String Phenomenology

Michael Ratz

2019

Based on collaborations with:
M. Blaszczyk, F. Brümmer, W. Buchmüller, M.–C. Chen, M. Fallbacher, M. Fischer, S. Groot Nibbelink, K. Hamaguchi, R. Kappl, T. Kobayashi, O. Lebedev, H.M. Lee, R. Mohapatra, A. Mütter, H.P. Nilles, B. Petersen, F. Plöger, S. Raby, S. Ramos–Sánchez, G. Ross, F. Ruehle, R. Schieren, K. Schmidt–Hoberg, C. Staudt,V. Takhistov, A. Trautner, M. Trapletti, P. Vaudrevange & A. Wingerter

Overview I

1. Outline & plan
   - Unification of forces
   - Standard model matter
   - Concept
   - Pati–Salam vs. Georgi–Glashow
   - SO(10)
   - Supersymmetric standard model
   - Doublet–triplet splitting vs. full generations
   - Anomaly–free symmetries, μ and unification
2. Grand unification
   - Kaluza–Klein compactification
   - Gauge symmetry breaking in extra dimensions
   - 2–dimensional orbifolds
   - What is an orbifold?
   - Limitations of orbifold GUTs
3. Orbifold GUTs
5. Stringy orbifolds
   - Objectives

Overview II

- Historical remarks

4. Local GUTs in strings
   - String embedding
   - From strings to the real world?
   - Orbifold compactifications of the heterotic string
   - Orbifold GUTs from heterotic orbifolds
   - Example: $\mathbb{Z}_6$–II orbifold
   - Gauge symmetry breaking
   - The ‘orbifold construction kit’
   - Short summary of orbifold construction
   - Couplings
   - Construction of orbifold models: summary
   - Hierarchy between Planck and weak scales
   - Hierarchically small $\langle \mathscr{W} \rangle$
   - Explicit string theory realization

6. Heterotic Orbifold Phenomenology

7. Orbifold Phenomenology

Overview III

Outline & Plan

Notes & Disclaimers

References:

Sometimes I will put references to the original works, sometimes to literature in which the things mentioned are explained well. You can click on the references to get dragged to the INSPIRE record. I apologize for having to suppress references. My selection of references does not imply a rating.

Notes & Disclaimers

Selection of topics:

String phenomenology is a vast field which is impossible to completely survey. These lectures focus on string constructions in which there is an explicit reference to strings. This does not mean that supergravity constructions are "bad", it is only my interpretation of *string* phenomenology. I will also restrict the discussion on constructions which are not immediately ruled out.

Outline

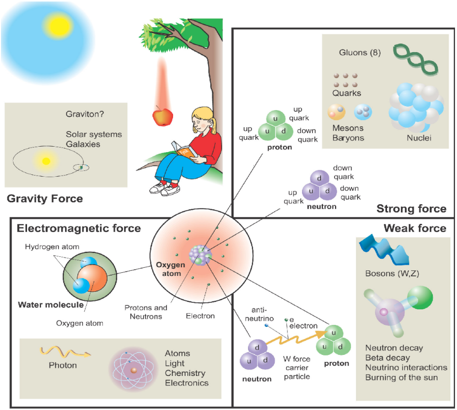
1. Introduction
2. Grand Unification in $D = 4$
3. Grand Unification in $D > 4$
4. Strings
5. String cosmology
6. Concluding remarks

The

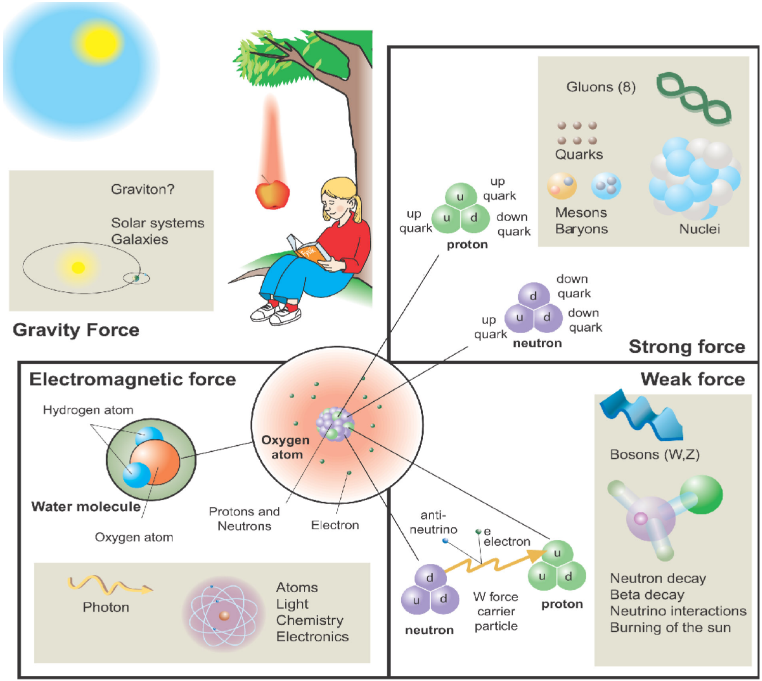
standard model

of particle phyics

is extremely successful in describing observation.



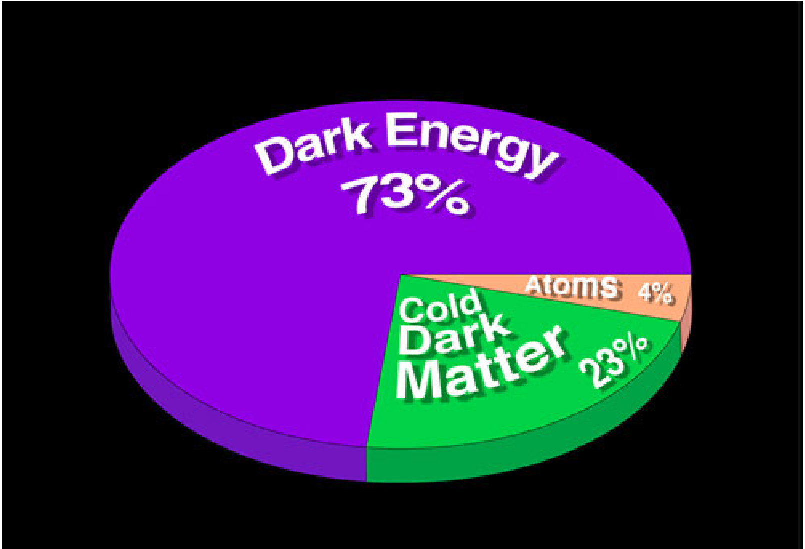
picture taken from http://www.nobelprize.org/



picture taken from http://www.nobelprize.org/

There are reasons to go beyond the standard model (SM):

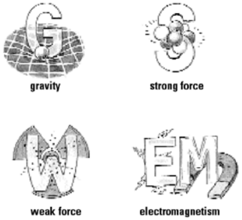
1. observational:
   - cold dark matter
   - baryon asymmetry of the universe



There are reasons to go beyond the standard model (SM):

1. observational:
   - cold dark matter
   - baryon asymmetry of the universe

2. theoretical: in the 'language' of the SM, quantum field theory, it is hard to describe gravitation

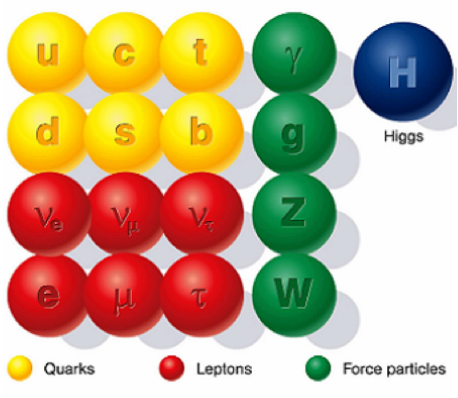


There are reasons to go beyond the standard model (SM):

1. observational:
   - cold dark matter
   - baryon asymmetry of the universe

2. theoretical: in the 'language' of the SM, quantum field theory, it is hard to describe gravitation

3. aesthetical: the structure of the SM is very 'peculiar'



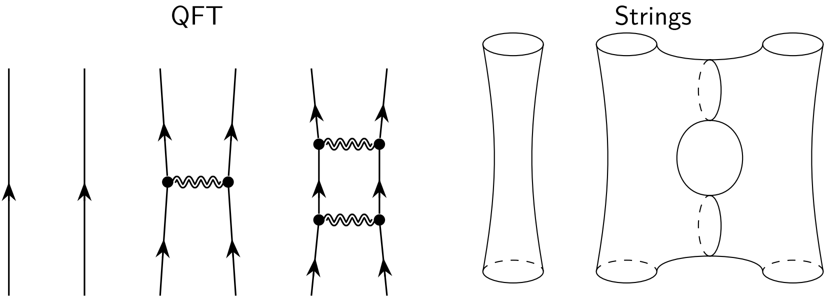
Why string model building

☞ Want to unify gauge theories, gravity and quantum effects

Why string model building

☞ Want to unify gauge theories, gravity and quantum effects

➥ Currently unique answer: strings



cf. Polchinski

Why strings?

cf. Polchinski

1. **Gravity**. Consistent description of gauge interactions and gravity.

Why strings?

cf. Polchinski

1. **Gravity**.
2. **Grand unification**. Unified description of all (standard model) gauge interactions.

Why strings?

cf. Polchinski

1. **Gravity**.
2. **Grand unification**.
3. **Supersymmetry**. After all, this is the PreSUSY summer school. 🙂

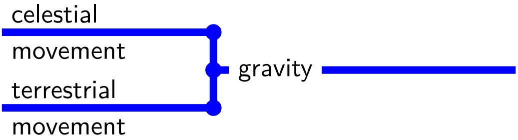
Why strings?

cf. Polchinski

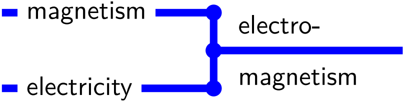
1. **Gravity**.
2. **Grand unification**.
3. **Supersymmetry**.
4. **No free parameters**. Isn't that really the aim of model building: reduce the number of free parameters? The standard model has at least 26 continuous parameters, even before adding something like an inflaton or dark matter candidate.

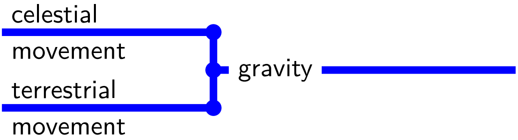
Unification of forces

Unification of all forces

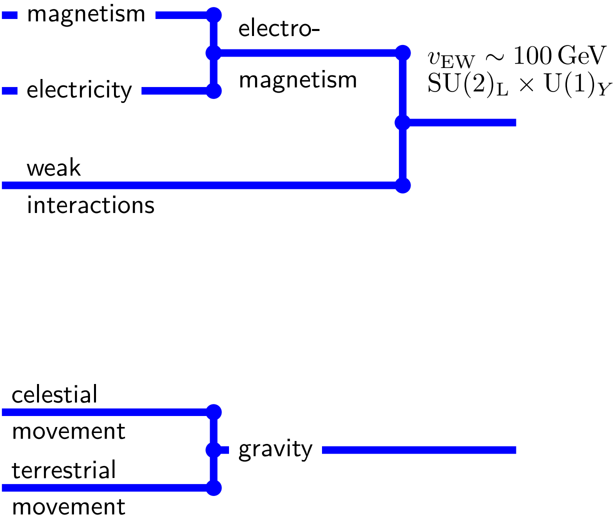


Unification of all forces

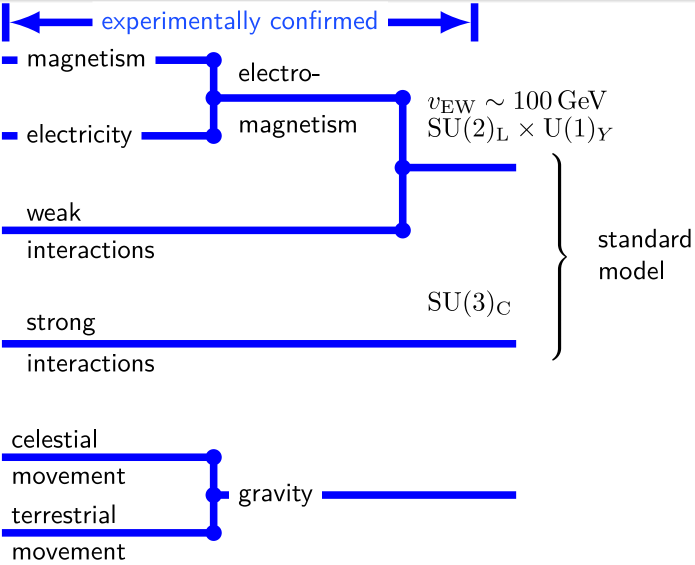




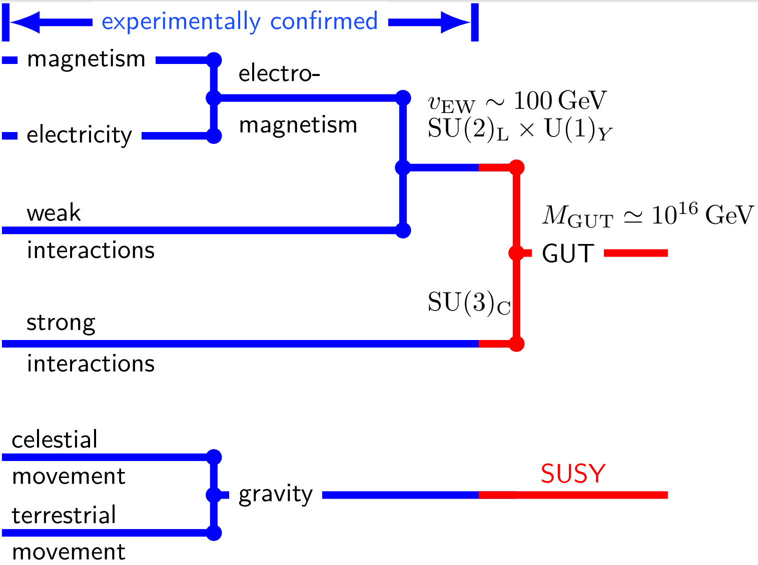
Unification of all forces



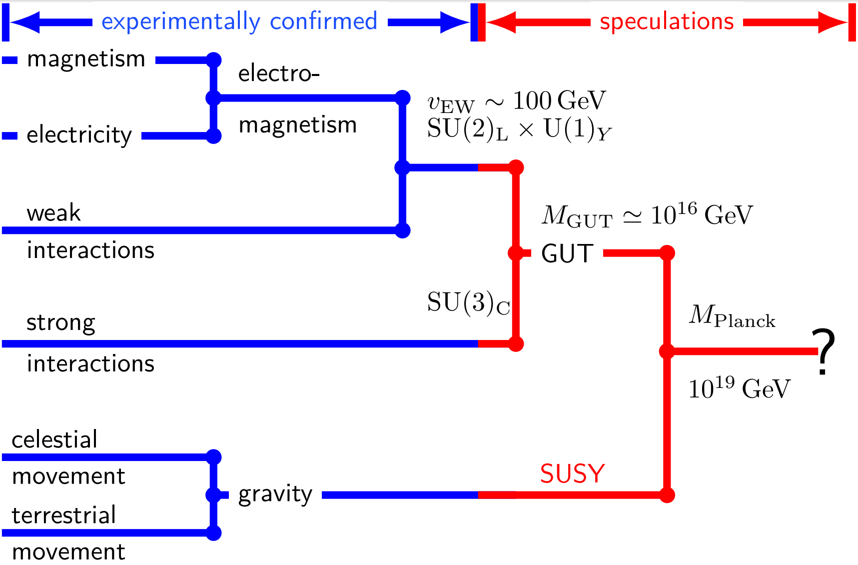
Unification of all forces



Unification of all forces

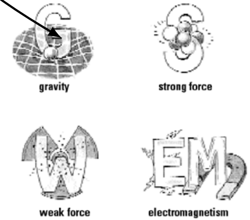


Unification of all forces



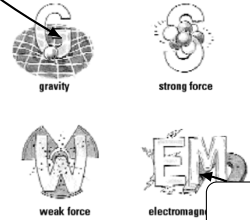
Forces in Nature

invariance
under local
coordinate
transformations



Forces in Nature

invariance under local coordinate transformations

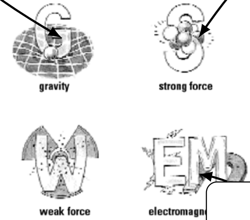


invariance under local U(1) rotation on an ‘internal circle’

Forces in Nature

invariance under local coordinate transformations

invariance under local SU(3) rotations

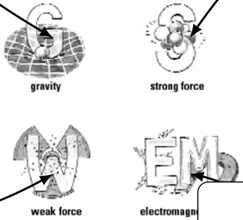


invariance under local U(1) rotation on an ‘internal circle’

Forces in Nature

invariance under local coordinate transformations

invariance under local SU(3) rotations



invariance under local SU(2) rotations

invariance under local U(1) rotation on an ‘internal circle’

Interactions

local U(1) rotation

$$\left(\begin{array}{c}\psi_1\\ \psi_2\end{array}\right) \to \left(\begin{array}{cc}\cos[\theta(x)] & \sin[\theta(x)]\\ -\sin[\theta(x)] & \cos[\theta(x)]\end{array}\right)\left(\begin{array}{c}\psi_1\\ \psi_2\end{array}\right)$$

or

$$\Psi \;\to\; \exp[\mathrm{i}\theta(x)]\,\Psi$$

e.g. electron

$$\psi_e \;\to\; \exp[\mathrm{i}\theta(x)\,q_e]\,\psi_e$$

Interactions

local SU(3) rotation : e.g. quark

$$\begin{pmatrix} \psi_q \\ \psi_q \\ \psi_q \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \qquad\qquad \begin{pmatrix} \psi_q \\ \psi_q \\ \psi_q \end{pmatrix}$$

Interactions

local SU(2) rotation : e.g. lepton

$$\begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix} \quad \to \quad \begin{pmatrix} * & * \\ * & * \end{pmatrix} \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix}$$

Interactions

local SU(5) rotation

$$\begin{pmatrix} \psi_q \\ \psi_q \\ \psi_q \\ \psi_\nu \\ \psi_e \end{pmatrix} \to \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix} \begin{pmatrix} \psi_q \\ \psi_q \\ \psi_q \\ \psi_\nu \\ \psi_e \end{pmatrix}$$

☞ all known (gauge) interactions can be unified in SU(5)

The structure

of the

standard model

hints at

unification

One generation of standard model matter

☞ left–handed quark doublets: $q_\mathrm{L} = \begin{array}{c} \left(\begin{array}{ccc} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{array} \right) \updownarrow \mathrm{SU}(2)_\mathrm{L} \\ \longleftrightarrow \\ \mathrm{SU}(3)_\mathrm{C} \end{array}$

One generation of standard model matter

☞ left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$

☞ left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix} \updownarrow \mathrm{SU}(2)_\mathrm{L}$

One generation of standard model matter

- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$
- right–handed u–type quarks: $u_\mathrm{R} = \begin{pmatrix} u_r & u_g & u_b \end{pmatrix}$

$$\xleftrightarrow{\hspace{3em}}$$
$$\mathrm{SU}(3)_\mathrm{C}$$

One generation of standard model matter

- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$
- right–handed u–type quarks: $u_\mathrm{R} = \begin{pmatrix} u_r & u_g & u_b \end{pmatrix}$
- right–handed d–type quarks: $d_\mathrm{R} = \begin{pmatrix} d_r & d_g & d_b \end{pmatrix}$

$$\xleftrightarrow{\hspace{3em}}$$
$$\mathrm{SU}(3)_\mathrm{C}$$

One generation of standard model matter

- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$
- right–handed u–type quarks: $u_\mathrm{R} = \begin{pmatrix} u_r & u_g & u_b \end{pmatrix}$
- right–handed d–type quarks: $d_\mathrm{R} = \begin{pmatrix} d_r & d_g & d_b \end{pmatrix}$
- right–handed lepton singlets: $e_\mathrm{R} = \begin{pmatrix} e \end{pmatrix} = \begin{pmatrix} e_\mathrm{R} \end{pmatrix}$

One generation of L–R symmetric matter

☞ left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$

☞ left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$

One generation of L–R symmetric matter

- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$
- right–handed u–type quarks: $u_\mathrm{R} = \begin{pmatrix} u_r & u_g & u_b \end{pmatrix}$

One generation of L–R symmetric matter

- ☞ left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- ☞ left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$
- ☞ right–handed u–type quarks: $u_\mathrm{R} = \begin{pmatrix} u_r & u_g & u_b \end{pmatrix}$
- ☞ right–handed d–type quarks: $d_\mathrm{R} = \begin{pmatrix} d_r & d_g & d_b \end{pmatrix}$

One generation of L–R symmetric matter

- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$

- left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$

- right–handed quark doublets: $q_\mathrm{R} = \underbrace{\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}}_{\mathrm{SU}(3)_\mathrm{C}} \updownarrow \mathrm{SU}(2)_\mathrm{R}$

One generation of L–R symmetric matter

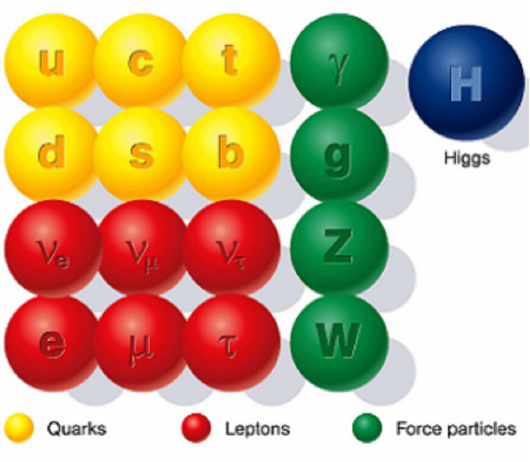
- left–handed quark doublets: $q_\mathrm{L} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- left–handed lepton doublets: $\ell_\mathrm{L} = \begin{pmatrix} \ell^\uparrow \\ \ell^\downarrow \end{pmatrix} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$
- right–handed quark doublets: $q_\mathrm{R} = \begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}$
- right–handed lepton doublets: $\ell_\mathrm{R} = \begin{pmatrix} \nu_\mathrm{R} \\ e_\mathrm{R} \end{pmatrix}$

new d.o.f.

Grand Unification

. . . in 4 dimensions

The standard model of particle physics



Pati–Salam vs. Georgi–Glashow

☞ Pati–Salam

$$\mathrm{SU}(2)_\mathrm{L}\updownarrow\underbrace{\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix}}_{\mathrm{SU}(3)_\mathrm{C}} \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$$

Pati–Salam vs. Georgi–Glashow

☞ Pati–Salam

$$\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix} \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix} \qquad \underbrace{\begin{pmatrix} \overline{q}_r^\uparrow & \overline{q}_g^\uparrow & \overline{q}_b^\uparrow \\ \overline{q}_r^\downarrow & \overline{q}_g^\downarrow & \overline{q}_b^\downarrow \end{pmatrix}}_{\mathrm{SU}(3)_\mathrm{C}} \begin{pmatrix} \nu_\mathrm{R} \\ e_\mathrm{R} \end{pmatrix} \updownarrow \mathrm{SU}(2)_\mathrm{R}$$

Pati–Salam vs. Georgi–Glashow

☞ Pati–Salam $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$

$$\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix} \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix} \qquad \begin{pmatrix} \overline{q}_r^\uparrow & \overline{q}_g^\uparrow & \overline{q}_b^\uparrow \\ \overline{q}_r^\downarrow & \overline{q}_g^\downarrow & \overline{q}_b^\downarrow \end{pmatrix} \begin{pmatrix} \nu_\mathrm{R} \\ e_\mathrm{R} \end{pmatrix}$$

$$(\mathbf{4}, \mathbf{2}, \mathbf{1})$$

Pati–Salam vs. Georgi–Glashow

☞ Pati–Salam $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$

$$\begin{pmatrix} q_r^{\uparrow} & q_g^{\uparrow} & q_b^{\uparrow} \\ q_r^{\downarrow} & q_g^{\downarrow} & q_b^{\downarrow} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix} \qquad \begin{pmatrix} \overline{q}_r^{\uparrow} & \overline{q}_g^{\uparrow} & \overline{q}_b^{\uparrow} \\ \overline{q}_r^{\downarrow} & \overline{q}_g^{\downarrow} & \overline{q}_b^{\downarrow} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{R}} \\ e_{\mathrm{R}} \end{pmatrix}$$

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \qquad (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$$

Pati–Salam vs. Georgi–Glashow

☞ Pati–Salam $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$

$$\begin{pmatrix} q_r^{\uparrow} & q_g^{\uparrow} & q_b^{\uparrow} \\ q_r^{\downarrow} & q_g^{\downarrow} & q_b^{\downarrow} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix} \qquad \begin{pmatrix} \overline{q}_r^{\uparrow} & \overline{q}_g^{\uparrow} & \overline{q}_b^{\uparrow} \\ \overline{q}_r^{\downarrow} & \overline{q}_g^{\downarrow} & \overline{q}_b^{\downarrow} \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{R}} \\ e_{\mathrm{R}} \end{pmatrix}$$

☞ Georgi–Glashow $\mathrm{SU}(5)$

$$\mathbf{10} = \begin{pmatrix} 0 & \overline{u}_b & -\overline{u}_g & q_r^{\uparrow} & q_r^{\downarrow} \\ \overline{u}_b & 0 & \overline{u}_r & q_g^{\uparrow} & q_g^{\downarrow} \\ \overline{u}_g & -\overline{u}_r & 0 & q_g^{\uparrow} & q_g^{\downarrow} \\ -q_r^{\uparrow} & -q_g^{\uparrow} & -q_b^{\uparrow} & 0 & \overline{e} \\ -q_r^{\downarrow} & -q_g^{\downarrow} & -q_b^{\downarrow} & -\overline{e} & 0 \end{pmatrix}$$

▸ details

Pati–Salam vs. Georgi–Glashow

☞ Pati–Salam $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$

$$\begin{pmatrix} q_r^\uparrow & q_g^\uparrow & q_b^\uparrow \\ q_r^\downarrow & q_g^\downarrow & q_b^\downarrow \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix} \qquad \begin{pmatrix} \overline{q}_r^\uparrow & \overline{q}_g^\uparrow & \overline{q}_b^\uparrow \\ \overline{q}_r^\downarrow & \overline{q}_g^\downarrow & \overline{q}_b^\downarrow \end{pmatrix} \begin{pmatrix} \nu_{\mathrm{R}} \\ e_{\mathrm{R}} \end{pmatrix}$$

☞ Georgi–Glashow $\mathrm{SU}(5)$

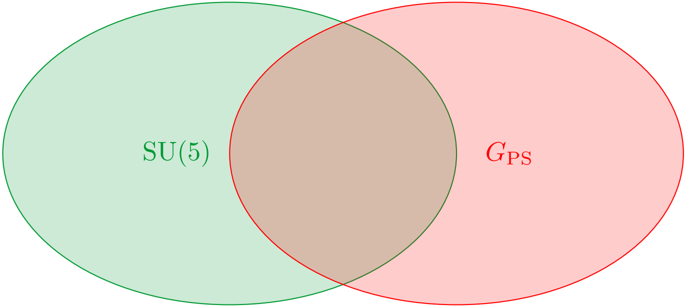
$$\mathbf{10} = \begin{pmatrix} 0 & \overline{u}_b & -\overline{u}_g & q_r^\uparrow & q_r^\downarrow \\ \overline{u}_b & 0 & \overline{u}_r & q_g^\uparrow & q_g^\downarrow \\ \overline{u}_g & -\overline{u}_r & 0 & q_g^\uparrow & q_g^\downarrow \\ -q_r^\uparrow & -q_g^\uparrow & -q_b^\uparrow & 0 & \overline{e} \\ -q_r^\downarrow & -q_g^\downarrow & -q_b^\downarrow & -\overline{e} & 0 \end{pmatrix} \qquad \overline{\mathbf{5}} = \begin{pmatrix} \nu_{\mathrm{L}} \\ e_{\mathrm{L}} \\ \overline{d}_r \\ \overline{d}_g \\ \overline{d}_b \end{pmatrix}$$

details

SO(10)

Asaka, Buchmüller & Covi [2001]

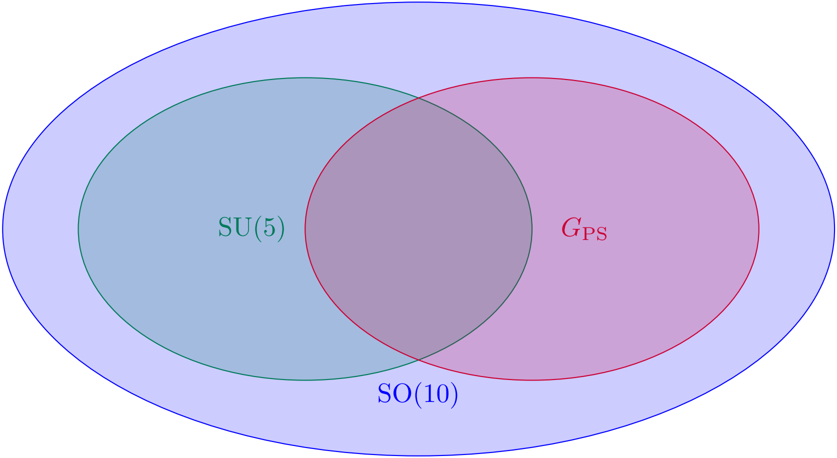
☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is $\mathrm{SO}(10)$



SO(10)

Asaka, Buchmüller & Covi [2001]

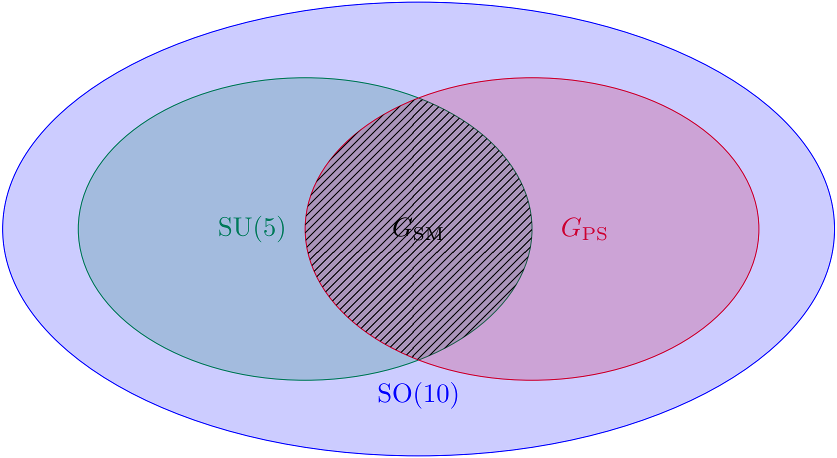
☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is $\mathsf{SO}(10)$



SO(10)

Asaka, Buchmüller & Covi [2001]

☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is SO(10)



SU(5)

$\mathrm{SU}(5)$ grand unified theory (GUT) ...

- ☞ explains charge quantization
- ☞ simplifies matter content

$$\text{SM generation} \;=\; \mathbf{10} + \overline{\mathbf{5}}$$

SU(5) and SO(10)

$\mathrm{SU}(5)$ grand unified theory (GUT) ...

- ☞ explains charge quantization
- ☞ simplifies matter content

$$\text{SM generation} \;=\; \mathbf{10} + \overline{\mathbf{5}}$$

further simplification of matter sector Fritzsch & Minkowski [1975]

$$\mathrm{SO}(10) \supset \mathrm{SU}(5)$$

$$\mathbf{16} = \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$$

$$= \text{SM generation with 'right–handed' neutrino}$$

SU(5) and SO(10)

$\mathrm{SU}(5)$ grand unified theory (GUT) ...

☞ explains charge quantization

☞ simplifies matter content

SM generation $= \mathbf{10} + \overline{\mathbf{5}}$

further simplification of matter sector Fritzsch & Minkowski [1975]

$$\mathrm{SO}(10) \supset \mathrm{SU}(5)$$
$$\mathbf{16} = \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$$

➥ once there is an electron, $\mathrm{SO}(10)$ tells us that there are also u and d quarks, i.e. protons and neutrons!

SU(5) and SO(10)

$\mathrm{SU}(5)$ grand unified theory (GUT) ...

- ☞ explains charge quantization
- ☞ simplifies matter content

  SM generation $= \mathbf{10} + \overline{\mathbf{5}}$

further simplification of matter sector

Fritzsch & Minkowski [1975]

$$\mathrm{SO}(10) \supset \mathrm{SU}(5)$$
$$\mathbf{16} = \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$$

- ➥ once there is an electron, $\mathrm{SO}(10)$ tells us that there are also u and d quarks, i.e. protons and neutrons!
- ☞ **However**: coupling strengths are measured to be different

SU(5) and SO(10)

$\mathrm{SU}(5)$ grand unified theory (GUT) ...

- explains charge quantization
- simplifies matter content

  SM generation $= \mathbf{10} + \overline{\mathbf{5}}$

further simplification of matter sector

Fritzsch & Minkowski [1975]

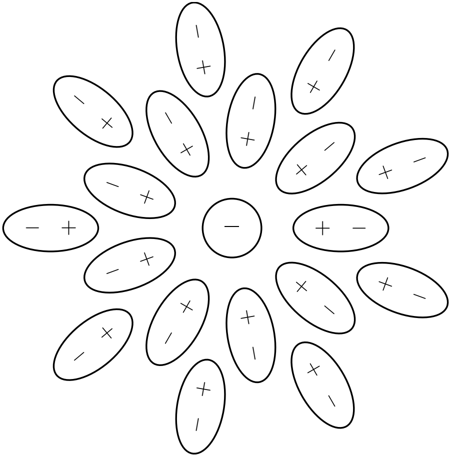
$$\mathrm{SO}(10) \supset \mathrm{SU}(5)$$
$$\mathbf{16} = \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$$

➥ once there is an electron, $\mathrm{SO}(10)$ tells us that there are also u and d quarks, i.e. protons and neutrons!

- **However**: coupling strengths are measured to be different
- **Rescue**: in quantum field theory couplings depend on energy scale (‘running couplings’)

Running couplings

☞ naïve picture: virtual particle–antiparticle–pairs screen charge



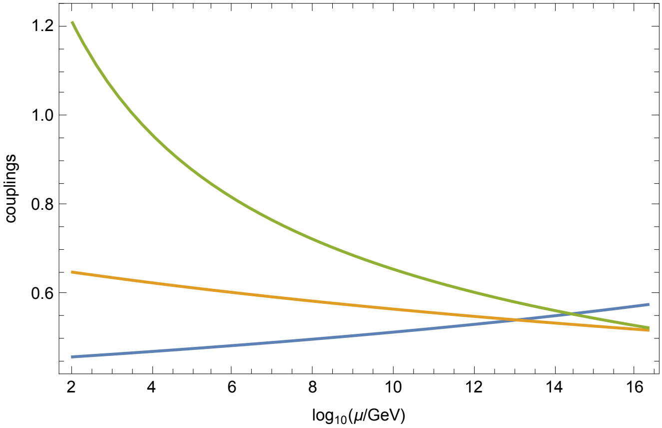
Running couplings

- naïve picture: virtual particle–antiparticle–pairs screen charge
- distance inversely proportional to energy

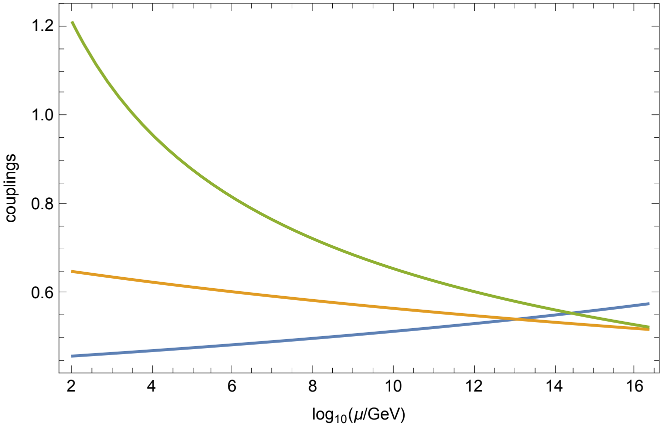
Running couplings

- ☞ naïve picture: virtual particle–antiparticle–pairs screen charge
- ☞ distance inversely proportional to energy
- ➥ couplings depend on energy/distance

Running couplings in the standard model

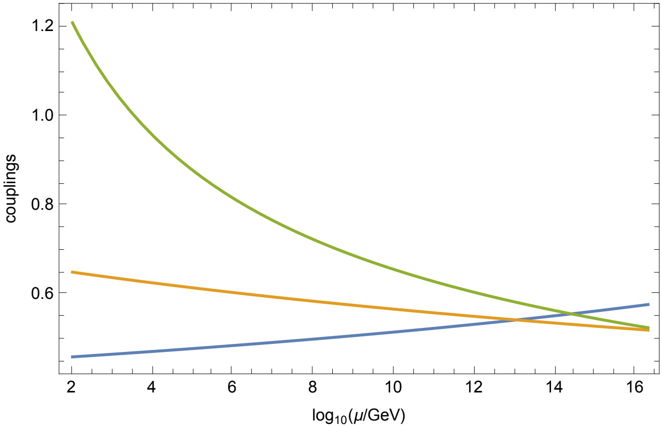


Running couplings in the standard model



☞ qualitatively nice: couplings approach each other

Running couplings in the standard model

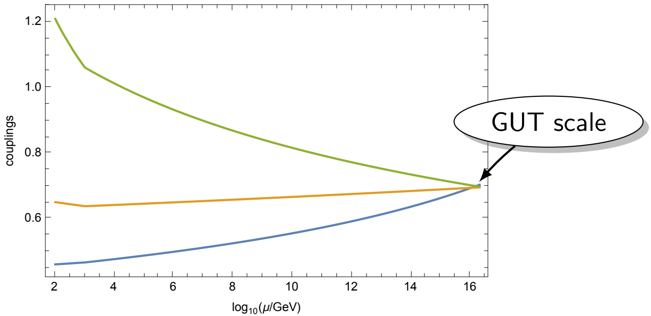


☞ qualitatively nice: couplings approach each other

☞ however: no (precision) unification

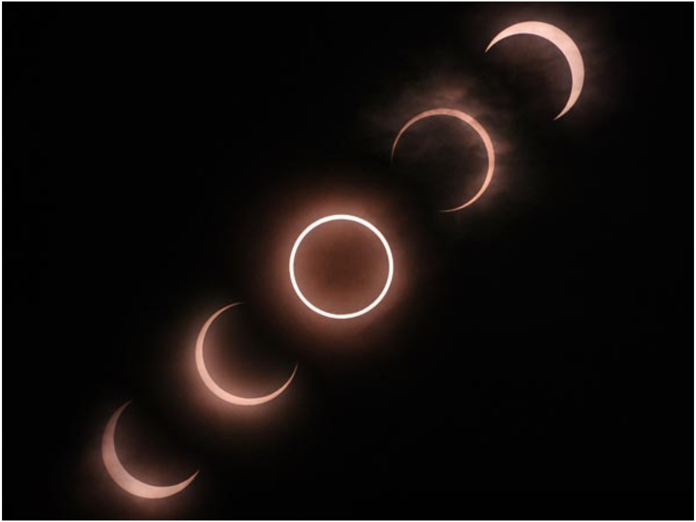
Running couplings in the MSSM

... gauge coupling unification in the (minimal) supersymmetric standard model



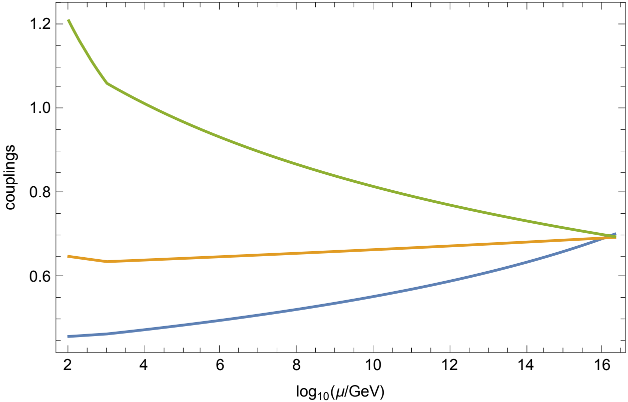
☞ **interpretation**: there is only one coupling at the fundamental level, the numerical difference between the couplings is due to quantum effects

Accidents in Nature



Why supersymmetry?

☞ gauge coupling unification



Why supersymmetry?

- gauge coupling unification
- supersymmetry stabilizes the electroweak scale against the GUT scale M_{GUT} $\curvearrowright$ solution of the hierarchy problem

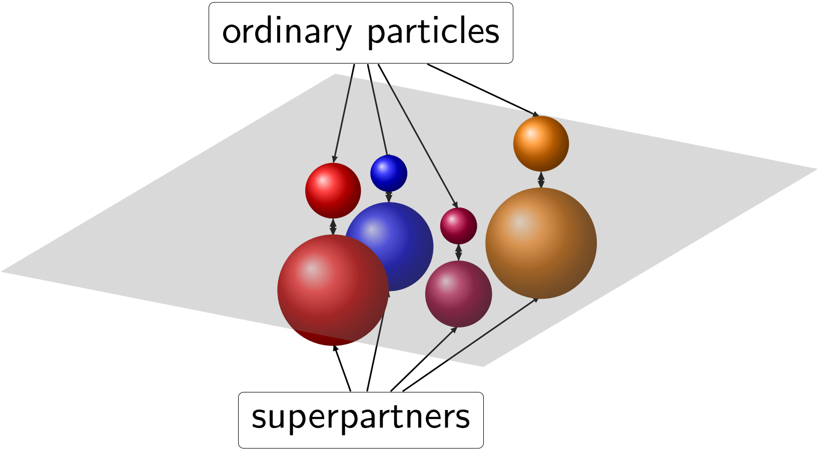
Why supersymmetry?

- gauge coupling unification
- supersymmetry stabilizes the electroweak scale against the GUT scale M_{GUT} $\curvearrowright$ solution of the hierarchy problem
- supersymmetry is the unique extension of the (Poincaré) symmetry of our space–time

Why supersymmetry?

- gauge coupling unification
- supersymmetry stabilizes the electroweak scale against the GUT scale M_{GUT} $\curvearrowright$ solution of the hierarchy problem
- supersymmetry is the unique extension of the (Poincaré) symmetry of our space–time
- supersymmetry provides the so–called lightest superpartner (LSP), a plausible candidate for **cold dark matter**

What is supersymmetry (SUSY)?



Where is SUSY?

Is SUSY for real?

. . . we may see . . .

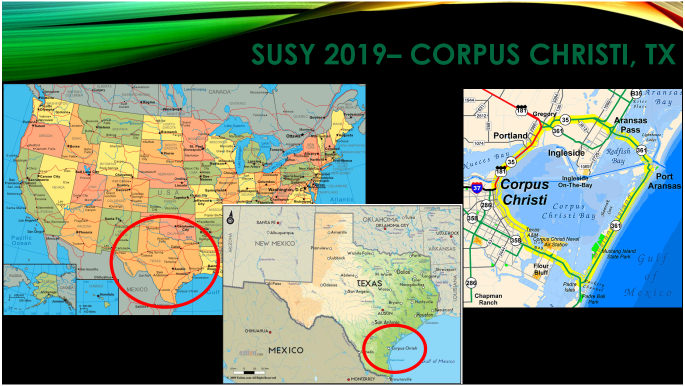
Is SUSY for real?

. . . we may see . . .

. . . or maybe not 🙂

Where is SUSY?

- Answer: in 2019 in Corpus Cristi (TX)



Grand unification and neutrino mass

☞ scale of grand unification $\sim 10^{16}\,\mathrm{GeV}$

Grand unification and neutrino mass

- scale of grand unification $\sim 10^{16}$ GeV

rather similar

- naïve see–saw scale $\sim 10^{14}$ GeV

Grand unification and neutrino mass

☞ scale of grand unification $\sim 10^{16}$ GeV

rather similar

☞ naïve see–saw scale $\sim 10^{14}$ GeV

➥ question: is there a relation between these scales?

SO(10)

☞ $\mathrm{SU}(5) \subset \mathrm{SO}(10)$

$\mathrm{SO}(10)$

- $\mathrm{SU}(5) \subset \mathrm{SO}(10)$
- 16–dimensional spinor representation of $\mathrm{SO}(10)$ contains full generation

$$\begin{array}{ccc} \mathrm{SO}(10) & \supset & \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \\ \mathbf{16} & \to & \mathbf{10}_1 \oplus \overline{\mathbf{5}}_{-3} \oplus \mathbf{1}_5 \end{array}$$

$q + u^C + e^C$

$\ell + d^C$

ν^C

SO(10)

- $\mathrm{SU}(5) \subset \mathrm{SO}(10)$
- 16–dimensional spinor representation of $\mathrm{SO}(10)$ contains full generation

$$\begin{array}{ccc} \mathrm{SO}(10) & \supset & \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \\ \mathbf{16} & \to & \mathbf{10}_1 \oplus \overline{\mathbf{5}}_{-3} \oplus \mathbf{1}_5 \end{array}$$

- $\mathbf{16}$–plet as the product of five two–dimensional spinors

$$\begin{aligned} \psi &= \psi^{(1)} \otimes \psi^{(2)} \otimes \psi^{(3)} \otimes \psi^{(4)} \otimes \psi^{(5)} \\ &= \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \\ &=: (\pm \pm \pm \pm \pm) \end{aligned}$$

SO(10) spinor

see Raby [2009]

		SO(10) GUT		
SM	$\mathrm{U}(1)_Y$	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(2)_\mathrm{L}$	
$\nu^\mathcal{C}$	0	$+\,+\,+$	$+\,+$	$\mathbf{1}$
$e^\mathcal{C}$	1	$+\,+\,+$	$-\,-$	$\mathbf{10}$
u_red	$\frac{1}{6}$	$-\,+\,+$	$+\,-$	
d_red		$-\,+\,+$	$-\,+$	
u_green		$+\,-\,+$	$+\,-$	
d_green		$+\,-\,+$	$-\,+$	
u_blue		$+\,+\,-$	$+\,-$	
d_blue		$+\,+\,-$	$-\,+$	
$u^\mathcal{C}_\mathrm{red}$	$-\frac{2}{3}$	$+\,-\,-$	$+\,+$	
$u^\mathcal{C}_\mathrm{green}$		$-\,+\,-$	$+\,+$	
$u^\mathcal{C}_\mathrm{blue}$		$-\,-\,+$	$+\,+$	
$d^\mathcal{C}_\mathrm{red}$	$\frac{1}{3}$	$+\,-\,-$	$-\,-$	$\overline{\mathbf{5}}$
$d^\mathcal{C}_\mathrm{green}$		$-\,+\,-$	$-\,-$	
$d^\mathcal{C}_\mathrm{blue}$		$-\,-\,+$	$-\,-$	
ν	$-\frac{1}{2}$	$-\,-\,-$	$+\,-$	
e		$-\,-\,-$	$-\,+$	

Higgs sector

- ☞ smallest $\mathrm{SO}(10)$ representation that contains the Higgs doublet: $\mathbf{10}$–plet

Higgs sector

- ☞ smallest $\mathrm{SO}(10)$ representation that contains the Higgs doublet: $\mathbf{10}$–plet
- ➥ get automatically two doublets (like in the MSSM)

Proton decay

☞ couplings between standard model matter and extra gauge bosons

$$
\begin{array}{rcl}
(\mathbf{3},\mathbf{2})_{1/6}\,(\mathbf{3},\mathbf{2})_{-5/6}\,(\mathbf{3},\mathbf{1})_{2/3} & : & \varepsilon_{ij}\,\varepsilon^{abc}\,\overline{u_a^{\mathcal{C}}}\,\gamma^\mu\,q_b^i\,(X_c^j)_\mu \\
(\mathbf{3},\mathbf{2})_{1/6}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{1},\mathbf{1})_{-1} & : & \varepsilon_{ij}\,\overline{e^{\mathcal{C}}}\,\gamma^\mu\,q_a^i\,(\overline{X}^j_{\;a})_\mu \\
(\mathbf{1},\mathbf{2})_{-1/2}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{3},\mathbf{1})_{-1/3} & : & \varepsilon_{ij}\,\overline{d_a^{\mathcal{C}}}\,\gamma^\mu\,\ell^i\,(\overline{X}^j_{\;a})_\mu
\end{array}
$$

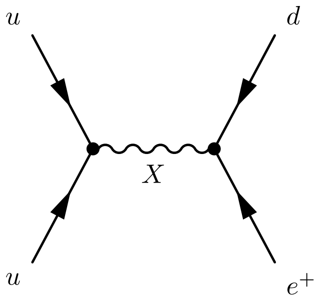
Proton decay

☞ couplings between standard model matter and extra gauge bosons

$$(\mathbf{3},\mathbf{2})_{1/6}\,(\mathbf{3},\mathbf{2})_{-5/6}\,(\mathbf{3},\mathbf{1})_{2/3} \quad : \quad \varepsilon_{ij}\,\varepsilon^{abc}\,\overline{u_a^{\mathcal{C}}}\,\gamma^\mu\,q_b^i\,(X_c^j)_\mu$$

$$(\mathbf{3},\mathbf{2})_{1/6}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{1},\mathbf{1})_{-1} \quad : \quad \varepsilon_{ij}\,\overline{e^{\mathcal{C}}}\,\gamma^\mu\,q_a^i\,(\overline{X}{}_a^j)_\mu$$

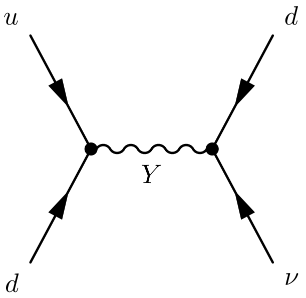
$$(\mathbf{1},\mathbf{2})_{-1/2}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{3},\mathbf{1})_{-1/3} \quad : \quad \varepsilon_{ij}\,\overline{d_a^{\mathcal{C}}}\,\gamma^\mu\,\ell^i\,(\overline{X}{}_a^j)_\mu$$



Proton decay

☞ couplings between standard model matter and extra gauge bosons

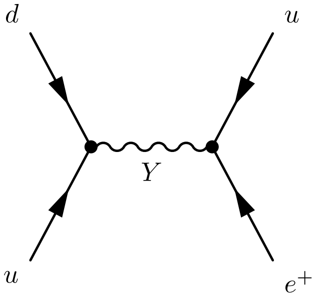
$$
\begin{array}{rcl}
(\mathbf{3},\mathbf{2})_{1/6}\,(\mathbf{3},\mathbf{2})_{-5/6}\,(\mathbf{3},\mathbf{1})_{2/3} & : & \varepsilon_{ij}\,\varepsilon^{abc}\,\overline{u_a^{\mathcal{C}}}\,\gamma^\mu\,q_b^i\,(X_c^j)_\mu \\
(\mathbf{3},\mathbf{2})_{1/6}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{1},\mathbf{1})_{-1} & : & \varepsilon_{ij}\,\overline{e^{\mathcal{C}}}\,\gamma^\mu\,q_a^i\,(\overline{X}{}_a^j)_\mu \\
(\mathbf{1},\mathbf{2})_{-1/2}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{3},\mathbf{1})_{-1/3} & : & \varepsilon_{ij}\,\overline{d_a^{\mathcal{C}}}\,\gamma^\mu\,\ell^i\,(\overline{X}{}_a^j)_\mu
\end{array}
$$



Proton decay

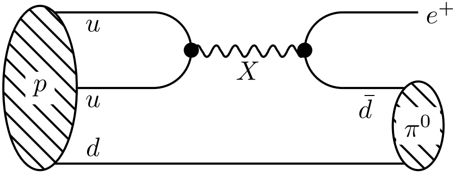
- couplings between standard model matter and extra gauge bosons

$$\begin{aligned}
(\mathbf{3},\mathbf{2})_{1/6}\,(\mathbf{3},\mathbf{2})_{-5/6}\,(\mathbf{3},\mathbf{1})_{2/3} &\quad:\quad \varepsilon_{ij}\,\varepsilon^{abc}\,\overline{u^{\mathcal{C}}_a}\,\gamma^\mu\,q^i_b\,(X^j_c)_\mu\\
(\mathbf{3},\mathbf{2})_{1/6}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{1},\mathbf{1})_{-1} &\quad:\quad \varepsilon_{ij}\,\overline{e^{\mathcal{C}}}\,\gamma^\mu\,q^i_a\,(\overline{X}^j_a)_\mu\\
(\mathbf{1},\mathbf{2})_{-1/2}\,(\overline{\mathbf{3}},\mathbf{2})_{5/6}\,(\mathbf{3},\mathbf{1})_{-1/3} &\quad:\quad \color{red}{\varepsilon_{ij}\,\overline{d^{\mathcal{C}}_a}\,\gamma^\mu\,\ell^i\,(\overline{X}^j_a)_\mu}
\end{aligned}$$



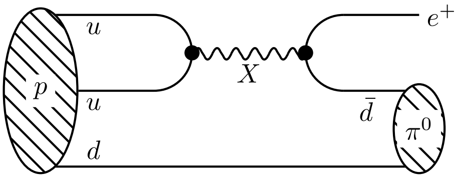
Gauge boson mediated proton decay

- process $p \ \rightarrow \ \pi^0 + e^+$
- proton life–time
  $\tau_p \ \gtrsim \ 10^{33}$ years



Gauge boson mediated proton decay

- process $p \;\rightarrow\; \pi^0 + e^+$
- proton life–time
  $\tau_p \;\gtrsim\; 10^{33}$ years



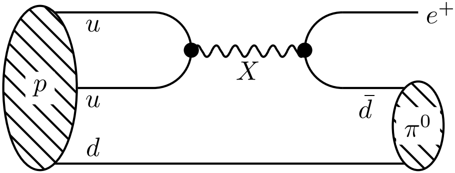
- proton decay rate

$$\Gamma_p \;\sim\; \alpha_2^2\,\frac{m_p^5}{M_X^4} \;\simeq\; 10^{-63}\cdot\left(\frac{10^{15}\,\text{GeV}}{M_X}\right)^4\;\text{GeV}$$

$$\simeq\; 10^{-37}\cdot\left(\frac{10^{15}\,\text{GeV}}{M_X}\right)^4\;\text{s}^{-1}$$

Gauge boson mediated proton decay

- process $p \rightarrow \pi^0 + e^+$
- proton life–time
  $\tau_p \gtrsim 10^{33}$ years



- proton decay rate

$$\Gamma_p \sim \alpha_2^2 \frac{m_p^5}{M_X^4} \simeq 10^{-63} \cdot \left(\frac{10^{15}\,\text{GeV}}{M_X}\right)^4 \text{GeV}$$

$$\simeq 10^{-37} \cdot \left(\frac{10^{15}\,\text{GeV}}{M_X}\right)^4 \text{s}^{-1}$$

- supersymmetric grand unification

$M_X \sim 2 \cdot 10^{16}$ GeV $\curvearrowright$ $\tau_p \simeq 10^{35}$ years

Supersymmetric grand unification

☞ gauge symmetry: $\mathrm{SO}(10)$

Supersymmetric grand unification

- ☞ gauge symmetry: $\mathrm{SO}(10)$
- ☞ matter: 3 $\mathbf{16}$–plets ψ_f

Supersymmetric grand unification

- gauge symmetry: $\mathrm{SO}(10)$
- matter: 3 $\mathbf{16}$–plets ψ_f
- branching

$$
\begin{aligned}
\mathrm{SO}(10) &\xrightarrow{\langle\mathbf{16}\rangle} \mathrm{SU}(5)\\
\mathbf{10} &\xrightarrow{\langle\mathbf{16}\rangle} \mathbf{5}\oplus\overline{\mathbf{5}}\\
\mathbf{16} &\xrightarrow{\langle\mathbf{16}\rangle} \mathbf{10}\oplus\overline{\mathbf{5}}\oplus\mathbf{1}\\
\mathbf{45} &\xrightarrow{\langle\mathbf{16}\rangle} \mathbf{24}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{1}
\end{aligned}
$$

Supersymmetric grand unification

- gauge symmetry: $\mathrm{SO}(10)$
- matter: 3 $\mathbf{16}$–plets ψ_f
- branching

$$\begin{aligned}
\mathrm{SO}(10) &\xrightarrow{\langle\mathbf{16}\rangle} \mathrm{SU}(5)\\
\mathbf{10} &\xrightarrow{\langle\mathbf{16}\rangle} \mathbf{5}\oplus\overline{\mathbf{5}}\\
\mathbf{16} &\xrightarrow{\langle\mathbf{16}\rangle} \mathbf{10}\oplus\overline{\mathbf{5}}\oplus\mathbf{1}\\
\mathbf{45} &\xrightarrow{\langle\mathbf{16}\rangle} \mathbf{24}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{1}
\end{aligned}$$

- representation content of some simple model

name	ψ_f	ϕ	χ	$\overline{\chi}$	H
$\mathrm{SO}(10)$ irrep	$\mathbf{16}$	$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{16}}$	$\mathbf{45}$

Fermion mass relations (I)

☞ at the renormalizable level there is only one type of Yukawa couplings

$$\mathscr{W}^{\mathrm{SO}(10)}_{\mathrm{Yukawa}} = Y^{fg}_{10}\,\psi_f\,\psi_g\,\phi$$

symmetric

Fermion mass relations (I)

☞ at the renormalizable level there is only one type of Yukawa couplings

$$\mathscr{W}_{\mathrm{Yukawa}}^{\mathrm{SO}(10)} \;=\; Y_{10}^{fg}\,\psi_f\,\psi_g\,\phi$$

➥ prediction at the renormalizable level

$$Y_u \;=\; Y_d \;=\; Y_e \quad \text{at } M_{\mathrm{GUT}}$$

Fermion mass relations (I)

☞ at the renormalizable level there is only one type of Yukawa couplings

$$\mathscr{W}_{\text{Yukawa}}^{\text{SO}(10)} \;=\; Y_{10}^{fg}\,\psi_f\,\psi_g\,\phi$$

➥ prediction at the renormalizable level

$$Y_u \;=\; Y_d \;=\; Y_e \quad \text{at } M_{\text{GUT}}$$

☞ consistent relation (large $\tan\beta$)

$$y_t \;=\; y_b \;=\; y_\tau \quad \text{at } M_{\text{GUT}}$$

Fermion mass relations (I)

☞ at the renormalizable level there is only one type of Yukawa couplings

$$\mathscr{W}_{\text{Yukawa}}^{\text{SO(10)}} = Y_{10}^{fg}\,\psi_f\,\psi_g\,\phi$$

➥ prediction at the renormalizable level

$$Y_u = Y_d = Y_e \quad \text{at } M_{\text{GUT}}$$

☞ consistent relation (large $\tan\beta$)

$$y_t = y_b = y_\tau \quad \text{at } M_{\text{GUT}}$$

☞ $\mathrm{SO}(10)$ relations inconsistent for light generations

Fermion mass relations (II)

☞ potential rescue: higher–dimensional couplings e.g. Pati [2006]

$$\mathscr{W}_{\mathrm{eff}} \supset \sum_{i=1}^{2} \frac{\lambda_{gf}^{(i)}}{M_{\mathrm{P}}} \psi_g \psi_f \phi H$$

Fermion mass relations (II)

☞ potential rescue: higher–dimensional couplings e.g. Pati [2006]

$$\mathscr{W}_{\mathrm{eff}} \;\supset\; \sum_{i=1}^{2} \frac{\lambda_{gf}^{(i)}}{M_{\mathrm{P}}}\, \psi_g\, \psi_f\, \phi\, H$$

☞ neutrino couplings

$$\mathscr{W}_{\mathrm{eff}} \;\supset\; \overline{\kappa}_{gf}\, \psi_g\, \psi_f\, \frac{\overline{\chi}\,\overline{\chi}}{M_{\mathrm{P}}} \xrightarrow{\overline{\chi}\to\langle\overline{\chi}\rangle} M_{fg}\, {\nu^{\mathcal{C}}}_f\, {\nu^{\mathcal{C}}}_g + \dots$$

Fermion mass relations (II)

☞ potential rescue: higher–dimensional couplings

e.g. Pati [2006]

$$\mathscr{W}_{\text{eff}} \supset \sum_{i=1}^{2} \frac{\lambda_{gf}^{(i)}}{M_{\text{P}}}\,\psi_g\,\psi_f\,\phi\,H$$

☞ neutrino couplings

$$\mathscr{W}_{\text{eff}} \supset \overline{\kappa}_{gf}\,\psi_g\,\psi_f\,\frac{\overline{\chi}\,\overline{\chi}}{M_{\text{P}}} \xrightarrow{\overline{\chi}\to\langle\overline{\chi}\rangle} M_{fg}\,{\nu^{\mathcal{C}}}_f\,{\nu^{\mathcal{C}}}_g+\dots$$

☞ effective mass terms

$$M^{fg} = \overline{\kappa}^{gf}\,\frac{M_{\text{GUT}}^2}{M_{\text{P}}} \sim \overline{\kappa}^{gf}\cdot 10^{14}\,\text{GeV}$$

Fermion mass relations (II)

☞ potential rescue: higher–dimensional couplings e.g. Pati [2006]

$$\mathscr{W}_\mathrm{eff} \supset \sum_{i=1}^{2} \frac{\lambda_{gf}^{(i)}}{M_\mathrm{P}} \psi_g \psi_f \phi H$$

☞ neutrino couplings

$$\mathscr{W}_\mathrm{eff} \supset \overline{\kappa}_{gf} \psi_g \psi_f \frac{\overline{\chi}\,\overline{\chi}}{M_\mathrm{P}} \xrightarrow{\overline{\chi}\to\langle\overline{\chi}\rangle} M_{fg} \nu^{\mathcal{C}}{}_f \nu^{\mathcal{C}}{}_g + \dots$$

☞ effective mass terms

$$M^{fg} = \overline{\kappa}^{gf} \frac{M_\mathrm{GUT}^2}{M_\mathrm{P}} \sim \overline{\kappa}^{gf} \cdot 10^{14}\,\text{GeV}$$

☞ reasonable light neutrino masses via see–saw

Neutrino masses in grand unification: see–saw

☞ allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\; (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$

'right-handed' neutrino = SM singlet

Neutrino masses in grand unification: see–saw

- ☞ allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\;$ (SM singlets) $\overline{\nu}\,\overline{\nu} + \dots$

- ☞ Higgs VEV: $\langle\overline{\mathbf{126}}\rangle$ ↷ mass term $M\,\overline{\nu}\,\overline{\nu}$
- ➥ expect: $\langle\overline{\mathbf{126}}\rangle \sim M_{\mathrm{GUT}} \simeq 2\cdot 10^{16}\,\mathrm{GeV}$

Neutrino masses in grand unification: see–saw

- ☞ allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\; (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$

- ☞ Higgs VEV: $\langle\overline{\mathbf{126}}\rangle \curvearrowright$ mass term $M\,\overline{\nu}\,\overline{\nu}$
- ☞ allowed coupling: $\mathbf{10}\,\mathbf{16}\,\mathbf{16} \;\to\; H_u\,L\,\overline{\nu} + \dots$

Neutrino masses in grand unification: see–saw

- allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\; (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$

- Higgs VEV: $\langle\overline{\mathbf{126}}\rangle$ ↷ mass term $M\,\overline{\nu}\,\overline{\nu}$
- allowed coupling: $\mathbf{10}\,\mathbf{16}\,\mathbf{16} \;\to\; H_u\,L\,\overline{\nu} + \dots$
- ➥ see–saw couplings: $\mathscr{W}_{\text{see}-\text{saw}} \;=\; y_\nu\,H_u\,L\,\overline{\nu} + M\,\overline{\nu}\,\overline{\nu}$

Minkowski [1977]
Gell-Mann, Ramond & Slansky [1979]
Yanagida [1979]

Neutrino masses in grand unification: see–saw

- ☞ allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\;$ (SM singlets) $\overline{\nu}\,\overline{\nu} + \dots$

- ☞ Higgs VEV: $\langle\overline{\mathbf{126}}\rangle \curvearrowright$ mass term $M\,\overline{\nu}\,\overline{\nu}$
- ☞ allowed coupling: $\mathbf{10}\,\mathbf{16}\,\mathbf{16} \;\to\; H_u\,L\,\overline{\nu} + \dots$
- ➥ see–saw couplings: $\mathscr{W}_{\text{see}-\text{saw}} \;=\; y_\nu\,H_u\,L\,\overline{\nu} + M\,\overline{\nu}\,\overline{\nu}$
- ➥ see–saw mass matrix

$$\mathscr{W}_{\text{see}-\text{saw}} \xrightarrow{H_u \to v} (\nu, \overline{\nu}) \begin{pmatrix} 0 & y_\nu\,v \\ y_\nu\,v & M \end{pmatrix} \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \simeq \frac{y_\nu^2\,v^2}{M}\,\nu\,\nu + M\,\overline{\nu}\,\overline{\nu}$$

Neutrino masses in grand unification: see–saw

- allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\; (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$

- Higgs VEV: $\langle\overline{\mathbf{126}}\rangle \curvearrowright$ mass term $M\,\overline{\nu}\,\overline{\nu}$
- allowed coupling: $\mathbf{10}\,\mathbf{16}\,\mathbf{16} \;\to\; H_u\,L\,\overline{\nu} + \dots$
- see–saw couplings: $\mathscr{W}_{\text{see}-\text{saw}} \;=\; y_\nu\,H_u\,L\,\overline{\nu} + M\,\overline{\nu}\,\overline{\nu}$
- see–saw mass matrix

$$\mathscr{W}_{\text{see}-\text{saw}} \xrightarrow{H_u\to v} (\nu,\overline{\nu})\begin{pmatrix} 0 & y_\nu\,v \\ y_\nu\,v & M\end{pmatrix}\begin{pmatrix}\nu\\ \overline{\nu}\end{pmatrix} \simeq \frac{y_\nu^2\,v^2}{M}\,\nu\,\nu + M\,\overline{\nu}\,\overline{\nu}$$

- expectation: $m_\nu \sim (100\,\mathrm{GeV})^2/10^{16}\,\mathrm{GeV} \sim 10^{-3}\,\mathrm{eV}$

Neutrino masses in grand unification: see–saw

- allowed coupling: $\overline{\mathbf{126}}\,\mathbf{16}\,\mathbf{16} \;\to\; (\text{SM singlets})\,\overline{\nu}\,\overline{\nu} + \dots$

- Higgs VEV: $\langle\overline{\mathbf{126}}\rangle \curvearrowright$ mass term $M\,\overline{\nu}\,\overline{\nu}$
- allowed coupling: $\mathbf{10}\,\mathbf{16}\,\mathbf{16} \;\to\; H_u\,L\,\overline{\nu} + \dots$
- see–saw couplings: $\mathscr{W}_{\text{see}-\text{saw}} \;=\; y_\nu\,H_u\,L\,\overline{\nu} + M\,\overline{\nu}\,\overline{\nu}$
- see–saw mass matrix

$$\mathscr{W}_{\text{see}-\text{saw}} \xrightarrow{H_u \to v} (\nu, \overline{\nu}) \begin{pmatrix} 0 & y_\nu\,v \\ y_\nu\,v & M \end{pmatrix} \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \simeq \frac{y_\nu^2\,v^2}{M}\,\nu\,\nu + M\,\overline{\nu}\,\overline{\nu}$$

- expectation: $m_\nu \sim (100\,\mathrm{GeV})^2/10^{16}\,\mathrm{GeV} \sim 10^{-3}\,\mathrm{eV}$

however:

$\overline{\mathbf{126}}$–plets not available in string models

Neutrino masses in Nature: oscillation experiments

☞ strong experimental evidence for neutrino oscillation on astro–physical scales

Neutrino masses in Nature: oscillation experiments

- ☞ strong experimental evidence for neutrino oscillation on astro–physical scales
- ☞ experiments: $\sqrt{\Delta m^2_\mathrm{atm}} \simeq 0.04\,\mathrm{eV}$ & $\sqrt{\Delta m^2_\mathrm{sol}} \simeq 0.008\,\mathrm{eV}$

Neutrino masses in Nature: oscillation experiments

- ☞ strong experimental evidence for neutrino oscillation on astro–physical scales
- ☞ experiments: $\sqrt{\Delta m^2_{\mathrm{atm}}} \simeq 0.04\,\mathrm{eV}$ & $\sqrt{\Delta m^2_{\mathrm{sol}}} \simeq 0.008\,\mathrm{eV}$
- ➥ neutrino masses hint at
  - see–saw
  - GUT structures
- ☞ Factor 10 discrepancy (. . . would need $M \sim 10^{15}\,\mathrm{GeV}$)

Neutrino masses in Nature: oscillation experiments

- ☞ strong experimental evidence for neutrino oscillation on astro–physical scales
- ☞ experiments: $\sqrt{\Delta m^2_\mathrm{atm}} \simeq 0.04\,\mathrm{eV}$ & $\sqrt{\Delta m^2_\mathrm{sol}} \simeq 0.008\,\mathrm{eV}$
- ➥ neutrino masses hint at
  - see–saw
  - GUT structures
- ☞ Factor 10 discrepancy (…would need $M \sim 10^{15}\,\mathrm{GeV}$)
- ➥ Rough (although not perfect) agreement

Grand unification: virtues & predictions

☞ GUTs explain charge quantization

Grand unification: virtues & predictions

- ☞ GUTs explain charge quantization
- ☞ in $\mathrm{SO}(10)$: understanding of the structure of SM matter

Grand unification: virtues & predictions

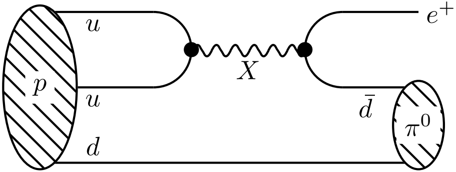
- ☞ GUTs explain charge quantization
- ☞ in $\mathrm{SO}(10)$: understanding of the structure of SM matter
- ☞ gauge coupling unification (...with supersymmetry)

Grand unification: virtues & predictions

- GUTs explain charge quantization
- in $\mathrm{SO}(10)$: understanding of the structure of SM matter
- gauge coupling unification (...with supersymmetry)
- $v_{\mathrm{EW}}^2/M_{\mathrm{GUT}}$: roughly the right scale of neutrino masses

Grand unification: virtues & predictions

- GUTs explain charge quantization
- in $\mathrm{SO}(10)$: understanding of the structure of SM matter
- gauge coupling unification (...with supersymmetry)
- $v_{\mathrm{EW}}^2/M_{\mathrm{GUT}}$: roughly the right scale of neutrino masses
- prediction: proton decay

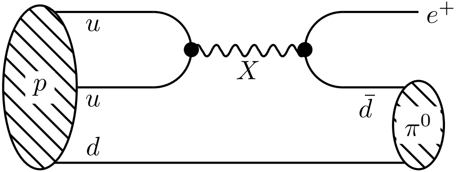


$\tau_p \sim 10^{35}$ years
for $M_X \simeq 2\cdot 10^{16}\,\mathrm{GeV}$

Grand unification: virtues & predictions

- GUTs explain charge quantization
- in $\mathrm{SO}(10)$: understanding of the structure of SM matter
- gauge coupling unification (. . . with supersymmetry)
- $v_{\mathrm{EW}}^2/M_{\mathrm{GUT}}$: roughly the right scale of neutrino masses
- prediction: proton decay

cf. talk by Marciano (2011)



$\tau_p \sim 10^{35}$ years
for $M_X \simeq 2 \cdot 10^{16}\,\mathrm{GeV}$

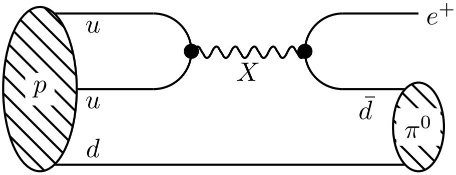
main prediction of GUTs:

matter unstable

Grand unification: virtues & predictions

- GUTs explain charge quantization
- in $\mathrm{SO}(10)$: understanding of the structure of SM matter
- gauge coupling unification (...with supersymmetry)
- $v_{\mathrm{EW}}^2/M_{\mathrm{GUT}}$: roughly the right scale of neutrino masses
- prediction: proton decay

cf. talk by Marciano (2011)



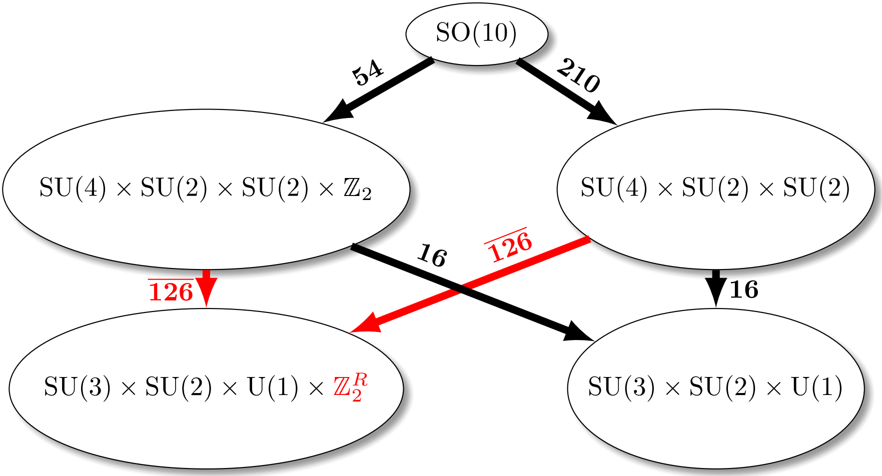
$\tau_p \sim 10^{35}$ years
for $M_X \simeq 2\cdot 10^{16}\,\mathrm{GeV}$

main prediction of GUTs:

matter unstable $\curvearrowright$ one day our universe will be empty

SO(10) breaking by Higgs mechanism

Mohapatra & Pal [1998]



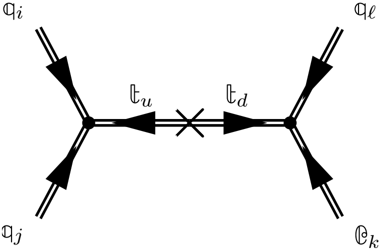
☞ GUT breaking by Higgs: need large Higgs representations ($\mathbf{54}$, $\overline{\mathbf{126}}$, $\mathbf{210}$) $\curvearrowright$ lot of 'junk' (which, however, can be paired up)

Doublet–triplet splitting

☞ two triplets contained in ϕ: $\phi \;=\; h_u + h_d + \underbrace{t_u}_{(\mathbf{3},\mathbf{1})_{-1/3}} + \underbrace{t_d}_{(\overline{\mathbf{3}},\mathbf{1})_{1/3}}$

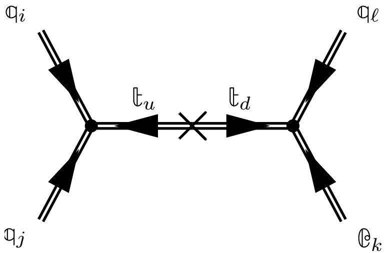
Doublet–triplet splitting

- two triplets contained in ϕ: $\phi \;=\; h_u + h_d + t_u + t_d$
- triplet exchange diagram

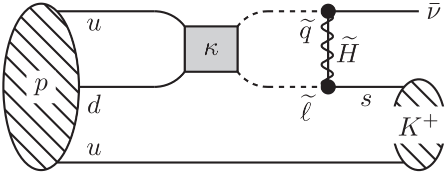


Doublet–triplet splitting

- two triplets contained in ϕ: $\phi = h_u + h_d + t_u + t_d$
- triplet exchange diagram

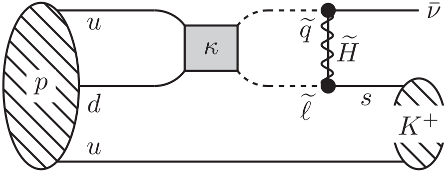


- dimension 5 proton decay operator



Proton decay

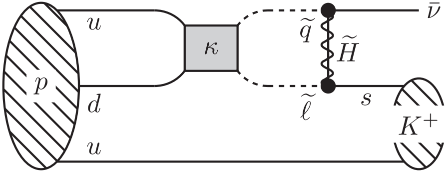
☞ dimension 5 proton decay operator



$$p \;\to\; \overline{\nu} + K^+$$

Proton decay

- dimension 5 proton decay operator

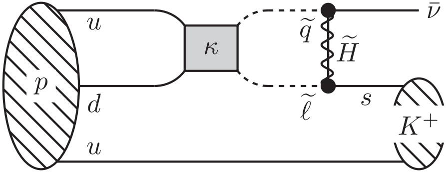


$$p \;\rightarrow\; \overline{\nu} + K^+$$

- constraint on triplet mass: $M_t \;\gg\; M_\mathrm{P}$

Proton decay

- dimension 5 proton decay operator

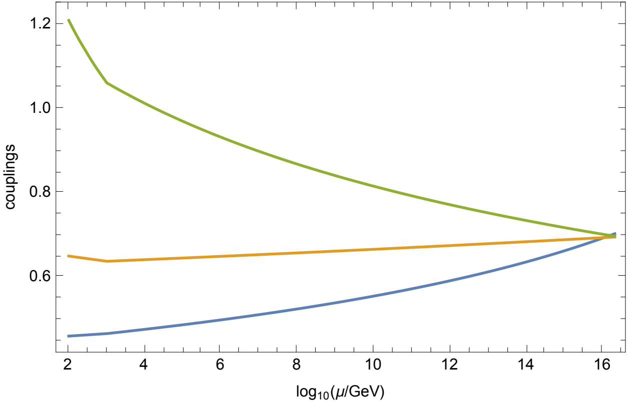


$$p \;\to\; \overline{\nu} + K^+$$

- constraint on triplet mass: $M_t \;\gg\; M_\mathrm{P}$
- possible loop–holes

Doublet–triplet splitting vs. full generations

- ☺ Gauge coupling unification: $M_{\text{GUT}} \sim 10^{16}\,\text{GeV}$ with SUSY



Doublet–triplet splitting vs. full generations

- ☺ Gauge coupling unification: $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$ with SUSY
- ☺ One generation of observed matter fits into **16** of $\mathrm{SO}(10)$

$$
\begin{array}{rcl}
\mathrm{SO}(10) & \to & \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_{\mathrm{Y}} = G_{\mathrm{SM}} \\
\mathbf{16} & \to & (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\
 & & \oplus\,(\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{0}
\end{array}
$$

Doublet–triplet splitting vs. full generations

☺ Gauge coupling unification: $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$ with SUSY

☺ One generation of observed matter fits into **16** of $\mathrm{SO}(10)$

$$\begin{aligned}\mathrm{SO}(10) &\to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_\mathrm{Y} = G_{\mathrm{SM}}\\ \mathbf{16} &\to (\mathbf{3},\mathbf{2})_{1/6}\oplus(\overline{\mathbf{3}},\mathbf{1})_{-2/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3}\\ &\quad\oplus(\mathbf{1},\mathbf{1})_{1}\oplus(\mathbf{1},\mathbf{2})_{-1/2}\oplus(\mathbf{1},\mathbf{1})_{0}\end{aligned}$$

☹ However: Higgs only as doublet(s):

$$\mathbf{10} \to \underbrace{(\mathbf{1},\mathbf{2})_{1/2}\oplus(\mathbf{1},\mathbf{2})_{-1/2}}_{\text{doublets: needed}}\oplus\underbrace{(\mathbf{3},\mathbf{1})_{-1/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3}}_{\text{triplets: excluded}}$$

Doublet–triplet splitting vs. full generations

☺ Gauge coupling unification: $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$ with SUSY

☺ One generation of observed matter fits into **16** of $\mathrm{SO}(10)$

$$\begin{array}{rcl} \mathrm{SO}(10) & \to & \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_\mathrm{Y} = G_\mathrm{SM} \\ \mathbf{16} & \to & (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\ & & \oplus\,(\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{0} \end{array}$$

☹ However: Higgs only as doublet(s):

$$\mathbf{10} \to (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3}$$

☞ A true solution to the problem requires a symmetry that forbids the μ term in the MSSM

Doublet–triplet splitting vs. full generations

☺ Gauge coupling unification: $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$ with SUSY

☺ One generation of observed matter fits into **16** of $\mathrm{SO}(10)$

$$\begin{aligned}\mathrm{SO}(10) &\to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_\mathrm{Y} = G_\mathrm{SM}\\ \mathbf{16} &\to (\mathbf{3},\mathbf{2})_{1/6}\oplus(\overline{\mathbf{3}},\mathbf{1})_{-2/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3}\\ &\quad\oplus(\mathbf{1},\mathbf{1})_{1}\oplus(\mathbf{1},\mathbf{2})_{-1/2}\oplus(\mathbf{1},\mathbf{1})_{0}\end{aligned}$$

☹ However: Higgs only as doublet(s):

$$\mathbf{10}\to(\mathbf{1},\mathbf{2})_{1/2}\oplus(\mathbf{1},\mathbf{2})_{-1/2}\oplus(\mathbf{3},\mathbf{1})_{-1/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3}$$

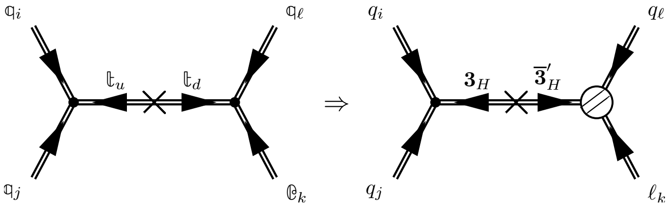
☞ A true solution to the problem requires a symmetry that forbids the μ term in the MSSM

☞ An appropriate μ term can then be generated by the Kim–Nilles and/or Giudice–Masiero mechanism(s) Kim & Nilles [1984] ; Giudice & Masiero [1988]

Dimension five proton decay

☞ Interesting solution: mass partner of triplet does not couple to SM matter (... requires extra Higgs multiplets)

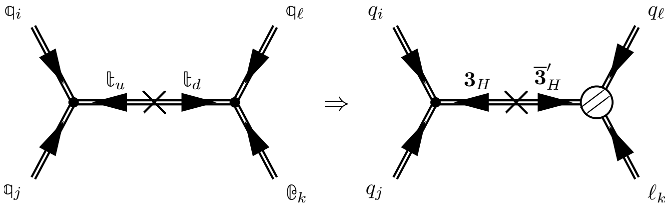
Babu & Barr [1993]



Dimension five proton decay

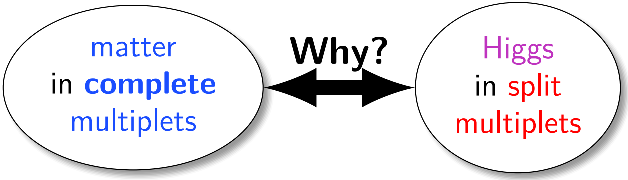
☞ Interesting solution: mass partner of triplet does not couple to SM matter (... requires extra Higgs multiplets)

Babu & Barr [1993]

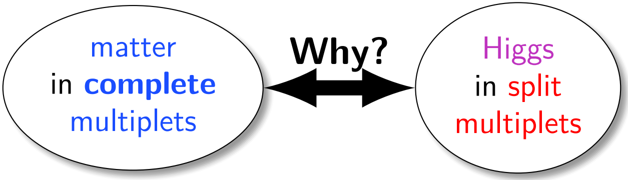


☞ suppression of $Q\,Q\,Q\,L$ also possible due to flavor symmetries

Doublet–triplet splitting in four dimensions



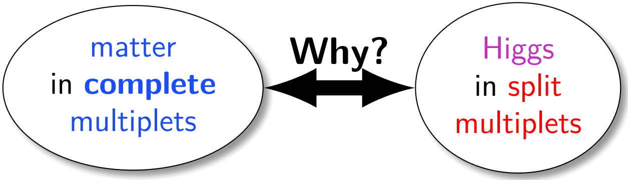
Doublet–triplet splitting in four dimensions



there exist proposals to solve the doublet–triplet splitting problem, e.g.

- ☞ Dimopoulos–Wilczek mechanism — Dimopoulos & Wilczek [1981]
- ☞ Missing partner mechanism — Masiero, Nanopoulos, Tamvakis & Yanagida [1982]
- ☞ . . .

Doublet–triplet splitting in four dimensions



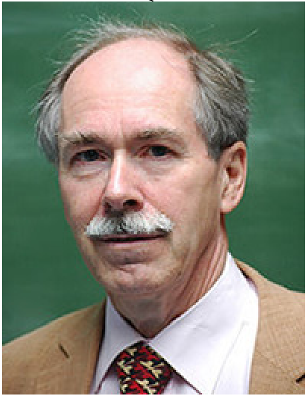
there exist proposals to solve the doublet–triplet splitting problem, e.g.

- ☞ Dimopoulos–Wilczek mechanism — Dimopoulos & Wilczek [1981]
- ☞ Missing partner mechanism — Masiero, Nanopoulos, Tamvakis & Yanagida [1982]
- ☞ ...

...however, a closer inspection shows that all of them have certain deficiencies

Doublet–triplet splitting in four dimensions

☞ 'Natural' solution of the doublet–triplet splitting problem requires symmetry that forbids Higgs mass μ



According to 't Hoofts 'naturalness' criteria: explaining a (supersymmetric) Higgs mass $\mu \ll M_{\mathrm{GUT}}$ requires a symmetry that forbids μ.

Doublet–triplet splitting in four dimensions

superpartners have different charges

- ☞ 'Natural' solution of the doublet–triplet splitting problem requires symmetry that forbids Higgs mass μ
- ☞ Only R symmetries can do the job

Hall, Nomura & Pierce [2002b] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a,b] ; Chen, M.R., Staudt & Vaudrevange [2012b]

Doublet–triplet splitting in four dimensions

☞ 'Natural' solution of the doublet–triplet splitting problem requires symmetry that forbids Higgs mass μ

☞ Only R symmetries can do the job and R parity does not

Hall, Nomura & Pierce [2002b] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a,b] ; Chen, M.R., Staudt & Vaudrevange [2012b]

- gauge coupling unification
- fermion masses (Yukawa couplings & neutrino mass operator)
- consistency with SU(5)

$\curvearrowright$ only R symmetries can forbid the μ term in the MSSM

. . . and R parity is not enough

Doublet–triplet splitting in four dimensions

- ‘Natural’ solution of the doublet–triplet splitting problem requires symmetry that forbids Higgs mass μ
- Only R symmetries can do the job and R parity does not

Hall, Nomura & Pierce [2002b] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a,b] ; Chen, M.R., Staudt & Vaudrevange [2012b]

- gauge coupling unification
- fermion masses (Yukawa couplings & neutrino mass operator)
- consistency with SU(5)

$\curvearrowright$ only R symmetries can forbid the μ term in the MSSM

. . . and R parity is not enough

- **However**: R symmetries are not available in 4D GUTs

details

Fallbacher, M.R. & Vaudrevange [2011]

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

if violated, gauge coupling unification will be spoiled

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} \rho \mod \eta \quad \text{for all } G$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

$\mathbb{Z}_N$ charge

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden (before ~~SUSY~~)

need to forbid the μ term to be able to appreciate the Kim–Nilles and/or Giudice–Masiero mechanisms Kim & Nilles [1984] ; Giudice & Masiero [1988]

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden (before ~~SUSY~~)

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden (before ~~SUSY~~)

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT

$$\begin{array}{rcl} \mathrm{SO}(10) & \to & \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_\mathrm{Y} = G_\mathrm{SM} \\ \mathbf{16} & \to & (\mathbf{3},\mathbf{2})_{1/6}\oplus(\overline{\mathbf{3}},\mathbf{1})_{-2/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3} \\ & & \oplus\,(\mathbf{1},\mathbf{1})_{1}\oplus(\mathbf{1},\mathbf{2})_{-1/2}\oplus(\mathbf{1},\mathbf{1})_{0} \end{array}$$

Anomaly–free symmetries, μ and unification

- Working assumptions:
  - (i) anomaly universality (allow for GS anomaly cancellation)
  - (ii) μ term forbidden (before ~~SUSY~~)
  - (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  - (iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT
- Can prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term

Anomaly–free symmetries, μ and unification

- Working assumptions:
  - (i) anomaly universality (allow for GS anomaly cancellation)
  - (ii) μ term forbidden (before ~~SUSY~~)
  - (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  - (iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT
- Can prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term
  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry

	q	$u^\mathcal{C}$	$d^\mathcal{C}$	ℓ	$e^\mathcal{C}$	h_u	h_d	$\nu^\mathcal{C}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

Anomaly–free symmetries, μ and unification

- Working assumptions:
  - (i) anomaly universality (allow for GS anomaly cancellation)
  - (ii) μ term forbidden (before ~~SUSY~~)
  - (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  - (iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT
- Can prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term
  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry
  3. R symmetries are not available in 4D GUTs

     uneaten parts of the Higgs that breaks the GUT symmetry cannot be paired up

$\mathbb{Z}_4^R$ summarized

Yukawa couplings ✓

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &+ Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &+ \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &+ \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\;& \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
& + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
& + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
& + \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}
$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\;& {\color{red}\mu}\, h_d h_u + \kappa_i\, \ell_i h_u \\
&+ Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
&+ \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
&+ \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + {\color{red}\kappa^{(1)}_{gfk\ell}}\, q_g q_f q_k \ell_\ell + {\color{red}\kappa^{(2)}_{gfk\ell}}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}
$$

$\mathcal{O}\left(\frac{m_{3/2}}{M_{\mathrm{P}}^2}\right)$

- R parity violating couplings forbidden
- μ term of the right size

order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

Planck units

- proton decay under control

Discussion

☞ A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ

Discussion

- ☞ A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ
- ☞ We learned that:
  1. only R symmetries can forbid the μ term
  2. anomaly matching requires the existence of split multiplets
  3. R symmetries are not available in 4D GUTs

Discussion

- A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ
- We learned that:
  1. only R symmetries can forbid the μ term
  2. anomaly matching requires the existence of split multiplets
  3. R symmetries are not available in 4D GUTs

bottom–line:
'Natural' solutions to the
μ and/or doublet–triplet splitting problems
are not available in four dimensions!

Discussion

- A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ
- We learned that:
  1. only R symmetries can forbid the μ term
  2. anomaly matching requires the existence of split multiplets
  3. R symmetries are not available in 4D GUTs

bottom–line:

'Natural' solutions to the
μ and/or doublet–triplet splitting problems
are not available in four dimensions!

➥ Need to go to extra dimensions/strings

Orbifold GUTs

Kaluza–Klein compactification

☞ start with a $4+1$–dimensional Minkowski space–time $\mathbb{M}^5$

Kaluza–Klein compactification

- start with a $4+1$–dimensional Minkowski space–time $\mathbb{M}^5$
- extra direction compactified on $\mathbb{S}^1$

$$x^5 \sim x^5 + 2\pi \underbrace{R}_{\text{radius}}$$

Kaluza–Klein compactification

- start with a $4+1$–dimensional Minkowski space–time $\mathbb{M}^5$
- extra direction compactified on $\mathbb{S}^1$

$$x^5 \;\sim\; x^5 + 2\pi\, R$$

- 5–dimensional metric

$$g_{MN} \;=\; \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\mu & g_{55} \end{array}\right)$$

4D vector

4D scalar

Kaluza–Klein expansion

☞ Fourier expansion of general field

$$\phi(x^0,\dots x^3,x^5) \;=\; \frac{1}{\sqrt{2\pi\,R}}\sum_{n=-\infty}^{\infty}\phi^{(n)}(x^0,\dots x^3)\cdot \mathrm{e}^{-\mathsf{i}\,n\,x^5/R}$$

Kaluza–Klein expansion

- Fourier expansion of general field

$$\phi(x^0,\dots x^3,x^5) \;=\; \frac{1}{\sqrt{2\pi\,R}}\sum_{n=-\infty}^{\infty}\phi^{(n)}(x^0,\dots x^3)\cdot \mathrm{e}^{-\mathrm{i}\,n\,x^5/R}$$

- Kaluza–Klein (KK) action

$$\begin{aligned} S_{\mathrm{KK}} &= \int \mathrm{d}^5x\,\frac{1}{2}\,\partial_M\phi(x,x^5)\,\partial^M\phi(x,x^5) - \frac{m_5^2}{2}\,\phi^2(x,x^5) \\ &= \int \mathrm{d}^5x\,\frac{1}{2}\left[\partial_\mu\phi(x,x^5)\,\partial^\mu\phi(x,x^5) - \frac{m_5^2}{2}\,\phi^2(x,x^5) - \big(\partial_5\phi(x,x^5)\big)^2\right]\end{aligned}$$

$$\int \mathrm{d}^5x \;:=\; \int \mathrm{d}^4x \int_0^{2\pi\,R} \mathrm{d}x^5$$

Kaluza–Klein expansion of real scalar field

$$\begin{aligned} S_{\mathrm{KK}} &= \int \mathsf{d}^4 x \sum_{m,n} \left(\int\limits_0^{2\pi R} \mathsf{d}x^5 \, \frac{1}{2\pi R} \, \mathrm{e}^{\mathsf{i}\,\frac{(m+n)}{R}\,x^5} \right) \\ &\qquad \cdot \frac{1}{2} \left[\partial_\mu \phi^{(m)}(x)\, \partial^\mu \phi^{(n)}(x) + \frac{m\,n}{R^2}\, \phi^{(m)}(x)\, \phi^{(n)}(x) \right] \\ &= \frac{1}{2} \int \mathsf{d}^4 x \sum_n \left[\partial_\mu \phi^{(-n)}\, \partial^\mu \phi^{(n)} - \frac{n^2}{R^2}\, \phi^{(-n)}\, \phi^{(n)} \right] \\ &= \int \mathsf{d}^4 x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)} \right)^* \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \left| \phi^{(n)} \right|^2 \right] \end{aligned}$$

$$\left(\phi^{(n)} \right)^* = \phi^{(-n)}$$

Kaluza–Klein expansion of real scalar field

$$
\begin{aligned}
S_{\mathrm{KK}} &= \int \mathrm{d}^4 x \sum_{m,n} \left(\int\limits_0^{2\pi R} \mathrm{d}x^5 \, \frac{1}{2\pi R} \, \mathrm{e}^{\mathrm{i}\,\frac{(m+n)}{R}\,x^5} \right) \\
&\qquad \cdot \frac{1}{2} \left[\partial_\mu \phi^{(m)}(x) \, \partial^\mu \phi^{(n)}(x) + \frac{m\,n}{R^2} \, \phi^{(m)}(x) \, \phi^{(n)}(x) \right] \\
&= \frac{1}{2} \int \mathrm{d}^4 x \sum_n \left[\partial_\mu \phi^{(-n)} \, \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \, \phi^{(-n)} \, \phi^{(n)} \right] \\
&= \int \mathrm{d}^4 x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)} \right)^* \, \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \, \left| \phi^{(n)} \right|^2 \right]
\end{aligned}
$$

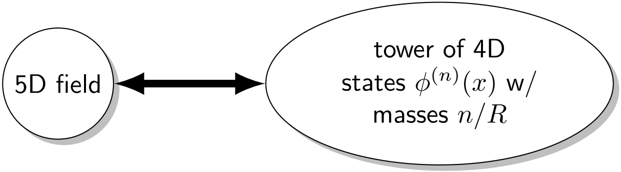
☞ orthogonality

$$
\int\limits_0^{2\pi R} \mathrm{d}x^5 \, \phi^{(m)\,*}(x^5) \, \phi^{(n)}(x^5) \;=\; \int\limits_0^{2\pi R} \mathrm{d}x^5 \, \frac{1}{2\pi R} \, \mathrm{e}^{\mathrm{i}\,\frac{(m+n)}{R}\,x^5} \;=\; \delta_{n,-m}
$$

Kaluza–Klein tower

☞ Kaluza–Klein action

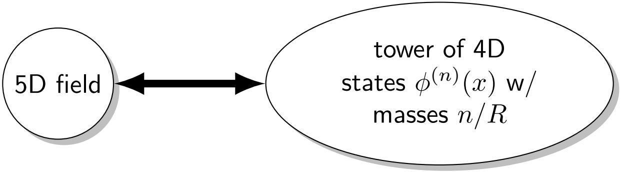
$$S_{\mathrm{KK}} \;=\; \int \mathrm{d}^4 x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)}\right)^* \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \left|\phi^{(n)}\right|^2 \right]$$



Kaluza–Klein tower

- Kaluza–Klein action

$$S_{\mathrm{KK}} = \int \mathrm{d}^4 x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)}\right)^* \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \left|\phi^{(n)}\right|^2 \right]$$



- generalization to higher dimensions with compactification radii $R_5, R_6 \dots$

$$m^2_{n_5,n_6,\cdots,n_d} = m_D^2 + \frac{n_5^2}{R_5^2} + \frac{n_6^2}{R_6^2} + \cdots + \frac{n_d^2}{R_d^2}$$

Graviton

☞ in $D = 5$

$$g_{MN} = \left(\begin{array}{c:c} g_{\mu\nu} & A_\mu \\ \hdashline A_\mu & \varphi \end{array}\right)$$

Graviton

☞ in $D = 5$

$$g_{MN} \;=\; \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\mu & \varphi \end{array}\right)$$

☞ zero–modes:

- 4D graviton
- a 4D vector
- a 4D scalar

Graviton

☞ in $D = 5$

$$g_{MN} \;=\; \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\mu & \varphi \end{array}\right)$$

☞ zero–modes:

- 4D graviton
- a 4D vector
- a 4D scalar

☞ in $D = (4 + d)$ dimensions:

- one KK tower of gravitons
- $(d - 1)$ KK towers of gauge fields
- $[\frac{1}{2}d(d + 1) - d]$ KK towers of scalars

Clifford algebra in D dimensions (I)

- Clifford algebra in D dimensions: $\left\{\Gamma^M,\Gamma^N\right\} \;=\; 2\eta^{MN}$

$$\eta^{MN} \;=\; \operatorname{diag}(1,-1,\cdots-1)$$

Clifford algebra in D dimensions (I)

- Clifford algebra in D dimensions: $\{\Gamma^M, \Gamma^N\} \;=\; 2\eta^{MN}$

- in $D = 2$

$$\Gamma^0_{\mathrm{2D}} \;=\; \sigma_x \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \Gamma^1_{\mathrm{2D}} \;=\; \mathrm{i}\,\sigma_y \;=\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Clifford algebra in D dimensions (I)

- Clifford algebra in D dimensions: $\{\Gamma^M, \Gamma^N\} \;=\; 2\eta^{MN}$

- in $D = 2$

$$\Gamma^0_{\mathrm{2D}} \;=\; \sigma_x \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \Gamma^1_{\mathrm{2D}} \;=\; \mathrm{i}\,\sigma_y \;=\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- in $D = 2k + 2$ dimensions

$$\Gamma^M_{(2k+2)\mathrm{D}} \;=\; \Gamma^M_{2k\mathrm{D}} \otimes \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{for } 0 \le M \le 2k-1$$

$$\Gamma^M_{(2k+2)\mathrm{D}} \;=\; \mathbb{1}_{2^k} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{for } M = 2k$$

$$\Gamma^M_{(2k+2)\mathrm{D}} \;=\; \mathbb{1}_{2^k} \otimes \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \qquad \text{for } M = 2k+1$$

Clifford algebra in D dimensions (II)

☞ analogue of γ_5 in four dimensions

$$\Gamma_{(2k+2)\mathrm{D}} \;=\; \mathsf{i}^{-k}\,\Gamma^0_{(2k+2)\mathrm{D}}\,\Gamma^1_{(2k+2)\mathrm{D}}\cdots\Gamma^{2k+1}_{(k+2)\mathrm{D}}$$

Clifford algebra in D dimensions (II)

☞ analogue of γ_5 in four dimensions

$$\Gamma_{(2k+2)\mathrm{D}} \;=\; \mathsf{i}^{-k}\,\Gamma^0_{(2k+2)\mathrm{D}}\,\Gamma^1_{(2k+2)\mathrm{D}}\cdots\Gamma^{2k+1}_{(k+2)\mathrm{D}}$$

➥ Dirac matrices in $D+1$ dimensions:
$\{\Gamma^0_{(2k+2)\mathrm{D}}\,\Gamma^1_{(2k+2)\mathrm{D}}\cdots\Gamma^{2k+1}_{(k+2)\mathrm{D}},\Gamma_{(2k+2)\mathrm{D}}\}$

Spinors in D dimensions

cf. Polchinski

D	Weyl	reality	Majorana	Majorana–Weyl
$8k$	✓	complex	✗	✗
$8k+1$	✗	real	✗	✗
$8k+2$	✓	real	✓	✓
$8k+3$	✗	real	✓	✗
$8k+4$	✓	complex	✓	✗
$8k+5$	✗	pseudo–real	✗	✗
$8k+6$	✓	pseudo–real	✗	✗
$8k+7$	✗	pseudo–real	✗	✗

Chiral fermions

☞ γ–matrices become large in higher dimensions

Chiral fermions

☞ γ–matrices become large in higher dimensions

➥ spinor representation becomes larger

Chiral fermions

☞ γ–matrices become large in higher dimensions

➥ spinor representation becomes larger

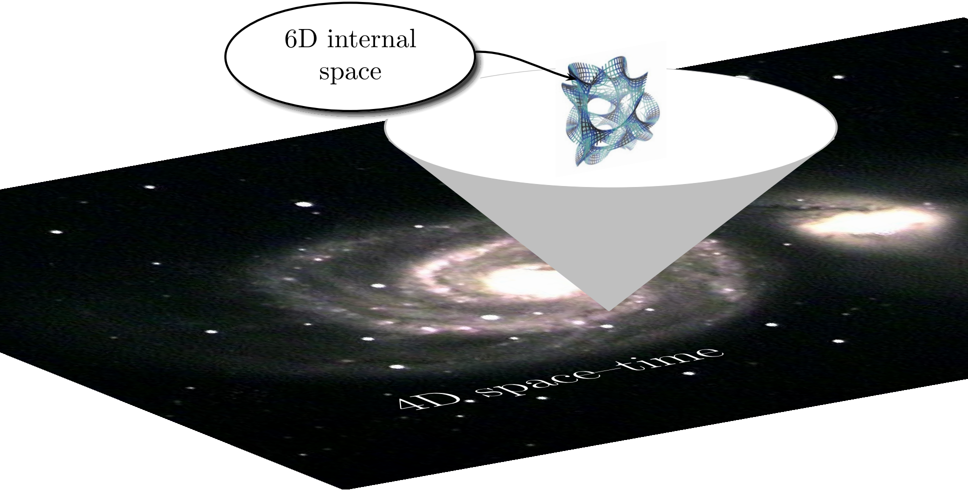
☞ smallest spinor representation in 5D is a 4D Dirac spinor

Chiral fermions

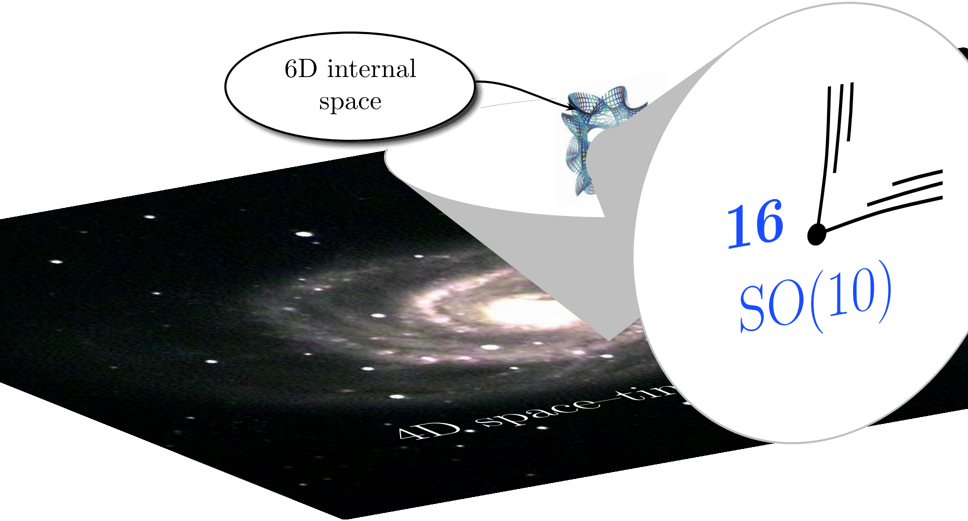
- ☞ γ–matrices become large in higher dimensions
- ➥ spinor representation becomes larger
- ☞ smallest spinor representation in 5D is a 4D Dirac spinor
- ➥ no–go for chiral theories from simple compactification on circle

(String) compactifications with local $\mathrm{SO}(10)$ GUTs

(String) compactifications with local SO(10) GUTs

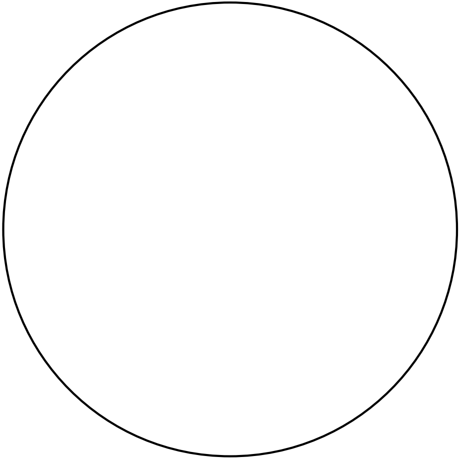


(String) compactifications with local SO(10) GUTs



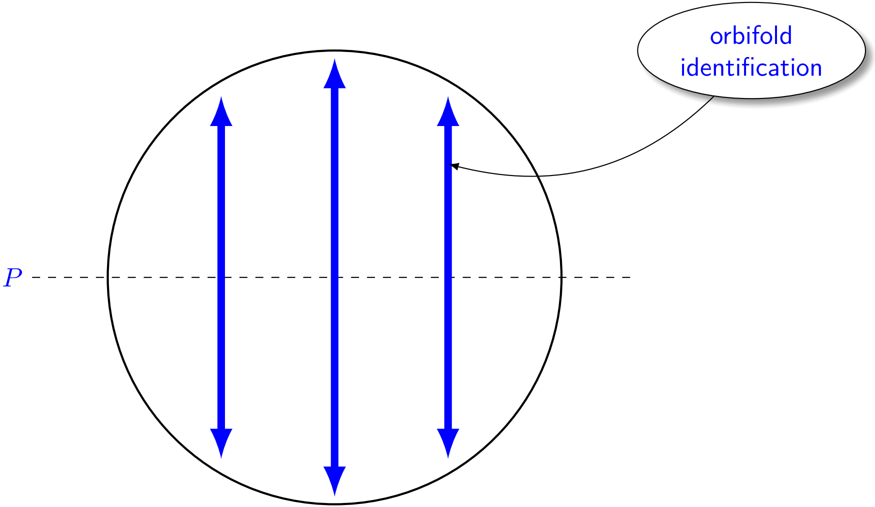
Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$

☞ $\mathbb{S}^1$



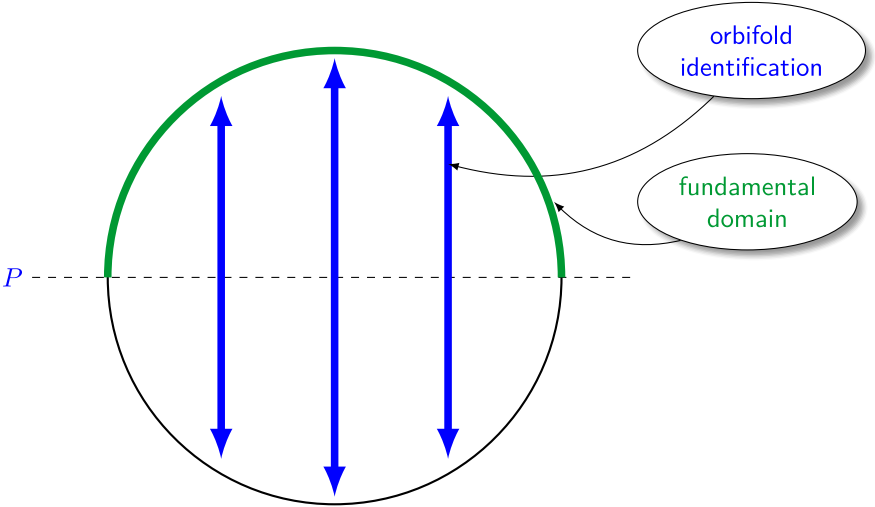
Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$

☞ $\mathbb{S}^1/\mathbb{Z}_2$ ($\mathbb{Z}_2$ reflection breaks to $N = 1$ supersymmetry)



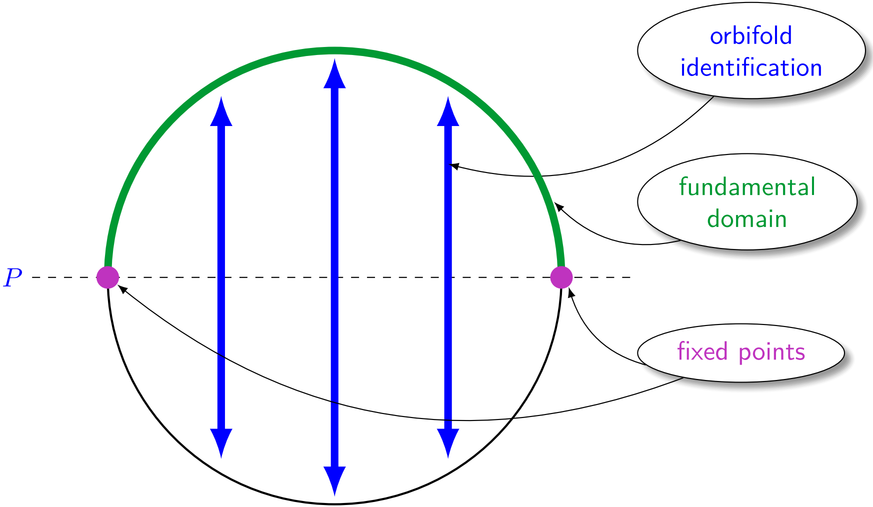
Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$

- $\mathbb{S}^1/\mathbb{Z}_2$



Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$

☞ $\mathbb{S}^1/\mathbb{Z}_2$



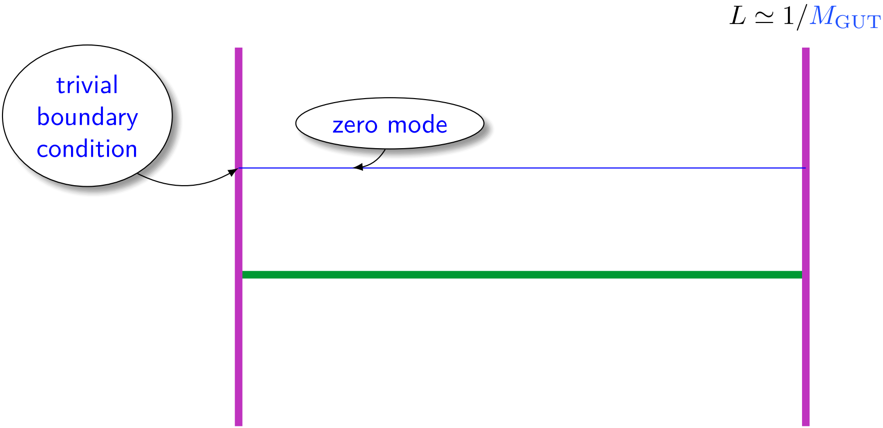
Symmetry breaking in extra dimensions

☞ Field theory on $\mathbb{M}^4 \times$ interval

$L \simeq 1/M_{\mathrm{GUT}}$

Symmetry breaking in extra dimensions

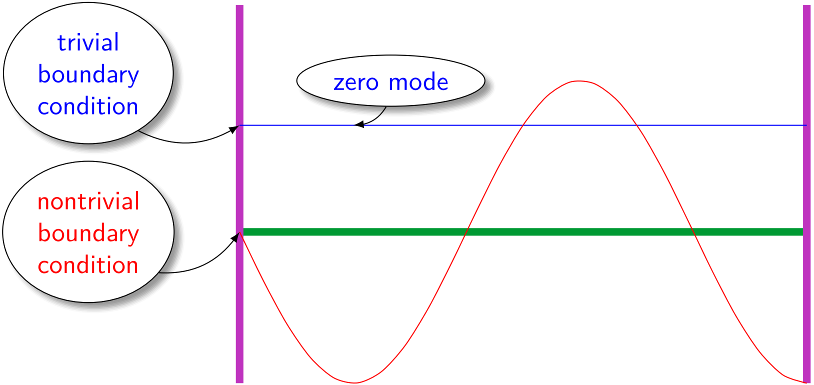
☞ Field theory on $\mathbb{M}^4 \times$ interval



Symmetry breaking in extra dimensions

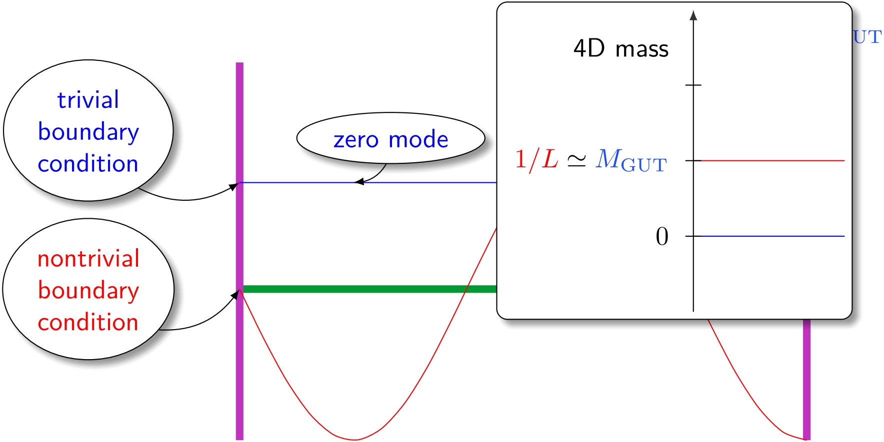
☞ Field theory on $\mathbb{M}^4 \times$ interval

$L \simeq 1/M_{\mathrm{GUT}}$



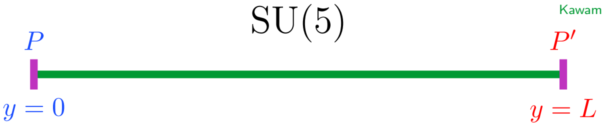
Symmetry breaking in extra dimensions

☞ Field theory on $\mathbb{M}^4 \times$ interval



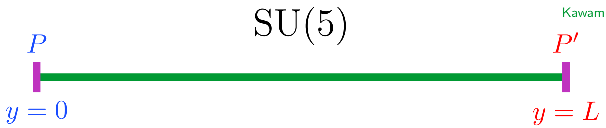
An example: Kawamura's model

Kawamura [2000, 2001]



An example: Kawamura's model

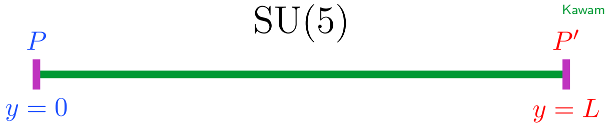
Kawamura [2000, 2001]



☞ Choose $P = \mathbb{1}$ and $P' = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$

An example: Kawamura's model

Kawamura [2000, 2001]

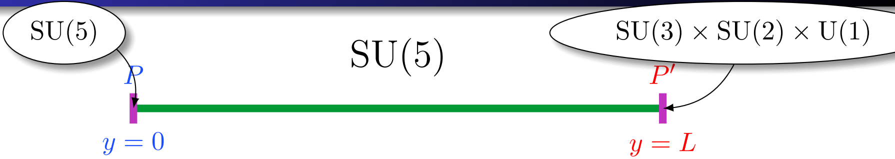


☞ Choose $P = \mathbb{1}$ and $P' = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$

☞ Boundary conditions for gauge fields

$$A_M(0) \; = \; P\, A_M(0)\, P^{-1} \quad \text{and} \quad A_M(L) \; = \; P'\, A_M(L)\, P'^{\,-1}$$

An example: Kawamura's model

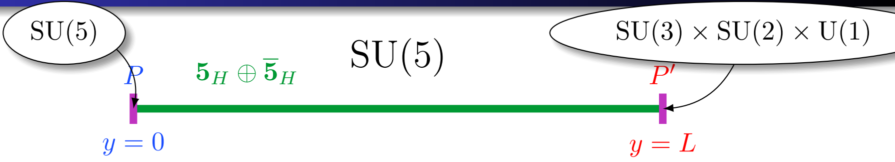


☞ Choose $P = \mathbb{1}$ and $P' = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$

☞ Boundary conditions for gauge fields

$$A_M(0) \;=\; P\,A_M(0)\,P^{-1} \quad \text{and} \quad A_M(L) \;=\; P'\,A_M(L)\,P'^{\,-1}$$

An example: Kawamura's model



- Choose $P = \mathbb{1}$ and $P' = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$

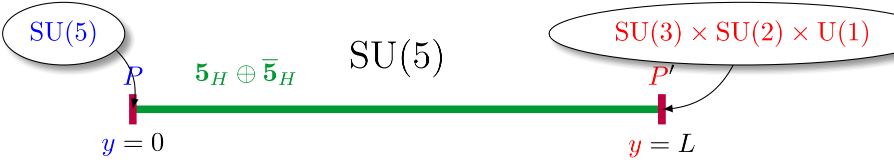
- Boundary conditions for gauge fields

$$A_M(0) \;=\; P\,A_M(0)\,P^{-1} \quad \text{and} \quad A_M(L) \;=\; P'\,A_M(L)\,P'^{\,-1}$$

- Higgs: $\mathbf{5}_H \oplus \overline{\mathbf{5}}_H$ in the bulk

$$\left.\begin{array}{rcl} H(0) & = & P\,H(0) \\ H(L) & = & P'\,H(L) \end{array}\right\} \Rightarrow \text{only doublet has zero-mode!}$$

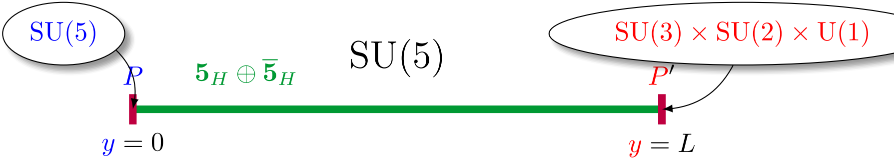
Kawamura model: mode expansion



☞ only nontrivial boundary condition at $y = L = \pi R/2$

$$\phi_\pm(y = L) \;=\; \pm\phi_\pm(y = L)$$

Kawamura model: mode expansion



☞ only nontrivial boundary condition at $y = L = \pi\, R/2$

$$\phi_\pm(y = L) \;=\; \pm\phi_\pm(y = L)$$

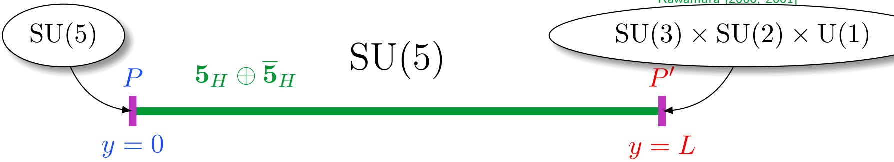
➥ Mode expansion

cf. Barbieri, Hall & Nomura [2001]

$$\begin{aligned}
\phi_+(x_\mu, y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n0}}\pi\, R}}\, \phi_+^{(n)}(x_\mu)\, \cos\left(\frac{2n\, y}{R}\right) \\
\phi_{+-}(x_\mu, y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi\, R}}\, \phi_-^{(n)}(x_\mu)\, \cos\left(\frac{(2n+1)\, y}{2R}\right)
\end{aligned}$$

Kawamura's model (cont'd)

Kawamura [2000, 2001]

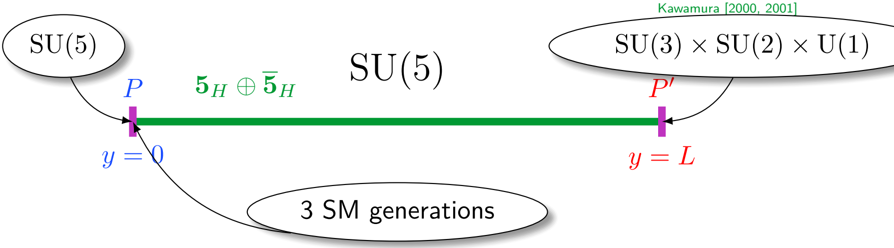


Features:

- **Local gauge groups**: $\mathrm{SU}(5)$ at $y = 0$ and G_{SM} at $y = L$
- Same mechanism breaks GUT and splits Higgs

  this point has been stressed early in the string literature — Witten (1985)

Kawamura's model (cont'd)

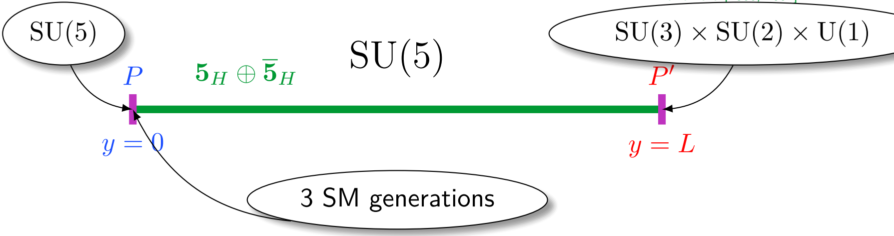


Features:

- **Local gauge groups**: $\mathrm{SU}(5)$ at $y = 0$ and G_{SM} at $y = L$
- Same mechanism breaks GUT and splits Higgs
- **Structure of SM matter**: matter placed at $\mathrm{SU}(5)$ fixed points has to appear in complete $\mathrm{SU}(5)$ representations

Kawamura's model (cont'd)

Kawamura [2000, 2001]



Features:

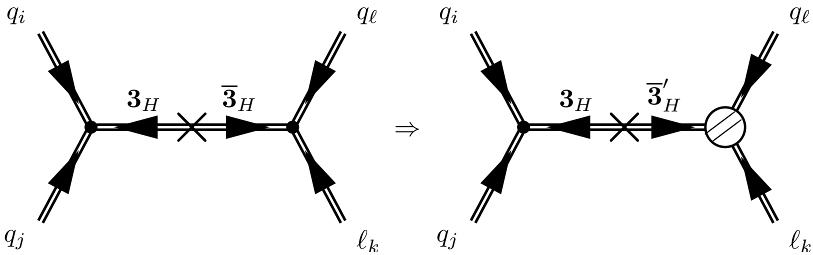
- **Local gauge groups**: $\mathrm{SU}(5)$ at $y=0$ and G_{SM} at $y=L$
- Same mechanism breaks GUT and splits Higgs
- **Structure of SM matter**: matter placed at $\mathrm{SU}(5)$ fixed points has to appear in complete $\mathrm{SU}(5)$ representations
- **Proton stability**: Higgs triplets get a Kaluza–Klein mass whereby the mass partner does not couple to SM matter

Altarelli & Feruglio [2001], Hall & Nomura [2001]

Proton decay

☞ Recall Babu–Barr mechanism

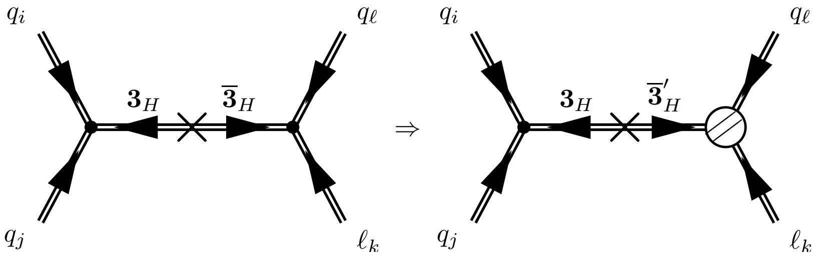
Babu & Barr [1993]



Proton decay

☞ Recall Babu–Barr mechanism

Babu & Barr [1993]



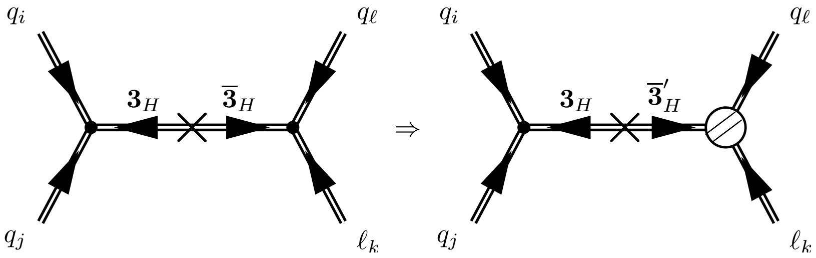
☞ This structure is automatic in orbifold GUTs

Altarelli & Feruglio [2001] ;Hall & Nomura [2001]

Proton decay

☞ Recall Babu–Barr mechanism

Babu & Barr [1993]



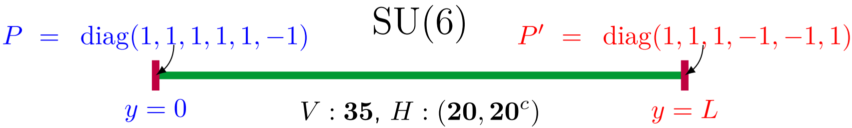
☞ This structure is automatic in orbifold GUTs

Altarelli & Feruglio [2001] ;Hall & Nomura [2001]

☞ The reason: the bulk fields come in hypermultiplets $H = (\phi, \phi^c)$ and the (bulk) mass marries a triplet $\mathbf{3}_H$ that couples to SM matter to an antitriplet $\overline{\mathbf{3}}'_H$ that does not

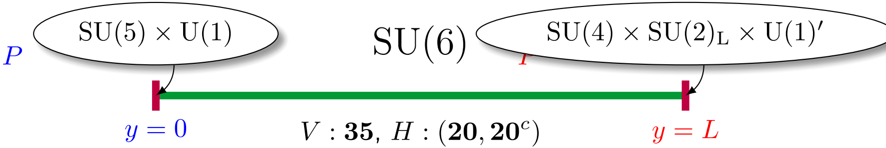
5D example & mode expansion

Burdman & Nomura [2003]



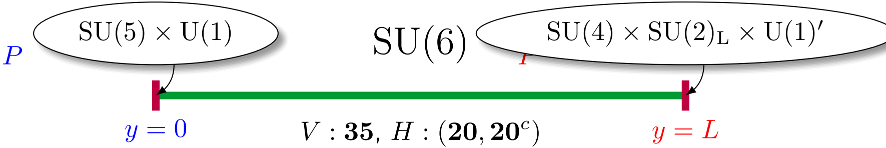
5D example & mode expansion

Burdman & Nomura [2003]



5D example & mode expansion

Burdman & Nomura [2003]

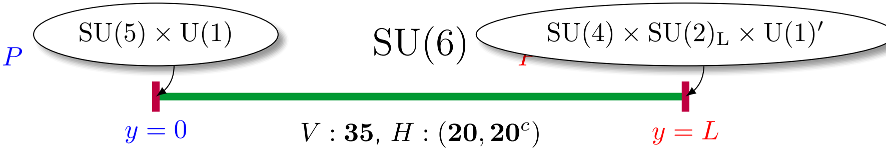


$$\begin{aligned}
\phi_{++}(x_\mu,x_5) &= \sum_{n=0}^{\infty}\frac{1}{\sqrt{2^{\delta_{n0}}\pi R}}\,\phi^{(2n)}_{++}(x_\mu)\,\cos\left(\frac{2n\,x_5}{R}\right)\\
\phi_{+-}(x_\mu,x_5) &= \sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi R}}\,\phi^{(2n+1)}_{+-}(x_\mu)\,\cos\left(\frac{(2n+1)\,x_5}{R}\right)\\
\phi_{-+}(x_\mu,x_5) &= \sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi R}}\,\phi^{(2n+1)}_{-+}(x_\mu)\,\sin\left(\frac{(2n+1)\,x_5}{R}\right)\\
\phi_{--}(x_\mu,x_5) &= \sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi R}}\,\phi^{(2n+2)}_{--}(x_\mu)\,\sin\left(\frac{(2n+2)\,x_5}{R}\right)
\end{aligned}$$

$x_5 = 0$ $x_5 = L$

5D example & mode expansion

Burdman & Nomura [2003]



☞ Adjoint scalar from 6D vector

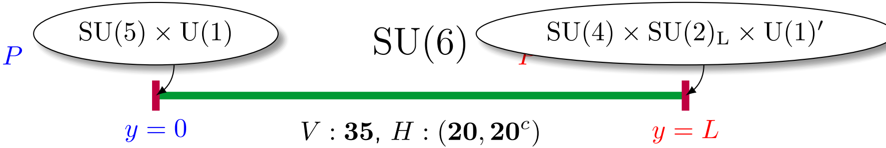
$$\Phi \;=\; \Phi^a\,\mathsf{T}_a \;=$$

$$\begin{pmatrix}
\Phi^{(--)}_{(\mathbf{8},\mathbf{1})_0} - \frac{1}{\sqrt{15}}\Phi^{(--)}_Y + \frac{1}{2\sqrt{15}}\Phi^{(--)}_\chi & \frac{1}{\sqrt{2}}\Phi^{(-+)}_{(\mathbf{3},\mathbf{2})_{-5/6}} & \frac{1}{\sqrt{2}}\Phi^{(+-)}_{(\mathbf{3},\mathbf{1})_{-1/3}} \\
\frac{1}{\sqrt{2}}\Phi^{(-+)}_{(\overline{\mathbf{3}},\mathbf{2})_{5/6}} & \Phi^{(--)}_{(\mathbf{1},\mathbf{3})} + \frac{3}{2\sqrt{15}}\Phi^{(--)}_Y + \frac{1}{2\sqrt{15}}\Phi^{(--)}_\chi & \frac{1}{\sqrt{2}}\Phi^{(++)}_{(\mathbf{1},\mathbf{2})_{1/2}} \\
\frac{1}{\sqrt{2}}\Phi^{(+-)}_{(\overline{\mathbf{3}},\mathbf{1})_{1/3}} & \frac{1}{\sqrt{2}}\Phi^{(++)}_{(\mathbf{1},\mathbf{2})_{-1/2}} & \frac{-5}{2\sqrt{15}}\Phi^{(--)}_{(\mathbf{1},\mathbf{1})_0}
\end{pmatrix}$$

➥ Only SM Higgs fields have zero modes

5D example & mode expansion

Burdman & Nomura [2003]



➥ Only SM Higgs fields have zero modes

☞ Group–theoretical intersection of $\mathrm{SU}(5)$ and $\mathrm{SU}(4)\times\mathrm{SU}(2)_\mathrm{L}$ in $\mathrm{SU}(6)$ is $G_\mathrm{SM}\times\mathrm{U}(1)$

2–dimensional orbifolds

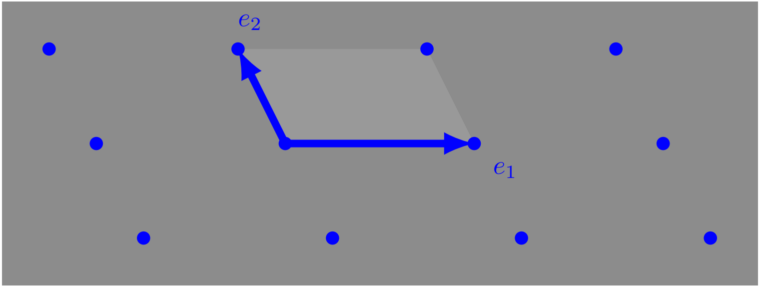
① start with $\mathbb{R}^2$

2–dimensional orbifolds

① start with $\mathbb{R}^2$

② compactify on a torus

- choose basis vectors e_a

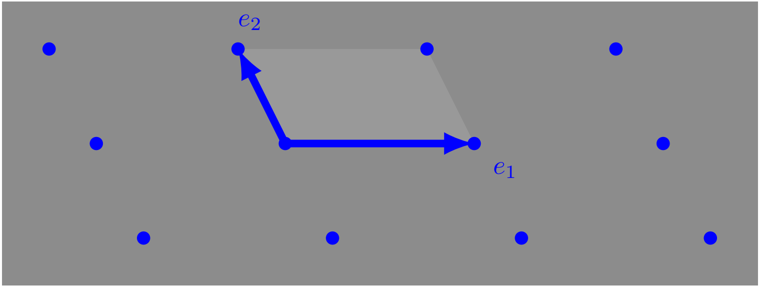


2–dimensional orbifolds

① start with $\mathbb{R}^2$

② compactify on a torus

- choose basis vectors e_a
- define torus lattice $\Lambda = \{n_a\, e_a;\ n_a \in \mathbb{Z}\}$

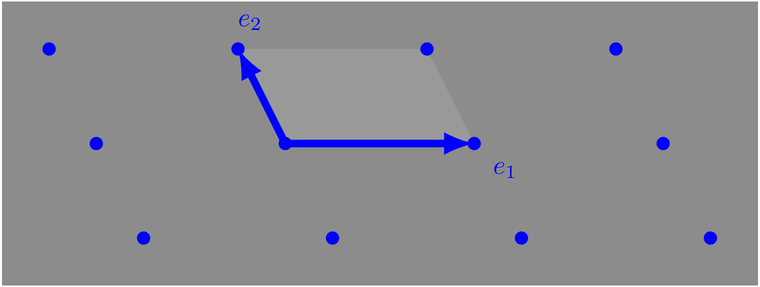


2–dimensional orbifolds

① start with $\mathbb{R}^2$

② compactify on a torus

- choose basis vectors e_a
- define torus lattice $\Lambda = \{n_a\, e_a ;\ n_a \in \mathbb{Z}\}$
- identify points differing by lattice vectors $\ell \in \Lambda$



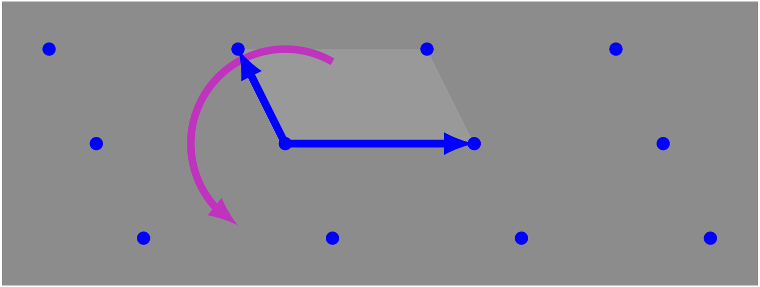
2–dimensional orbifolds

① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

- choose discrete rotation θ which maps Λ onto itself



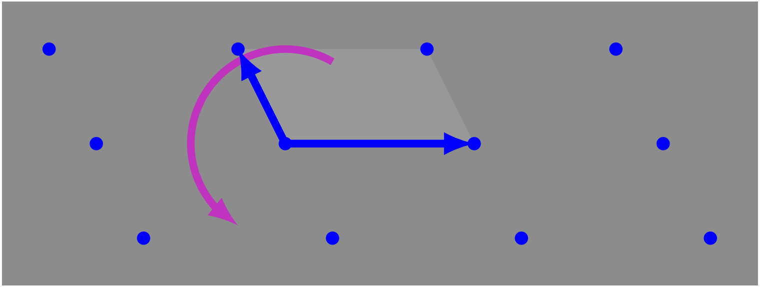
2–dimensional orbifolds

① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

- choose discrete rotation θ which maps Λ onto itself
- identify points related by θ



2–dimensional orbifolds

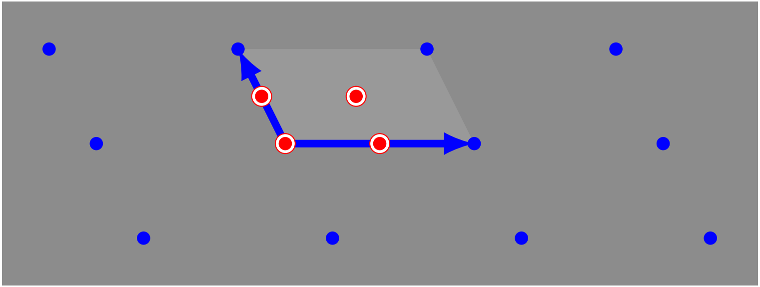
① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

④ identify fixed points : $\theta\, f \;=\; f + \ell\,, \quad \ell \in \Lambda$

- correspondence $f \leftrightarrow (\theta, \ell)$



2–dimensional orbifolds

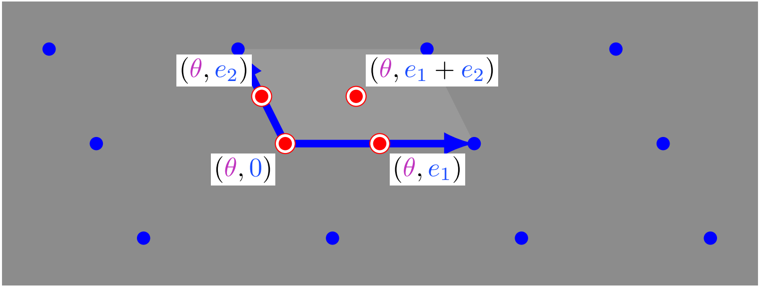
① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

④ identify fixed points : $\theta\, f \;=\; f + \ell\,,\quad \ell \in \Lambda$

- correspondence $f \leftrightarrow (\theta, \ell)$
- ℓ is only determined up to translations $\lambda \in (\mathbb{1} - \theta)\,\Lambda$



2–dimensional orbifolds

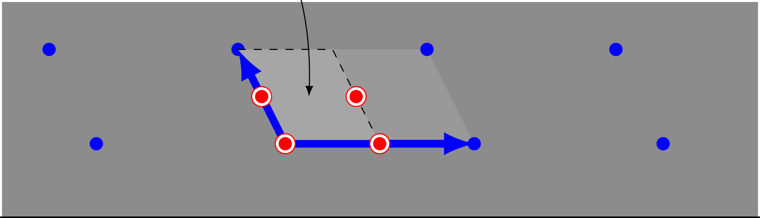
① start with $\mathbb{R}^2$

② compactify on a torus

③ mod out a $\mathbb{Z}_N$ symmetry of the lattice

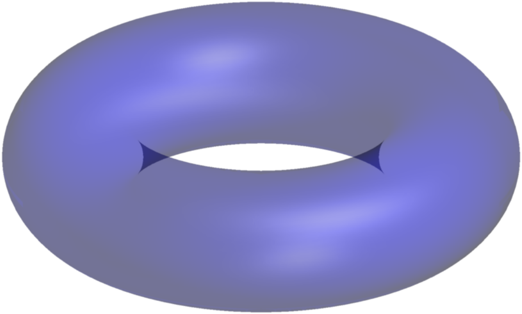
④ identify fixed points : $\theta\, f \;=\; f + \ell\,, \quad \ell \in \Lambda$

$$\begin{pmatrix}\text{fundamental domain}\\ \text{of the orbifold}\end{pmatrix} = \frac{1}{N} \times \begin{pmatrix}\text{fundamental domain}\\ \text{of the torus}\end{pmatrix}$$



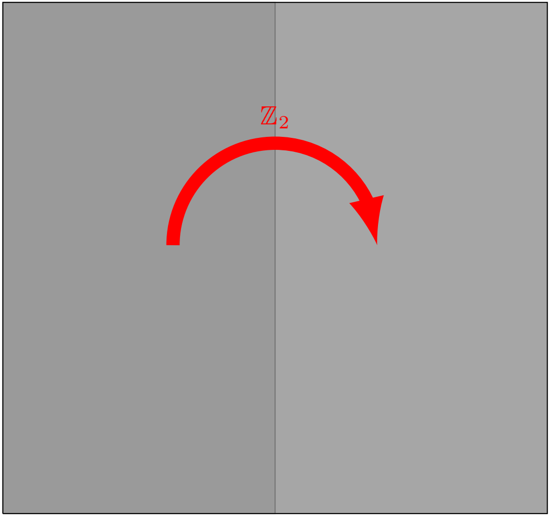
$\mathbb{Z}_2$ orbifold pillow

☞ Starting point: torus

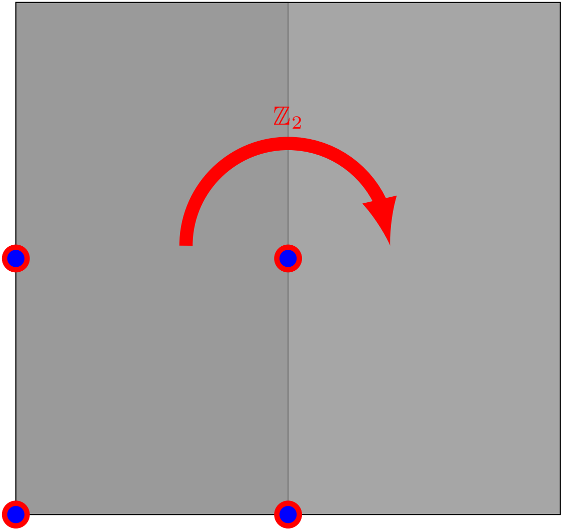


$\mathbb{Z}_2$ orbifold pillow

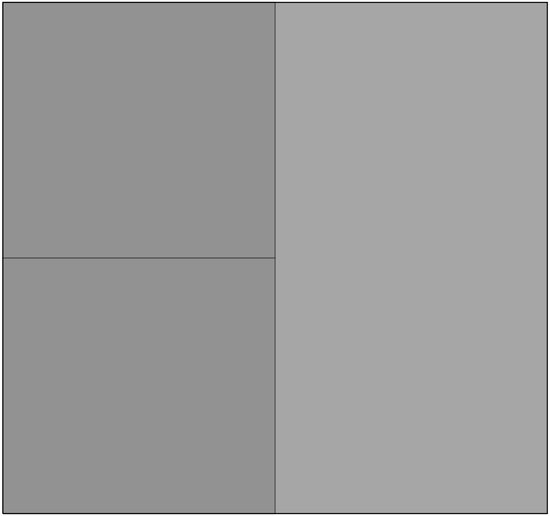
$\mathbb{Z}_2$ orbifold pillow



$\mathbb{Z}_2$ orbifold pillow

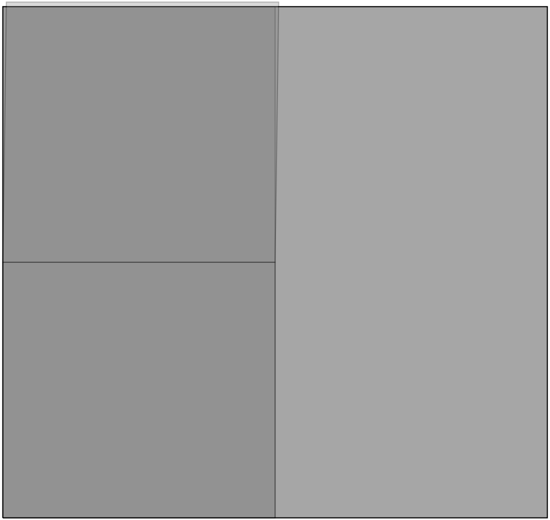


$\mathbb{Z}_2$ orbifold pillow



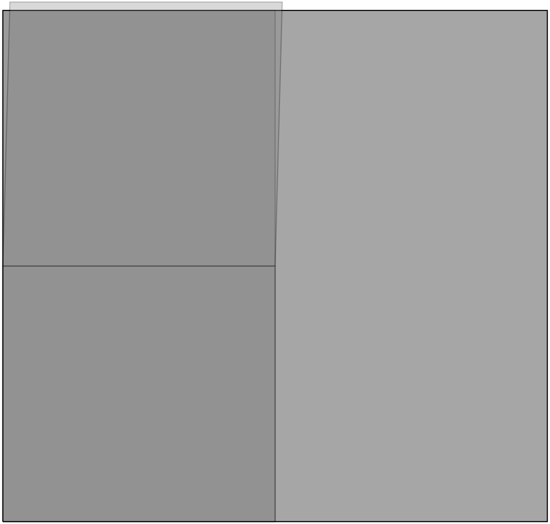
▸ back

$\mathbb{Z}_2$ orbifold pillow



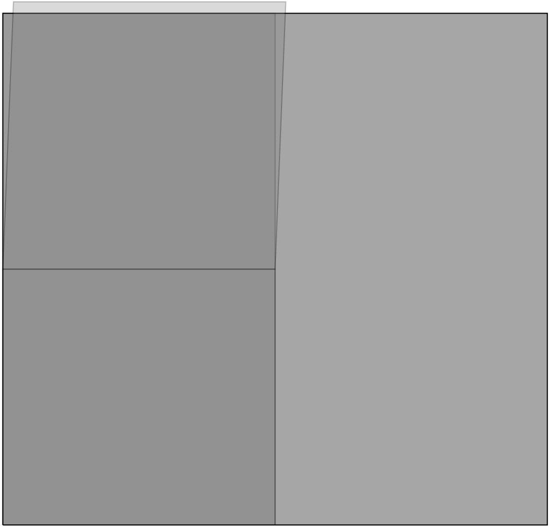
back

$\mathbb{Z}_2$ orbifold pillow



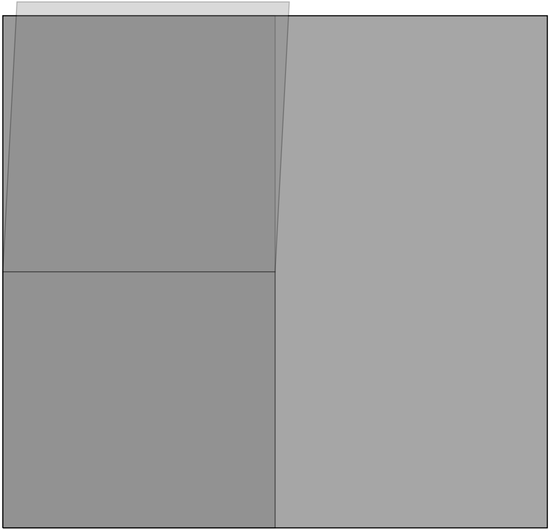
back

$\mathbb{Z}_2$ orbifold pillow



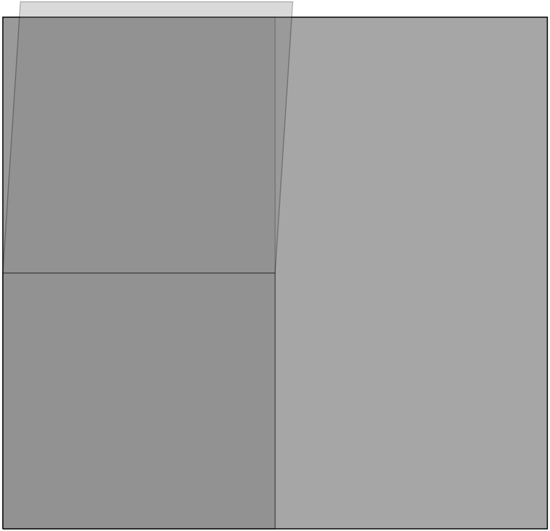
back

$\mathbb{Z}_2$ orbifold pillow



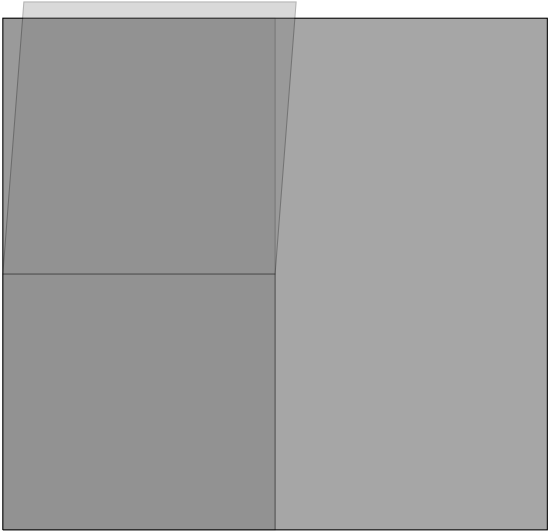
back

$\mathbb{Z}_2$ orbifold pillow



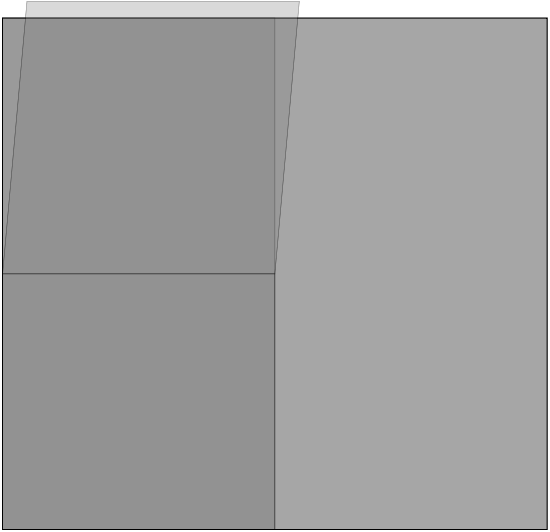
▸ back

$\mathbb{Z}_2$ orbifold pillow



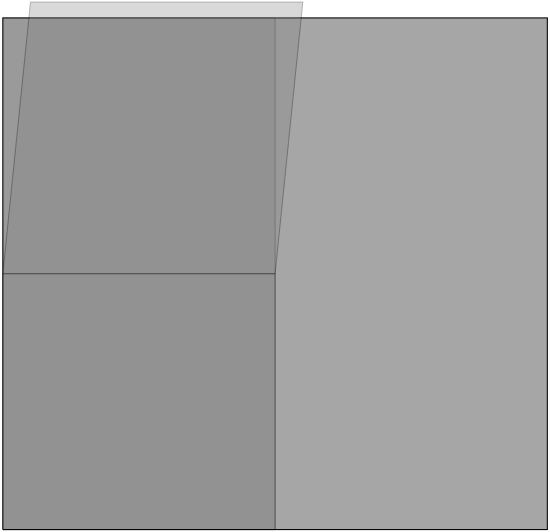
▸ back

$\mathbb{Z}_2$ orbifold pillow



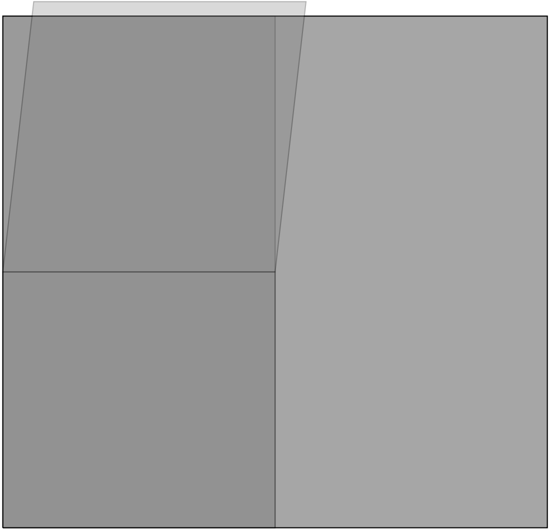
back

$\mathbb{Z}_2$ orbifold pillow



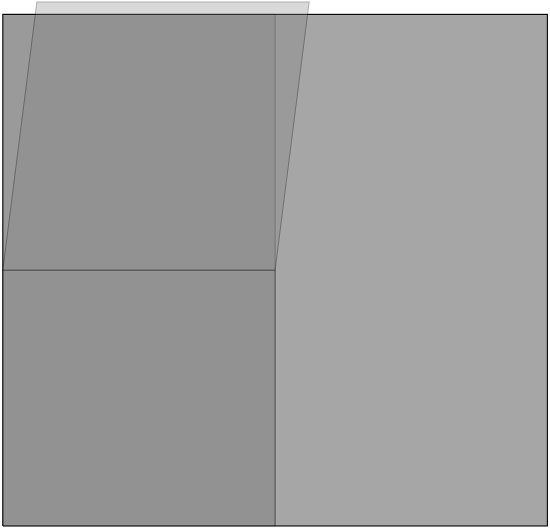
back

$\mathbb{Z}_2$ orbifold pillow



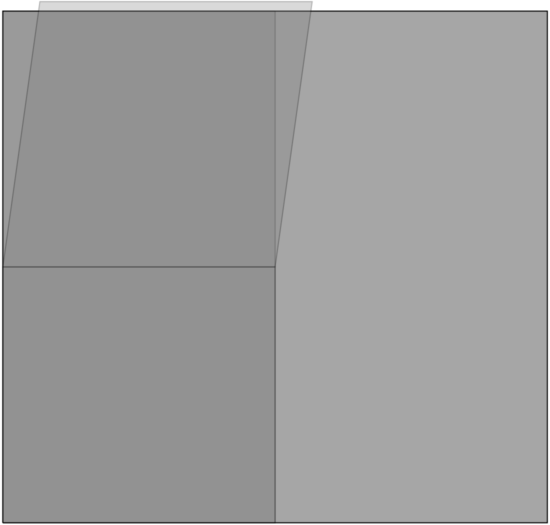
▶ back

$\mathbb{Z}_2$ orbifold pillow



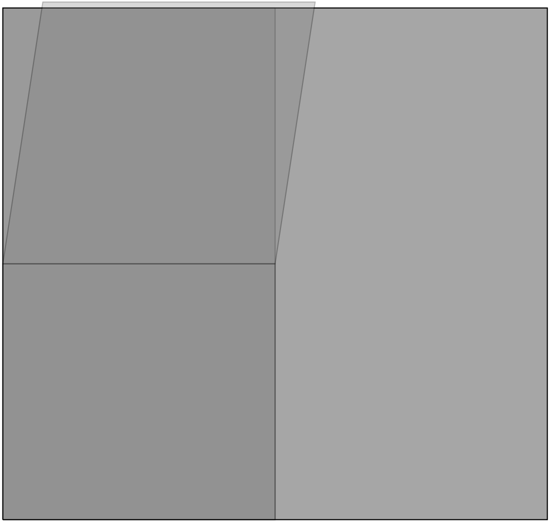
▸ back

$\mathbb{Z}_2$ orbifold pillow



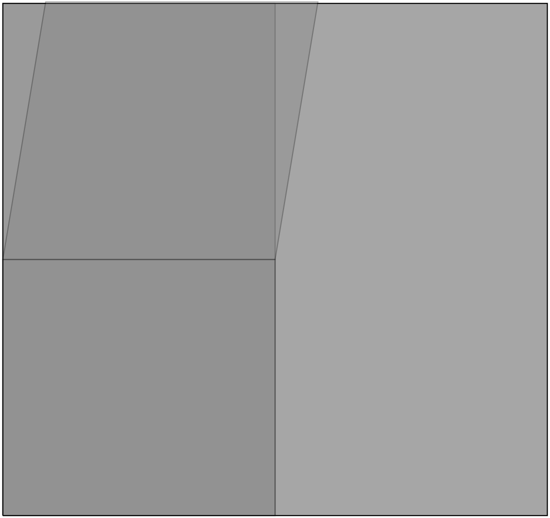
back

$\mathbb{Z}_2$ orbifold pillow



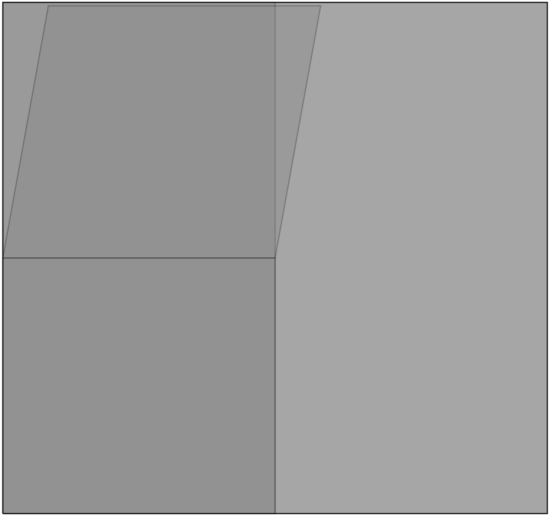
back

$\mathbb{Z}_2$ orbifold pillow



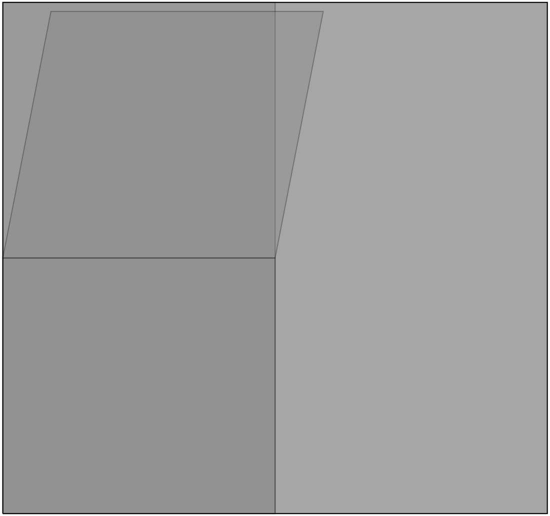
back

$\mathbb{Z}_2$ orbifold pillow



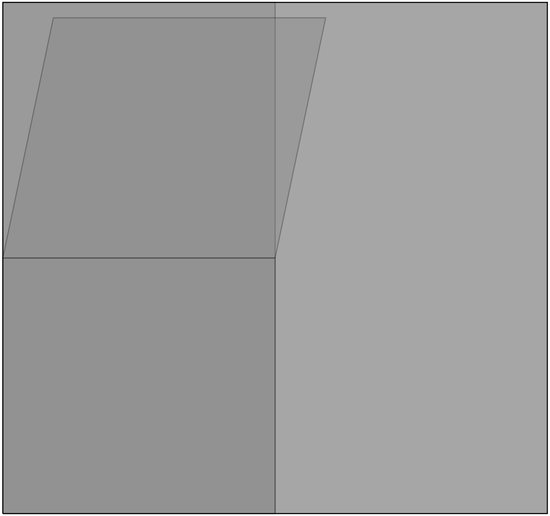
back

$\mathbb{Z}_2$ orbifold pillow



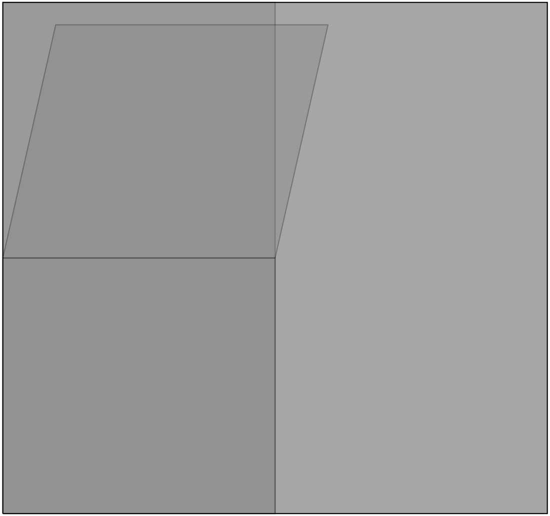
▸ back

$\mathbb{Z}_2$ orbifold pillow



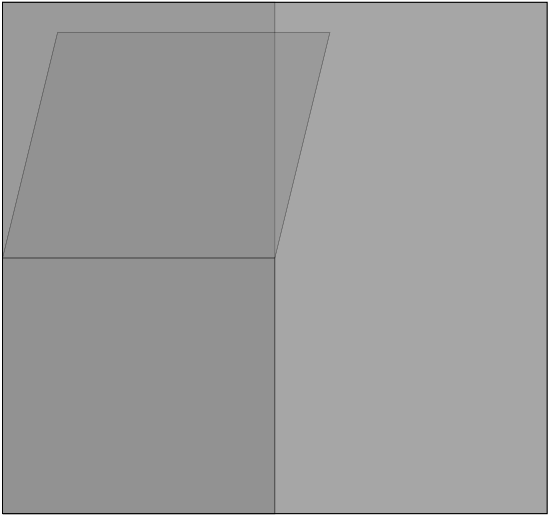
back

$\mathbb{Z}_2$ orbifold pillow



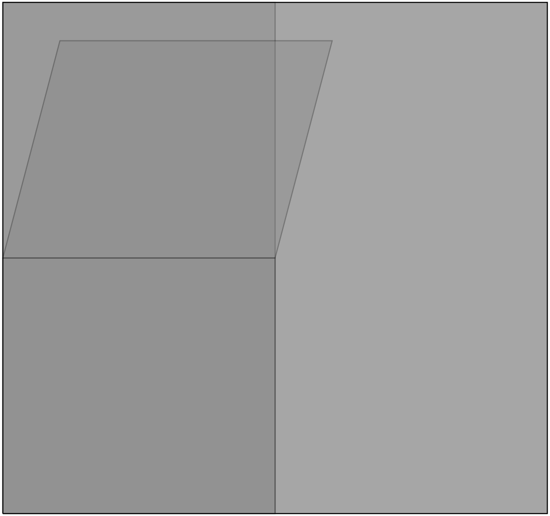
back

$\mathbb{Z}_2$ orbifold pillow



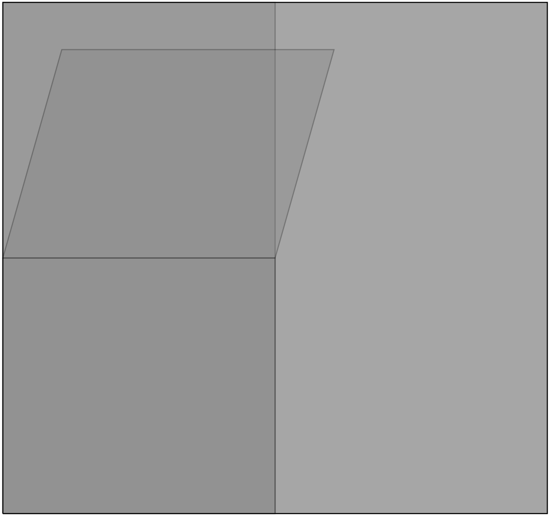
back

$\mathbb{Z}_2$ orbifold pillow



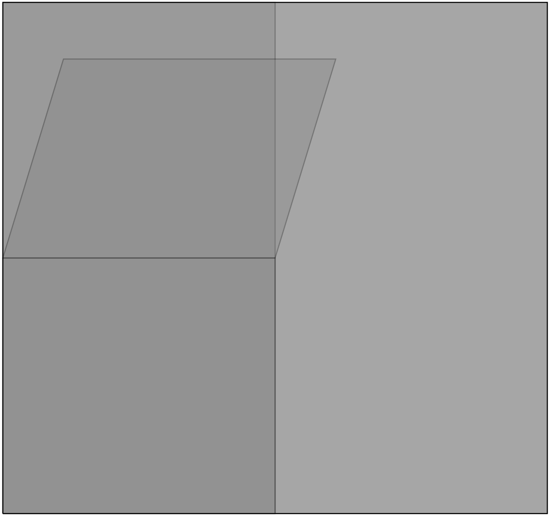
back

$\mathbb{Z}_2$ orbifold pillow



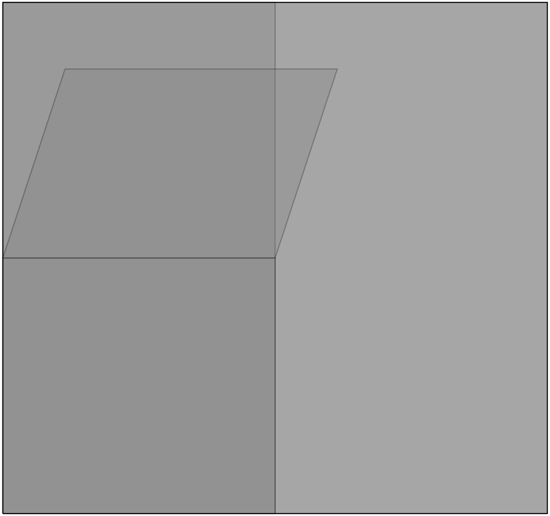
back

$\mathbb{Z}_2$ orbifold pillow



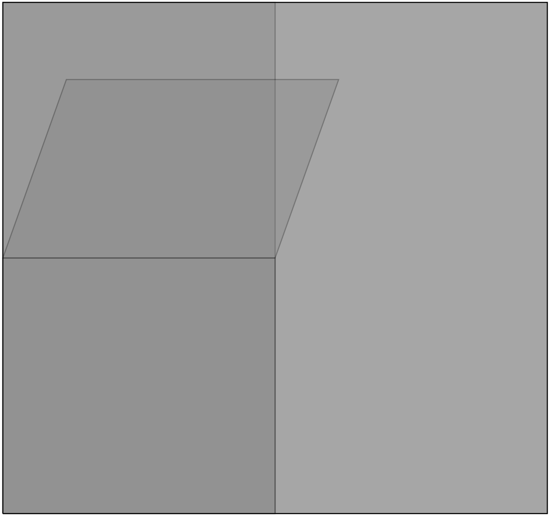
back

$\mathbb{Z}_2$ orbifold pillow



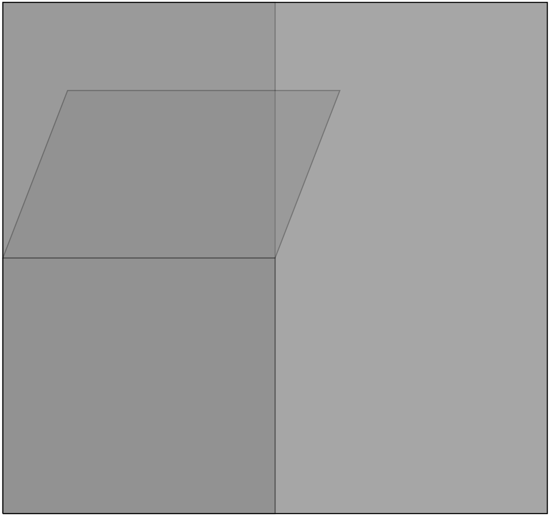
▸ back

$\mathbb{Z}_2$ orbifold pillow



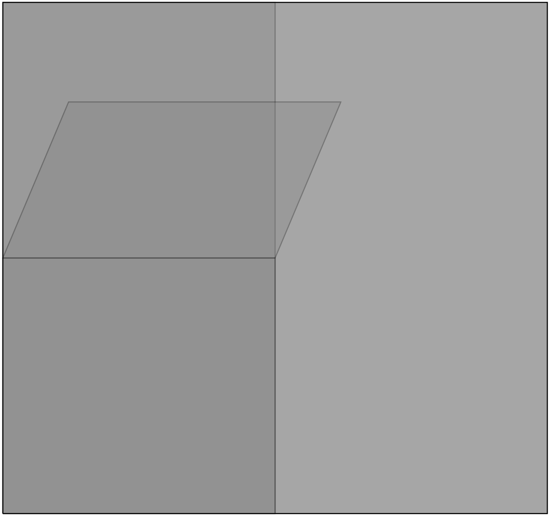
back

$\mathbb{Z}_2$ orbifold pillow



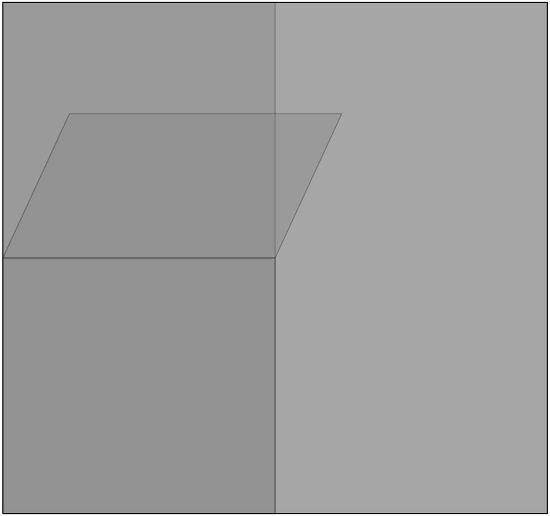
back

$\mathbb{Z}_2$ orbifold pillow



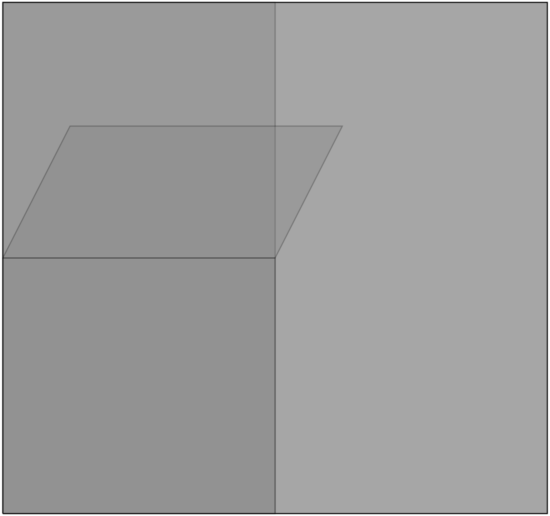
back

$\mathbb{Z}_2$ orbifold pillow



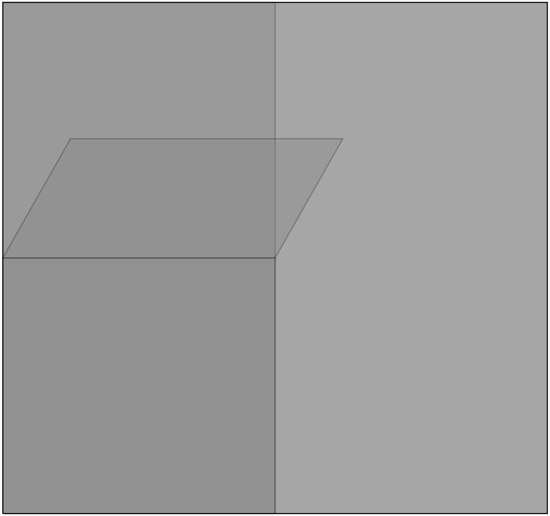
back

$\mathbb{Z}_2$ orbifold pillow



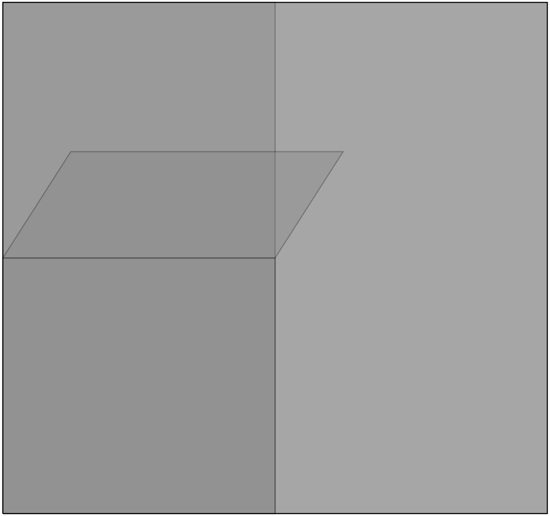
back

$\mathbb{Z}_2$ orbifold pillow



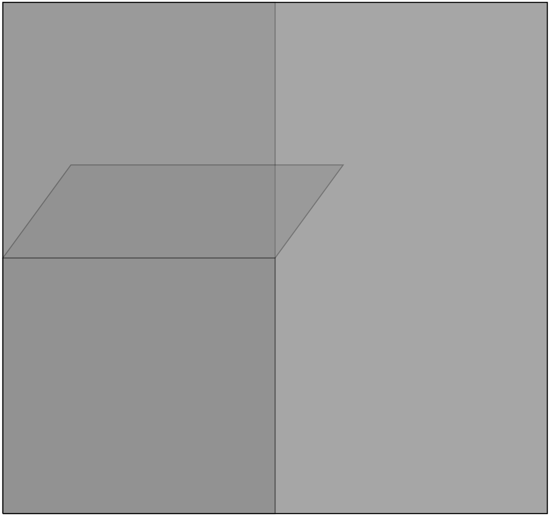
back

$\mathbb{Z}_2$ orbifold pillow



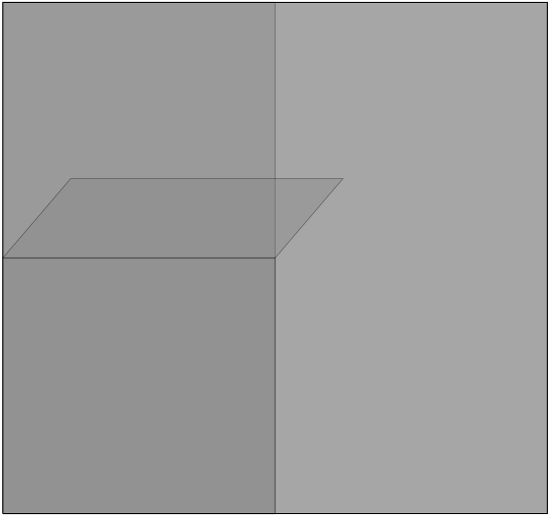
▸ back

$\mathbb{Z}_2$ orbifold pillow



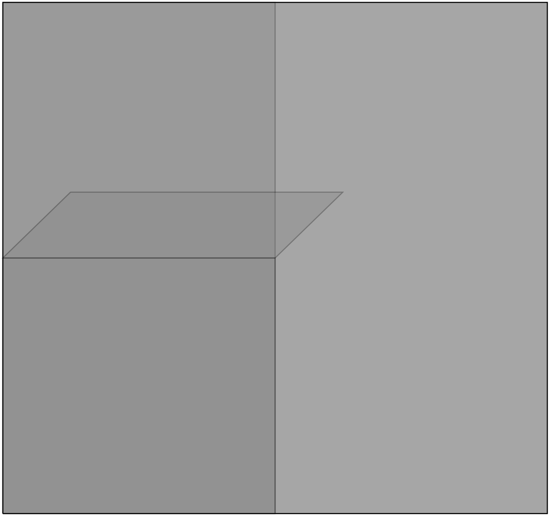
back

$\mathbb{Z}_2$ orbifold pillow



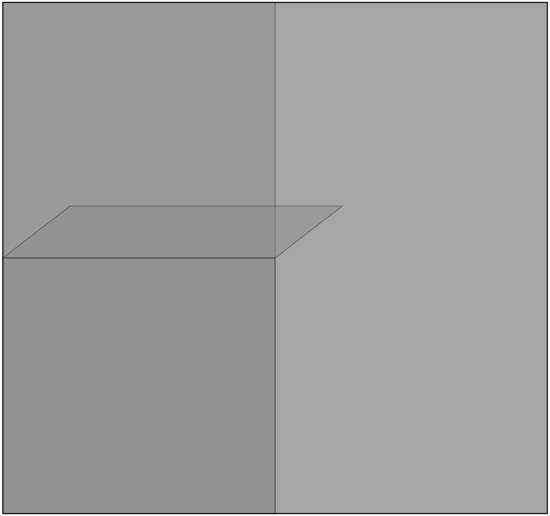
▸ back

$\mathbb{Z}_2$ orbifold pillow



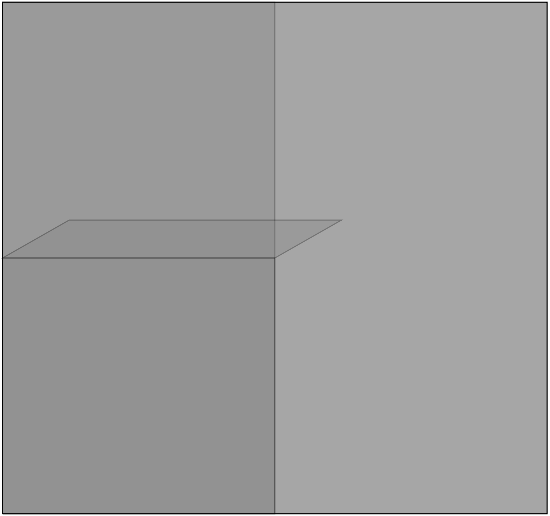
back

$\mathbb{Z}_2$ orbifold pillow



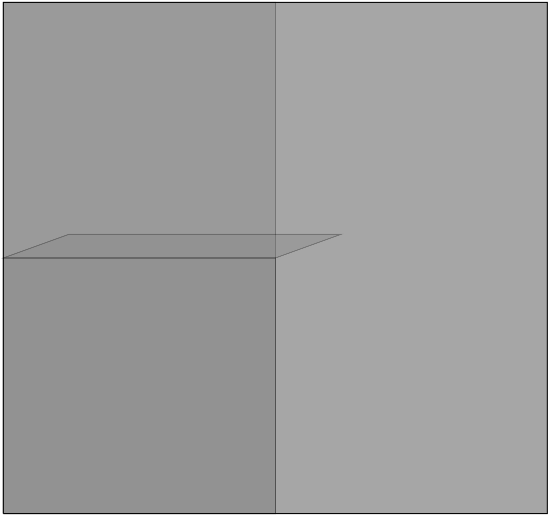
back

$\mathbb{Z}_2$ orbifold pillow



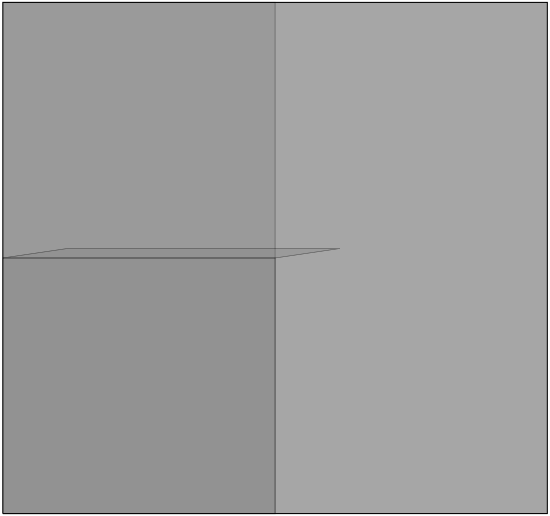
back

$\mathbb{Z}_2$ orbifold pillow



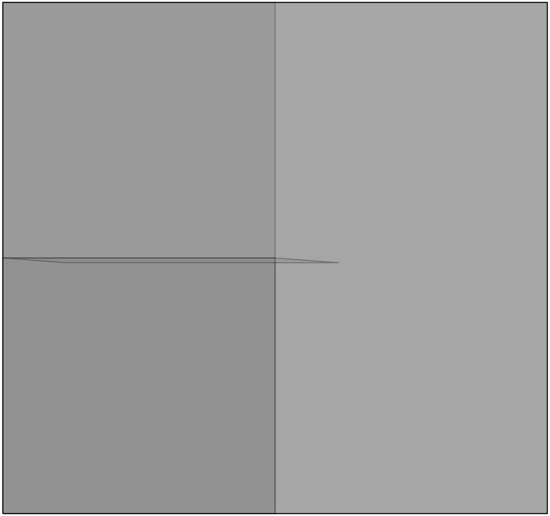
▸ back

$\mathbb{Z}_2$ orbifold pillow



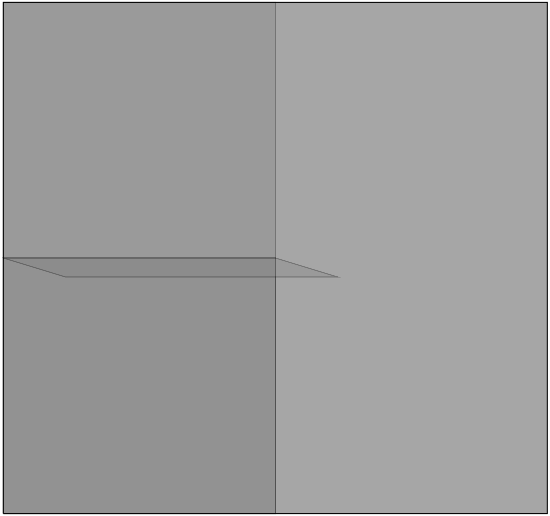
back

$\mathbb{Z}_2$ orbifold pillow



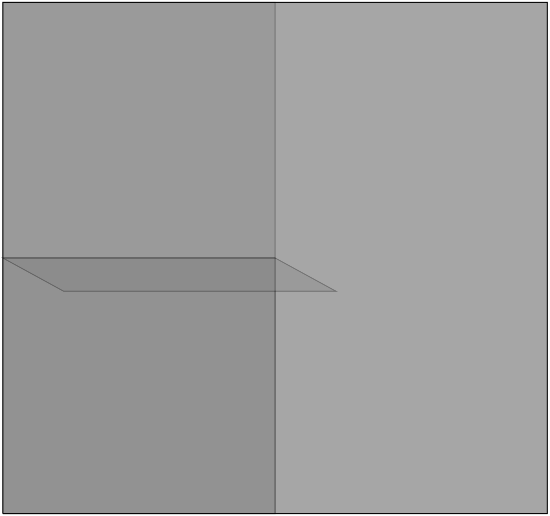
back

$\mathbb{Z}_2$ orbifold pillow



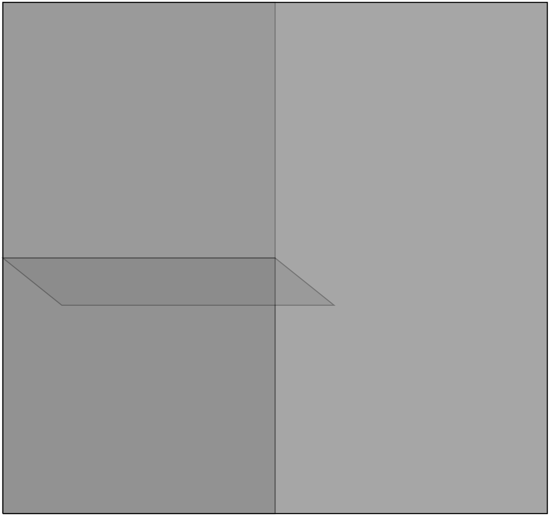
▸ back

$\mathbb{Z}_2$ orbifold pillow



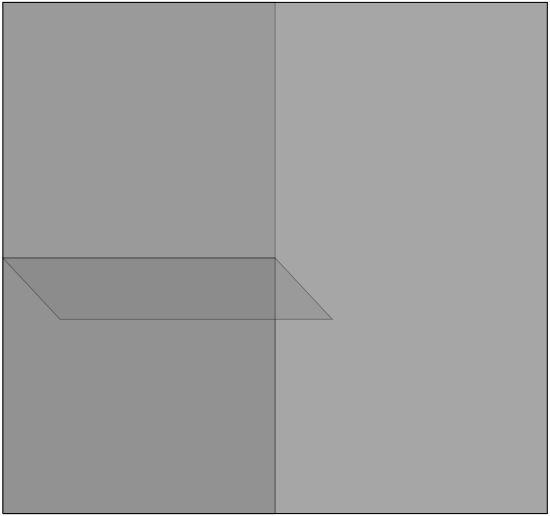
back

$\mathbb{Z}_2$ orbifold pillow



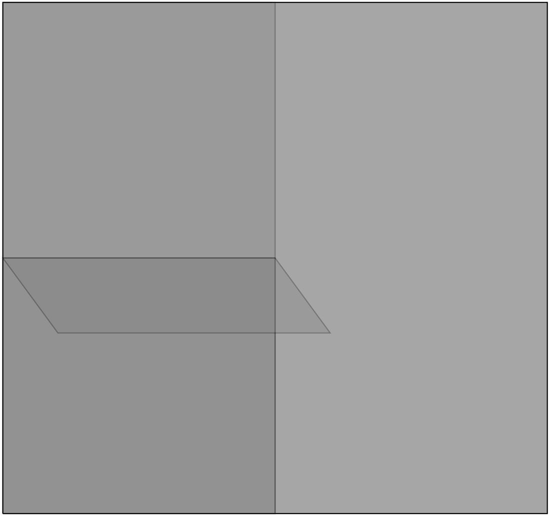
back

$\mathbb{Z}_2$ orbifold pillow



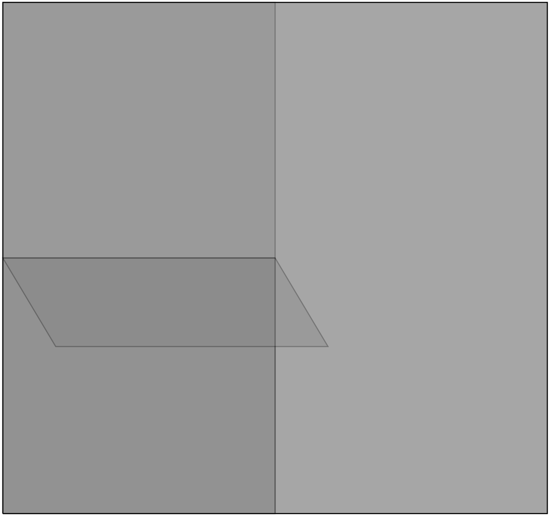
back

$\mathbb{Z}_2$ orbifold pillow



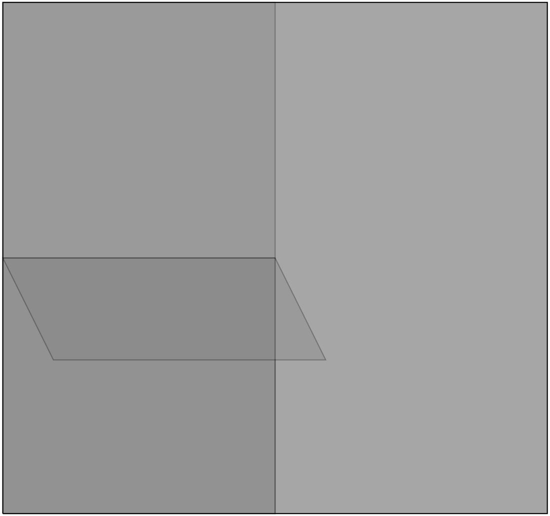
back

$\mathbb{Z}_2$ orbifold pillow



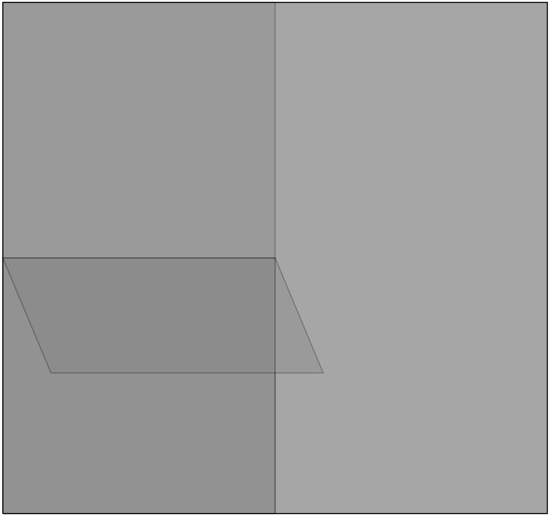
back

$\mathbb{Z}_2$ orbifold pillow



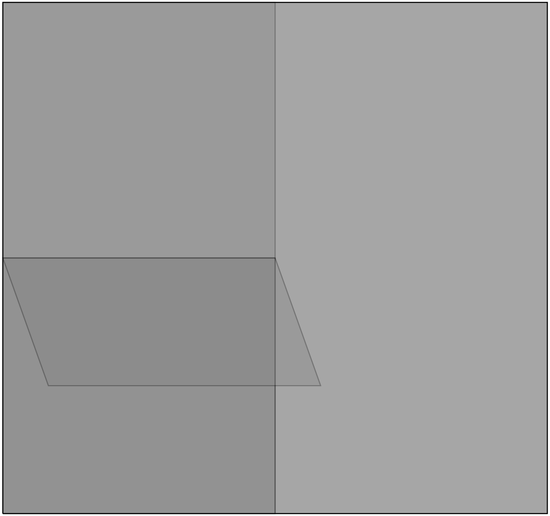
back

$\mathbb{Z}_2$ orbifold pillow



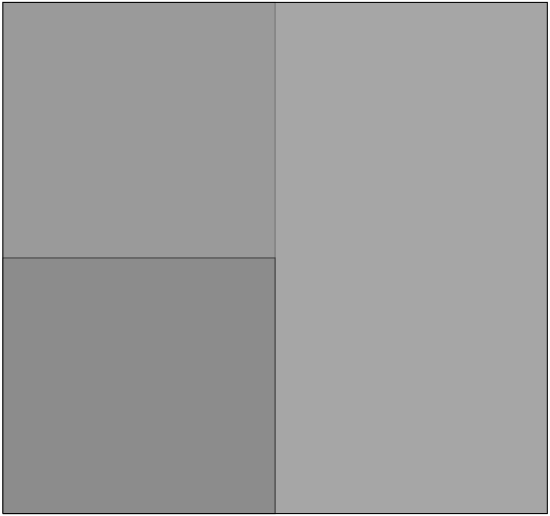
back

$\mathbb{Z}_2$ orbifold pillow



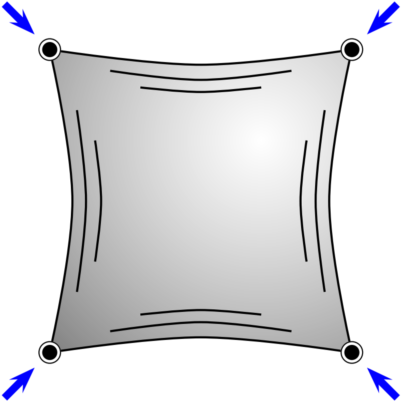
back

$\mathbb{Z}_2$ orbifold pillow



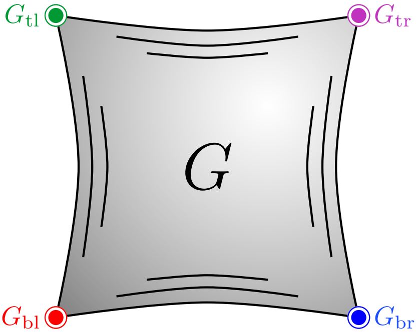
back

$\mathbb{Z}_2$ orbifold pillow



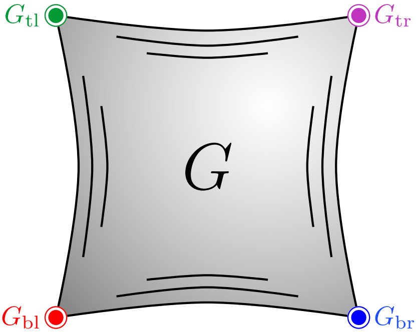
☞ An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points

$\mathbb{Z}_2$ orbifold pillow



- An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points
- 'Bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points

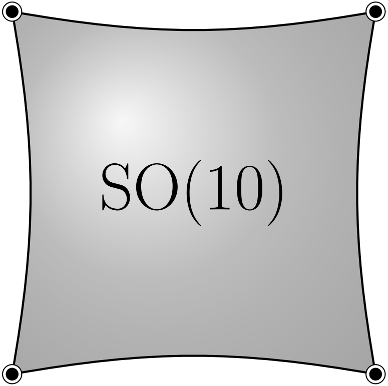
$\mathbb{Z}_2$ orbifold pillow



- An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points
- 'Bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points
- Low–energy gauge group : $G_{\mathrm{low-energy}} = G_{\mathrm{bl}} \cap G_{\mathrm{br}} \cap G_{\mathrm{tl}} \cap G_{\mathrm{tr}}$

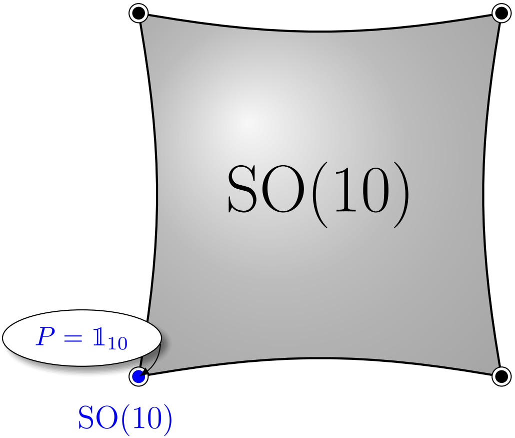
A 6D example

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



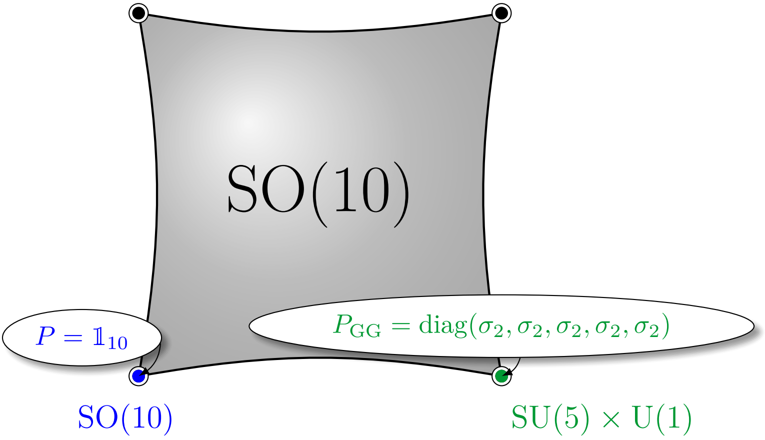
A 6D example

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



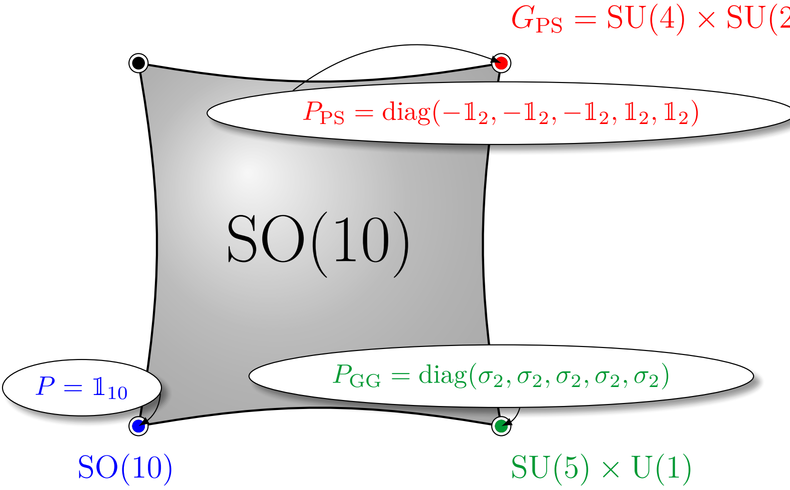
A 6D example

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



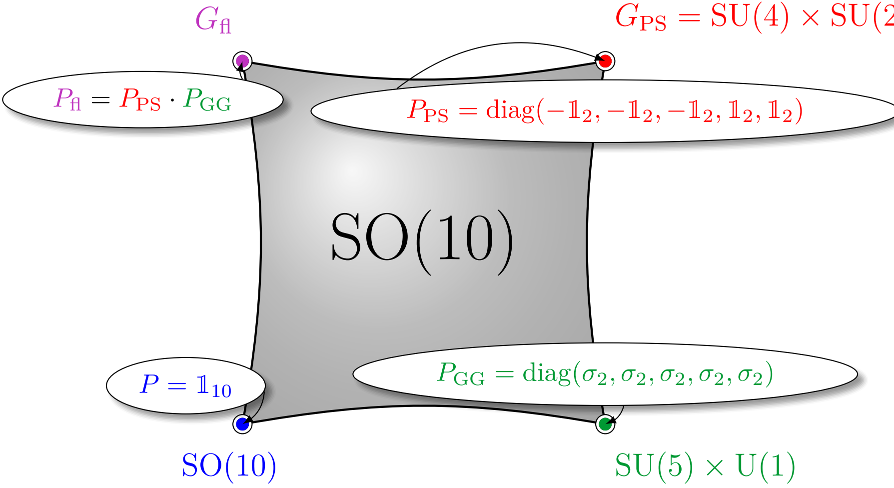
A 6D example

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



A 6D example

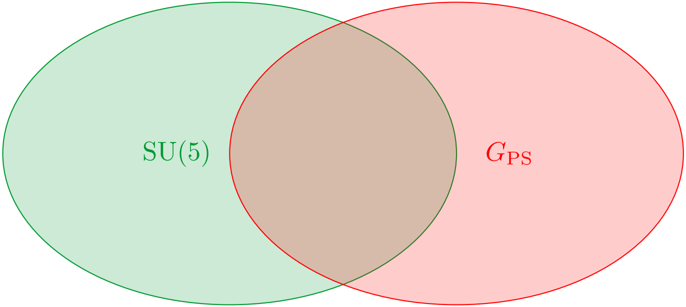
Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



SO(10)

Asaka, Buchmüller & Covi [2001]

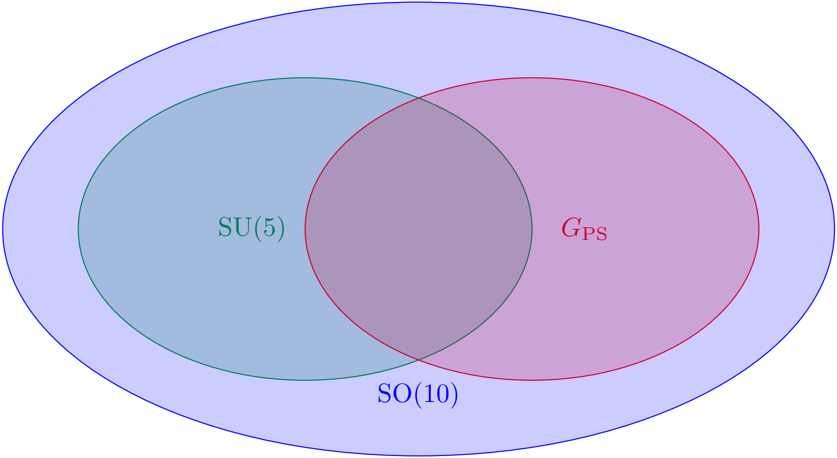
☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is SO(10)



SO(10)

Asaka, Buchmüller & Covi [2001]

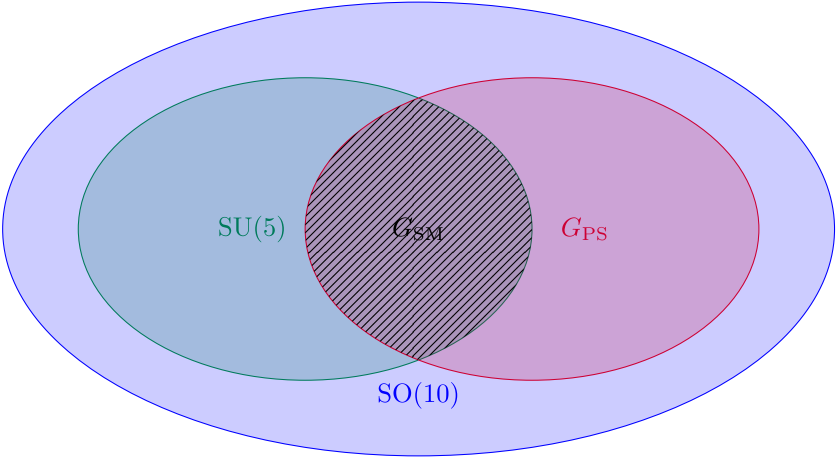
☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is SO(10)



SO(10)

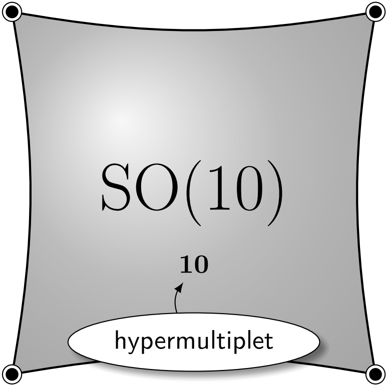
Asaka, Buchmüller & Covi [2001]

☞ smallest group containing both $\mathrm{SU}(5)$ and $G_{\mathrm{PS}} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ is SO(10)



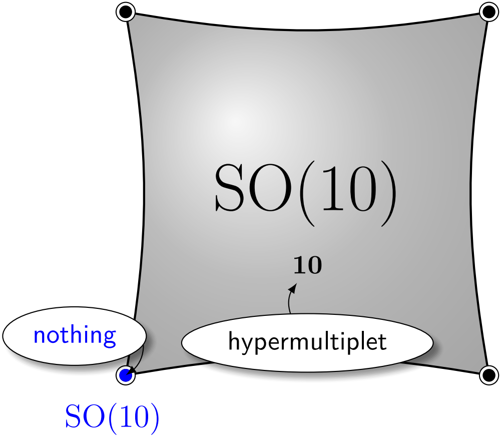
Matter in the ABC model

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



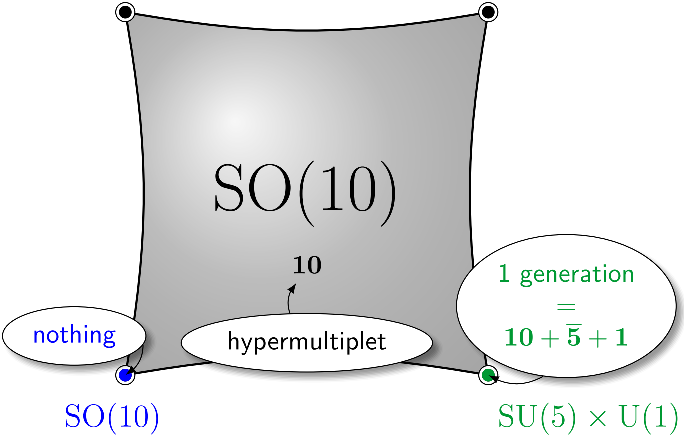
Matter in the ABC model

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



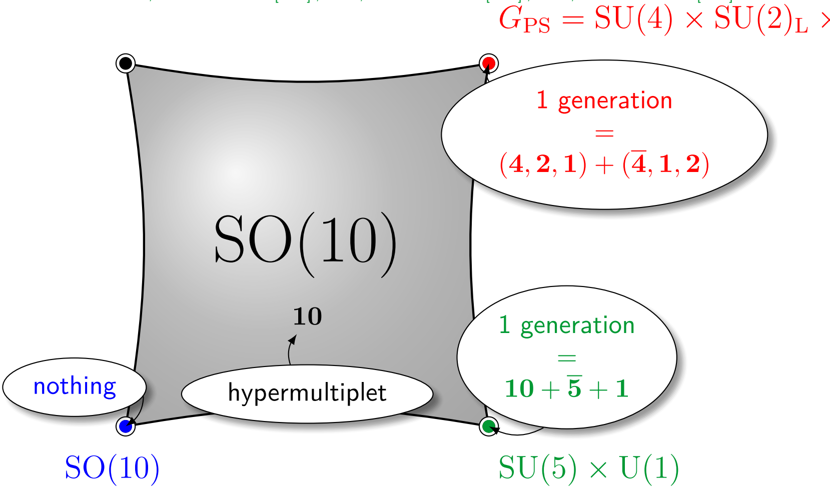
Matter in the ABC model

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



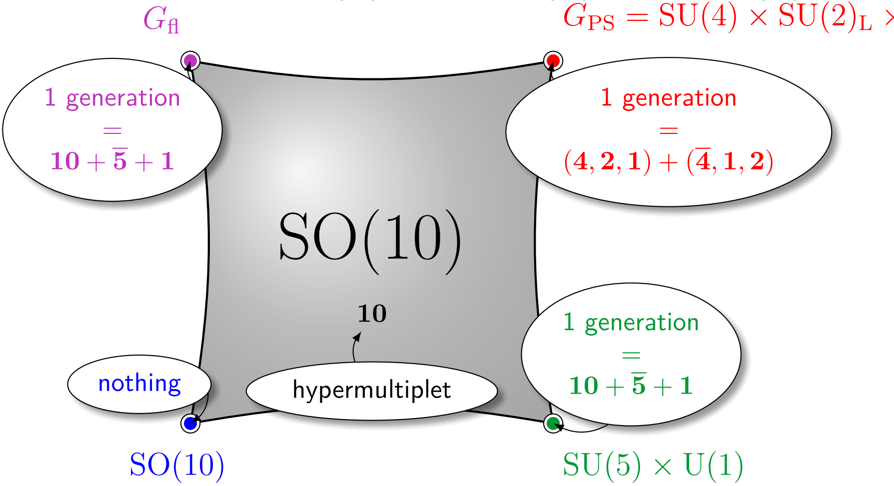
Matter in the ABC model

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



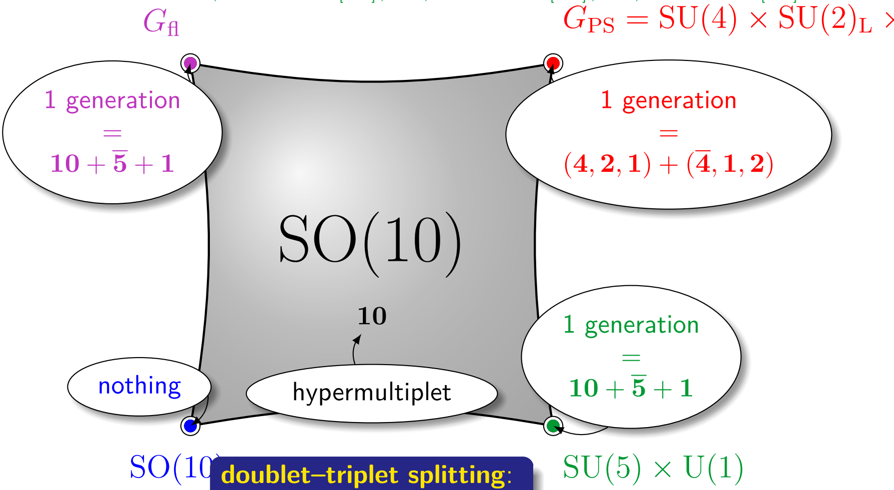
Matter in the ABC model

Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



Matter in the ABC model

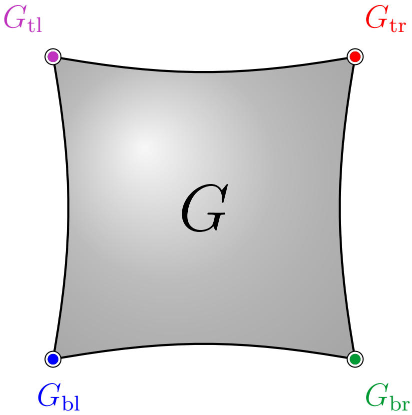
Asaka, Buchmüller & Covi [2001] ; Asaka, Buchmüller & Covi [2002] ; Asaka, Buchmüller & Covi [2003]



doublet–triplet splitting:

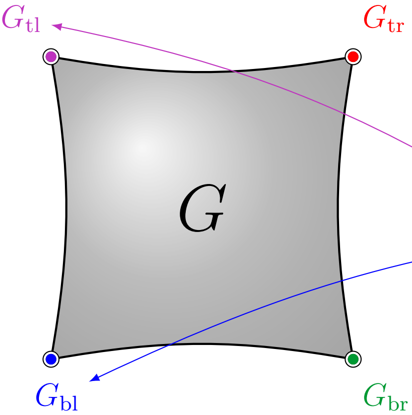
only 2 $\mathrm{SU}(2)$ doublets survive projections

Lessons from 6D orbifold GUTs



☞ non–trivial gauge group topography

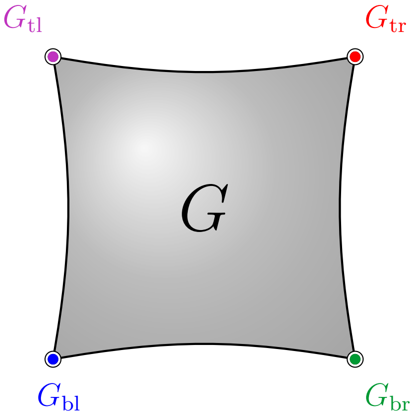
Lessons from 6D orbifold GUTs



- non–trivial gauge group topography
- low–energy gauge group is the intersection of local gauge groups

$$G_{\mathrm{low-energy}} = G_{\mathrm{tl}} \cap G_{\mathrm{tr}} \cap G_{\mathrm{bl}} \cap G_{\mathrm{br}}$$

Lessons from 6D orbifold GUTs

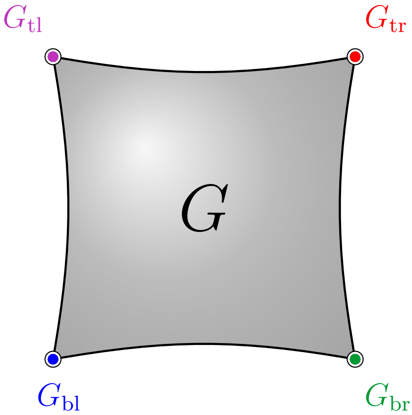


- non–trivial gauge group topography
- low–energy gauge group is the intersection of local gauge groups

$$G_{\text{low-energy}} = G_{\rm tl} \cap G_{\rm tr} \cap G_{\rm bl} \cap G_{\rm br}$$

- localized matter comes in complete representations of the local gauge group

Lessons from 6D orbifold GUTs



- non–trivial gauge group topography
- low–energy gauge group is the intersection of local gauge groups

$$G_{\text{low-energy}} = G_{\mathrm{tl}} \cap G_{\mathrm{tr}} \cap G_{\mathrm{bl}} \cap G_{\mathrm{br}}$$

- localized matter comes in complete representations of the local gauge group
- bulk fields appear in split multiplets

(Many) open questions

? Is there an explanation for the representations?

. . . anomalies are not that constraining

(Many) open questions

- ? Is there an explanation for the representations?
- ? What are the couplings?

(Many) open questions

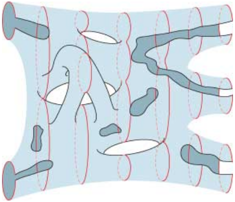
- ? Is there an explanation for the representations?
- ? What are the couplings?
- ? To which extent can we trust the higher–dimensional field-theoretical (i.e. nonrenormalizable) calculations?

(Many) open questions

- ? Is there an explanation for the representations?
- ? What are the couplings?
- ? To which extent can we trust the higher–dimensional field-theoretical (i.e. nonrenormalizable) calculations?
- ? Why do we observe matter in the form of complete $\mathbf{16}$–plets?

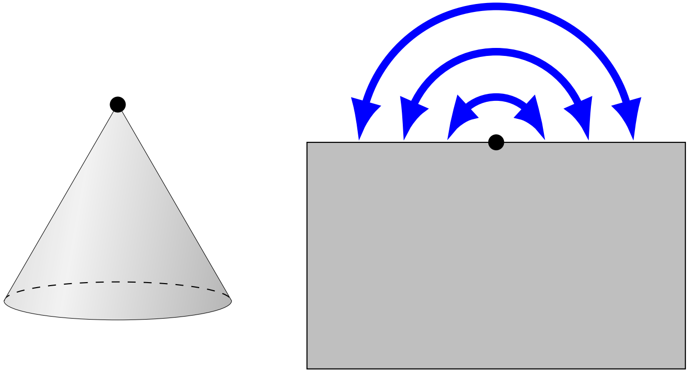
(Many) open questions

- ? Is there an explanation for the representations?
- ? What are the couplings?
- ? To which extent can we trust the higher–dimensional field-theoretical (i.e. nonrenormalizable) calculations?
- ? Why do we observe matter in the form of complete $\mathbf{16}$–plets?
- ☞ Answer



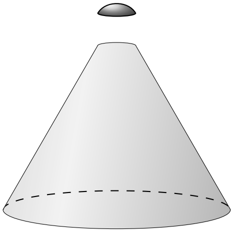
Why strings?

??? What is the field content of states living at the fixed points



Why strings?

??? What is the field content of states living at the fixed points



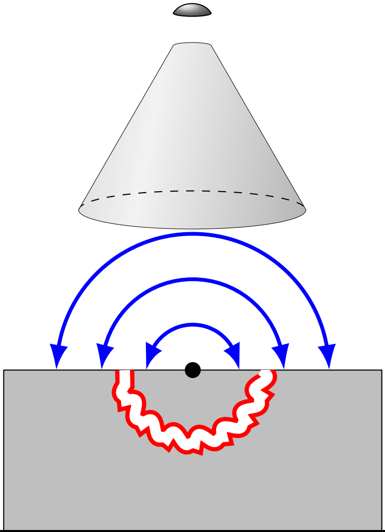
Field–theoretic method :

1. replace conical singularities by smooth manifolds with the same asymptotic behavior
2. calculate zero-modes (via index theorem)

. . . technically quite challenging . . .

Why strings?

??? What is the field content of states living at the fixed points



Field–theoretic method :

1. replace conical singularities by smooth manifolds with the same asymptotic behavior
2. calculate zero-modes (via index theorem)

. . . technically quite challenging . . .

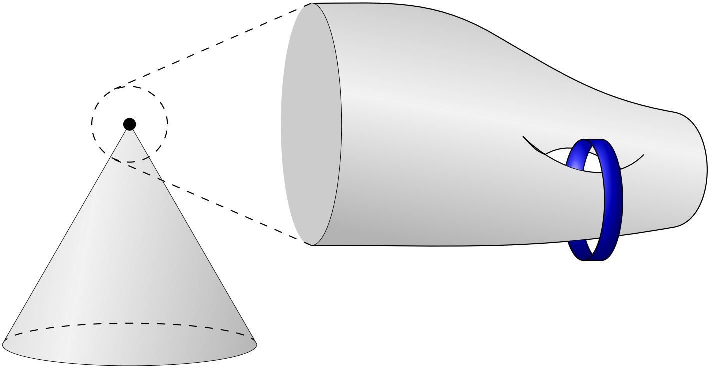
String–theorist's method

1. consider strings 'encircling' the fixed points
2. calculate their spectrum

. . . (technically) rather simple . . .

Why strings

??? What are the fields sitting at the fixed points?



Why strings

??? What are the fields sitting at the fixed points?

☞ Stringy and index methods (seem to) yield the same results

Walton [1988] ; Erler [1994]

Why strings

??? What are the fields sitting at the fixed points?

- ☞ Stringy and index methods (seem to) yield the same results

  Walton [1988] ; Erler [1994]

- ➥ 'String theory as a tool'

Why strings

??? What are the fields sitting at the fixed points?

☞ Stringy and index methods (seem to) yield the same results

Walton [1988] ; Erler [1994]

➥ 'String theory as a tool'

☺ Many important features:

- consistency
- calculability
- ...

Orbifold compactifications

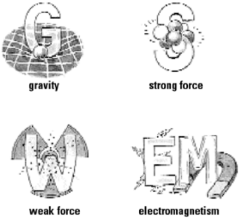
of the

Heterotic string

Main objective of string phenomenology

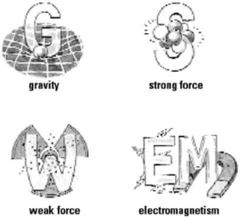
Why strings?

☞ UV complete description of gauge interactions and gravity



Why strings?

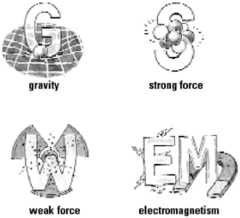
- ☞ UV complete description of gauge interactions and gravity



- ☞ More or less unique ("only game in town")

Why strings?

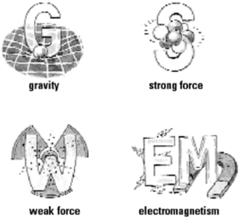
- ☞ UV complete description of gauge interactions and gravity



- ☞ More or less unique ("only game in town")
- ☞ Predictive framework

Why strings?

- ☞ UV complete description of gauge interactions and gravity



- ☞ More or less unique ("only game in town")
- ☞ Predictive framework. . . in principle

A few words of caution

CiteIbanez:1987dw

☞ In his lecture notes, Ibáñez writes

"We should remark at this point that the naive expectation that it should be possible to accomodate easily the standard model inside such a big group like $\mathrm{E}_8 \times \mathrm{E}_8$ *has no basis. The standard model includes quite a number of delicate properties which are not so easy to reproduce. Of course,* $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ *is contained in* $\mathrm{E}_8 \times \mathrm{E}_8$*, but* $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ *<u>is not</u> the standard model. The standard model is something more sophisticated. Since the younger generations are nowadays jumping directly from Newtonian dynamics to conformal field theory, it could be worthwhile to list some of the standard model properties which one would like to find in a realistic superstring."*

Ibáñez' wish list

1. chirality   Ibáñez [1987]

Ibáñez' wish list

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$

Ibáñez [1987]

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{30}$ years)

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.21$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$
3. $N=1$ supersymmetry in $d=4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim 100\,\mathrm{GeV}$ nor fermions $\lesssim 40\,\mathrm{GeV}$

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\,1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}\,100\,\mathrm{GeV}$

Ibáñez' wish list

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$

Ibáñez [1987]

9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\,1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}\,100\,\mathrm{GeV}$
10. no flavour–changing neutral currents

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\,1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}\,100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. massless (or so) left–handed neutrino

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2 \theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\,1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}\,100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. ~~massless~~ (or so) left–handed neutrino

Ibáñez' wish list

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2 \theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$

Ibáñez [1987]

9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. ~~massless~~ (or so) left–handed neutrino
12. weak CP violation exists

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2 \theta_{\mathrm{W}} \simeq 0.2\cancel{1}3$, $M_X \simeq M_{\mathrm{P}} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\,1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}\,100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. ~~massless~~ (or so) left–handed neutrino
12. weak CP violation exists
13. potentially realistic Yukawa couplings (fermion masses)

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
3. $N = 1$ supersymmetry in $d = 4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\, 1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}\, 100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. ~~massless~~ (or so) left–handed neutrino
12. weak CP violation exists
13. potentially realistic Yukawa couplings (fermion masses)
14. $\mathrm{SU}(2) \times \mathrm{U}(1)$ breaking feasible

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$
3. $N=1$ supersymmetry in $d=4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W} \simeq 0.2\cancel{1}3$, $M_X \simeq M_\mathrm{P} \simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}\,1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. ~~massless~~ (or so) left–handed neutrino
12. weak CP violation exists
13. potentially realistic Yukawa couplings (fermion masses)
14. $\mathrm{SU}(2)\times\mathrm{U}(1)$ breaking feasible
15. small supersymmetry breaking

Ibáñez' wish list

Ibáñez [1987]

1. chirality
2. gauge group contains (and can be broken to) $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$
3. $N=1$ supersymmetry in $d=4$
4. contains standard quark-lepton families
5. contains Weinberg-Salam doublets
6. three quark-lepton generations
7. proton is sufficiently stable ($\tau_p \gtrsim 10^{3\cancel{0}4}$ years)
8. correct prediction $\sin^2\theta_\mathrm{W}\simeq 0.2\cancel{1}3$, $M_X\simeq M_\mathrm{P}\simeq 10^{18}\,\mathrm{GeV}$
9. no exotic gauge boson with mass $\lesssim \cancel{100\,\mathrm{GeV}}1\,\mathrm{TeV}$ nor fermions $\lesssim \cancel{40}100\,\mathrm{GeV}$
10. no flavour–changing neutral currents
11. ~~massless~~ (or so) left–handed neutrino
12. weak CP violation exists
13. potentially realistic Yukawa couplings (fermion masses)
14. $\mathrm{SU}(2)\times\mathrm{U}(1)$ breaking feasible
15. small supersymmetry breaking
16. ...

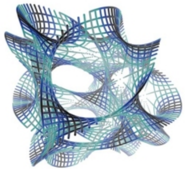
String embedding

String compactifications

- Violin: needs to be constructed in such a way that the oscillating strings produce the right sounds

String compactifications

☞ Violin: needs to be constructed in such a way that the oscillating strings produce the right sounds



☞ String compactification: twist the string in such a way that the excitations carry the quantum numbers of the standard model particles

From strings to the real world?

☞ Many popular attempts to connect strings with observation:

- heterotic orbifolds
- intersecting D–branes
- Calabi–Yau compactifications
- F–theory
- ...

From strings to the real world?

☞ Many popular attempts to connect strings with observation:

- heterotic orbifolds
- intersecting D–branes
- Calabi–Yau compactifications
- F–theory
- ...

☞ Only the first two are true string models
(but the others are believed to relate to string compactifications)

From strings to the real world?

☞ Many popular attempts to connect strings with observation:

- heterotic orbifolds
- intersecting D–branes
- Calabi–Yau compactifications
- F–theory
- ...

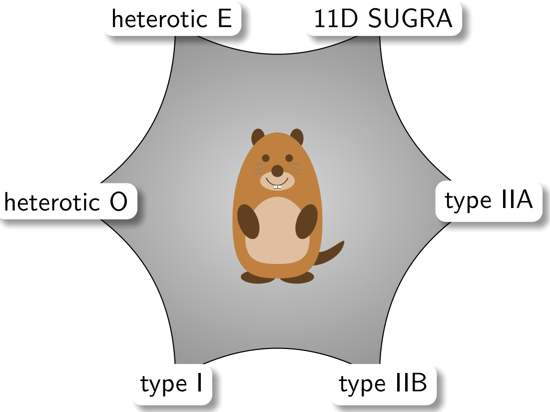
☞ Only the first two are true string models
(but the others are believed to relate to string compactifications)

main theme of the rest of this talk:

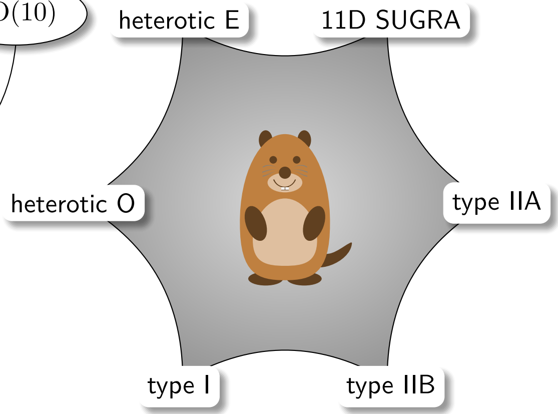
orbifold compactifications of the heterotic string

Ingredients for cooking up string models

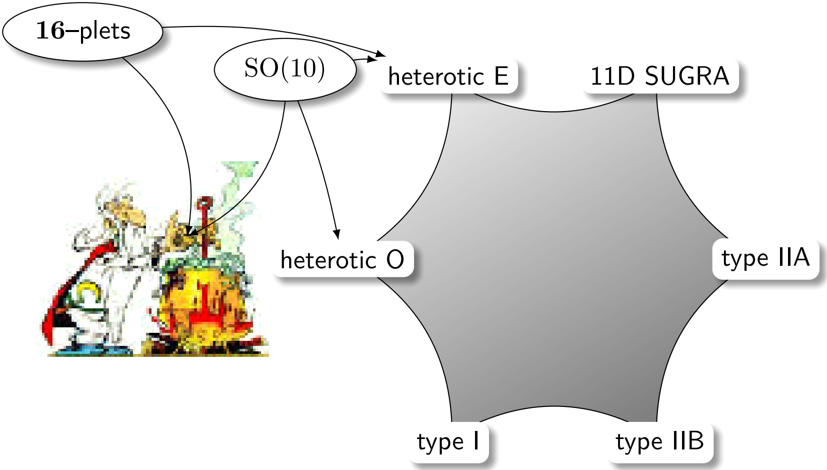
Ingredients for cooking up string models



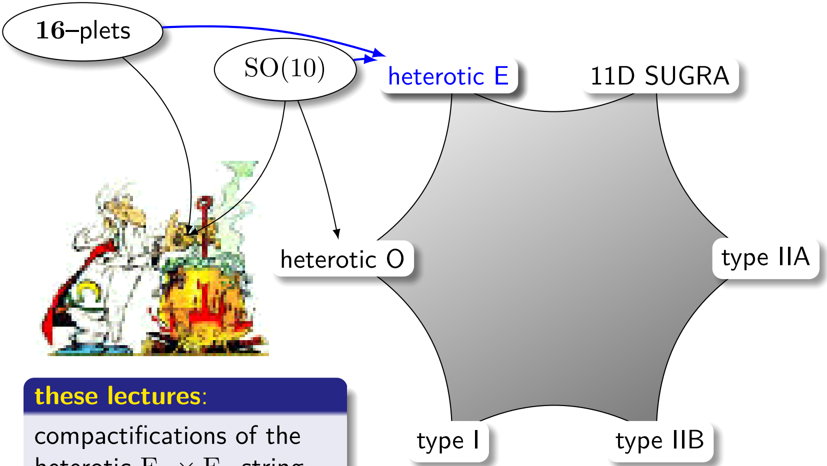
Ingredients for cooking up string models



Ingredients for cooking up string models



Ingredients for cooking up string models



these lectures:

compactifications of the heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ string

History of heterotic model building

- ☞ Calabi–Yau compactification with ‘exact’ MSSM spectrum

  Pokorski & Ross [1999] ,...

- ☞ Free fermionic construction with ‘exact’ MSSM spectrum

  Cleaver, Faraggi & Nanopoulos [1999] ,...

History of heterotic model building

- ☞ Calabi–Yau compactification with 'exact' MSSM spectrum

  Pokorski & Ross [1999] ,...

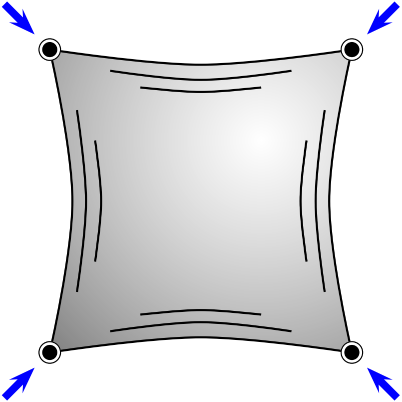
- ☞ Free fermionic construction with 'exact' MSSM spectrum

  Cleaver, Faraggi & Nanopoulos [1999] ,...

- ☞ Heterotic string revival after 2004

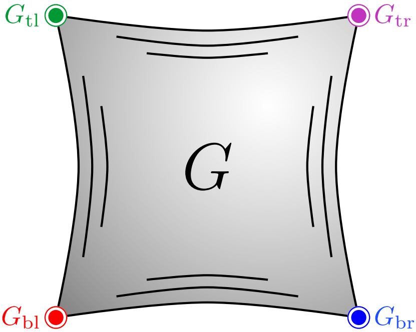
  Kobayashi, Raby & Zhang [2004] ,...

What is an orbifold?



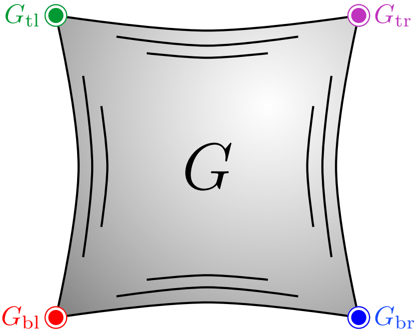
☞ An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points

What is an orbifold?



- An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points
- 'Bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points

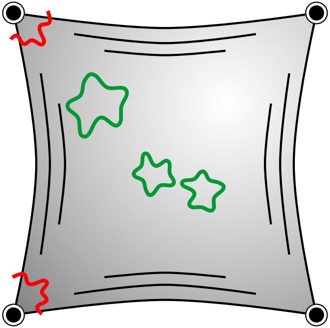
What is an orbifold?



- An orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points
- 'Bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points
- Low–energy gauge group : $G_{\mathrm{low-energy}} = G_{\mathrm{bl}} \cap G_{\mathrm{br}} \cap G_{\mathrm{tl}} \cap G_{\mathrm{tr}}$

Strings on orbifolds

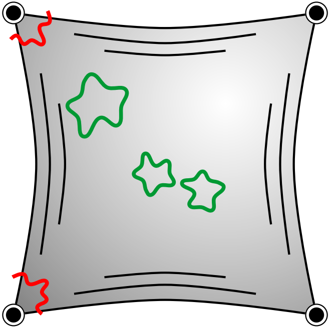
heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
'twisted' sectors = strings which are only closed on the orbifold	'brane fields' (hard to understand in field–theoretical framework)



☞ ('Brane') Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry

Strings on orbifolds

heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
'twisted' sectors = strings which are only closed on the orbifold	'brane fields' (hard to understand in field–theoretical framework)

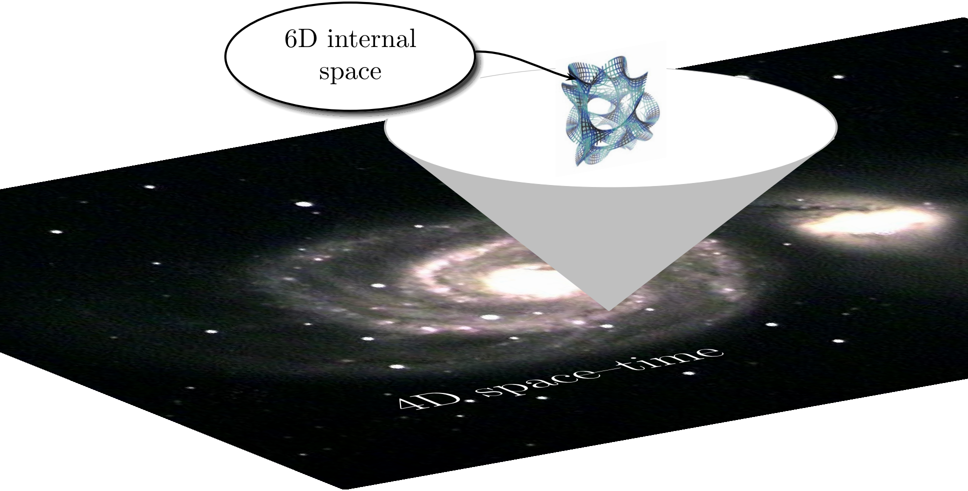


☞ ('Brane') Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry

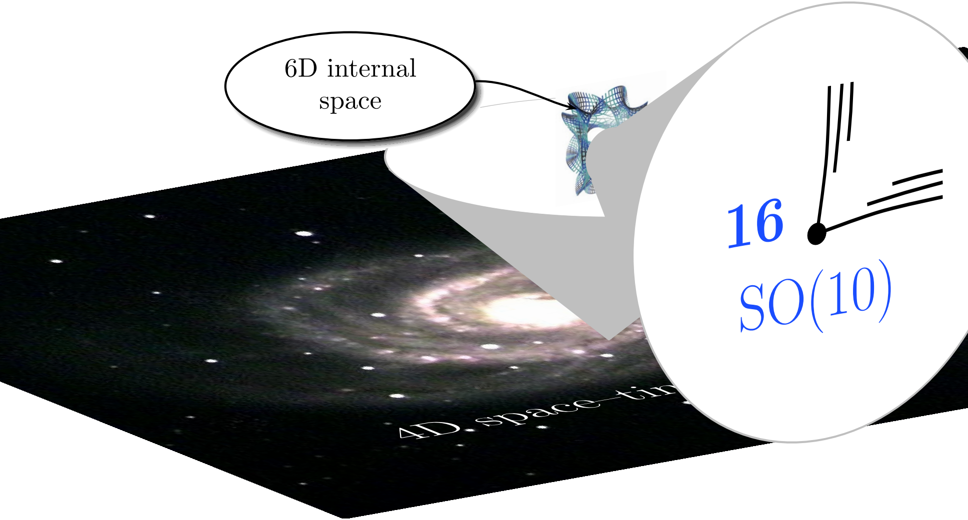
➥ E.g. if the electron lives at a point with SO(10) symmetry also u and d quarks live there

String compactifications with local $\mathrm{SO}(10)$ GUTs

String compactifications with local $\mathrm{SO}(10)$ GUTs



String compactifications with local SO(10) GUTs



Orbifold compactifications

of the

heterotic string

Goals

☞ Want to derive orbifold GUTs from strings

Goals

- ☞ Want to derive orbifold GUTs from strings
- ☞ Want to understand fields living at fixed points

Goals

- ☞ Want to derive orbifold GUTs from strings
- ☞ Want to understand fields living at fixed points
- ☞ Want a realistic model

Goals

- ☞ Want to derive orbifold GUTs from strings
- ☞ Want to understand fields living at fixed points
- ☞ Want a realistic model
- ☞ Want to understand curious things (e.g. strong CP problem)

Goals

- ☞ Want to derive orbifold GUTs from strings
- ☞ Want to understand fields living at fixed points
- ☞ Want a realistic model
- ☞ Want to understand curious things (e.g. strong CP problem)
- ☞ Want to make non–trivial and testable predictions

Higher–dimensional GUTs vs. heterotic orbifolds

top–down

→ Orbifold compactifications of the heterotic string

Dixon, Harvey, Vafa & Witten [1985, 1986]
Ibáñez, Nilles & Quevedo [1987b] ;...

- has UV completion
- automatically consistent
- explain representations

bottom–up

→ Orbifold GUTs

Kawamura [2000, 2001]
Altarelli & Feruglio [2001] ;...

- simple geometrical interpretation
- shares many features with 4D GUTs

combine both approaches

implement field–theoretic GUTs in orbifold compactifications of the heterotic string

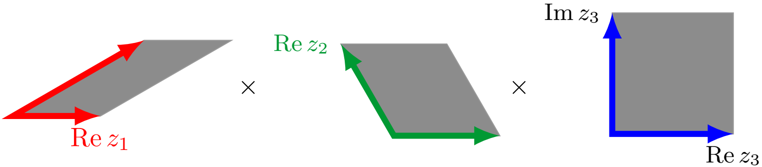
Kobayashi, Raby & Zhang [2004]
Förste, Nilles, Vaudrevange & Wingerter [2004]
Buchmüller, Hamaguchi, Lebedev & M.R. [2005]
⋮

Example: compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold $(\mathbb{Z}_6 - \mathrm{II})$

Kobayashi, Raby & Zhang [2004] ; Kobayashi, Raby & Zhang [2005]

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)} \;:=\; \text{root lattice of Lie algebra of } \mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$$

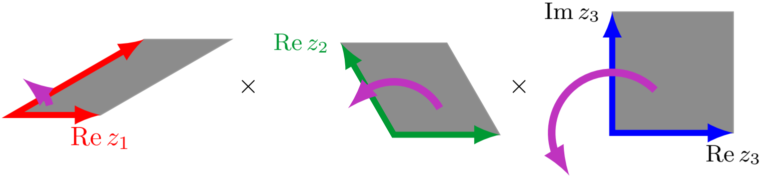


Example: compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold $(\mathbb{Z}_6-\mathrm{II})$

Kobayashi, Raby & Zhang [2004] ; Kobayashi, Raby & Zhang [2005]

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)} \quad := \quad \text{root lattice of Lie algebra of } \mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$$



The $\mathbb{Z}_6$ action on $\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)}$ is

$$z_i \to \mathrm{e}^{2\pi\mathrm{i}\, {v_6}^i}\, z_i \qquad \text{with} \qquad v_6 \;=\; \frac{1}{6}(-1,-2,3)$$

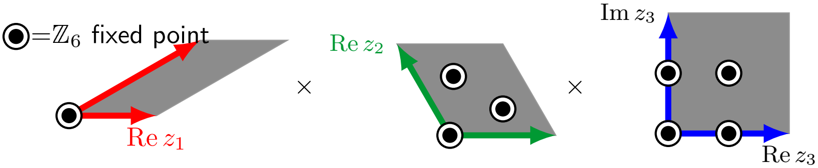
☞ **Note**: $\sum_i v_i = 0 \leftrightarrow \mathcal{N}=1$ supersymmetry in 4D

Example: compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold $(\mathbb{Z}_6 - \mathrm{II})$

Kobayashi, Raby & Zhang [2004] ; Kobayashi, Raby & Zhang [2005]

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)} \;:=\; \text{root lattice of Lie algebra of } \mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$$



The $\mathbb{Z}_6$ action on $\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)}$ is

$$z_i \to \mathrm{e}^{2\pi\mathrm{i}\, v_6{}^i}\, z_i \qquad \text{with} \qquad v_6 \;=\; \frac{1}{6}(-1,-2,3)$$

and has $\mathbb{Z}_k$ $(k=2,3,6)$ fixed points:

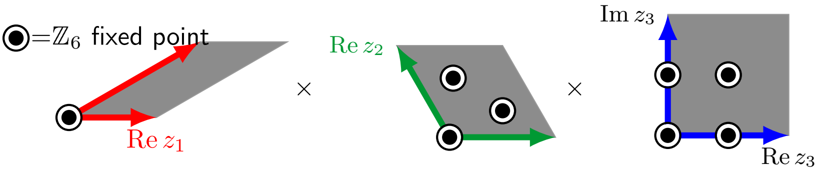
$$z^i_{\mathbb{Z}_k\mathrm{f.p.}} - \mathrm{e}^{2\pi\mathrm{i}\,\frac{6}{k}v_6{}^i}\, z^i_{\mathbb{Z}_k\mathrm{f.p.}} \in \Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)}$$

Example: compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold $(\mathbb{Z}_6 - \mathrm{II})$

Kobayashi, Raby & Zhang [2004] ; Kobayashi, Raby & Zhang [2005]

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)} \;:=\; \text{root lattice of Lie algebra of } \mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$$



twist action is embedded into the gauge degrees of freedom

$$X^I \;\to\; X^I + \pi\, V_6^I \qquad (\text{where } 6V_6 \in \Lambda_{\mathrm{E}_8\times\mathrm{E}_8})$$

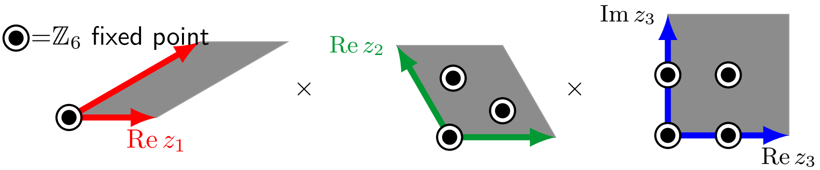
left–movers

Example: compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold $(\mathbb{Z}_6-\mathrm{II})$

Kobayashi, Raby & Zhang [2004] ; Kobayashi, Raby & Zhang [2005]

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)} \;:=\; \text{root lattice of Lie algebra of } \mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$$



torus translations are associated to Wilson lines, e.g.

$$z_3 \;\to\; z_3+1 \quad\leftrightarrow\quad X^I \to X^I + \pi\, W_2 \quad \text{(where } 2W_2 \in \Lambda_{\mathrm{E}_8\times\mathrm{E}_8}\text{)}$$

Bosonic coordinates and $\mathrm{E}_8 \times \mathrm{E}_8$

☞ Fermionic/bosonic coordinates

$$\begin{aligned} \widetilde{\psi}^i(\sigma_-) &= \mathrm{e}^{-2\mathrm{i}\,H^i(\sigma_-)} \quad (i = 1\dots 4) \\ \widetilde{\lambda}^I(\sigma_+) &= \mathrm{e}^{2\mathrm{i}\,X^I(\sigma_+)} \quad (I = 1\dots 16) \end{aligned}$$

Bosonic coordinates and $\mathrm{E}_8 \times \mathrm{E}_8$

- Fermionic/bosonic coordinates

$$\begin{aligned} \widetilde{\psi}^i(\sigma_-) &= \mathrm{e}^{-2\mathrm{i}\, H^i(\sigma_-)} \quad (i = 1 \dots 4) \\ \widetilde{\lambda}^I(\sigma_+) &= \mathrm{e}^{2\mathrm{i}\, X^I(\sigma_+)} \quad (I = 1 \dots 16) \end{aligned}$$

- X^I are compactified on a 16–dimensional torus represented by the $\mathrm{E}_8 \times \mathrm{E}_8$ root lattice

$$\Lambda_{\mathrm{E}_8} \;:\; p = (n_1, \dots, n_8) \quad \text{or} \quad \left(n_1 + \frac{1}{2}, \dots, n_8 + \frac{1}{2}\right)$$

$$n_i \in \mathbb{Z} \text{ with } \sum_{i=1}^{8} n_i = 0 \mod 2$$

Bosonic coordinates and $\mathrm{E}_8 \times \mathrm{E}_8$

- Fermionic/bosonic coordinates

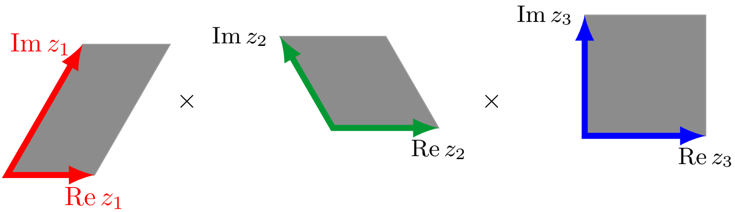
$$
\begin{aligned}
\widetilde{\psi}^i(\sigma_-) &= \mathrm{e}^{-2\mathrm{i}\,H^i(\sigma_-)} \quad (i = 1\dots 4) \\
\widetilde{\lambda}^I(\sigma_+) &= \mathrm{e}^{2\mathrm{i}\,X^I(\sigma_+)} \quad (I = 1\dots 16)
\end{aligned}
$$

- X^I are compactified on a 16–dimensional torus represented by the $\mathrm{E}_8 \times \mathrm{E}_8$ root lattice

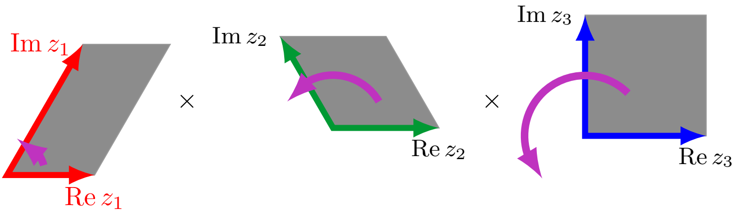
$$
\Lambda_{\mathrm{E}_8} \ :\ p = (n_1, \dots, n_8) \quad \text{or} \quad \left(n_1 + \frac{1}{2}, \dots, n_8 + \frac{1}{2}\right)
$$

➥ $\mathrm{E}_8 \times \mathrm{E}_8$ gauge symmetry

$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists

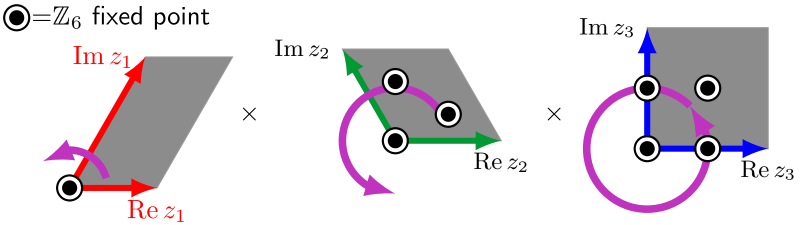


$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists



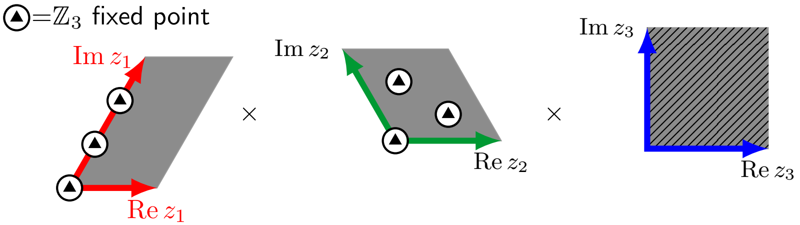
☞ $\mathbb{Z}_6$ twist: $v_6 = \frac{1}{6}(1, 2, -3)$

$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists



- $\mathbb{Z}_6$ twist: $v_6 = \frac{1}{6}(1, 2, -3)$
- $3 \cdot 4 = 12$ $\mathbb{Z}_6$ fixed points

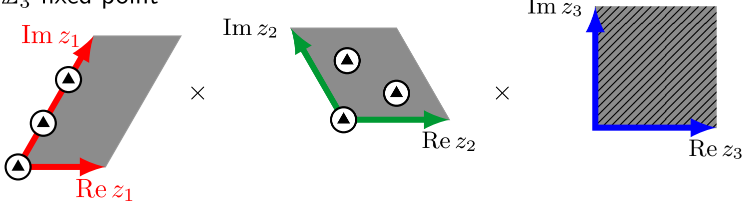
$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists



$\mathbb{Z}_3$ subtwist: $v_3 = 2v_6 = \frac{1}{3}(1, 2, -3)$

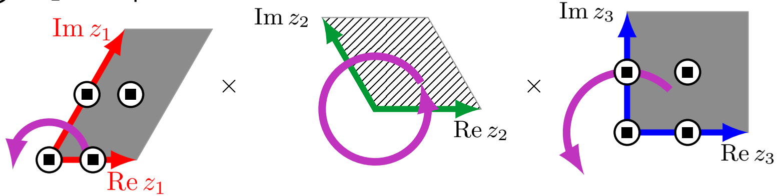
$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists

▲ $=\mathbb{Z}_3$ fixed point



$\mathbb{Z}_3$ subtwist: $v_3 = 2v_6 = \frac{1}{3}(1, 2, -3)$

■ $=\mathbb{Z}_2$ fixed point



☞ $\mathbb{Z}_2$ subtwist: $v_2 = 3v_6 = \frac{1}{2}(1, 2, -3)$

Heterotic spectrum: L$\otimes$R

10D:

left–moving vacuum

- gravity: $\widetilde{\alpha^{M}_{-1}}\,|0\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R}$
- gauge: $\widetilde{\alpha^{I}_{-1}}\,|0\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R} \oplus |p\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R}$

Heterotic spectrum: L⊗R

10D:

- gravity: $\widetilde{\alpha^{M}_{-1}}\,|0\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}$
- gauge: $\widetilde{\alpha^{I}_{-1}}\,|0\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}\oplus|p\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}$

$p\in\Lambda_{\mathrm{E}_8\times\mathrm{E}_8}$

$$\Lambda_{\mathrm{E}_8}\ :\ p=(n_1,\dots,n_8)\quad\text{or}\quad\left(n_1+\frac{1}{2},\dots,n_8+\frac{1}{2}\right)$$

$$n_i\in\mathbb{Z}\ \text{with}\ \sum_{i=1}^{8}n_i=0\mod 2$$

Heterotic spectrum: L⊗R

oscillator

10D:

- gravity: $\widetilde{\alpha^M_{-1}}\,|0\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R}$ $\quad q \in \Lambda^*_{\mathrm{SO}(8)}$
- gauge: $\widetilde{\alpha^I_{-1}}\,|0\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R} \oplus |p\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R}$

- (Dual) $\mathrm{SO}(8)$ lattice

$$\Lambda^*_{\mathrm{SO}(8)} \;:\; q = (n_1, n_2, n_3, n_4) \quad \text{with} \begin{cases} n_i \text{ integer with } \sum_i n_i \text{ odd} \\ \text{or} \\ n_i \text{ half–integer with } \sum_i n_i \text{ even} \end{cases}$$

Heterotic spectrum: L⊗R

10D:

- gravity: $\widetilde{\alpha^M_{-1}}\,|0\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}$
- gauge: $\widetilde{\alpha^I_{-1}}\,|0\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}\oplus|p\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}$

orbifold compactification

4D:

- 4D gravity: $\mathrm{e}^{\mathrm{i}\,\gamma_\mu}\,\widetilde{\alpha}^\mu_{-1}\,|0\rangle_\mathrm{L}\otimes\mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_\mathrm{R}$

$$\gamma_\mu+\gamma_q \;=\; 0 \mod 2\pi$$

Heterotic spectrum: L⊗R

10D:

- gravity: $\widetilde{\boldsymbol{\alpha}^{M}_{-1}}\,|0\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}$
- gauge: $\widetilde{\boldsymbol{\alpha}^{I}_{-1}}\,|0\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}\oplus|p\rangle_\mathrm{L}\otimes|q\rangle_\mathrm{R}$

orbifold compactification

4D:

- 4D gravity: $\mathrm{e}^{\mathrm{i}\,\gamma_\mu}\,\widetilde{\boldsymbol{\alpha}}^{\mu}_{-1}\,|0\rangle_\mathrm{L}\otimes\mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_\mathrm{R}$
- geometric moduli: $\mathrm{e}^{\mathrm{i}\,\gamma_i}\,\widetilde{\boldsymbol{\alpha}}^{i}_{-1}\,|0\rangle_\mathrm{L}\otimes\mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_\mathrm{R}$

$$\gamma_i+\gamma_q\;=\;0\mod 2\pi$$

Heterotic spectrum: L⊗R

10D:

- gravity: $\widetilde{\boldsymbol{\alpha}^{M}_{-1}}\,|0\rangle_{\mathrm{L}} \otimes |q\rangle_{\mathrm{R}}$
- gauge: $\widetilde{\boldsymbol{\alpha}^{I}_{-1}}\,|0\rangle_{\mathrm{L}} \otimes |q\rangle_{\mathrm{R}} \oplus |p\rangle_{\mathrm{L}} \otimes |q\rangle_{\mathrm{R}}$

orbifold compactification

4D:

- 4D gravity: $\mathrm{e}^{\mathrm{i}\,\gamma_\mu}\,\widetilde{\boldsymbol{\alpha}}^{\mu}_{-1}\,|0\rangle_{\mathrm{L}} \otimes \mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_{\mathrm{R}}$
- geometric moduli: $\mathrm{e}^{\mathrm{i}\,\gamma_i}\,\widetilde{\boldsymbol{\alpha}}^{i}_{-1}\,|0\rangle_{\mathrm{L}} \otimes \mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_{\mathrm{R}}$
- untwisted sector: $\mathrm{e}^{\mathrm{i}\,\gamma_p}\,|p\rangle_{L} \otimes \mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_{\mathrm{R}}$

$$\gamma_p + \gamma_q = 0 \mod 2\pi$$

Heterotic spectrum: L⊗R

10D:

- gravity: $\widetilde{\alpha^M_{-1}}\,|0\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R}$
- gauge: $\widetilde{\alpha^I_{-1}}\,|0\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R} \oplus |p\rangle_\mathrm{L} \otimes |q\rangle_\mathrm{R}$

orbifold compactification

4D:

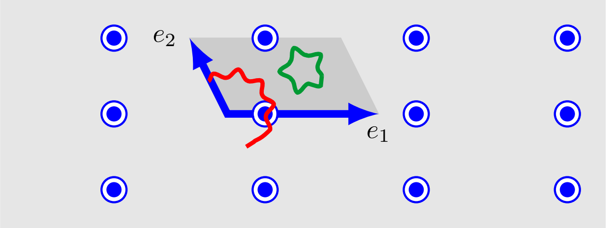
- 4D gravity: $\mathrm{e}^{\mathrm{i}\,\gamma_\mu}\,\widetilde{\alpha}^\mu_{-1}\,|0\rangle_\mathrm{L} \otimes \mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_\mathrm{R}$
- geometric moduli: $\mathrm{e}^{\mathrm{i}\,\gamma_i}\,\widetilde{\alpha}^i_{-1}\,|0\rangle_\mathrm{L} \otimes \mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_\mathrm{R}$
- untwisted sector: $\mathrm{e}^{\mathrm{i}\,\gamma_p}\,|p\rangle_L \otimes \mathrm{e}^{\mathrm{i}\,\gamma_q}\,|q\rangle_\mathrm{R}$
- twisted sector: $\mathrm{e}^{\mathrm{i}\,\gamma'_p}\,|p'\rangle_\mathrm{L} \otimes \mathrm{e}^{\mathrm{i}\,\gamma'_q}\,|q'\rangle_\mathrm{R}$

What are the light states of an orbifold?

☞ Boundary conditions

$$X^i(\sigma+\pi) \;=\; (\theta^k\,X)^i(\sigma) + m_\alpha\,e_\alpha^i$$

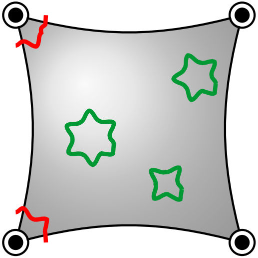
. . . can either be twisted $(1 \leq k \leq N-1)$ or untwisted $(k=0)$



What are the light states of an orbifold?

☞ Boundary conditions

$$X^i(\sigma+\pi) \;=\; (\theta^k\,X)^i(\sigma) + m_\alpha\, e_\alpha^i$$

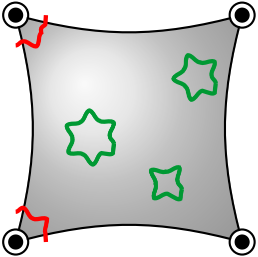


heterotic string	field theory
$k = 0$ untwisted sector = strings closed on the torus	4D gauge fields and extra components of gauge fields
T_k twisted sector = strings which are only closed on the orbifold	'brane fields' (hard to understand in field-theoretical framework)

What are the light states of an orbifold?

- Boundary conditions

$$X^i(\sigma+\pi) \;=\; (\theta^k\, X)^i(\sigma) + m_\alpha\, e_\alpha^i$$



heterotic string	field theory
$k = 0$ untwisted sector = strings closed on the torus	4D gauge fields and extra components of gauge fields
T_k twisted sector = strings which are only closed on the orbifold	'brane fields' (hard to understand in field-theoretical framework)

- Fields living at fixed point ('brane') with a certain symmetry appear as complete multiplet of that symmetry

Untwisted or bulk states

☞ Untwisted sector = bulk states : $k = 0$

$$v_6 \cdot q + V_6 \cdot p \;=\; 0 \quad \text{and} \quad W_n \cdot p \;=\; 0$$

$q \in \Lambda^*_{\mathrm{SO}(8)}$

Untwisted or bulk states

- Untwisted sector = bulk states : $k = 0$

$p \in \Lambda_{\mathrm{E}_8 \times \mathrm{E}_8}$

$$v_6 \cdot q + V_6 \cdot p \;=\; 0 \quad \text{and} \quad W_n \cdot p \;=\; 0$$

$6\,V_6 \in \Lambda_{\mathrm{E}_8 \times \mathrm{E}_8}$ $\quad$ $n\,W_n \in \Lambda_{\mathrm{E}_8 \times \mathrm{E}_8}$

Untwisted or bulk states

- Untwisted sector = bulk states : $k = 0$

$$v_6 \cdot q + V_6 \cdot p \;=\; 0 \quad \text{and} \quad W_n \cdot p \;=\; 0$$

- (Dual) $\mathrm{SO}(8)$ lattice

$$\Lambda^*_{\mathrm{SO}(8)} \;:\; q = (n_1, n_2, n_3, n_4) \quad \text{with} \quad \begin{cases} n_i \text{ integer with } \sum_i n_i \text{ odd} \\ \text{or} \\ n_i \text{ half–integer with } \sum_i n_i \text{ even} \end{cases}$$

1. 4D gauge fields: $q = (\pm 1; 0^3)$ $\left(\text{superpartner: } q = \pm\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\right)$
   $\curvearrowright v_6 \cdot q = 0$

$$\left\{\begin{array}{rcl} V \cdot p & = & 0 \\ W_n \cdot p & = & 0 \end{array}\right\} \iff \left\{\begin{array}{c} V_f \cdot p \;=\; 0 \\ \text{for all fixed} \\ \text{points } f \end{array}\right\}$$

Untwisted or bulk states

☞ Untwisted sector $=$ bulk states : $k=0$

$$v_6 \cdot q + V_6 \cdot p \;=\; 0 \quad \text{and} \quad W_n \cdot p \;=\; 0$$

☞ (Dual) $\mathrm{SO}(8)$ lattice

$$\Lambda^*_{\mathrm{SO}(8)} \;:\; q = (n_1, n_2, n_3, n_4) \quad \text{with} \quad \begin{cases} n_i \text{ integer with } \sum_i n_i \text{ odd} \\ \text{or} \\ n_i \text{ half–integer with } \sum_i n_i \text{ even} \end{cases}$$

1. 4D gauge fields: $q = (\pm 1; 0^3)$ $\Big($superpartner: $q = \pm\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\Big)$
   $\curvearrowright v_6 \cdot q = 0$

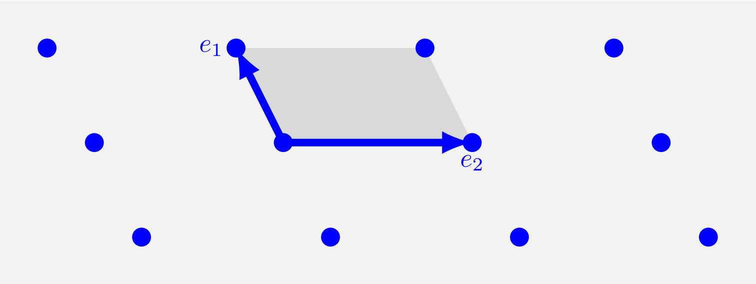
$$\left\{ \begin{array}{rcl} V \cdot p & = & 0 \\ W_n \cdot p & = & 0 \end{array} \right\} \Longleftrightarrow \left\{ \begin{array}{c} V_f \cdot p \;=\; 0 \\ \text{for all fixed} \\ \text{points } f \end{array} \right\}$$

2. untwisted chiral matter $v_6 \cdot q \neq 0$
   $=$ extra components of gauge fields

Twisted states ('brane fields')

☞ Correspondence

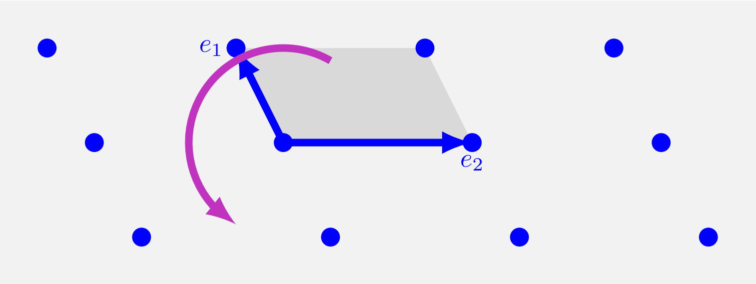
$$\left\{\begin{array}{c}\text{inequivalent}\\ \text{fixed points}\end{array}\right\} \leftrightarrow \left\{\begin{array}{c}\text{conjugacy}\\ \text{classes}\\ (\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Twisted states ('brane fields')

☞ Correspondence

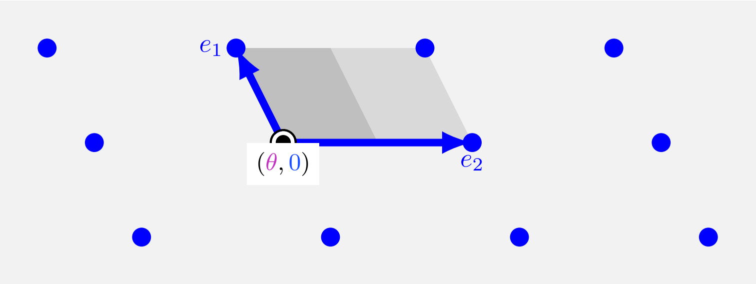
$$\left\{\begin{array}{c}\text{inequivalent}\\ \text{fixed points}\end{array}\right\}\leftrightarrow\left\{\begin{array}{c}\text{conjugacy}\\ \text{classes}\\ (\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Twisted states ('brane fields')

- Correspondence

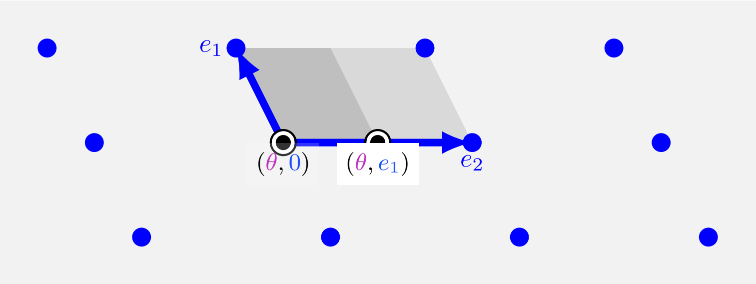
$$\left\{\begin{array}{c}\text{inequivalent}\\\text{fixed points}\end{array}\right\}\leftrightarrow\left\{\begin{array}{c}\text{conjugacy}\\\text{classes}\\(\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Twisted states ('brane fields')

☞ Correspondence

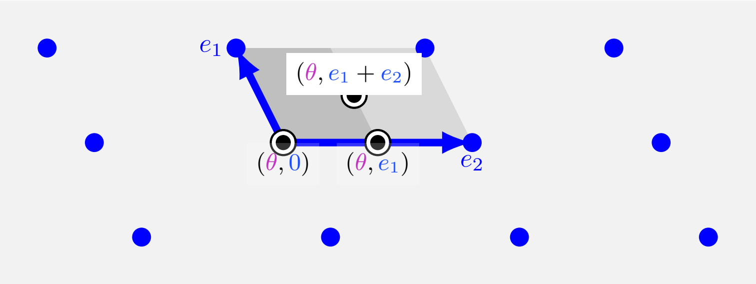
$$\left\{\begin{array}{c}\text{inequivalent}\\\text{fixed points}\end{array}\right\}\leftrightarrow\left\{\begin{array}{c}\text{conjugacy}\\\text{classes}\\(\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Twisted states ('brane fields')

☞ Correspondence

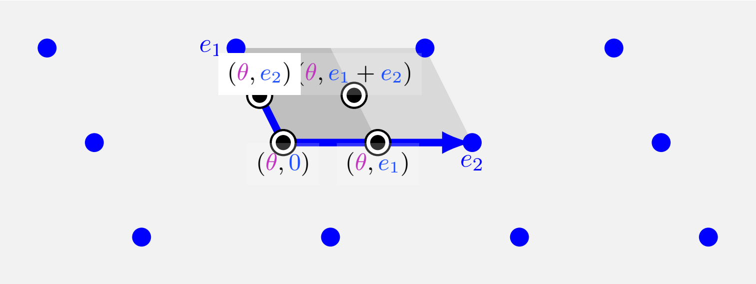
$$\left\{\begin{array}{c}\text{inequivalent}\\ \text{fixed points}\end{array}\right\}\leftrightarrow\left\{\begin{array}{c}\text{conjugacy}\\ \text{classes}\\ (\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Twisted states ('brane fields')

- Correspondence

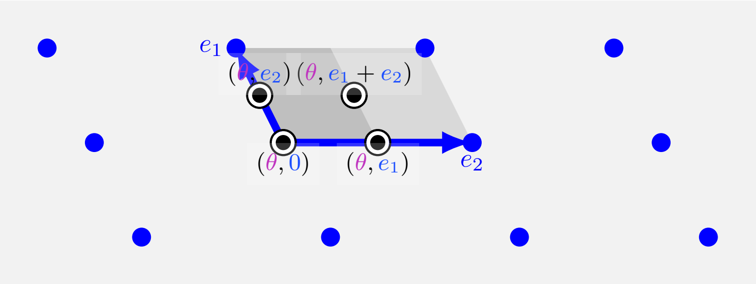
$$\left\{\begin{array}{c}\text{inequivalent}\\ \text{fixed points}\end{array}\right\} \leftrightarrow \left\{\begin{array}{c}\text{conjugacy}\\ \text{classes}\\ (\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Twisted states ('brane fields')

☞ Correspondence

$$\left\{\begin{array}{c}\text{inequivalent}\\ \text{fixed points}\end{array}\right\} \leftrightarrow \left\{\begin{array}{c}\text{conjugacy}\\ \text{classes}\\ (\theta^k, m_\alpha\, e_\alpha)\end{array}\right\}$$



Space group

☞ Lattice translations and discrete rotations can be combined to the space group $\mathcal{S}$

$$\mathcal{S} \ni (\theta^k, \ell) = (\theta^k, n_a\, e_a^i)$$

$\ell \in \Lambda$

Space group

- Lattice translations and discrete rotations can be combined to the space group $\mathcal{S}$

$$\mathcal{S} \ni (\theta^k, \ell) = (\theta^k, n_a\, e_a^i)$$

- Multiplication law

$$\left(\theta^{k^{(1)}}, \ell^{(1)}\right)\left(\theta^{k^{(2)}}, \ell^{(2)}\right) = \left(\theta^{k^{(1)}+k^{(2)}}, \theta^{k^{(1)}}\, \ell^{(2)} + \ell^{(1)}\right)$$

Space group

☞ Lattice translations and discrete rotations can be combined to the space group $\mathcal{S}$

$$\mathcal{S} \ni \left(\theta^k, \ell\right) = \left(\theta^k, n_a\, e_a^i\right)$$

☞ Multiplication law

$$\left(\theta^{k^{(1)}}, \ell^{(1)}\right) \left(\theta^{k^{(2)}}, \ell^{(2)}\right) = \left(\theta^{k^{(1)}+k^{(2)}}, \theta^{k^{(1)}}\, \ell^{(2)} + \ell^{(1)}\right)$$

☞ Conjugacy classes

$$\left(\theta^k, \ell\right) \simeq \left(\theta^k, \ell + \lambda\right) \quad \text{with } \lambda \in \left(\mathbb{1} - \theta^k\right) \Lambda$$

Local gauge embedding

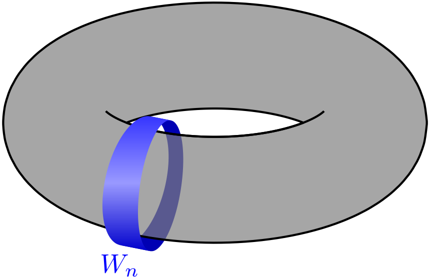
☞ Local gauge embedding at fixed point f

$$V_f^I \;=\; k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$

Local gauge embedding

- Local gauge embedding at fixed point f

$$V_f^I \;=\; k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$



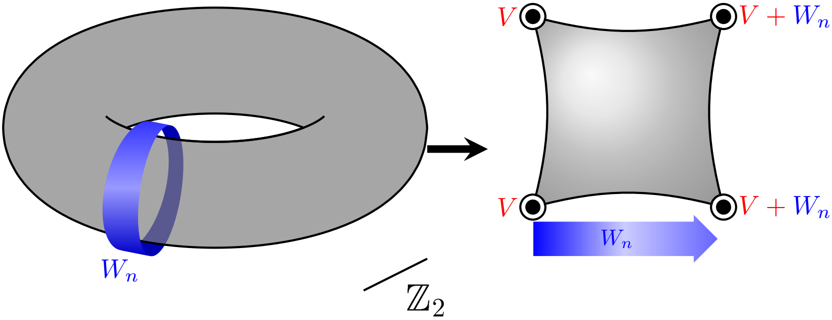
- Local gauge shifts

Groot Nibbelink, Hillenbach, Kobayashi & Walter [2004]

Local gauge embedding

☞ Local gauge embedding at fixed point f

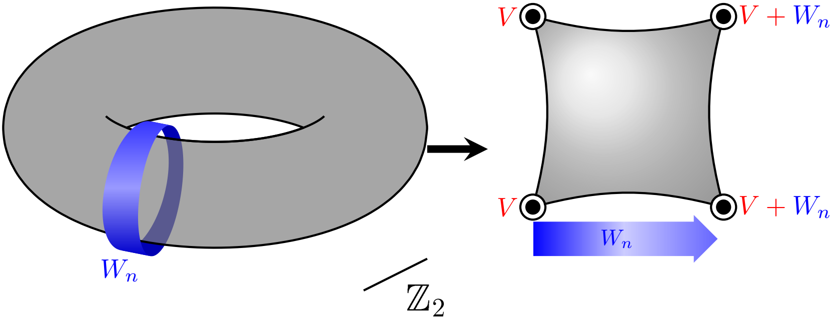
$$V_f^I \;=\; k\, V_N^I + m_\alpha\, W_{n\alpha}^I$$



Local gauge embedding

☞ Local gauge embedding at fixed point f

$$V_f^I \;=\; k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$



➥ Local gauge shifts

Groot Nibbelink, Hillenbach, Kobayashi & Walter [2004]

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations

$$
\begin{aligned}
\frac{1}{8}m_{\mathrm{R}}^2 &= \frac{1}{2}(q + k\, v_N)^2 - \frac{1}{2} + \delta c^{(k)} + \omega_i^{(k)}\, N_{fi} + \overline{\omega}_i^{(k)}\, N_{fi}^* \stackrel{!}{=} 0 \\
\frac{1}{8}m_{\mathrm{L}}^2 &= \frac{1}{2}(p + V_f)^2 - 1 + \delta c^{(k)} + \omega_i^{(k)}\, \widetilde{N}_{fi} + \overline{\omega}_i^{(k)}\, \widetilde{N}_{fi}^* \stackrel{!}{=} 0
\end{aligned}
$$

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations

$$\begin{aligned}\frac{1}{8}m_{\mathrm{R}}^2 &= \frac{1}{2}(q+k\,v_N)^2-\frac{1}{2}+\delta c^{(k)}+\omega_i^{(k)}\,N_{fi}+\overline{\omega}_i^{(k)}\,N_{fi}^* \stackrel{!}{=} 0\\ \frac{1}{8}m_{\mathrm{L}}^2 &= \frac{1}{2}(p+V_f)^2-1+\delta c^{(k)}+\omega_i^{(k)}\,\widetilde{N}_{fi}+\overline{\omega}_i^{(k)}\,\widetilde{N}_{fi}^* \stackrel{!}{=} 0\end{aligned}$$

$$\omega_i^{(k)} = (k\,v_N)_i \mod 1$$
so that $0<\omega_i^{(k)}\leq 1$

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations

$$\begin{aligned}\frac{1}{8}m_\mathrm{R}^2 &= \frac{1}{2}(q+k\,v_N)^2-\frac{1}{2}+\delta c^{(k)}+\omega_i^{(k)}\,N_{fi}+\overline{\omega}_i^{(k)}\,N_{fi}^* \stackrel{!}{=} 0\\ \frac{1}{8}m_\mathrm{L}^2 &= \frac{1}{2}(p+V_f)^2-1+\delta c^{(k)}+\omega_i^{(k)}\,\widetilde{N}_{fi}+\overline{\omega}_i^{(k)}\,\widetilde{N}_{fi}^* \stackrel{!}{=} 0\end{aligned}$$

$$\overline{\omega}_i^{(k)} = (-k\,v_N)_i \mod 1$$
so that $0<\overline{\omega}_i^{(k)}\leq 1$

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations

$$\begin{aligned}
\frac{1}{8} m_{\mathrm{R}}^2 &= \frac{1}{2}(q + k\, v_N)^2 - \frac{1}{2} + \delta c^{(k)} + \omega_i^{(k)}\, N_{fi} + \overline{\omega}_i^{(k)}\, N_{fi}^* \stackrel{!}{=} 0 \\
\frac{1}{8} m_{\mathrm{L}}^2 &= \frac{1}{2}(p + V_f)^2 - 1 + \delta c^{(k)} + \omega_i^{(k)}\, \widetilde{N}_{fi} + \overline{\omega}_i^{(k)}\, \widetilde{N}_{fi}^* \stackrel{!}{=} 0
\end{aligned}$$

$$\delta c^{(k)} = \frac{1}{2} \sum_i \omega_i^{(k)} \left(1 - \omega_i^{(k)}\right)$$

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations

$$\begin{aligned}\frac{1}{8}m_{\mathrm{R}}^2 &= \frac{1}{2}(q+k\,v_N)^2-\frac{1}{2}+\delta c^{(k)}+\omega_i^{(k)}\,N_{fi}+\overline{\omega}_i^{(k)}\,N_{fi}^* \stackrel{!}{=} 0\\ \frac{1}{8}m_{\mathrm{L}}^2 &= \frac{1}{2}(p+V_f)^2-1+\delta c^{(k)}+\omega_i^{(k)}\,\widetilde{N}_{fi}+\overline{\omega}_i^{(k)}\,\widetilde{N}_{fi}^* \stackrel{!}{=} 0\end{aligned}$$

$$\left.\begin{array}{c}N_{fi}\\ \widetilde{N}_{fi}\end{array}\right\}\text{ oscillator numbers of }\left\{\begin{array}{c}\text{left-}\\ \text{right-}\end{array}\right\}\text{movers in } z_i \text{ directions}$$

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations

$$\begin{aligned}
\frac{1}{8} m_{\mathrm{R}}^2 &= \frac{1}{2}(q + k\, v_N)^2 - \frac{1}{2} + \delta c^{(k)} + \omega_i^{(k)}\, N_{fi} + \overline{\omega}_i^{(k)}\, N_{fi}^* \stackrel{!}{=} 0 \\
\frac{1}{8} m_{\mathrm{L}}^2 &= \frac{1}{2}(p + V_f)^2 - 1 + \delta c^{(k)} + \omega_i^{(k)}\, \widetilde{N}_{fi} + \overline{\omega}_i^{(k)}\, \widetilde{N}_{fi}^* \stackrel{!}{=} 0
\end{aligned}$$

$$\left.\begin{array}{c} N_{fi}^* \\ \widetilde{N}_{fi}^* \end{array}\right\} \text{ oscillator numbers of } \left\{\begin{array}{c} \text{left-} \\ \text{right-} \end{array}\right\} \text{movers in } \overline{z}_i \text{ directions}$$

Recipe for calculating the massless spectrum

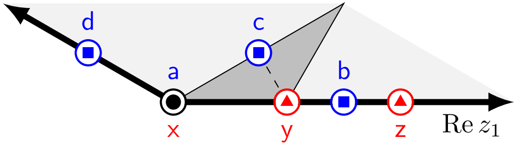
1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations
2. For each state $f \leftrightarrow (\theta^k, m_\alpha\, e_\alpha)$ identify commuting elements $(\theta^{\overline{k}}, \overline{m}_\alpha\, e_\alpha)$ and subject the states to invariance conditions

$$\sum_i \overline{k}\, v_i^i \left(\tilde{N}_i - \tilde{N}_i^*\right) - \overline{k}\, v \cdot (q + k\, v_N) + (\overline{k} V_{\overline{f}} + \overline{m}_\alpha\, W_\alpha) \cdot (p + V_f) \stackrel{!}{=} 0 \mod 1$$

Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations
2. For each state $f \leftrightarrow (\theta^k, m_\alpha\, e_\alpha)$ identify commuting elements $(\theta^{\overline{k}}, \overline{m}_\alpha\, e_\alpha)$ and subject the states to invariance conditions

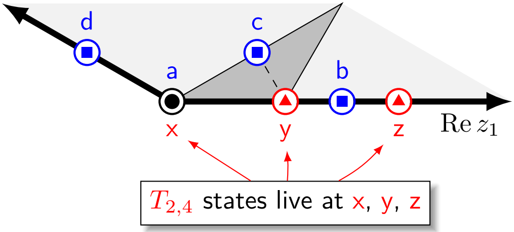
☞ Note: for higher twisted states on non–prime orbifolds one has to form linear combinations e.g. Kobayashi & Ohtsubo [1994]



Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations
2. For each state $f \leftrightarrow (\theta^k, m_\alpha\, e_\alpha)$ identify commuting elements $(\theta^{\overline{k}}, \overline{m}_\alpha\, e_\alpha)$ and subject the states to invariance conditions

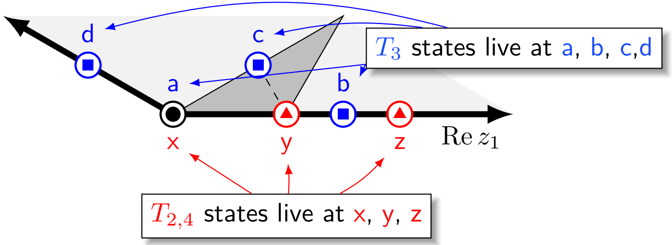
☞ Note: for higher twisted states on non–prime orbifolds one has to form linear combinations e.g. Kobayashi & Ohtsubo [1994]



Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations
2. For each state $f \leftrightarrow (\theta^k, m_\alpha\, e_\alpha)$ identify commuting elements $(\theta^{\overline{k}}, \overline{m}_\alpha\, e_\alpha)$ and subject the states to invariance conditions

☞ Note: for higher twisted states on non–prime orbifolds one has to form linear combinations e.g. Kobayashi & Ohtsubo [1994]

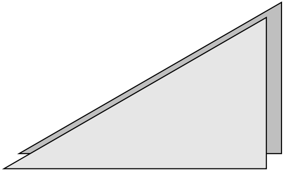


Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations
2. For each state $f \leftrightarrow (\theta^k, m_\alpha\, e_\alpha)$ identify commuting elements $(\theta^{\overline{k}}, \overline{m}_\alpha\, e_\alpha)$ and subject the states to invariance conditions

☞ Note: for higher twisted states on non–prime orbifolds one has to form linear combinations

e.g. Kobayashi & Ohtsubo [1994]



Recipe for calculating the massless spectrum

1. For each fixed point (or conjugacy class $(\theta^k, m_\alpha\, e_\alpha)$) solve the mass equations
2. For each state $f \leftrightarrow (\theta^k, m_\alpha\, e_\alpha)$ identify commuting elements $(\theta^{\overline{k}}, \overline{m}_\alpha\, e_\alpha)$ and subject the states to invariance conditions

☞ Note: for higher twisted states on non–prime orbifolds one has to form linear combinations

e.g. Kobayashi & Ohtsubo [1994]

☞ Automatization

▸ Orbifolder

Nilles, Ramos-Sánchez, Vaudrevange & Wingerter [2012]

Quantum numbers of massless spectrum

$\mathrm{E}_8 \times \mathrm{E}_8$ momentum

☞ Quantum numbers

$$|\text{state}\rangle = |f; q_{\mathrm{sh}}, p_{\mathrm{sh}}\rangle \equiv |q + k\, v_N\rangle \otimes |p + V_f\rangle$$

Quantum numbers of massless spectrum

- Quantum numbers

$$|\text{state}\rangle = |f; q_{\text{sh}}, p_{\text{sh}}\rangle \equiv |q + k\, v_N\rangle \otimes |p + V_f\rangle$$

SO(8) momentum

Quantum numbers of massless spectrum

☞ Quantum numbers

$$|\text{state}\rangle \;=\; |f; q_{\text{sh}}, p_{\text{sh}}\rangle \;\equiv\; |q + k\,v_N\rangle \otimes |p + \underbrace{V_f}_{\text{local shift}}\rangle$$

Quantum numbers of massless spectrum

- Quantum numbers

$$|\text{state}\rangle \;=\; |f; q_{\mathrm{sh}}, p_{\mathrm{sh}}\rangle \;\equiv\; |q + k\,v_N\rangle \otimes |p + V_f\rangle$$

fixed point

Quantum numbers of massless spectrum

☞ Quantum numbers

$$|\text{state}\rangle = |f; q_{\text{sh}}, p_{\text{sh}}\rangle \equiv |q + k\, v_N\rangle \otimes |p + V_f\rangle$$

☞ $\mathrm{SO}(8)$ momenta

$$q = (n_1, n_2, n_3, n_4) \quad \text{where} \begin{cases} n_i \text{ integer with } \sum_i n_i \text{ odd} \\ \text{or} \\ n_i \text{ half-integer with } \sum_i n_i \text{ even} \end{cases}$$

Quantum numbers of massless spectrum

☞ Quantum numbers

$$|\text{state}\rangle \;=\; |f; q_{\mathrm{sh}}, p_{\mathrm{sh}}\rangle \;\equiv\; |q + k\, v_N\rangle \otimes |p + V_f\rangle$$

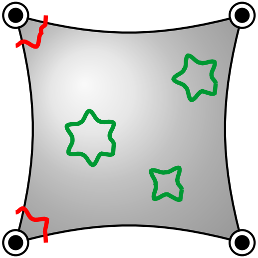
☞ $\mathrm{SO}(8)$ momenta

$$q = (n_1, n_2, n_3, n_4) \quad \text{where} \begin{cases} n_i \text{ integer with } \sum_i n_i \text{ odd} \\ \text{or} \\ n_i \text{ half-integer with } \sum_i n_i \text{ even} \end{cases}$$

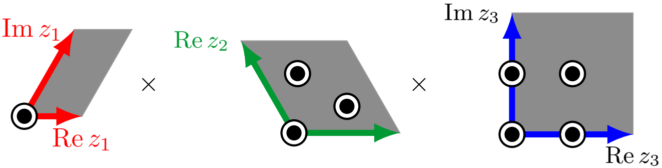
☞ E_8 momenta: $p = (p, p')$ where

$$p, p' \;=\; (n_1, \dots, n_8) \;\text{ or }\; \left(n_1 + \frac{1}{2}, \dots, n_8 + \frac{1}{2}\right)$$

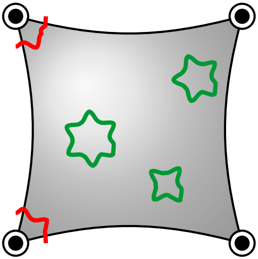
Localization of twisted states



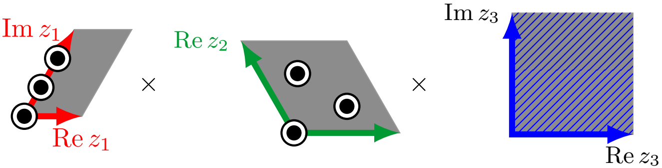
k	interpretation
$k = 0$	bulk fields
$k = 1, 5$	fields live on points in 6D compact space
$k = 2, 3, 4$	fields live on 2-dimensional planes in 6D compact space



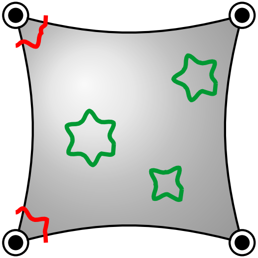
Localization of twisted states



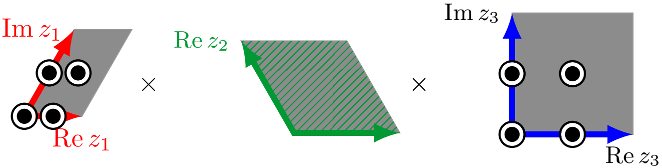
k	interpretation
$k = 0$	bulk fields
$k = 1, 5$	fields live on points in 6D compact space
$k = 2, 3, 4$	fields live on 2-dimensional planes in 6D compact space



Localization of twisted states

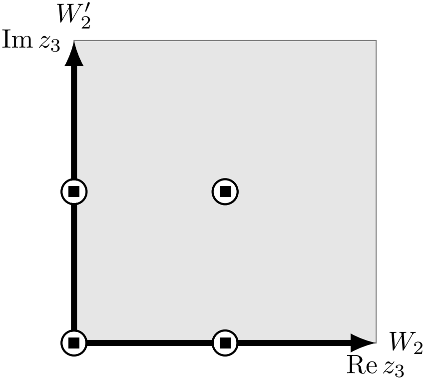


k	interpretation
$k = 0$	bulk fields
$k = 1, 5$	fields live on points in 6D compact space
$k = 2, 3, 4$	fields live on 2-dimensional planes in 6D compact space



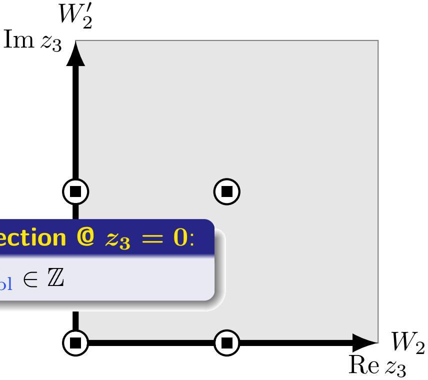
Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)



Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)



projection @ $z_3 = 0$:

$p \cdot V_{\mathrm{bl}} \in \mathbb{Z}$

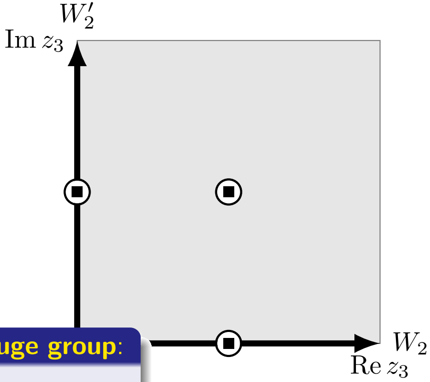
local shift

gauge group generator

$$\mathrm{e}^{2\pi\mathrm{i}\,p\,\cdot\,V_{\mathrm{bl}}} A_\mu^p(x;\dots,\mathrm{e}^{2\pi\mathrm{i}/2}z_3) = A_\mu^p(x;\dots,z_3)$$

Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)

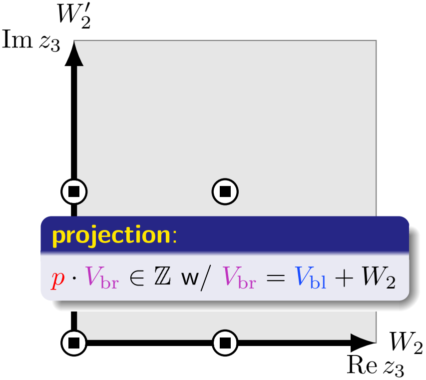


emerging *local* gauge group:

$G_{\mathrm{bl}} \subset \mathrm{E}_8 \times \mathrm{E}_8$

Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)

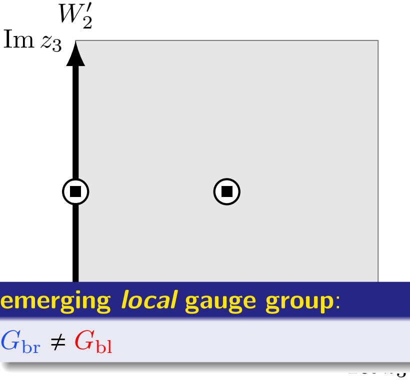


projection:

$p \cdot V_{\mathrm{br}} \in \mathbb{Z}$ w/ $V_{\mathrm{br}} = V_{\mathrm{bl}} + W_2$

Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)

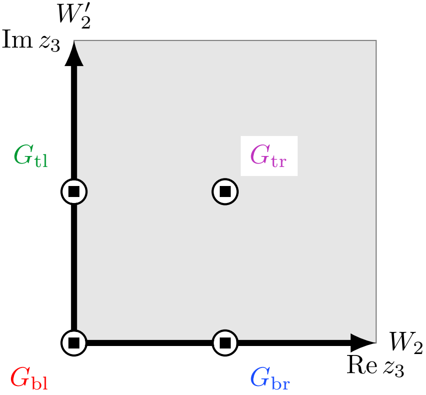


emerging *local* gauge group:

$G_{\mathrm{br}} \neq G_{\mathrm{bl}}$

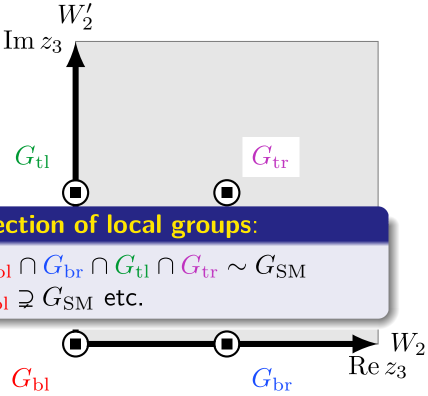
Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)



Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)



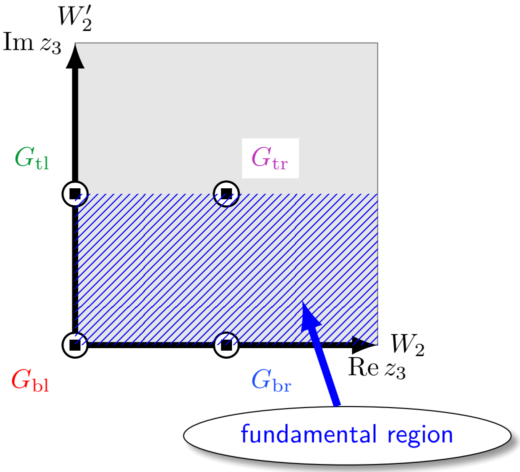
intersection of local groups:

e.g. $G_{\mathrm{bl}} \cap G_{\mathrm{br}} \cap G_{\mathrm{tl}} \cap G_{\mathrm{tr}} \sim G_{\mathrm{SM}}$
but $G_{\mathrm{bl}} \supsetneq G_{\mathrm{SM}}$ etc.

G_{bl} etc. : 'local GUTs'

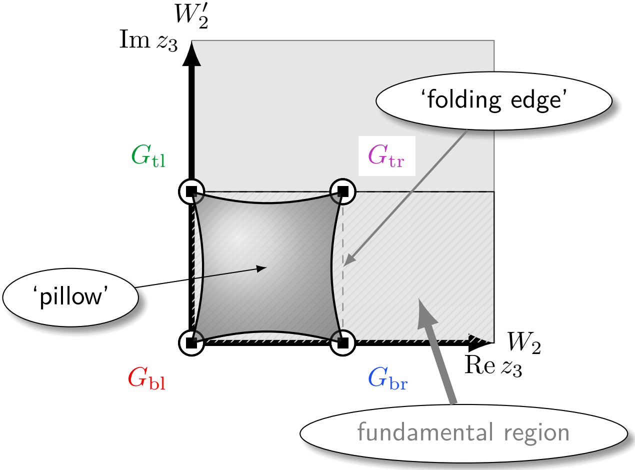
Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)

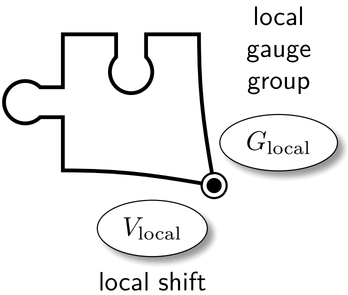


Local gauge symmetry (breaking)

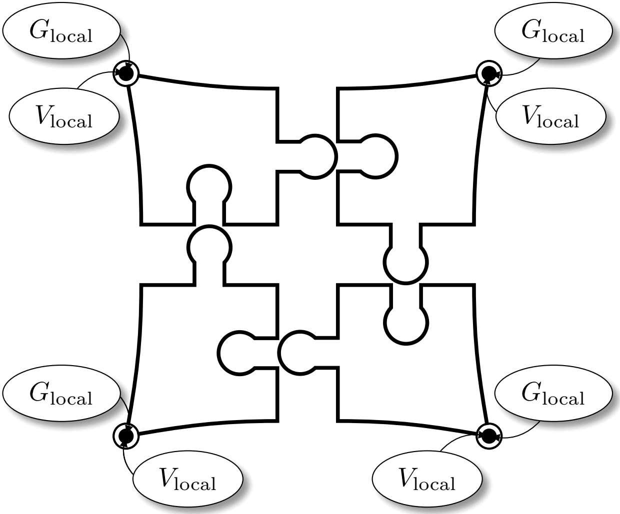
Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)



The ‘orbifold construction kit’

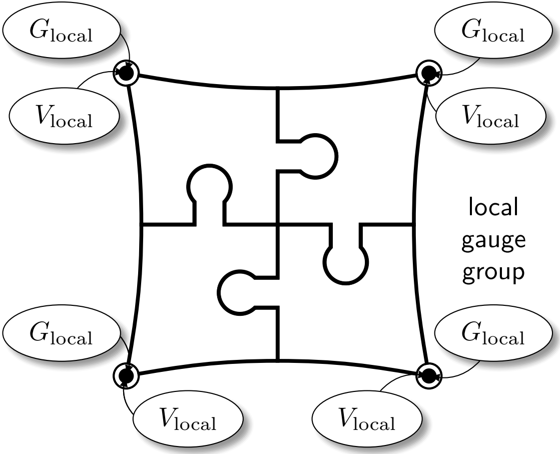


The 'orbifold construction kit'



simplest possibility: consider identical corners

The ‘orbifold construction kit’

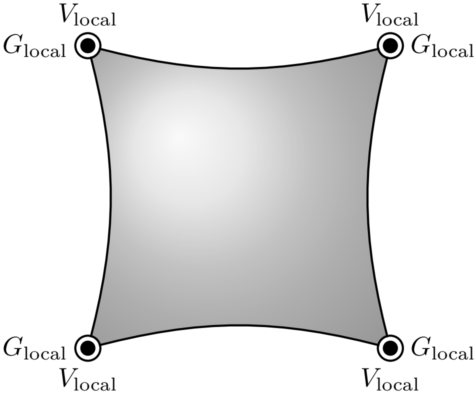


local gauge group

local shift

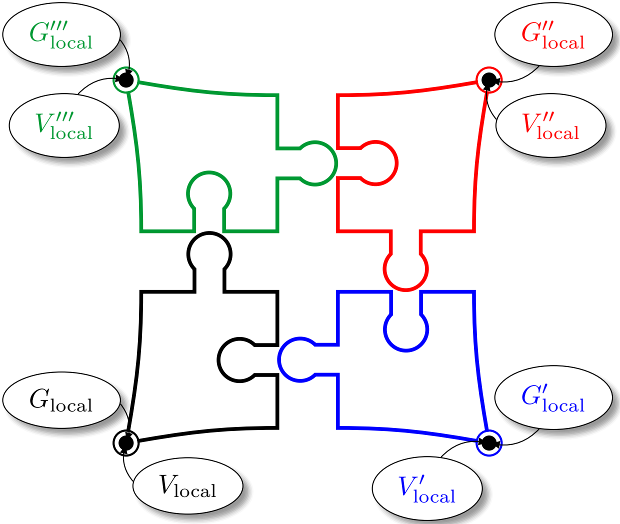
the combination corresponds to an

The ‘orbifold construction kit’



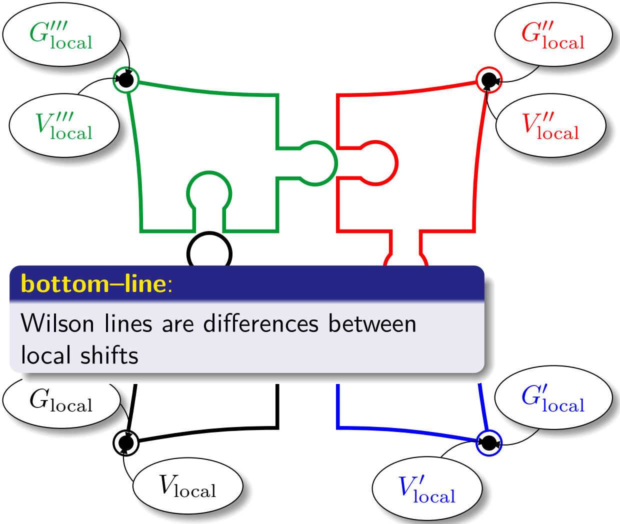
orbifold without Wilson lines

The 'orbifold construction kit'



orbifold with Wilson lines

The ‘orbifold construction kit’

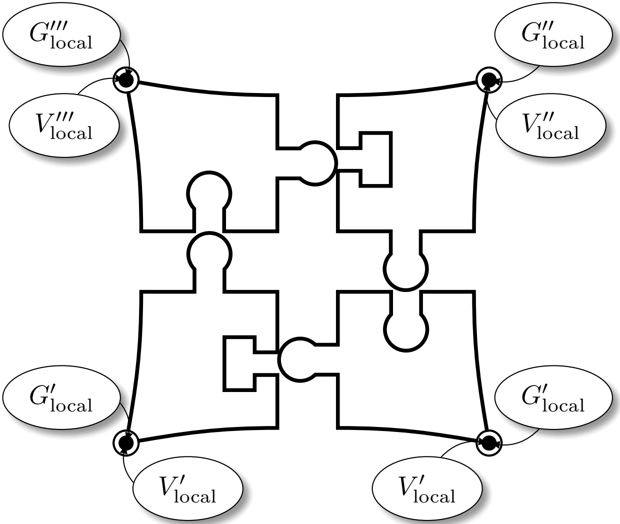


bottom–line:

Wilson lines are differences between local shifts

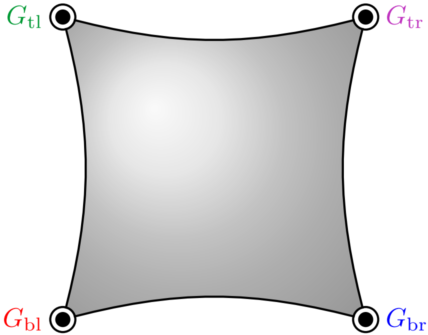
orbifold with Wilson lines

The 'orbifold construction kit'



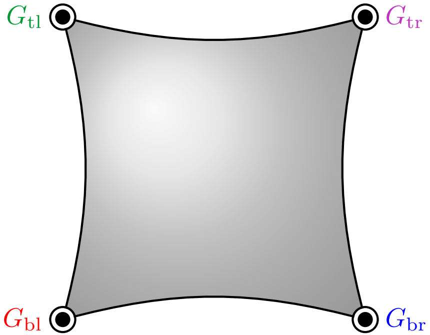
strong consistency requirements

Summary of crucial features of orbifolds



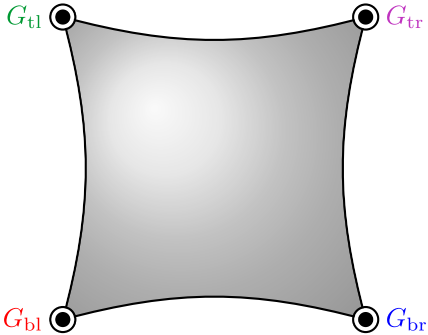
☞ orbifold can be envisaged as a manifold which is smooth everywhere except for special (orbifold fixed) points

Summary of crucial features of orbifolds



- orbifold can be envisaged as a manifold which is smooth everywhere except for special (orbifold fixed) points
- 'bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points

Summary of crucial features of orbifolds

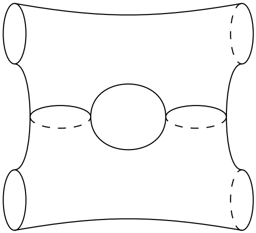


- orbifold can be envisaged as a manifold which is smooth everywhere except for special (orbifold fixed) points
- 'bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points
- low–energy gauge group : $G_{\rm low-energy} = G_{\rm bl} \cap G_{\rm br} \cap G_{\rm tl} \cap G_{\rm tr}$

Couplings

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ General recipe: couplings in the effective theory from matching string amlitudes



Couplings

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

- ☞ General recipe: couplings in the effective theory from matching string amlitudes
- ☞ Orbifolds: CFT allows to compute correlators between states

Couplings

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

- ☞ General recipe: couplings in the effective theory from matching string amlitudes
- ☞ Orbifolds: CFT allows to compute correlators between states
- ☞ Two steps:
  1. identify non–vanishing couplings

Couplings

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

- General recipe: couplings in the effective theory from matching string amlitudes
- Orbifolds: CFT allows to compute correlators between states
- Two steps:
  1. identify non–vanishing couplings
  2. evaluate coupling strengths

Superpotential couplings

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ Order N superpotential couplings

$$\mathscr{W} \supset y\,\Psi^{(1)}\,\Psi^{(2)}\ldots\Psi^{(N)}$$

determined by correlators between 2 fermions and $N-2$ bosons

$$y \propto \left\langle \underbrace{\boldsymbol{V}_{-1/2}^{(1)}\,\boldsymbol{V}_{-1/2}^{(2)}}_{\text{fermionic vertex operators}}\,\underbrace{\boldsymbol{V}_{-1}^{(1)}\,\boldsymbol{V}_{0}^{(1)}\,\ldots\,\boldsymbol{V}_{0}^{(N-3)}}_{\text{bosonic vertex operators}} \right\rangle$$

Superpotential couplings

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ Order N superpotential couplings

$$\mathscr{W} \supset y\,\Psi^{(1)}\,\Psi^{(2)}\ldots\Psi^{(N)}$$

determined by correlators between 2 fermions and $N-2$ bosons

$$y \propto \left\langle \boldsymbol{V}^{(1)}_{-1/2}\,\boldsymbol{V}^{(2)}_{-1/2}\,\boldsymbol{V}^{(1)}_{-1}\,\boldsymbol{V}^{(1)}_{0}\,\ldots\,\boldsymbol{V}^{(N-3)}_{0}\right\rangle$$

"ghost charges"

Superpotential couplings

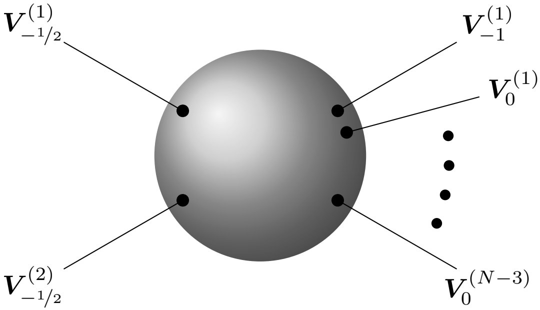
Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

- Order N superpotential couplings

$$\mathscr{W} \supset y\,\Psi^{(1)}\,\Psi^{(2)}\dots\Psi^{(N)}$$

determined by correlators between 2 fermions and $N-2$ bosons

$$y \propto \left\langle \boldsymbol{V}_{-1/2}^{(1)}\,\boldsymbol{V}_{-1/2}^{(2)}\,\boldsymbol{V}_{-1}^{(1)}\,\boldsymbol{V}_{0}^{(1)}\dots\boldsymbol{V}_{0}^{(N-3)}\right\rangle$$



Vertex operators

$$\boldsymbol{V}_{-1}^{(f)} = \mathrm{e}^{-\phi}\,\mathrm{e}^{2\mathrm{i}\,(q+k\,v_N)\cdot H}\,\mathrm{e}^{2\mathrm{i}\,(p+V_f)\cdot X}\prod_{i=1}^{3}\left(\bar\partial Z^i\right)^{\widetilde{N}_{f\,i}}\left(\bar\partial Z^{*\,i}\right)^{\widetilde{N}^*_{f\,i}}\,\sigma_f$$

$$\boldsymbol{V}_{0}^{(f)} = \mathrm{e}^{2\mathrm{i}\,(q+k\,v_N)\cdot H}\,\mathrm{e}^{2\mathrm{i}\,(p+V_f)\cdot X}\prod_{i=1}^{3}\left(\bar\partial Z^i\right)^{\widetilde{N}_{f\,i}}\left(\bar\partial Z^{*\,i}\right)^{\widetilde{N}^*_{f\,i}} \times\sum_{j=1}^{3}\left(\mathrm{e}^{2\mathrm{i}\,H^j}\,\partial Z^j+\mathrm{e}^{-2\mathrm{i}\,H^j}\,\partial Z^{*\,j}\right)\,\sigma_f$$

$$\boldsymbol{V}_{-1/2}^{(f)} = \mathrm{e}^{-\phi/2}\,\mathrm{e}^{2\mathrm{i}\,(q+k v_N)\cdot H}\,\mathrm{e}^{2\mathrm{i}(p+V_f)\cdot X}\prod_{i=1}^{3}\left(\bar\partial Z^i\right)^{\widetilde{N}_{f\,i}}\left(\bar\partial Z^{*\,i}\right)^{\widetilde{N}^*_{f\,i}}\,\sigma_f$$

bosonized superconformal ghost

twist field

Step 1: selection rules

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ Correlator factorizes

Step 1: selection rules

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ Correlator factorizes

$$\left\langle \prod_r \mathrm{e}^{2\mathrm{i}\,(p+V_f)_{(r)}\cdot X} \right\rangle \stackrel{!}{\neq} 0 \qquad \curvearrowright \quad \text{gauge invariance}$$

Step 1: selection rules

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ Correlator factorizes

$$\left\langle \prod_r \mathrm{e}^{2\mathrm{i}\,(p+V_f)_{(r)}\cdot X} \right\rangle \stackrel{!}{\neq} 0 \qquad \curvearrowright \quad \text{gauge invariance}$$

$$\left\langle \prod_r \sigma^{(r)} \right\rangle \stackrel{!}{\neq} 0 \qquad \curvearrowright \quad \text{space group selection rule}$$

Step 1: selection rules

Hamidi & Vafa [1987], Dixon, Friedan, Martinec & Shenker [1987]

☞ Correlator factorizes

$$\left\langle \prod_r \mathrm{e}^{2\mathrm{i}\,(p+V_f)_{(r)}\cdot X} \right\rangle \stackrel{!}{\neq} 0 \qquad \curvearrowright \quad \text{gauge invariance}$$

$$\left\langle \prod_r \sigma^{(r)} \right\rangle \stackrel{!}{\neq} 0 \qquad \curvearrowright \quad \text{space group selection rule}$$

$$\left\langle \prod_r \mathsf{rest}_{(r)} \right\rangle \stackrel{!}{\neq} 0 \qquad \curvearrowright \quad H\text{–momentum conservation}$$

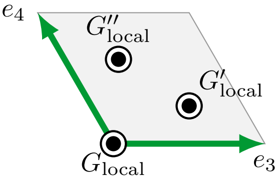
Gauge invariance

$$\sum_r (p + V_f)_{(r)} \;=\; 0$$

Gauge invariance

$$\sum_r (p + V_f)_{(r)} \;=\; 0$$

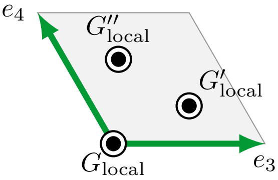
☞ Consider states localized at points with local gauge symmetries G_local, G'_local, G''_local, ...



Gauge invariance

$$\sum_r (p + V_f)_{(r)} \ = \ 0$$

- Consider states localized at points with local gauge symmetries G_{local}, G'_{local}, $G''_{\text{local}}, \dots$



- Relevant gauge symmetry

$$G_{\text{intersection}} \ = \ G_{\text{local}} \cap G'_{\text{local}} \cap G''_{\text{local}} \cap \dots \ \subset \ \mathrm{E}_8 \times \mathrm{E}_8$$

Space group selection rule

- Coupling only allowed if

$$\prod_r \left(\theta^{k^{(r)}}, \ell^{(r)}\right) \simeq (\mathbb{1}, 0)$$

$$\sum_r \left(k_{(r)}\, n_2^{(r)}, k_{(r)}\, n_2'^{(r)}, k_{(r)}\, n_3^{(r)}\right) = (0,0,0) \mod (2,2,3) \quad \text{etc.}$$

'localization quantum numbers'

Space group selection rule

☞ Coupling only allowed if

$$\prod_r \left(\theta^{k^{(r)}}, \ell^{(r)}\right) \simeq (\mathbb{1}, 0)$$

$$\sum_r \left(k_{(r)}\, n_2^{(r)}, k_{(r)}\, n_2'^{(r)}, k_{(r)}\, n_3^{(r)}\right) = (0,0,0) \mod (2,2,3) \quad \text{etc.}$$

'localization quantum numbers'

➥ Discrete $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries

Space group selection rule

- Coupling only allowed if

$$\prod_r \left(\theta^{k^{(r)}}, \ell^{(r)}\right) \simeq (\mathbb{1}, 0)$$

$$\sum_r \left(k_{(r)}\, n_2^{(r)}, k_{(r)}\, n_2'^{(r)}, k_{(r)}\, n_3^{(r)}\right) = (0,0,0) \mod (2,2,3) \quad \text{etc.}$$

'localization quantum numbers'

- Discrete $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries

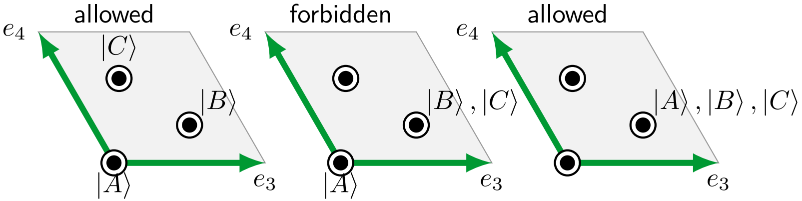
Space group selection rule

- Coupling only allowed if

$$\prod_r \left(\theta^{k^{(r)}}, \ell^{(r)}\right) \simeq (\mathbb{1}, 0)$$

- ➥ Discrete $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries

- Example: $\mathrm{SU}(3)$ plane

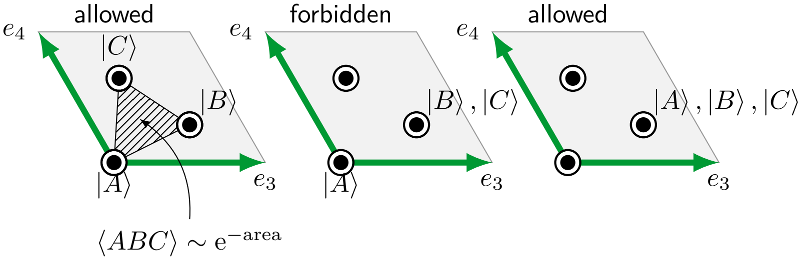


Space group selection rule

- Coupling only allowed if

$$\prod_r \left(\theta^{k^{(r)}}, \ell^{(r)}\right) \simeq (\mathbb{1}, 0)$$

- ➥ Discrete $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries
- Example: $\mathrm{SU}(3)$ plane



Couplings: summary

☞ gauge invariance

Couplings: summary

- ☞ gauge invariance
- ☞ space group selection rules ($\curvearrowright$ discrete symmetries)

Couplings: summary

- gauge invariance
- space group selection rules ($\curvearrowright$ discrete symmetries)
- H–momentum conservation ($\curvearrowright$ discrete R symmetries)

Couplings: summary

- gauge invariance
- space group selection rules ($\curvearrowright$ discrete symmetries)
- H–momentum conservation ($\curvearrowright$ discrete R symmetries)

main message:

couplings are **predicted!**

one **cannot** 'invent' couplings

Construction of orbifold models: summary

10D theory

$E_8 \times E_8$

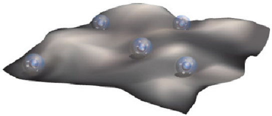
specify:

- geometry
- gauge embedding

effective 4D theory

get:

- spectrum
- interactions



note: couplings are moduli–dependent

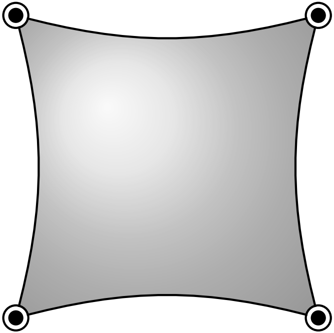
R symmetries

- ☞ Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$

R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

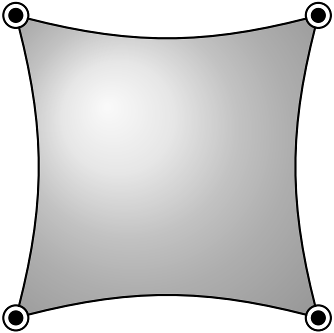
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

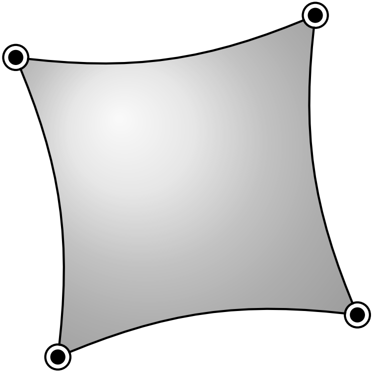
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

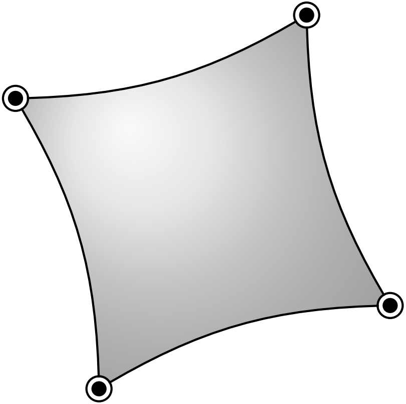
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

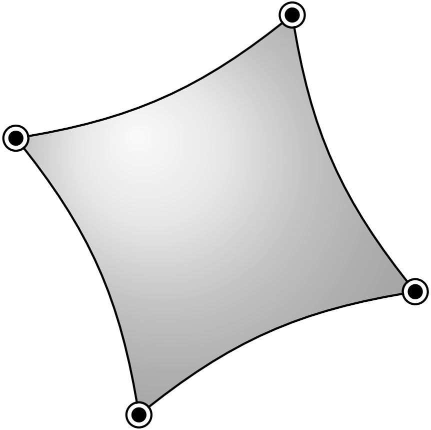
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

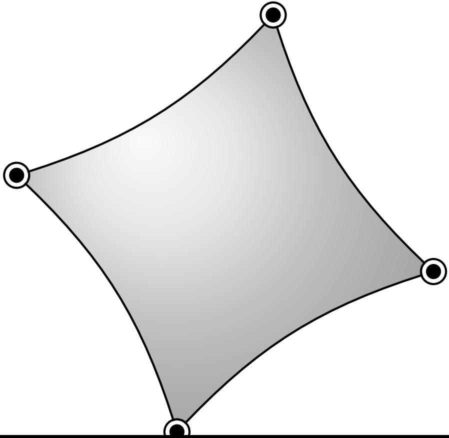
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

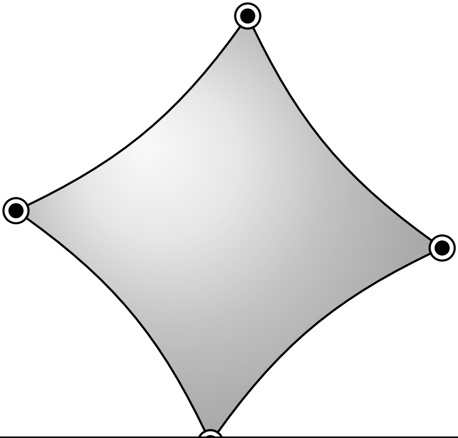
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

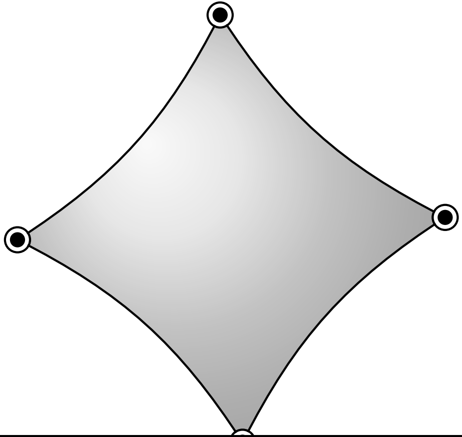
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

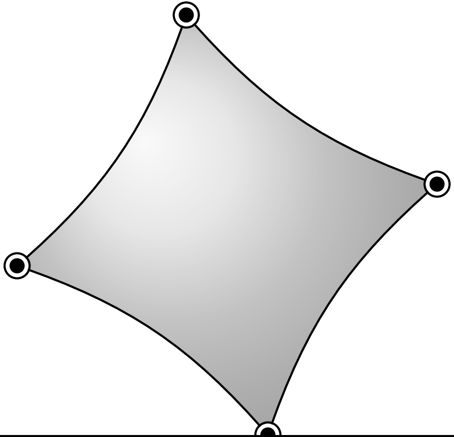
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

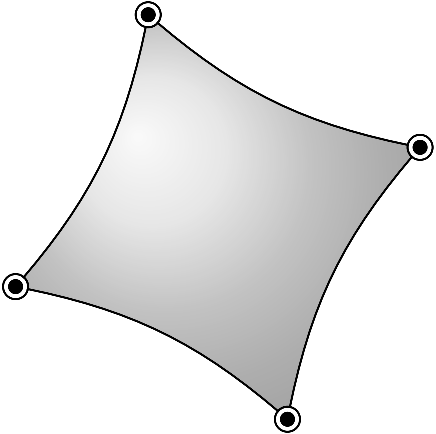
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

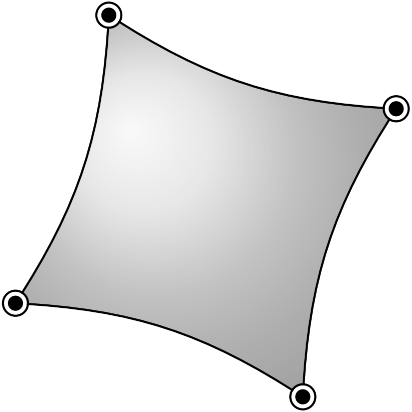
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

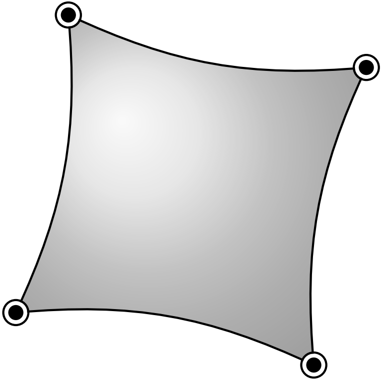
back



R symmetries

- ☞ Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- ☞ Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

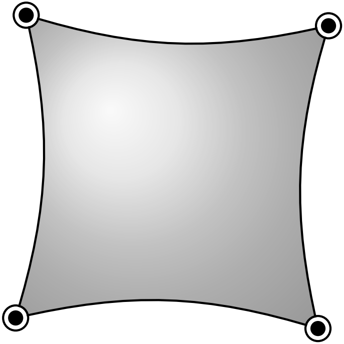
▸ back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

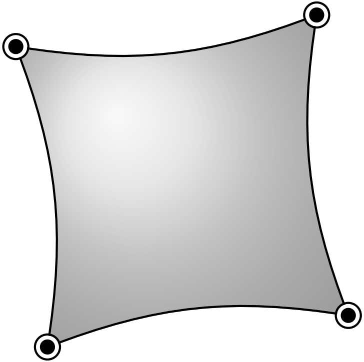
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

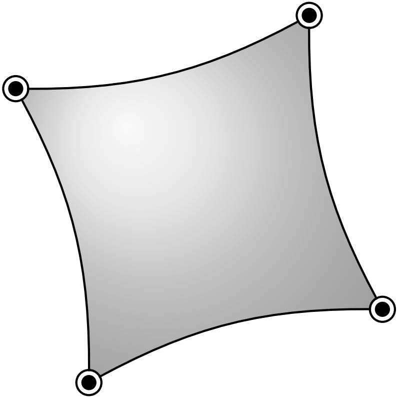
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

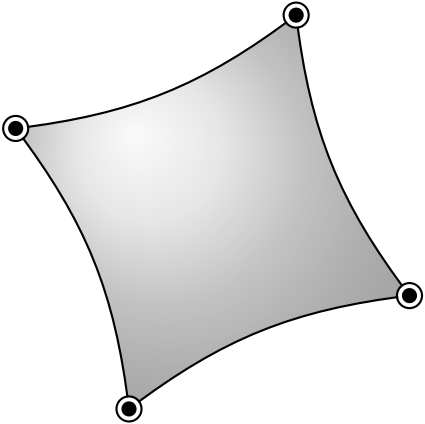
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

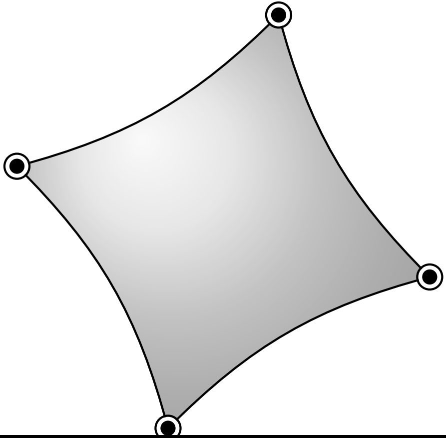
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

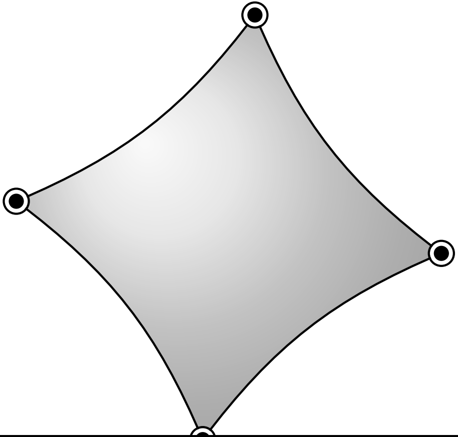
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

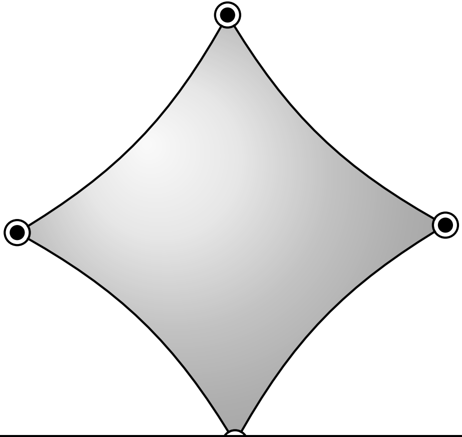
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

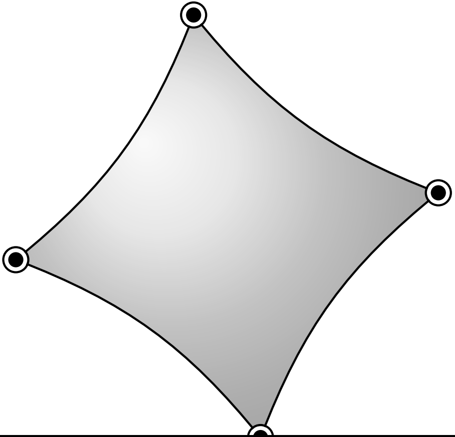
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

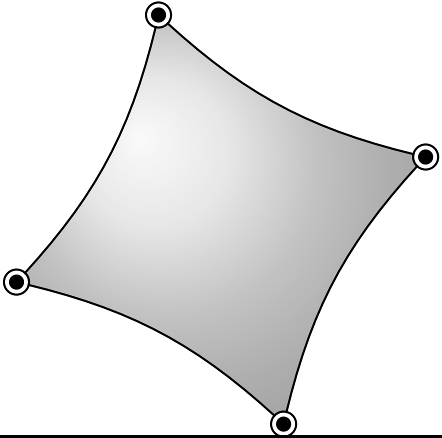
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

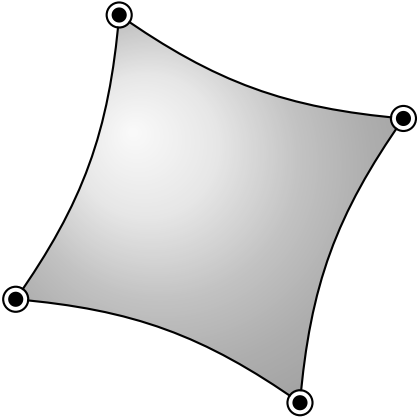
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

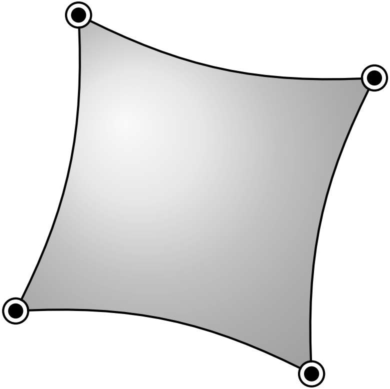
back



R symmetries

- ☞ Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- ☞ Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

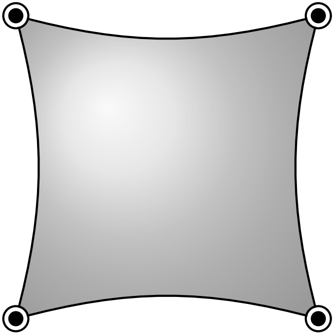
back



R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

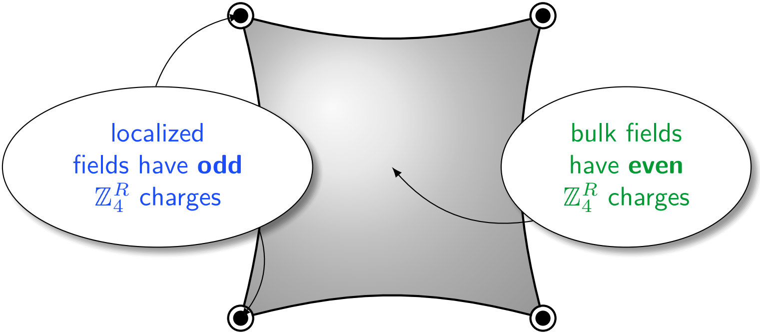
back



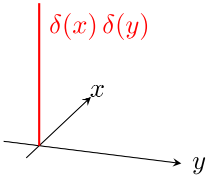
R symmetries

- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

back

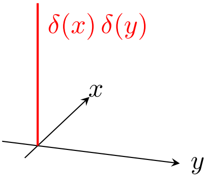


Interlude: Field theory vs. string theory



☞ In field theory it is hard to figure out whether or not localized states transform non–trivially under discrete rotations in compact dimensions

Interlude: Field theory vs. string theory



- In field theory it is hard to figure out whether or not localized states transform non–trivially under discrete rotations in compact dimensions
- In string theory it follows from H–momentum conservation that localized (twisted) states have odd $\mathbb{Z}_4^R$ charges while bulk (untwisted) have even $\mathbb{Z}_4^R$ charges

Large Hierarchies in Nature

Large hierarchies in Nature

- ☞ Observed hierarchy: $M_{\mathrm{P}}/m_W \sim 10^{17}$

Large hierarchies in Nature

☞ Observed hierarchy: $M_{\mathrm{P}}/m_W \sim 10^{17}$

Why?

Large hierarchies in Nature

- Observed hierarchy: $M_{\mathrm{P}}/m_W \sim 10^{17}$

Why?

- Compelling answer: scale of supersymmetry breakdown set by dimensional transmutation Witten [1981]

$$\Lambda \ \sim \ M_{\mathrm{P}} \, \exp\left(-b/g^2\right)$$

Large hierarchies in Nature

- Observed hierarchy: $M_\mathrm{P}/m_W \sim 10^{17}$

Why?

- Compelling answer: scale of supersymmetry breakdown set by dimensional transmutation Witten [1981]

$$\Lambda \sim M_\mathrm{P}\,\exp\left(-b/g^2\right)$$

$\mathcal{O}(10)$

RG invariant scale

$\mathcal{O}(1)$ coupling

Large hierarchies in Nature

- Observed hierarchy: $M_{\mathrm{P}}/m_W \sim 10^{17}$

Why?

- Compelling answer: scale of supersymmetry breakdown set by dimensional transmutation — Witten [1981]

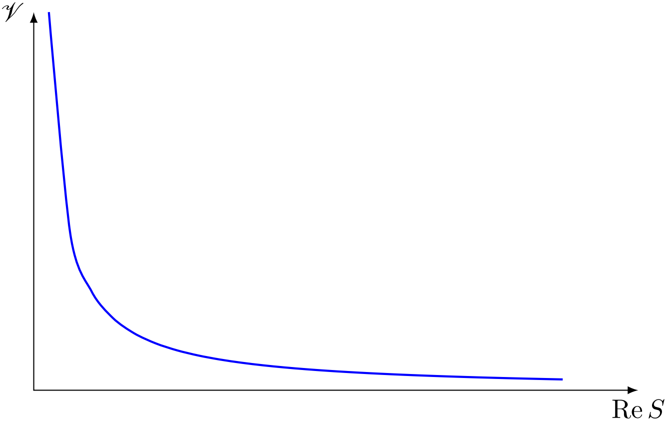
$$\Lambda \sim M_{\mathrm{P}} \exp\left(-b/g^2\right)$$

➥ hierarchically small gravitino mass (‘gaugino condensation’) — Nilles [1982]

$$m_W \sim m_{3/2} \sim \frac{\Lambda^3}{{M_{\mathrm{P}}}^2}$$

Problem with string theory realization

☞ **However**: embedding into string theory $\curvearrowright$ run–away problem
Dine & Seiberg [1985]



Moduli fixing and non–perturbative terms

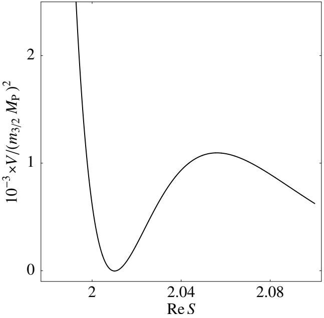
There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

Moduli fixing and non–perturbative terms

There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

- Race–track
  Krasnikov [1987] ; . . .

use several gaugino condensates



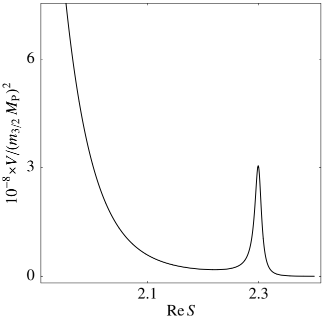
Moduli fixing and non–perturbative terms

There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

- Race–track
- Kähler stabilization

Casas [1996] ; Binétruy, Gaillard & Wu [1997] ; ...

non–perturbative corrections
to the Kähler potential



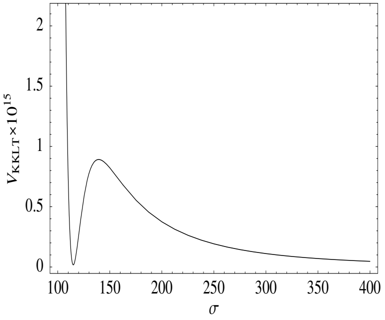
Moduli fixing and non–perturbative terms

There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

- Race–track
- Kähler stabilization
- Flux compactification

e.g. Kachru, Kallosh, Linde & Trivedi [2003]

e.g. KKLT proposal



Moduli fixing and non–perturbative terms

There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

- Race–track
- Kähler stabilization
- Flux compactification
- etc. . . .

?

Constant + exponential scheme

non–perturbative

☞ KKLT type proposal: $\mathscr{W}_{\text{eff}} = c + A\,\mathrm{e}^{-a\,S}$

constant

Constant + exponential scheme

☞ KKLT type proposal: $\mathscr{W}_{\mathrm{eff}} \;=\; c + A\,\mathrm{e}^{-a\,S}$

☞ Gravitino mass

$$m_{3/2} \;\sim\; |c|$$

Constant + exponential scheme

- KKLT type proposal: $\mathscr{W}_{\text{eff}} \;=\; c + A\,\mathrm{e}^{-a\,S}$

- Gravitino mass

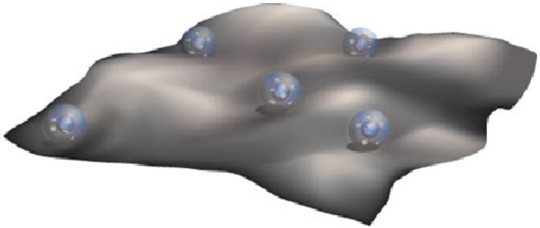
$$m_{3/2} \;\sim\; |c| \quad \xrightarrow{m_{3/2} \overset{!}{\simeq} \text{TeV}} \quad |c| \;\sim\; 10^{-15}$$

Constant + exponential scheme

- KKLT type proposal: $\mathscr{W}_{\text{eff}} = c + A\,\mathrm{e}^{-a\,S}$
- Gravitino mass

$$m_{3/2} \sim |c| \quad \xrightarrow{m_{3/2} \overset{!}{\simeq} \mathrm{TeV}} \quad |c| \sim 10^{-15}$$

- Philosophy of flux compactifications: many vacua, in some of them c might be small by accident



www.nature.com

Constant + exponential scheme

- KKLT type proposal: $\mathscr{W}_{\text{eff}} = c + A\,\mathrm{e}^{-a\,S}$

- Gravitino mass

$$m_{3/2} \sim |c| \quad \xrightarrow{m_{3/2} \overset{!}{\simeq} \text{TeV}} \quad |c| \sim 10^{-15}$$

- Philosophy of flux compactifications: many vacua, in some of them c might be small by accident

- Alternative proposal: hierarchically small expectation of the perturbative superpotential due to approximate $\mathrm{U}(1)_R$ symmetry

order of $\mathrm{U}(1)_R$ breaking

$$c \to \langle \mathscr{W}_{\text{pert}} \rangle \sim \langle \phi \rangle^N \quad \text{with} \quad N = \mathcal{O}(10)$$

typical VEV < 1

Hierarchically small $\langle\mathscr{W}\rangle$

Two observations:

1. in the presence of an exact $\mathrm{U}(1)_R$ symmetry

$$\frac{\partial\mathscr{W}}{\partial\phi_i}=0 \quad \curvearrowright \quad \langle\mathscr{W}\rangle=0$$

Hierarchically small $\langle \mathscr{W} \rangle$

Two observations:

1. in the presence of an exact $\mathrm{U}(1)_R$ symmetry

$$\frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \quad \curvearrowright \quad \langle \mathscr{W} \rangle = 0$$

fields

superpotential

Hierarchically small $\langle \mathscr{W} \rangle$

Two observations:

1. in the presence of an exact $\mathrm{U}(1)_R$ symmetry

$$\frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \quad \curvearrowright \quad \langle \mathscr{W} \rangle = 0$$

2. for approximate R symmetries

$$\langle \mathscr{W} \rangle \sim \langle \phi \rangle^N$$

typical field VEV

order of explicit $\mathrm{U}(1)_R$ breaking terms

$\langle\mathscr{W}\rangle = 0$ because of $\mathrm{U}(1)_R$ (I)

aim: show that

$$\frac{\partial\mathscr{W}}{\partial\phi_i} = 0 \qquad \curvearrowright \qquad \langle\mathscr{W}\rangle = 0$$

$\langle \mathscr{W} \rangle = 0$ because of $\mathrm{U}(1)_R$ (I)

aim: show that

$$\frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \qquad \curvearrowright \qquad \langle \mathscr{W} \rangle = 0$$

Consider a superpotential

$$\mathscr{W} = \sum c_{n_1 \cdots n_M} \phi_1^{n_1} \cdots \phi_M^{n_M}$$

with an exact R symmetry

$$\mathscr{W} \to \mathrm{e}^{2\mathrm{i}\,\alpha} \mathscr{W} \,, \quad \phi_j \to \phi_j' = \mathrm{e}^{\mathrm{i}\, r_j\, \alpha} \phi_j$$

where each monomial in $\mathscr{W}$ has total R charge 2

$\langle \mathscr{W} \rangle = 0$ because of $\mathrm{U}(1)_R$ (II)

Consider a field configuration $\langle \phi_i \rangle$ with

$$F_i = \frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \quad \text{at } \phi_j = \langle \phi_j \rangle$$

Under an infinitesimal $\mathrm{U}(1)_R$ transformation, the superpotential transforms nontrivially

$$\mathscr{W}(\phi_j) \to \mathscr{W}(\phi_j') = \mathscr{W}(\phi_j) + \sum_i \frac{\partial \mathscr{W}}{\partial \phi_i} \Delta\phi_i$$

$\langle\mathscr{W}\rangle = 0$ because of $\mathrm{U}(1)_R$ (II)

Consider a field configuration $\langle\phi_i\rangle$ with

$$F_i \;=\; \frac{\partial\mathscr{W}}{\partial\phi_i} \;=\; 0 \quad \text{at } \phi_j = \langle\phi_j\rangle$$

Under an infinitesimal $\mathrm{U}(1)_R$ transformation, the superpotential transforms nontrivially

$$\mathscr{W}(\phi_j) \;\to\; \mathscr{W}(\phi_j') \;=\; \mathscr{W}(\phi_j) + \sum_i \cancel{\frac{\partial\mathscr{W}}{\partial\phi_i}}\,\Delta\phi_i$$

$\langle \mathscr{W} \rangle = 0$ because of $\mathrm{U}(1)_R$ (II)

Consider a field configuration $\langle \phi_i \rangle$ with

$$F_i = \frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \quad \text{at } \phi_j = \langle \phi_j \rangle$$

Under an infinitesimal $\mathrm{U}(1)_R$ transformation, the superpotential transforms nontrivially

$$\mathscr{W}(\phi_j) \to \mathscr{W}(\phi_j') = \mathscr{W}(\phi_j) + \sum_i \cancel{\frac{\partial \mathscr{W}}{\partial \phi_i}} \Delta\phi_i \stackrel{!}{=} \mathrm{e}^{2\mathrm{i}\,\alpha}\, \mathscr{W}$$

This is only possible if $\langle \mathscr{W} \rangle = 0\,!$

bottom–line:

$$\frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \qquad \curvearrowright \qquad \langle \mathscr{W} \rangle = 0$$

Comments

1. Statement $\langle \mathscr{W} \rangle = 0$ holds regardless of whether $\mathrm{U}(1)_R$ is unbroken (where it is trivial) or broken

Comments

1. Statement $\langle\mathscr{W}\rangle = 0$ holds regardless of whether $\mathrm{U}(1)_R$ is unbroken $\bigl($where it is trivial$\bigr)$ or broken

2. Relation to Nelson–Seiberg theorem

   Nelson & Seiberg [1994]

$$\left\{\begin{array}{c}\text{setting without}\\ \text{supersymmetric}\\ \text{ground state}\end{array}\right\} \xrightarrow{\text{requires}} \mathrm{U}(1)_R \text{ symmetry}$$

Comments

1. Statement $\langle \mathscr{W} \rangle = 0$ holds regardless of whether $\mathrm{U}(1)_R$ is unbroken (where it is trivial) or broken

2. Relation to Nelson–Seiberg theorem Nelson & Seiberg [1994]

$$\left\{ \begin{array}{c} \text{setting without} \\ \text{supersymmetric} \\ \text{ground state} \end{array} \right\} \begin{array}{c} \xrightarrow{\text{requires}} \\ \xleftarrow{\text{does not imply}} \end{array} \mathrm{U}(1)_R \text{ symmetry}$$

Comments

1. Statement $\langle \mathscr{W} \rangle = 0$ holds regardless of whether $\mathrm{U}(1)_R$ is unbroken (where it is trivial) or broken

2. Relation to Nelson–Seiberg theorem

Nelson & Seiberg [1994]

$$\left\{\begin{array}{c}\text{setting without}\\ \text{supersymmetric}\\ \text{ground state}\end{array}\right\} \begin{array}{c}\xrightarrow{\text{requires}}\\ \xleftarrow{\text{does not imply}}\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!/\!/\!/\!/\!/\!/\!/\!/\!/\end{array} \mathrm{U}(1)_R \text{ symmetry}$$

3. in local SUSY : $\dfrac{\partial \mathscr{W}}{\partial \phi_i} = 0$ and $\langle \mathscr{W} \rangle = 0$ imply $D_i \mathscr{W} = 0$

(That is, a $\mathrm{U}(1)_R$ symmetry implies Minkowski solutions.)

Comments

1. Statement $\langle\mathscr{W}\rangle = 0$ holds regardless of whether $\mathrm{U}(1)_R$ is unbroken (where it is trivial) or broken

2. Relation to Nelson–Seiberg theorem

   Nelson & Seiberg [1994]

   $$\left\{\begin{array}{c}\text{setting without}\\ \text{supersymmetric}\\ \text{ground state}\end{array}\right\} \quad \begin{array}{c}\xrightarrow{\text{requires}}\\[1em] \xleftarrow{\text{does not imply}}\end{array} \quad \mathrm{U}(1)_R \text{ symmetry}$$

3. in local SUSY : $\dfrac{\partial\mathscr{W}}{\partial\phi_i} = 0$ and $\langle\mathscr{W}\rangle = 0$ imply $D_i\mathscr{W} = 0$

   (That is, a $\mathrm{U}(1)_R$ symmetry implies Minkowski solutions.)

4. in 'no-scale' type settings

   Weinberg [1989]

$$\boxed{\begin{array}{c}\text{solutions of}\\ \text{global SUSY}\\ F \text{ term eq.'s}\end{array}} \quad = \quad \boxed{\begin{array}{c}\text{stationary points}\\ \text{of supergravity}\\ \text{scalar potential}\end{array}}$$

Approximate R symmetries

☞ Consider now the case of an **approximate** R symmetry, i.e. explicit R symmetry breaking terms appear at order N in the fields ϕ_i

Approximate R symmetries

- Consider now the case of an **approximate** R symmetry, i.e. explicit R symmetry breaking terms appear at order N in the fields ϕ_i
- This allows us to avoid certain problems:
  - for a continuous $\mathrm{U}(1)_R$ symmetry we would have
    - a supersymmetric ground state with $\langle \mathscr{W} \rangle = 0$ and $\mathrm{U}(1)_R$ spontaneously broken
    - a problematic R Goldstone boson

Approximate R symmetries

- Consider now the case of an **approximate** R symmetry, i.e. explicit R symmetry breaking terms appear at order N in the fields ϕ_i
- This allows us to avoid certain problems:
  - for a continuous $\mathrm{U}(1)_R$ symmetry we would have
    - a supersymmetric ground state with $\langle \mathscr{W} \rangle = 0$ and $\mathrm{U}(1)_R$ spontaneously broken
    - a problematic R Goldstone boson
  - however, for an **approximate** $\mathrm{U}(1)_R$ symmetry one has
    - Goldstone boson massive and harmless
    - a nontrivial VEV of $\mathscr{W}$ at order N in ϕ VEVs

$$\langle \mathscr{W} \rangle \;\sim\; \langle \phi \rangle^N$$

Approximate R symmetries

- Consider now the case of an **approximate** R symmetry, i.e. explicit R symmetry breaking terms appear at order N in the fields ϕ_i
- This allows us to avoid certain problems:
  - for a continuous $\mathrm{U}(1)_R$ symmetry we would have
    - a supersymmetric ground state with $\langle \mathscr{W} \rangle = 0$ and $\mathrm{U}(1)_R$ spontaneously broken
    - a problematic R Goldstone boson
  - however, for an **approximate** $\mathrm{U}(1)_R$ symmetry one has
    - Goldstone boson massive and harmless
    - a nontrivial VEV of $\mathscr{W}$ at order N in ϕ VEVs

$$\langle \mathscr{W} \rangle \sim \langle \phi \rangle^N$$

- Such **approximate** $\mathrm{U}(1)_R$ symmetries can be a consequence of discrete $\mathbb{Z}_N^R$ symmetries

Approximate R symmetries

- Consider now the case of an **approximate** R symmetry, i.e. explicit R symmetry breaking terms appear at order N in the fields ϕ_i

- This allows us to avoid certain problems:
  - for a continuous $\mathrm{U}(1)_R$ symmetry we would have
    - a supersymmetric ground state with $\langle\mathscr{W}\rangle = 0$ and $\mathrm{U}(1)_R$ spontaneously broken
    - a problematic R Goldstone boson
  - however, for an **approximate** $\mathrm{U}(1)_R$ symmetry one has
    - Goldstone boson massive and harmless
    - a nontrivial VEV of $\mathscr{W}$ at order N in ϕ VEVs

$$\langle\mathscr{W}\rangle \sim \langle\phi\rangle^N$$

- Such **approximate** $\mathrm{U}(1)_R$ symmetries can be a consequence of discrete $\mathbb{Z}_N^R$ symmetries

- Confirmed in various field–theoretic examples

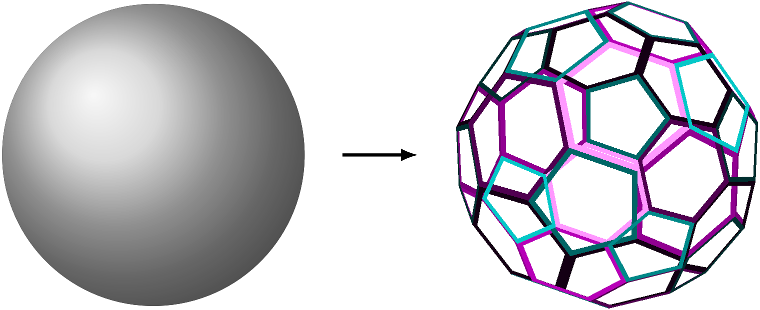
Explicit

string theory

realization

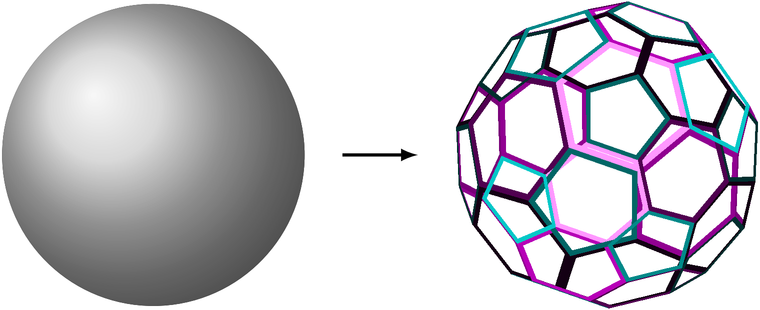
Origin of high–power discrete R symmetries

☞ Discrete R symmetries arise as remnants of Lorentz symmetries of compact space



Origin of high–power discrete R symmetries

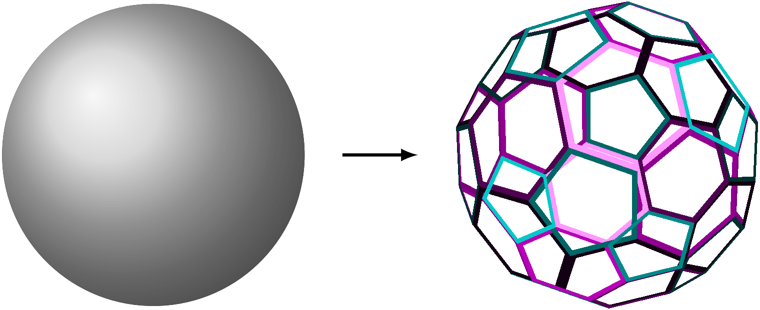
- Discrete R symmetries arise as remnants of Lorentz symmetries of compact space



- Orbifolds break $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ Lorentz symmetry of compact space to discrete subgroups

Origin of high–power discrete R symmetries

- Discrete R symmetries arise as remnants of Lorentz symmetries of compact space



- Orbifolds break $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ Lorentz symmetry of compact space to discrete subgroups
- For example: a $\mathbb{Z}_2$ orbifold plane leads to $\mathbb{Z}_4^R$ symmetry

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

☞ Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV
- R symmetry breaking terms appear at order 9

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV
- R symmetry breaking terms appear at order 9
- $D_a = 0$ as well as global $F_i = 0$ at order 9 explicitly solved

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV
- R symmetry breaking terms appear at order 9
- $D_a = 0$ as well as global $F_i = 0$ at order 9 explicitly solved
- Search for solutions $|s_i| < 1$, and find/argue that they exist

Note: in order to prove the existence a full understanding of coupling coefficients is required

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV
- R symmetry breaking terms appear at order 9
- $D_a = 0$ as well as global $F_i = 0$ at order 9 explicitly solved
- Search for solutions $|s_i| < 1$, and find/argue that they exist

- All fields acquire positive m^2
(no flat directions; not destroyed by supergravity corrections)

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV
- R symmetry breaking terms appear at order 9
- $D_a = 0$ as well as global $F_i = 0$ at order 9 explicitly solved
- Search for solutions $|s_i| < 1$, and find/argue that they exist
- All fields acquire positive m^2
  (no flat directions; not destroyed by supergravity corrections)
- Superpotential VEV $\langle \mathscr{W} \rangle \sim \langle s_i \rangle^9 \ll 1$ (as expected)

An example

Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets s_i getting a VEV
- R symmetry breaking terms appear at order 9
- $D_a = 0$ as well as global $F_i = 0$ at order 9 explicitly solved
- Search for solutions $|s_i| < 1$, and find/argue that they exist
- All fields acquire positive m^2
  (no flat directions; not destroyed by supergravity corrections)
- Superpotential VEV $\langle \mathscr{W} \rangle \sim \langle s_i \rangle^9 \ll 1$ (as expected)

bottom–line:

straightforward embedding in heterotic orbifolds

General picture

☞ The more fields are switched on, the lower N we obtain examples:

- model with 23 fields $\curvearrowright$ $N = 9$
- model with 7 fields $\curvearrowright$ $N = 26$

General picture

☞ The more fields are switched on, the lower N we obtain examples:

- model with 23 fields $\curvearrowright$ $N = 9$
- model with 7 fields $\curvearrowright$ $N = 26$

☞ Suppressed s_i in accord with scale set by Fayet–Iliopoulos term (i.e. $\langle s_i \rangle \sim 0.3$)

General picture

- The more fields are switched on, the lower N we obtain examples:
  - model with 23 fields $\curvearrowright N = 9$
  - model with 7 fields $\curvearrowright N = 26$
- Suppressed s_i in accord with scale set by Fayet–Iliopoulos term (i.e. $\langle s_i \rangle \sim 0.3$)
- One approximate Goldstone mode η

  $m_\eta \sim \langle \mathscr{W} \rangle / \langle s \rangle^2$ ... somewhat heavier than the gravitino

General picture

- ☞ The more fields are switched on, the lower N we obtain examples:
  - model with 23 fields $\curvearrowright$ $N = 9$
  - model with 7 fields $\curvearrowright$ $N = 26$
- ☞ Suppressed s_i in accord with scale set by Fayet–Iliopoulos term (i.e. $\langle s_i \rangle \sim 0.3$)
- ☞ One approximate Goldstone mode η

  $m_\eta \sim \langle \mathscr{W} \rangle / \langle s \rangle^2$ ... somewhat heavier than the gravitino
- ☞ In most examples: all other s_i fields acquire masses $\gg m_\eta$
  i.e. isolated points in s_i space with $F_i = D_a = 0$

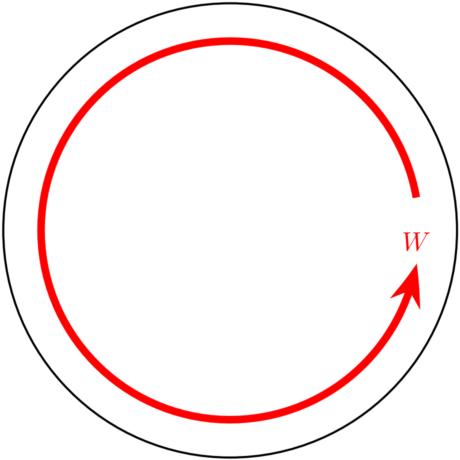
General picture

- The more fields are switched on, the lower N we obtain examples:
  - model with 23 fields $\curvearrowright$ $N = 9$
  - model with 7 fields $\curvearrowright$ $N = 26$
- Suppressed s_i in accord with scale set by Fayet–Iliopoulos term (i.e. $\langle s_i \rangle \sim 0.3$)
- One approximate Goldstone mode η

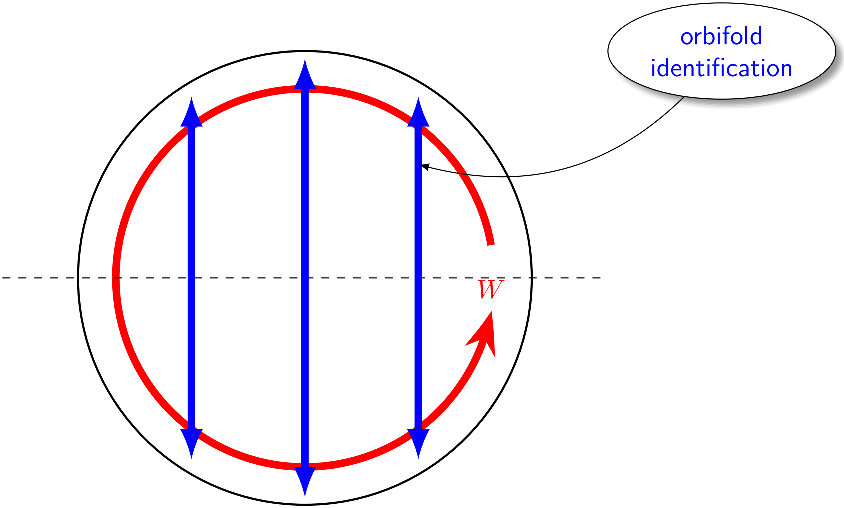
  $m_\eta \sim \langle \mathscr{W} \rangle / \langle s \rangle^2$ ...somewhat heavier than the gravitino
- In most examples: all other s_i fields acquire masses $\gg m_\eta$ i.e. isolated points in s_i space with $F_i = D_a = 0$
- Minima survive supergravity corrections

Non–local
GUT breaking

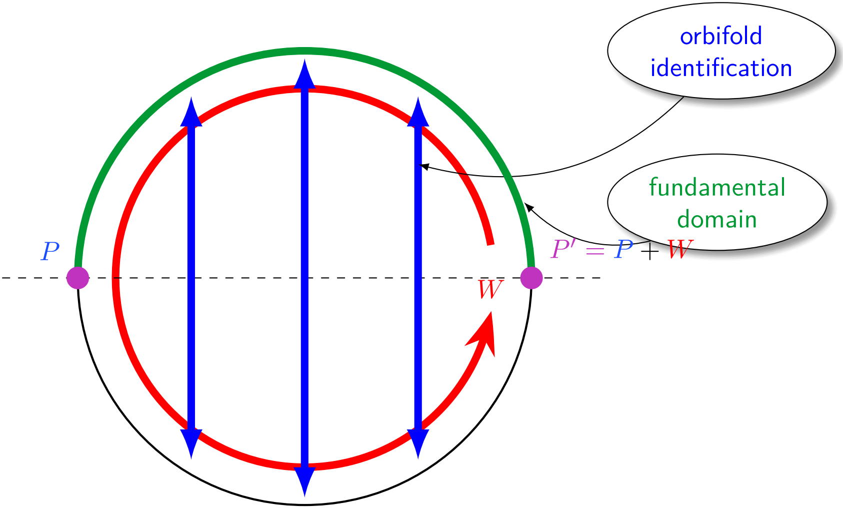
Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$



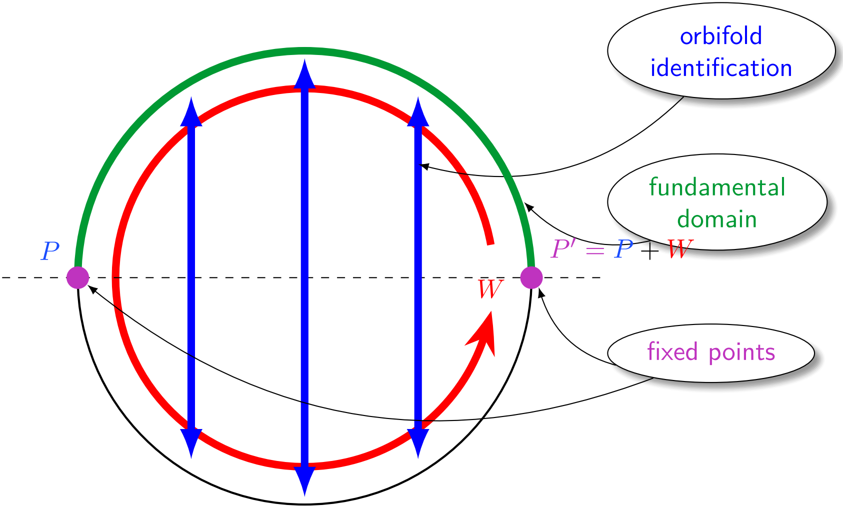
Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$



Simplest example : the orbifold $\mathbb{S}^{\mathbb{1}}/\mathbb{Z}_2$



Simplest example : the orbifold $\mathbb{S}^1/\mathbb{Z}_2$



Orbifolds & Wilson lines

Ibáñez, Nilles & Quevedo [1987b] ; Hall, Murayama & Nomura [2002a]

skip

☞ Local gauge embedding at fixed point f

$$V_f^I = k\, V_N^I + m_\alpha\, W_{n\alpha}^I$$

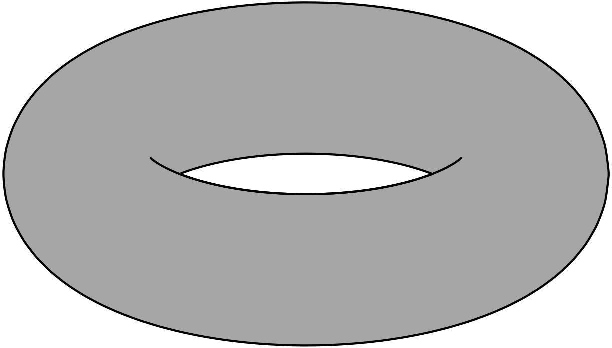
Orbifolds & Wilson lines

Ibáñez, Nilles & Quevedo [1987b] ; Hall, Murayama & Nomura [2002a]

skip

- Local gauge embedding at fixed point f

$$V_f^I \;=\; k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$



- Upshot: so–called discrete Wilson lines are differences between local shifts (and *not* Wilson lines in the usual sense)

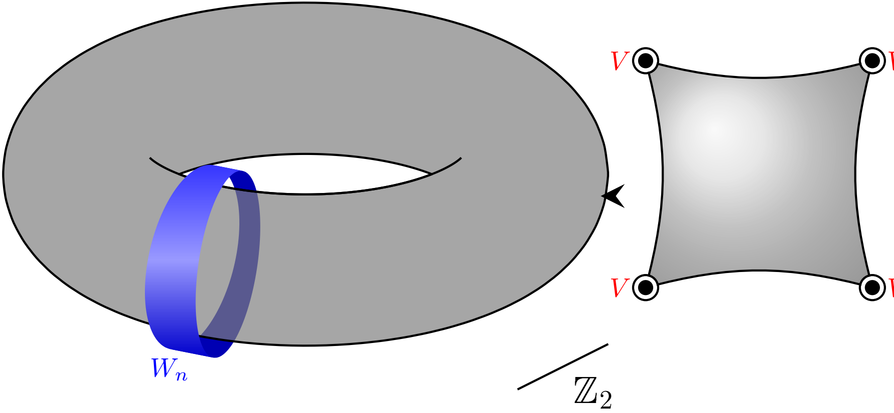
Orbifolds & Wilson lines

skip

Ibáñez, Nilles & Quevedo [1987b] ; Hall, Murayama & Nomura [2002a]

- Local gauge embedding at fixed point f

$$V_f^I = k\, V_N^I + m_\alpha\, W_{n\alpha}^I$$



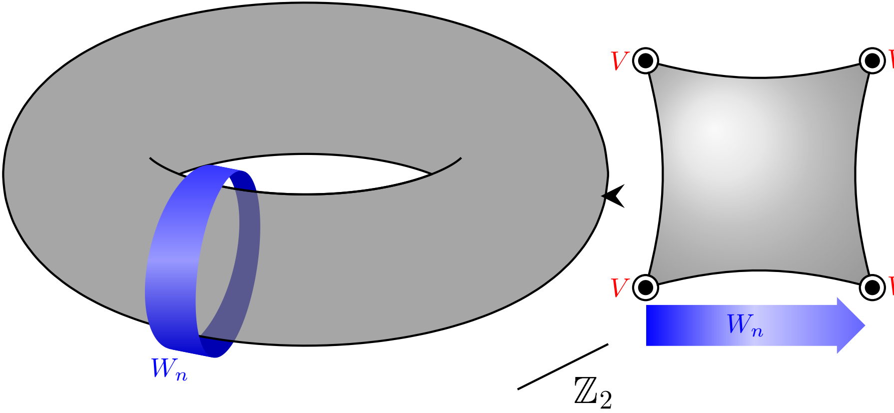
Orbifolds & Wilson lines

Ibáñez, Nilles & Quevedo [1987b] ; Hall, Murayama & Nomura [2002a]

skip

- Local gauge embedding at fixed point f

$$V_f^I = k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$



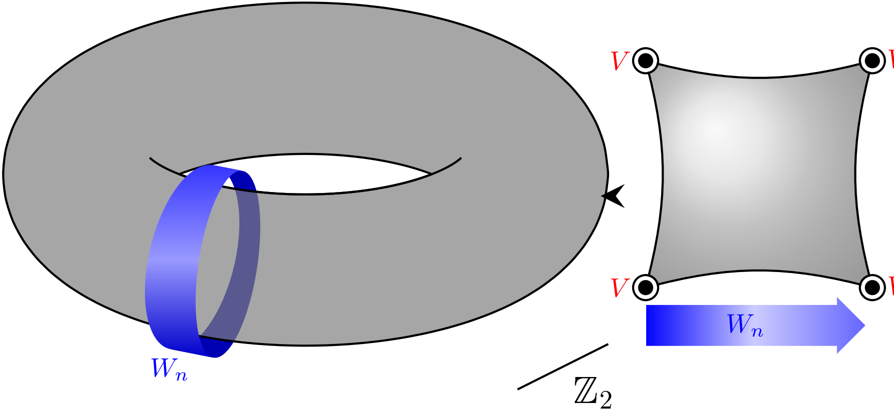
Orbifolds & Wilson lines

Ibáñez, Nilles & Quevedo [1987b] ; Hall, Murayama & Nomura [2002a]

skip

☞ Local gauge embedding at fixed point f

$$V_f^I = k\, V_N^I + m_\alpha\, W_{n\alpha}^I$$



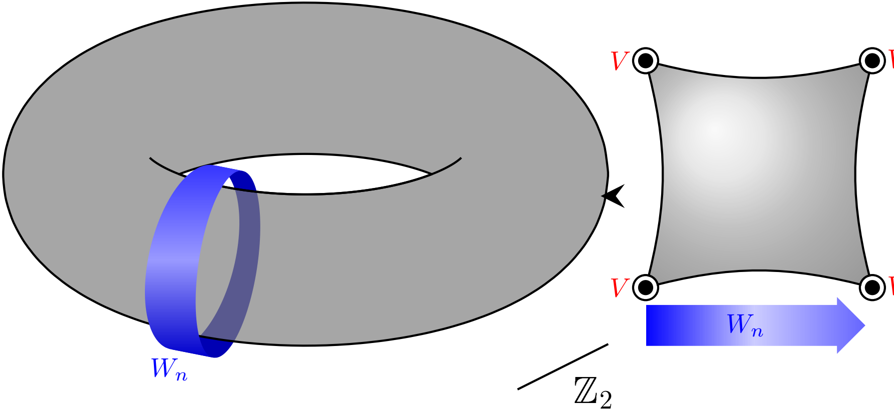
Orbifolds & Wilson lines

Ibáñez, Nilles & Quevedo [1987b] ; Hall, Murayama & Nomura [2002a]

▸ skip

☞ Local gauge embedding at fixed point f

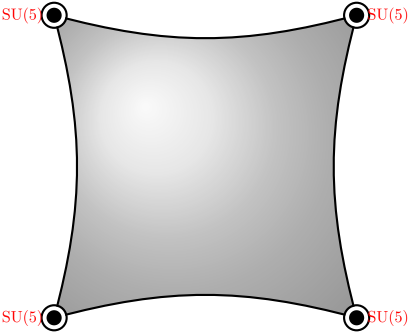
$$V_f^I \;=\; k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$



☞ Upshot: so–called discrete Wilson lines are differences between local shifts (and not Wilson lines in the usual sense)

Local vs. non–local GUT breaking

Hall, Murayama & Nomura [2002a] ; Hebecker [2004]

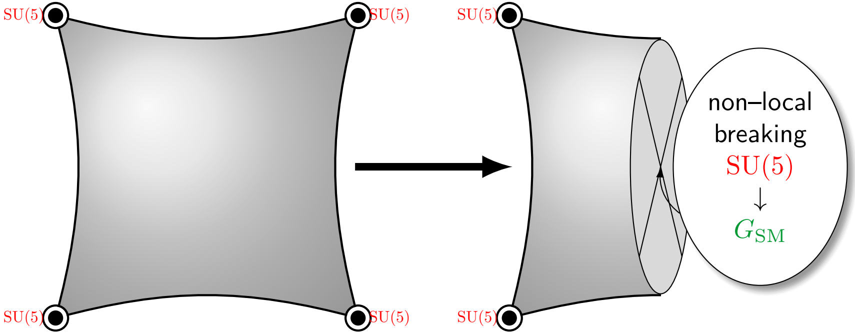


1. step: construct $\mathbb{T}^2/\mathbb{Z}_2$ orbifold which breaks $\mathrm{SU}(6)$ locally to $\mathrm{SU}(5)$

$$\mathbb{Z}_2 \quad : \quad (x_5, x_6) \;\to\; (-x_5, -x_6)$$

Local vs. non–local GUT breaking

Hall, Murayama & Nomura [2002a] ; Hebecker [2004]



❶ step: construct $\mathbb{T}^2/\mathbb{Z}_2$ orbifold which breaks $\mathrm{SU}(6)$ locally to $\mathrm{SU}(5)$

❷ step: mod out a freely acting $\mathbb{Z}_2'$ symmetry which breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$

$$\mathbb{Z}_2' \quad : \quad (x_5, x_6) \;\to\; (-x_5 + \pi\, R_5, -x_6 + \pi\, R_6)$$

Non–local GUT breaking in heterotic orbifolds

Fischer, M.R., Torrado & Vaudrevange [2013b] $\longrightarrow$ talk by M. Fischer

☞ **Complete** classification of (symmetric) heterotic orbifolds

✏ more detailled analysis of non–Abelian orbifolds

Konopka [2013] ; Fischer, Ramos-Sánchez & Vaudrevange [2013a] $\longrightarrow$ talk by S. Ramos–Sánchez

✏ recent progress in asymmetric orbifolds

Beye, Kobayashi & Kuwakino [2013]

Non–local GUT breaking in heterotic orbifolds

Fischer, M.R., Torrado & Vaudrevange [2013b] $\longrightarrow$ talk by M. Fischer

- ☞ **Complete** classification of (symmetric) heterotic orbifolds
- ☞ 31 geometries with non–trivial fundamental groups (after orbifolding!) with point groups $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$

  - ✏ 38 additional geometries with non–trivial fundamental groups in non–Abelian orbifolds

    Fischer, Ramos-Sánchez & Vaudrevange [2013a] $\longrightarrow$ talk by S. Ramos–Sánchez

  - ✏ some models are non–chiral but chirality may be achieved by adding fluxes

    Groot Nibbelink & Vaudrevange [2013] $\longrightarrow$ talk by S. Groot–Nibbelink

  - ✏ recent analysis of $\mathbb{Z}_2 \times \mathbb{Z}_4$ models w/ local GUT breaking

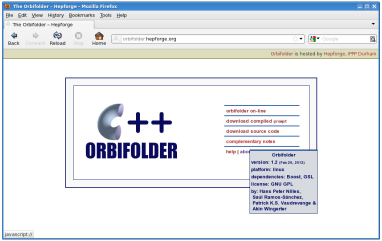
    Pena, Nilles & Oehlmann [2012] $\longrightarrow$ talk by P. Oehlmann

Non–local GUT breaking in heterotic orbifolds

Fischer, M.R., Torrado & Vaudrevange [2013b] $\longrightarrow$ talk by M. Fischer

- **Complete** classification of (symmetric) heterotic orbifolds
- 31 geometries with non–trivial fundamental groups (after orbifolding!) with point groups $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$
- Geometries online and ready to use

http://einrichtungen.ph.tum.de/T30e/codes/ClassificationOrbifolds/



http://orbifolder.hepforge.org

Nilles, Ramos-Sánchez, Vaudrevange & Wingerter [2012]

Non–local GUT breaking in heterotic orbifolds

Fischer, M.R., Torrado & Vaudrevange [2013b] $\longrightarrow$ talk by M. Fischer

- **Complete** classification of (symmetric) heterotic orbifolds
- 31 geometries with non–trivial fundamental groups (after orbifolding!) with point groups $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$
- Geometries online and ready to use with the `C++ orbifolder`

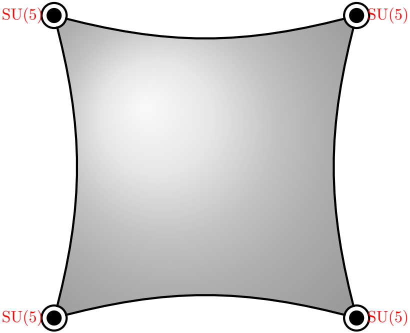
➥ Many promising models w/ non–local GUT breaking

Fischer et al. (in preparation)

An example

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

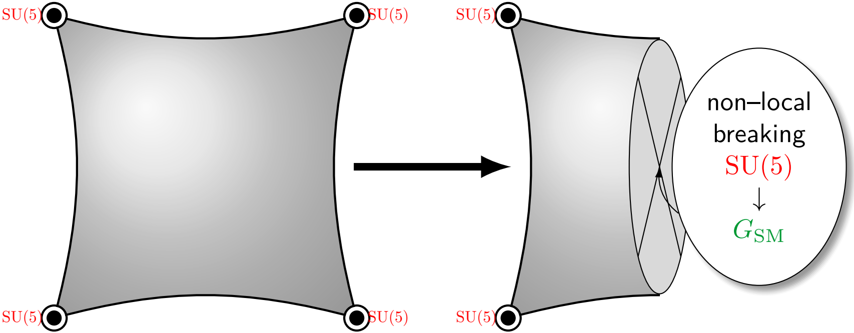
Blaszczyk, Groot Nibbelink, M.R., Ruehle, Trapletti, et al. [2010] ; Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

Blaszczyk, Groot Nibbelink, M.R., Ruehle, Trapletti, et al. [2010] ; Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry
2. step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:
   - breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$
   - reduces the number of generations to 3

analogous mechanism in CY MSSMs Bouchard & Donagi [2006]
Braun, He, Ovrut & Pantev [2005]

Main features

1. GUT symmetry breaking non–local
$\curvearrowright$ (almost) no 'logarithmic running above the GUT scale'
Hebecker & Trapletti [2005] ; Anandakrishnan & Raby [2013]

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
   $\curvearrowright$ complete blow–up without breaking SM gauge symmetry in principle possible

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$
4. massless spectrum

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
3	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$
4. massless spectrum

   spectrum $=$ $3 \times$ generation $+$ vector-like

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
5	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-2/3}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

$\mathbb{Z}_4^R$: discriminate between

matter with $\mathbb{Z}_4^R$ charge 1

and

Higgs/exotics with $\mathbb{Z}_4^R$ charge 0 or 2

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
5	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-2/3}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

☞ Many other good features:

- no fractionally charged exotics (i.e. all SM fields come from $\mathrm{SU}(5)$ representations)
- non–trivial full–rank Yukawa couplings
- gauge–top unification
- SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

$\mathbb{Z}_4^R$ summarized

Yukawa couplings ✓

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &+ Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &+ \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &+ \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &+ Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &+ \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &+ \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\; & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
& + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
& + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
& + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}
$$

$\mathcal{O}\left(\frac{m_{3/2}}{M_{\mathrm{P}}^2}\right)$

- R parity violating couplings forbidden
- μ term of the right size

order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

Planck units

- proton decay under control

$\mathbb{Z}_4^R$ summarized

Yukawa couplings ✓

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &+ Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &+ \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &+ \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\; & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
& + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
& + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
& + \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}
$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &\quad + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &\quad + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &\quad + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

$\mathcal{O}\left(\frac{m_{3/2}}{M_\mathrm{P}^2}\right)$

- R parity violating couplings forbidden
- μ term of the right size

  order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

  Planck units
- proton decay under control

$\mathbb{Z}_4^R$ summarized

Yukawa couplings ✓

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &\quad + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &\quad + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &\quad + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator ✓

☞ allowed superpotential terms have R charge $2 \mod 4$

$\mathbb{Z}_4^R$ summarized

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ &\quad + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ &\quad + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ &\quad + \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

☞ $\mathbb{Z}_4^R$ has an unbroken $\mathbb{Z}_2$ matter parity subgoup

$\mathbb{Z}_4^R$ summarized

$\mathcal{O}(m_{3/2})$

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \; = \;& \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
&+ Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
&+ \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
&+ \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}
$$

$\mathcal{O}\left(\frac{m_{3/2}}{M_{\mathrm{P}}^2}\right)$

- R parity violating couplings forbidden
- μ term of the right size

order parameter of R symmetry breaking $= \langle \mathscr{W} \rangle \simeq m_{3/2}$

Planck units

- proton decay under control

Expectations

and

Tests

Soft masses

- ☞ General expressions for soft masses

Soni & Weldon [1983]

$$\begin{aligned} M_a &= \frac{1}{2}(\mathrm{Re}\, f_a)^{-1} F^m \partial_m f_a \\ m_\alpha^2 &= m_{3/2}^2 - \overline{F}^{\overline{m}} F^n \partial_{\overline{m}} \partial_n \ln K_\alpha \\ A_{\alpha\beta\gamma} &= F^m \Big[\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(K_\alpha K_\beta K_\gamma) \Big] \end{aligned}$$

Soft masses

☞ General expressions for soft masses

Soni & Weldon [1983]

gaugino masses

gauge–kinetic function

$$
\begin{aligned}
M_a &= \frac{1}{2}(\mathrm{Re}\, f_a)^{-1} F^m \partial_m f_a \\
m_\alpha^2 &= m_{3/2}^2 - \overline{F}^{\overline{m}} F^n \partial_{\overline{m}} \partial_n \ln K_\alpha \\
A_{\alpha\beta\gamma} &= F^m \Big[\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(K_\alpha K_\beta K_\gamma)\Big]
\end{aligned}
$$

F–terms

“normalization”

Soft masses

☞ General expressions for soft masses

Soni & Weldon [1983]

$$\begin{aligned} M_a &= \frac{1}{2}(\mathrm{Re}\, f_a)^{-1} F^m \partial_m f_a \\ m_\alpha^2 &= m_{3/2}^2 - \overline{F}^{\overline{m}} F^n \partial_{\overline{m}} \partial_n \ln K_\alpha \\ A_{\alpha\beta\gamma} &= F^m \Big[\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(K_\alpha K_\beta K_\gamma) \Big] \end{aligned}$$

Dynamical supersymmetry breaking

☞ Basic idea: explain a hierarchically small scale of supersymmetry breakdown

Witten [1981]

Dynamical supersymmetry breaking

- ☞ Basic idea: explain a hierarchically small scale of supersymmetry breakdown
  Witten [1981]
- ☞ However, if one demands that there be no supersymmetric ground state, the models become very "special"
  e.g. Shadmi & Shirman [2000]

Dynamical supersymmetry breaking

- Basic idea: explain a hierarchically small scale of supersymmetry breakdown
  Witten [1981]
- However, if one demands that there be no supersymmetric ground state, the models become very "special"
  e.g. Shadmi & Shirman [2000]
- However, if you "only" demand metastable SUSY breaking vacua, very simple models do it. E.g. $\mathrm{SU}(N_c)$ with N_f superfields transforming as $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$
  Intriligator, Seiberg & Shih [2006]

Dynamical supersymmetry breaking

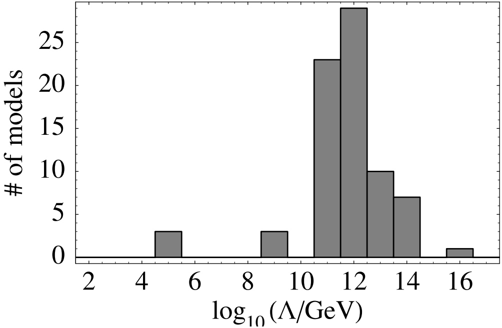
- Basic idea: explain a hierarchically small scale of supersymmetry breakdown
  Witten [1981]
- However, if one demands that there be no supersymmetric ground state, the models become very "special"
  e.g. Shadmi & Shirman [2000]
- However, if you "only" demand metastable SUSY breaking vacua, very simple models do it. E.g. $\mathrm{SU}(N_c)$ with N_f superfields transforming as $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$
  Intriligator, Seiberg & Shih [2006]
- E.g. $\mathrm{SU}(N_c)$ gauge symmetry with N_f chiral superfields transforming as $\boldsymbol{N_c}$–plets and $\overline{\boldsymbol{N_c}}$–plets, where $N_c + 1 < N_f < \frac{3}{2}$

Dynamical supersymmetry breaking

- Basic idea: explain a hierarchically small scale of supersymmetry breakdown
  Witten [1981]
- However, if one demands that there be no supersymmetric ground state, the models become very "special"
  e.g. Shadmi & Shirman [2000]
- However, if you "only" demand metastable SUSY breaking vacua, very simple models do it. E.g. $\mathrm{SU}(N_c)$ with N_f superfields transforming as $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$
  Intriligator, Seiberg & Shih [2006]
- E.g. $\mathrm{SU}(N_c)$ gauge symmetry with N_f chiral superfields transforming as $\boldsymbol{N_c}$–plets and $\overline{\boldsymbol{N_c}}$–plets, where $N_c + 1 < N_f < \frac{3}{2}$
- Interestingly almost every explicit MSSM model derived from heterotic orbifolds does have an appropriate hidden sector

Top–down motivation for low–energy SUSY

☞ Distribution of the (naive) scale of gaugino condensation

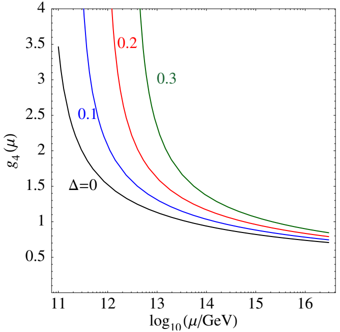


☞ Recall: relation between Λ and $m_{3/2}$ Nilles [1982]

$$m_{3/2} \sim \frac{\Lambda^3}{M_\mathrm{P}^2}$$

Top–down motivation for low–energy SUSY

☞ Distribution of the (naive) scale of gaugino condensation



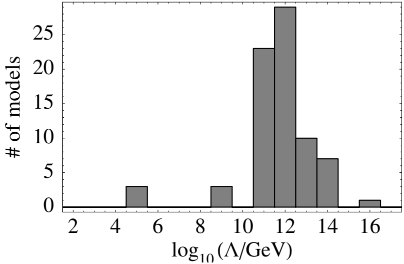
☞ Hidden sector usually **stronger** coupled

Ibáñez & Nilles [1986] ;Dixon, Kaplunovsky & Louis [1991] ;Mayr & Stieberger [1993]

$$g_{\mathrm{vis/hid}}^{-2} = \underbrace{\mathrm{Re}\,S}_{\text{dilaton}} \pm \varepsilon\, \underbrace{\mathrm{Re}\,T}_{\text{Kähler modulus}} + \dots =: \mathrm{Re}\,S \pm \Delta$$

Top–down motivation for low–energy SUSY

☞ Distribution of the (naive) scale of gaugino condensation



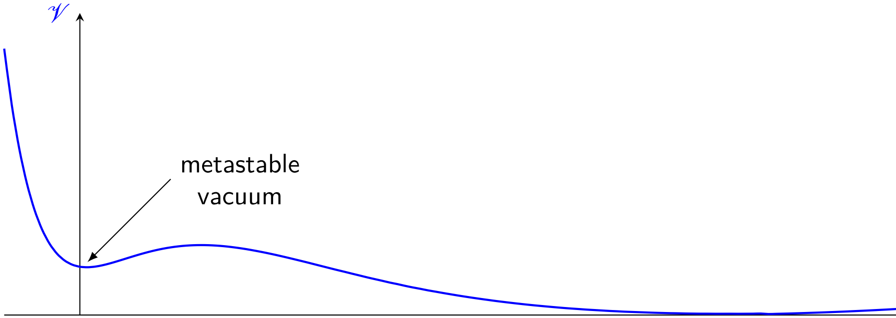
☞ Hidden sector usually **stronger** coupled

Result:

intermediate scale supersymmetry breaking is statistically favored

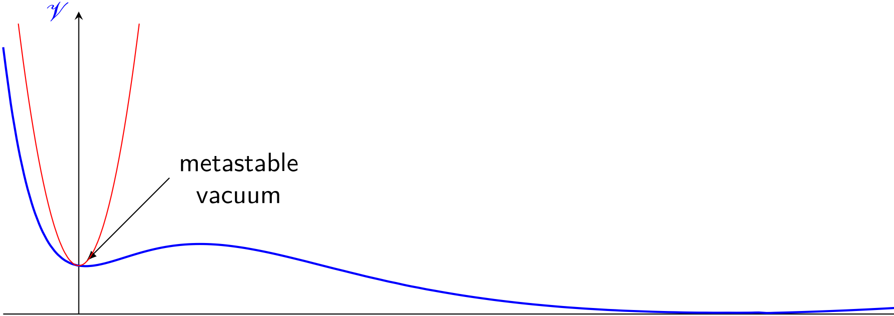
Metastable SUSY breaking

Intriligator, Seiberg & Shih [2006]



Metastable SUSY breaking

Intriligator, Seiberg & Shih [2006]



☞ Early universe settles in metastable vacuum

e.g. Abel, Durnford, Jaeckel & Khoze [2008]

(Non)perturbative supersymmetry breaking

☞ if supersymmetry is unbroken at tree level, it remains unbroken to all orders

(Non)perturbative supersymmetry breaking

- ☞ if supersymmetry is unbroken at tree level, it remains unbroken to all orders
- ☞ scenarios in which supersymmetry gets broken at the nonperturbative level exist

(Non)perturbative supersymmetry breaking

- if supersymmetry is unbroken at tree level, it remains unbroken to all orders
- scenarios in which supersymmetry gets broken at the nonperturbative level exist
- simple models with metastable supersymmetry breaking vacua

Mirage mediation

☞ Hidden sector + one modulus stabilized nonperturbatively

Lebedev, Nilles & M.R. [2006]

$$\begin{aligned} M_a &= (0 \text{ or } 1) \times \frac{F^T}{T_0 + \overline{T}_0} + \text{anomaly} \\ m_\alpha^2 &= m_{3/2}^2 + n_\alpha \frac{|F^T|^2}{(T_0 + \overline{T}_0)^2} - \xi_\alpha |F^C|^2 + \text{anomaly} \\ A_{\alpha\beta\gamma} &= -\frac{F^T}{T_0 + \overline{T}_0} \left[3 + n_\alpha + n_\beta + n_\gamma\right] + \text{anomaly} \end{aligned}$$

Mirage mediation

$\partial f/\partial T$

- Hidden sector + one modulus stabilized nonperturbatively

Lebedev, Nilles & M.R. [2006]

$$
\begin{aligned}
M_a &= (0 \text{ or } 1) \times \frac{F^T}{T_0 + \overline{T}_0} + \text{anomaly} \\
m_\alpha^2 &= m_{3/2}^2 + n_\alpha \frac{|F^T|^2}{(T_0 + \overline{T}_0)^2} - \xi_\alpha |F^C|^2 + \text{anomaly} \\
A_{\alpha\beta\gamma} &= -\frac{F^T}{T_0 + \overline{T}_0} \left[3 + n_\alpha + n_\beta + n_\gamma\right] + \text{anomaly}
\end{aligned}
$$

Implications for the LHC

- Al(most al)l moduli fixed in a supersymmetric way in MSSM vacua with residual (discrete and/or approximate) R symmetries

  Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]

- Approximate R symmetries can explain an effective small constant in the superpotential

  Kappl, Nilles, Ramos-Sánchez, M.R., Schmidt-Hoberg & Vaudrevange [2009]

- Approximate/discrete R symmetries provide us with a solution to the μ problem

  Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010] ;
  Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; ...

- Approximate/discrete R symmetries provide us with a solution to the proton decay problems of the MSSM

  Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; ...

Implications for the LHC

- Al(most al)l moduli fixed in a supersymmetric way in MSSM vacua with residual (discrete and/or approximate) R symmetries
- ➥ Scenario with ~~SUSY~~ by 'matter field' X + dilaton S

stabilized with large mass from Coleman-Weinberg potential

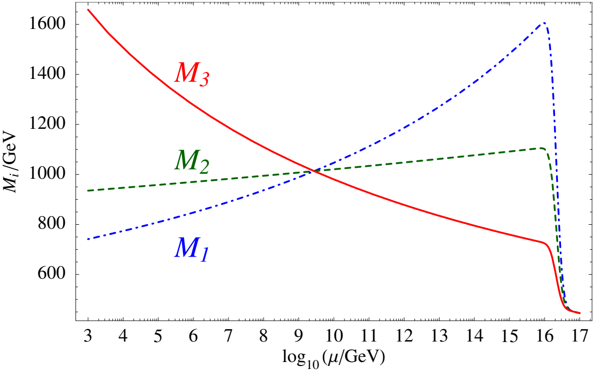
$m_S \gg m_{3/2} \sim 10\dots100\,\mathrm{TeV}$

Implications for the LHC

☞ Al(most al)l moduli fixed in a supersymmetric way in MSSM vacua with residual (discrete and/or approximate) R symmetries

➥ Scenario with ~~SUSY~~ by 'matter field' X + dilaton S

➥ Mirage pattern for gaugino masses + heavy sfermions



Implications for the LHC

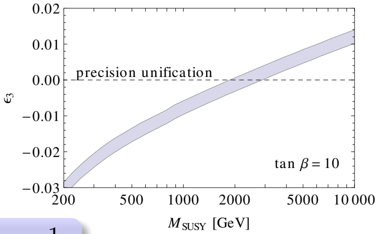
- ☞ Al(most al)l moduli fixed in a supersymmetric way in MSSM vacua with residual (discrete and/or approximate) R symmetries
- ➡ Scenario with ~~SUSY~~ by 'matter field' X + dilaton S
- ➡ Mirage pattern for gaugino masses + heavy sfermions
- ➡ Yields natural scenario for precision gauge unification (PGU)

Carena, Clavelli, Matalliotakis, Nilles & Wagner [1993] . . . Raby, M.R. & Schmidt-Hoberg [2010]
Krippendorf, Nilles, M.R. & Winkler [2013]

$$\epsilon_3 = \frac{g_3^2(M_{\mathsf{GUT}}) - g_{1,2}^2(M_{\mathsf{GUT}})}{g_{1,2}^2(M_{\mathsf{GUT}})}$$

$$M_{\mathsf{SUSY}} = \frac{m_{\widetilde{W}}^{32/19}\, m_{\widetilde{h}}^{12/19}\, m_H^{3/19}}{m_{\widetilde{g}}^{28/19}}\, X_{\mathsf{sfermion}}$$

$X_{\mathsf{sfermion}} \sim 1$



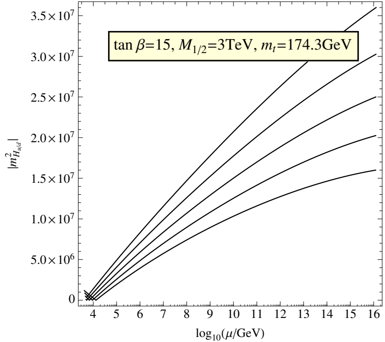
Focus point

☞ Geometric properties of ingredients of top–Yukawa coupling entail 'focus point' Krippendorf, Nilles, M.R. & Winkler [2012]

Focus point

- Geometric properties of ingredients of top–Yukawa coupling entail 'focus point'

Krippendorf, Nilles, M.R. & Winkler [2012]



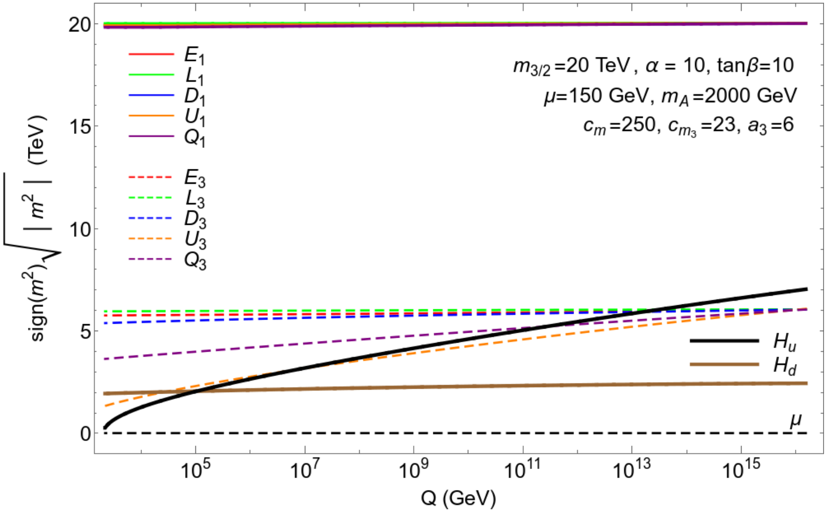
- H_u, Q_L & t_R bulk fields
- ➥ Coinciding boundary conditions at high scale
- ➥ 'Focus point'

Feng, Matchev & Moroi [2000]

Results from a more recent analysis

☞ Running of the soft masses

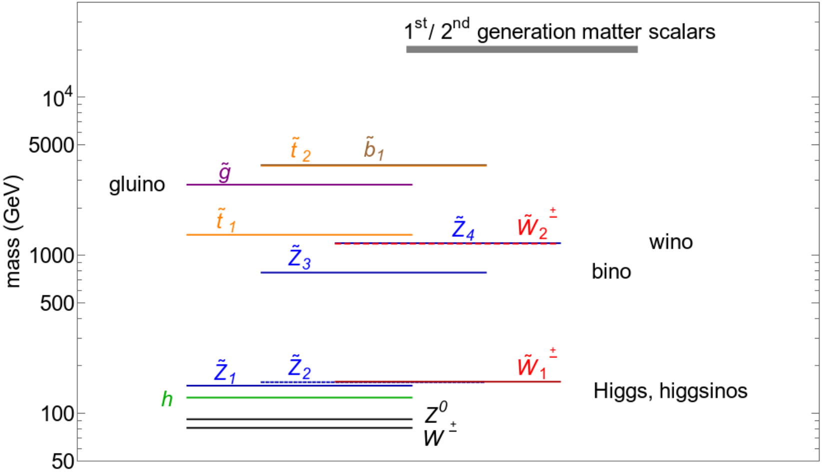
Baer, Barger, Savoy, Serce & Tata [2017]



Results from a more recent analysis

☞ Sample spectrum

Baer, Barger, Savoy, Serce & Tata [2017]



Results from a more recent analysis

Baer, Barger, Savoy, Serce & Tata [2017]

- Sample spectrum
- Amazingly low fine–tunig: $\Delta_{\mathrm{EW}} < 20$ possible

Results from a more recent analysis

Baer, Barger, Savoy, Serce & Tata [2017]

- Sample spectrum
- Amazingly low fine–tunig: $\Delta_{\mathrm{EW}} < 20$ possible
- Perhaps hard to verify at the LHC

$\mathcal{CP}$ violation

from

strings

First 3 family models from stringy orbifolds

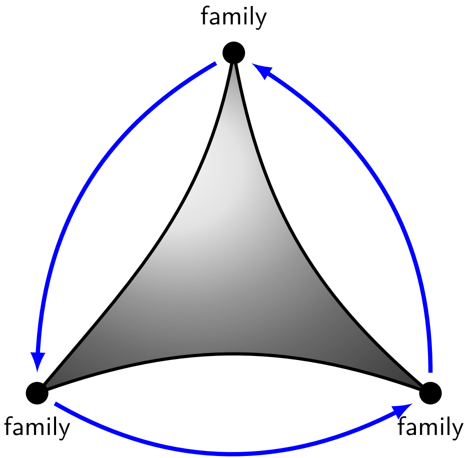
Ibáñez, Kim, Nilles, & Quevedo [1987a]

☞ Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold

First 3 family models from stringy orbifolds

Ibáñez, Kim, Nilles, & Quevedo [1987a]

- Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold
  - three generations may live on equivalent fixed points
  - permutation symmetry of fixed points/families

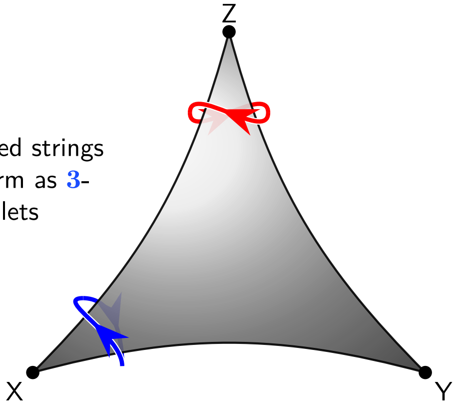


First 3 family models from stringy orbifolds

Ibáñez, Kim, Nilles, & Quevedo [1987a]

- Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold
  - three generations may live on equivalent fixed points
  - permutation symmetry of fixed points/families
  - ➥ flavor/family symmetry

localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets



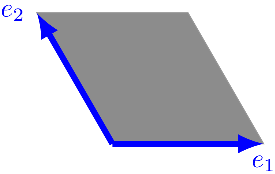
$\Delta(54)$

from a

$\mathbb{Z}_3$ orbifold plane

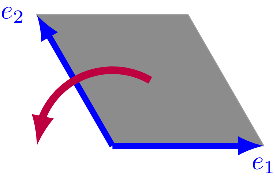
$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



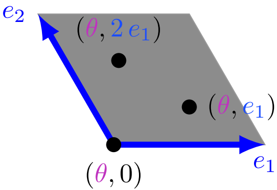
$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



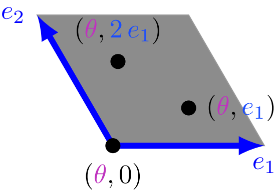
$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



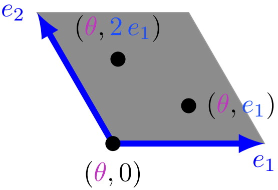
Hamidi & Vafa [1987]
Dixon, Friedan, Martinec & Shenker [1987]

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n \;=\; 3\times(\text{integer}) \quad \wedge \quad \sum_{j=1}^{n} m_1^{(j)} \;=\; 0 \mod 3$$

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



$$\begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \to \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} c|(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

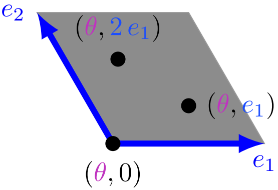
Hamidi & Vafa [1987]
Dixon, Friedan, Martinec & Shenker [1987]

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n = 3\times(\text{integer}) \quad \wedge \quad \sum_{j=1}^{n} m_1^{(j)} = 0 \mod 3$$

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



$$\begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} c|(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

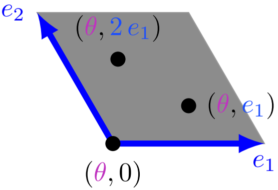
Hamidi & Vafa [1987]
Dixon, Friedan, Martinec & Shenker [1987]

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n = 3 \times (\text{integer}) \quad \wedge \quad \sum_{j=1}^{n} m_1^{(j)} = 0 \mod 3$$

$\mathbb{T}^2/\mathbb{Z}_3$ orbifold

Kobayashi, Nilles, Plöger, Raby & M.R. [2007]



Hamidi & Vafa [1987]
Dixon, Friedan, Martinec & Shenker [1987]

☞ coupling between n localized states $|(\theta, m^{(j)}\,e_1)\rangle$ only allowed if

$$n \;=\; 3\times(\text{integer}) \quad\wedge\quad \sum_{j=1}^{n} m_1^{(j)} \;=\; 0 \mod 3$$

➥ flavor symmetry

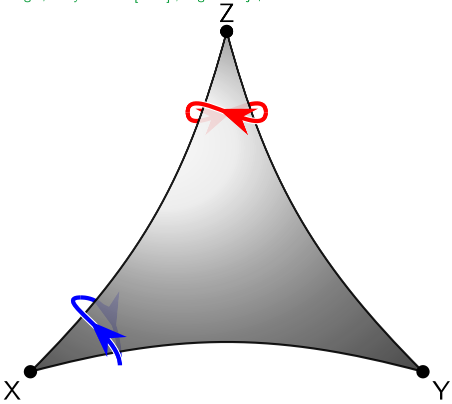
$$S_3 \cup (\mathbb{Z}_3\times\mathbb{Z}_3) \;=\; S_3 \ltimes (\mathbb{Z}_3\times\mathbb{Z}_3) \;=\; \Delta(54)$$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

☞ $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry

Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]

localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets



$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]
- explicit model Carballo-Perez, Peinado & Ramos-Sánchez [2016]

#	irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\overline{u}_i$
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\overline{d}_i$
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\overline{e}_i$
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\overline{\nu}_i$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry

  Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]

- explicit model

  Carballo-Perez, Peinado & Ramos-Sánchez [2016]

- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry
  Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]
- explicit model
  Carballo-Perez, Peinado & Ramos-Sánchez [2016]
- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$
- $\Delta(54)$ is type I group: $\curvearrowright$ $\mathcal{CP}$ violation for free?

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry

  Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]

- explicit model

  Carballo-Perez, Peinado & Ramos-Sánchez [2016]

- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$
- $\Delta(54)$ is type I group: $\curvearrowright \mathcal{CP}$ violation for free?
- not that simple! if the representation content is very special, one *can* impose a $\mathcal{CP}$ transformation

$$\exists\ \text{out} \ : \ \mathbf{3}_i \xleftrightarrow{\text{out}} \overline{\mathbf{3}}_i \quad \text{and} \quad \mathbf{1}_i \xleftrightarrow{\text{out}} \overline{\mathbf{1}}_i$$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry

  Kobayashi, Nilles, Plöger, Raby & M.R. [2007] ; Olguin-Trejo, Pérez-Martínez & Ramos-Sánchez [2018]

- explicit model

  Carballo-Perez, Peinado & Ramos-Sánchez [2016]

- quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$
- $\Delta(54)$ is type I group: $\curvearrowright$ $\mathcal{CP}$ violation for free?
- not that simple! if the representation content is very special, one *can* impose a $\mathcal{CP}$ transformation
- at the massless level, only 3- and 1–dimensional representations occur $\curvearrowright$ a class–inverting outer automorphism exists $\curvearrowright$ a $\mathcal{CP}$ candidate exists

$\mathcal{CP}$ violation

in the

$\mathbb{Z}_3$ orbifold

$\mathcal{CP}$ violation from strings

☞ however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

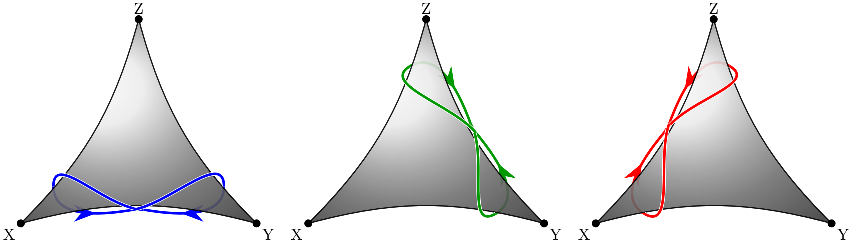
Nilles, M.R., Trautner & Vaudrevange [2018]

$\mathcal{CP}$ violation from strings

- however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

  Nilles, M.R., Trautner & Vaudrevange [2018]

- doublets $\mathbf{2}_1$, $\mathbf{2}_3$ and $\mathbf{2}_4$ correspond to linear combinations of strings that wind around two different fixed points in opposite directions

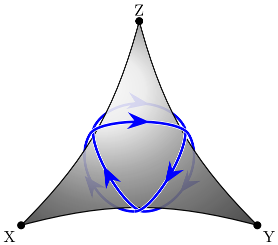


$\mathcal{CP}$ violation from strings

- however, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

Nilles, M.R., Trautner & Vaudrevange [2018]

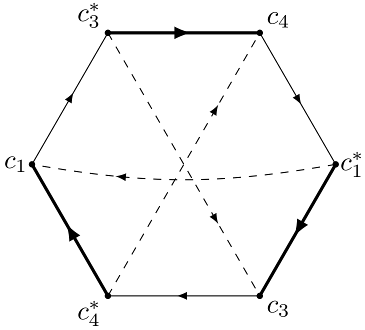
- doublets $\mathbf{2}_1$, $\mathbf{2}_3$ and $\mathbf{2}_4$ correspond to linear combinations of strings that wind around two different fixed points in opposite directions
- doublet $\mathbf{2}_2$



$\mathcal{CP}$ violation from strings

☞ doublets save the day

Nilles, M.R., Trautner & Vaudrevange [2018]

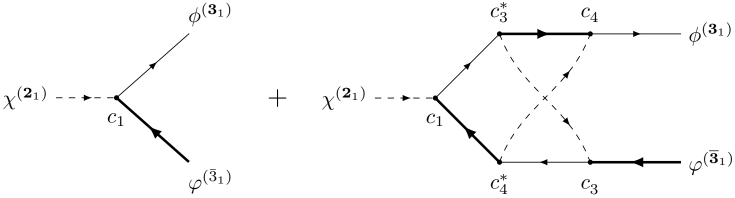


- we follow invariant approach — Bernabéu, Branco & Gronau [1986]
- super powerful tool: `Susyno` — Fonseca [2012]

$\mathcal{CP}$ violation from strings

- doublets save the day
- physical $\not{\mathcal{CP}}$ in doublet decay

Nilles, M.R., Trautner & Vaudrevange [2018]



$\mathcal{CP}$ violation from strings

- doublets save the day

Nilles, M.R., Trautner & Vaudrevange [2018]

- physical $\cancel{\mathcal{CP}}$ in doublet decay
- phenomenological implications not worked out

$\mathcal{CP}$ violation from strings

- doublets save the day

Nilles, M.R., Trautner & Vaudrevange [2018]

- physical $\not{\mathcal{CP}}$ in doublet decay
- phenomenological implications not worked out

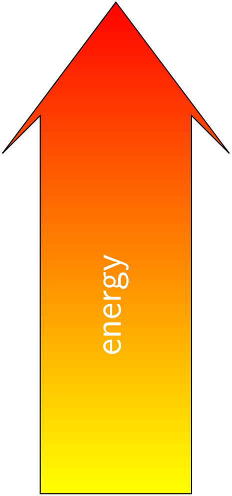
bottom–line:

$\mathcal{CP}$ violation can come from group theory in UV complete settings in which the origin of the flavor group is fully understood

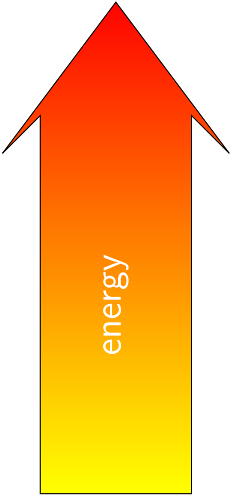
String

cosmology

Overview



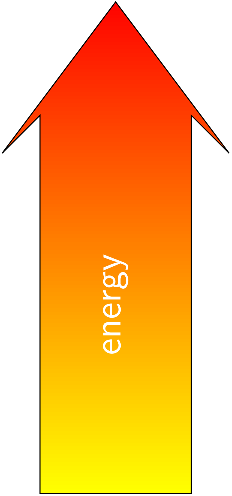
Overview



1 MeV

Nucleosynthesis

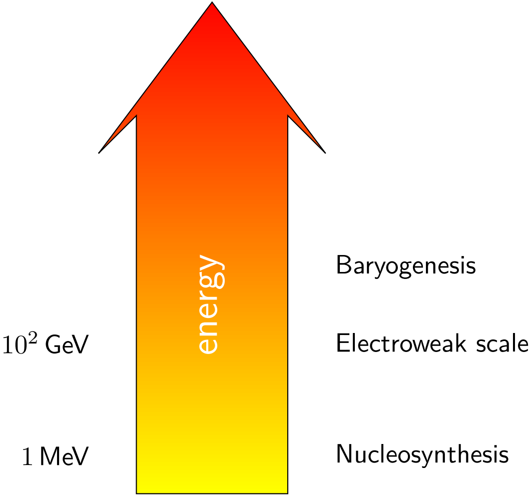
Overview



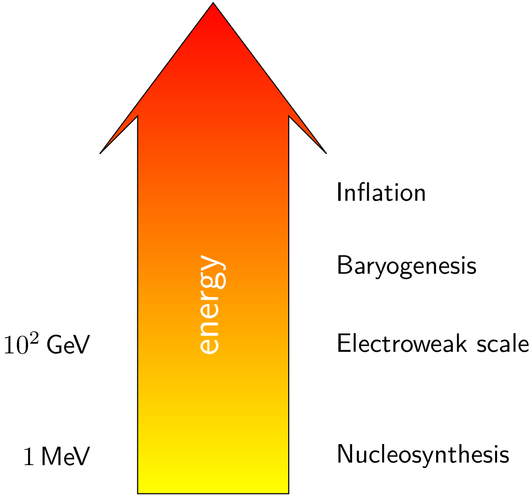
10^2 GeV — Electroweak scale

1 MeV — Nucleosynthesis

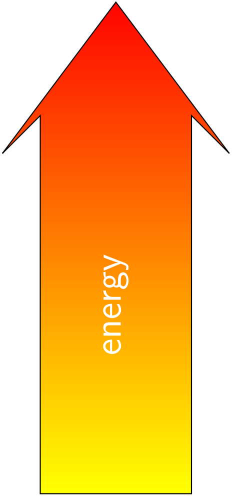
Overview


energy
Baryogenesis
10^2 GeV
Electroweak scale
1 MeV
Nucleosynthesis

Overview

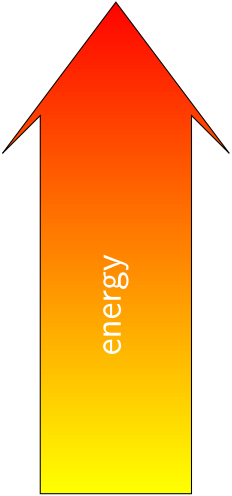

energy
Inflation
Baryogenesis
10^2 GeV
Electroweak scale
1 MeV
Nucleosynthesis

Overview



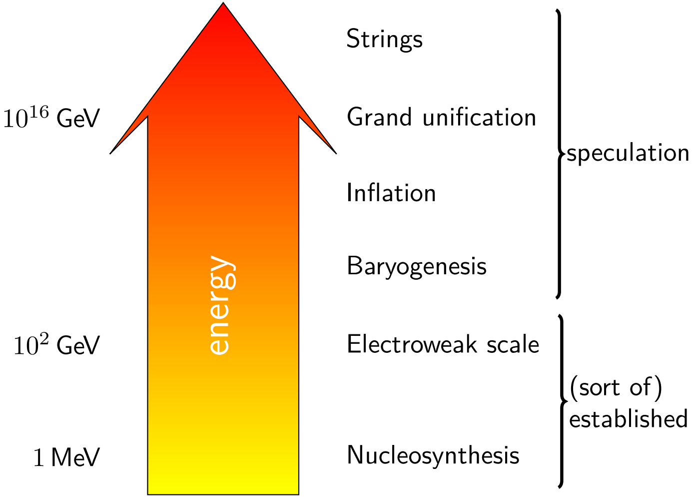
	Strings
10^{16} GeV	Grand unification
	Inflation
	Baryogenesis
10^2 GeV	Electroweak scale
1 MeV	Nucleosynthesis

Overview



Energy		
	Strings	
10^{16} GeV	Grand unification	
	Inflation	
	Baryogenesis	
10^{2} GeV	Electroweak scale	(sort of) established
1 MeV	Nucleosynthesis	(sort of) established

Overview


energy
10^{16} GeV
10^{2} GeV
1 MeV
Strings
Grand unification
Inflation
Baryogenesis
Electroweak scale
Nucleosynthesis
speculation
(sort of)
established

Moduli

☞ At tree–level string compactifications have many flat directions
. . . after all, the MSSM has several D–flat directions

Moduli

- At tree–level string compactifications have many flat directions
- In all known constructions there are some scalars that have masses of the order of the gravitino mass $m_{3/2}$

Moduli

- At tree–level string compactifications have many flat directions
- In all known constructions there are some scalars that have masses of the order of the gravitino mass $m_{3/2}$
- Gauge and Yukawa couplings depend on these moduli

$$g = g(X) \quad \text{and} \quad y = y(X)$$

gauge coupling

Yukawa coupling

Moduli

- At tree–level string compactifications have many flat directions
- In all known constructions there are some scalars that have masses of the order of the gravitino mass $m_{3/2}$
- Gauge and Yukawa couplings depend on these moduli

$$g = g(X) \quad \text{and} \quad y = y(X)$$

modulus

Moduli

- At tree–level string compactifications have many flat directions
- In all known constructions there are some scalars that have masses of the order of the gravitino mass $m_{3/2}$
- Gauge and Yukawa couplings depend on these moduli

$$g = g(X) \quad \text{and} \quad y = y(X)$$

- What can one say about these moduli in the early universe?

Thermal corrections to moduli potentials

- Free energy of a supersymmetric $\mathrm{SU}(N_c)$ gauge theory with N_f $\boldsymbol{N_c}+\overline{\boldsymbol{N_c}}$–plets

e.g. Laine & Vuorinen [2016]

$$\mathcal{F}(g,T) = -\frac{\pi^2 T^4}{24}\left\{\alpha_0+\alpha_2 g^2+\mathcal{O}(g^3)\right\}$$

$$\alpha_2 = -\tfrac{3}{8\pi^2}\,(N_c^2-1)(N_c+3N_f)$$

Thermal corrections to moduli potentials

- Free energy of a supersymmetric $\mathrm{SU}(N_c)$ gauge theory with N_f $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$–plets

e.g. Laine & Vuorinen [2016]

$$\mathcal{F}(g,T) \;=\; -\frac{\pi^2 T^4}{24}\left\{\alpha_0+\alpha_2 g^2+\mathcal{O}(g^3)\right\}$$

- Additional contribution from Yukawa interactions

Lillard, M.R., Tait & Trojanowski [2018]

$$\Delta\mathcal{F} \;=\; \frac{5\,|y|^2}{576}\,T^4$$

Thermal corrections to moduli potentials

☞ Free energy of a supersymmetric $\mathrm{SU}(N_c)$ gauge theory with N_f $\boldsymbol{N_c}+\overline{\boldsymbol{N_c}}$–plets

e.g. Laine & Vuorinen [2016]

$$\mathcal{F}(g,T) \;=\; -\frac{\pi^2 T^4}{24}\left\{\alpha_0+\alpha_2 g^2+\mathcal{O}(g^3)\right\}$$

☞ Additional contribution from Yukawa interactions

Lillard, M.R., Tait & Trojanowski [2018]

$$\Delta\mathcal{F} \;=\; \frac{5\,|y|^2}{576}\,T^4$$

➥ Moduli get pushed to values at which g and y become smaller

Buchmüller, Hamaguchi, Lebedev & M.R. [2004]

Thermal corrections to moduli potentials

- Free energy of a supersymmetric $\mathrm{SU}(N_c)$ gauge theory with N_f $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$–plets

e.g. Laine & Vuorinen [2016]

$$\mathcal{F}(g,T) \;=\; -\frac{\pi^2 T^4}{24}\left\{\alpha_0 + \alpha_2 g^2 + \mathcal{O}(g^3)\right\}$$

- Additional contribution from Yukawa interactions

Lillard, M.R., Tait & Trojanowski [2018]

$$\Delta\mathcal{F} \;=\; \frac{5\,|y|^2}{576}\,T^4$$

➥ Moduli get pushed to values at which g and y become smaller

Buchmüller, Hamaguchi, Lebedev & M.R. [2004]

- Calculable minimal displacement of the moduli values from their zero–temperature minimum of the potential

Thermal corrections to moduli potentials

☞ Free energy of a supersymmetric $\mathrm{SU}(N_c)$ gauge theory with N_f $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$–plets

e.g. Laine & Vuorinen [2016]

$$\mathcal{F}(g,T) \;=\; -\frac{\pi^2 T^4}{24}\left\{\alpha_0 + \alpha_2 g^2 + \mathcal{O}(g^3)\right\}$$

☞ Additional contribution from Yukawa interactions

Lillard, M.R., Tait & Trojanowski [2018]

$$\Delta\mathcal{F} \;=\; \frac{5\,|y|^2}{576}\,T^4$$

➥ Moduli get pushed to values at which g and y become smaller

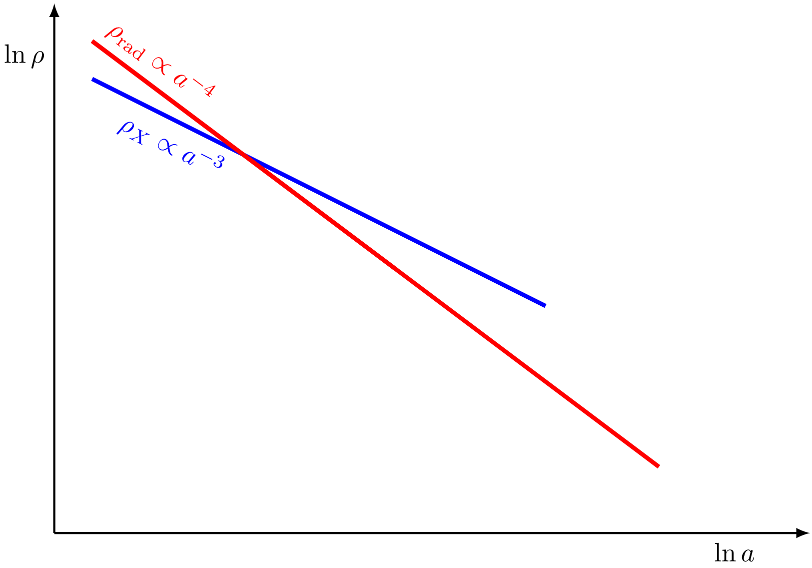
Buchmüller, Hamaguchi, Lebedev & M.R. [2004]

☞ Calculable minimal displacement of the moduli values from their zero–temperature minimum of the potential

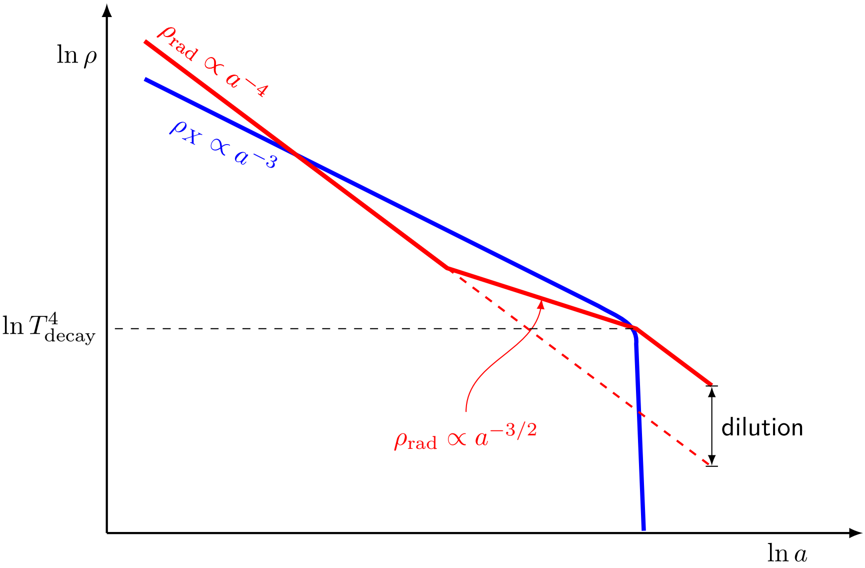
☞ Multidimensional moduli potentials more complex

Kane & Winkler [2019]

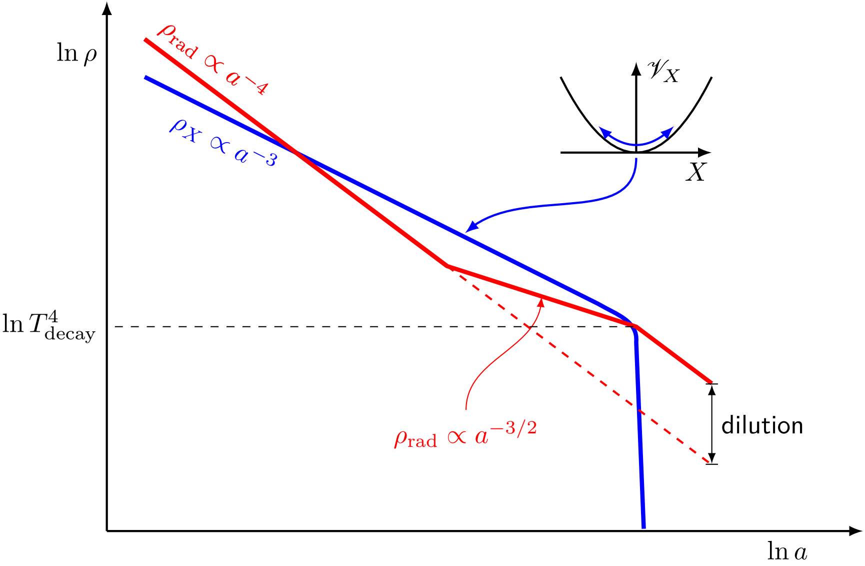
Moduli problems



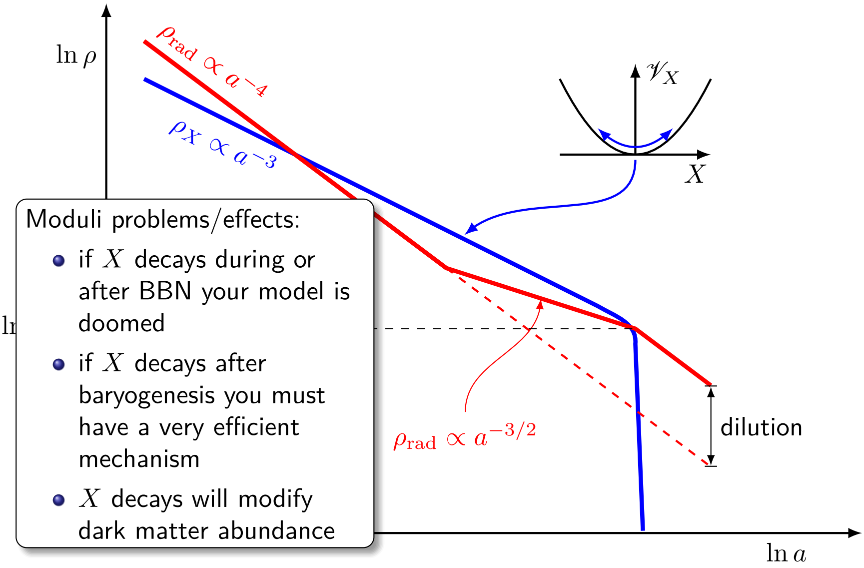
Moduli problems



Moduli problems



Moduli problems



Moduli problems/effects:

- if X decays during or after BBN your model is doomed
- if X decays after baryogenesis you must have a very efficient mechanism
- X decays will modify dark matter abundance

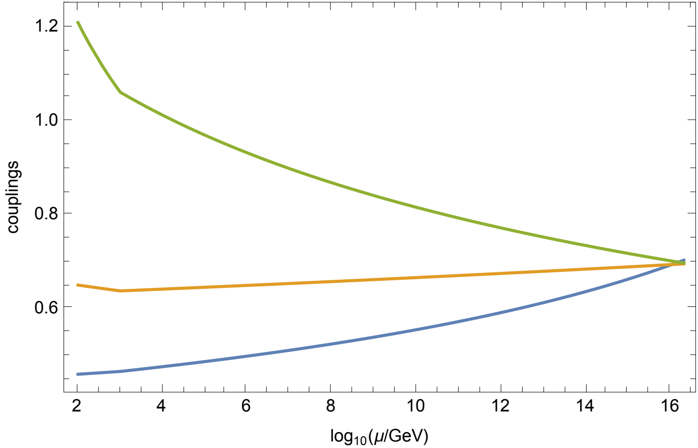
Summary

and

Outlook

Summary — GUTs

☺ GUT symmetry gives rise to gauge coupling unification



Summary — GUTs

- ☺ GUT symmetry gives rise to gauge coupling unification
- ☺ GUTs explain charge quantization

Summary — GUTs

- ☺ GUT symmetry gives rise to gauge coupling unification
- ☺ GUTs explain charge quantization
- ☺ GUTs explain structure of matter

$$\begin{aligned} \mathrm{SO}(10) &\supset \mathrm{SU}(5) \\ \mathbf{16} &= \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1} \\ &= \text{SM generation with 'right–handed' neutrino} \end{aligned}$$

Summary — GUTs

☺ GUT symmetry gives rise to gauge coupling unification

☺ GUTs explain charge quantization

☺ GUTs explain structure of matter

☹ However: doublet–triplet splitting:

$$\mathbf{10} \to \underbrace{(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2}}_{\text{doublets: needed}} \oplus \underbrace{(\mathbf{3},\mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3}}_{\text{triplets: excluded}}$$

Summary — GUTs

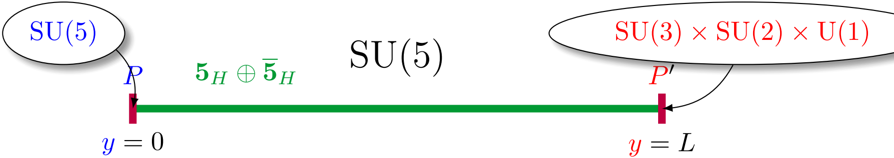
- ☺ GUT symmetry gives rise to gauge coupling unification
- ☺ GUTs explain charge quantization
- ☺ GUTs explain structure of matter
- ☹ However: doublet–triplet splitting:

$$\mathbf{10} \to (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3}$$

- ☹ Natural solutions to the doublet–triplet splitting problem not available in 4D GUTs

Summary — Orbifold GUTs

☺ Orbifold GUTs allow one to solve the doublet–triplet splitting problem



Summary — Orbifold GUTs

- ☺ Orbifold GUTs allow one to solve the doublet–triplet splitting problem
- ☺ Orbifold GUTs allow for the intuitive understanding of the simultaneous existence of complete and split multiplets due to localization

Summary — Orbifold GUTs

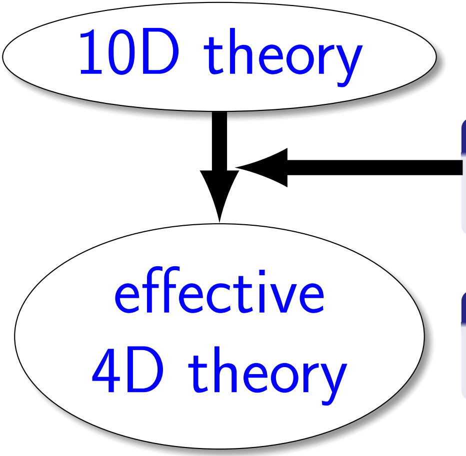
- ☺ Orbifold GUTs allow one to solve the doublet–triplet splitting problem
- ☺ Orbifold GUTs allow for the intuitive understanding of the simultaneous existence of complete and split multiplets due to localization
- ☹ Orbifold GUTs can only be effective theories

Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity

Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions



$$\mathrm{E}_8 \times \mathrm{E}_8$$

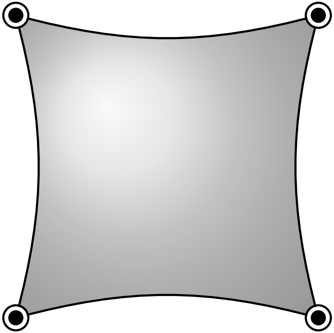
specify:
- geometry
- gauge embedding

get:
- spectrum
- interactions

Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpepetation

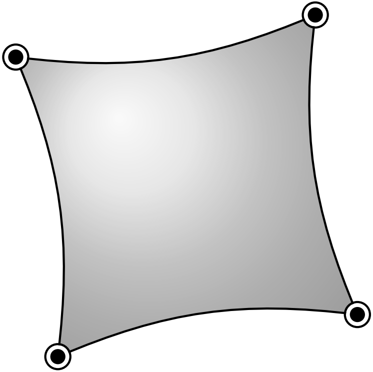
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

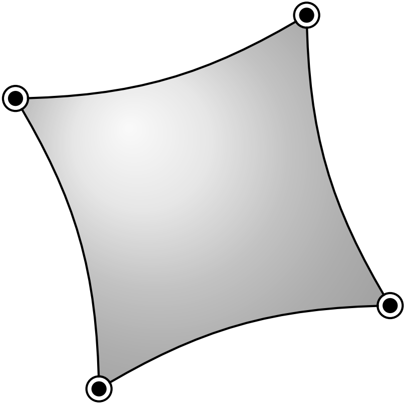
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

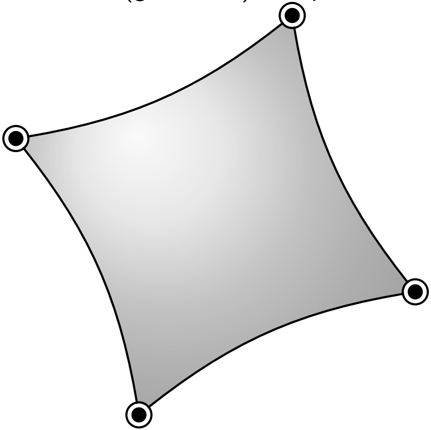
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpepetation

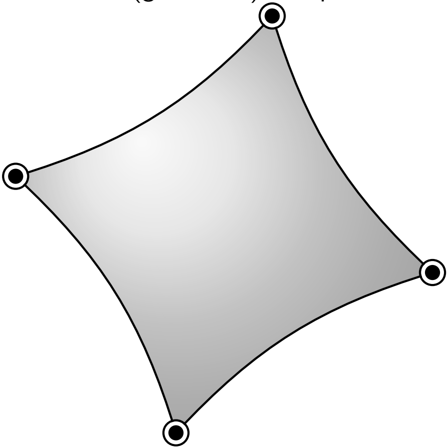
▸ back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

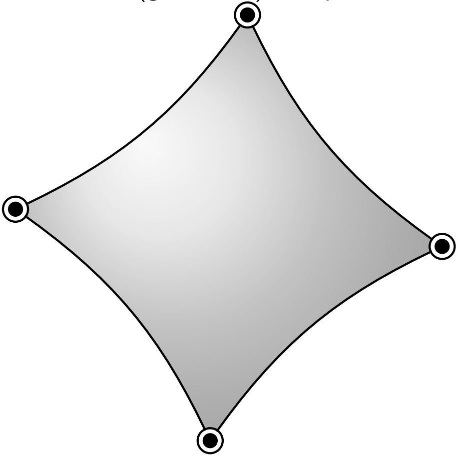
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

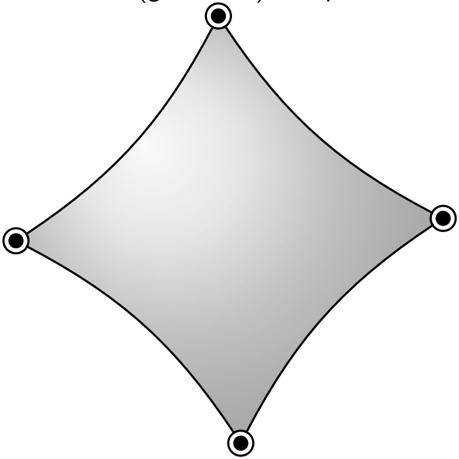
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

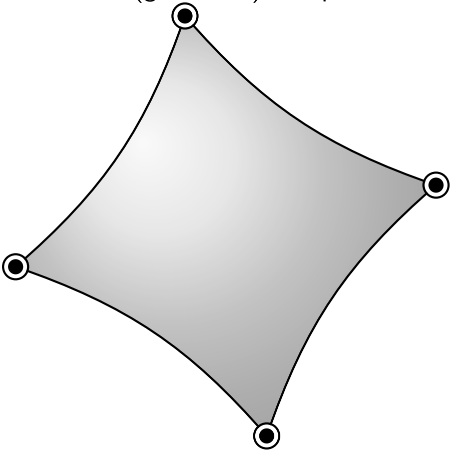
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpepetation

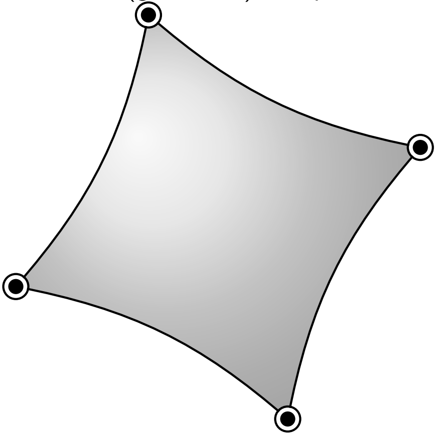
▸ back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

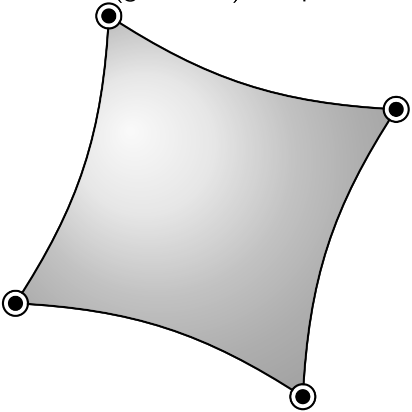
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

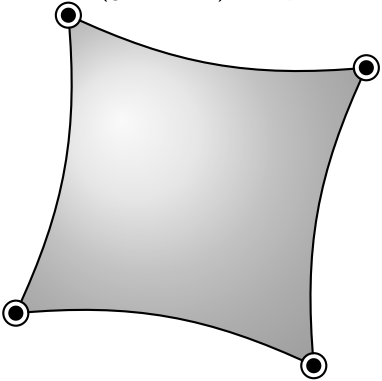
▸ back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

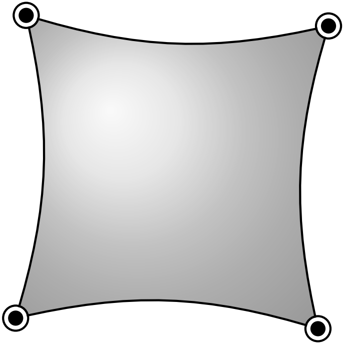
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

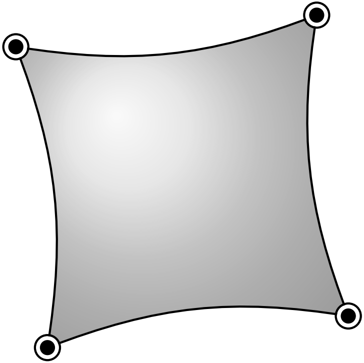
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

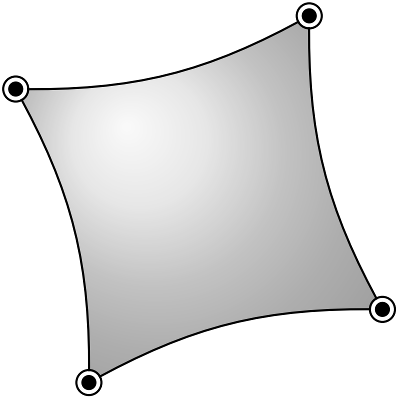
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

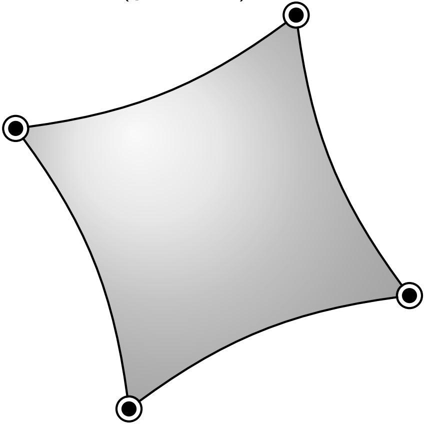
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

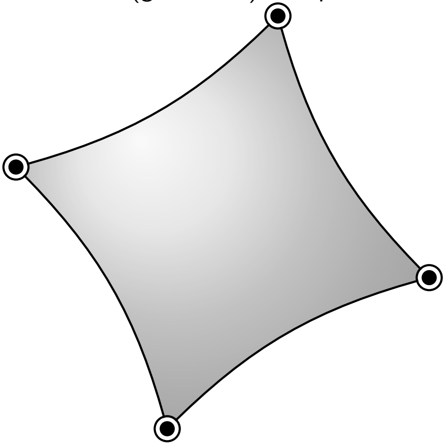
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

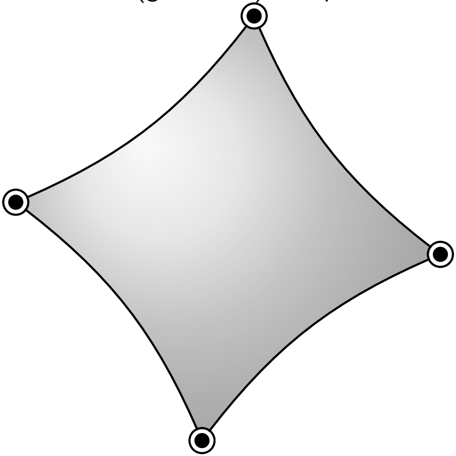
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

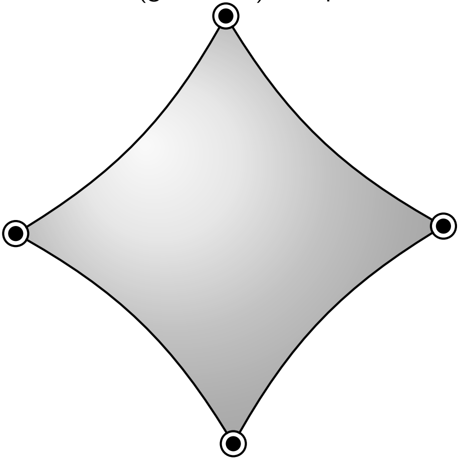
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpepetation

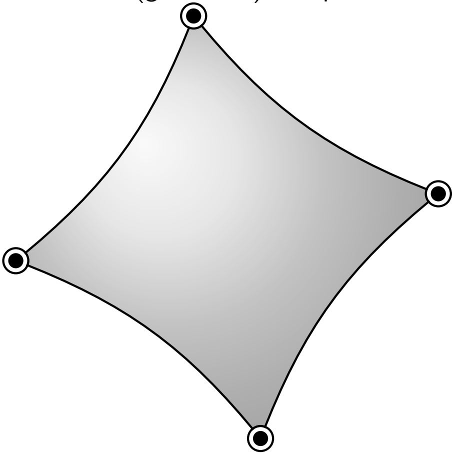
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpepetation

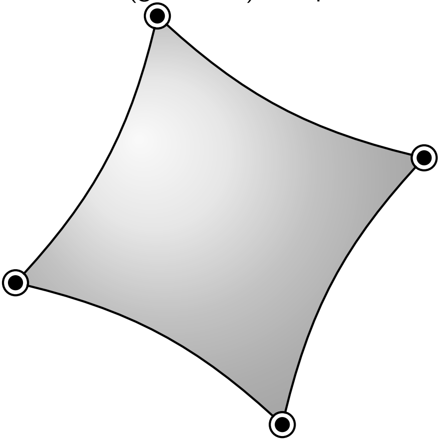
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

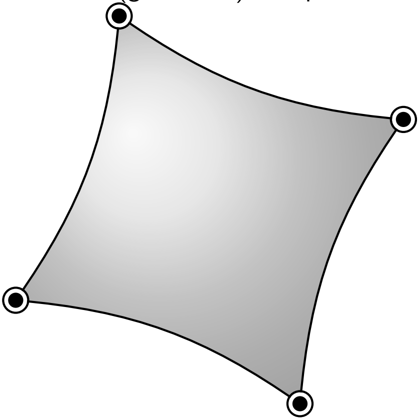
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

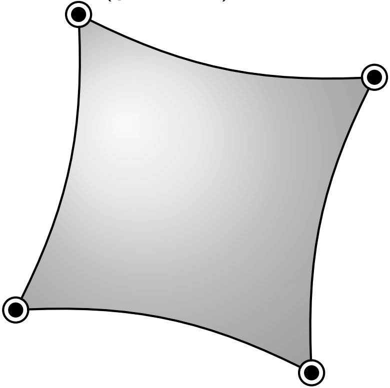
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpepetation

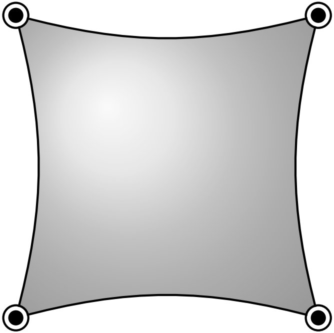
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

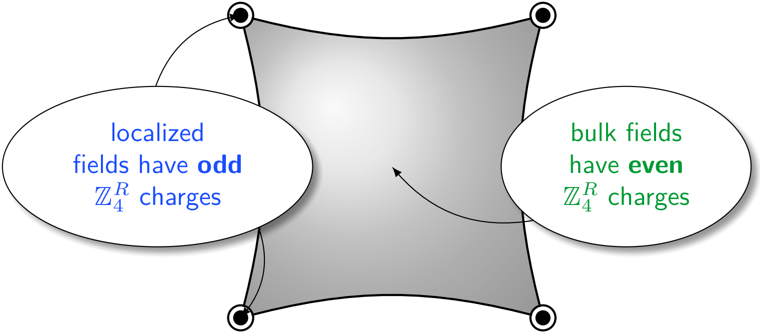
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

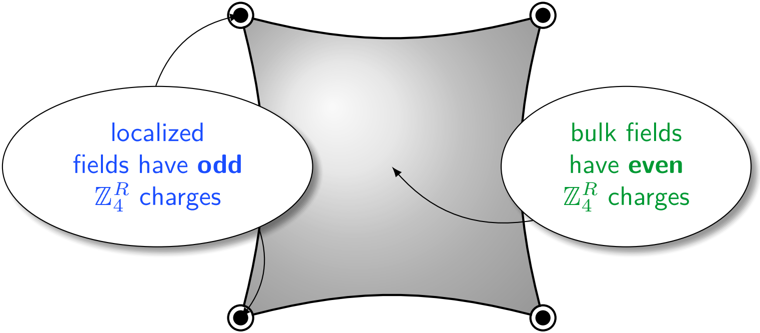
back



Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

back

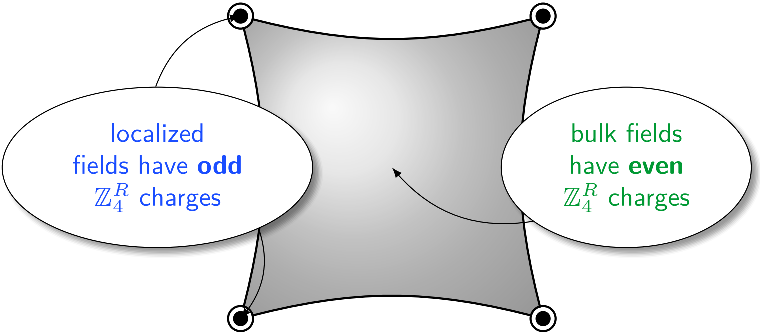


- ☺ Explicit globally consistent models come very close to the MSSM

Summary — Heterotic orbifolds

- ☺ String theory promises a consistent description of quantum gravity
- ☺ Heterotic orbifolds allow one to compute spectrum & interactions
- ☺ Symmetries have a clear (geometric) interpetation

back



- ☺ Explicit globally consistent models come very close to the MSSM
- ☹ No fully realistic model obtained so far

Features

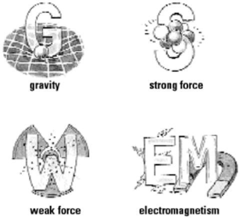
❶ $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$

No exotics

Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007a]

Features

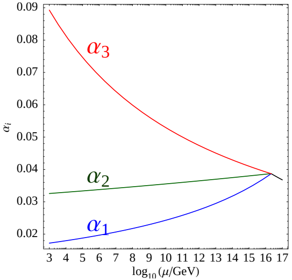
1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\text{hid}}$



Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007a]

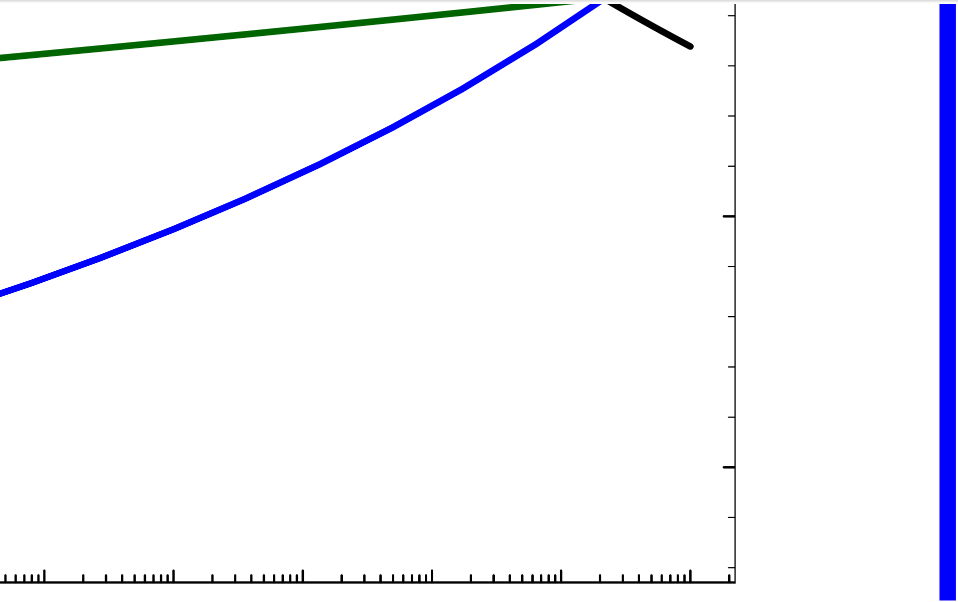
Features

1. $3 \times \mathbf{16}$ + Higgs + nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
   precision gauge unification (PGU)
   from non–local GUT breaking



Raby, M.R. & Schmidt-Hoberg [2010], Krippendorf, Nilles, M.R. & Winkler [2013]

Features



Features

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_\mathrm{hid}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$

$\cancel{\bar{u}\,\bar{d}\,\bar{d}}$ $\cancel{q\,\bar{d}\,\ell}$

$\cancel{\ell\,\ell\,\bar{e}}$ $\cancel{\ell\,H_u}$

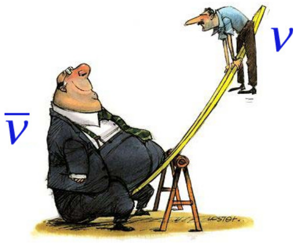
$\curvearrowright$ proton long–lived

$\curvearrowright$ DM stable

Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007b], Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]

Features

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$
5. See–saw

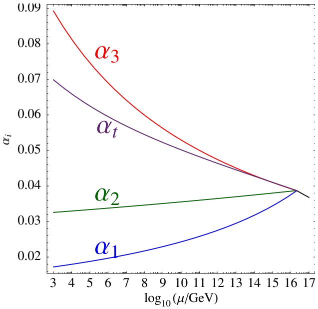


$\curvearrowright$ suppressed ν masses

Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez & M.R. [2007]

Features

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\text{hid}}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$
5. See–saw
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen

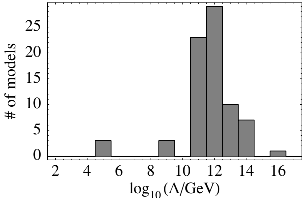


$\curvearrowright$ realistic top mass

Hosteins, Kappl, M.R. & Schmidt-Hoberg [2009]

Features

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$
5. See–saw
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen
7. 'Realistic' hidden sector
   scale of hidden sector strong dynamics is consistent with TeV–scale soft masses



Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange & Wingerter [2007a]

Features & “stringy surprises”

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$
5. See–saw
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen
7. ‘Realistic’ hidden sector
8. Solution to the μ problem

$\mu \sim \langle \mathscr{W} \rangle$

$\langle \mathscr{W} \rangle \ll M_{\mathrm{P}}^3$ from approximate $\mathrm{U}(1)_R$ symmetries

$\curvearrowright$ light Higgs

Kappl, Nilles, Ramos-Sánchez, M.R., Schmidt-Hoberg & Vaudrevange [2009], Brümmer, Kappl, M.R. & Schmidt-Hoberg [2010]

Features & "stringy surprises"

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. Unification
4. R parity & $\mathbb{Z}_4^R$

that's what we searched for. . .

5. See–saw
6. $y_t \simeq g$ @ M_{GUT} & potentially realistic flavor structures à la Froggatt-Nielsen
7. 'Realistic' hidden sector
8. Solution to the μ problem

. . . that's what we got 'for free'

"stringy surprises"

Orbifolds and smooth compactifications

heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ string

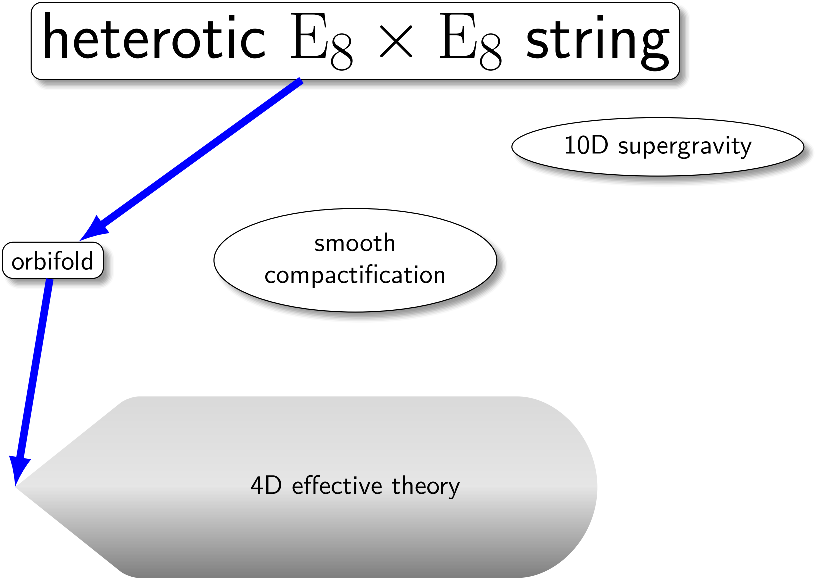
10D supergravity

orbifold

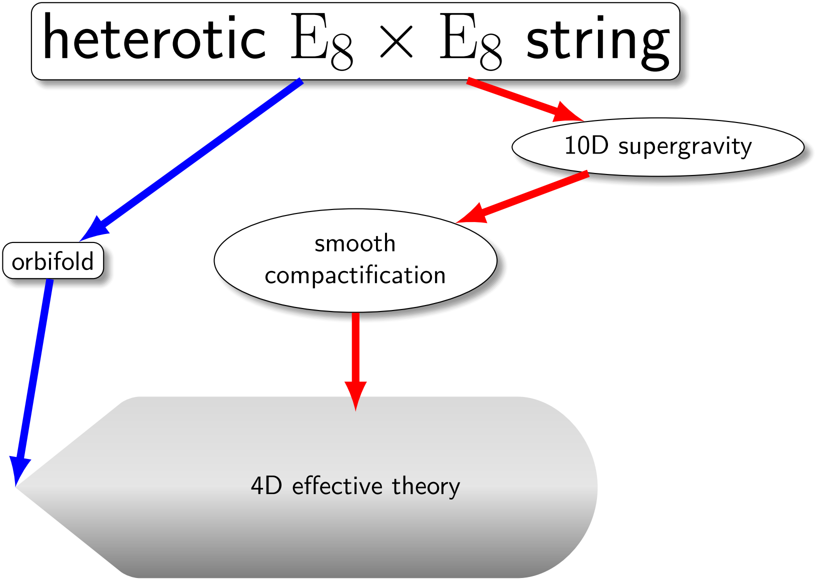
smooth compactification

4D effective theory

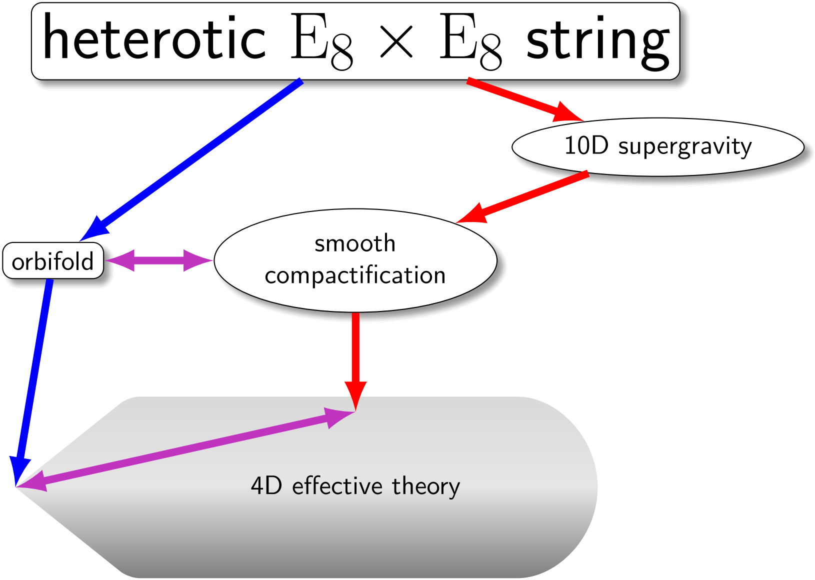
Orbifolds and smooth compactifications



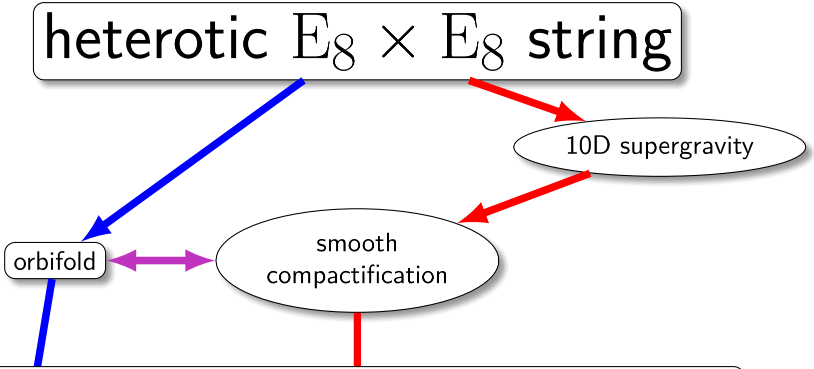
Orbifolds and smooth compactifications



Orbifolds and smooth compactifications



Orbifolds and smooth compactifications



"It is conceivable that one of the lessons of orbifolds will turn out to be that the moduli space of conformally invariant sigma models is "better" than that of the corresponding manifolds, and that the conformal sigma models remain smooth in limits (such as the orbifold limit) in which the corresponding manifolds become singular."

Dixon, Harvey, Vafa & Witten [1986]

Thank you

very much!

Appendix

$G_{\mathrm{SM}} \subset \mathrm{SU}(5)$ (I)

☞ $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ 'fit' into $\mathrm{SU}(5)$

$$\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * \\ * & * \end{pmatrix} \to \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$$

$G_{\mathrm{SM}} \subset \mathrm{SU}(5)$ (I)

☞ $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ 'fit' into $\mathrm{SU}(5)$

$$\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * \\ * & * \end{pmatrix} \rightarrow \begin{pmatrix} \color{red}{*} & \color{red}{*} & \color{red}{*} & * & * \\ \color{red}{*} & \color{red}{*} & \color{red}{*} & * & * \\ \color{red}{*} & \color{red}{*} & \color{red}{*} & * & * \\ * & * & * & \color{red}{*} & \color{red}{*} \\ * & * & * & \color{red}{*} & \color{red}{*} \end{pmatrix}$$

$G_{\mathrm{SM}} \subset \mathrm{SU}(5)$ (I)

- $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ 'fit' into $\mathrm{SU}(5)$

$$\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * \\ * & * \end{pmatrix} \to \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$$

- d–type quarks and lepton doublets can be combined to $\mathrm{SU}(5)$ $\overline{\mathbf{5}}$–plet

$$\overline{\mathbf{5}} = \psi_i = \begin{pmatrix} d^\mathcal{C}_\mathrm{red} \\ d^\mathcal{C}_\mathrm{green} \\ d^\mathcal{C}_\mathrm{blue} \\ \ell^\uparrow \\ \ell^\downarrow \end{pmatrix}$$

Standard model matter in $\mathrm{SU}(5)$ (I)

- quark doublets, u–type quarks and lepton singlets can be combined to $\mathrm{SU}(5)$ $\mathbf{10}$–plet

$$\mathbf{10} \;=\; \chi_{ij} \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u^{\mathcal{C}}_{\mathrm{blue}} & -u^{\mathcal{C}}_{\mathrm{green}} & q^{\uparrow}_{\mathrm{red}} & q^{\downarrow}_{\mathrm{red}} \\ -u^{\mathcal{C}}_{\mathrm{blue}} & 0 & u^{\mathcal{C}}_{\mathrm{red}} & q^{\uparrow}_{\mathrm{green}} & q^{\downarrow}_{\mathrm{green}} \\ u^{\mathcal{C}}_{\mathrm{green}} & -u^{\mathcal{C}}_{\mathrm{red}} & 0 & q^{\uparrow}_{\mathrm{blue}} & q^{\downarrow}_{\mathrm{blue}} \\ -q^{\uparrow}_{\mathrm{red}} & -q^{\uparrow}_{\mathrm{green}} & -q^{\uparrow}_{\mathrm{blue}} & 0 & e^{\mathcal{C}} \\ -q^{\downarrow}_{\mathrm{red}} & -q^{\downarrow}_{\mathrm{green}} & -q^{\downarrow}_{\mathrm{blue}} & -e^{\mathcal{C}} & 0 \end{pmatrix}$$

Standard model matter in $\mathrm{SU}(5)$ (I)

- quark doublets, u–type quarks and lepton singlets can be combined to $\mathrm{SU}(5)$ **10**–plet

$$\mathbf{10} \;=\; \chi_{ij} \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u^{\mathcal{C}}_{\text{blue}} & -u^{\mathcal{C}}_{\text{green}} & q^{\uparrow}_{\text{red}} & q^{\downarrow}_{\text{red}} \\ -u^{\mathcal{C}}_{\text{blue}} & 0 & u^{\mathcal{C}}_{\text{red}} & q^{\uparrow}_{\text{green}} & q^{\downarrow}_{\text{green}} \\ u^{\mathcal{C}}_{\text{green}} & -u^{\mathcal{C}}_{\text{red}} & 0 & q^{\uparrow}_{\text{blue}} & q^{\downarrow}_{\text{blue}} \\ -q^{\uparrow}_{\text{red}} & -q^{\uparrow}_{\text{green}} & -q^{\uparrow}_{\text{blue}} & 0 & e^{\mathcal{C}} \\ -q^{\downarrow}_{\text{red}} & -q^{\downarrow}_{\text{green}} & -q^{\downarrow}_{\text{blue}} & -e^{\mathcal{C}} & 0 \end{pmatrix}$$

- transformation of **10**–plet

$$\chi \;\to\; U\cdot\chi\cdot U^T$$

$\mathrm{SU}(5)$ matrix

Standard model matter in $\mathrm{SU}(5)$ (II)

☞ specialize to $\mathrm{SU}(5)$ transformations of the type

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

$\mathrm{SU}(3)_\mathrm{C}$ matrix

$\mathrm{SU}(2)_\mathrm{L}$ matrix

Standard model matter in $\mathrm{SU}(5)$ (II)

☞ specialize to $\mathrm{SU}(5)$ transformations of the type

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

☞ short–hand notation

$$\mathbf{10} = \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

Standard model matter in $\mathrm{SU}(5)$ (II)

☞ specialize to $\mathrm{SU}(5)$ transformations of the type

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

☞ short–hand notation

$$\mathbf{10} = \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

☞ transformation of u–type quarks

$$\begin{pmatrix} 0 & u^{\mathcal{C}}_{\mathrm{blue}} & -u^{\mathcal{C}}_{\mathrm{green}} \\ -u^{\mathcal{C}}_{\mathrm{blue}} & 0 & u^{\mathcal{C}}_{\mathrm{red}} \\ u^{\mathcal{C}}_{\mathrm{green}} & -u^{\mathcal{C}}_{\mathrm{red}} & 0 \end{pmatrix} \to U_3 \cdot \begin{pmatrix} 0 & u^{\mathcal{C}}_{\mathrm{blue}} & -u^{\mathcal{C}}_{\mathrm{green}} \\ -u^{\mathcal{C}}_{\mathrm{blue}} & 0 & u^{\mathcal{C}}_{\mathrm{red}} \\ u^{\mathcal{C}}_{\mathrm{green}} & -u^{\mathcal{C}}_{\mathrm{red}} & 0 \end{pmatrix} \cdot U_3^T$$

Standard model matter in $\mathrm{SU}(5)$ (II)

☞ specialize to $\mathrm{SU}(5)$ transformations of the type

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

☞ short–hand notation

$$\mathbf{10} = \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

☞ transformation of u–type quarks

$$\begin{pmatrix} 0 & u^{\mathcal{C}}_{\mathrm{blue}} & -u^{\mathcal{C}}_{\mathrm{green}} \\ -u^{\mathcal{C}}_{\mathrm{blue}} & 0 & u^{\mathcal{C}}_{\mathrm{red}} \\ u^{\mathcal{C}}_{\mathrm{green}} & -u^{\mathcal{C}}_{\mathrm{red}} & 0 \end{pmatrix} \to U_3 \cdot \begin{pmatrix} 0 & u^{\mathcal{C}}_{\mathrm{blue}} & -u^{\mathcal{C}}_{\mathrm{green}} \\ -u^{\mathcal{C}}_{\mathrm{blue}} & 0 & u^{\mathcal{C}}_{\mathrm{red}} \\ u^{\mathcal{C}}_{\mathrm{green}} & -u^{\mathcal{C}}_{\mathrm{red}} & 0 \end{pmatrix} \cdot U_3^T$$

➥ u–type quarks transform as $\overline{\mathbf{3}}$–plets

Standard model matter in $\mathrm{SU}(5)$ (III)

☞ transformation of quark doublets

$$\left(\begin{array}{cc} q^\uparrow_\mathrm{red} & q^\downarrow_\mathrm{red} \\ q^\uparrow_\mathrm{green} & q^\downarrow_\mathrm{green} \\ q^\uparrow_\mathrm{blue} & q^\downarrow_\mathrm{blue} \end{array}\right) \quad\to\quad U_3\cdot\left(\begin{array}{cc} q^\uparrow_\mathrm{red} & q^\downarrow_\mathrm{red} \\ q^\uparrow_\mathrm{green} & q^\downarrow_\mathrm{green} \\ q^\uparrow_\mathrm{blue} & q^\downarrow_\mathrm{blue} \end{array}\right)\cdot U_2^T$$

Standard model matter in $\mathrm{SU}(5)$ (III)

☞ transformation of quark doublets

$$\begin{pmatrix} q^\uparrow_{\mathrm{red}} & q^\downarrow_{\mathrm{red}} \\ q^\uparrow_{\mathrm{green}} & q^\downarrow_{\mathrm{green}} \\ q^\uparrow_{\mathrm{blue}} & q^\downarrow_{\mathrm{blue}} \end{pmatrix} \quad\to\quad U_3 \cdot \begin{pmatrix} q^\uparrow_{\mathrm{red}} & q^\downarrow_{\mathrm{red}} \\ q^\uparrow_{\mathrm{green}} & q^\downarrow_{\mathrm{green}} \\ q^\uparrow_{\mathrm{blue}} & q^\downarrow_{\mathrm{blue}} \end{pmatrix} \cdot U_2^T$$

➥ quark doublets transform as $(\mathbf{3},\mathbf{2})$ under $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L}$

Standard model matter in $\mathrm{SU}(5)$ (III)

☞ transformation of quark doublets

$$\left(\begin{array}{cc} q^{\uparrow}_{\mathrm{red}} & q^{\downarrow}_{\mathrm{red}} \\ q^{\uparrow}_{\mathrm{green}} & q^{\downarrow}_{\mathrm{green}} \\ q^{\uparrow}_{\mathrm{blue}} & q^{\downarrow}_{\mathrm{blue}} \end{array}\right) \quad\rightarrow\quad U_3\cdot\left(\begin{array}{cc} q^{\uparrow}_{\mathrm{red}} & q^{\downarrow}_{\mathrm{red}} \\ q^{\uparrow}_{\mathrm{green}} & q^{\downarrow}_{\mathrm{green}} \\ q^{\uparrow}_{\mathrm{blue}} & q^{\downarrow}_{\mathrm{blue}} \end{array}\right)\cdot U_2^T$$

➥ quark doublets transform as $(\mathbf{3},\mathbf{2})$ under $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}$

☞ transformation of lepton singlets

$$\left(\begin{array}{cc} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{array}\right) \quad\rightarrow\quad U_2\cdot\left(\begin{array}{cc} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{array}\right)\cdot U_2^T$$

Standard model matter in $\mathrm{SU}(5)$ (III)

☞ transformation of quark doublets

$$\begin{pmatrix} q^\uparrow_{\text{red}} & q^\downarrow_{\text{red}} \\ q^\uparrow_{\text{green}} & q^\downarrow_{\text{green}} \\ q^\uparrow_{\text{blue}} & q^\downarrow_{\text{blue}} \end{pmatrix} \quad\to\quad U_3 \cdot \begin{pmatrix} q^\uparrow_{\text{red}} & q^\downarrow_{\text{red}} \\ q^\uparrow_{\text{green}} & q^\downarrow_{\text{green}} \\ q^\uparrow_{\text{blue}} & q^\downarrow_{\text{blue}} \end{pmatrix} \cdot U_2^T$$

➥ quark doublets transform as $(\mathbf{3},\mathbf{2})$ under $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L}$

☞ transformation of lepton singlets

$$\begin{pmatrix} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{pmatrix} \quad\to\quad U_2 \cdot \begin{pmatrix} 0 & e^{\mathcal{C}} \\ -e^{\mathcal{C}} & 0 \end{pmatrix} \cdot U_2^T$$

➥ $e^{\mathcal{C}}$ transform as singlets

Unification of matter

☞ $\mathrm{SU}(5)$ representations $\overline{\mathbf{5}}$ and $\mathbf{10}$ contain precisely one generation of standard model matter

$$\left.\begin{array}{c} d^C \\ \ell \end{array}\right\} \to \overline{\mathbf{5}} \quad \text{and} \quad \left.\begin{array}{c} q \\ u^C \\ e^C \end{array}\right\} \to \mathbf{10}$$

Hypercharge (I)

☞ hypercharge is $\mathrm{SU}(5)$ generator that commutes with the generators of the $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ subgroups

$$\mathsf{t}_Y = \mathcal{N}\begin{pmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1/2 & \\ & & & & 1/2 \end{pmatrix}$$

Hypercharge (I)

- hypercharge is $\mathrm{SU}(5)$ generator that commutes with the generators of the $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ subgroups

$$\mathsf{t}_Y = \mathcal{N}\begin{pmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1/2 & \\ & & & & 1/2 \end{pmatrix}$$

- G_SM maximal subgroup of $\mathrm{SU}(5)$

$$\mathrm{SU}(5) \supset \mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y = G_\mathrm{SM}$$

Hypercharge (II)

☞ infinitesimal t_Y transformations of $\overline{\mathbf{5}}$–plet

$$-\mathsf{t}_Y \begin{pmatrix} d^{\mathcal{C}}_{\text{red}} \\ d^{\mathcal{C}}_{\text{green}} \\ d^{\mathcal{C}}_{\text{blue}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3}\, d^{\mathcal{C}}_{\text{red}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\text{green}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\text{blue}} \\ -\frac{1}{2}\, \ell^{\uparrow} \\ -\frac{1}{2}\, \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, d^{\mathcal{C}}_{\text{red}} \\ Q_Y\, d^{\mathcal{C}}_{\text{green}} \\ Q_Y\, d^{\mathcal{C}}_{\text{blue}} \\ Q_Y\, \ell^{\uparrow} \\ Q_Y\, \ell^{\downarrow} \end{pmatrix}$$

normalization constant

Hypercharge (II)

☞ infinitesimal t_Y transformations of $\overline{\mathbf{5}}$–plet

$$-\mathsf{t}_Y \begin{pmatrix} d^{\mathcal{C}}_{\mathrm{red}} \\ d^{\mathcal{C}}_{\mathrm{green}} \\ d^{\mathcal{C}}_{\mathrm{blue}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{red}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{green}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{blue}} \\ -\frac{1}{2}\, \ell^{\uparrow} \\ -\frac{1}{2}\, \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, d^{\mathcal{C}}_{\mathrm{red}} \\ Q_Y\, d^{\mathcal{C}}_{\mathrm{green}} \\ Q_Y\, d^{\mathcal{C}}_{\mathrm{blue}} \\ Q_Y\, \ell^{\uparrow} \\ Q_Y\, \ell^{\downarrow} \end{pmatrix}$$

☞ infinitesimal transformation of $\mathbf{10}$–plet

$$\begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \to \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

with

$$\Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} = \mathsf{t}_Y \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \mathsf{t}_Y^T$$
$$= \mathcal{N} \begin{pmatrix} -\frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \mathcal{N} \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Hypercharge (II)

☞ infinitesimal t_Y transformations of $\overline{\mathbf{5}}$–plet

$$-\mathsf{t}_Y \begin{pmatrix} d^{\mathcal{C}}_{\text{red}} \\ d^{\mathcal{C}}_{\text{green}} \\ d^{\mathcal{C}}_{\text{blue}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3}\, d^{\mathcal{C}}_{\text{red}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\text{green}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\text{blue}} \\ -\frac{1}{2}\, \ell^{\uparrow} \\ -\frac{1}{2}\, \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, d^{\mathcal{C}}_{\text{red}} \\ Q_Y\, d^{\mathcal{C}}_{\text{green}} \\ Q_Y\, d^{\mathcal{C}}_{\text{blue}} \\ Q_Y\, \ell^{\uparrow} \\ Q_Y\, \ell^{\downarrow} \end{pmatrix}$$

☞ infinitesimal transformation of $\mathbf{10}$–plet

$$\begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \to \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

with

$$\Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} = \mathsf{t}_Y \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \mathsf{t}_Y^T$$

$$= \mathcal{N} \begin{pmatrix} \left(-\frac{1}{3} - \frac{1}{3}\right) u^{\mathcal{C}} & \left(-\frac{1}{3} + \frac{1}{2}\right) q \\ -\left(-\frac{1}{3} + \frac{1}{2}\right) q^T & \left(\frac{1}{2} + \frac{1}{2}\right) e^{\mathcal{C}} \end{pmatrix}$$

Hypercharge (II)

☞ infinitesimal t_Y transformations of $\overline{\mathbf{5}}$–plet

$$-\mathsf{t}_Y \begin{pmatrix} d^{\mathcal{C}}_{\mathrm{red}} \\ d^{\mathcal{C}}_{\mathrm{green}} \\ d^{\mathcal{C}}_{\mathrm{blue}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{red}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{green}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{blue}} \\ -\frac{1}{2}\, \ell^{\uparrow} \\ -\frac{1}{2}\, \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, d^{\mathcal{C}}_{\mathrm{red}} \\ Q_Y\, d^{\mathcal{C}}_{\mathrm{green}} \\ Q_Y\, d^{\mathcal{C}}_{\mathrm{blue}} \\ Q_Y\, \ell^{\uparrow} \\ Q_Y\, \ell^{\downarrow} \end{pmatrix}$$

☞ infinitesimal transformation of $\mathbf{10}$–plet

$$\begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \to \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

with

$$\Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} = \mathsf{t}_Y \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \mathsf{t}_Y^T$$

$$= \mathcal{N} \begin{pmatrix} -\frac{2}{3}\, u^{\mathcal{C}} & \frac{1}{6}\, q \\ -\frac{1}{6}\, q^T & 1\, e^{\mathcal{C}} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, u^{\mathcal{C}} & Q_Y\, q \\ -Q_Y\, q^T & Q_Y\, e^{\mathcal{C}} \end{pmatrix}$$

Hypercharge (II)

☞ infinitesimal t_Y transformations of $\overline{\mathbf{5}}$–plet

$$-\mathsf{t}_Y \begin{pmatrix} d^{\mathcal{C}}_{\mathrm{red}} \\ d^{\mathcal{C}}_{\mathrm{green}} \\ d^{\mathcal{C}}_{\mathrm{blue}} \\ \ell^{\uparrow} \\ \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{red}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{green}} \\ \frac{1}{3}\, d^{\mathcal{C}}_{\mathrm{blue}} \\ -\frac{1}{2}\, \ell^{\uparrow} \\ -\frac{1}{2}\, \ell^{\downarrow} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, d^{\mathcal{C}}_{\mathrm{red}} \\ Q_Y\, d^{\mathcal{C}}_{\mathrm{green}} \\ Q_Y\, d^{\mathcal{C}}_{\mathrm{blue}} \\ Q_Y\, \ell^{\uparrow} \\ Q_Y\, \ell^{\downarrow} \end{pmatrix}$$

☞ infinitesimal transformation of $\mathbf{10}$–plet

$$\begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \to \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix}$$

with

$$\Delta \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} = \mathsf{t}_Y \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} u^{\mathcal{C}} & q \\ -q^T & e^{\mathcal{C}} \end{pmatrix} \mathsf{t}_Y^T$$

$$= \mathcal{N} \begin{pmatrix} -\frac{2}{3}\, u^{\mathcal{C}} & \frac{1}{6}\, q \\ -\frac{1}{6}\, q^T & 1\, e^{\mathcal{C}} \end{pmatrix} = \mathcal{N} \begin{pmatrix} Q_Y\, u^{\mathcal{C}} & Q_Y\, q \\ -Q_Y\, q^T & Q_Y\, e^{\mathcal{C}} \end{pmatrix}$$

➥ standard model hypercharges get reproduced!

Hypercharge (III)

☞ $\mathrm{SU}(5)$ explains charge quantization

Hypercharge (III)

- $\mathrm{SU}(5)$ explains charge quantization
- normalization for $\mathrm{SU}(5)$ generators

$$\mathrm{Tr}\left(\mathsf{T}_a\,\mathsf{T}_b\right) \;=\; \frac{1}{2}\delta_{ab}$$

Hypercharge (III)

- $\mathrm{SU}(5)$ explains charge quantization
- normalization for $\mathrm{SU}(5)$ generators

$$\mathrm{Tr}\left(\mathsf{T}_a\,\mathsf{T}_b\right) \;=\; \frac{1}{2}\delta_{ab}$$

- impose

$$\mathrm{Tr}(\mathsf{t}_Y\,\mathsf{t}_Y) \;=\; \mathcal{N}^2\cdot(3/9+2/4) \;=\; \mathcal{N}^2\cdot\frac{5}{6} \;\stackrel{!}{=}\; \frac{1}{2} \quad\curvearrowright\quad \mathcal{N} \;=\; \sqrt{\frac{3}{5}}$$

Hypercharge (III)

- $\mathrm{SU}(5)$ explains charge quantization
- normalization for $\mathrm{SU}(5)$ generators

$$\mathrm{Tr}\left(\mathsf{T}_a\,\mathsf{T}_b\right) \;=\; \frac{1}{2}\delta_{ab}$$

- impose

$$\mathrm{Tr}(\mathsf{t}_Y\,\mathsf{t}_Y) \;=\; \mathcal{N}^2\cdot(3/9+2/4) \;=\; \mathcal{N}^2\cdot\frac{5}{6} \;\stackrel{!}{=}\; \frac{1}{2} \quad\curvearrowright\quad \mathcal{N} \;=\; \sqrt{\frac{3}{5}}$$

- normalization can be absorbed in redefinition of the coupling strength g_1

back

Discrete symmetries

and

Grand Unification

- anomaly cancellation
- consistency with unification
- unique $\mathbb{Z}_4^R$ symmetry
- no–go theorems in 4D

Prejudices and assumptions

Assumptions:

- ☞ $\mathrm{SO}(10)$ unification of matter is not an accident
- ☞ μ term is forbidden by a symmetry
- ☞ symmetries need to be anomaly–free

Important ingredient :

- ☞ Green–Schwarz anomaly cancellation

▸ GUTs

Anomaly freedom

Anomaly freedom

$+$

Gauge unification

$+$

Green–Schwarz
anomaly cancellation

Anomaly freedom

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ + \\ \text{Gauge unification}\\ + \\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

Anomaly freedom

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ + \\ \text{Gauge unification}\\ + \\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$\begin{aligned} A_{G^2-\mathbb{Z}_N} &= \sum_f \ell^{(f)} \cdot q^{(f)} \\ A_{\mathrm{grav}^2-\mathbb{Z}_N} &= \sum_m q^{(m)} \end{aligned}$$

Anomaly freedom

Anomaly freedom
+
Gauge unification
+
Green–Schwarz anomaly cancellation

→ "Anomaly universality"

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

sum over all representations of G

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

sum over all fermions

Anomaly freedom

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ +\\ \text{Gauge unification}\\ +\\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

Dynkin index

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

discrete charges

Anomaly freedom

Anomaly freedom
+
Gauge unification
+
Green–Schwarz
anomaly cancellation

$\rightarrow$ "Anomaly universality"

traditional anomaly freedom:

all A coefficients vanish

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Ibáñez & Ross [1991]
Banks & Dine [1992]

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} 0 \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} 0 \mod \eta$$

Anomaly freedom

Anomaly freedom
+
Gauge unification
+
Green–Schwarz anomaly cancellation

$\Bigg\}$ → "Anomaly universality"

universal shift due to GS saxion

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} \rho \mod \eta$$

$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

traditional anomaly freedom:

all A coefficients vanish

⬇ ⬇ ⬇ ⬇ ⬇

anomaly "universality":

$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = A_{\mathrm{SU}(2)^2-\mathbb{Z}_N}$ if $\mathrm{SU}(3)\times\mathrm{SU}(2) \subset \mathrm{SU}(5)$ or E_8

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly universality (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT

☞ Will prove:

1. assuming (i) & $\mathrm{SU}(5)$ relations:
   ↷ only R symmetries can forbid the μ term

Anomaly–free symmetries, μ and unification

- Working assumptions:
  (i) anomaly universality (allow for GS anomaly cancellation)
  (ii) μ term forbidden at perturbative level
  (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  (iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT
- Will prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term
  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry

Anomaly–free symmetries, μ and unification

- Working assumptions:
  - (i) anomaly universality (allow for GS anomaly cancellation)
  - (ii) μ term forbidden at perturbative level
  - (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  - (iv) compability with $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT
- Will prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term
  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry
  3. R symmetries are not available in 4D GUTs

It has to be an R symmetry

Hall, Nomura & Pierce [2002b] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

charge of g^{th} $\overline{\mathbf{5}}$–plet

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^{g} + q_{\overline{\mathbf{5}}}^{g}\right)$$

Higgs charges

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^{g} + q_{\overline{\mathbf{5}}}^{g}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

charge of g^{th} $\mathbf{10}$–plet

It has to be an R symmetry

Hall, Nomura & Pierce [2002b] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right)$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

- Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0$

$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

It has to be an R symmetry

Hall, Nomura & Pierce [2002b] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011b]

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right)$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

- Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0$

$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

bottom–line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

Only discrete R symmetries may do the job

- ☞ Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries
- ☞ There are no anomaly–free continuous R symmetries in the MSSM

  Chamseddine & Dreiner [1996]

- ➥ Only remaining option: discrete R symmetries

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

☞ Powerful tool: anomaly matching

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = \underbrace{A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}}_{\text{matter}} + \overbrace{A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}}^{\text{extra}} + \underbrace{5q_\theta}_{\text{gauginos}}$$

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \frac{1}{2}\cdot 2\cdot 2\cdot q_\theta$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \frac{1}{2}\cdot 2\cdot 3\cdot q_\theta$$

SM gauginos

universal

extra gauginos from X, Y bosons

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \cancel{\frac{1}{2}\cdot 2\cdot 2\cdot q_\theta}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \cancel{\frac{1}{2}\cdot 2\cdot 3\cdot q_\theta}$$

- Assume now that some mechanism eliminates the extra gauginos

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \cancel{\frac{1}{2}\cdot 2\cdot 2\cdot q_\theta}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \cancel{\frac{1}{2}\cdot 2\cdot 3\cdot q_\theta}$$

- Assume now that some mechanism eliminates the extra gauginos

➥ Extra stuff must be non–universal (split multiplets)

't Hooft anomaly matching for R symmetries

't Hooft [1976] ; Csáki & Murayama [1998]

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$\begin{aligned} A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} &= A^{\mathrm{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \cancel{\frac{1}{2}\cdot 2\cdot 2\cdot q_\theta} \\ A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} &= A^{\mathrm{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \cancel{\frac{1}{2}\cdot 2\cdot 3\cdot q_\theta} \end{aligned}$$

bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

$\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- ☞ Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
  i.e. quarks and leptons have universal charge q

$\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
  i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

R charge of superspace coordinate θ

superpotential has R charge $2q_\theta$

$$\int \mathrm{d}^2\theta\, \mathscr{W} \subset \mathscr{L}$$

$\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
  i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

$\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$ i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

- u–type Yukawa and Weinberg operator requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M$$

SO(10) implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
  i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} \; = \; 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} \; = \; 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} \; = \; 0 \mod M$$

- u–type Yukawa and Weinberg operator requires that

$$2q + q_{H_u} \; = \; 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} \; = \; 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} \; = \; 0 \mod M$$

$\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$ i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

- u–type Yukawa and Weinberg operator requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} = 0 \mod M$$

bottom–line:

$$q_{H_u} = q_{H_d} = 0 \mod M \;\&\; q = q_\theta \mod M$$

Unique $\mathbb{Z}_4^R$ symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

☞ We know already that $\left\{\begin{array}{l} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} \ = \ q_{H_d} \ = \ 0 \mod M \end{array}\right.$

Unique $\mathbb{Z}_4^R$ symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$

Babu, Gogoladze & Wang [2003]

- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
  $M = 2$ does not work since this is not an R symmetry

Unique $\mathbb{Z}_4^R$ symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- We know already that $\left\{ \begin{array}{l} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{array} \right.$

Babu, Gogoladze & Wang [2003]

- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$

Unique $\mathbb{Z}_4^R$ symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$

Babu, Gogoladze & Wang [2003]

- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$
- However: these are only trivial extensions (as far as the MSSM is concerned)

Chen, M.R. & Takhistov [2014]

Unique $\mathbb{Z}_4^R$ symmetry

Lee, Raby, M.R., Ross, Schieren, et al. [2011a] ; Chen, Fallbacher, Omura, M.R. & Staudt [2012a]

- We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$

Babu, Gogoladze & Wang [2003]

- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$
- However: these are only trivial extensions (as far as the MSSM is concerned)

Lee, Raby, M.R., Ross, Schieren, et al. [2011a]

bottom-line:

unique symmetry : $\mathbb{Z}_4^R$ w/ $q = q_\theta = 1$ & $q_{H_u} = q_{H_d} = 0$

Unique $\mathbb{Z}_4^R$ symmetry & GS anomaly cancellation

☞ Anomaly coefficients

$$\begin{aligned} A_{\mathrm{SU}(3)^2-\mathbb{Z}_4^R} &= 6q - 3q_\theta = 1q_\theta \mod 4/2 \\ A_{\mathrm{SU}(2)^2-\mathbb{Z}_4^R} &= 6q + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 5q_\theta = 1q_\theta \mod 4/2 \end{aligned}$$

➥ Consistent with anomaly universality

bottom-line:

$\mathbb{Z}_4^R$ is anomaly–free via non–trivial GS mechanism

Automatic absence of $Q\,Q\,Q\,L$ operators

- Consider family–independent $\mathbb{Z}_M^R$ symmetry
- Conditions for usual MSSM Yukawa couplings

$$\begin{aligned} 2q_{\mathbf{10}} + q_{H_u} &= q_{\mathscr{W}} \quad \mod M \\ q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + q_{H_d} &= q_{\mathscr{W}} \quad \mod M \end{aligned}$$

$$\curvearrowright\; 3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + \underbrace{q_{H_u} + q_{H_d}}_{=0} \;=\; 2q_{\mathscr{W}} \mod M \;=\; 0 \mod M$$

bottom–line:

- compatibility w/ $\mathrm{SU}(5)$
- Giudice–Masiero term
- anomaly freedom

$\Big\}\ \curvearrowright$ ~~dimension five proton decay~~

GS anomaly cancellation vs. nonperturbative terms

- ☞ GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)

GS anomaly cancellation vs. nonperturbative terms

- GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)
- As $a = \mathrm{Im}\, S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, non–invariant superpotential terms can be made invariant by multiplying them by $\mathrm{e}^{-b\, S}$

GS anomaly cancellation vs. nonperturbative terms

- GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)
- As $a = \mathrm{Im}\, S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, non–invariant superpotential terms can be made invariant by multiplying them by $\mathrm{e}^{-b\,S}$
- Main example

$\mu\, H_u\, H_d$ forbidden

R charge 0

but

$B\,\mathrm{e}^{-b\,S}\, H_u\, H_d$ allowed (for appropriate b)

R charge 2

GS anomaly cancellation vs. nonperturbative terms

- GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)
- As $a = \operatorname{Im} S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, non–invariant superpotential terms can be made invariant by multiplying them by $\mathrm{e}^{-b\,S}$
- Main example

  $\mu\, H_u\, H_d$ forbidden

but

  $B\,\mathrm{e}^{-b\,S}\, H_u\, H_d$ allowed (for appropriate b)

bottom–line:

holomorphic $\mathrm{e}^{-b\,S}$ terms appear to violate $\mathbb{Z}_M^R$ symmetry

R symmetry breaking vs. supersymmetry breaking

- ☞ The order parameter of R symmetry breaking is the expectation value of the superpotential $\langle \mathscr{W} \rangle$

R symmetry breaking vs. supersymmetry breaking

- ☞ The order parameter of R symmetry breaking is the expectation value of the superpotential $\langle \mathscr{W} \rangle$
- ➥ But $m_{3/2} = \langle \mathscr{W} \rangle / M_{\mathrm{P}}^2$

R symmetry breaking vs. supersymmetry breaking

- The order parameter of R symmetry breaking is the expectation value of the superpotential $\langle \mathscr{W} \rangle$
- But $m_{3/2} = \langle \mathscr{W} \rangle / M_\mathrm{P}^2$
- Cancellation of the vacuum energy

$$\sum_i |D_i \mathscr{W}|^2 - 3\frac{|\mathscr{W}|^2}{M_\mathrm{P}^2} \stackrel{!}{=} 0$$

F–terms

R symmetry breaking vs. supersymmetry breaking

☞ The order parameter of R symmetry breaking is the expectation value of the superpotential $\langle \mathscr{W} \rangle$

➥ But $m_{3/2} = \langle \mathscr{W} \rangle / M_\mathrm{P}^2$

➥ Cancellation of the vacuum energy

$$\sum_i |D_i \mathscr{W}|^2 - 3 \frac{|\mathscr{W}|^2}{M_\mathrm{P}^2} \stackrel{!}{=} 0$$

bottom–line:

R symmetry breaking tied to supersymmetry breaking

Proton hexality

Dreiner, Luhn & Thormeier [2006] ; Dreiner, Luhn, Murayama & Thormeier [2008]

☞ combine $\mathbb{Z}_2^R$ and baryon triality B_3

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_2^R$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

Proton hexality

Dreiner, Luhn & Thormeier [2006] ; Dreiner, Luhn, Murayama & Thormeier [2008]

- combine $\mathbb{Z}_2^R$ and baryon triality B_3

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_2^R$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

- good and not so good features

Proton hexality

Dreiner, Luhn & Thormeier [2006] ; Dreiner, Luhn, Murayama & Thormeier [2008]

☞ combine $\mathbb{Z}_2^R$ and baryon triality B_3

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_2^R$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

☞ good and not so good features

☺ forbids dimension–4 & 5 proton decay

☺ allows Yukawa couplings & effective neutrino operator

☺ anomaly–free

Proton hexality

Dreiner, Luhn & Thormeier [2006] ; Dreiner, Luhn, Murayama & Thormeier [2008]

☞ combine $\mathbb{Z}_2^R$ and baryon triality B_3

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_2^R$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

☞ good and not so good features

☺ forbids dimension–4 & 5 proton decay

☺ allows Yukawa couplings & effective neutrino operator

☺ anomaly–free

☹ not consistent with grand unification

☹ does not address the μ problem

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a]

☞ unique symmetry that probibits proton decay operators and is consistent with grand unification

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a]

- ☞ unique symmetry that probibits proton decay operators and is consistent with grand unification

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

- ☞ requires Green–Schwarz anomaly cancellation $\curvearrowright$ broken

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a]

☞ unique symmetry that probibits proton decay operators and is consistent with grand unification

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

☞ requires Green–Schwarz anomaly cancellation $\curvearrowright$ broken

☞ order parameter of $\mathbb{Z}_4^R$ breaking: gravitino mass $m_{3/2}$

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a]

- unique symmetry that probibits proton decay operators and is consistent with grand unification

	q	$u^\mathcal{C}$	$d^\mathcal{C}$	ℓ	$e^\mathcal{C}$	h_u	h_d	$\nu^\mathcal{C}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

- requires Green–Schwarz anomaly cancellation $\curvearrowright$ broken
- order parameter of $\mathbb{Z}_4^R$ breaking: gravitino mass $m_{3/2}$
- $\mathbb{Z}_2^R \subset \mathbb{Z}_4^R$ remains unbroken

$\mathbb{Z}_4^R$ summarized

Babu, Gogoladze & Wang [2003] ; Lee, Raby, M.R., Ross, Schieren, et al. [2011a]

- unique symmetry that probibits proton decay operators and is consistent with grand unification

	q	$u^{\mathcal{C}}$	$d^{\mathcal{C}}$	ℓ	$e^{\mathcal{C}}$	h_u	h_d	$\nu^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

- requires Green–Schwarz anomaly cancellation $\curvearrowright$ broken
- order parameter of $\mathbb{Z}_4^R$ breaking: gravitino mass $m_{3/2}$
- $\mathbb{Z}_2^R \subset \mathbb{Z}_4^R$ remains unbroken
- can be explained as discrete remnant of the Lorentz group in extra dimensions

$\mathbb{Z}_4^R$ summarized

Yukawa couplings

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} = {} & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\ & + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\ & + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\ & + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell\end{aligned}$$

effective neutrino mass operator

$\mathbb{Z}_4^R$ summarized

$$
\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\;& \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
& + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
& + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
& + \kappa_{gf}\, h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}
$$

forbidden by $\mathbb{Z}_4^{\mathrm{R}}$

$\mathbb{Z}_4^R$ summarized

$$\mathcal{O}(m_{3/2})$$

$$\begin{aligned}
\mathscr{W}_{\text{gauge invariant}} \;=\; & \mu\, h_d h_u + \kappa_i\, \ell_i h_u \\
& + Y_e^{gf}\, \ell_g h_d e^{\mathcal{C}}{}_f + Y_d^{gf}\, q_g h_d d^{\mathcal{C}}{}_f + Y_u^{gf}\, q_g h_u u^{\mathcal{C}}{}_f \\
& + \lambda_{gfk}\, \ell_g \ell_f e^{\mathcal{C}}{}_k + \lambda'_{gfk}\, \ell_g q_f d^{\mathcal{C}}{}_k + \lambda''_{gfk}\, u^{\mathcal{C}}{}_g d^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k \\
& + \kappa_{gf}\, h_u \ell_g\, h_u \ell_f + \kappa^{(1)}_{gfk\ell}\, q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell}\, u^{\mathcal{C}}{}_g u^{\mathcal{C}}{}_f d^{\mathcal{C}}{}_k e^{\mathcal{C}}{}_\ell
\end{aligned}$$

$$\mathcal{O}\left(\frac{m_{3/2}}{M_{\mathrm{P}}^2}\right)$$

- ☞ R parity violating couplings forbidden
- ☞ μ term of the right size and proton decay under control

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

- anomaly freedom
- consistency with SU(5)

$\curvearrowright$ only R symmetries can forbid the μ term in the MSSM

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

- anomaly freedom
- consistency with SU(5)

$\curvearrowright$ only R symmetries can forbid the μ term in the MSSM

☞ **However**: R symmetries are not available in 4D SUSY GUTs

Fallbacher, M.R. & Vaudrevange [2011]

R symmetries vs. 4D GUTs

- We have seen that only R symmetries can forbid the μ term

  - anomaly freedom
  - consistency with SU(5)

  $\curvearrowright$ only R symmetries can forbid the μ term in the MSSM

- **However**: R symmetries are not available in 4D SUSY GUTs

  Fallbacher, M.R. & Vaudrevange [2011]

- Assumptions:

  (i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$

  (ii) GUT symmetry breaking is spontaneous

  (iii) Only finite number of fields

R symmetries vs. 4D GUTs

- We have seen that only R symmetries can forbid the μ term

  - anomaly freedom
  - consistency with SU(5)

  $\curvearrowright$ only R symmetries can forbid the μ term in the MSSM

- **However**: R symmetries are not available in 4D SUSY GUTs

  Fallbacher, M.R. & Vaudrevange [2011]

- Assumptions:

  (i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$

  (ii) GUT symmetry breaking is spontaneous

  (iii) Only finite number of fields

- One can prove that it is impossible to get low–energy effective theory with both:

  1. just the MSSM field content
  2. residual R symmetries

The basic argument

☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_\mathrm{SM}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a **24**–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

R charge 0

The basic argument

☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus \underbrace{(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}}_{\text{get eaten}} \oplus (\mathbf{1},\mathbf{1})_0$$

The basic argument

☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

extra massless states

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

- Introducing extra $\mathbf{24}$–plets with R charge 2 does not help because this would lead to massless $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$ representations

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

- Introducing extra $\mathbf{24}$–plets with R charge 2 does not help because this would lead to massless $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$ representations
- Iterating this argument shows that with a finite number of $\mathbf{24}$–plets one will always have massless exotics

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

- Introducing extra $\mathbf{24}$–plets with R charge 2 does not help because this would lead to massless $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$ representations
- Iterating this argument shows that with a finite number of $\mathbf{24}$–plets one will always have massless exotics
- Loophole for infinitely many $\mathbf{24}$–plets

cf. Goodman & Witten [1986]

Generalizing the basic argument

☞ It is possible to generalize the basic argument to

- arbitrary $\mathrm{SU}(5)$ representations
- larger GUT groups $G \supset \mathrm{SU}(5)$
- singlet extensions of the MSSM

▸ back

for details see Fallbacher, M.R. & Vaudrevange [2011]

References I

Steven Abel, Callum Durnford, Joerg Jaeckel & Valentin V. Khoze. Dynamical breaking of U(1)(R) & supersymmetry in a metastable vacuum. *Phys. Lett.*, B661:201–209, 2008. doi: 10.1016/j.physletb.2008.01.065.

Guido Altarelli & Ferruccio Feruglio. SU(5) grand unification in extra dimensions & proton decay. *Phys. Lett.*, B511:257–264, 2001. doi: 10.1016/S0370-2693(01)00650-5.

Archana Anandakrishnan & Stuart Raby. SU(6) GUT Breaking on a Projective Plane. *Nucl. Phys.*, B868:627–651, 2013. doi: 10.1016/j.nuclphysb.2012.12.001.

T. Asaka, W. Buchmüller & L. Covi. Gauge unification in six-dimensions. *Phys. Lett.*, B523:199–204, 2001. doi: 10.1016/S0370-2693(01)01324-7.

T. Asaka, W. Buchmüller & L. Covi. Exceptional coset spaces & unification in six dimensions. *Phys. Lett.*, B540:295–300, 2002.

References II

T. Asaka, W. Buchmüller & L. Covi. Quarks & leptons between branes & bulk. *Phys. Lett.*, B563:209–216, 2003.

K. S. Babu & Stephen M. Barr. Natural suppression of higgsino mediated proton decay in supersymmetric so(10). *Phys. Rev.*, D48:5354–5364, 1993.

K.S. Babu, Ilia Gogoladze & Kai Wang. Natural R parity, mu term & fermion mass hierarchy from discrete gauge symmetries. *Nucl. Phys.*, B660:322–342, 2003. doi: 10.1016/S0550-3213(03)00258-X.

Howard Baer, Vernon Barger, Michael Savoy, Hasan Serce & Xerxes Tata. Superparticle phenomenology from the natural mini-landscape. *JHEP*, 06:101, 2017. doi: 10.1007/JHEP06(2017)101.

Tom Banks & Michael Dine. Note on discrete gauge anomalies. *Phys. Rev.*, D45:1424–1427, 1992. doi: 10.1103/PhysRevD.45.1424.

Riccardo Barbieri, Lawrence J. Hall & Yasunori Nomura. A constrained standard model from a compact extra dimension. *Phys. Rev.*, D63: 105007, 2001.

References III

J. Bernabéu, G.C. Branco & M. Gronau. CP RESTRICTIONS ON QUARK MASS MATRICES. *Phys. Lett.*, B169:243–247, 1986. doi: 10.1016/0370-2693(86)90659-3.

Florian Beye, Tatsuo Kobayashi & Shogo Kuwakino. Gauge Symmetries in Heterotic Asymmetric Orbifolds. *Nucl. Phys.*, B875:599–620, 2013. doi: 10.1016/j.nuclphysb.2013.07.018.

Pierre Binétruy, Mary K. Gaillard & Yi-Yen Wu. Supersymmetry breaking & weakly vs. strongly coupled string theory. *Phys. Lett.*, B412: 288–295, 1997.

Michael Blaszczyk, Stefan Groot Nibbelink, Michael Ratz, Fabian Ruehle, Michele Trapletti, et al. A Z2xZ2 standard model. *Phys. Lett.*, B683: 340–348, 2010. doi: 10.1016/j.physletb.2009.12.036.

Vincent Bouchard & Ron Donagi. An SU(5) heterotic standard model. *Phys. Lett.*, B633:783–791, 2006.

Volker Braun, Yang-Hui He, Burt A. Ovrut & Tony Pantev. A heterotic standard model. *Phys. Lett.*, B618:252–258, 2005.

References IV

Felix Brümmer, Rolf Kappl, Michael Ratz & Kai Schmidt-Hoberg. Approximate R-symmetries & the mu term. *JHEP*, 04:006, 2010. doi: 10.1007/JHEP04(2010)006.

Wilfried Buchmüller, Koichi Hamaguchi, Oleg Lebedev & Michael Ratz. Dilaton destabilization at high temperature. *Nucl. Phys.*, B699: 292–308, 2004.

Wilfried Buchmüller, Koichi Hamaguchi, Oleg Lebedev & Michael Ratz. Dual models of gauge unification in various dimensions. *Nucl. Phys.*, B712:139–156, 2005.

Wilfried Buchmüller, Koichi Hamaguchi, Oleg Lebedev, Saul Ramos-Sánchez & Michael Ratz. Seesaw neutrinos from the heterotic string. *Phys. Rev. Lett.*, 99:021601, 2007.

Gustavo Burdman & Yasunori Nomura. Unification of Higgs & gauge fields in five dimensions. *Nucl. Phys.*, B656:3–22, 2003.

References V

Nana Geraldine Cabo Bizet, Tatsuo Kobayashi, Damian Kaloni Mayorga Pena, Susha L. Parameswaran, Matthias Schmitz & Ivonne Zavala. R-charge Conservation & More in Factorizable & Non-Factorizable Orbifolds. *JHEP*, 05:076, 2013. doi: 10.1007/JHEP05(2013)076.

Brenda Carballo-Perez, Eduardo Peinado & Saul Ramos-Sánchez. $\Delta(54)$ flavor phenomenology & strings. *JHEP*, 12:131, 2016. doi: 10.1007/JHEP12(2016)131.

Marcela S. Carena, L. Clavelli, D. Matalliotakis, Hans Peter Nilles & C.E.M. Wagner. Light gluinos & unification of couplings. *Phys. Lett.*, B317:346–353, 1993. doi: 10.1016/0370-2693(93)91006-9.

J. A. Casas. The generalized dilaton supersymmetry breaking scenario. *Phys. Lett.*, B384:103–110, 1996.

Ali H. Chamseddine & Herbert K. Dreiner. Anomaly free gauged R symmetry in local supersymmetry. *Nucl. Phys.*, B458:65–89, 1996. doi: 10.1016/0550-3213(95)00583-8.

References VI

Mu-Chun Chen, Maximilian Fallbacher, Yuji Omura, Michael Ratz & Christian Staudt. 2012a. in preparation.

Mu-Chun Chen, Michael Ratz, Christian Staudt & Patrick K.S. Vaudrevange. The mu Term & Neutrino Masses. *Nucl. Phys.*, B866: 157–176, 2012b. doi: 10.1016/j.nuclphysb.2012.08.018.

Mu-Chun Chen, Michael Ratz & Volodymyr Takhistov. R parity violation from discrete R symmetries. *Nucl.Phys.*, B891:322–345, 2014. doi: 10.1016/j.nuclphysb.2014.12.011.

G. B. Cleaver, A. E. Faraggi & D. V. Nanopoulos. String derived MSSM & M-theory unification. *Phys. Lett.*, B455:135–146, 1999.

Csaba Csáki & Hitoshi Murayama. Discrete anomaly matching. *Nucl. Phys.*, B515:114–162, 1998.

S. Dimopoulos & Frank Wilczek. Incomplete multiplets in supersymmetric unified models. 1981. Santa Barbara Print-81-0600.

References VII

Michael Dine & Nathan Seiberg. Couplings & Scales in Superstring Models. *Phys. Rev. Lett.*, 55:366, 1985. doi: 10.1103/PhysRevLett.55.366.

Lance J. Dixon, Jeffrey A. Harvey, C. Vafa & Edward Witten. Strings on orbifolds. *Nucl. Phys.*, B261:678–686, 1985.

Lance J. Dixon, Jeffrey A. Harvey, C. Vafa & Edward Witten. Strings on orbifolds. 2. *Nucl. Phys.*, B274:285–314, 1986.

Lance J. Dixon, Daniel Friedan, Emil J. Martinec & Stephen H. Shenker. The Conformal Field Theory of Orbifolds. *Nucl. Phys.*, B282:13–73, 1987.

Lance J. Dixon, Vadim Kaplunovsky & Jan Louis. Moduli dependence of string loop corrections to gauge coupling constants. *Nucl. Phys.*, B355:649–688, 1991.

Herbi K. Dreiner, Christoph Luhn & Marc Thormeier. What is the discrete gauge symmetry of the MSSM? *Phys. Rev.*, D73:075007, 2006. doi: 10.1103/PhysRevD.73.075007.

References VIII

Herbi K. Dreiner, Christoph Luhn, Hitoshi Murayama & Marc Thormeier. Proton Hexality from an Anomalous Flavor U(1) & Neutrino Masses: Linking to the String Scale. *Nucl. Phys.*, B795:172–200, 2008. doi: 10.1016/j.nuclphysb.2007.11.014.

Jens Erler. Anomaly cancellation in six-dimensions. *J. Math. Phys.*, 35: 1819–1833, 1994.

Maximilian Fallbacher, Michael Ratz & Patrick K.S. Vaudrevange. No-go theorems for R symmetries in four-dimensional GUTs. *Phys. Lett.*, B705:503–506, 2011. doi: 10.1016/j.physletb.2011.10.063.

Jonathan L. Feng, Konstantin T. Matchev & Takeo Moroi. Multi - TeV scalars are natural in minimal supergravity. *Phys. Rev. Lett.*, 84: 2322–2325, 2000. doi: 10.1103/PhysRevLett.84.2322.

Maximilian Fischer, Saúl Ramos-Sánchez & Patrick K. S. Vaudrevange. Heterotic non-Abelian orbifolds. *JHEP*, 1307:080, 2013a. doi: 10.1007/JHEP07(2013)080.

References IX

Maximilian Fischer, Michael Ratz, Jesus Torrado & Patrick K.S. Vaudrevange. Classification of symmetric toroidal orbifolds. *JHEP*, 1301:084, 2013b. doi: 10.1007/JHEP01(2013)084.

Renato M. Fonseca. Calculating the renormalisation group equations of a SUSY model with Susyno. *Comput. Phys. Commun.*, 183:2298–2306, 2012. doi: 10.1016/j.cpc.2012.05.017.

Stefan Förste, Hans Peter Nilles, Patrick K. S. Vaudrevange & Akin Wingerter. Heterotic brane world. *Phys. Rev.*, D70:106008, 2004.

Harald Fritzsch & Peter Minkowski. Unified interactions of leptons & hadrons. *Ann. Phys.*, 93:193–266, 1975.

Murray Gell-Mann, Pierre Ramond & Richard Slansky. Complex spinors & unified theories. In P. van Nieuwenhuizen & D. Z. Freedman, editors, *Supergravity*, page 315. North Holland, Amsterdam, 1979.

G.F. Giudice & A. Masiero. A Natural Solution to the mu Problem in Supergravity Theories. *Phys. Lett.*, B206:480–484, 1988. doi: 10.1016/0370-2693(88)91613-9.

References X

Mark W. Goodman & Edward Witten. Global Symmetries in Four Dimensions & Higher Dimensions. *Nucl. Phys.*, B271:21, 1986.

Stefan Groot Nibbelink & Patrick K.S. Vaudrevange. Schoen manifold with line bundles as resolved magnetized orbifolds. *JHEP*, 1303:142, 2013. doi: 10.1007/JHEP03(2013)142.

Stefan Groot Nibbelink, Mark Hillenbach, Tatsuo Kobayashi & Martin G. A. Walter. Localization of heterotic anomalies on various hyper surfaces of T(6)/Z(4). *Phys. Rev.*, D69:046001, 2004.

Lawrence J. Hall & Yasunori Nomura. Gauge unification in higher dimensions. *Phys. Rev.*, D64:055003, 2001. doi: 10.1103/PhysRevD.64.055003.

Lawrence J. Hall, Hitoshi Murayama & Yasunori Nomura. Wilson lines & symmetry breaking on orbifolds. *Nucl. Phys.*, B645:85–104, 2002a. doi: 10.1016/S0550-3213(02)00816-7.

References XI

Lawrence J. Hall, Yasunori Nomura & Aaron Pierce. R symmetry & the mu problem. *Phys. Lett.*, B538:359–365, 2002b. doi: 10.1016/S0370-2693(02)02043-9.

Shahram Hamidi & Cumrun Vafa. Interactions on Orbifolds. *Nucl. Phys.*, B279:465, 1987.

A. Hebecker. Grand unification in the projective plane. *JHEP*, 01:047, 2004.

A. Hebecker & M. Trapletti. Gauge unification in highly anisotropic string compactifications. *Nucl. Phys.*, B713:173–203, 2005.

Pierre Hosteins, Rolf Kappl, Michael Ratz & Kai Schmidt-Hoberg. Gauge-top unification. *JHEP*, 07:029, 2009. doi: 10.1088/1126-6708/2009/07/029.

Luis E. Ibáñez. The search for a standard model SU(3) x SU(2) x U(1) superstring: An introduction to orbifold constructions. 1987. Based on lectures given at the XVII GIFT Seminar on Strings & Superstrings, El Escorial, Spain, Jun 1-6, 1987 & Mt. Sorak Symposium, Korea, Jul 1987 & ELAF '87, La Plata, Argentina, Jul 6-24, 1987.

References XII

Luis E. Ibáñez & Hans Peter Nilles. Low-energy remnants of superstring anomaly cancellation terms. *Phys. Lett.*, B169:354, 1986.

Luis E. Ibáñez & Graham G. Ross. Discrete gauge symmetry anomalies. *Phys. Lett.*, B260:291–295, 1991.

Luis E. Ibáñez, Jihn E. Kim, Hans Peter Nilles & F. Quevedo. Orbifold compactifications with three families of SU(3) x SU(2) x U(1)**n. *Phys. Lett.*, B191:282–286, 1987a.

Luis E. Ibáñez, Hans Peter Nilles & F. Quevedo. Orbifolds & Wilson lines. *Phys. Lett.*, B187:25–32, 1987b.

Kenneth Intriligator, Nathan Seiberg & David Shih. Dynamical SUSY breaking in meta-stable vacua. *JHEP*, 04:021, 2006.

Shamit Kachru, Renata Kallosh, Andrei Linde & Sandip P. Trivedi. De Sitter vacua in string theory. *Phys. Rev.*, D68:046005, 2003. doi: 10.1103/PhysRevD.68.046005.

Gordon Kane & Martin Wolfgang Winkler. Deriving the Inflaton in Compactified M-theory with a De Sitter Vacuum. 2019.

References XIII

Rolf Kappl, Hans Peter Nilles, Sául Ramos-Sánchez, Michael Ratz, Kai Schmidt-Hoberg & Patrick K.S. Vaudrevange. Large hierarchies from approximate R symmetries. *Phys. Rev. Lett.*, 102:121602, 2009. doi: 10.1103/PhysRevLett.102.121602.

Rolf Kappl, Bjoern Petersen, Stuart Raby, Michael Ratz, Roland Schieren & Patrick K.S. Vaudrevange. String-derived MSSM vacua with residual R symmetries. *Nucl. Phys.*, B847:325–349, 2011. doi: 10.1016/j.nuclphysb.2011.01.032.

Yoshiharu Kawamura. Gauge symmetry reduction from the extra space S(1)/Z(2). *Prog. Theor. Phys.*, 103:613–619, 2000.

Yoshiharu Kawamura. Triplet doublet splitting, proton stability & extra dimension. *Prog. Theor. Phys.*, 105:999–1006, 2001. doi: 10.1143/PTP.105.999.

Jihn E. Kim & Hans Peter Nilles. The mu Problem & the Strong CP Problem. *Phys. Lett.*, B138:150, 1984.

References XIV

Tatsuo Kobayashi & Noriyasu Ohtsubo. Geometrical aspects of Z_N orbifold phenomenology. *Int. J. Mod. Phys.*, A9:87–126, 1994.

Tatsuo Kobayashi, Stuart Raby & Ren-Jie Zhang. Constructing 5d orbifold grand unified theories from heterotic strings. *Phys. Lett.*, B593:262–270, 2004.

Tatsuo Kobayashi, Stuart Raby & Ren-Jie Zhang. Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a Z(6) orbifold. *Nucl. Phys.*, B704:3–55, 2005.

Tatsuo Kobayashi, Hans Peter Nilles, Felix Plöger, Stuart Raby & Michael Ratz. Stringy origin of non-Abelian discrete flavor symmetries. *Nucl. Phys.*, B768:135–156, 2007.

Sebastian J.H. Konopka. Non Abelian orbifold compactifications of the heterotic string. *JHEP*, 1307:023, 2013. doi: 10.1007/JHEP07(2013)023.

References XV

N. V. Krasnikov. On Supersymmetry Breaking in Superstring Theories. *Phys. Lett.*, B193:37–40, 1987. doi: 10.1016/0370-2693(87)90452-7.

Sven Krippendorf, Hans Peter Nilles, Michael Ratz & Martin Wolfgang Winkler. The heterotic string yields natural supersymmetry. *Phys. Lett.*, B712:87–92, 2012. doi: 10.1016/j.physletb.2012.04.043.

Sven Krippendorf, Hans Peter Nilles, Michael Ratz & Martin Wolfgang Winkler. Hidden SUSY from precision gauge unification. *Phys. Rev.*, D88:035022, 2013. doi: 10.1103/PhysRevD.88.035022.

Mikko Laine & Aleksi Vuorinen. Basics of Thermal Field Theory. *Lect. Notes Phys.*, 925:pp.1–281, 2016. doi: 10.1007/978-3-319-31933-9.

Oleg Lebedev, Hans Peter Nilles & Michael Ratz. de Sitter vacua from matter superpotentials. *Phys. Lett.*, B636:126–131, 2006. doi: 10.1016/j.physletb.2006.03.046.

Oleg Lebedev, Hans-Peter Nilles, Stuart Raby, Saúl Ramos-Sánchez, Michael Ratz, Patrick K. S. Vaudrevange & Akin Wingerter. Low Energy Supersymmetry from the Heterotic Landscape. *Phys. Rev. Lett.*, 98:181602, 2007a. doi: 10.1103/PhysRevLett.98.181602.

References XVI

Oleg Lebedev, Hans Peter Nilles, Stuart Raby, Saúl Ramos-Sánchez, Michael Ratz, Patrick K. S. Vaudrevange & Akin Wingerter. The heterotic road to the MSSM with R parity. *Phys. Rev.*, D77:046013, 2007b.

Hyun Min Lee, Stuart Raby, Michael Ratz, Graham G. Ross, Roland Schieren, et al. A unique Z_4^R symmetry for the MSSM. *Phys. Lett.*, B694:491–495, 2011a. doi: 10.1016/j.physletb.2010.10.038.

Hyun Min Lee, Stuart Raby, Michael Ratz, Graham G. Ross, Roland Schieren, et al. Discrete R symmetries for the MSSM & its singlet extensions. *Nucl. Phys.*, B850:1–30, 2011b. doi: 10.1016/j.nuclphysb.2011.04.009.

Benjamin Lillard, Michael Ratz, M. P. Tait, Tim & Sebastian Trojanowski. The Flavor of Cosmology. *JCAP*, 1807(07):056, 2018. doi: 10.1088/1475-7516/2018/07/056.

A. Masiero, Dimitri V. Nanopoulos, K. Tamvakis & T. Yanagida. Naturally massless higgs doublets in supersymmetric SU(5). *Phys. Lett.*, B115:380, 1982.

References XVII

P. Mayr & S. Stieberger. Threshold corrections to gauge couplings in orbifold compactifications. *Nucl. Phys.*, B407:725–748, 1993.

Peter Minkowski. mu → e gamma at a Rate of One Out of 1-Billion Muon Decays? *Phys. Lett.*, B67:421, 1977. doi: 10.1016/0370-2693(77)90435-X.

R. N. Mohapatra & P. B. Pal. Massive neutrinos in physics & astrophysics. Second edition. *World Sci. Lect. Notes Phys.*, 60:1–397, 1998.

Ann E. Nelson & Nathan Seiberg. R symmetry breaking versus supersymmetry breaking. *Nucl. Phys.*, B416:46–62, 1994. doi: 10.1016/0550-3213(94)90577-0.

Hans Peter Nilles. Dynamically broken supergravity & the hierarchy problem. *Phys. Lett.*, B115:193, 1982.

References XVIII

Hans Peter Nilles, Saúl Ramos-Sánchez, Patrick K.S. Vaudrevange & Akin Wingerter. The Orbifolder: A Tool to study the Low Energy Effective Theory of Heterotic Orbifolds. *Comput.Phys.Commun.*, 183: 1363–1380, 2012. doi: 10.1016/j.cpc.2012.01.026. 29 pages, web page http://projects.hepforge.org/orbifolder/.

Hans Peter Nilles, Saúl Ramos-Sánchez, Michael Ratz & Patrick K.S. Vaudrevange. A note on discrete R symmetries in $\mathbb{Z}_6$-II orbifolds with Wilson lines. *Phys. Lett.*, B726:876–881, 2013. doi: 10.1016/j.physletb.2013.09.041.

Hans Peter Nilles, Michael Ratz, Andreas Trautner & Patrick K. S. Vaudrevange. $\mathcal{CP}$ Violation from String Theory. 2018.

Yessenia Olguin-Trejo, Ricardo Pérez-Martínez & Saul Ramos-Sánchez. Charting the flavor landscape of MSSM-like Abelian heterotic orbifolds. 2018.

Jogesh C. Pati. Grand unification as a bridge between string theory & phenomenology. *Int. J. Mod. Phys.*, D15:1677–1698, 2006. doi: 10.1142/S0218271806009030.

References XIX

Damian Kaloni Mayorga Pena, Hans Peter Nilles & Paul-Konstantin Oehlmann. A Zip-code for Quarks, Leptons & Higgs Bosons. 2012.

Witold Pokorski & Graham G. Ross. Flat directions, string compactification & 3 generation models. *Nucl. Phys.*, B551:515–548, 1999.

J. Polchinski. *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge, UK: Univ. Pr. (1998) 402 p.

Stuart Raby. SUSY GUT Model Building. *Eur. Phys. J.*, C59:223–247, 2009. doi: 10.1140/epjc/s10052-008-0736-x.

Stuart Raby, Michael Ratz & Kai Schmidt-Hoberg. Precision gauge unification in the MSSM. *Phys. Lett.*, B687:342–348, 2010. doi: 10.1016/j.physletb.2010.03.060.

Yael Shadmi & Yuri Shirman. Dynamical supersymmetry breaking. *Rev. Mod. Phys.*, 72:25–64, 2000. doi: 10.1103/RevModPhys.72.25.

References XX

Sanjeev K. Soni & H. Arthur Weldon. Analysis of the Supersymmetry Breaking Induced by N=1 Supergravity Theories. *Phys. Lett.*, B126: 215, 1983. doi: 10.1016/0370-2693(83)90593-2.

Gerard 't Hooft. Symmetry Breaking Through Bell-Jackiw Anomalies. *Phys. Rev. Lett.*, 37:8–11, 1976. doi: 10.1103/PhysRevLett.37.8.

Mark A. Walton. The heterotic string on the simplest calabi-yau manifold & its orbifold limits. *Phys. Rev.*, D37:377, 1988.

Steven Weinberg. The cosmological constant problem. *Rev. Mod. Phys.*, 61:1–23, 1989. doi: 10.1103/RevModPhys.61.1.

Edward Witten. Dynamical Breaking of Supersymmetry. *Nucl. Phys.*, B188:513, 1981. doi: 10.1016/0550-3213(81)90006-7.

T. Yanagida. Horizontal gauge symmetry & masses of neutrinos. In O. Sawada & A. Sugamoto, editors, *Proceedings of the Workshop on The Unified Theory & the Baryon Number in the Universe*, page 95. KEK, Tsukuba, Japan, 1979.