α_s *from non-strange hadronic* τ *decays*

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in Collaboration with D. Boito, M. Golterman, K. Maltman and J. Osborne.

Trento, February 11, 2019

Papers

1) O. Cata, M. Golterman and S. Peris, “Duality violations and spectral sum rules,” JHEP **0508**, 076 (2005).

2) O. Cata, M. Golterman and S. Peris, “Unraveling duality violations in hadronic tau decays,” Phys. Rev.**D77**, 093006 (2008).

3) O. Cata, M. Golterman and S. Peris, “Possible duality violations in tau decay and their impact on the determination of alpha(s),” Phys. Rev.**D79**, 053002 (2009).

4) D. Boito, O. Cata, M. Golterman, M. Jamin, K. Maltman, J. Osborne and S. Peris, “A new determination of α_s from hadronic τ decays,” Phys. Rev. **D84**, 113006 (2011).

5) D. Boito, M. Golterman, M. Jamin, A. Mahdavi, K. Maltman, J. Osborne and S. Peris, “An Updated determination of α_s from τ decays,” Phys. Rev. **D85**, 093015 (2012).

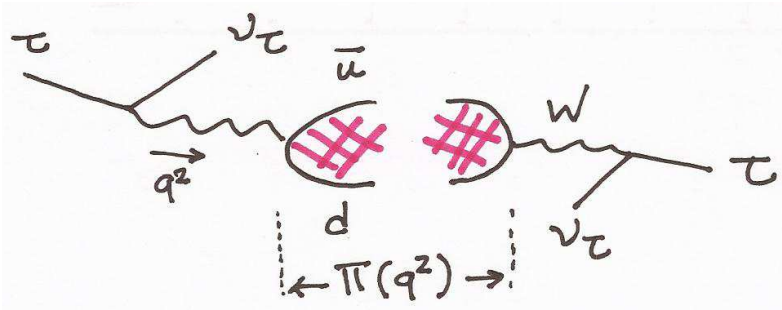
6) D. Boito, M. Golterman, K. Maltman, J. Osborne and S. Peris, “Strong coupling from the revised ALEPH data for hadronic τ decays,” Phys. Rev. **D91**, no. 3, 034003 (2015).

7) S. Peris, D. Boito, M. Golterman and K. Maltman, “The case for duality violations in the analysis of hadronic τ decays,” Mod. Phys. Lett. **A31**, no. 30, 1630031 (2016).

8) D. Boito, M. Golterman, K. Maltman and S. Peris, “Strong coupling from hadronic τ decays: A critical appraisal,” Phys. Rev. **D95**, no. 3, 034024 (2017).

9) D. Boito, I. Caprini, M. Golterman, K. Maltman and S. Peris, “Hyperasymptotics and quark-hadron duality violations in QCD,” Phys. Rev. **D97**, no. 5, 054007 (2018).

QCD in τ decay

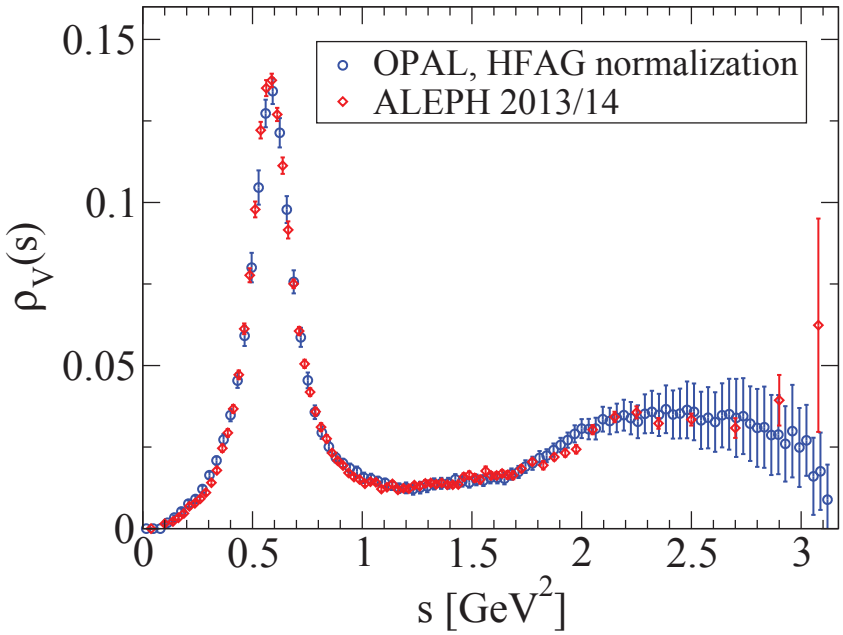


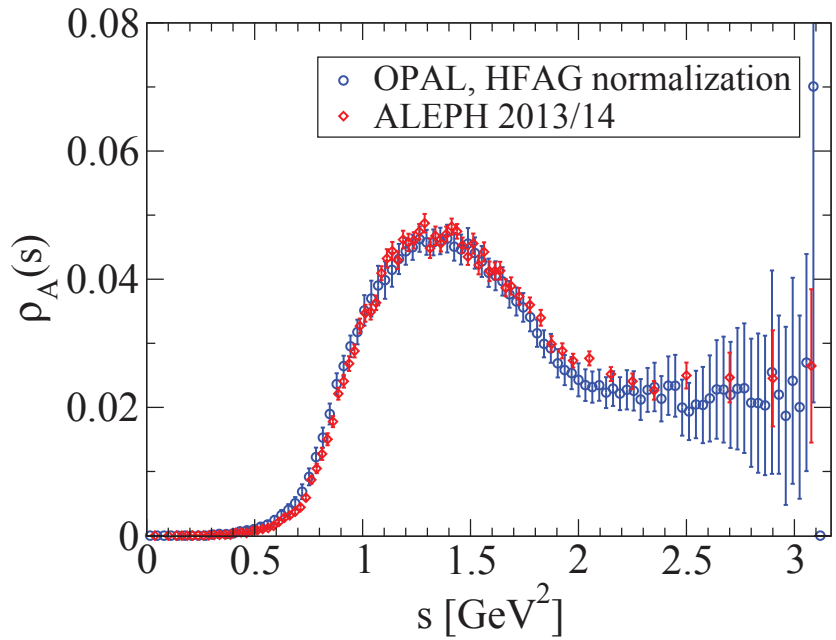
$$w_T(s;s_0) = \left(1+2\frac{s}{s_0}\right)\underbrace{\left(1-\frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$$w_L(s;s_0) = 2\left(\frac{s}{s_0}\right)\underbrace{\left(1-\frac{s}{s_0}\right)^2}_{\text{doubly pinched}}$$

$$\boxed{s_0 = m_\tau^2} \qquad \rho_{V,A} = \frac{1}{\pi}\mathrm{Im}\Pi_{V,A}$$

$$\frac{\Gamma(\tau\to\nu_\tau \mathrm{H}_{ud}(\gamma))}{\Gamma[\tau\to\nu_\tau e\bar{\nu}_e(\gamma)]} = 12\pi^2|V_{ud}|^2 S_{EW}\int_0^{s_0}\frac{ds}{s_0}\left[w_T(s;s_0)\rho_{V+A}^{(1+0)}(s) - w_L(s;s_0)\rho_A^{(0)}(s)\right]$$



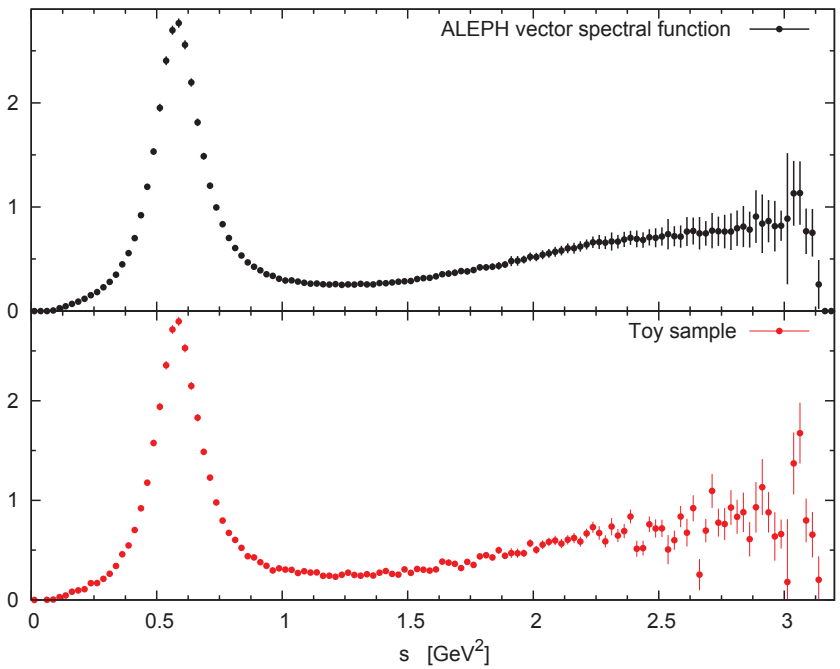


Since 2014: New correlation matrices

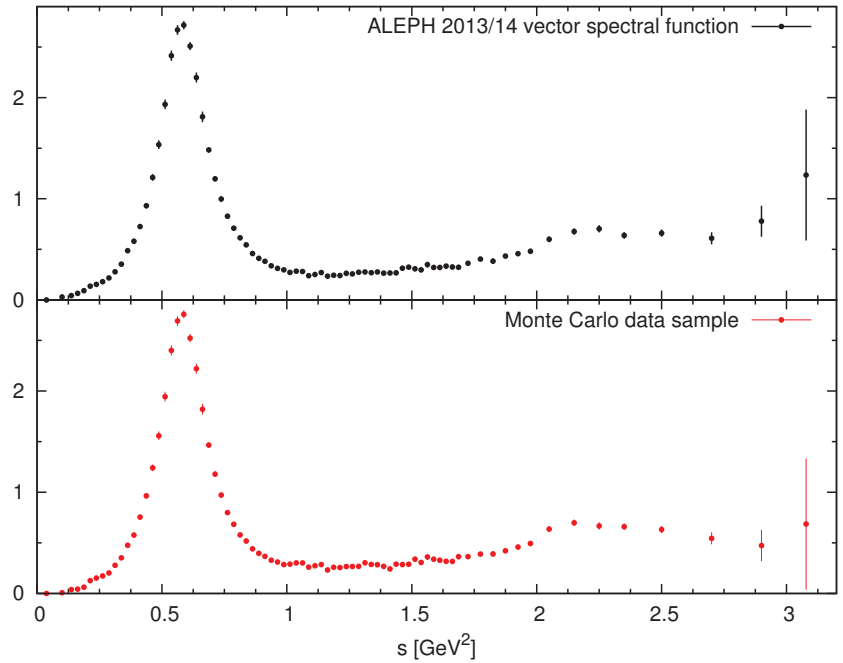
Davier et al. '14

- Pseudo-data analysis (Boito et al. '11):

2011



2014



PROBLEM !

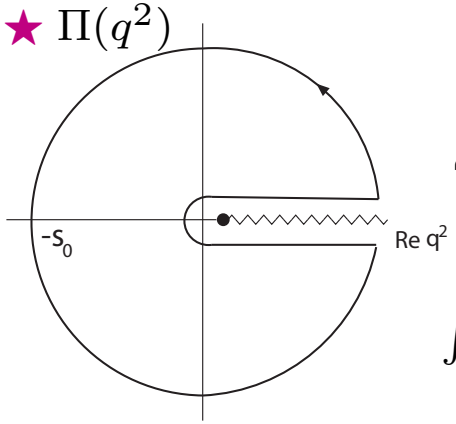
OK!

Warning: results based on $t < 2014$ data, unreliable.

Theoretical Foundations (I)

Shankar '77; Braaten-Narison-Pich '92

★ $\Pi(q^2)$



"Cauchy's Theorem" $(z = q^2\ ;\ w = \text{polynomial})$:

$$\int_0^{s_0} dt\, w(t)\ \underbrace{\frac{1}{\pi}\mathrm{Im}\Pi(t)}_{exp.} = \frac{-1}{2i\pi}\oint_{|z|=s_0} dz\, w(z)\ \underbrace{\Pi(z)}_{\Pi_{\rm OPE}(z)+\Pi_{DV}(z)}$$

$$= \frac{-1}{2i\pi}\oint_{|z|=s_0} dz\, w(z)\ \underbrace{\Pi_{\rm OPE}(z)}_{\mathcal{O}(\alpha_s^5)+condensates} - \int_{s_0}^{\infty} dt\, w(t)\ \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(t)$$

(Cata-Golterman-S.P. '05, '08, '09)

Note,

- $w(t) \sim t^N \Leftrightarrow$ contribution from $\Pi_{OPE} \sim \frac{C_{2N+2}}{s_0^{N+1}}$ $\quad$ (C_N, condensates).
- $\Pi(z) = \Pi_{OPE}(z) + \Pi_{DV}(z)$. $\quad$ OPE convergent $\Leftrightarrow \Pi_{DV} = 0$.
- $\Pi_{\rm OPE}$ expected asymptotic : $\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(t) \sim\ e^{-\gamma t} \times (oscillation), \quad t \to \infty$.

(Notice $\frac{1}{\pi}\mathrm{Im}\Pi_{DV}(t)$ is on the Minkowski axis.)

$\Rightarrow$ use polynomial with zero $w(s_0) = 0$ ("pinching")

Warning: more pinching $\Rightarrow$ higher degree polynomial $\Rightarrow$ higher condensates to consider.

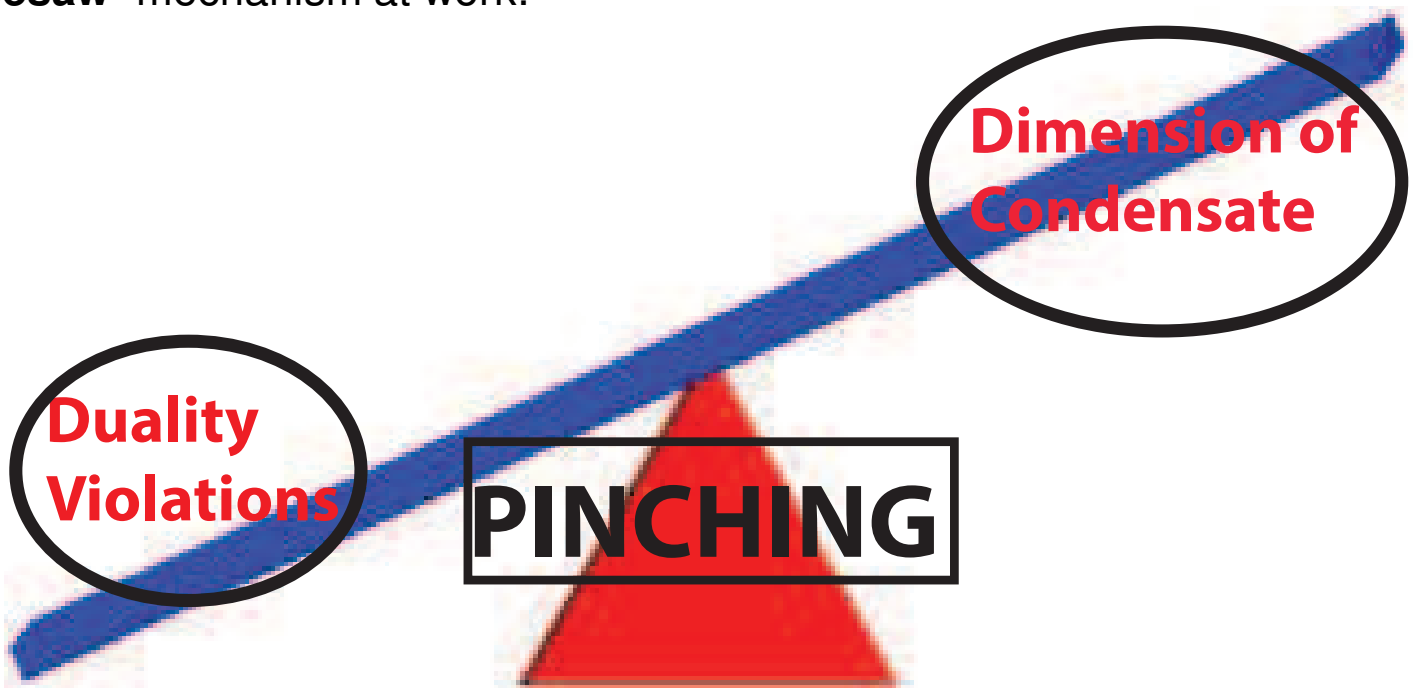
Main Theoretical Message:

(Maltman-Yavin '08, Boito et al. '11)

★ No free lunch. With pinching one has a **price to pay**:

It is not possible to simultaneously **suppress DVs** and **condensates**

★ "**Seesaw**" mechanism at work:



Theoretical Foundations (II)

- Asymptotic Regge-like spectrum for $N_c = \infty$ in chiral limit:

-Masses

$$M^2(n) = n\,\Lambda^2_{QCD} + b\log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right) \quad , \quad n \gg 1\,.$$

-Decay constants

$$F(n) \propto 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right) \quad , \quad n \gg 1\,.$$

- When $N_c = 3,\ n \gg 1$ ($d = 2$ QCD, strings, phenomenology):

Blok et al. '98; Shifman et al. '08; Masjuan et al. '12

$$\frac{\Gamma}{M} \sim \frac{a}{N_c} + \ldots$$

Boito et al. '17

leads to ($\mathrm{Re}\, q^2 > 0\,,\ \mathrm{Im}\, q^2 > 0$)

$$\frac{1}{\pi}\mathrm{Im}\,\Pi_{DV}(q^2) \sim \mathrm{e}^{-2\pi\frac{a}{N_c}\frac{q^2}{\Lambda^2_{QCD}}} \sin\left[\frac{2\pi}{\Lambda^2_{QCD}}\left(q^2 - c - b\log\frac{q^2}{\Lambda^2_{QCD}}\right)\right]\left(1 + \mathcal{O}\left(\frac{1}{N_c}; \frac{1}{q^2}; \frac{1}{\log q^2}\right)\right)$$

$$\equiv e^{-\delta-\gamma q^2}\ \sin\left(\alpha + \beta\, q^2\right) \quad (\text{up to logs})$$

(OK OPE and Poggio, Quinn and Weinberg '76)

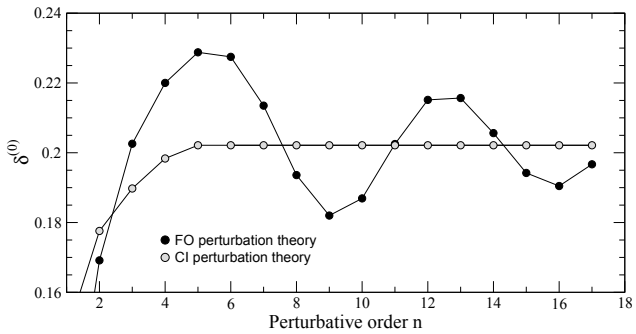
Pert. Theory: CIPT vs. FOPT

Caprini-Fischer '09; Menke '09; Descotes-Malaescu '09; Cvetic '10,...

★Partial integration: $\oint_{|z|=s_0} dz\; w(z)\; \Pi(z) = \oint_{|z|=s_0} dz\; \widetilde{w}(z)\; \underbrace{\left(-z\frac{d}{dz}\Pi(z)\right)}_{\text{Adler's D(z)}}$

★ $\mathrm{D}(z) = \frac{1}{4\pi^2}\sum_{n,k}\; c_{n,k}\; \underline{\alpha_s(\mu)^n\; \log^k\left(-z/\mu^2\right)}$, what μ ?

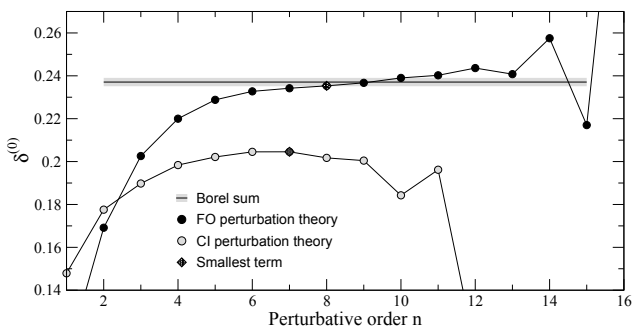
★ Since $z = s_0\; \mathrm{e}^{i\phi}$, two ($\sim$ extreme) choices:



• CIPT: $\mu^2 = -z.$

Good *if* no important cancellations between Adler function series and running of α_s (e.g. if Adler function stops at finite order)

LeDiberder-Pich '92, Pivovarov '92



• FOPT: $\mu^2 = s_0$.

Good *if* there are cancellations between Adler function series and the running coupling (due to the leading IR renormalon contribution).

Beneke-Jamin '08, Beneke-Boito-Jamin '13

The Truncated-OPE Strategy

(LeDiberder-Pich '92)

★ τ width depends on α_s and $C_{6,8}$ $\Rightarrow$ more input needed :

- Use 5 pinched weights

$$w_{kl}(y) = (1+2y)\,(1-y)^{2+k}\,y^l \quad , \quad y = s/s_0, \quad s_0 = m_\tau^2 \text{ (only)}$$

with $(k,l) = \{(0,0),(1,0),(1,1),(1,2),(1,3)\}$.

- Fit to extract 4 parameters : α_s and $C_{D=4,6,8}$.

- Neglect (unknown) OPE condensates $C_{D=10,12,14,16}$:

$$\Pi_{OPE} \sim \frac{C_D}{m_\tau^D} \sim 0 \quad (D = 10, 12, 14, 16, \sim \text{OPE convergent})$$

- Assume (unknown) Duality Violations also negligible.

- May use V and A, but assume $V+A$ more reliable (see later).

(Davier et al. '14)

$$\langle \frac{\alpha_s}{\pi} GG \rangle = (-0.5 \pm 0.3) \times 10^{-2}\ \text{GeV}^4\ , \qquad V\ ,$$

$$(-2.0 \pm 0.3) \times 10^{-2}\ \text{GeV}^4\ , \qquad V+A\ .$$

The DV Strategy

(Boito et al. '11 and '12)

- Do not use $w(y)$ with a term linear in y. (Beneke et al. '13)
- No neglect of any contributing condensate. Let the data speak.
- Do not assume that Duality Violations are zero. Let the data speak.

  For $s \geq s_{min}$ (Regge/asymp. series):

$$\frac{1}{\pi}\mathrm{Im}^{V,A}_{DV}(s) = \mathrm{e}^{-\delta_{V,A}-\gamma_{V,A}\,s}\sin\left(\alpha_{V,A}+\beta_{V,A}s\right)$$

  c.f. Truncated-OPE assumption $\Leftrightarrow$ $\mathrm{e}^{-\delta_{V,A}} = 0$.

- Fit to $\alpha_s, C_{D=6,8}$ and DV parameters with 3 weights (avoid high-orders in OPE):

$$w_0 = 1, w_2 = 1-y^2 \quad \text{and} \quad w_3 = (1-y)^2(1+2y)$$

  Use all data for $s_0 \geq s_{min}$, (s_{min} to be determined by the fit as well).

- Use V and A.
- Make many checks: Weinberg sum rules, etc...

We did lots of other fits as well...

Fits :

- V channel, $w_0 = 1$.
- V and A channels, $w_0 = 1$.
- V channel, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V and A channels, $w_0 = 1$ and $w_2 = 1 - y^2$.
- V channel, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1-y)^2(1+2y)$.
- V and A channels, $w_0 = 1, w_2 = 1 - y^2$ and $w_3 = (1-y)^2(1+2y)$.

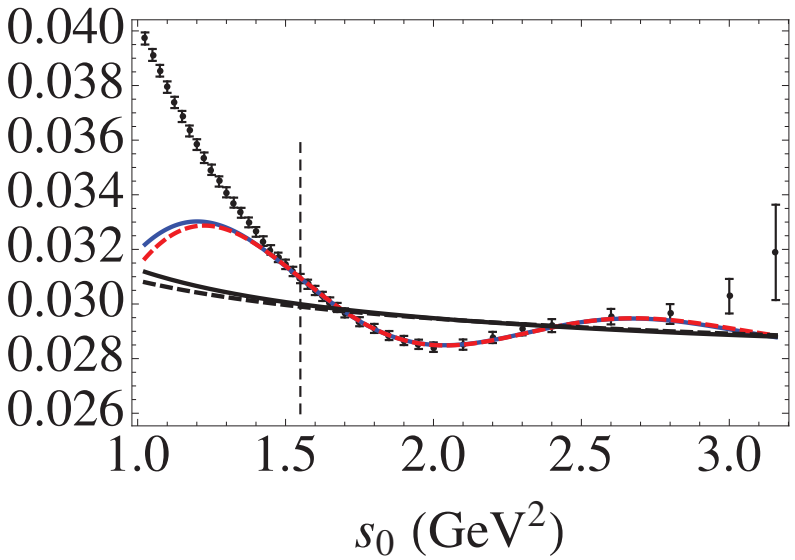
Consistent results in all cases

Example: Fit to $w_0 = 1$, V channel (I).

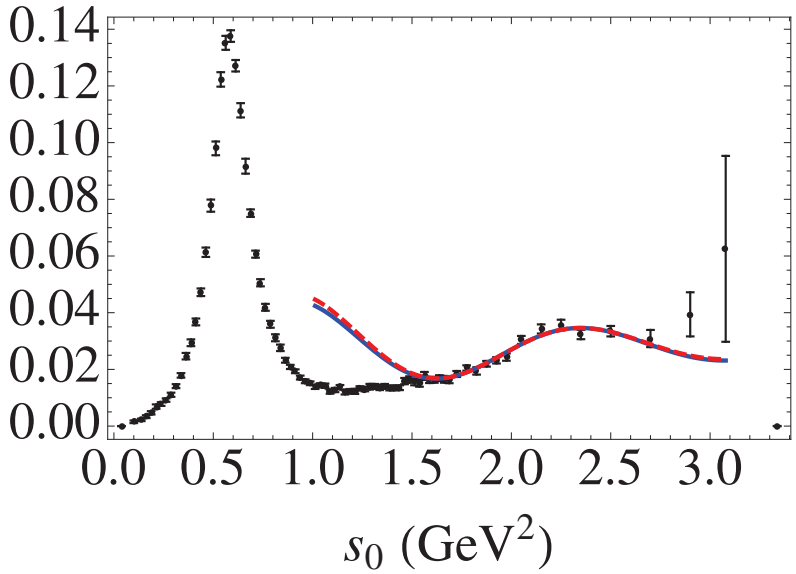
$s_{min} = 1.55\ \mathrm{GeV}^2$, $\chi^2/dof = 24.5/16$ $(p = 8\%)$ (This is FOPT, CIPT similar)

<u>curves</u>: red=CIPT blue =FOPT black =no DV

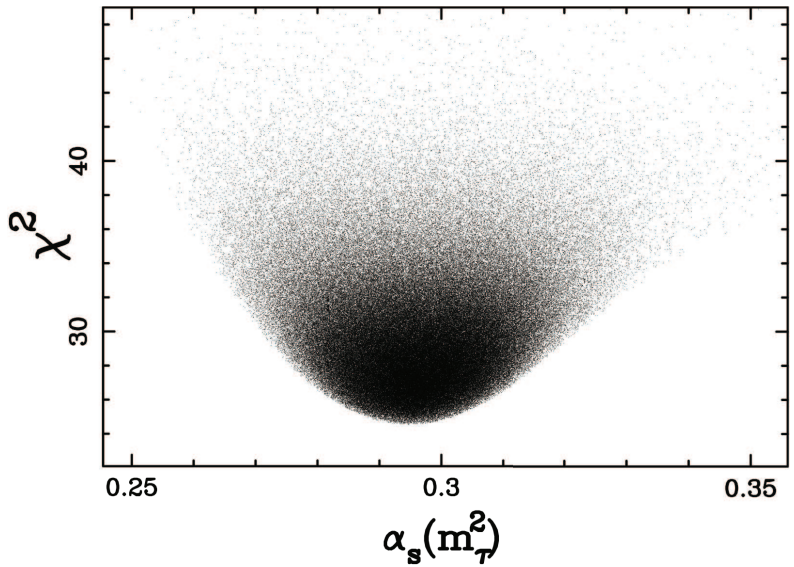
$w_0 = 1$ spectral integral

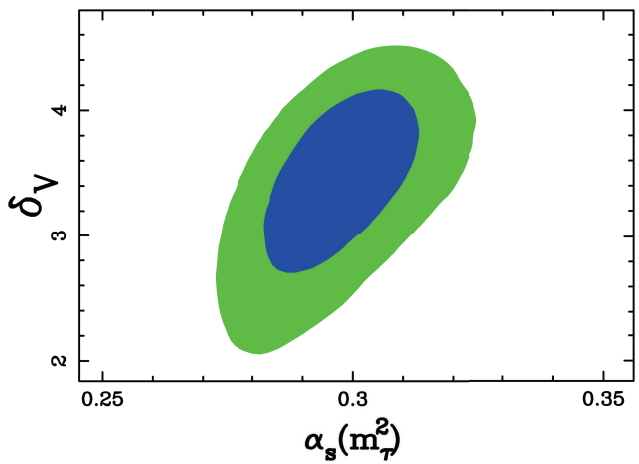


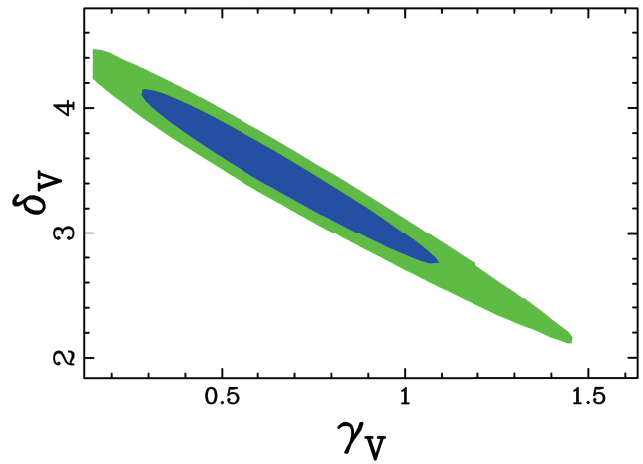
ALEPH's V spectrum



Example: Fit to $w_0 = 1$, V channel (II).





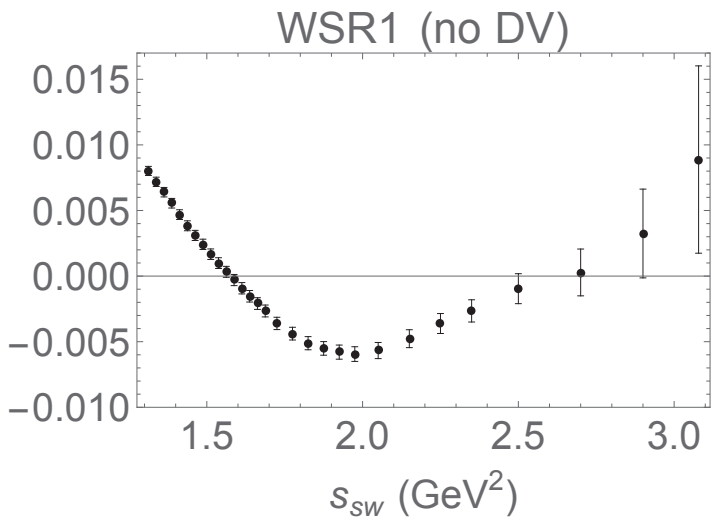


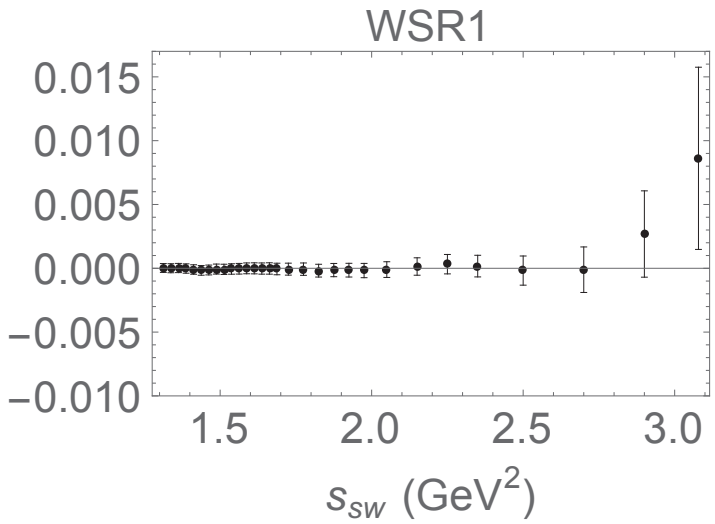
(68% and 95% contour plots), FOPT.

Clearly $DVs \neq 0$.

Classic Tests

Weinberg sum rule: $\int_0^\infty ds \left(\rho_V^{(1)}(s) - \rho_A^{(1)}(s)\right) - 2f_\pi^2 = 0$



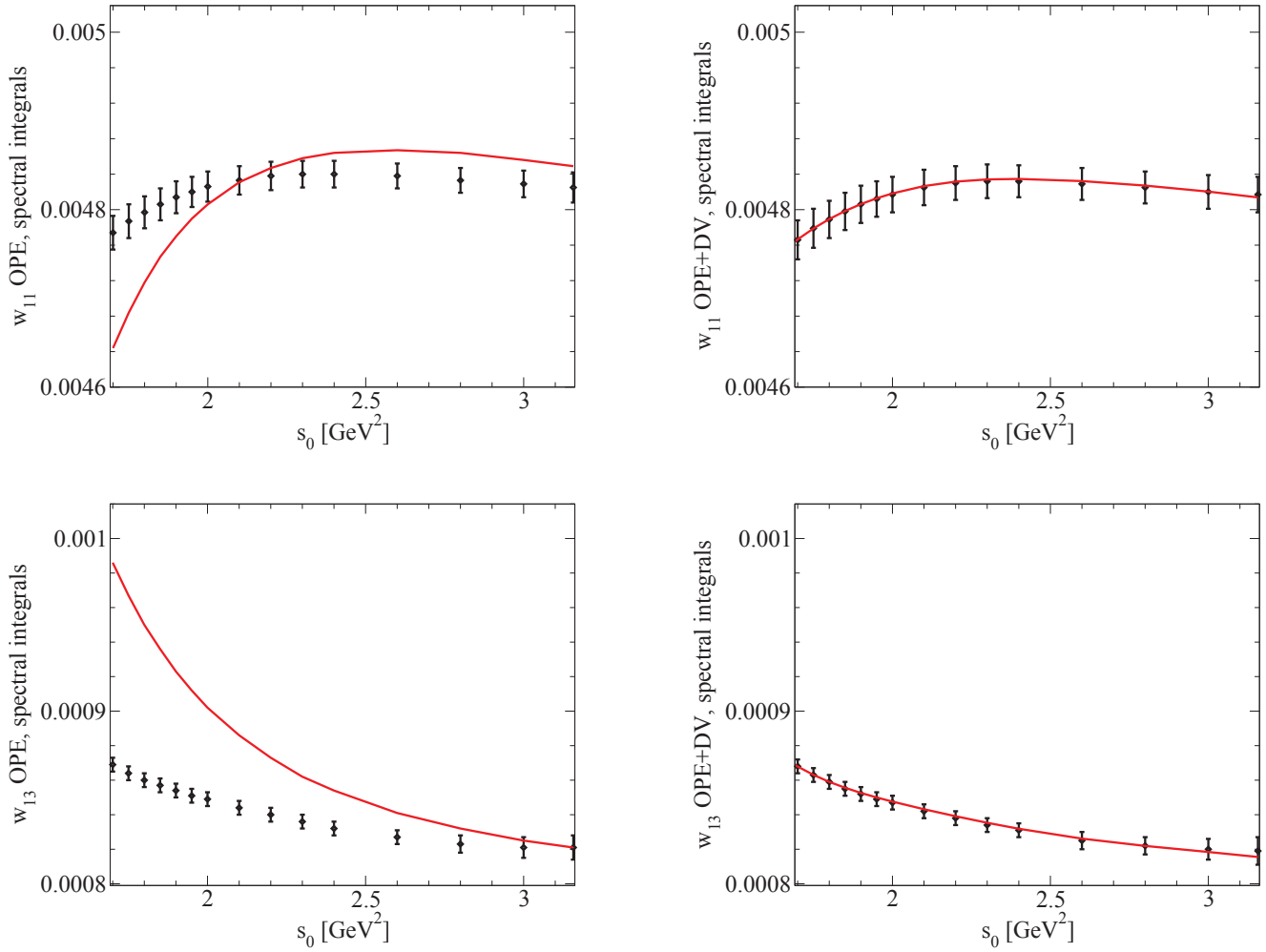


etc...

Further Tests

Truncated-OPE vs. DV strategies in FESRs ($V+A$, CIPT): w_{11}, w_{13}, etc... (many more)

Truncated-OPE | **with DV**



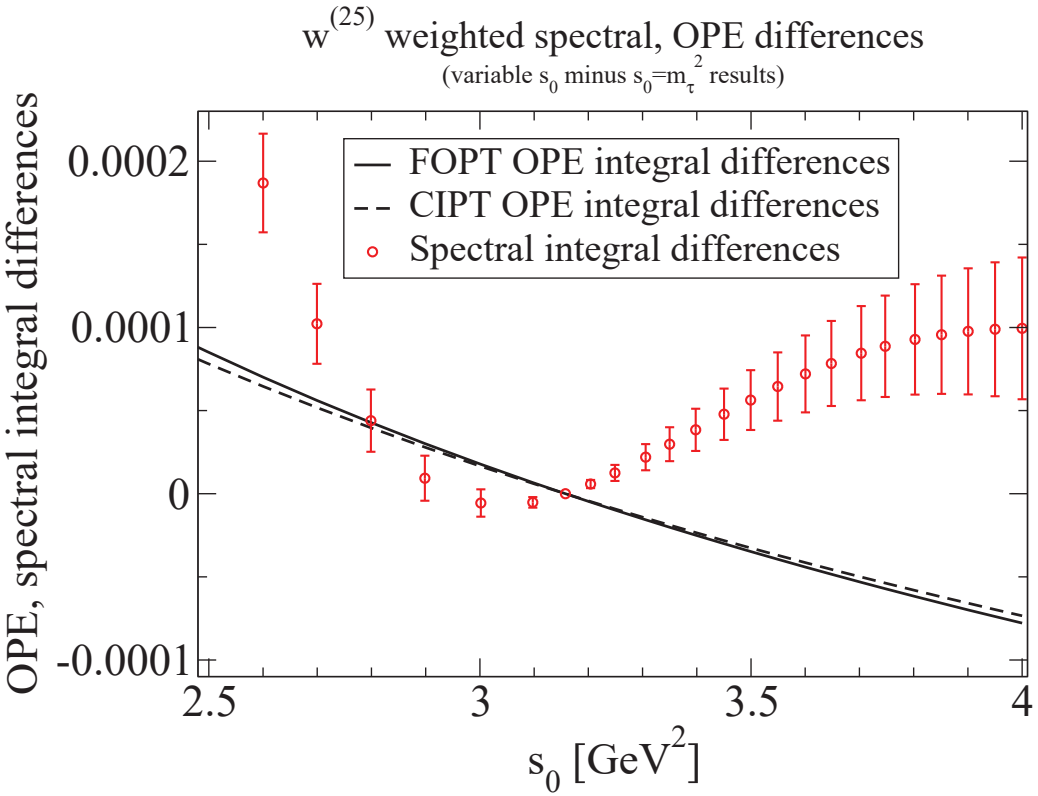
Tests using e^+e^- data: above m_τ !

(e^+e^- data compilation from Keshavarzi, Nomura, Teubner '18)

- If truncated-OPE assumptions good for $s_0 = m_\tau^2$, should work even better for $s_0 > m_\tau^2$.
- In $SU(3)$ limit: $\quad Q = T_3 + Y \quad \Rightarrow \quad \langle QQ \rangle = \frac{4}{3}\langle T_3 T_3 \rangle$

Truncated-OPE method for $\mathrm{w}^{(2,n)}(y) = 1 - (n+2)y^{n+1} + (n+1)y^{n+2}$ (doubly pinched)

$n = 1, \ldots 5$, "optimal" weights for $\alpha_s, C_6, C_8, C_{10}$ with $C_{12} = C_{14} = C_{16} = 0$ (Pich and Rodriguez-Sanchez '16)



(WORK IN PROGRESS)

Note $s_0 > m_\tau^2$!

Results

DV strategy results with **(ALEPH data)**: $\delta\alpha_s(M_Z) \simeq \left(\frac{\alpha_s(M_Z)}{\alpha_s(m_\tau)}\right)^2 \delta\alpha(m_\tau)$

(FOPT)	$\alpha_s(m_\tau) = 0.296 \pm 0.010 \longrightarrow \alpha_s(m_Z) = 0.1155 \pm 0.0014$
(CIPT)	$\alpha_s(m_\tau) = 0.310 \pm 0.014 \longrightarrow \alpha_s(m_Z) = 0.1174 \pm 0.0019$

- N.B. Truncated-OPE strategy produces a shift, i.e. (Davier et al. '14; Pich-Rodriguez Sanchez '16)

$$\Delta\alpha_s(m_\tau) \sim +0.02 \qquad \underline{\text{higher}}$$

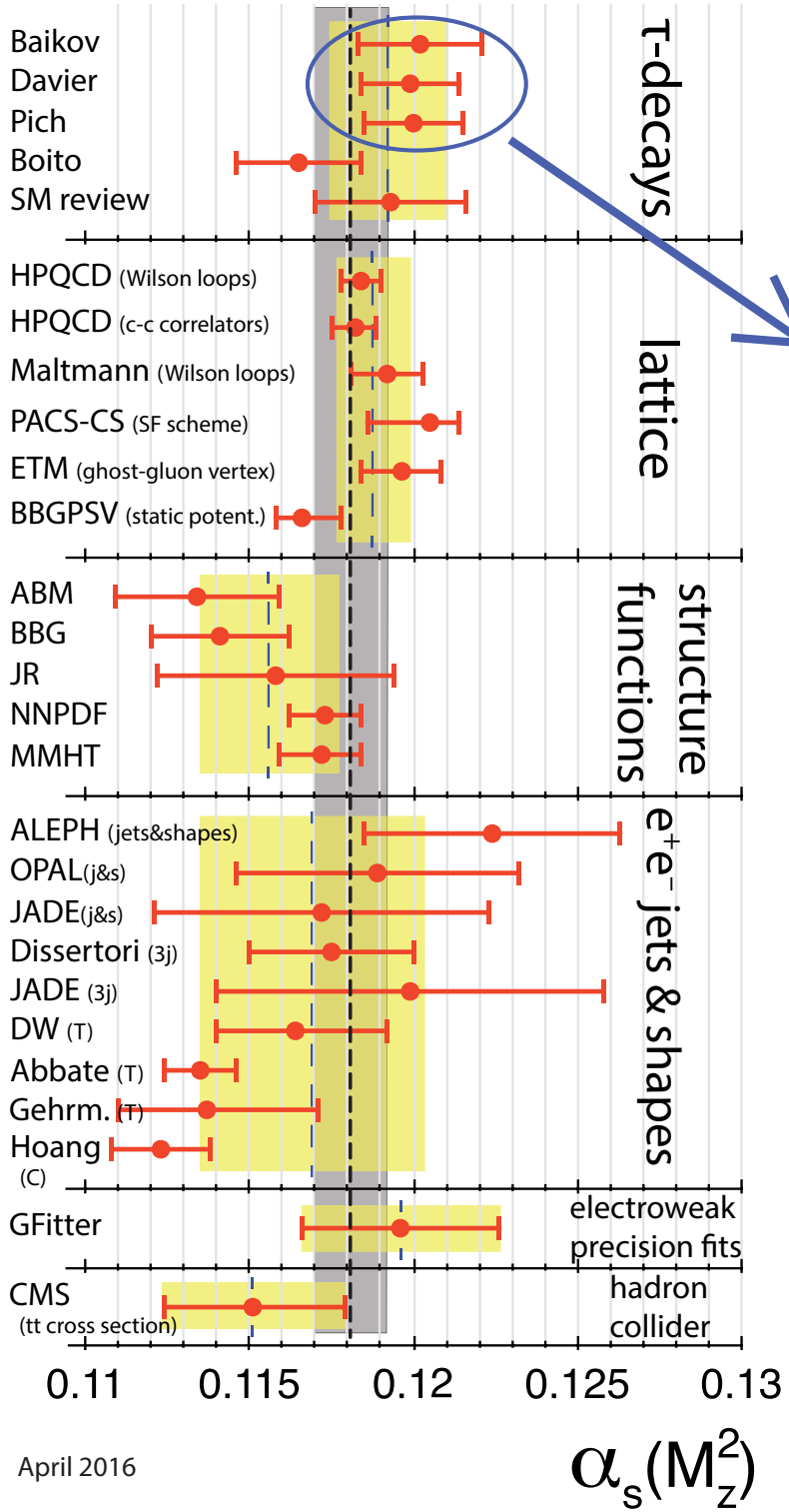
- DV strategy results with **ALEPH + OPAL** data:

$\alpha_s(m_Z) = 0.1165 \pm 0.0012$ (FOPT) $\qquad \alpha_s(m_Z) = 0.1185 \pm 0.0015$ (CIPT)

(Current PDG 2018 world average: $\alpha_s(m_Z) = 0.1181 \pm 0.0011$)

.

α_s Overview (PDG '18)



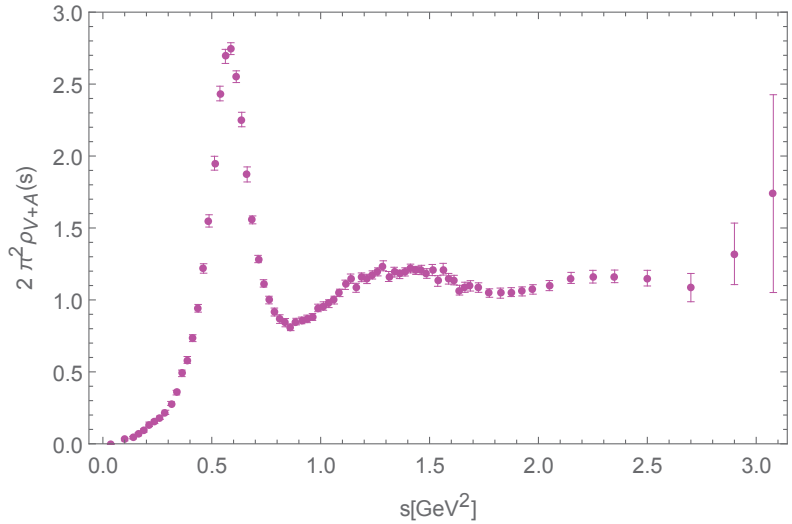
Used same ALEPH non perturbative corrections: α_s not really independent.

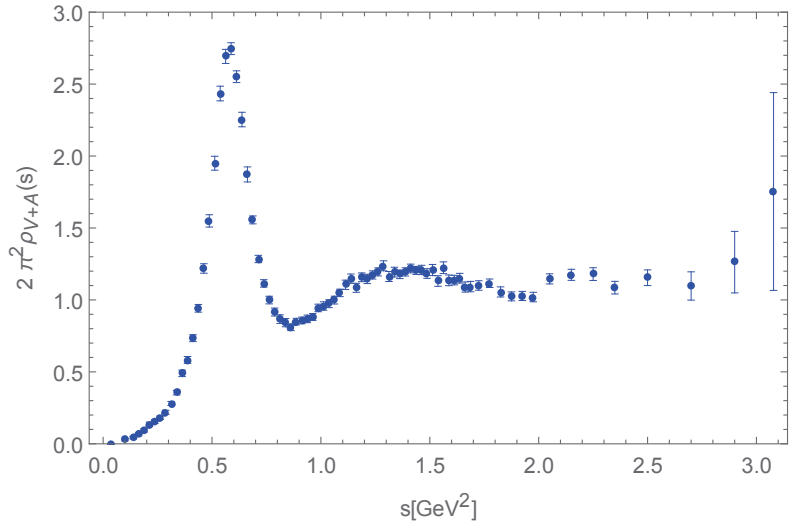
New truncated-OPE reanalysis

(Pich, Rodriguez Sanchez '16)

- Same conclusions as before (not a surprise), and same s_0 dependence problems.

All criticisms raised about DV strategy addressed in Boito et al. Phys. Rev. **D95** (2017) 034024.





- Pseudo-data analysis in $V+A$:

input CIPT, $\alpha_s(m_\tau)^{fake} = 0.312$, DV parameters, and same Cov. matrix as experiment.

- Truncated-OPE fits are good and consistent (χ^2, p-value, etc...)

but

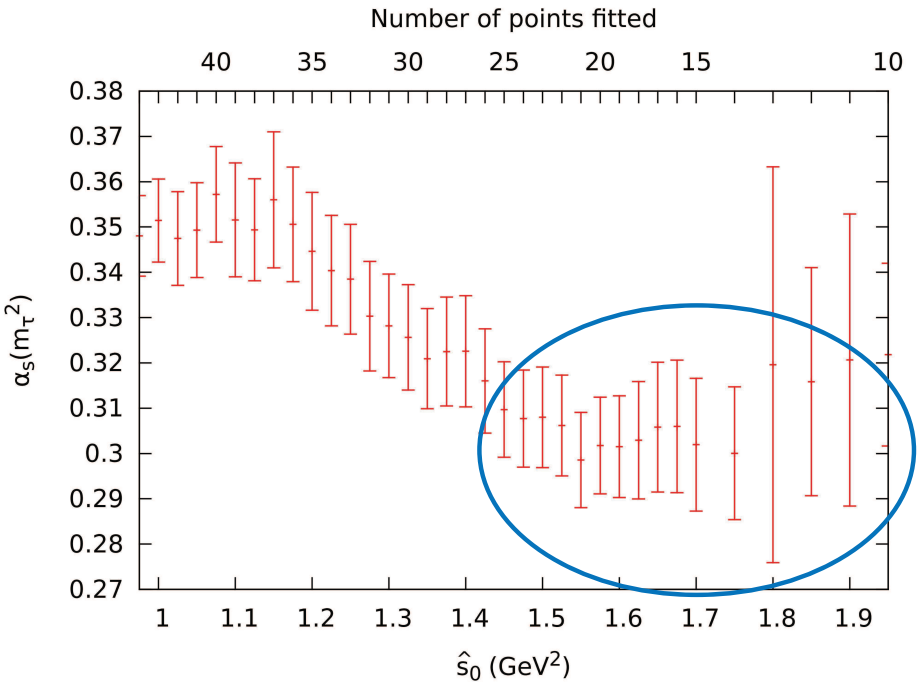
they produce a syst. error $\Delta\alpha_s(m_\tau) \sim +0.02$ with deviation from exact value $\sim 5-7\sigma$.

$\Rightarrow$ Conclusion: Truncated-OPE strategy unreliable

New truncated-OPE reanalysis (II)

(Pich, Rodriguez Sanchez '16)

- Example: when DVs are included one obtains (FOPT, V channel)



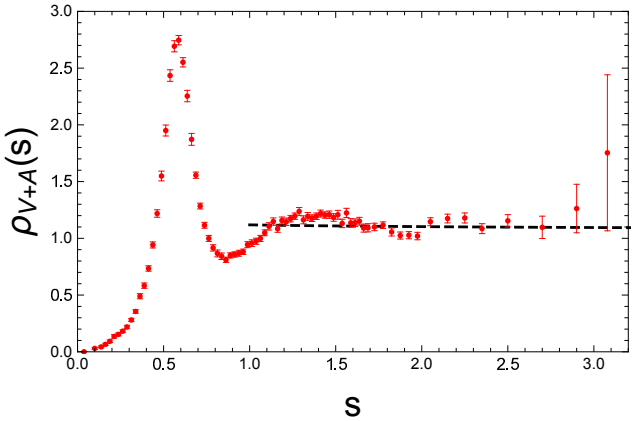
$\Leftarrow$ Stability as $\widehat{s}_0 = s_{min}$ increases, as expected for the DV ansatz. (So this is actually nice...)

DV Strategy obtains correct answer: confirmed by pseudo-data analysis.

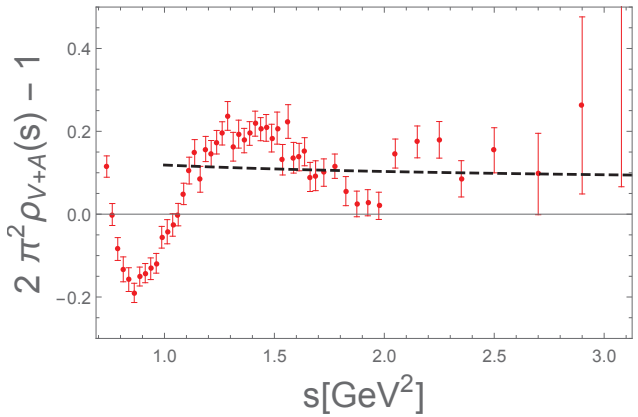
Conclusions and Outlook

- Roughly, perturbative corrections are an order of magnitude larger than nonperturbative corrections, but the latter are important to determine α_s <u>even</u> in $V + A$.

- DVs are clearly visible in the data. (Dashed line= Pert. Theory)



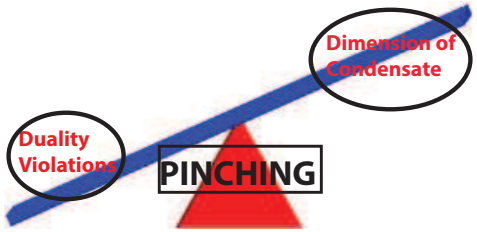
$\Longrightarrow$



(DVs are not a question of principle, they exist in practice.)

- Pinching does not allow a simultaneous reduction of DVs **<u>and</u>** higher-dim condensates

(unlike what has been assumed so far in the Truncated-OPE method).



This introduces a significant systematic error.

Conclusions and Outlook (II)

- The new strategy using the asymptotic DV form passes all known tests, experimental and theoretical, performing better than the Truncated-OPE Strategy.

- New α_s determination from e^+e^- data in good agreement with DV-strategy result from τ data. However, because of higher energy, DVs in e^+e^- marginal.

($\rightarrow$ M. Golterman's talk)

- Better data (Babar and Belle ?) would help significantly.

SUMMARY

- Using **Aleph + Opal** data, we get:

$\alpha_s(m_Z) = 0.1165 \pm 0.0012$ (FOPT) $\qquad$ $\alpha_s(m_Z) = 0.1185 \pm 0.0015$ (CIPT)

(Current PDG 2018 world average: $\alpha_s(m_Z) = 0.1181 \pm 0.0011$)

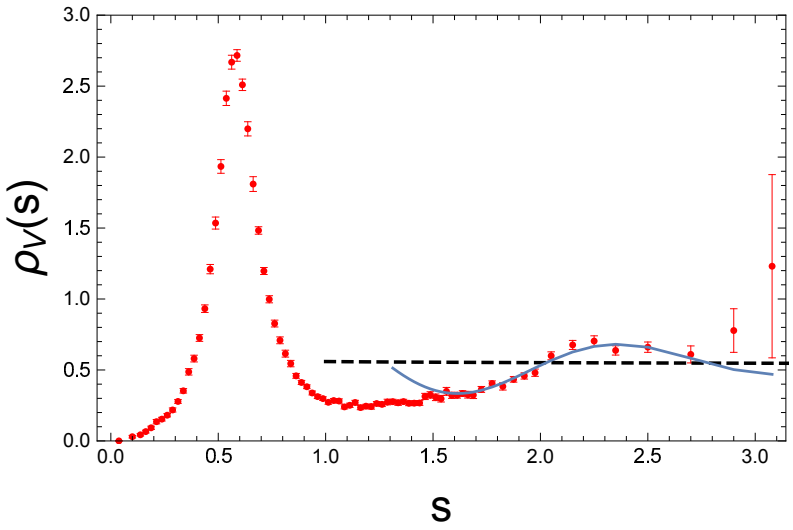
.

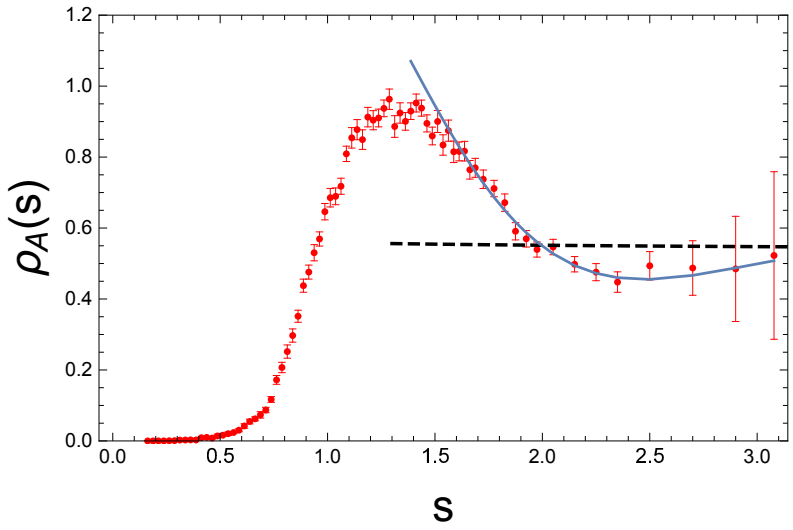
THANK YOU !

BACK-UP SLIDES

Spectral Functions

V and A spectral functions well represented by PT + asymptotic DV form:





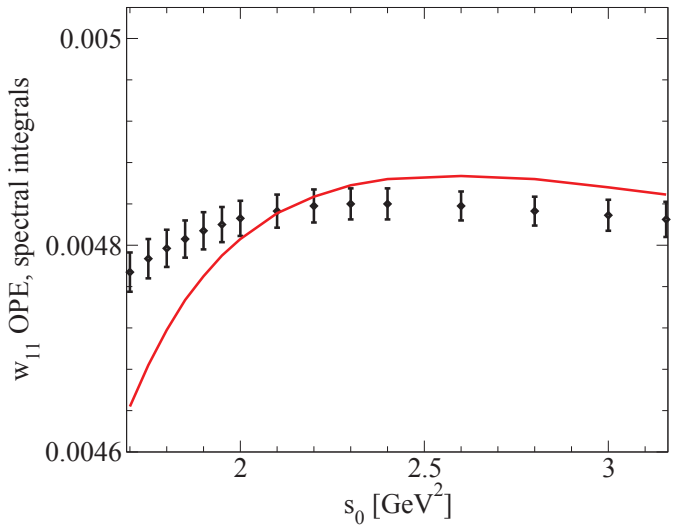
Tests: w_{11}, w_{12}, w_{13}

Looking only at $s = m_\tau^2$ potentially misleading. (Maltman-Yavin '08).

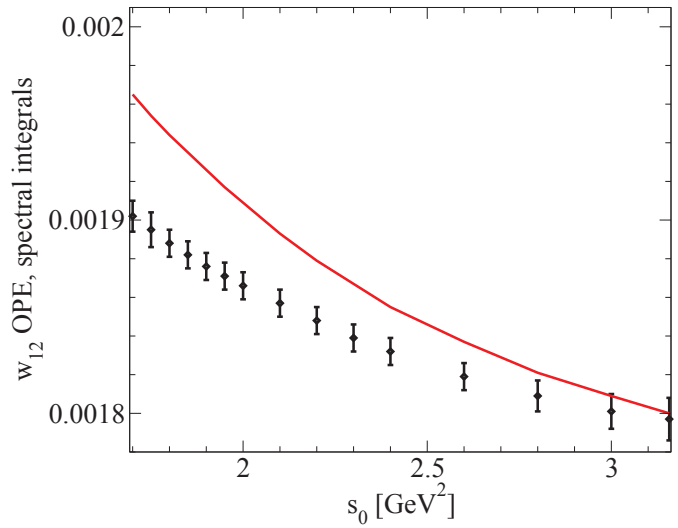
(Davier et al. '14)

CIPT, V+A:

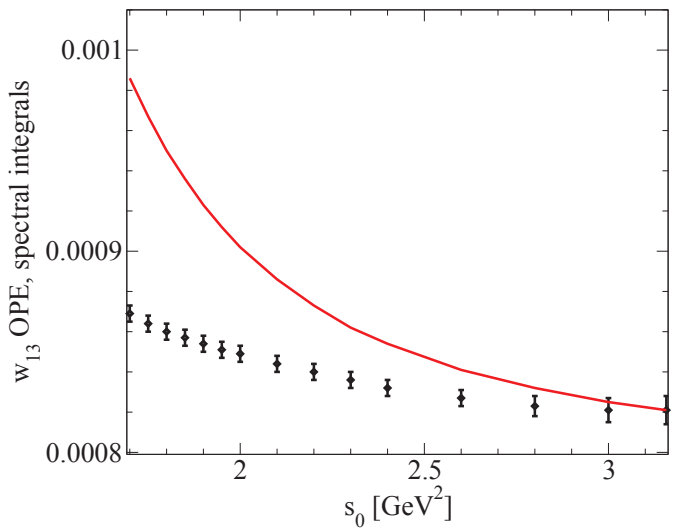
w_{11}



w_{12}



w_{13}



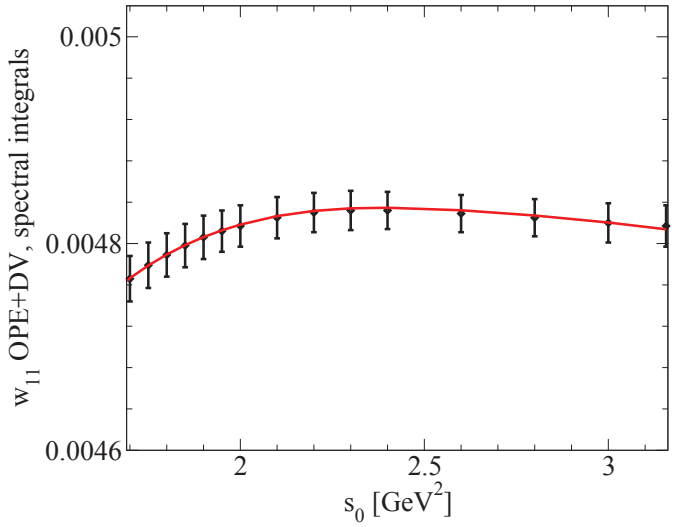
Tests: w_{11}, w_{12}, w_{13}

$D > 8$ condensates vital !

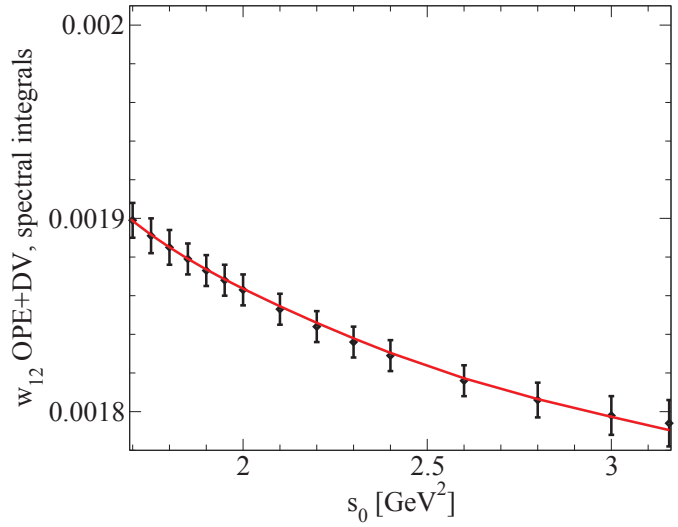
(Boito et al. '15)

CIPT, V+A:

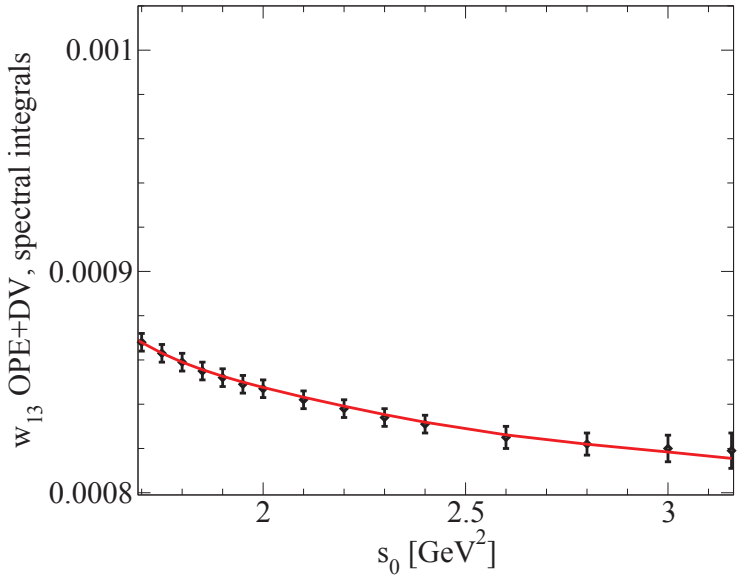
w_{11}



w_{12}



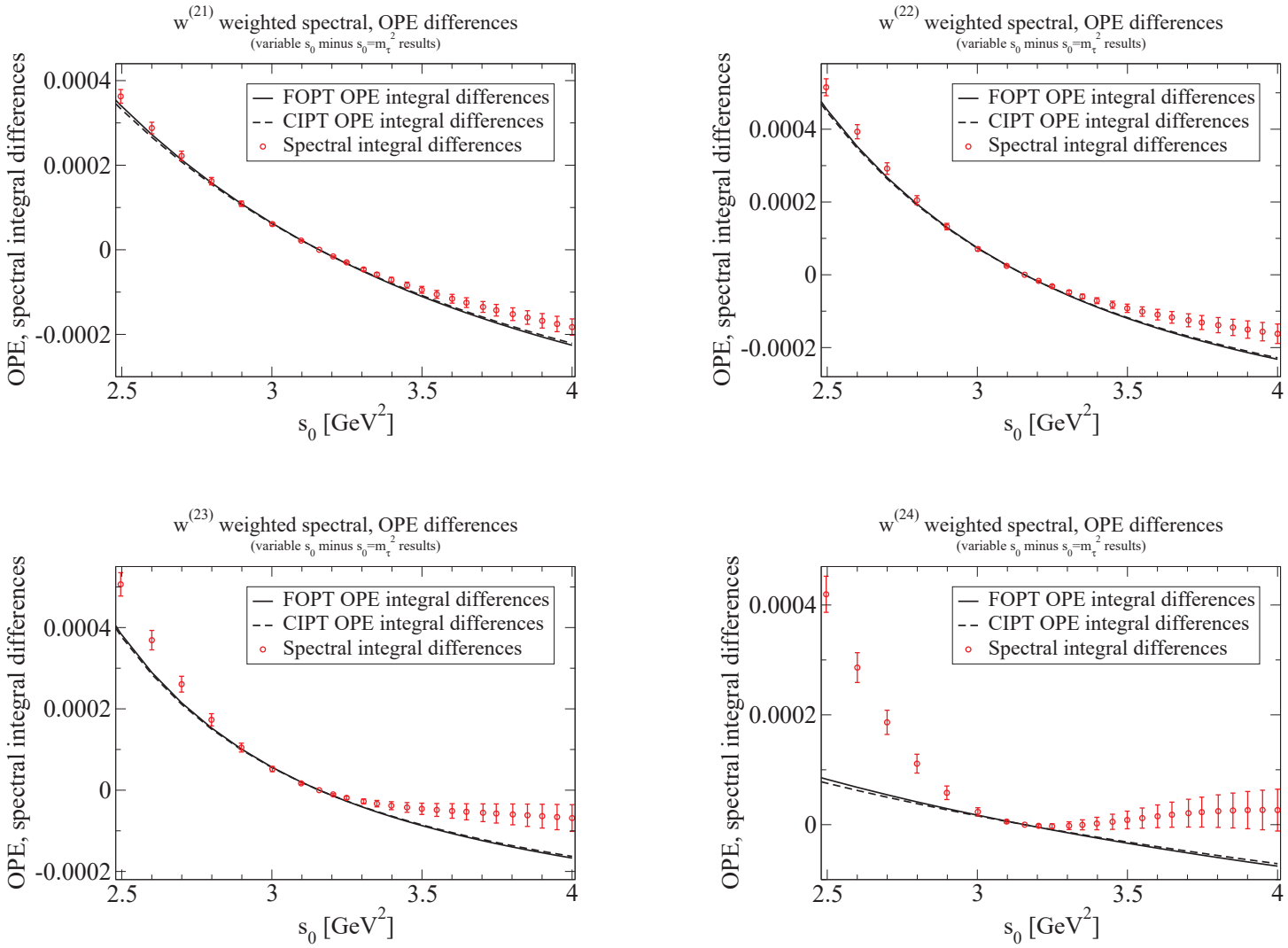
w_{13}



Additional consistency checks on use of asymptotic DV+OPE form.

.

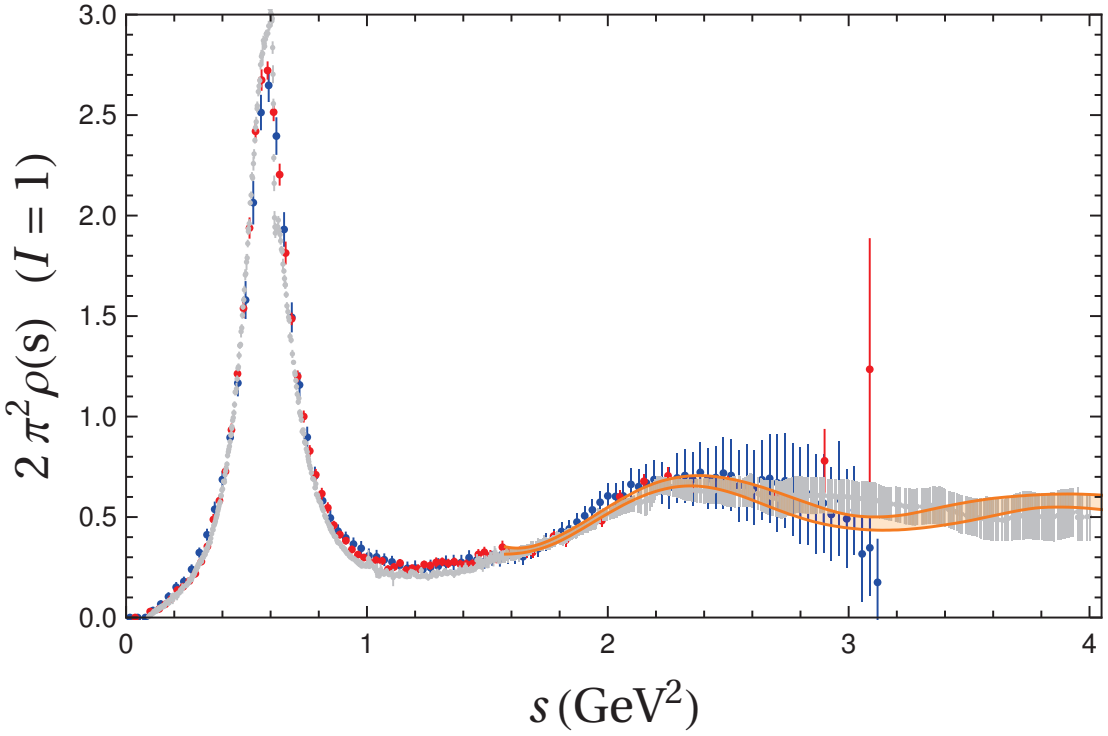
More tests using e^+e^- data: above m_τ !



WORK IN PROGRESS

Comparing DVs in τ and $I = 1$ e^+e^- data

WORK IN PROGRESS...



Blue: OPAL's τ data.

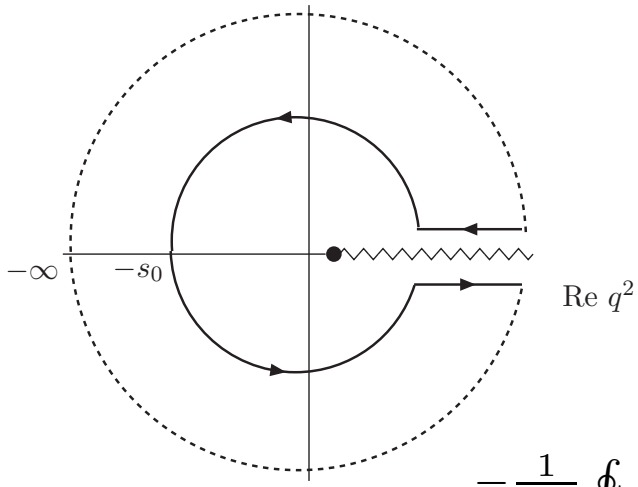
Red:ALEPH's τ data.

Grey: e^+e^-, $(I = 1)$.

Orange: ALEPH's τ data, extrapolated with DVs.

Theoretical Foundations

- $\Pi_{DV}(s) \to 0$, as $s \to \infty$. Then:



(Cata-Golterman-S.P. '05)

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\; w(z)\;\Pi_{DV}(z) = -\underbrace{\int_{s_0}^{\infty} ds}_{\text{extrapolation!}}\;\; w(s)\;\; \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s)$$

Towards a Theory of DVs ? (I)

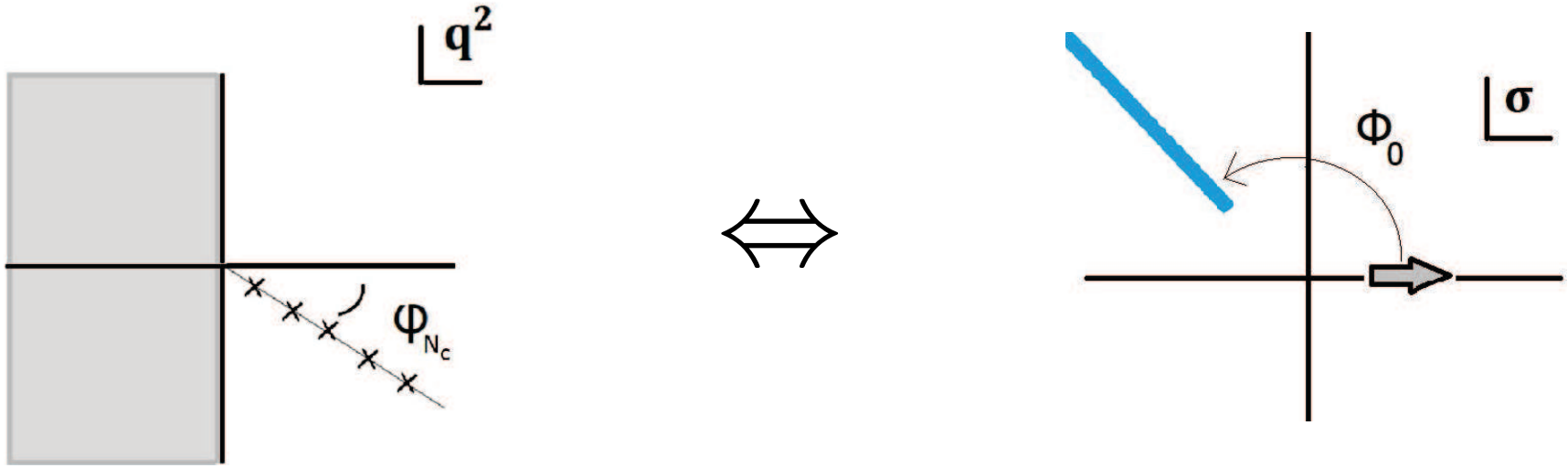
Boito, Caprini, Golterman, Maltman, SP '17

-A theory of DVs requires NP input.

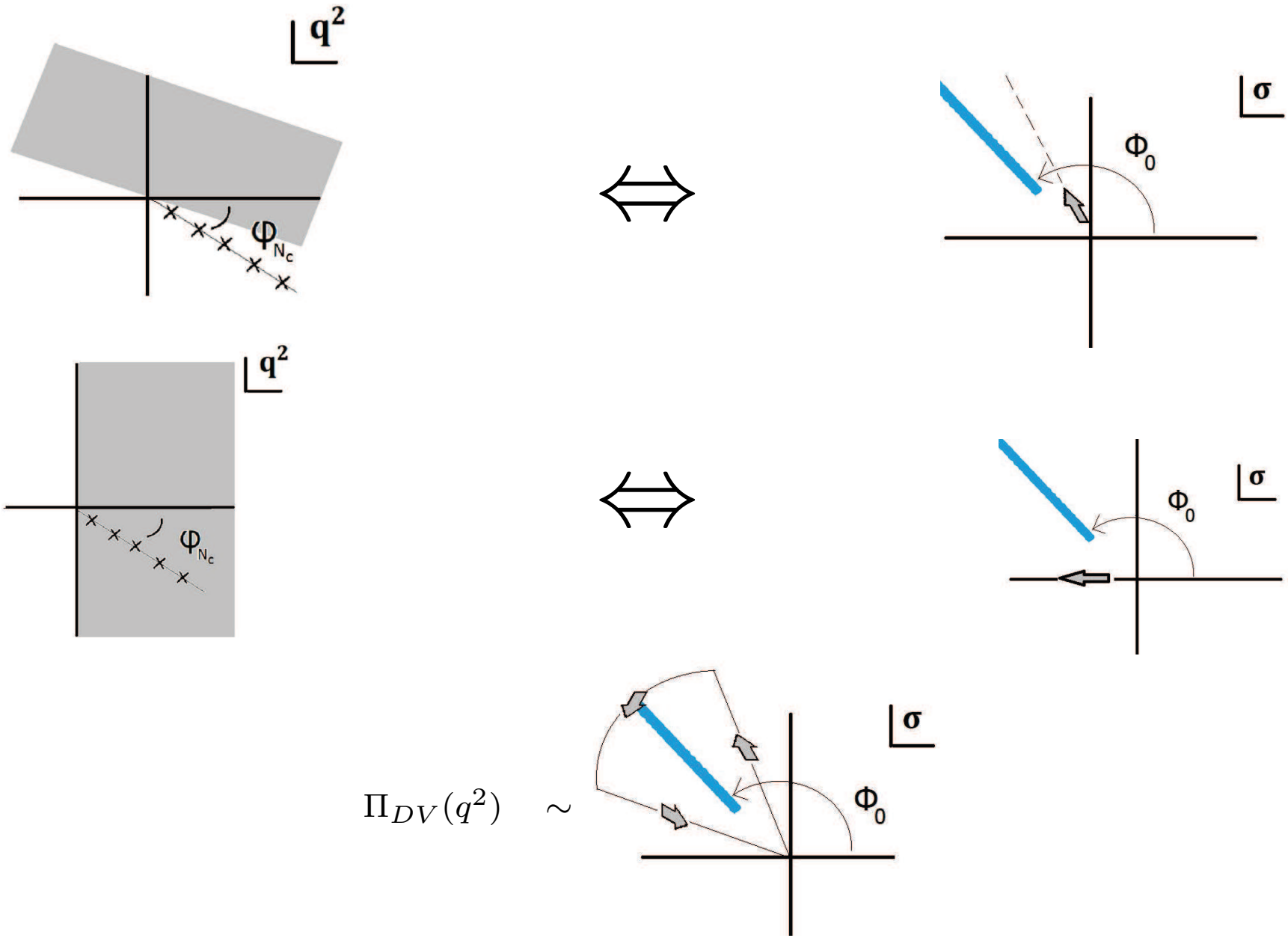
-Let's start with (exact !) Disp. Rel. of Adler's function ($q^2 < 0$, Euclidean):

$$\mathcal{A}(q^2) = -q^2 \int_0^\infty d\sigma \, \mathrm{e}^{\sigma q^2} \, \sigma \mathcal{B}(\sigma) \quad , \quad \mathcal{B}(\sigma) = \int_0^\infty dt \, \rho(t) \, \mathrm{e}^{-\sigma t}$$

$\sigma q^2 = (\sigma > 0, q^2 < 0) = (\sigma < 0, q^2 > 0)$, i.e. rotate $\sigma \Leftrightarrow q^2$ analytic continuation.



Towards a Theory of DVs ? (II)



OPE + DV ($N_c = \infty$)

-Asymptotic Regge behavior ($\Lambda_{QCD} = 1$, Regge slope):

$$M^2(n) = n + b\log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right)$$

$$F(n) = 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right)$$

produces a Dirichlet series

$$\sigma\mathcal{B}(\sigma) = \sum_n^\infty F(n)\,\mathrm{e}^{-\sigma M^2(n)} = \underbrace{1 + \mathcal{O}\left(\log^{k_1}\sigma\right)}_{Pert.\ Theory} + \underbrace{\mathcal{O}\left(\sigma^{k_2}\log^{k_3}\sigma\right)}_{Power\ Corrections} + \underbrace{\mathcal{O}\left(\frac{1}{(\sigma-\hat{\sigma})^{1+\gamma}}\right)}_{Duality\ Violations}$$

and leads to

$$\mathcal{A}(q^2) = \underbrace{1 + \mathcal{O}\left(\frac{1}{\log^{p_1}(-q^2)}\right)}_{Pert.\ Theory} + \underbrace{\mathcal{O}\left(\frac{\log^{p_2}(-q^2)}{(q^2)^{p_3}}\right)}_{Power\ Corrections} + \underbrace{\mathcal{O}\left(\mathrm{e}^{\hat{\sigma}q^2}(-q^2)^{\gamma}\right)}_{Duality\ Violations}$$

Towards a Theory of DVs ? (III)

- Asymptotic Regge-like spectrum ($\Lambda_{QCD} = 1$, Regge slope) for $N_c = \infty$ in chiral limit:

-Masses

$$M^2(n) = n + b\log n + c + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_1}}, \frac{1}{n^{\lambda_1}(\log n)^{\nu_2}}\right) \quad , \quad n \gg 1 \, .$$

-Decay constants

$$F(n) \propto 1 + \mathcal{O}\left(\frac{1}{(\log n)^{\nu_3}}, \frac{1}{n^{\lambda_2}(\log n)^{\nu_4}}\right) \quad , \quad n \gg 1 \, .$$

- When $N_c = 3$ ($d = 2$ QCD, strings, phenomenology):

Blok et al. '98; Shifman et al. '08; Masjuan et al. '12

$$\varphi_{N_c} = -\frac{\Gamma}{M} \sim -\frac{a}{N_c} + \ldots$$

we have a branch point in σ plane at

$$\sigma_0 = 2\pi \, \mathrm{e}^{i\Phi_0} + \ldots \quad , \quad \Phi_0 = \frac{a}{N_c} + \frac{\pi}{2} + \ldots$$

Boito et al. '17

producing ($\mathrm{Re}\, q^2 > 0 \, , \, \mathrm{Im} q^2 > 0$)

$$\Pi_{DV}(q^2) \sim \mathrm{e}^{-2\pi q^2 \frac{a}{N_c}} \, \mathrm{e}^{i2\pi\left(q^2 - c - b\log q^2\right)} \left(1 + \mathcal{O}\left(\frac{1}{N_c}; \frac{1}{q^2}; \frac{1}{\log q^2}\right)\right)$$

(OK Poggio,Quinn and Weinberg '76)

Towards a Theory of DVs ? (IV)

Agreement DVs in tau decay $\Leftrightarrow$ Regge spectrum.

Fits of meson spectrum to radial trajectories:

Anisovich et al. '00; Klempt et al. '12; Masjuan et al. '12;

$$\Lambda^2 = 1.35(4)\ \mathrm{GeV}^2 \qquad , \qquad \frac{\Gamma}{M} = 0.12(8) \simeq \frac{a}{N_c}$$

leading to

$$\beta_V = \frac{2\pi}{\Lambda^2} = 4.7(2)\ \mathrm{GeV}^2 \qquad , \qquad \gamma_V = \frac{2\pi}{\Lambda^2}\frac{a}{N_c} = 0.6(4)\mathrm{GeV}^{-2}$$

to be compared with results from fits to τ decay:

$$\beta_V = 4.2(5)\ \mathrm{GeV}^2 \qquad , \qquad \gamma_V = 0.7(3)\ \mathrm{GeV}^{-2}$$

coincidence ?,... Notice the 2π.