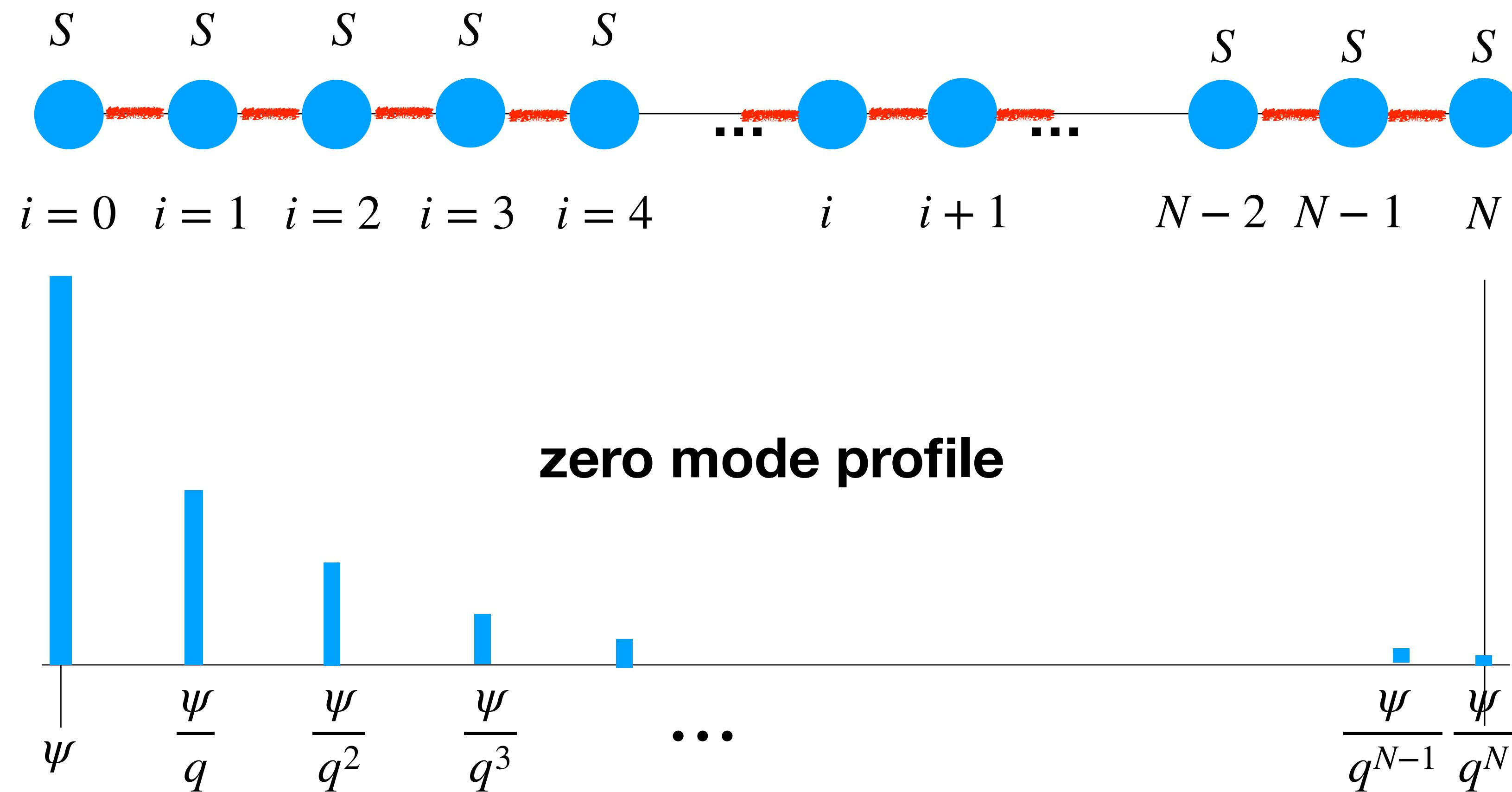
# CW theory space



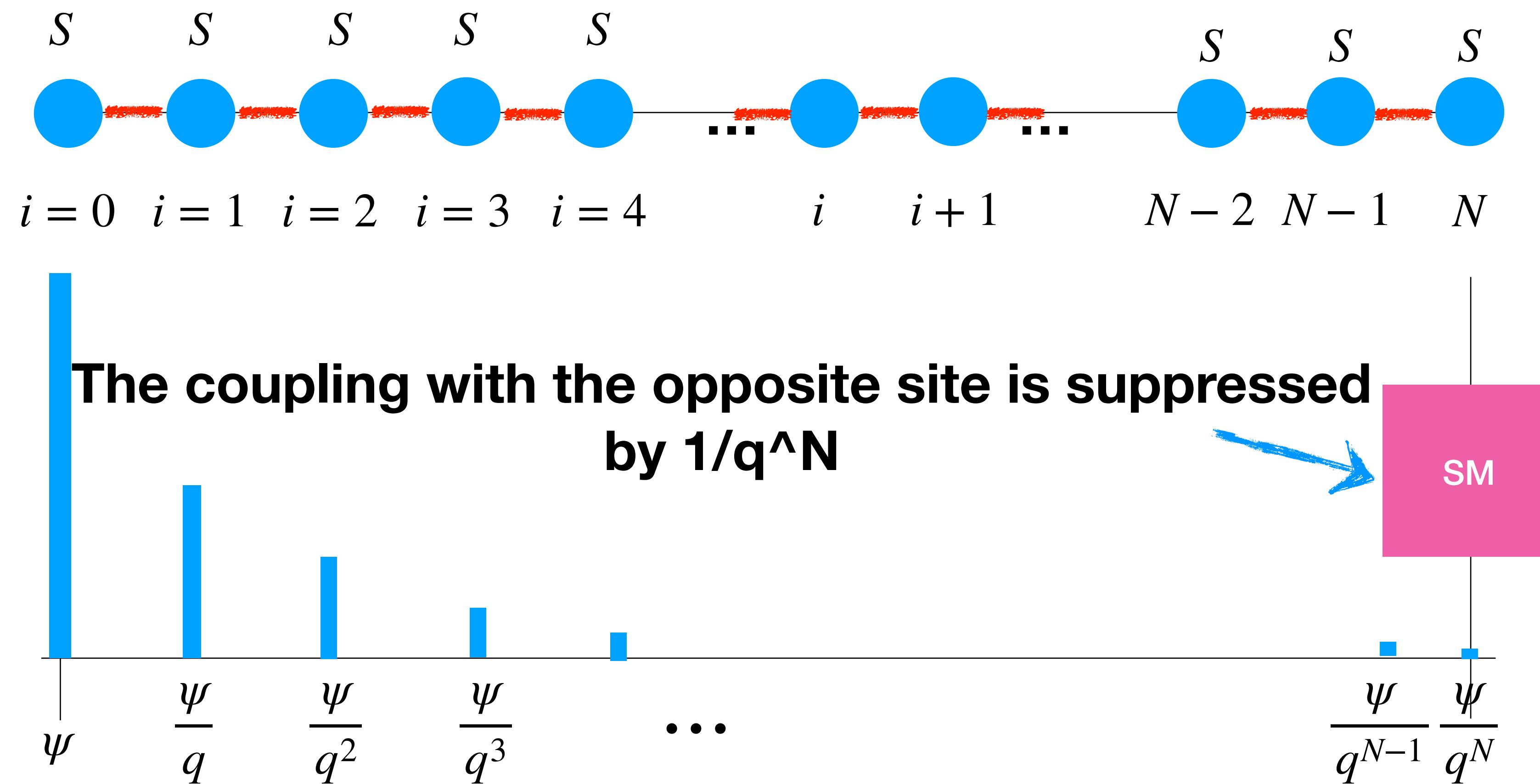
**Symmetry breaking by N-links:**  $\mathcal{G} \supset S^{N+1} \rightarrow S_0$   
**(asymmetric)**

**1 massless particle remains:**  $P_0 = \sum_{i=0}^N C_i P_0^i$        $C_i \approx \frac{1}{q^i}$

# CW theory space



# CW theory space



<b>CW</b>	<b>s=0</b>	<b>s=1/2</b>	<b>s=1</b>	<b>s=2</b>
<b>symmetry S</b>	Shift	Chiral	gauge	4D diffeomorphism
<b>CW model</b>	axion	neutrino mass, flavor	tiny gauge coupling	CW graviton, naturalness problem

# Scalar CW potential

Introduce **(N+1)-‘gear fields’ in CW sector**

$$\{\phi_0, \phi_1, \phi_2, \phi_3, \dots, \phi_N\}$$

$$V_{\text{CW}}(\phi_i) = \sum_{i=0}^N \frac{1}{2} m_{\text{CW}}^2 (\phi_i - q\phi_{i+1})^2$$

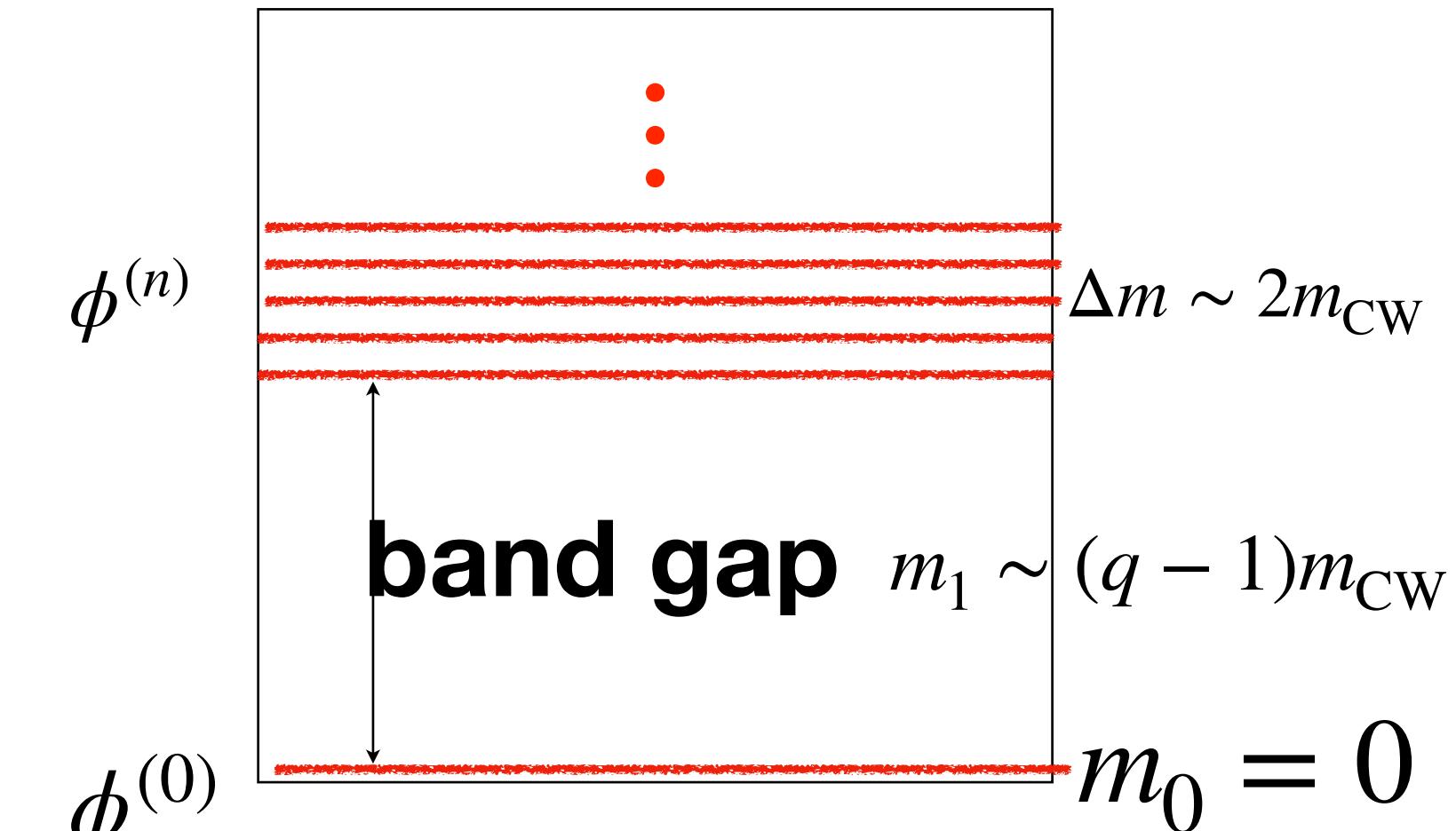
**Shift symmetry:**  $\phi_i \rightarrow \phi_i + q^{-i}C$



**massless mode**  $m_0^2 = 0$

# Scalar CW potential

## Spectrum of CW gears

$$\mathcal{M}_\phi^2 = m_{\text{CW}}^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1+q^2 & -q \\ & & & & -q & q^2 \end{pmatrix}$$


$$m_0 = 0$$

$$m_n^2 = m_{\text{CW}}^2 \left( q^2 - 1 - 2q \cos \frac{n\pi}{N+1} \right)$$

$$\phi^{(0)} \approx \sum_{i=0}^N \frac{\phi_N}{q^N}$$

**KK spectrum of extra dimension,  
or Deconstructed 5D theory.**

# Continuum limit

$$\mathcal{L}_{\text{CW}} = - \sum_{i=0}^N \frac{1}{2} \left( \partial_\mu \phi_i \right)^2 - \sum_{i=0}^N \frac{1}{2} m_i^2 \left( \phi_i - q \phi_{i+1} \right)^2$$

$N \rightarrow \infty$



$$\begin{aligned}\phi_{\textcolor{red}{i}}(x) &\rightarrow \Phi(x, \textcolor{red}{y}) \\ \phi_{i+1}(x) - \phi_i(x) &\rightarrow \partial_y \Phi(x, \textcolor{red}{y}) \\ q - 1 &\rightarrow k, m_{\textcolor{red}{i}}^2 \rightarrow m(\textcolor{red}{y})^2\end{aligned}$$

$$\mathcal{L}_{\text{CW}} = - \int dy \frac{1}{2} \left( \partial_\mu \Phi(x, y) \right)^2 + \frac{1}{2} m(y)^2 \left( \partial_y \Phi(x, y) - k \Phi(x, y) \right)^2$$

# Extra dimension

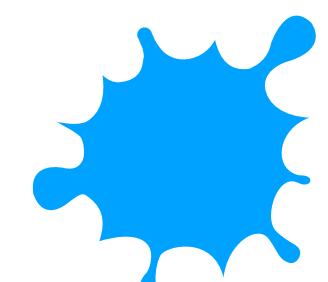
$$\mathcal{L}_{\text{CW}} = - \int dy \frac{1}{2} \left( \partial_\mu \Phi(x, y) \right)^2 + \frac{1}{2} m(y)^2 \left( \partial_y \Phi(x, y) - k \Phi(x, y) \right)^2$$

**Field redefinition**

$$\Phi(x, y) = e^{ky} \Psi(x, y)$$

$$\mathcal{L}_{\text{CW}} = - \int dy \frac{1}{2} e^{2ky} \left[ \left( \partial_\mu \Psi(x, y) \right)^2 + \frac{1}{2} m(y)^2 \left( \partial_y \Phi(x, y) \right)^2 \right]$$

“CW geometry”

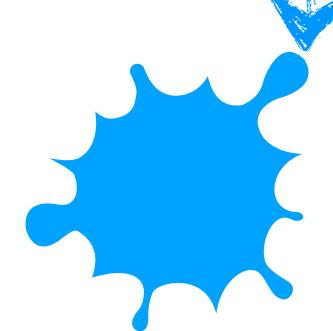


$$ds^2 = \left( \frac{e^{2ky}}{m^2(y)} \right)^{2/3} (m(y)^2 dx^2 + dy^2)$$

# Linear dilaton+Warped geometry

$$ds^2 = \left( \frac{e^{2ky}}{m^2(y)} \right)^{2/3} (m(y)^2 dx^2 + dy^2)$$

$$k = \frac{1}{2} k_{\text{LD}} - p \quad m(y) = e^{-py}$$



$$ds^2 = e^{\frac{3}{2}k_{\text{LD}}y} (e^{-2py} dx^2 + dy^2)$$

address **Hierarchy Problem**  
á la Randall-Sundrum/LED/Linear dilaton (little string)

# CW dictionary

Discrete ( $N=\text{finite}$ )	Continuum ( $N \rightarrow \infty$ )
1D-Lattice in theory space	5th-dimension
CW gears	KK modes
interaction with the boundary site	localized interaction on a brane at the boundary of compact dimension

# CW axion

[Choi, Kim, Yun 14], [Farina, et.al. 16],[Coy, Frigerio, Ibe 17],[Agrawal, Fan, Reece 18], [Long 18], [Bonnefoy, Dudas, Pokorski 19], [Bae, Kost, Shin 19]...

$$\mathcal{L} = \frac{\pi_N}{16\pi^2 f} \tilde{G}_{\mu\nu} G^{\mu\nu}$$

$$= \frac{\pi_0}{16\pi^2 q^N f} \tilde{G}_{\mu\nu} G^{\mu\nu}$$

**CW:**  $\frac{1}{f} \rightarrow \frac{1}{q^N f}$

# CW graviton

[Ibanez, Montero 17], [Hong, Kim, Shin 18], [Terresi 18], [Im, Nilles, Olechowski 19]

## Pauli-Fierz theory of massive graviton with Gears

$$\mathcal{L} = -\frac{m^2}{2} \sum_{j=0}^{N-1} \left( \left[ h_j^{\mu\nu} - q h_{j+1}^{\mu\nu} \right]^2 - \left[ \eta_{\mu\nu} (h_j^{\mu\nu} - q h_{j+1}^{\mu\nu}) \right]^2 \right)$$

$$-\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \rightarrow -\frac{1}{M_P} \tilde{h}_0^{\mu\nu} T_{\mu\nu} \quad M_P = \frac{q^N M_N}{\mathcal{N}_0}$$

# CW inflation

[Kegagias, Riotto 16]

**A large field inflation**

$$V_{inf} = m^3 \phi_{inf}$$

**The problem:**  $\left(\frac{m}{M_P}\right)^3 \sim 10^{-10}$

**The CW solution:**  $\frac{1}{q^N} \left(\frac{m}{M_P}\right)^3 \sim 10^{-10}$

$$V(\pi_1, \dots, \pi_N) = \frac{M_1^2}{2} \sum_{i=0}^{N-1} (\pi_i - q \pi_{i+1})^2 + M_2^3 \pi_N$$

# CW Higgs inflation

[SCP, Shin 18]

**The ‘Higgs inflation’**

$$S = \int d^4x \sqrt{-g} \left( -\frac{M_P^2 + \xi\phi^2}{2} R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{4}\phi^4 \right)$$

**The problem:**  $\lambda = 4.4 \times 10^{-10} \xi^2$

**The CW solution:**  $\lambda = \frac{\lambda}{q^{4N}} \xi^2$

$$K(\phi_i) \equiv \sum_{i=1}^{N+1} \xi_i \phi_i^2,$$

$$V_{CW}(\phi_i) = \sum_{i=1}^N \frac{m^2}{2} (\phi_{i+1} - q\phi_i)^2$$

# CW mechanisms for neutrino mass

[SCP, Shin 17]

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- CW Dirac mass, Type-0

$$m_\nu \approx \left( \frac{y}{q^N} \right) v$$

# CW mechanisms for neutrino mass

[SCP, Shin 17]

- CW Dirac mass, Type-0
- CW EW scale seesaw, Type-1

$$m_\nu \approx \left( \frac{y}{q^N} \right) v$$

$$m_\nu \approx \left( \frac{y}{q^N} \right)^2 \frac{v^2}{m_M}$$

# CW mechanisms for neutrino mass

[SCP, Shin 17]

- CW Dirac mass, Type-0
- CW EW scale seesaw, Type-1
- CW Inverse seesaw, Type-ia

$$m_\nu \approx \left( \frac{y}{q^N} \right) v$$
$$m_\nu \approx \left( \frac{y}{q^N} \right)^2 \frac{v^2}{m_M}$$
$$m_\nu \approx m_M \left( \frac{y}{q^N} \right)^2 \frac{v^2}{m_D^2}$$

# CW mechanisms for neutrino mass

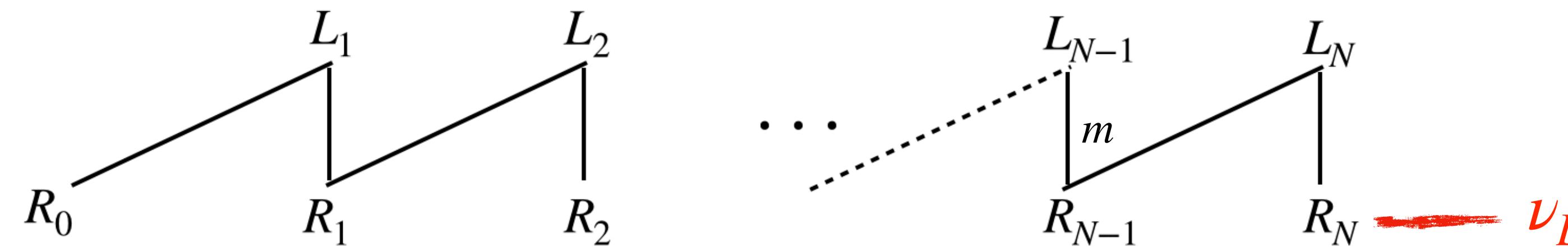
[SCP, Shin 17]

- CW Dirac mass, Type-0
- CW EW scale seesaw, Type-1
- CW Inverse seesaw, Type-ia
- CW Inverse seesaw, Type-ib

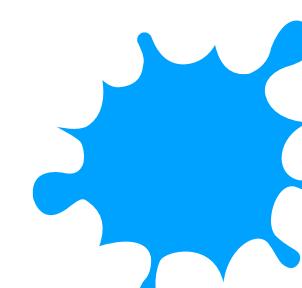
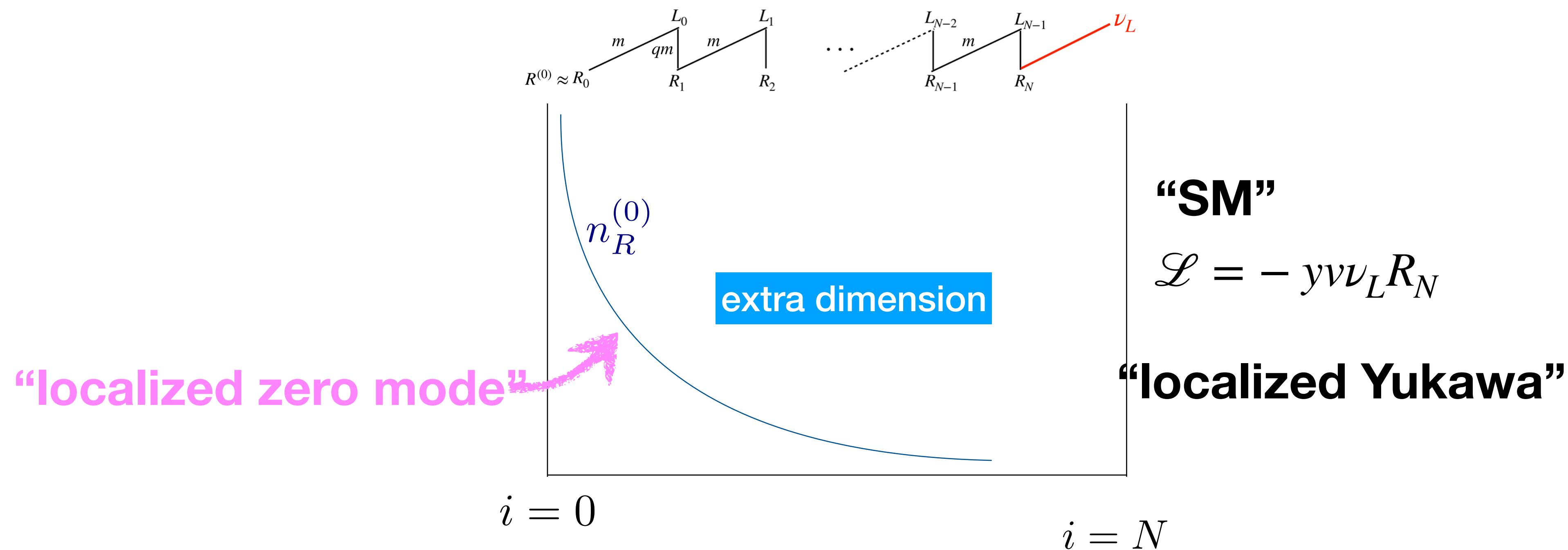
$$m_\nu \approx \left( \frac{y}{q^N} \right) v$$
$$m_\nu \approx \left( \frac{y}{q^N} \right)^2 \frac{v^2}{m_M}$$
$$m_\nu \approx m_M \left( \frac{y}{q^N} \right)^2 \frac{v^2}{m_D^2}$$
$$m_\nu \approx \left( \frac{m_M}{q^{2N}} \right) \frac{y^2 v^2}{m_D^2 + y^2 v^2}$$

# Type-0 CW diagram

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i + h.c.) - y H \bar{\ell}_L R_N + h.c.$$



# Type-0 CW



$$m_\nu \approx \left(\frac{y}{q^N}\right) v$$

(cf) Grossmann, Neubert (2000)

$$y \approx 1, q = 3, N = 25$$

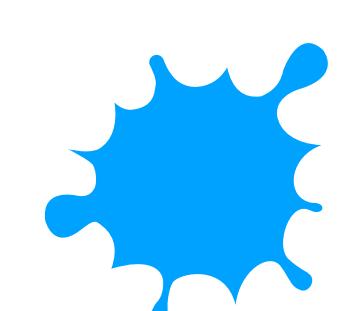
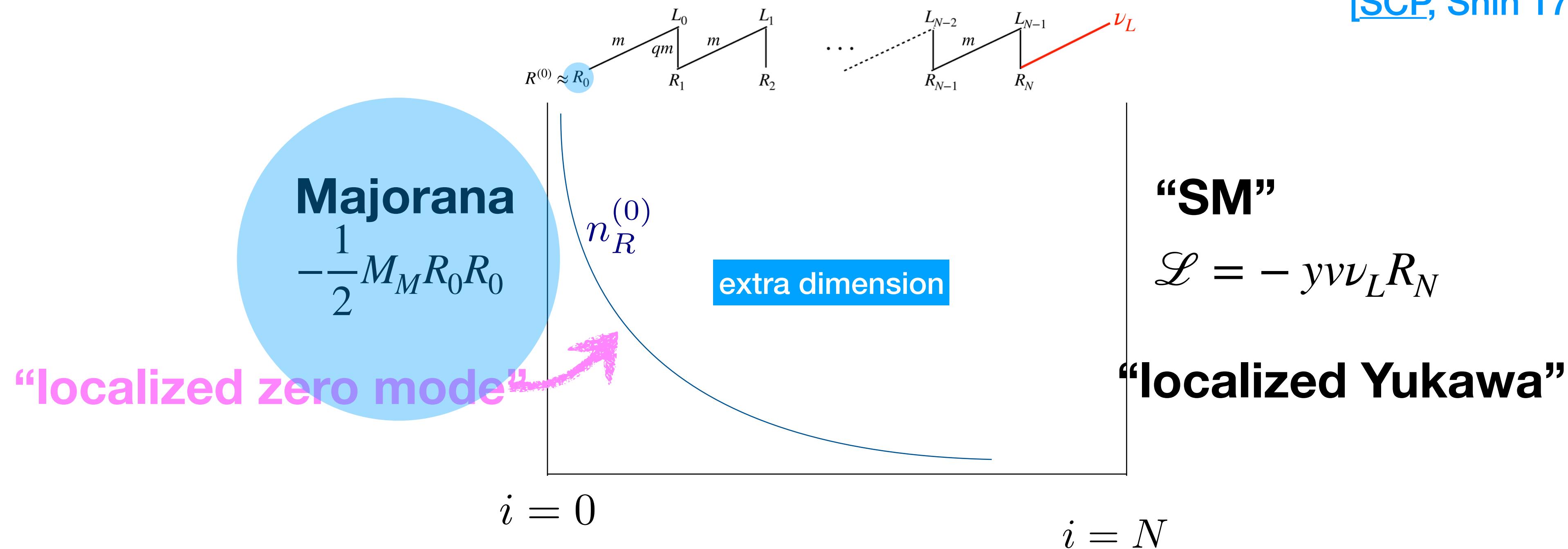
# Type-1 CW diagram

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i + h.c.) - y H \bar{\ell}_L R_N + h.c. - \frac{1}{2} m_M R_0 R_0$$



# Type-1 CW ‘seesaw’

[SCP, Shin 17]



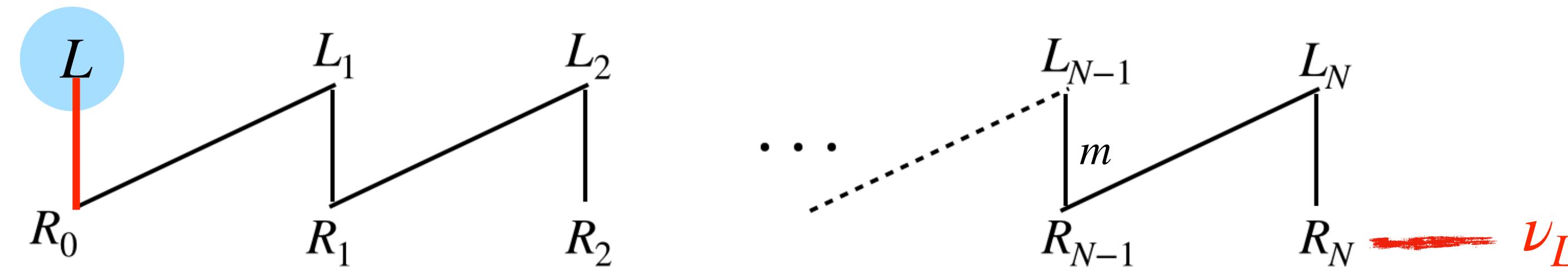
$$m_\nu \approx \left(\frac{y}{q^N}\right)^2 \frac{v^2}{m_M}$$

Unsuppressed Majorana mass,  
suppressed Yukawa

# Type-ia diagram

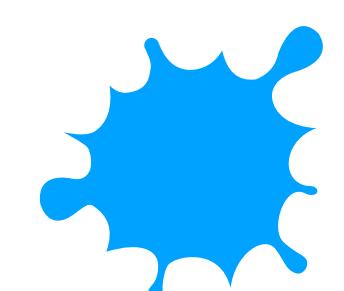
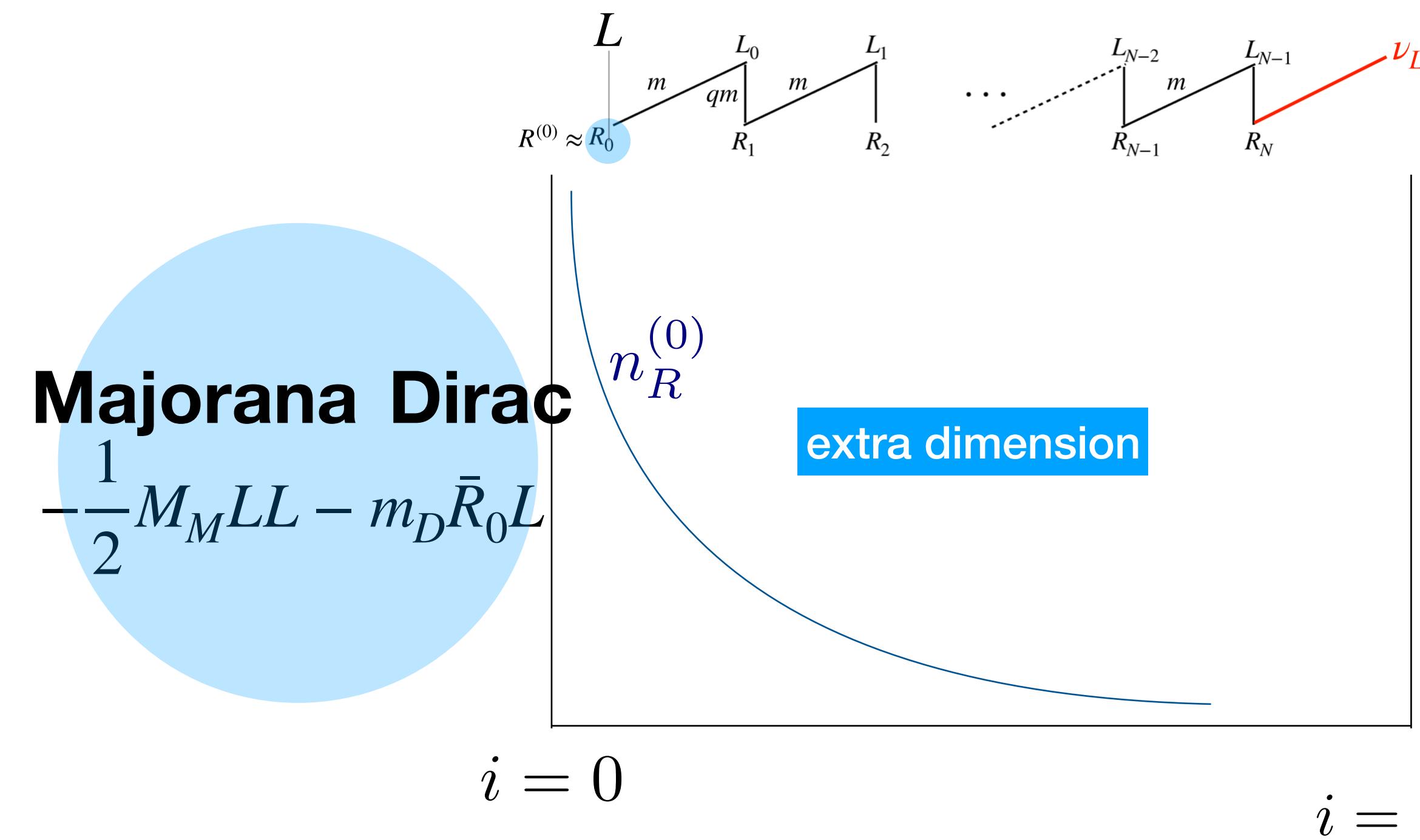
$$\mathcal{L} = \mathcal{L}_{\text{kin}} - m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i + h.c.) - y H \bar{\ell}_L R_N + h.c. - \frac{1}{2} m_M L L - m_D \bar{L} R_0$$

“inverse seesaw”



# Type-ia CW

[SCP, Shin 17]



$$m_\nu \approx m_M \left( \frac{y}{q^N} \right)^2 \frac{\nu^2}{m_D^2}$$

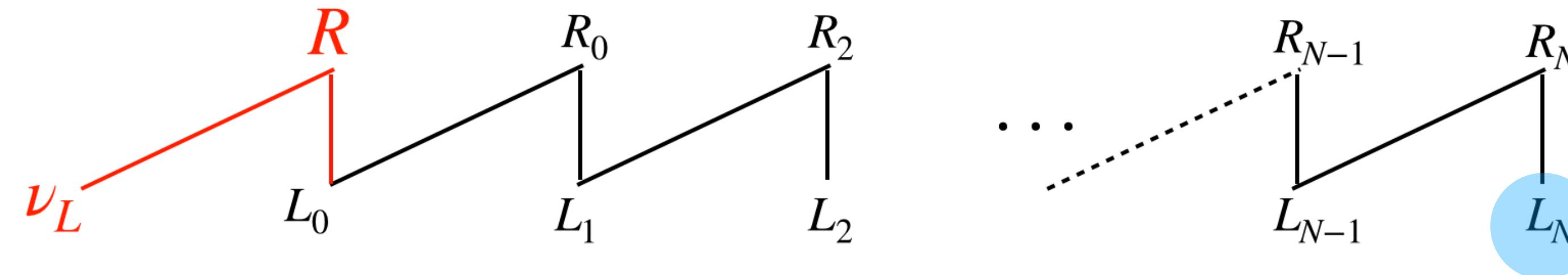
**“SM”**  
 $\mathcal{L} = - y \nu \nu_L R_N$   
**“localized Yukawa”**

**Inverse seesaw-like**  
w/ suppressed Yukawa,  
Unsuppressed Majorana  
(cf) [SCP, Wang, Yanagida 10]

# Type-ib diagram

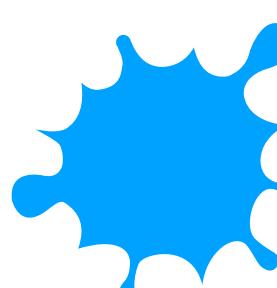
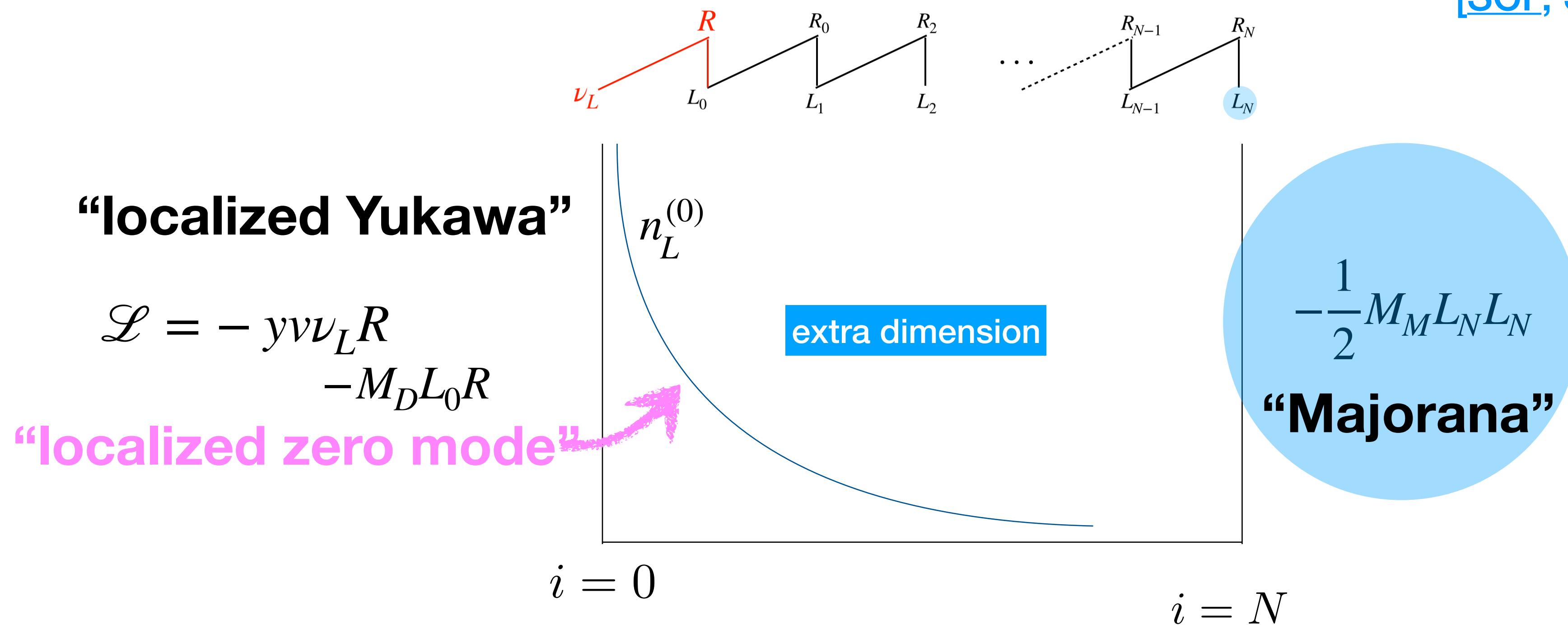
$$\mathcal{L} = \mathcal{L}_{\text{kin}} - m \sum_{i=1}^N (\bar{L}_i R_{i-1} - q \bar{L}_i R_i + h.c.) - y H \bar{\ell}_L R_N + h.c. - \frac{1}{2} m_M L_N L_N - m_D \bar{L}_0 R$$

**“inverse seesaw”**



# Type-ib CW

[SCP, Shin 17]

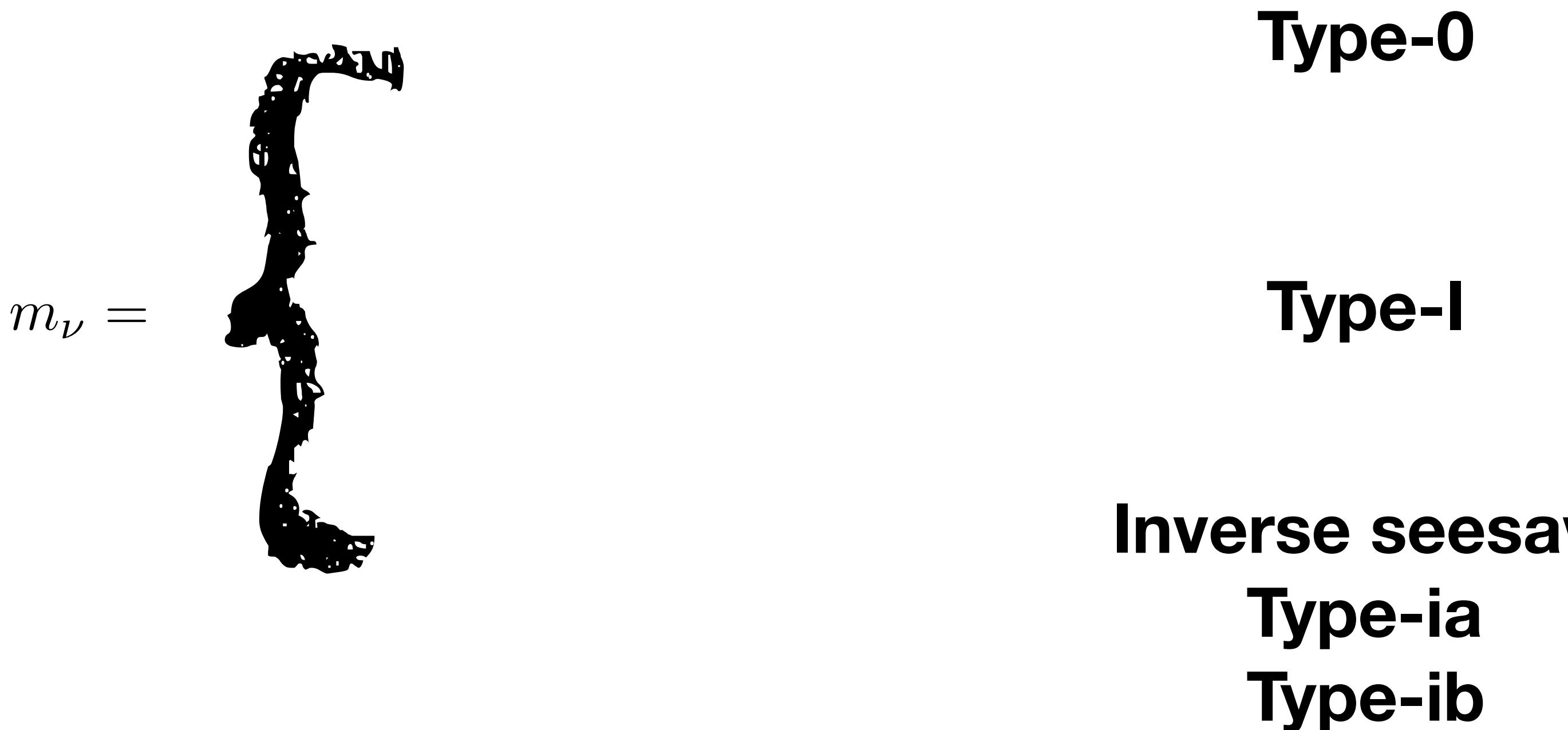


$$m_\nu \approx \left(\frac{m_M}{q^{2N}}\right) \frac{y^2 v^2}{m_D^2 + y^2 v^2}$$

**Inverse seesaw-like**  
w/ unsuppressed Yukawa,  
Suppressed Majorana

(cf) [SCP, Wang, Yanagida 10]  
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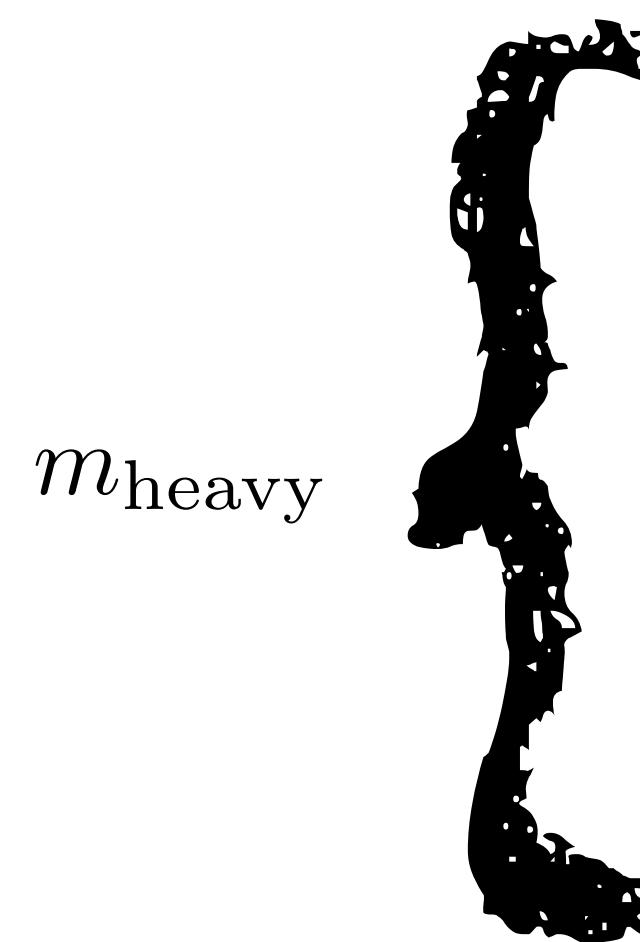
# CW models for neutrino mass



# CW models for neutrino mass

$$m_\nu = \begin{cases} \left(\frac{y}{q^N}\right) v & \text{Type-0} \\ \left(\frac{y}{q^N}\right)^2 \frac{v^2}{m_M} & \text{Type-I} \\ \left(\frac{m_M}{q^{2N}}\right) \frac{y^2 v^2}{m_D^2 + y^2 v^2} & \text{Inverse seesaw} \\ & \text{Type-ia} \\ & \text{Type-ib} \end{cases}$$

# heavy modes



$m_{\text{heavy}}$

Type I :  $M_{\text{I}} = m_M$

Type Ia :  $M_{\text{Ia}\pm} = M_D \pm \frac{m_M}{2} + \frac{m_M^2}{8M_D} \pm \frac{y_{\text{eff}}^4 v^4 m_M^3}{2M_D^6}$ ,

Type Ib :  $M_{\text{Ib}\pm} = \sqrt{M_D^2 + \lambda^2 v^2} \pm \frac{m_M^{\text{eff}} M_D^2}{2(M_D^2 + \lambda^2 v^2)}$ .

**with unsuppressed Yukawa coupling!**

# Summary

- **CW mechanisms** are newly proposed to ‘solve’ naturalness problems using ‘CW gears’ and ‘localized interactions’.

$$\lambda \rightarrow \frac{\lambda}{q^N} \quad q > 1, N \sim 10$$

- **CW phenomenology** is rich with gear fields ( $N>1$ ) at CW scale, that often leads to observational consequences.

# Discussion

CW	$s=0$	$s=1/2$	$s=1$	$s=3/2$	$s=2$
symmetry $S$	Shift	Chiral	gauge	SUSY?	4D diffeomorphism
CW model	axion	neutrino mass, flavor	tiny gauge coupling	??	CW graviton, naturalness problem

# 25th International Summer Institute on Phenomenology of Elementary Particle Physics and Cosmology (SI2019)



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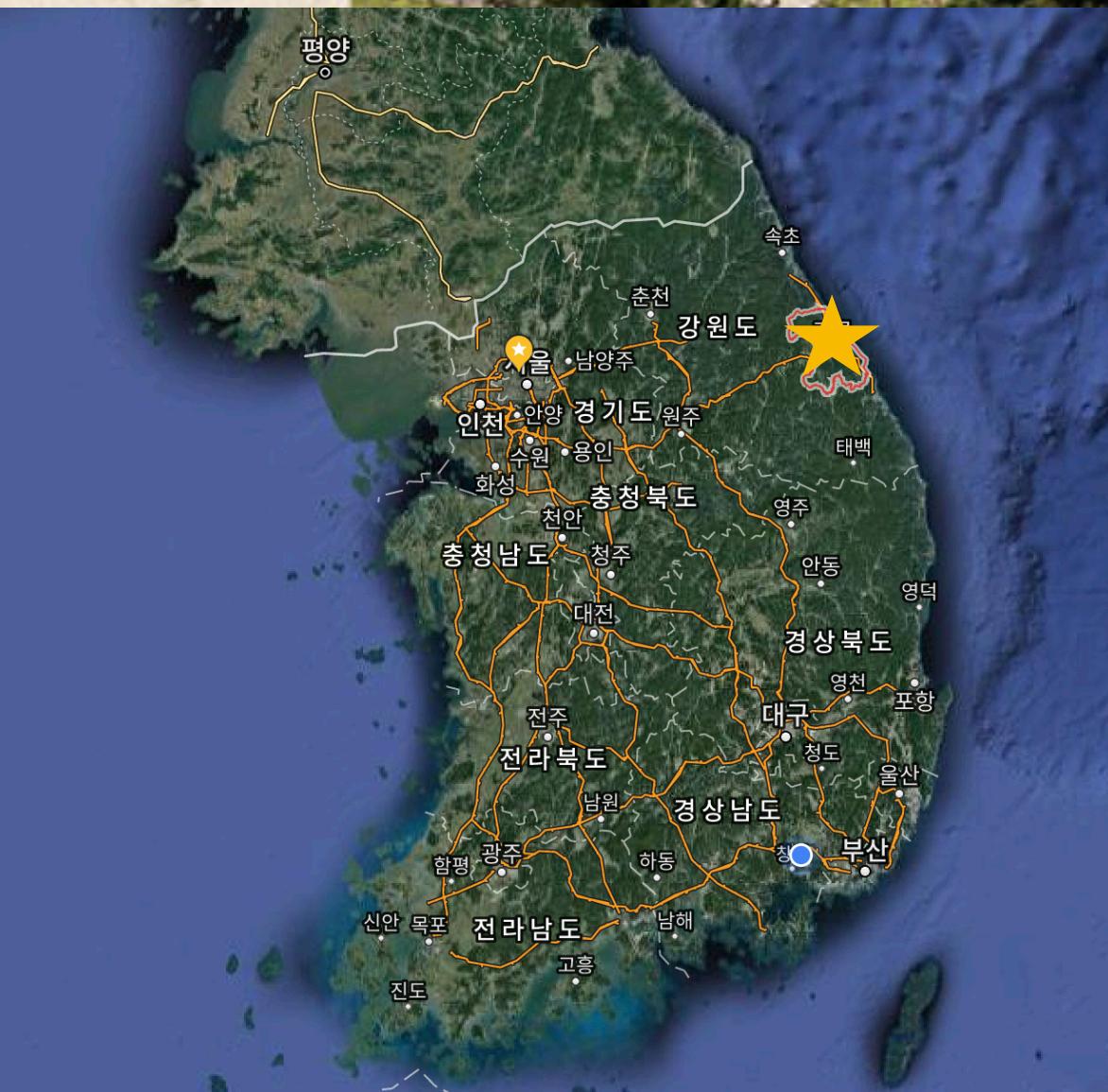
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Hitoshi Murayama (overview),  
Misao Sasaki(inflation and cosmology),  
Chang Sub Shin (BSM theories)  
Carsten Rott (Multi-messenger physics)

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## 강릉(江陵市, Gangneung) Lakai Sandpine Resort

