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Cosmology

Lecture 2

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pre-SUSY2019 Summer School

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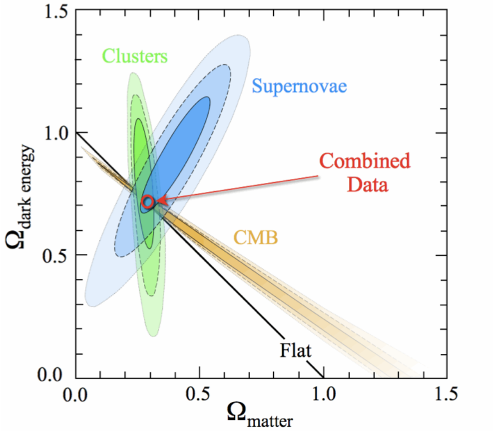
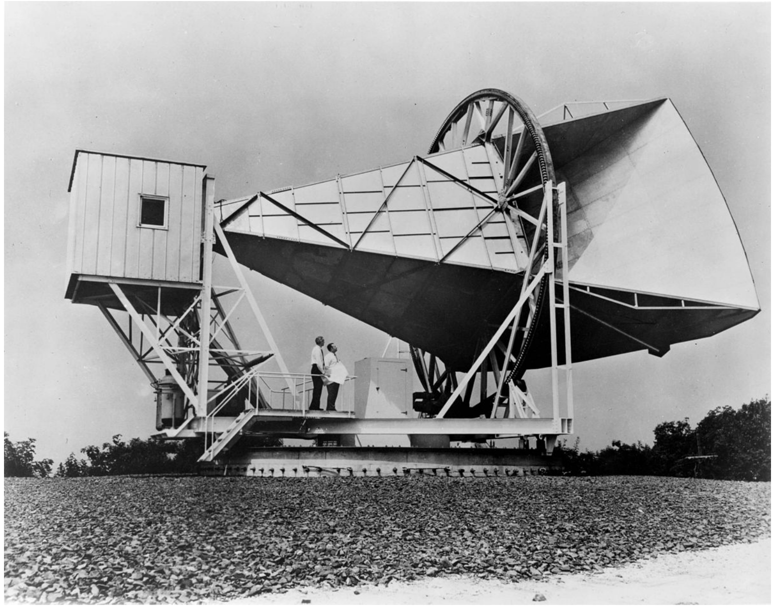


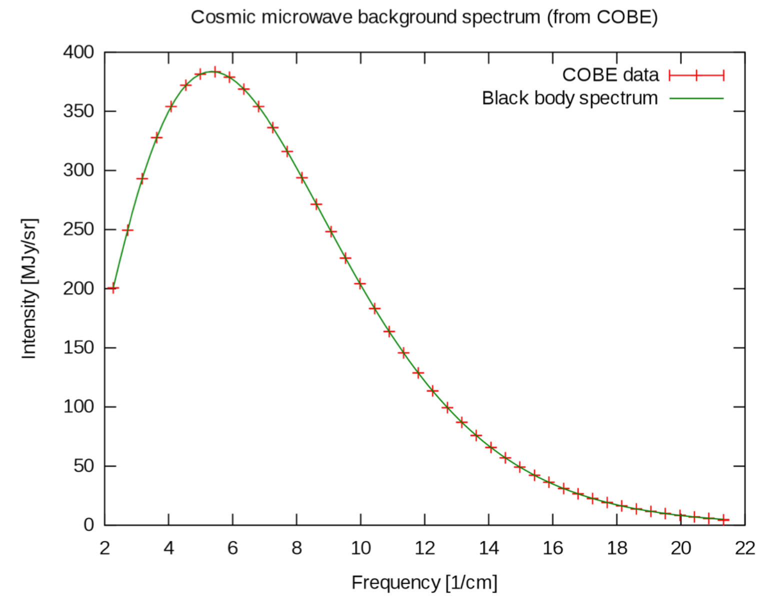
fundamental tool to pinpoint cosmological parameters is the **cosmic microwave background** – relic photons from the early universe



the Holmdel Horn Antenna (Penzias, Wilson, 1964)

key observation: universe filled with a gas of photons with a **black-body spectrum** at a temperature of 2.7K, or **$2.4x10^{-4}$ eV** in natural units the observed **specific intensity** (ergs/cm^2/sec/sr/Hz)

$$I_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT_0} - 1}$$



of course, the photons are **NOT** in thermal equilibrium any more, but their **momentum distribution** indicates that they once were...

this, and the fact that eventually, as $a \rightarrow 0$ densities diverge, tells us that **once** the universe contained species in **thermal equilibrium**...

let's explore the **thermodynamics** of the universe

$$(\hbar = c = 1 = k_B = 1)$$

$$f(\vec{p}) = \left[\exp\left(\frac{E-\mu}{T}\right) \pm 1\right]^{-1} \qquad E(p) = (p^2 + m^2)^{1/2}$$

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3p,$$

$$\rho = \frac{g}{(2\pi)^3} \int f(\vec{p})\, E(\vec{p})\, d^3p$$

$$P = \frac{g}{(2\pi)^3} \int f(\vec{p})\, \frac{|\vec{p}|^2}{3E}\, d^3p$$

If **expansion timescale** of the Universe is **long** compared with the timescales for **reactions** that maintain thermal equilibrium, then fluids are in thermal equilibrium with **adiabatic** changes, meaning the **entropy** per comoving volume will be constant.

Second law of thermodynamics (per comoving volume):

$$TdS = d(\rho V) + PdV = d[(\rho + P)V] - VdP.$$

$$dP = (\rho + P)(dT/T)$$

$$dS = \frac{1}{T}d[(\rho + P)V] - (\rho + P)V\frac{dT}{T^2} = d\left[\frac{(\rho + P)V}{T} + \text{constant}\right]$$

$$S = a^3(\rho + P)/T.$$

$$s \equiv S/V = (\rho + p)/T \propto a^{-3}$$

entropy *density*

An important **limit** we will use later on: $T \gg m$ and $T \gg \mu$

$$\rho = \begin{cases} (\pi^2/30)gT^4 & \text{bosons} \\ (7/8)(\pi^2/30)gT^4 & \text{fermions} \end{cases}$$

$$n = \begin{cases} [\zeta(3)/\pi^2]gT^3 & \text{bosons} \\ (3/4)[\zeta(3)/\pi^2]gT^3 & \text{fermions} \end{cases}$$

$$\rho = \frac{g}{(2\pi)^3} \int f(\vec{p})\, E(\vec{p})\, d^3p,$$

$$P = \frac{g}{(2\pi)^3} \int f(\vec{p})\, \frac{|\vec{p}|^2}{3E}\, d^3p,$$

$$P = \frac{1}{3}\rho$$

…also **entropy** density (*s=(ρ +P)/T*)scales like T^3, and ***a ~ 1/T***

$$\rho = \begin{cases} (\pi^2/30)gT^4 & \text{bosons} \\ (7/8)(\pi^2/30)gT^4 & \text{fermions} \end{cases}$$

if the universe is filled with **multiple relativistic** species, the **total** radiation and pressure density can be cast as

$$\rho_R = (\pi^2/30)g_* T^4, \qquad P_R = \rho_R/3,$$

$$g_* \equiv \sum_{i=\text{bosons}} g_i (T_i/T)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i (T_i/T)^4.$$

T_i are **different** if the species are not in statistical ("**kinetic**") equilibrium!

Quick **shortcut** that will be useful later on: assume **flat** universe (k=0),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$H^2 = \frac{8\pi G}{3}\rho$$

$$M_P = \sqrt{\frac{1}{8\pi G}}$$

$$\rho_{\rm rad} = \frac{\pi^2}{30} g_* T^4, \quad g_* \simeq 106.75$$

$$H = \sqrt{\frac{\pi^2 g_*}{3 \cdot 30}} \frac{T^2}{M_P} \simeq 3.4 \frac{T^2}{M_P}$$

similarly for **entropy** density

$$s = (2\pi^2/45) g_{*s} T^3$$

$$g_{*s} \equiv \sum_{i=\text{bosons}} g_i (T_i/T)^3 + (7/8) \sum_{i=\text{fermions}} g_i (T_i/T)^3$$

…so for an adiabatic universe, $g_{*s}T^3a^3$ is a **constant**!

…back to radiation energy density and ***$g_*(T)$***

$$\rho_R = (\pi^2/30) g_* T^4$$

$$g_* \equiv \sum_{i=\text{bosons}} g_i (T_i/T)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i (T_i/T)^4.$$

…***$g_*(T)$*** is very useful in model building!
E.g. if there exist new "**dark**" degrees of freedom (e.g. light mediators) they should contribute to the **effective number of degrees of freedom**, which is something we measure!

First, let's calculate the effective number of relativistic degrees of freedom in the **late universe**

Two species: **photons** and **neutrinos** and
Issue: they have **different temperatures!**

As we will see shortly, neutrinos decouple around a temperature of **1 MeV**

After that, **electrons and positrons** annihilate, and "heat up*" the photons (but not that neutrinos), so need to correct for the **mismatch** between electron and neutrino temperatures

*not true, they just slow down the temperature drop with *1/a*

Now using **conservation** of **entropy**,
and the fact that entropy scales like gT^3,
assuming instantaneous decays ($a_0=a_1$)
(0=after, 1=before)

$$\left(\frac{g_0}{g_1}\right)^{\frac{1}{3}} = \frac{T_1}{T_0}, \qquad \frac{T_\nu}{T_\gamma} = \left(\frac{2}{2+2\times 7/8+2\times 7/8}\right)^{\frac{1}{3}} = \left(\frac{4}{11}\right)^{\frac{1}{3}}.$$

e^- e^+ (71%)

the contribution of neutrinos to the radiation energy density can be then **parameterized** by the effective number of **neutrinos species**, times this correction for the neutrino vs photon temperature

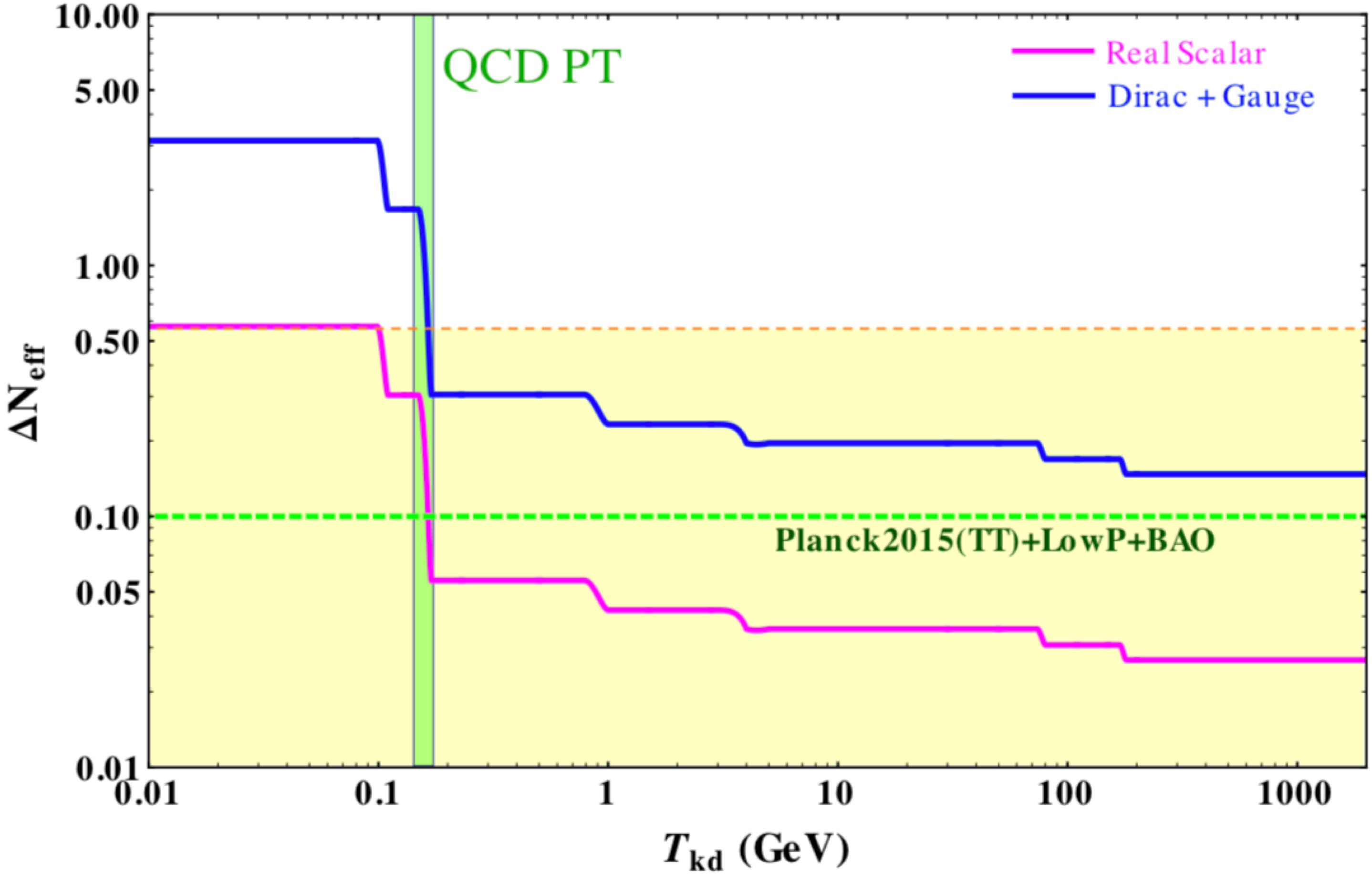
$$1+\frac{7}{8}N_\nu\left(\frac{4}{11}\right)^{\frac{4}{3}}$$

plugging in the numbers, g_* today should be 3.36, equivalent to ***N_ν ~ 3***

…in fact the SM prediction is N_ν~3.046 since neutrinos are **not completely decoupled** at electron-positron annihilation

N_ν impacts several "**late-universe**" observables:

- **Big Bang Nucleosynthesis** $N_\nu = 3.14^{+0.70}_{-0.65}$ at 68%
- **Baryon Acoustic Oscillations** (SN+WMAP) $N_\nu = 4.34^{+0.88}_{-0.86}$ at 68%
- **CMB** (Planck) $N_\nu = 3.15 \pm 0.23.$



1505.04192, Chacko+

Second important **limit** we will use later on: $m \gg T$

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3p,$$

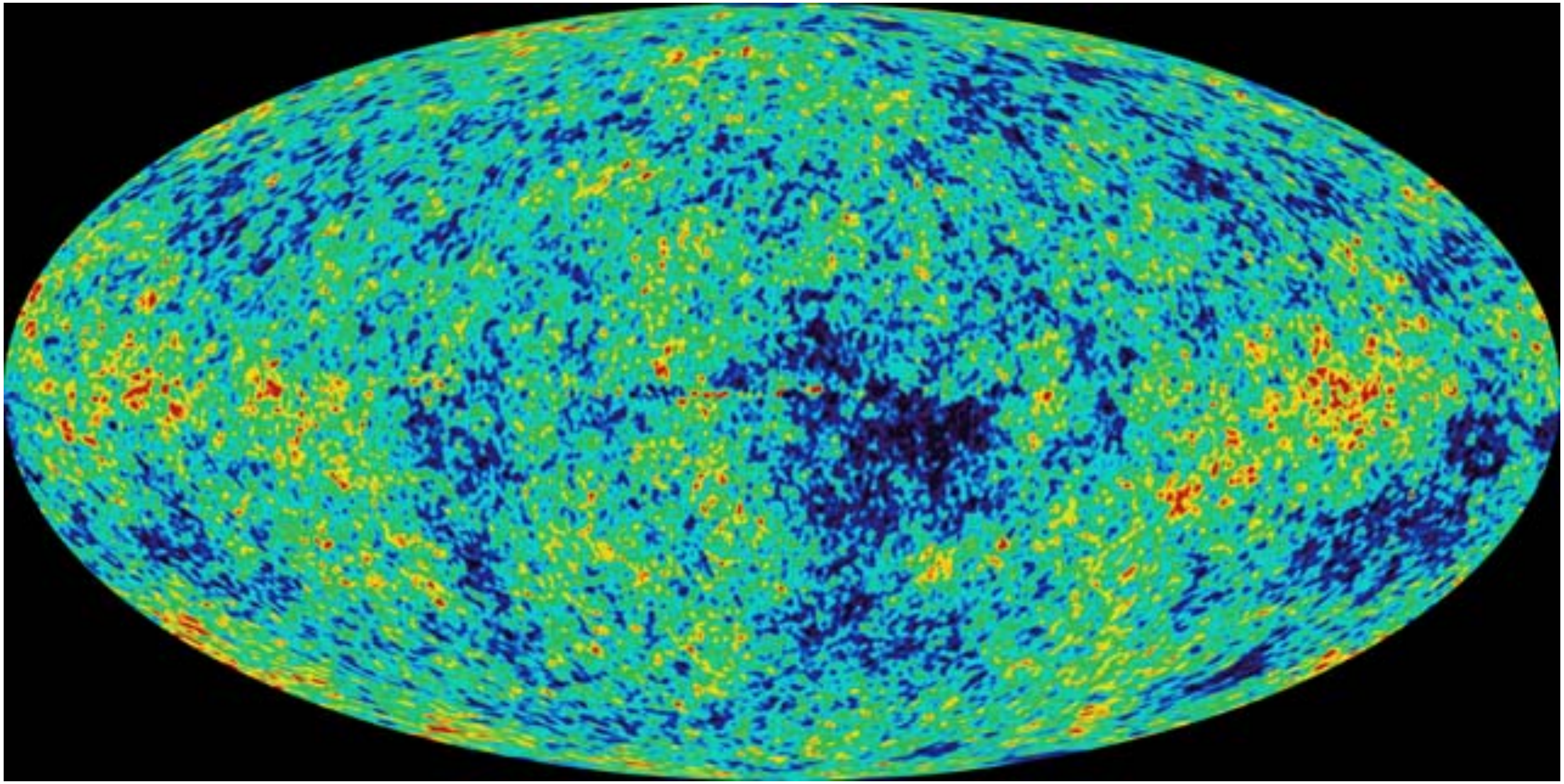
$$f(\vec{p}) = \left[\exp\left(\frac{E-\mu}{T}\right) \pm 1\right]^{-1}$$

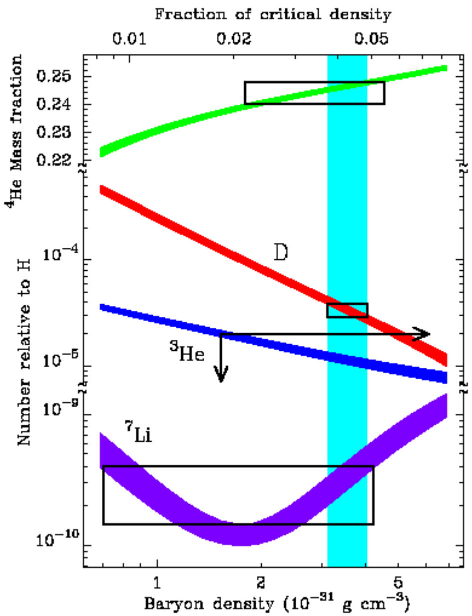
$$\rho = \frac{g}{(2\pi)^3} \int f(\vec{p})\, E(\vec{p})\, d^3p$$

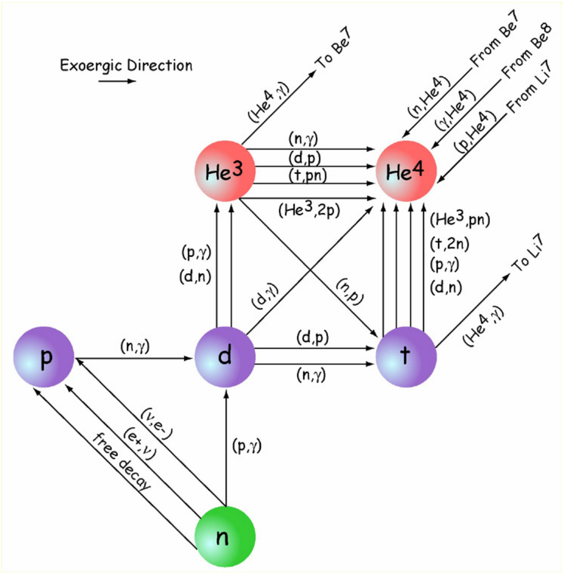
$$P = \frac{g}{(2\pi)^3} \int f(\vec{p})\, \frac{|\vec{p}|^2}{3E}\, d^3p$$

$$n = g\left(\frac{mT}{2\pi}\right)^{2/3} e^{-(m-\mu)/T}, \qquad \rho = mn, \qquad P = nT \ll \rho.$$

The Universe's **Thermodynamics** provides a successful framework for the **origin of species** in the early universe: **thermal decoupling**







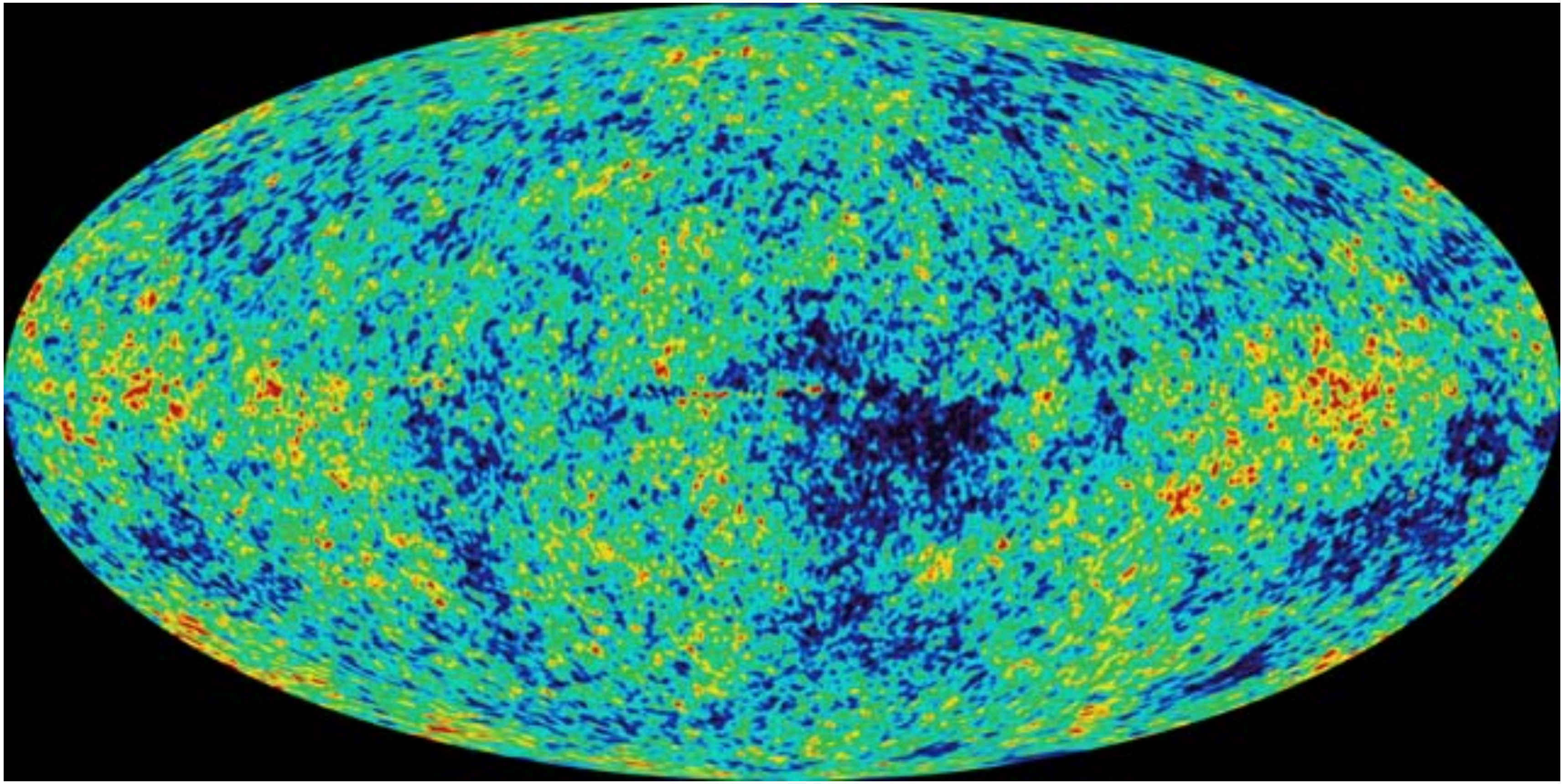
A successful **synergy** of ***statistical mechanics***,

general relativity, and of ***nuclear and particle physics***

making **predictions** testable to exquisite accuracy

with ***astronomical*** observations!

Event	time t
Inflation	10^{-34} s (?)
Baryogenesis	?
EW phase transition	20 ps
QCD phase transition	20 μs
Dark matter freeze-out	?
Neutrino decoupling	1 s
Electron-positron annihilation	6 s
Big Bang nucleosynthesis	3 min
Matter-radiation equality	60 kyr
Recombination	260–380 kyr
Photon decoupling	380 kyr
Reionization	100–400 Myr
Dark energy-matter equality	9 Gyr
Present	13.8 Gyr



CMB decouples because free electrons bind with protons to form Hydrogen atoms (**recombination**) when temperatures are around **13.6 eV**

n_e, n_p, and n_H

$$e^- + p \leftrightarrow H \qquad \mu_p + \mu_e = \mu_H$$

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} \exp \left(\frac{\mu_i - m_i}{T} \right)$$

$$n_H = g_h \left(\frac{m_H T}{2\pi} \right)^{3/2} \exp \left(\frac{\mu_p + \mu_e - m_H}{T} \right) = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi} \right)^{-3/2} e^{B/T}$$

$$B \equiv m_p + m_e - m_H = 13.6 \text{ eV} \qquad m_p \simeq m_H$$

$$n_H = g_h \left(\frac{m_H T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_p + \mu_e - m_H}{T}\right) = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

$$g_p = g_e = 2 \qquad g_H = 4 \qquad n_p = n_e$$

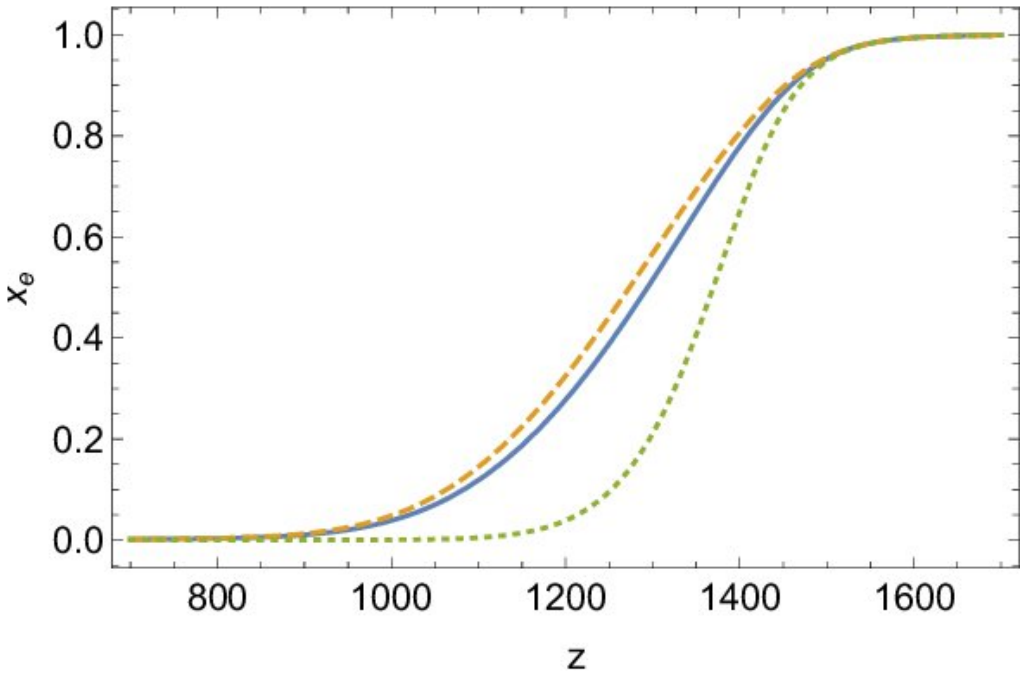
ionization **fraction**
(fraction of free electrons)

$$X_e \equiv n_p / n_H \qquad n_H = (1 - X_e) n_p$$

$$\frac{(1 - X_e^{\rm eq})}{(X_e^{\rm eq})^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}} \eta \left(\frac{T}{m_e}\right)^{-3/2} e^{B/T}$$

$$T(z) = T_0(1 + z)$$

Saha's equation and its solution



$$\frac{(1 - X_e^{\text{eq}})}{(X_e^{\text{eq}})^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}}\eta\left(\frac{T}{m_e}\right)^{-3/2} e^{B/T}$$

Saha’s equation describes recombination when the **reaction** leading to bound hydrogen atoms is in **equilibrium**

in reality, n_e=n_p are **dropping** exponentially, so the process ends when it goes **out of equilibrium**

Let’s investigate the process of “**freeze-out**” from the thermal bath, and actually calculate when **photons** decouple from **matter**

Key **idea** of thermal decoupling:
if the **reaction** keeping a species in equilibrium
is **faster** than the **expansion rate** of the universe,
the reaction is in **statistical equilibrium**;
if it's **slower**, the species **decouples** ("freeze-out")

$$\Gamma \ll H(T) \qquad \qquad \Gamma(T_{\mathrm{t.o.}}) \sim H(T_{\mathrm{t.o.}})$$

the **reaction rate** (from definition of cross section!)

$$\Gamma = n \cdot \sigma \cdot v$$

when do **CMB** photons decouple?

key process: **Thompson** scattering off of **free electrons** (X_e)

free electron number density are fixed by
(1) electron-positron **asymmetry** and (2) **recombination** (i.e. X_e)

$$\Gamma = n_e \, \sigma_T \, v_e$$

(we will see later v_e is not too small)

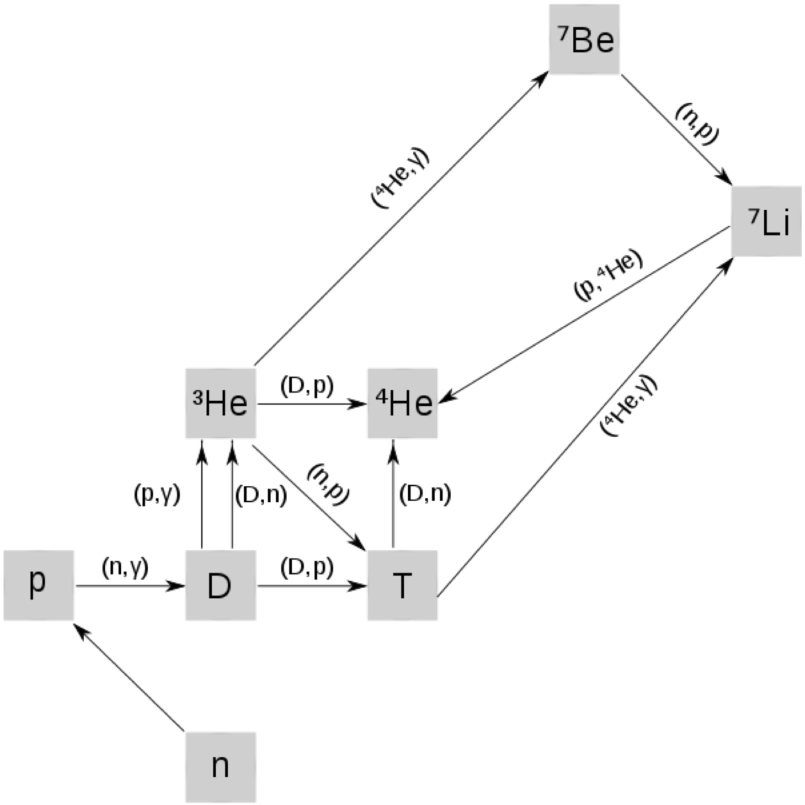
$$\sigma_T = 6.625 \times 10^{-25} \ \mathrm{cm}^2$$

$$H \simeq \frac{T^2}{M_P} \sim X_e \left(\frac{n_e}{s}\right) T^3 \sigma_T \simeq 0.01 \cdot 10^{-10} \cdot (6 \times 10^{-25}) \cdot 2.5 \times 10^{27} \cdot T^3$$

$$T \simeq \frac{3 \times 10^{-11} \ \mathrm{GeV}}{X_e} \simeq 0.3 \ \mathrm{eV}$$

redshifted to today, this **agrees** with the CMB black body spectrum!

Another (earlier) thermal **decoupling** process:
Big Bang **Nucleosynthesis**



$$p + n \longrightarrow {}^2H + \gamma$$
$$p + {}^2H \longrightarrow {}^3He + \gamma$$
$${}^2H + {}^2H \longrightarrow {}^3He + n$$
$${}^2H + {}^2H \longrightarrow {}^3H + p$$
$${}^3He + {}^2H \longrightarrow {}^4He + p$$
$${}^3H + {}^2H \longrightarrow {}^4He + n$$

will focus on one key prediction: **Helium4** mass fraction **~ 25%**

Early BBN: neutron decoupling $T \gg \mathrm{MeV}$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e$$

$$\mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\nu_e + n \leftrightarrow p + e^-$$

$$\frac{n}{p} \equiv \frac{n_n}{n_p} = \exp\left[-Q/T + (\mu_e - \mu_\nu)/T\right]$$

$$Q \equiv m_n - m_p = 1.293\ \mathrm{MeV}$$

$$\Gamma_{pe\leftrightarrow\nu n} = \begin{cases} \tau_n^{-1}(T/m_e)^3 e^{-Q/T} & T \ll Q, m_e \\ \frac{7\pi^4\tau_n^{-1}}{30\lambda_0}\left(\frac{T}{m_e}\right)^5 \simeq G_F^2 T^5 & T \gg Q, m_e \end{cases}$$

$$T \gtrsim m_e,\ (\Gamma/H) \simeq (T/0.8\,\mathrm{MeV})^3$$

$$\left(\frac{n}{p}\right)_{\rm freezeout} = e^{-Q/T_f} \simeq \frac{1}{6}$$

- all neutrons “find” protons and (via *D*) **form *^{4}He*** nuclei
- 1 in 5 neutrons **decay** → *n/p~1/7*

$$X_4 = \frac{4n_{^4He}}{n_N} = \frac{4(n_n/2)}{n_n + n_p} = \frac{2(n/p)}{1+(n/p)} \simeq 0.25$$

Go further up in **temperature** and back in **time**!

(Standard Model) neutrino freeze-out (**hot** thermal relic)

language definition: **hot** = relativistic at $T_{f.o}$

cold = *v<c=1*. (actually not by much,

key reaction: neutrino-antineutrino (back-)conversion to electron-positron pairs

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

$$n(T_\nu) \cdot \sigma(T_\nu) = H(T_\nu) \qquad \sigma \sim G_F^2 T_\nu^2$$

suppose this is a hot relic… $n \sim T_\nu^3$

$$T_\nu^3 G_F^2 T_\nu^2 = T_\nu^2 / M_P$$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \text{GeV} \sim 1\ \text{MeV}$$

happy about **two** things in particular:

1. **hot** relic assumption works! $T_\nu \gg m_\nu$

2. **Fermi** effective theory OK! $T_\nu \ll m_W$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \mathrm{GeV} \sim 1\ \mathrm{MeV}$$

now, how do we calculate the **relic** thermal **abundance** of this prototypical hot relic?

Introduce ***Y=n/s*** (ratio of number and entropy **density**, $V=a^3$)

If universe is iso-entropic (i.e. adiabatic), ***$s \times a^3 = S$*** is conserved

$Y \sim n\, a^3$ is thus ~ **comoving number density**, and (without entropy injection)

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$n_{\text{today}} = s_{\text{today}} \times Y_{\text{today}} = s_{\text{today}} \times Y_{\text{freeze-out}}$$

$$\rho_{\nu,\text{today}} = m_\nu \times Y_{\text{freeze-out}} \times s_{\text{today}}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\text{crit}}} h^2 \simeq \frac{m_\nu}{91.5 \text{ eV}}$$

Cowsik-Mc-Clelland limit

That was **fun**! Let's see if it works for something else...

Try **proton-antiproton** freeze-out:
what's the **relic** matter **abundance** in a baryon-symmetric Universe?

$$\sigma \sim \Lambda_{\rm QCD}^{-2}$$

$$n\ \sigma = H \rightarrow T^3\ \Lambda^{-2} = T^2/M_P \rightarrow T = \Lambda^2/M_P$$

doesn't quite work, we're way **outside**
the regime of validity for **hot relics**, since T<<<<<<<m_p ...

Need to work out the case of **cold relics**, which looks nastier by eye

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$