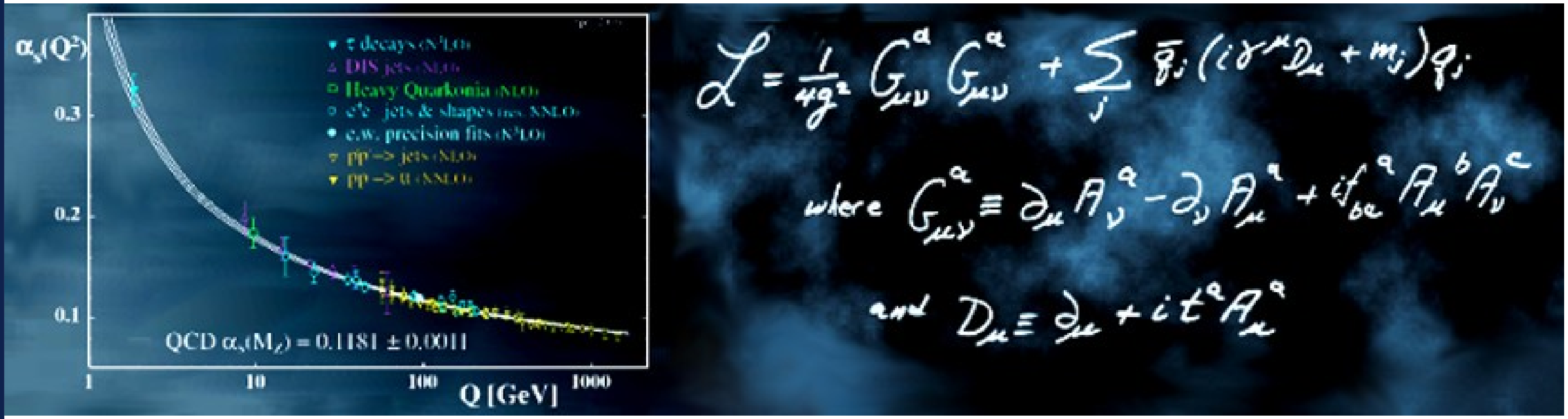


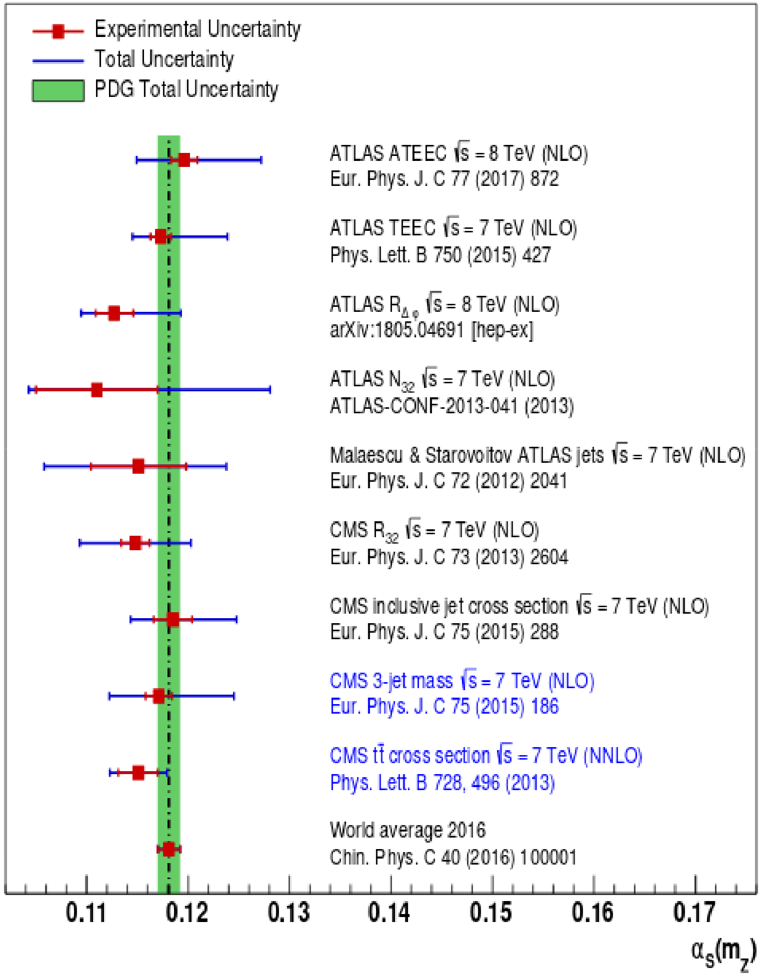
Determination of $\alpha_s(m_Z)$ from the Z-boson transverse momentum

Stefano Camarda

alphas-2019
Trento, Italy
11-15 February 2019

Measure $\alpha_s(m_Z)$ at hadron colliders

- Measurements of α_s at hadron colliders allow probing the strong coupling at high q^2
- However they generally suffer from large uncertainties, and do not provide a competitive determination of $\alpha_s(m_Z)$
- Only a few of them have the required NNLO accuracy of the predictions to enter the PDG average ($t\bar{t}$ inclusive cross section, and recently jets, V+jet)
- *Can we do better?*



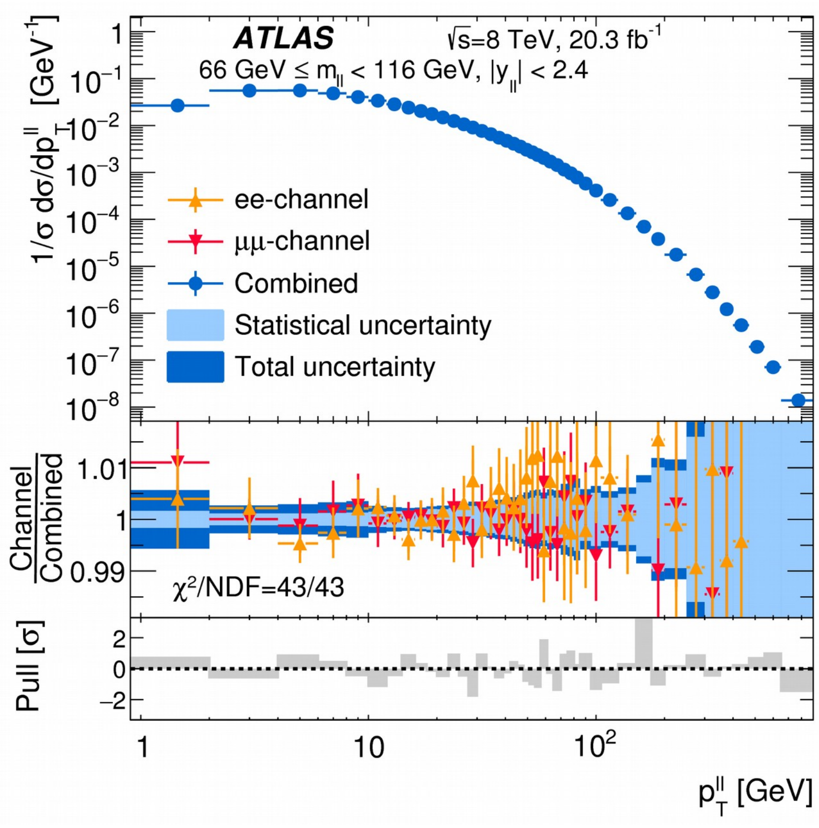
Desirable features for a measurement of $\alpha_s(m_Z)$ [PDG]

- Large observable's sensitivity to $\alpha_s(m_Z)$ compared to the experimental precision
- High accuracy of the theory prediction
- Small size of non-perturbative QCD effects

Measure $\alpha_s(m_Z)$ from semi-inclusive DY

- Measuring Λ_{MS} from semi-inclusive (radiation inhibited) DY cross sections was first proposed in
Nucl. Phys. B 349 (1991) 635-654

- Use Monte Carlo parton showers to determine Λ_{MC} and convert to Λ_{MS}



Resummation arguments show that a set of universal QCD corrections can be absorbed in coherent parton showers by applying the Catani-Marchesini-Webber (CMW) rescaling of the $\overline{MS}$ value of Λ_{QCD}

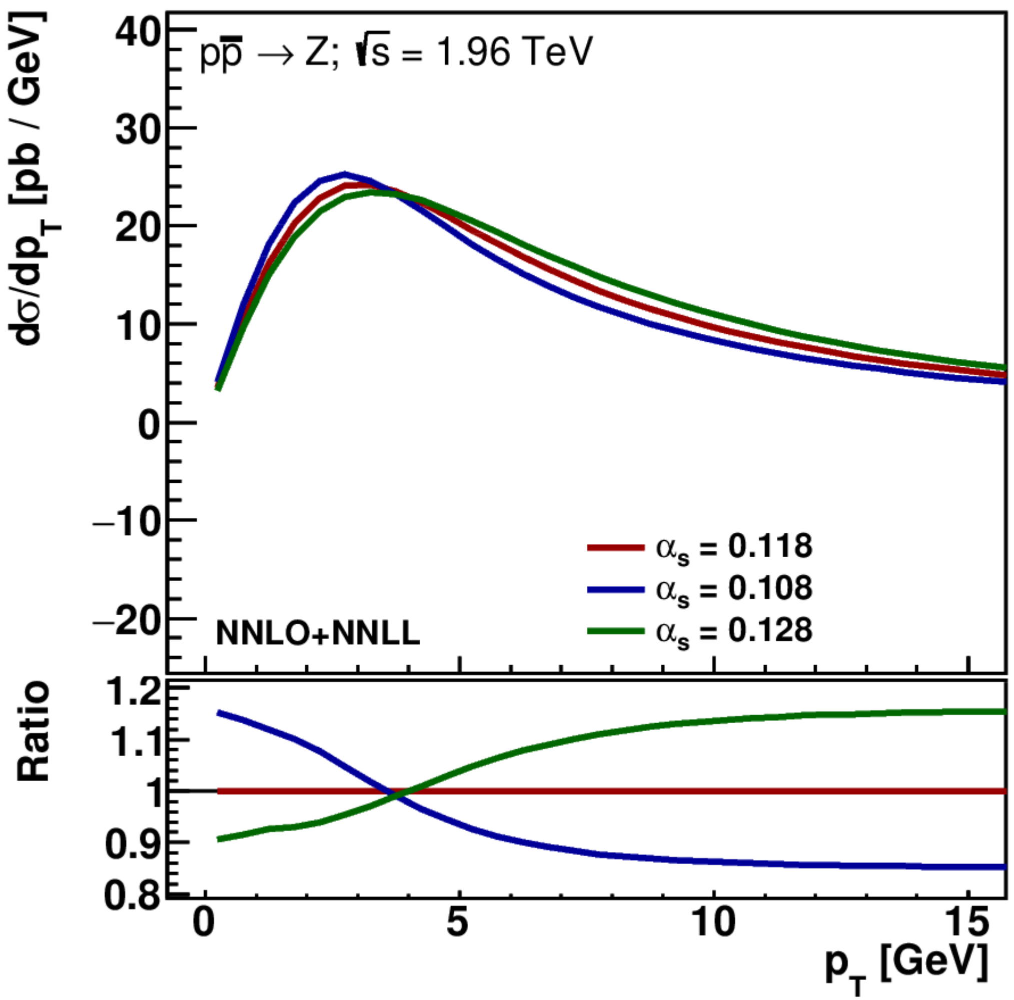
$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} C_F \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

$$\alpha_s^{(MC)} = \alpha_s^{(\overline{MS})}\left(1 + K\frac{\alpha_s^{(\overline{MS})}}{2\pi}\right)$$

- The Z p_T distribution at small transverse momentum is one of such semi-inclusive observables

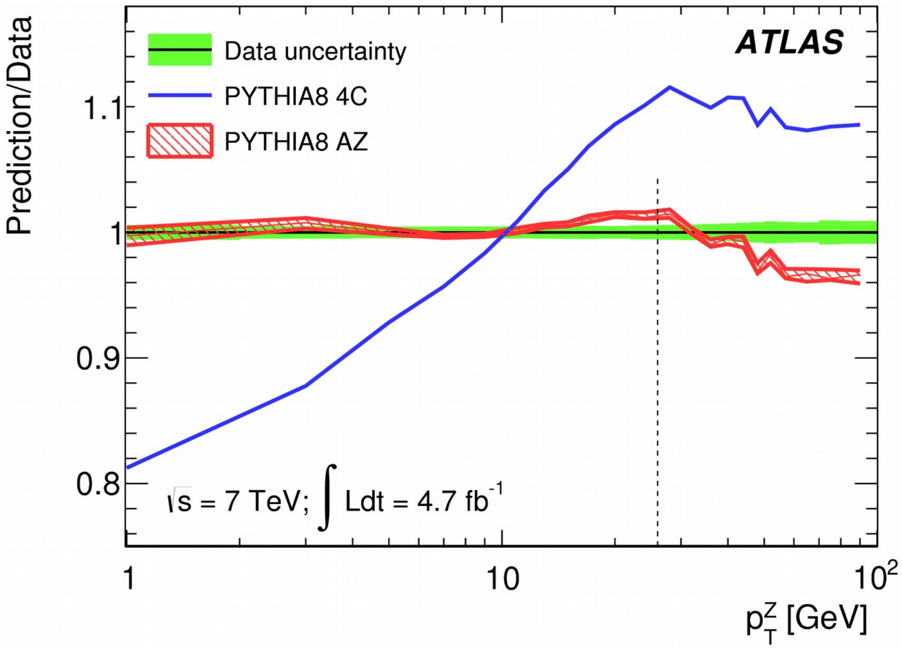
Measure $\alpha_s(m_Z)$ from the Z p_T distribution

- The recoil of Z bosons produced in hadron collisions is mainly due to QCD initial-state radiation
- The Sudakov factor is responsible for the existence of a Sudakov peak in the Z-boson transverse-momentum distribution, at transverse-momentum values of approximately 4 GeV
- The position of the peak is sensitive to the value of the strong-coupling constant



Measure $\alpha_s(m_Z)$ with MC parton showers

- ATLAS result on Pythia 8 MC tuning to the Z p_T distribution can be interpreted as a measurement of $\alpha_s(m_Z)$



	PYTHIA8
Tune Name	AZ
Primordial k_T [GeV]	1.71 ± 0.03
ISR $\alpha_S^{ISR}(m_Z)$	0.1237 ± 0.0002
ISR cut-off [GeV]	0.59 ± 0.08
χ^2_{min}/dof	$45.4/32$

$$a_s^{CMW}(m_Z) = 0.124 \rightarrow a_s(m_Z) = 0.116$$

- Naive result missing important theory uncertainties as PDFs and missing higher order corrections
- However this simple exercise already shows:
  - Great experimental sensitivity (0.2%)
  - Relatively small non-perturbative QCD uncertainties (primordial k_T and shower cut-off are fitted simultaneously with α_s)

Challenges

- Analytic predictions of Z p_T including resummation of large $\log(p_T/m)$ contributions are available since long time → desirable to use such analytic predictions to achieve higher accuracy
- Very precise Z p_T measurements (~2% at the Tevatron ~0.5% at the LHC) require high numerical precision of theory predictions
- Large correlations between $\alpha_s(m_Z)$ and non the perturbative Sudakov form factor would spoil the measurement
- At the LHC, significant heavy-flavour initiated production (6% of cc → Z and 3% of bb → Z) introduce additional uncertainties

DYTurbo project

- Optimised version of DYNNLO, DYqT, DYRes with improvements in

Software	Numerical integration
Code profiling	Quadrature with interpolating functions
Loop vectorisation	Factorisation of integrals
Hoisting	Analytic integration
Loop unrolling	
Multi-threading	

- Achieved significant enhancement in time performance for a given numerical precision
- The main application is the measurement of the W mass at the LHC, Other applications: PDF fits including qt-resummation for cross-section predictions, $\sin^2\theta_W$, $\alpha_s(m_Z)$
- Two main modes of operation: Vegas integration and Quadrature rules based on interpolating functions

Drell-Yan cross section

- DYTurbo can compute the DY cross section at

Fixed order
with qt-subtraction

$$d\sigma^{V}_{(N)NLO} = d\sigma^{virt}_{(N)NLO} - d\sigma^{CT(FO)}_{(N)LO} + d\sigma^{V+jet}_{(N)LO}$$

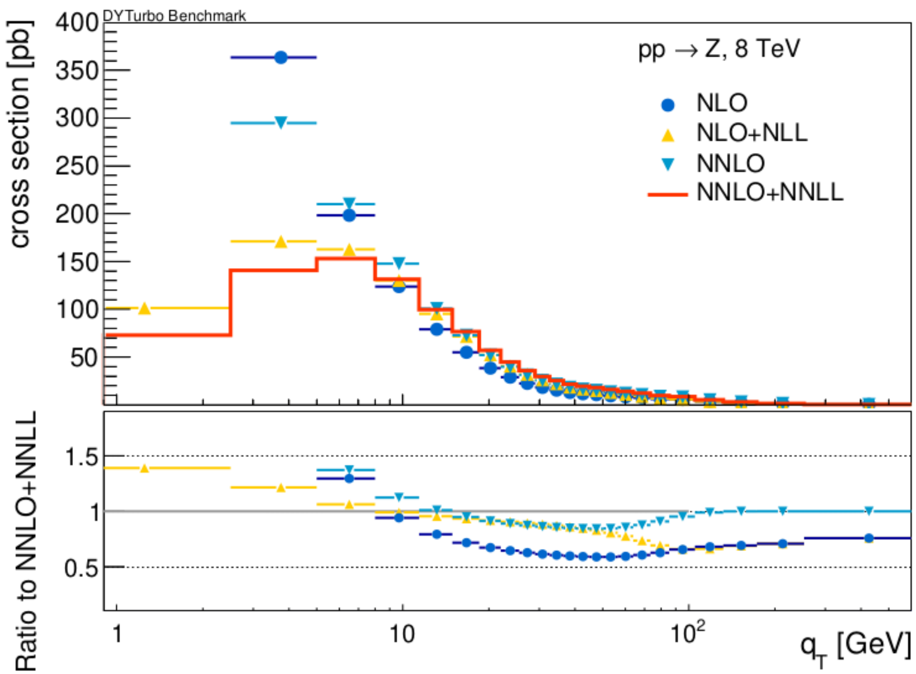
→ Fast predictions already implemented with NNLOJET + APPLfast

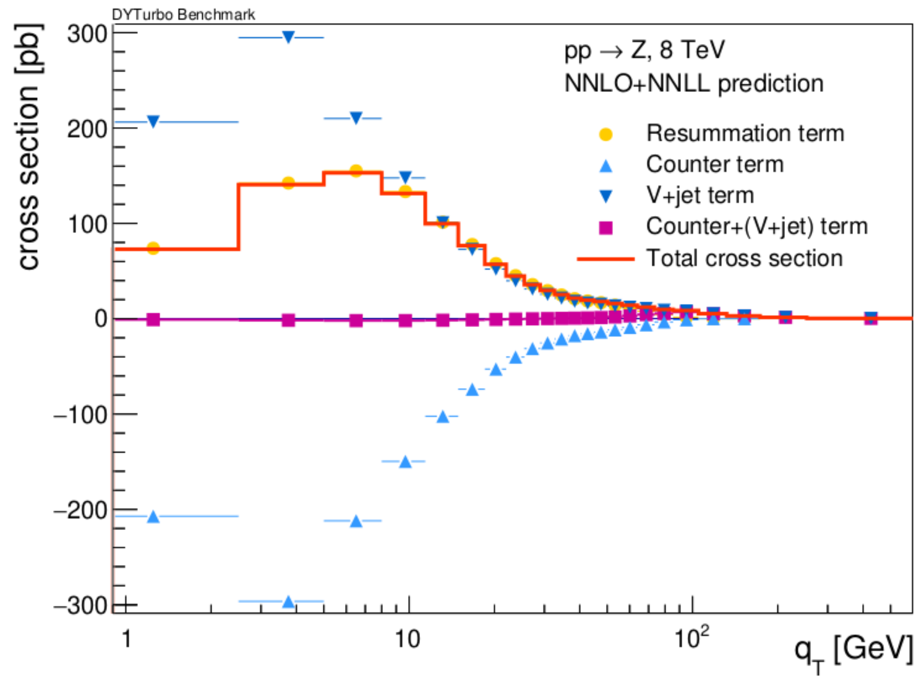
Fixed order
+qt-resummation

$$d\sigma^{V}_{(N)NLO+(N)NLL} = d\sigma^{res}_{(N)NLL} - d\sigma^{CT(res)}_{(N)LO} + d\sigma^{V+jet}_{(N)LO}$$

→ Main motivation for DYTurbo

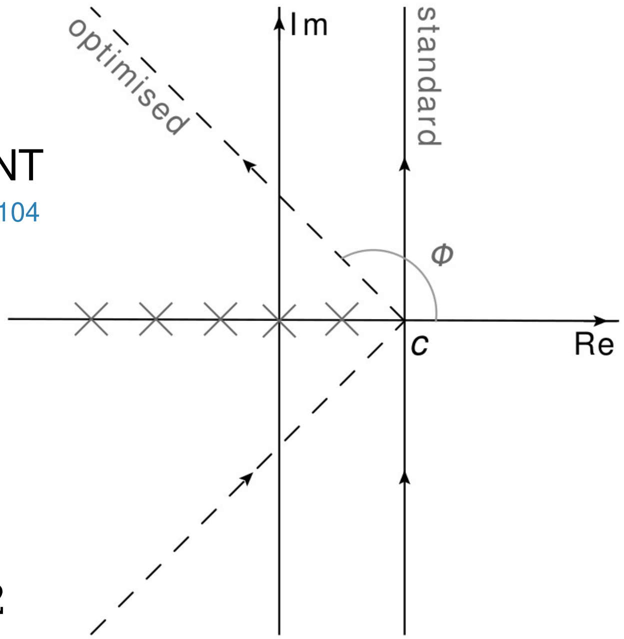
b-space resummation formalism of *Bozzi, Catani, de Florian, Ferrera, Grazzini*





Numerical integration improvements

- Fast x-space to Mellin-space integral transform of PDFs based on Gauss-Legendre quadrature
  → coefficients in Mellin space evaluated through ANCONT
  Comput.Phys.Commun. 133 (2000) 76-104



- Acceptance corrections evaluated by factorising the integration over lepton-decay angular variables in the LO cross section

$$\sigma_{LO} = a + b\cos(\theta) + c\cos^2(\theta) \rightarrow a\,\theta_0 + b\,\theta_1 + c\,\theta_2$$

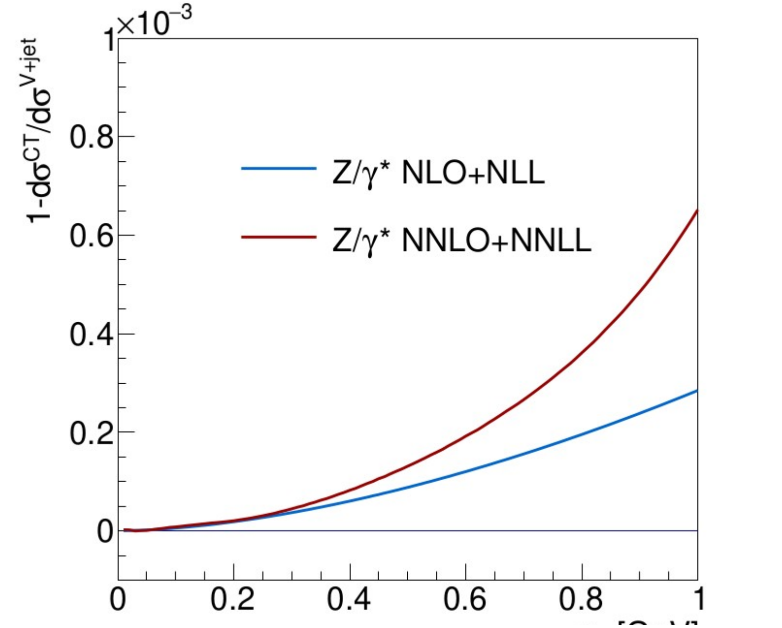
$$\left\{ \begin{aligned} \theta_0 = \int \mathrm{d}\Omega\, \Theta_K \,, \theta_1 &= \int \mathrm{d}\Omega\, \Theta_K \, \cos\theta_\ell \,, \\ \theta_2 &= \int \mathrm{d}\Omega\, \Theta_K \, \cos^2\theta_\ell \end{aligned} \right.$$

- Analytic integration over q_T of inverse Fourier transform

$$\int \mathrm{d}x\, x J_0(x) = x J_1(x) \quad \Rightarrow \quad \int_{q_\mathrm{T}^0}^{q_\mathrm{T}^1} \mathrm{d}q_\mathrm{T}\, 2q_\mathrm{T}\, \mathcal{W}(q_\mathrm{T}, m) = \frac{m^2}{s} \int_0^\infty \mathrm{d}b\, \frac{1}{2} \left[q_\mathrm{T}^1 J_1(bq_\mathrm{T}^1) - q_\mathrm{T}^0 J_1(bq_\mathrm{T}^0) \right] \tilde{\mathcal{W}}(b, m)$$

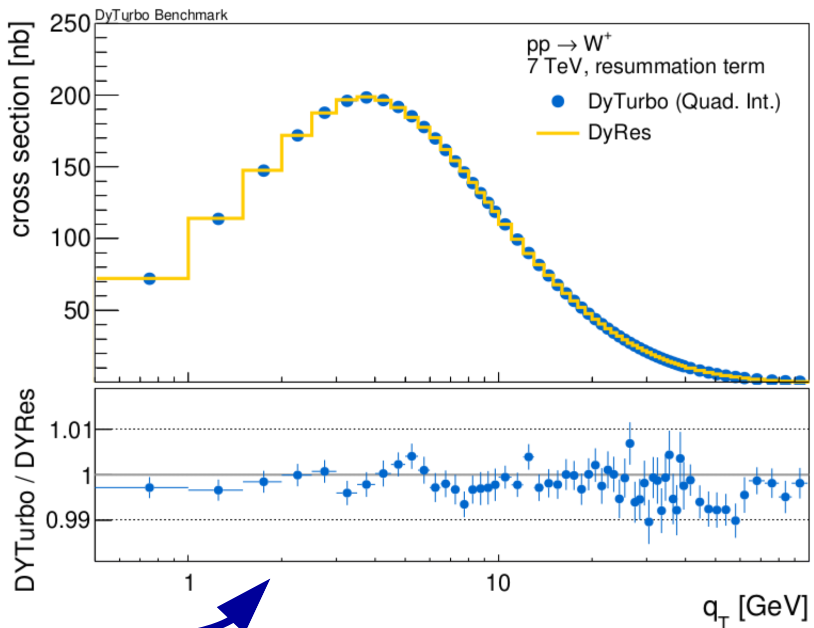
Closure tests and benchmark

- Matching conditions implies relation between the terms which can be used to test their numerical precision



$$\lim_{q_T \to 0} 1 - d\sigma^{CT(res)}/d\sigma^{V+jet} = 0$$

→ tested at 10^{-5}



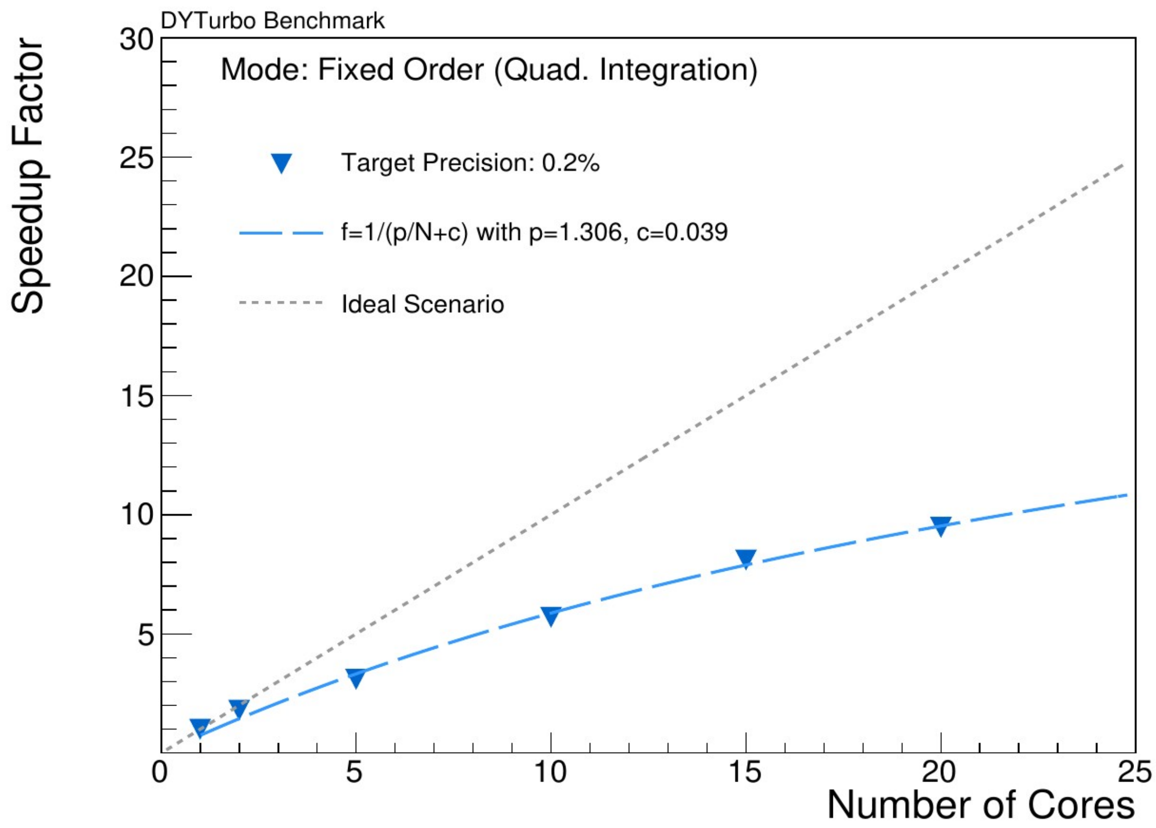
- DYTurbo predictions fare benchmarked with DYRes at NNLL, and with other programs at NNLO

	SHERPA	DYNNLO	FEWZ	DYTurbo (Quad.)
$\sigma(pp \to W^+ \to l^+\nu)$ [pb]	3204 ± 4	3191 ± 7	3207 ± 2	3196 ± 7
$\sigma(pp \to W^- \to l^-\nu)$ [pb]	2252 ± 3	2243 ± 6	2238 ± 1	2248 ± 4
$\sigma(pp \to Z/\gamma \to l^+l^-)$ [pb]	502.0 ± 0.6	502.4 ± 0.4	504.6 ± 0.1	502.8 ± 1.0

Small differences between FEWZ and the other predictions are expected due to phase space with p_T^l symmetric cuts, and different subtraction scheme

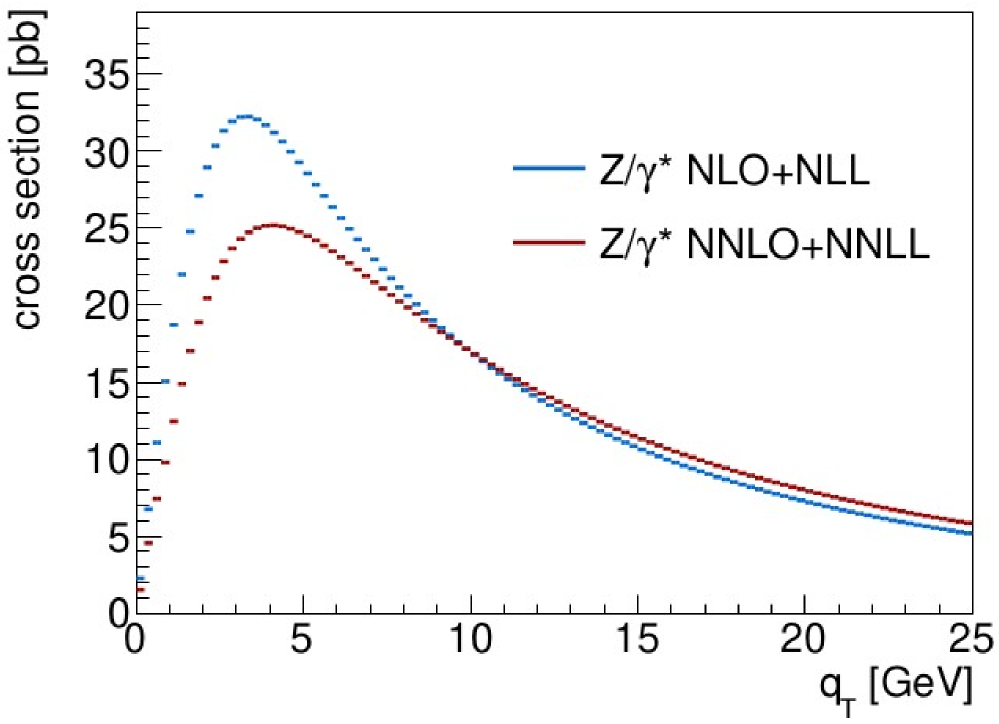
Time performance

- Measured the performance of the multi-threading implementation



- Good efficiency (1/p ~ 77%) thanks to small overhead
- c ~ 4% is the nonparallelisable fraction
- Larger gain from multi-threading when the required precision is higher

Example calculation



- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

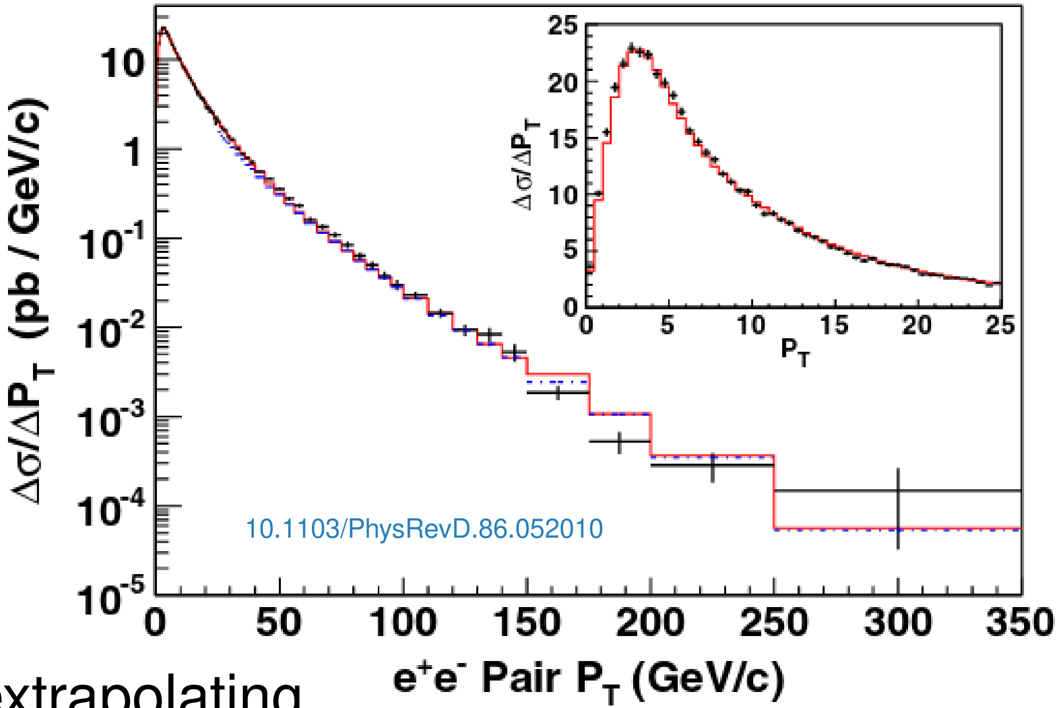
Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

- The most demanding calculation is V+jet

→ can use APPLgrid/FASTnlo for this term

CDF measurement of Z-boson p_T

- Fast NNLO+NNLL predictions allow measuring of $\alpha_s(m_Z)$ from the Sudakov region of the Z-boson p_T distribution → $<p_T>$ ~ 10 GeV
- The CDF measurement of Z-boson p_T is ideal for testing the extraction of $\alpha_s(m_Z)$ with DYTurbo predictions



- Angular coefficients method allows extrapolating the measurement to full-lepton phase space with small theoretical uncertainties

$$\frac{d\sigma}{dpdq} = \frac{d^3\sigma}{dp_T dy dm} \sum_i A_i(y, p_T, m) P_i(\cos\theta, \phi)$$

→ allows fast predictions

→ avoid theoretical uncertainties on Z polarisation

- $p\bar{p}$ collisions

→ reduced contribution from heavy-flavour-initiated production compared to pp collisions

$b\bar{b}$ → Z: 0.4%
$c\bar{c}$ → Z: 1.3%

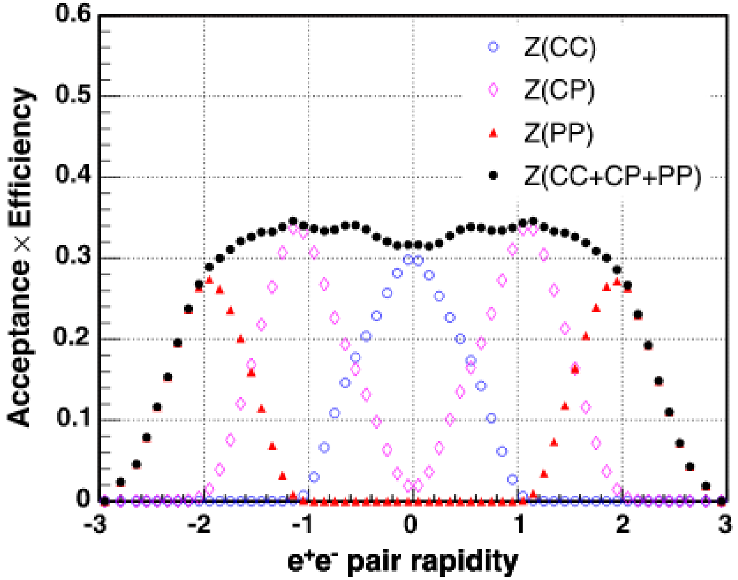
CDF measurement of Z p_T

- Measurement performed in the electron channel, with CC, CF, FF

  → small extrapolation to full rapidity range:

  $|\eta^e| < 2.8 \rightarrow y_{max} \sim 3.1$

- Low pileup data with good electron resolution



$$\sigma/E = 14\%/\sqrt{E_T}$$

Central → σ = 0.9 GeV at E_T = 45 GeV

$$\sigma/E = 16\%/\sqrt{E} \oplus 1\%$$

Forward → σ = 1.1 GeV at E_T = 45 GeV

→ allows fine p_T bins (0.5 GeV) with relatively small bin-to-bin correlations

Non-perturbative form factor

- qt-resummed cross section in b-space have the general from:

$$\frac{d\hat{\sigma}_V^{(\mathrm{res.})}}{dq_T^2}(q_T, M) = \frac{M^2}{\hat{s}} \int_0^\infty db \, \frac{b}{2} \, J_0(bq_T) \, \mathcal{W}^V(b, M)$$

$$\mathcal{W}_N^V(b, M) = \mathcal{H}_N^V(\alpha_S) \times \exp\{\mathcal{G}(\alpha_S, L)_N\}, \; L = log(M^2 b^2)$$

perturbative Sudakov form factor

Sensitivity to α_s

- The Sudakov form factor is modified by including a non-perturbative term: $S(b) \rightarrow S(b) \cdot S_{NP}(b)$
- The general form of $S_{NP}(b)$ is mass and centre-of-mass energy dependent, see for instance the BLNY parameterisation Phys. Rev. D 67, 073016 (2003)

$$S_{NP} = exp[-(g_1 + g_2 ln(Q^2/Q_0^2) + g_3 ln(100 x_a x_b)) b^2]$$

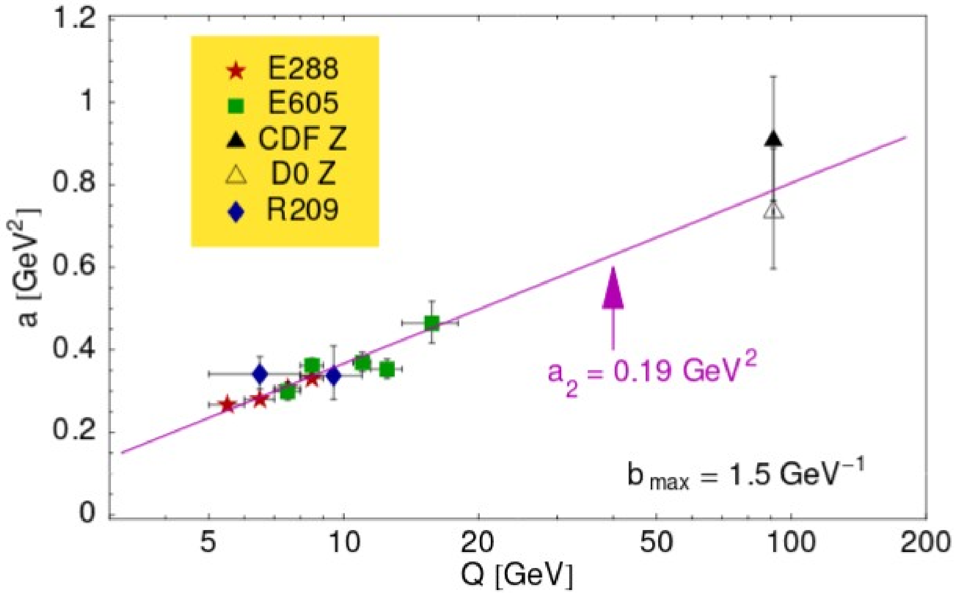
- At fixed $Q = m_Z$, and for one value of sqrt(s), the form of $S_{NP}(b)$ can be simplified with $g = g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(100\, x_a\, x_b)$

b-space Landau prescription

- g is generally determined from the data, its value depends on the chosen prescription to avoid the Landau pole in b-space

$\rho_L = \exp\left[1/(2b_0\alpha_s(\mu))\right]$

Konychev, Nadolsky, PLB 633, 710 (2006)

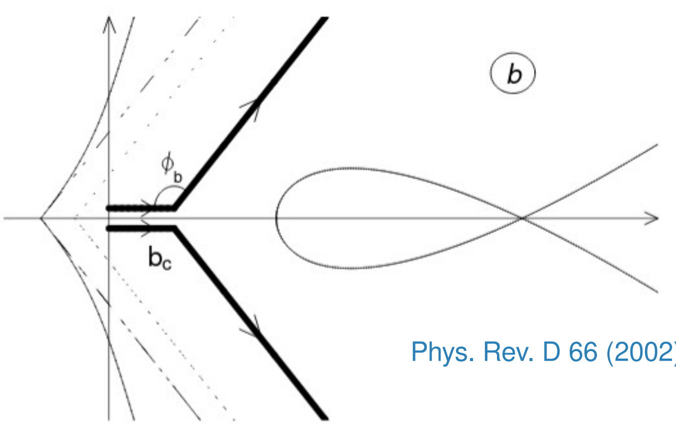


b* prescription (CSS)

$$b_\star = \frac{b}{1 + b^2/b_{\text{lim}}^2}$$

Minimal prescription

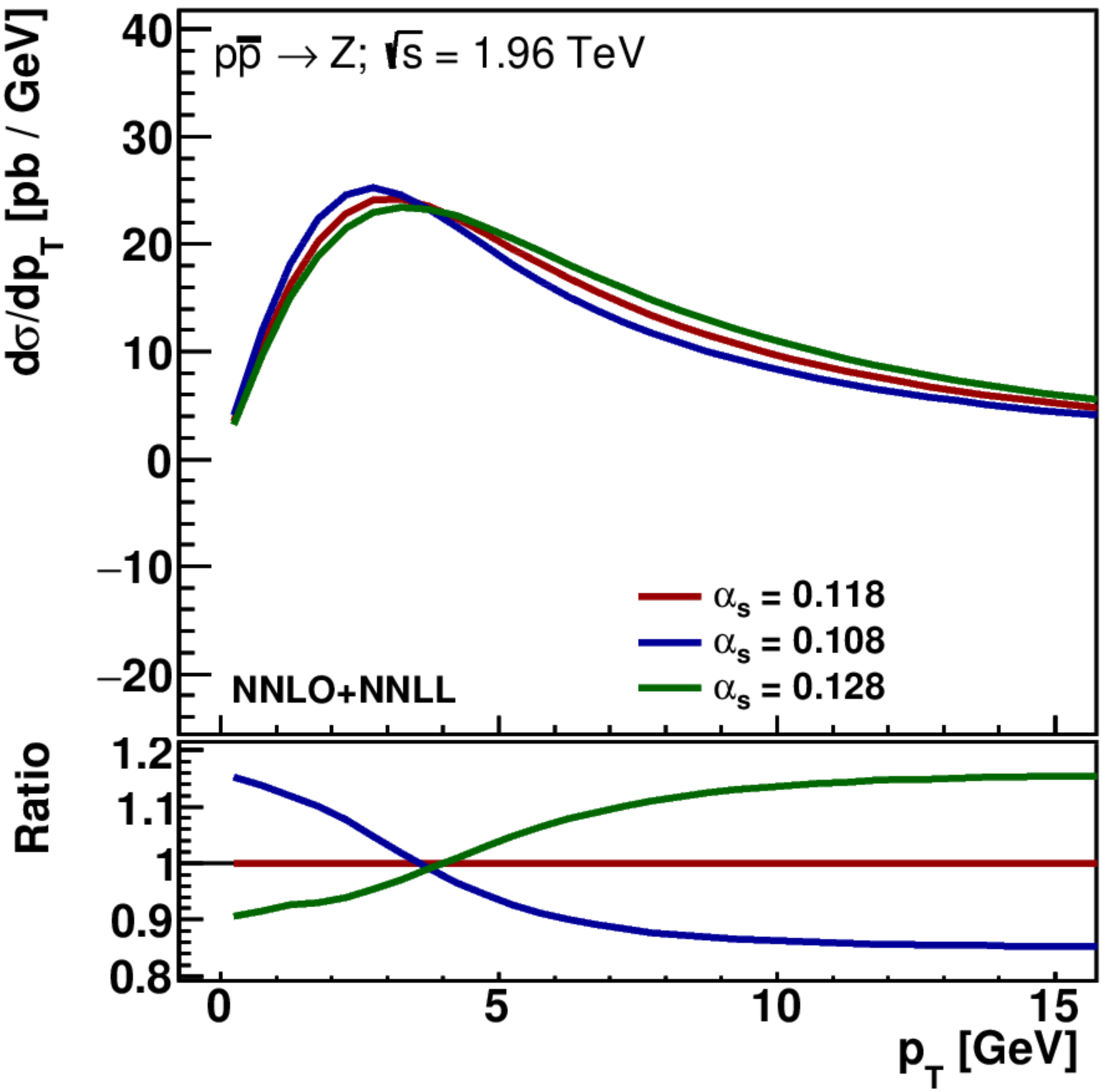
avoid the Landau pole by bending the integration contour in the complex plane



Phys. Rev. D 66 (2002) 014011

- We choose the b* prescription with $b_{\text{lim}} = \rho_L$ → numerically equivalent to the minimal prescription
- We consider also $b_{\text{lim}} = \rho_L/2$ as systematic uncertainty

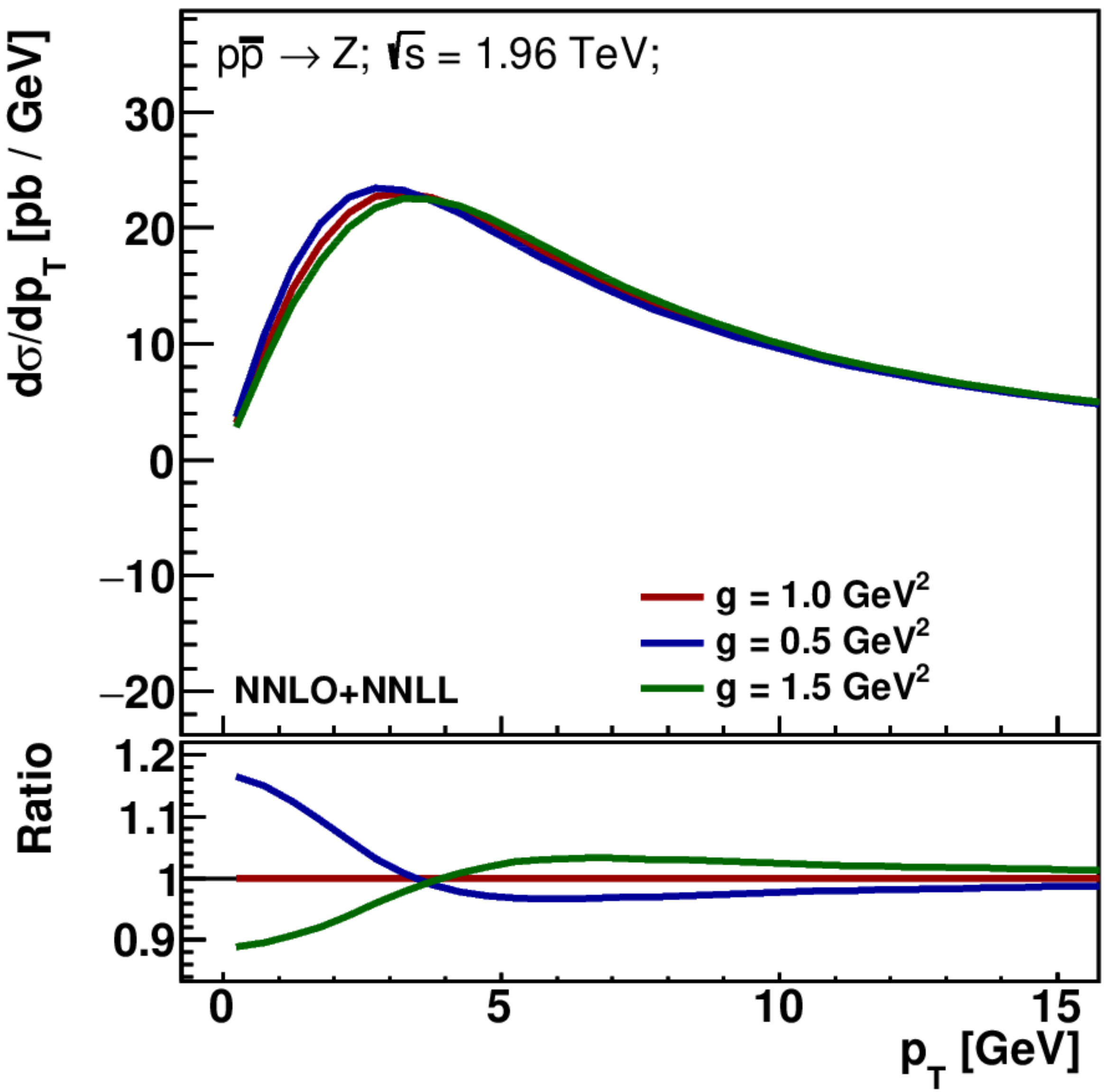
Sensitivity to $\alpha_s(m_Z)$



- The sensitivity of the Z-boson p_T distribution to $\alpha_s(m_Z)$ mainly comes from the position of the Sudakov peak
- Typical recoil scale:

  $\langle p_T \rangle$ ~ 10 GeV

Sensitivity to g



The sensitivity of the Z-boson p_T distribution to g also comes from the position of the Sudakov peak

The scale of the non-perturbative smearing is given by the primordial $\langle k_T \rangle$.

Fourier transform of SNP(b):

$$e^{-k_T^2/(4g_{NP})}$$

$$\Downarrow$$

DESY-PROC-2012-02/136

$$\langle k_T^2 \rangle = 4g_{NP}$$

g ~ 0.8 GeV^2 → $\langle k_T \rangle$ ~ 1.8 GeV

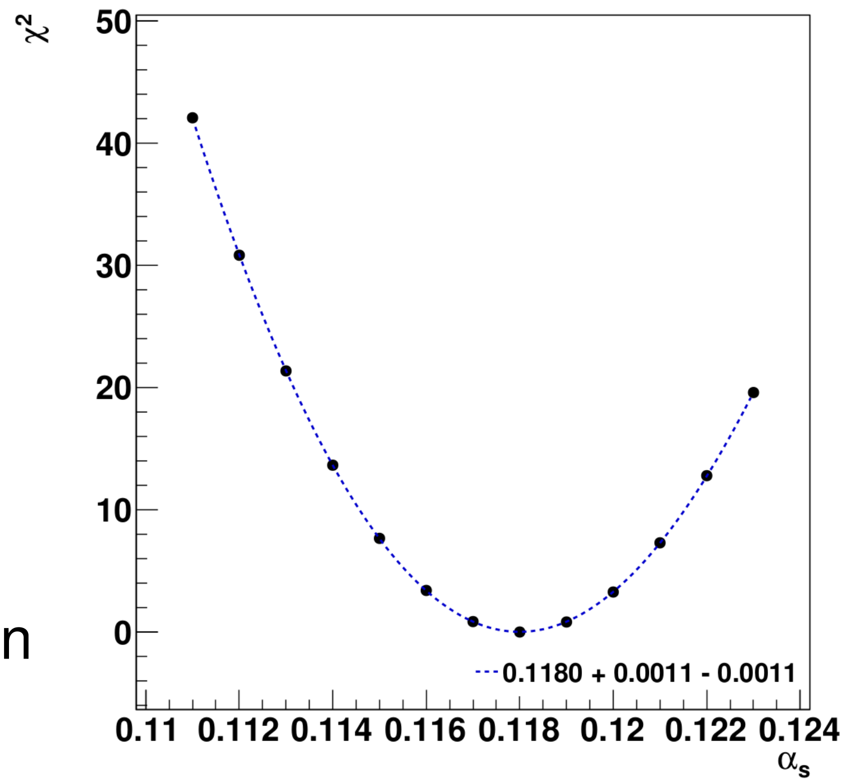
→ Possible to disentangle the perturbative Sudakov, governed by α_s, from the non-perturbative one, determined by g, thanks to their different scale

Methodology for the $\alpha_s(m_Z)$ determination

- DYTurbo interfaced to xFitter
- Define a χ^2 with experimental and PDFs theoretical uncertainties

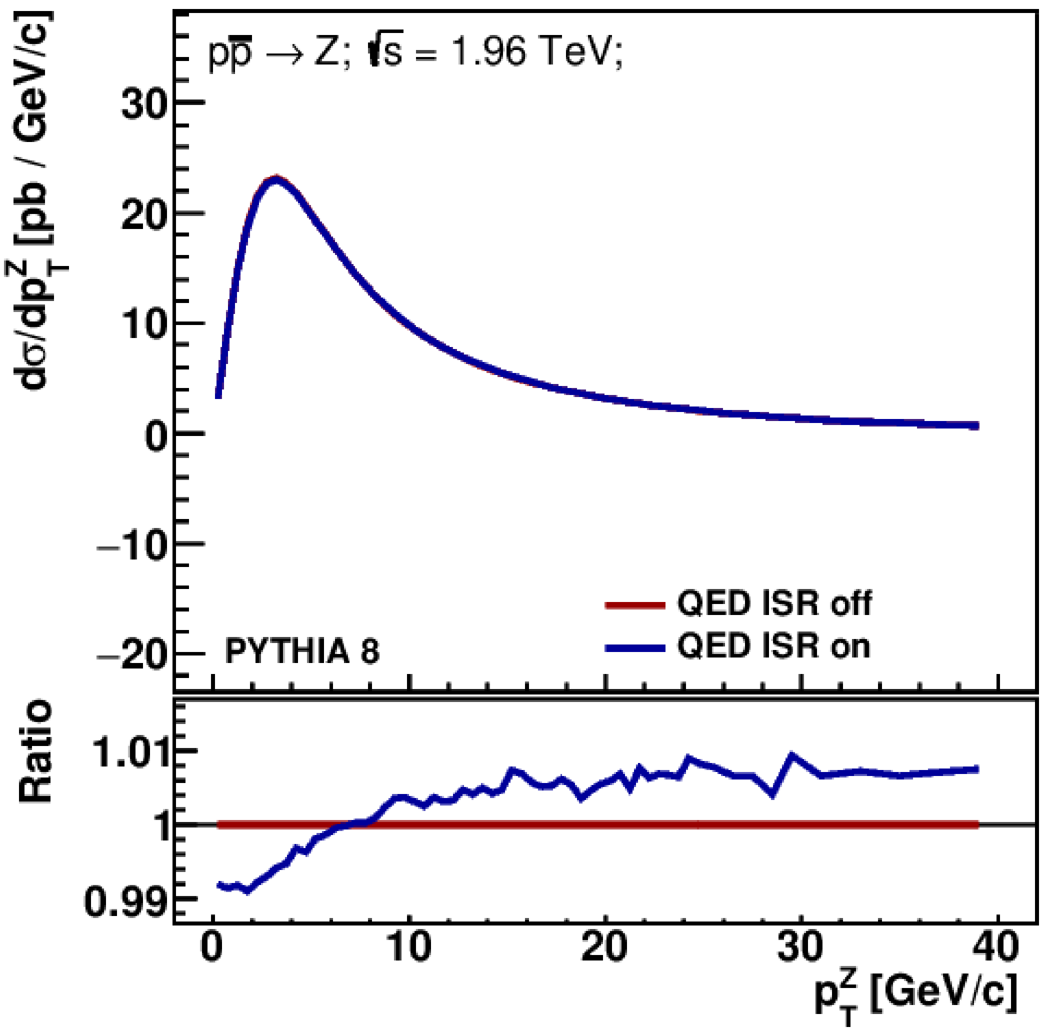
$$\chi^2(\boldsymbol{\beta}_{\mathbf{exp}}, \boldsymbol{\beta}_{\mathbf{th}}) = \sum_{i=1}^{N_{\rm data}} \frac{\left(\sigma_i^{\rm exp} + \sum_j \Gamma_{ij}^{\rm exp}\beta_{j,\rm exp} - \sigma_i^{\rm th} - \sum_k \Gamma_{ik}^{\rm th}\beta_{k,\rm th}\right)^2}{\Delta_i^2} + \sum_j \beta_{j,\rm exp}^2 + \sum_k \beta_{k,\rm th}^2$$

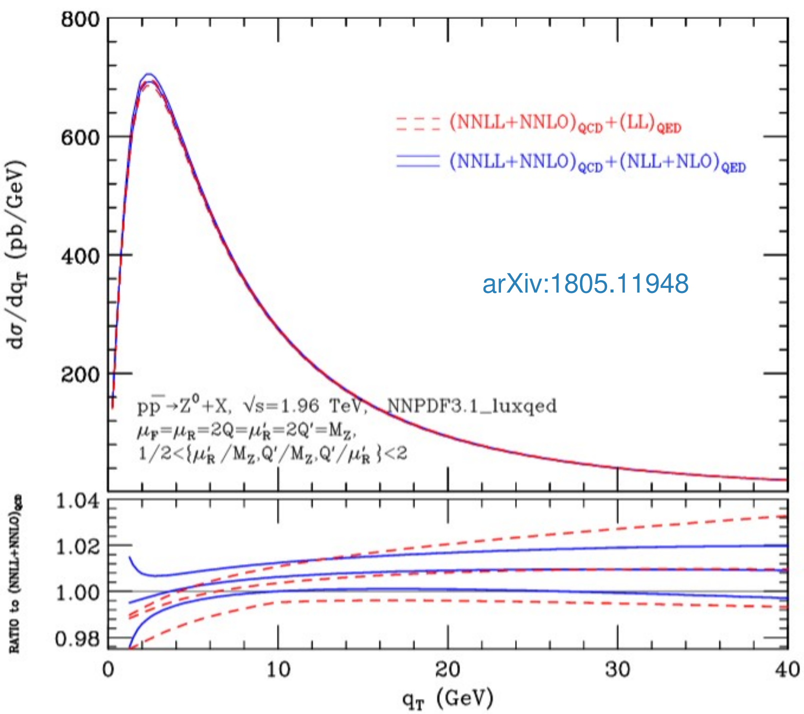
- The non-perturbative form factor is added as unconstrained nuisance parameter ($\beta = 0$) → left free in the fit
- Fit the region $p_T < m_Z$
- Evaluate $\chi^2(\alpha_s)$ with α_s variations as provided in LHAPDF



QED ISR correction

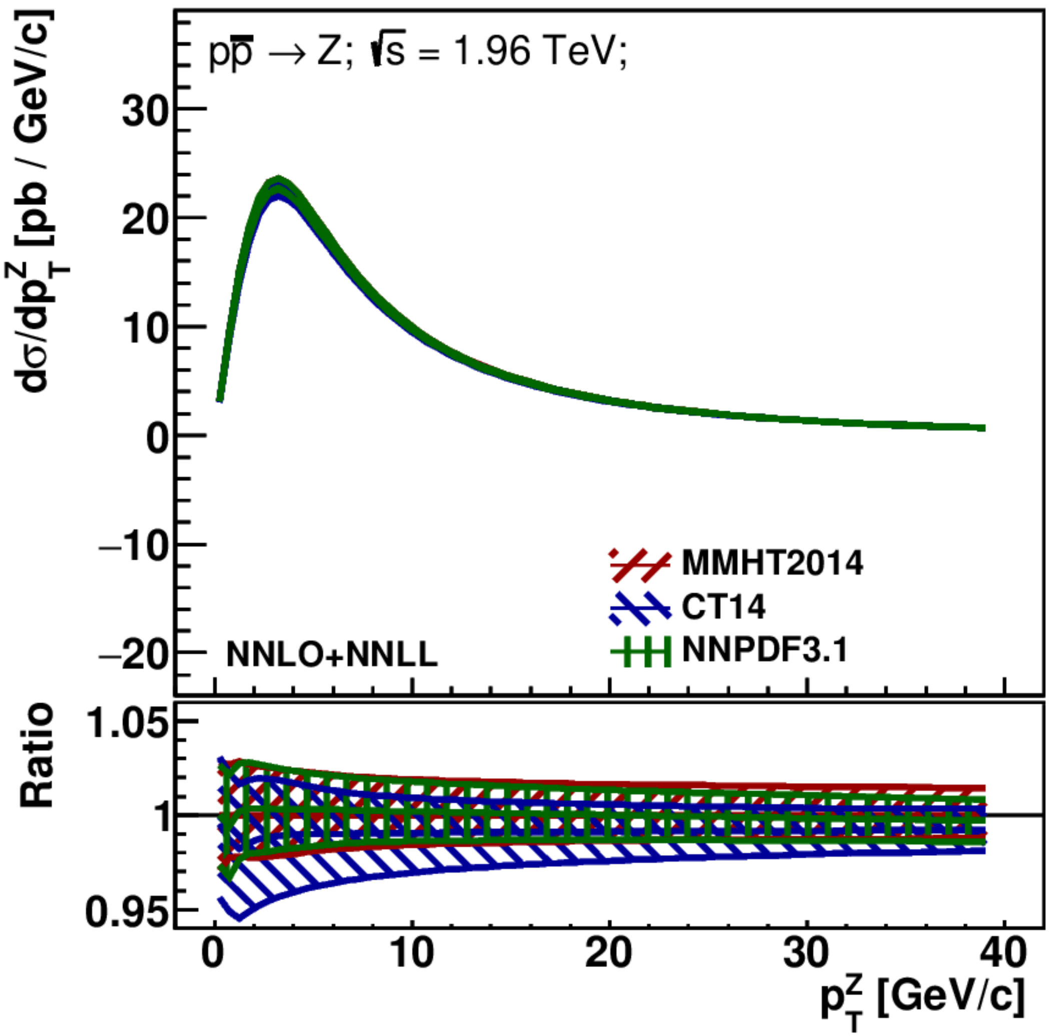
- QED ISR estimated with Pythia 8, and applied as a multiplicative correction
- Correction to the Z-boson p_T at the level of 1%
- Effect on $\alpha_s(m_Z)$: $\Delta\alpha_s = -0.0004$
- Comparable to corrections obtained with QED qt-resummation





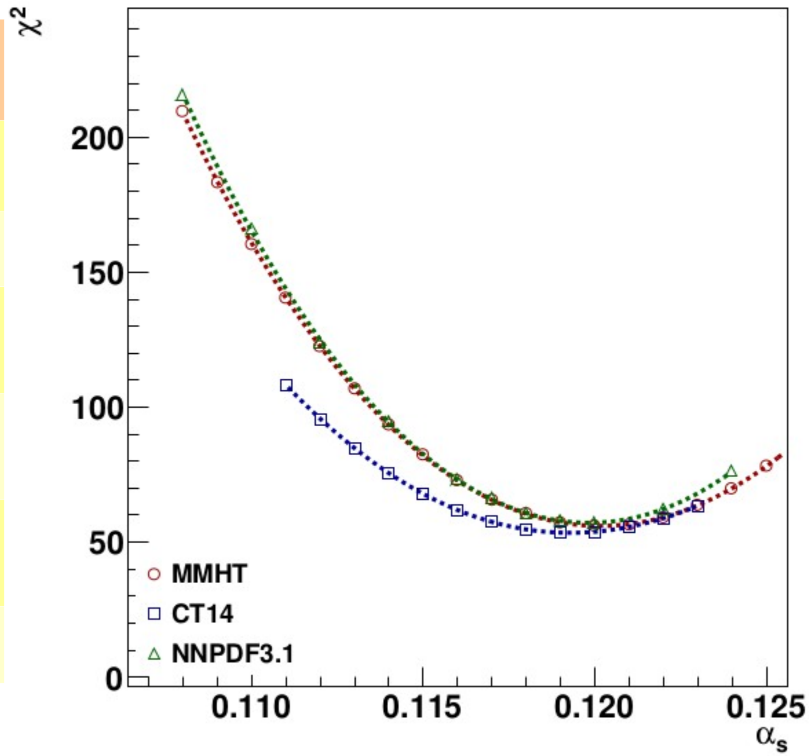
PDF uncertainties

- PDF sets: CT14, MMHT, NNPDF3.1
- PDF uncertainties profiled in the fit
- Small uncertainties at the level of 2 – 4 % on Z-boson p_T
- PDF uncertainty on $\alpha_s(m_Z)$ ~ 0.0007



Simultaneous fit of $\alpha_s(m_Z)$ and g

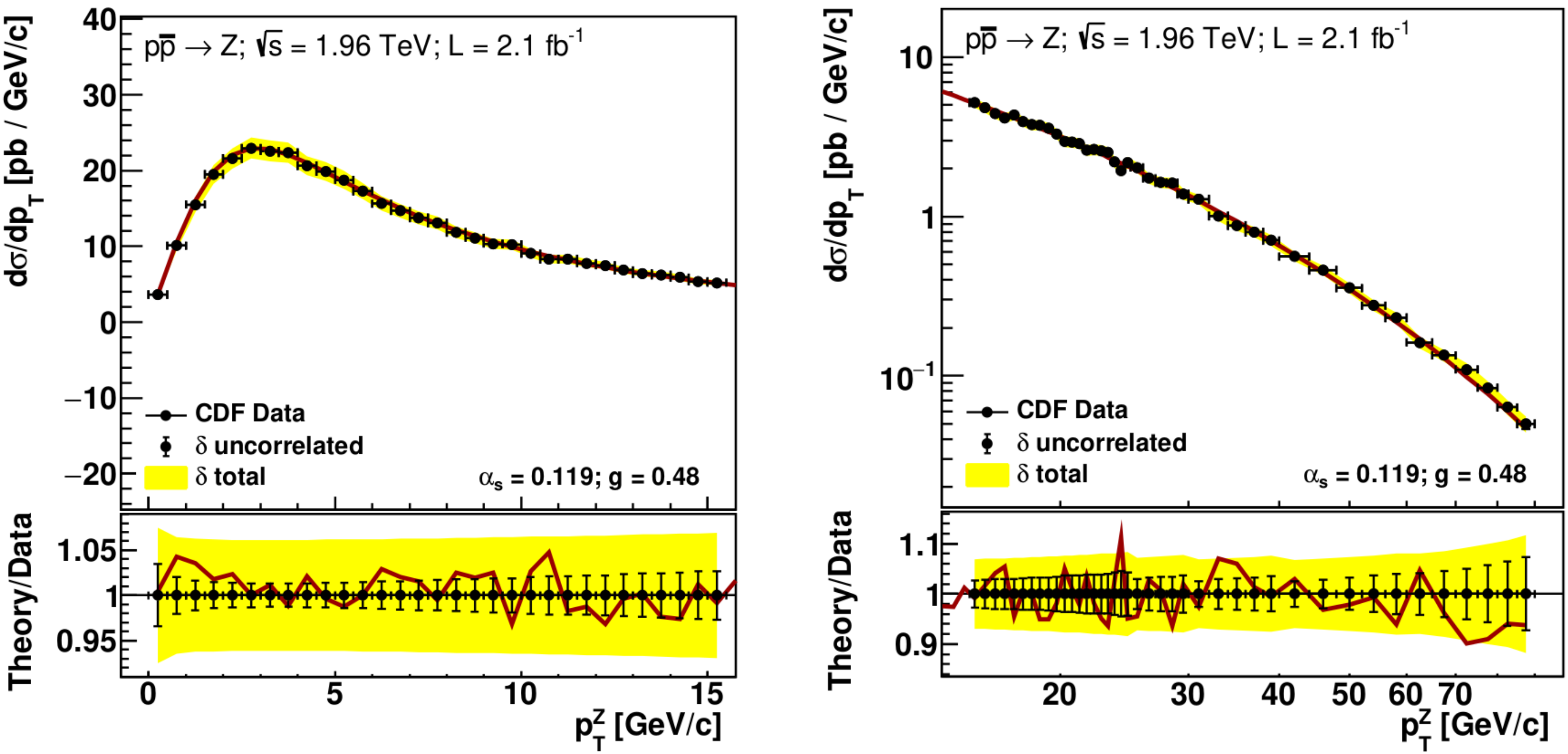
	MMHT	CT14	NNPDF3.1
$\alpha_s(m_Z)$	0.1202	0.1193	0.1198
Stat. unc.	0.0008	0.0008	0.0007
Syst. unc.	0.0002	0.0003	0.0002
PDF unc.	0.0006	0.0007	0.0006
g (GeV^2)	0.48 ± 0.07	0.51 ± 0.07	0.35 ± 0.08
χ^2/dof	56/71	54/71	57/71



- CT14 has the smallest χ^2

  → used as central result

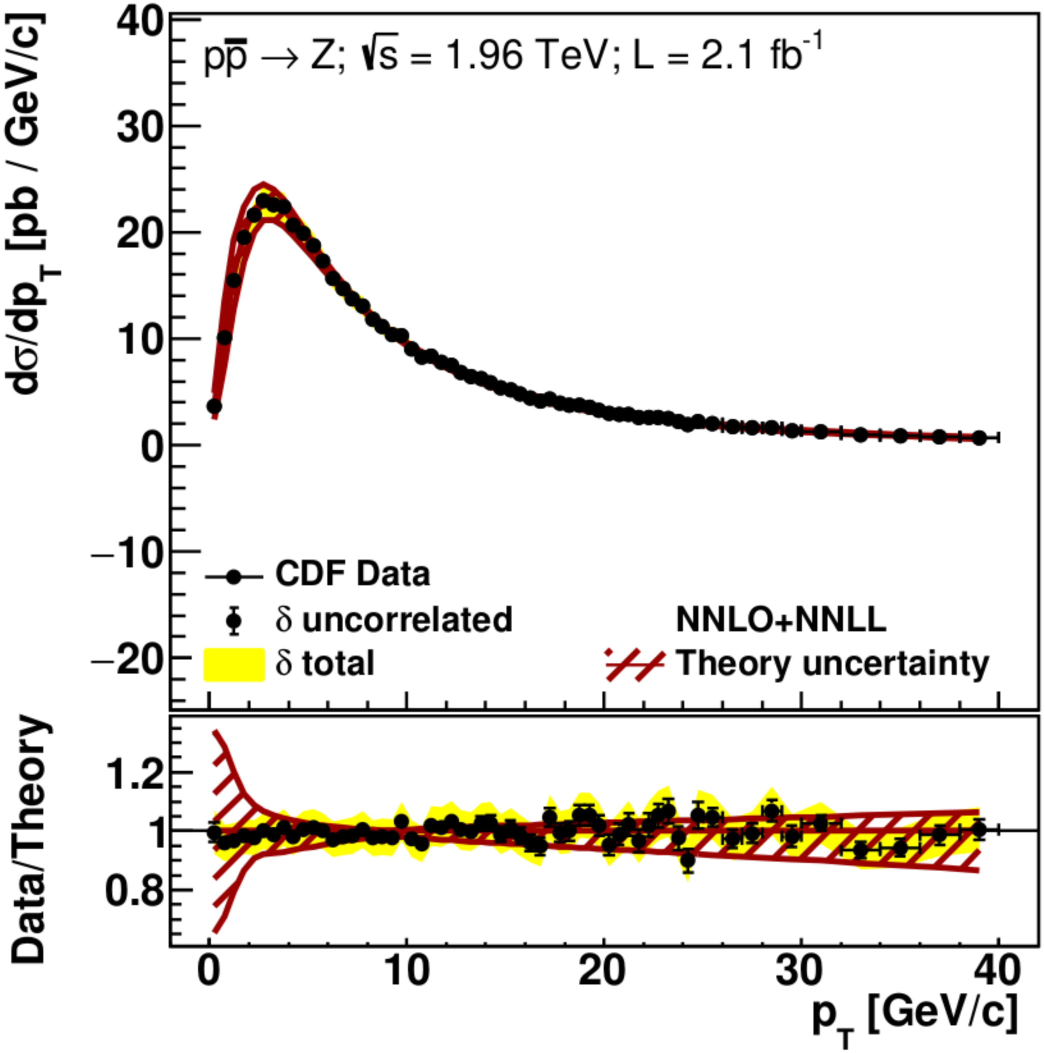
Post fit predictions



- Very good agreement of postfit predictions with data at low and intermediate p_T

QCD scale variations

- Predictions depend on three QCD scales: renormalisation, factorisation, and resummation scales



- Central value:

  $\mu_R = \mu_F = \mu_{RES} = m_Z/2$

- Uncertainty from all possible combinations of factor of 2 variations, excluding any factor of 4 variation

QCD scale variations

μ_R	μ_F	μ_{RES}	$\alpha_s(m_Z)$	g(GeV^2)	χ^2
0.5	0.5	0.5	0.1193	0.51	53.5
0.5	0.5	1	0.1122	0.62	150.6
0.5	0.5	0.25	0.1201	0.60	52.6
0.5	1	0.5	0.1168	0.61	55.1
0.5	1	1	0.1195	0.74	54.4
0.5	0.25	0.5	0.1228	0.23	76.4
0.5	0.25	0.25	0.1223	0.53	53.7
1	0.5	0.5	0.1205	0.51	61.4
1	0.5	1	0.1177	0.55	287.6
1	1	0.5	0.1180	0.68	53.3
1	1	1	0.1182	0.51	76.1
0.25	0.5	0.5	0.1171	1.12	55.0
0.25	0.5	0.25	0.1166	0.71	53.7
0.25	0.25	0.5	0.1196	1.10	65.0
0.25	0.25	0.25	0.1186	0.64	54.4

Envelope:

$\alpha_s = 0.1193 \; +0.0035 \; -0.0027$

$g = 0.51 \; +0.61 \; -0.28$ (GeV^2)

QCD scale variations

μ_R	μ_F	μ_{RES}	$\alpha_s(m_Z)$	$g(GeV^2)$	χ^2
0.5	0.5	0.5	0.1193	0.51	53.5
0.5	0.5	1	0.1122	0.62	150.6
0.5	0.5	0.25	0.1201	0.60	52.6
0.5	1	0.5	0.1168	0.61	55.1
0.5	1	1	0.1195	0.74	54.4
0.5	0.25	0.5	0.1228	0.23	76.4
0.5	0.25	0.25	0.1223	0.53	53.7
1	0.5	0.5	0.1205	0.51	61.4
1	0.5	1	0.1177	0.55	287.6
1	1	0.5	0.1180	0.68	53.3
1	1	1	0.1182	0.51	76.1
0.25	0.5	0.5	0.1171	1.12	55.0
0.25	0.5	0.25	0.1166	0.71	53.7
0.25	0.25	0.5	0.1196	1.10	65.0
0.25	0.25	0.25	0.1186	0.64	54.4

- Some scale variations have unreasonably large χ^2 → measurement more precise than theory predictions
- Is it possible to use the high accuracy of the data to further constraint the theory uncertainties?
- The same scale choice is used for the full p_T range 0 – 90 GeV. Should consider a partial decorrelation of scale variations across the p_T spectrum?

Resummation theoretical uncertainties

- Uncertainty related to the matching between resummation and fixed order prediction

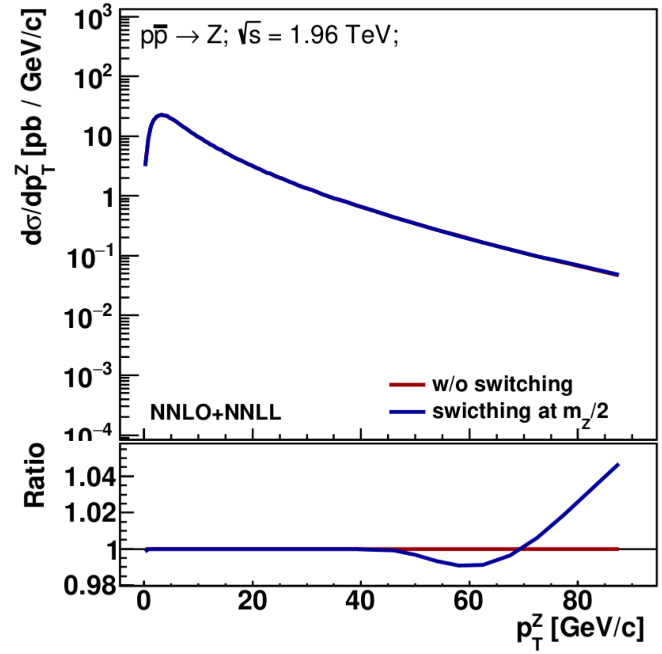
  → switch off resummation contribution above $p_T = m_Z/2$

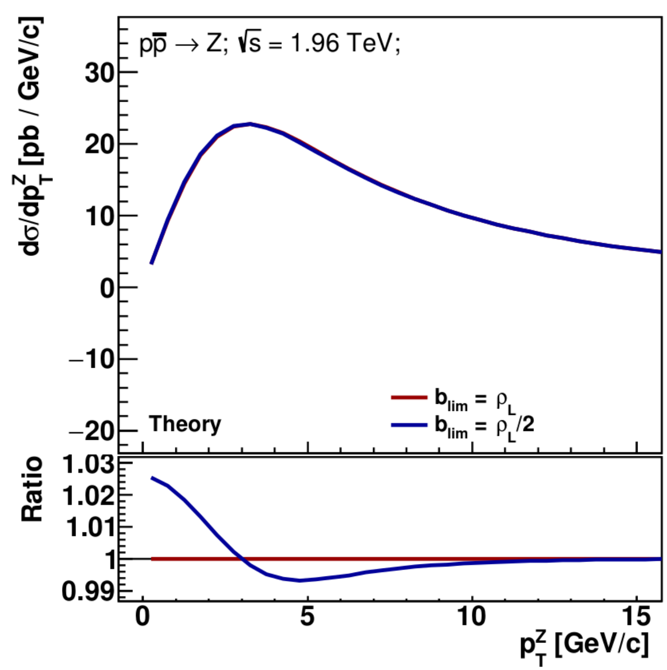
  → $\Delta\alpha_s = +\ 0.0001$

- b-space prescription to avoid the Landau pole: b* with $b_{lim} = \rho_L/2$

  → $\Delta\alpha_s = -\ 0.0008$

  → $\Delta g = +\ 0.18\ (GeV^2)$





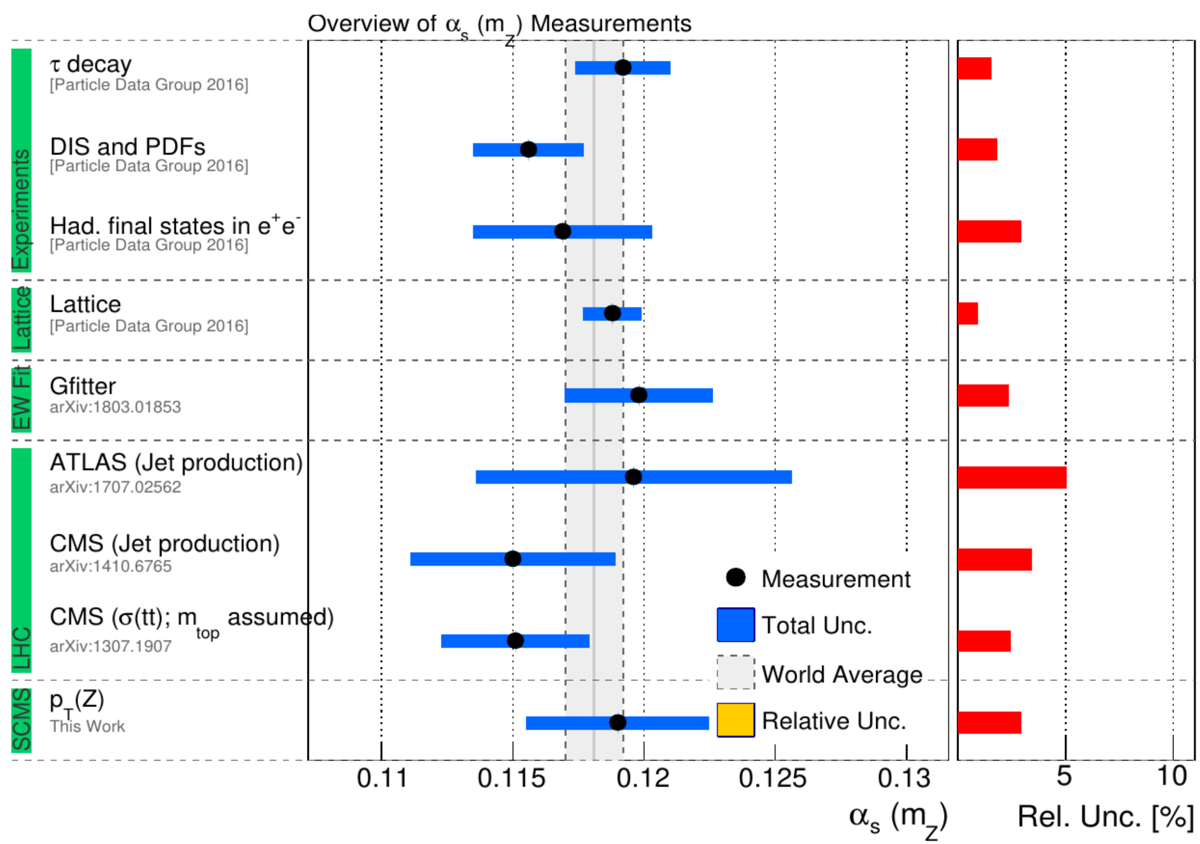
Result

$$\alpha_s = 0.119 + 0.004 - 0.003$$

$$g = 0.51 + 0.64 - 0.34 \text{ GeV}^2$$

	$\alpha_s(m_Z)$	$g(\text{GeV}^2)$
Exp. unc.	0.0009	0.07
PDF unc.	0.0007	
Scale var.	+0.0035 –0.0027	+0.61 –0.28
Res. th. unc.	0.0008	0.18

- Measurement in agreement with the world average
- Uncertainty comparable to other LHC determinations

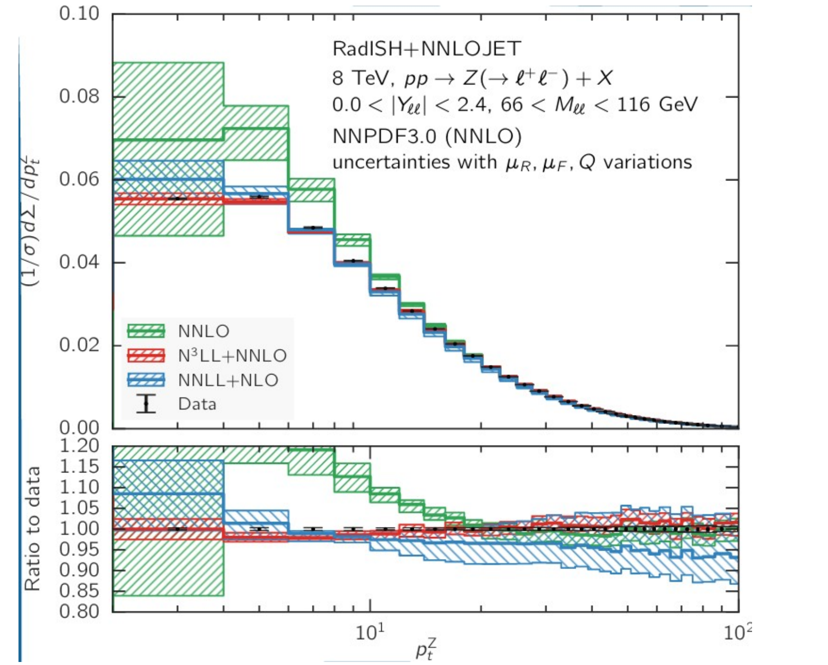


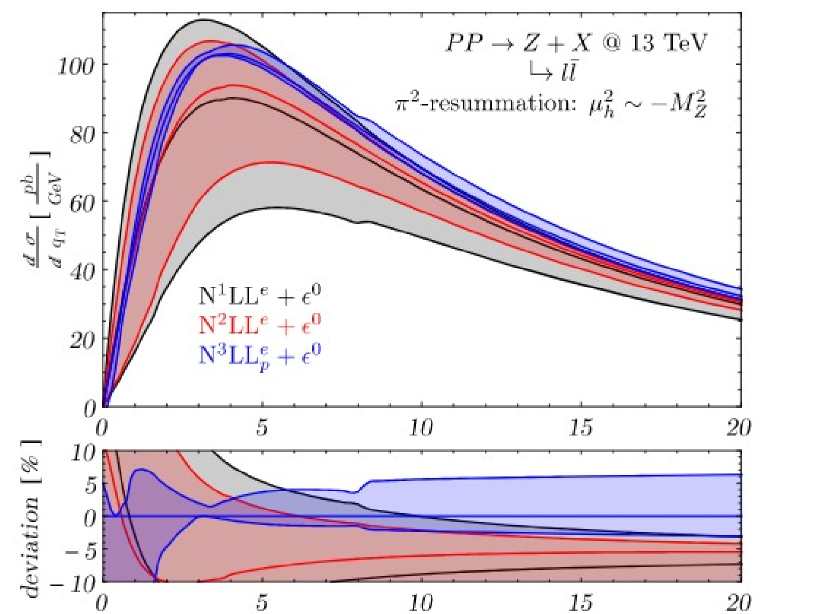
Open questions

- At which scale?
  - The main sensitivity to α_s comes from the perturbative Sudakov form factor, i.e. from the p_T recoil. The dominant scale is related to $\langle p_T \rangle \sim 10$ GeV
- At which order?
  - The NNLO+NNLL prediction is:
    - NNLO accurate for the total cross section
    - NLO accurate for the p_T distribution
  - α_s runs at:
    - 3 loops for the $q^2 \sim m_Z^2$ dependence
    - 2 loops for the impact-parameter (p_T) dependence

Prospects with NNLO+N³LL predictions

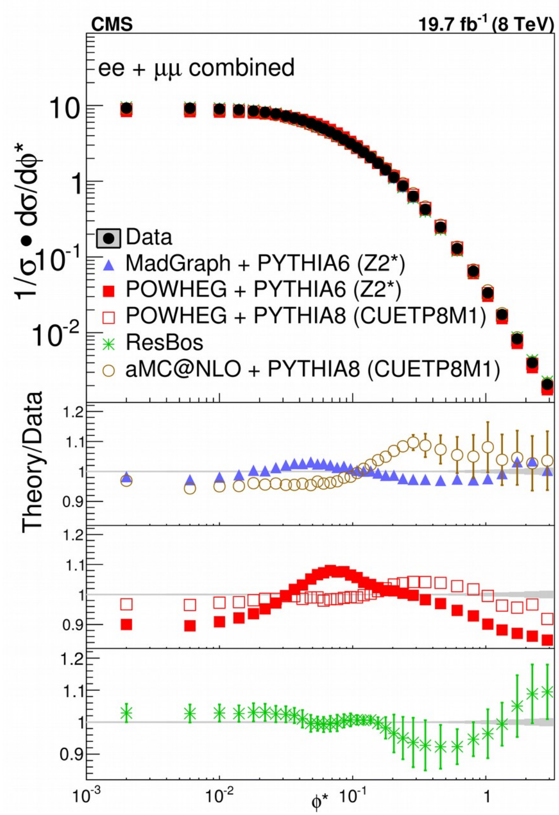
- Result dominated by missing higher order uncertainties (scale variations), at the level of 3%.
- qt-resummation at N³LL is now available (Radish, Cute), and Z pT at NNLO (NNLOJET)
- Expected a factor of 3 – 5 reduction of scale variations when using such higher order predictions

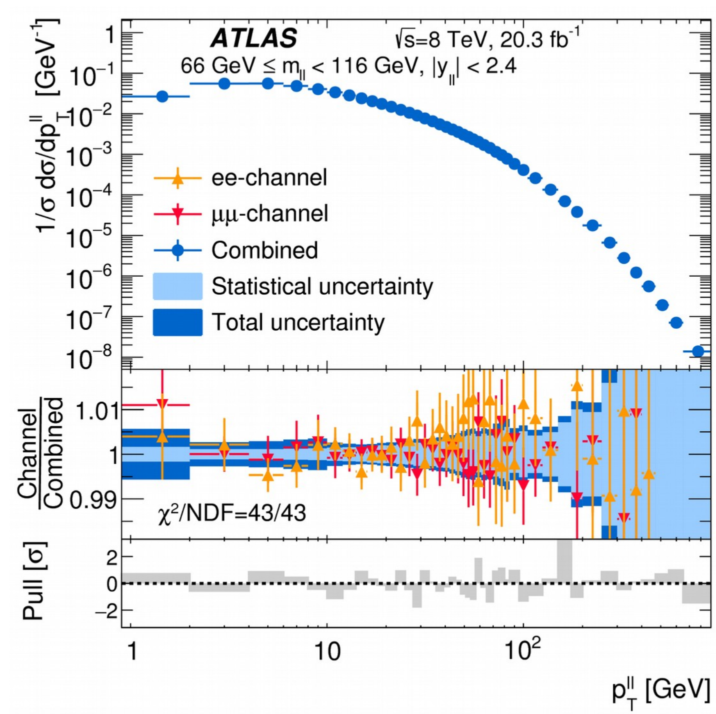




Prospects with LHC data

- LHC measurement of Z p_T are significantly more precise than Tevatron:
  - ATLAS 7 TeV data yields 0.2% exp. unc. on α_s
  - 3 times smaller uncertainties with 8 TeV ATLAS/CMS
  - Likely to reach a few 10^{-4} with full Run 2 data sample





	$\alpha_s(m_Z)$
Exp. unc.	< 0.0001
PDF unc.	0.0005
Scale var.	0.0010
Res. th. unc.	0.0005
Heavy flavour	0.0005 ?
Total expected	0.0013

For measuring α_s at LHC need to address open issues with heavy-flavour-initiated production
→ HF schemes for DY

Summary and conclusions

- Performed a preliminary measurent of $\alpha_s(m_Z)$ at NNLO+NNLL by fitting the CDF Z p_T distribution in the Sudakov region $p_T < m_Z$
- To this end the DYTurbo program was developed, which allows to compute fast and numerically precise Drell-Yan predictions
- Prospects to reach 1% by analyzing LHC data and using higher order predictions
- Need to address the issue of large contribution from heavy-flavour initiated production at the LHC

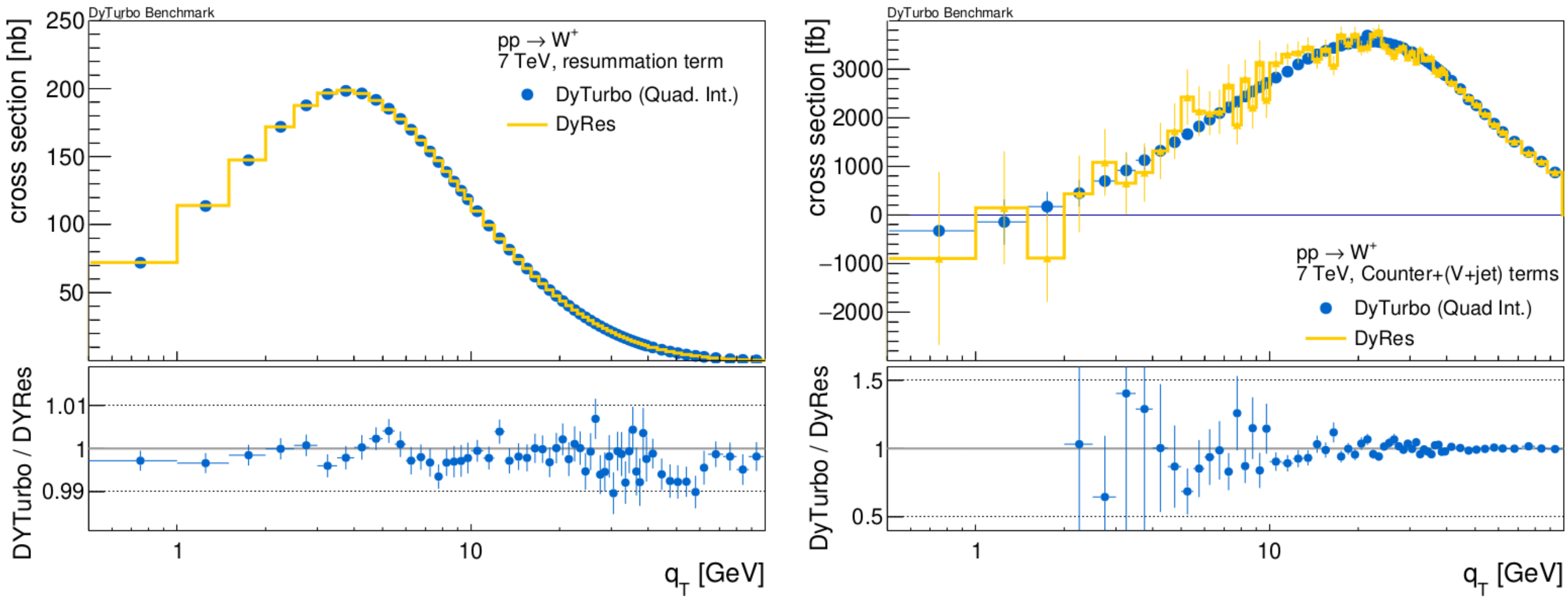
$\alpha_s(m_Z)$ from Z p_T collaborators: M. Schott and A. Glazov

DYTurbo collaborators: M. Boonekamp, G. Bozzi, S. Catani, L. Cieri, J. Cuth, G. Ferrera, D. de Florian, A. Glazov, M. Grazzini, M. G. Vincter, M. Schott

BACKUP

Benchmark results

- The predictions from DYTurbo are benchmarked against DYRes for qt-resummation, and with other programs for the NNLO

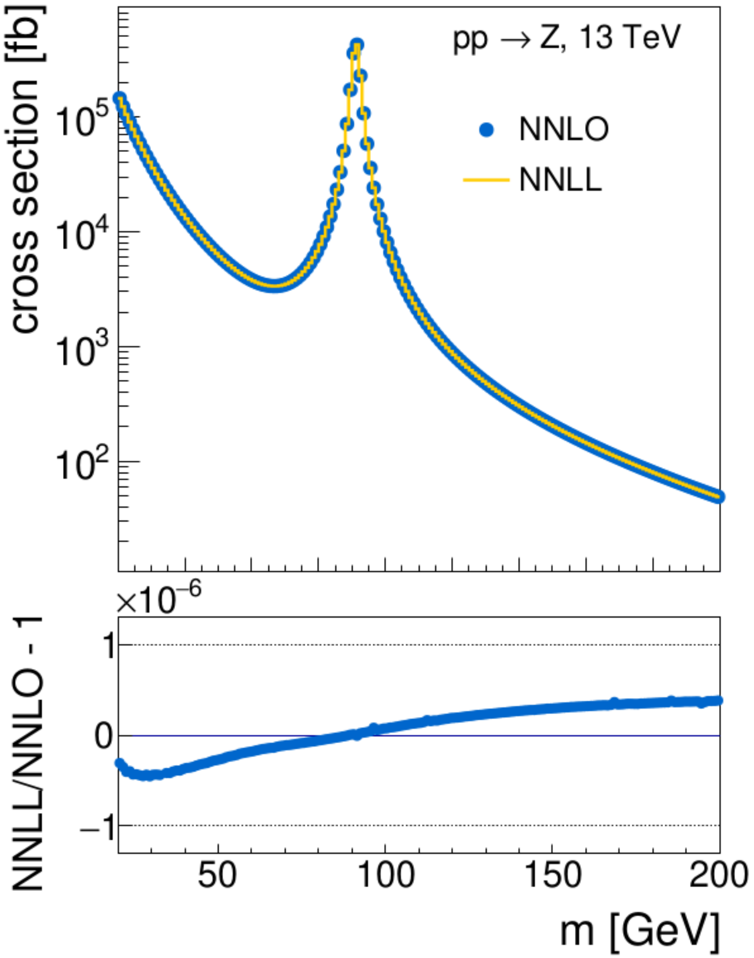


	SHERPA	DYNNLO	FEWZ	DYTurbo (Quad.)
$\sigma(pp \to W^+ \to l^+\nu)$ [pb]	3204 ± 4	3191 ± 7	3207 ± 2	3196 ± 7
$\sigma(pp \to W^- \to l^-\nu)$ [pb]	2252 ± 3	2243 ± 6	2238 ± 1	2248 ± 4
$\sigma(pp \to Z/\gamma \to l^+l^-)$ [pb]	502.0 ± 0.6	502.4 ± 0.4	504.6 ± 0.1	502.8 ± 1.0

Small differences between FEWZ and the other predictions are expected due to phase space with p_T^l symmetric cuts, and different subtraction scheme

Closure tests of numerical precision

- Matching conditions implies relation between the terms which can be used to test their numerical precision



$$\int_0^{\infty} \mathrm{d}q_\mathrm{T}\, \mathrm{d}\sigma^{\mathrm{res}} = \mathrm{d}\sigma^{\mathrm{virt}}$$

→ tested at 10^{-6}