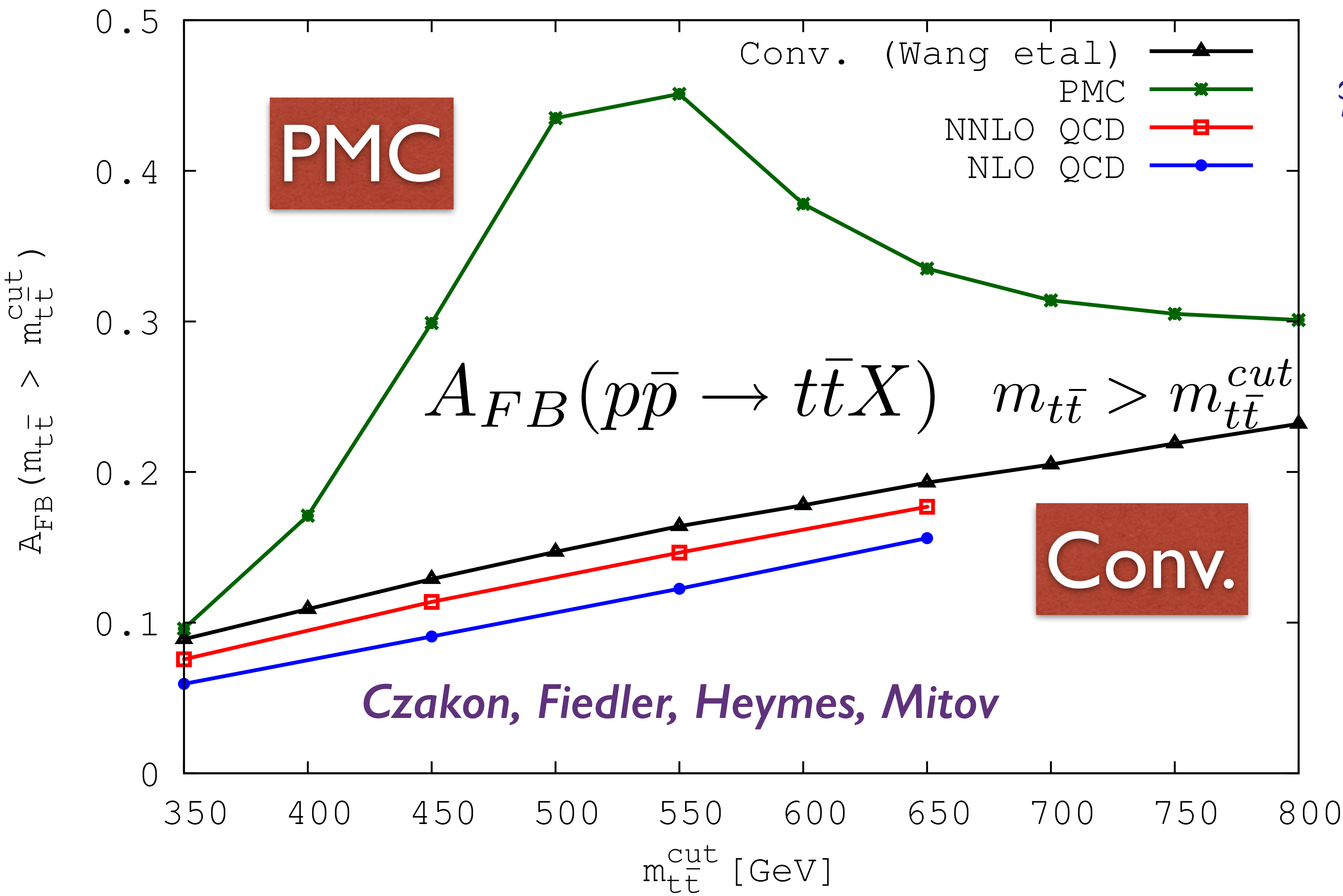
The QCD coupling at all scales and the elimination of renormalization scale uncertainties

The Principle of Maximum Conformality (PMC)

BLM: G. Peter Lepage
Paul Mackenzie

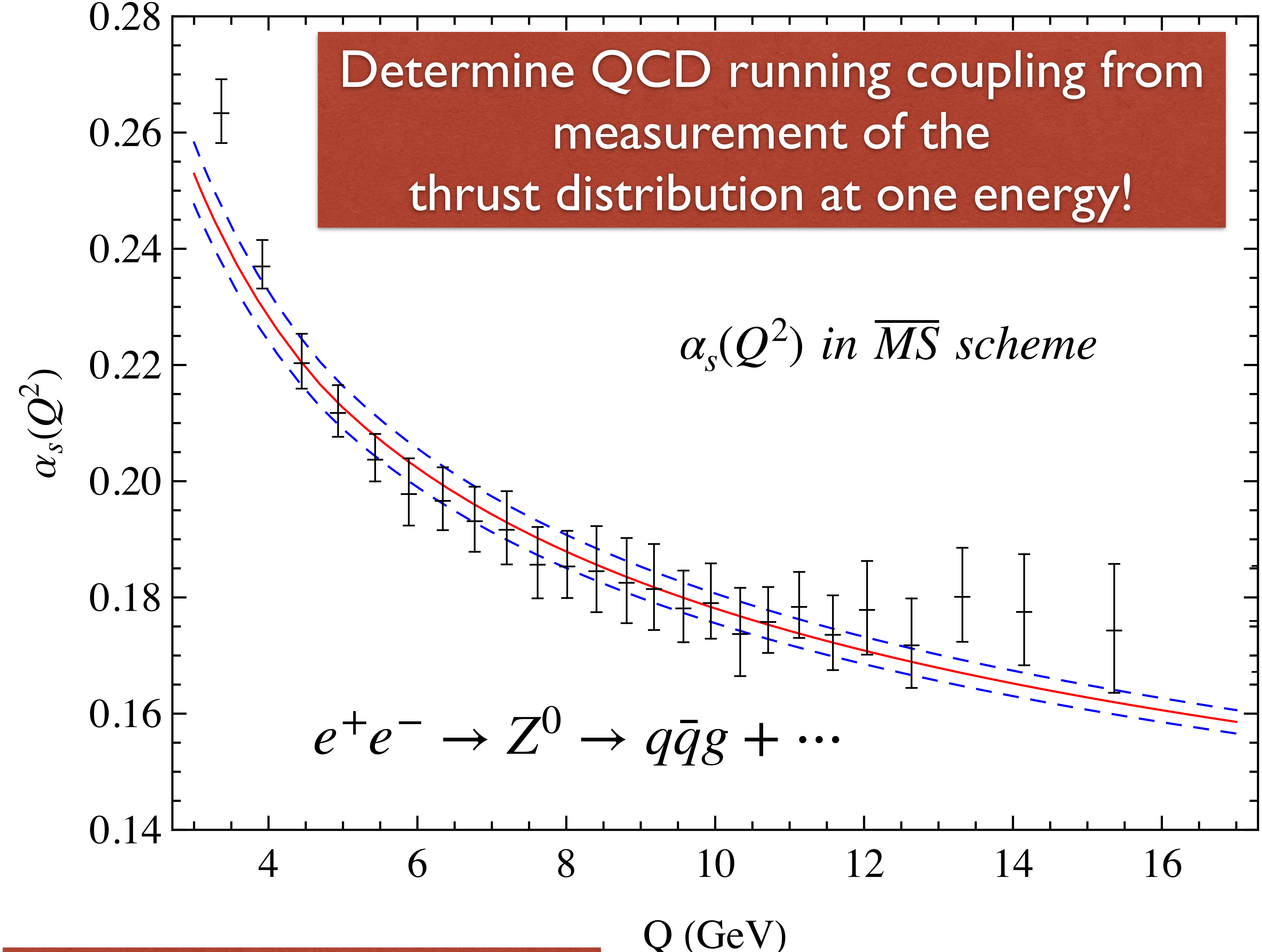
PMC: Leonardo di Giustino,
Xing-Gang Wu
Matin Mojaza



Stan Brodsky
SLAC

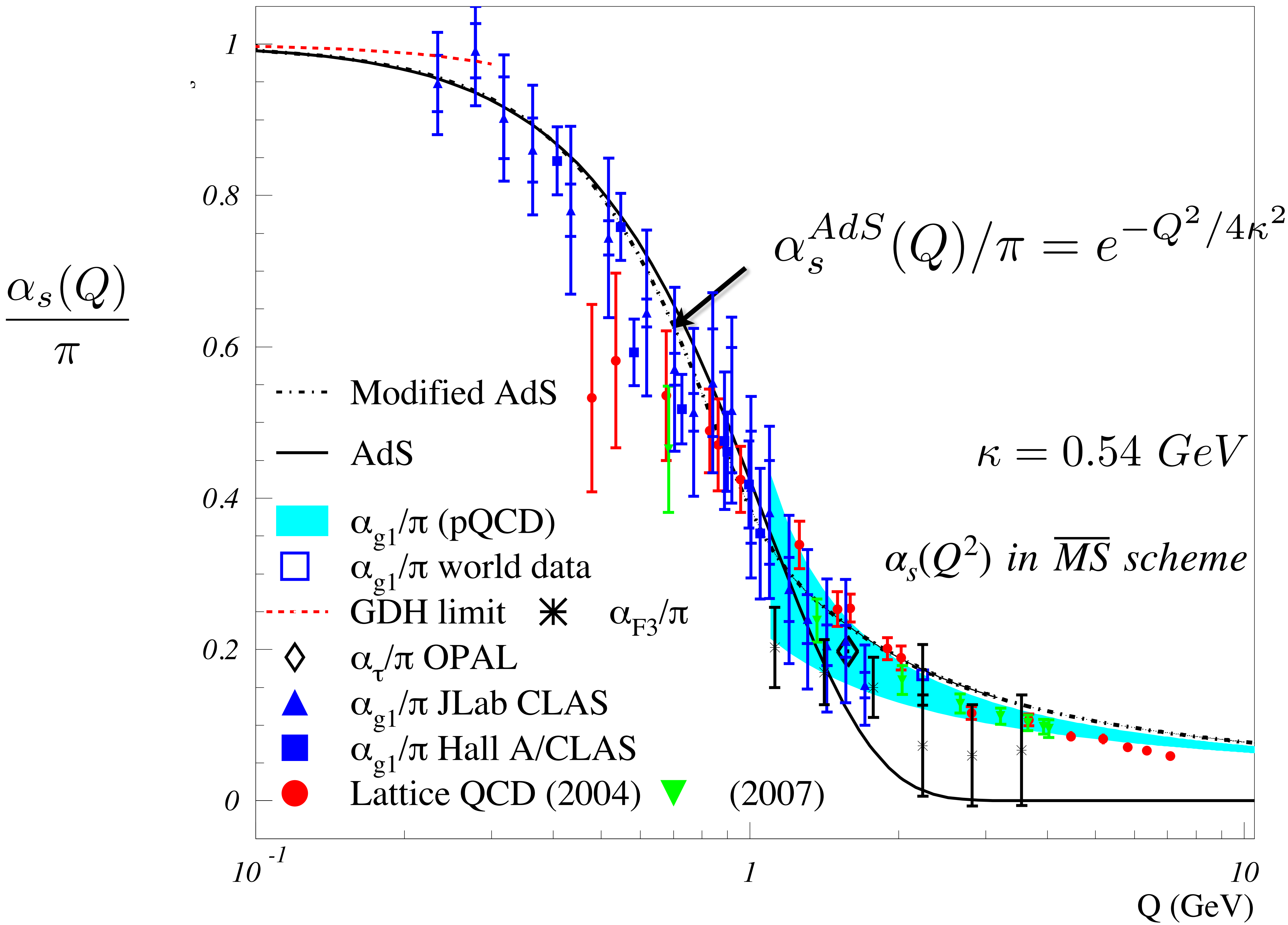
*ECT**
February 12, 2018

α_s Workshop



S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB

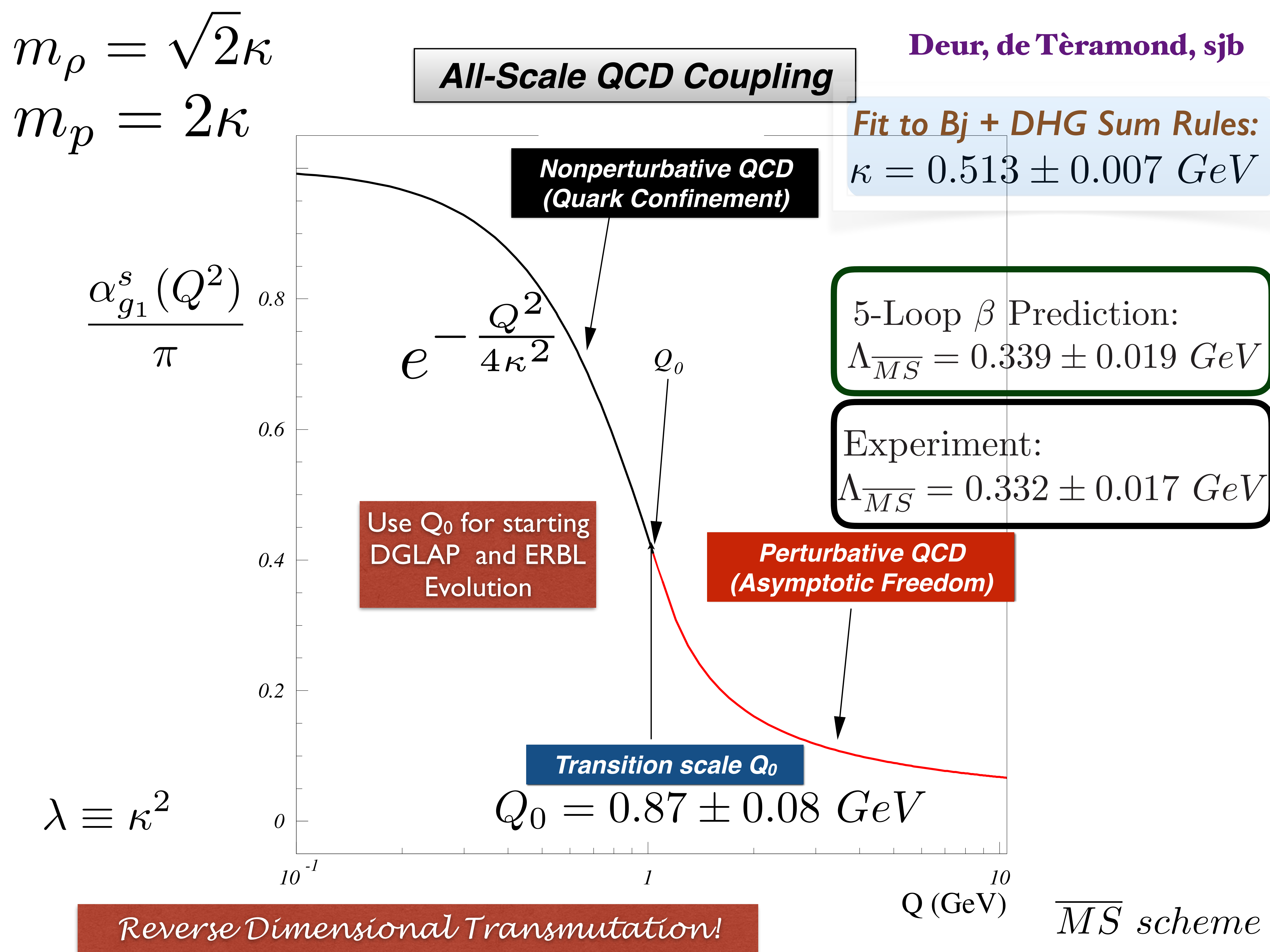
QCD Coupling defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb



$m_\rho = \sqrt{2}\kappa$
$m_p = 2\kappa$
All-Scale QCD Coupling
Deur, de Tèramond, sjb
Fit to Bj + DHG Sum Rules:
$\kappa = 0.513 \pm 0.007\ GeV$
Nonperturbative QCD
(Quark Confinement)
$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$
$e^{-\frac{Q^2}{4\kappa^2}}$
Q_0
5-Loop β Prediction:
$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$
Experiment:
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$
0.8
0.6
0.4
0.2
0
Use Q_0 for starting DGLAP and ERBL Evolution
Perturbative QCD
(Asymptotic Freedom)
Transition scale Q_0
$Q_0 = 0.87 \pm 0.08\ GeV$
$\lambda \equiv \kappa^2$
10^{-1}
1
10
Q (GeV)
$\overline{MS}$ scheme
Reverse Dimensional Transmutation!

Goals

- **Test QCD to maximum precision at the LHC**
- **Maximize sensitivity to new physics**
- **Obtain high precision determination of $\alpha_s(Q^2)$ and other parameters**
- **Determine renormalization scales without ambiguity**
- **Eliminate scheme dependence**

Predictions for physical observables cannot depend on theoretical conventions, such as the renormalization scheme or the initial scale choice

- *Principle of Maximum Conformality (PMC)*

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

Stanley J. Brodsky
Institute for Advanced Study, Princeton, New Jersey 08540
*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**

G. Peter Lepage
Institute for Advanced Study, Princeton, New Jersey 08540
*and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Stanley J. Brodsky†

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu‡

Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

(Received 13 January 2013; published 10 May 2013)

Principle of Maximum Conformality (PMC)

Setting the Renormalization Scale in QCD: The Principle of Maximum Conformality

Stanley J. Brodsky (SLAC & Southern Denmark U., CP3-Origins), Leonardo Di Giustino (SLAC)..

Published in **Phys.Rev. D86 (2012) 085026**

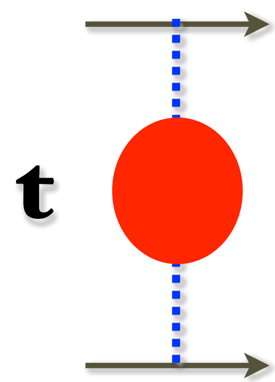
s

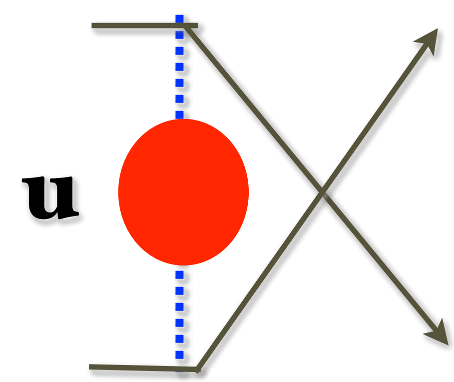
Features of BLM/PMC

- *Predictions are scheme-independent at every order*
- *Matches conformal series*
- *No n! Renormalon growth of pQCD series*
- *New scale appears at each order; n_F determined at each order - matches virtuality of quark loops*
- *Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)*
- *Rigorous: Satisfies all Renormalization Group Principles*
- *Realistic Estimate of Higher-Order Terms*
- *Reduces to standard QED scale* $N_C \to 0$
- *GUT: Must use the same scale setting procedure for QED, QCD*
- *Eliminates unnecessary theory error*
- *Maximal sensitivity to new physics*
- *Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)*
- *Reduces to BLM at NLO: Example: BFKL intercept* **(Fadin, Kim, Lipatov, Pivovarov, sjb)**

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$





$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

Gell-Mann--Low Effective Charge

- **Dressed Photon Propagator sums all β (vacuum polarization) contributions, proper and improper**

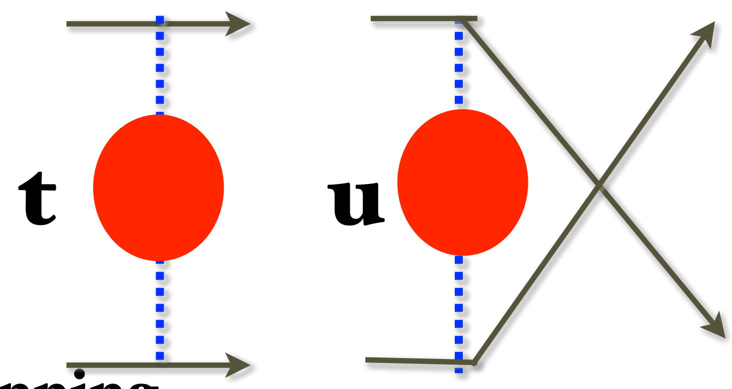
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_o)}{1 - \Pi(t_0)}$$

- **Initial Scale Choice t_o is Arbitrary!**

- **Any renormalization scheme can be used** $\alpha(t) \to \alpha_{\overline{MS}}(e^{-\frac{5}{3}}t)$

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



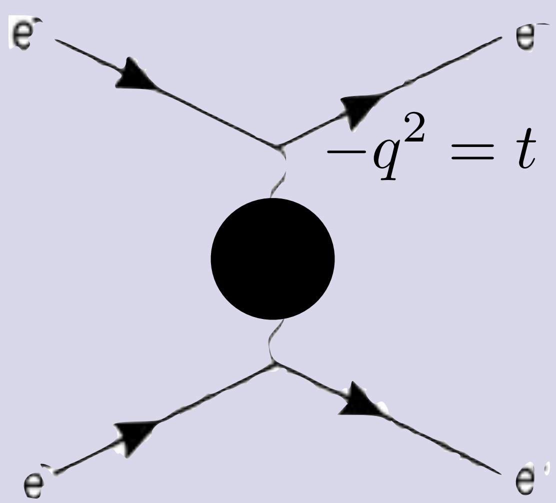
- **No renormalization scale ambiguity!**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
  - **Two separate physical scales: t, u = photon virtuality**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

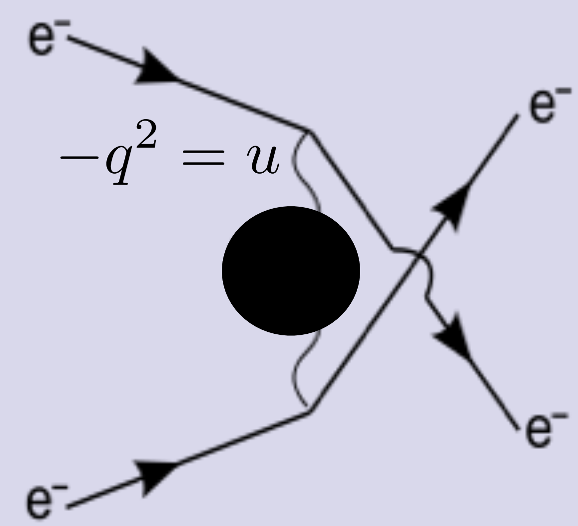
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Example: ee-scattering



$$\mathcal{M}_{ee \to ee} = \frac{8\pi s}{t}\alpha(t) + \frac{8\pi s}{u}\alpha(u)$$

Two separate scales;
one for each skeleton graph.



For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

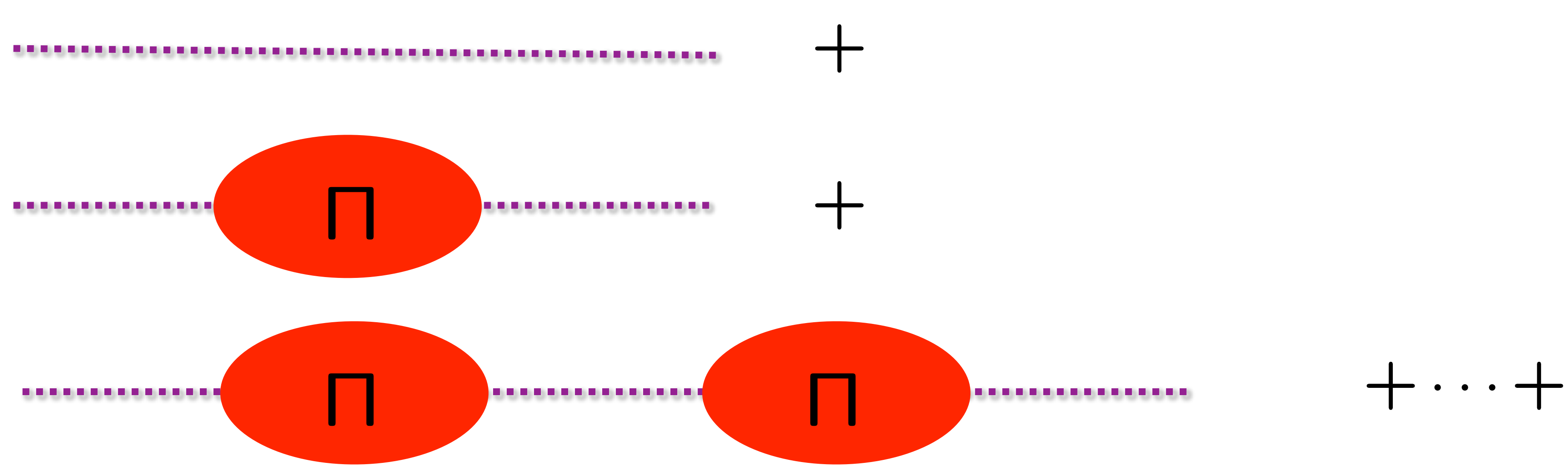
$$\log \frac{\mu^2_{\overline{MS}}}{m_\ell^2} = 6 \int_0^1 dx\, x(1-x) \log \frac{m_\ell^2 + Q^2 x(1-x)}{m_\ell^2}, \quad \xrightarrow{Q^2 \gg m_\ell^2} \log \frac{Q^2}{m_\ell^2} - \frac{5}{3}$$

$$\alpha_{\overline{MS}}(e^{-5/3} q^2) = \alpha_{GM-L}(q^2).$$

QED Running Coupling

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

All-orders lepton-loop corrections to dressed photon propagator



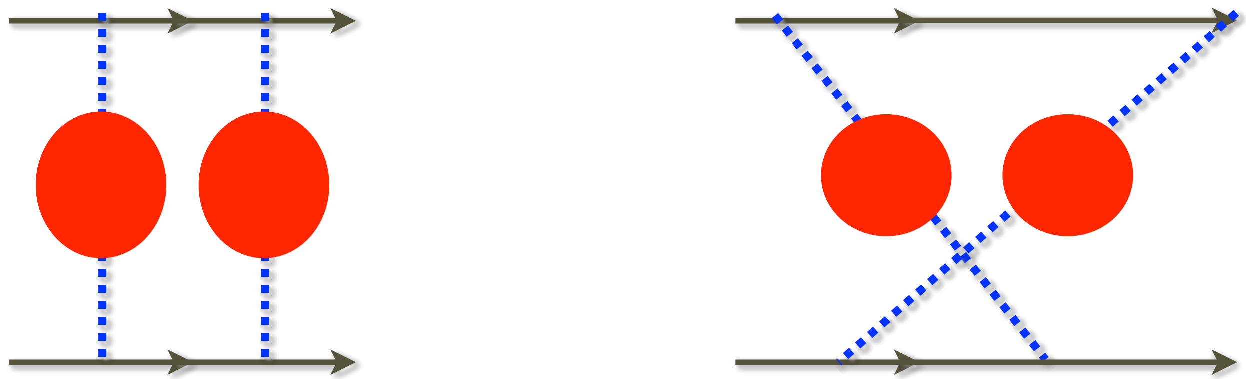
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

***Initial* scale t_o is arbitrary -- Variation gives RGE Equations**

Physical renormalization scale t not arbitrary!

Electron-Electron Scattering in QED

New renormalization scale at each order of pQED



Each "skeleton" graph has its own renormalization scale

Renormalization scheme independent at each order

Independent of initial scale μ_0

Abelian theory is the analytic limit QCD at Nc = 0

Lessons from QED

- **No Renormalization Scale Ambiguity**
- **Dressed Photon Propagator sums all β terms**
- **New Scale at Every Order, Every Skeleton Graph**
- **effective number of flavors n_f determined**
- **Predictions are scheme independent**
- **QCD becomes Abelian QED in Zero Color Limit** $N_C \to 0$

Can use $\overline{MS}$ scheme in QED; answers are scheme independent
Analytic extension: coupling is complex for time-like argument

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

All β (vacuum polarization) terms summed by the running coupling $\alpha(Q^2)$

BLM-PMC Scale Setting

$$\beta_0 = 11 - \frac{2}{3} n_f$$

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2}\beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right],$$

Conformal coefficient - independent of β

where

$$Q^* = Q \exp(3A_{VP}),$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B .$$

The term $33A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

Use skeleton expansion:

Gardi, Grunberg, Rathsman, sjb

BLM/PMC: Set Scales

$$a(Q) \equiv \frac{\alpha_s(Q)}{\pi}$$

such to absorb all 'renormalon-terms', i.e. non-conformal terms

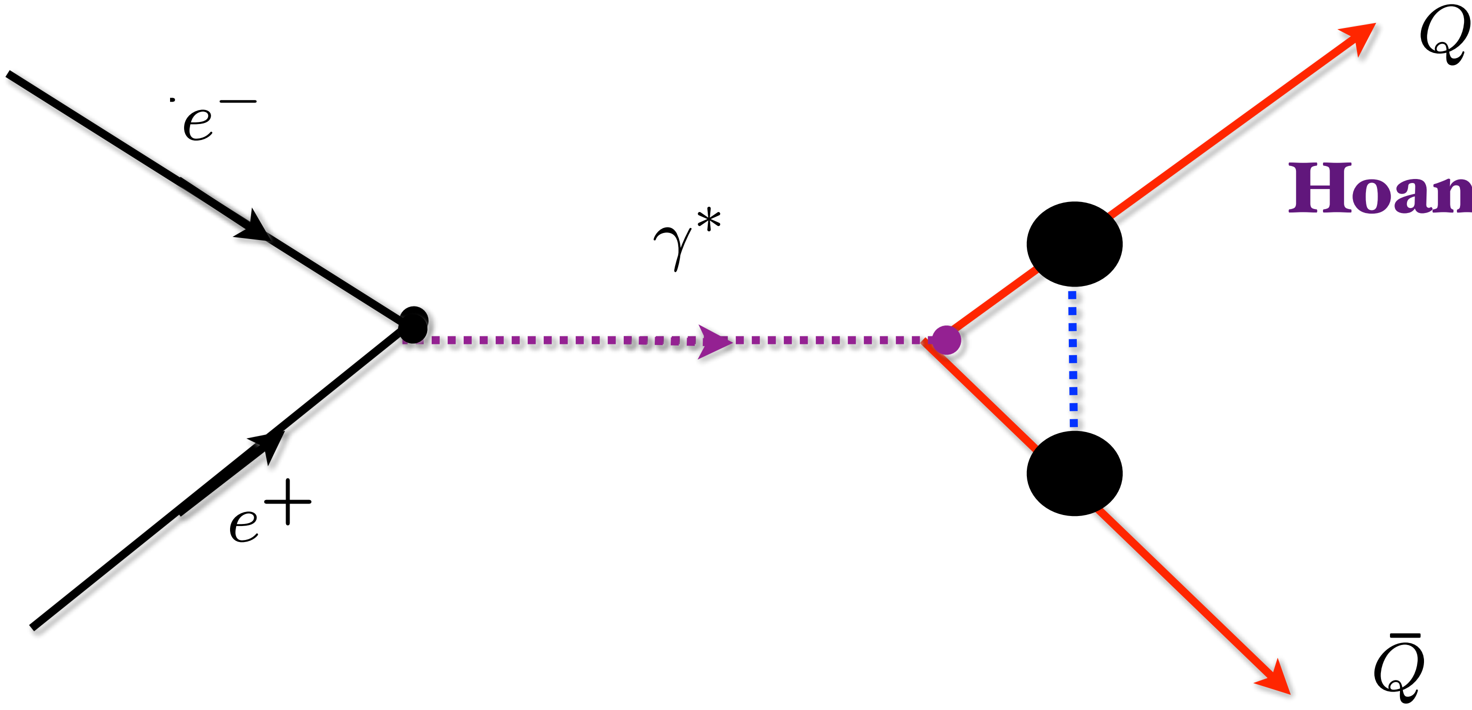
$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

How do we identify the β terms at all orders?

BLM: Use n_f dependence of β_0 and β_1



Hoang, Kuhn, Teubner, sjb

small scale

$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(sv^2)}{4v}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Principle of Maximum Conformality (PMC)

- **Subtract extra constant δ in dimensional regularization. Defines new scheme R_δ**

$$\log 4\pi - \gamma_E - \delta \qquad \overline{MS} : \delta = 0$$

(δ: Arbitrary constant!)

- **Coefficients of δ identify β terms !**
- **Shift β terms to argument of running coupling $\alpha_s(Q_n^2)$ at each order n (analogous to all-orders vacuum polarization summation in QED)**
- **Resulting PQCD series matches β= 0 conformal series!**
- **scheme-independent predictions at each computed order!**
- **almost independent of initial scale μ_0**

M. Mojaza, L. di Giustino, Xing-Gang Wu, sjb

M. Mojaza, Xing-Gang Wu, sjb

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$ -scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\mathrm{MS}} \ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS} \qquad \mu^2 = \mu^2_{\overline{\mathrm{MS}}} \exp(\ln 4\pi - \gamma_E) \ , \quad \mu^2_{\delta_2} = \mu^2_{\delta_1} \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \mathrm{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

↓

Choose μ_R^{init}*; arbitrary initial renormalization scale*

↓

Identify $\{\beta_i^R\}$ *– terms using* δ*- terms*

through the PMC – BLM correspondence principle

↓ *order by order*

Shift scale of α_s *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\}$ *– terms*

↓

Conformal Series

Result is independent of $\mu_R^{\rm init}$ *and scheme at fixed order*

Principle of Maximum Conformality

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at $N_C=0$

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

δ-Scheme automatically identifies β-terms!

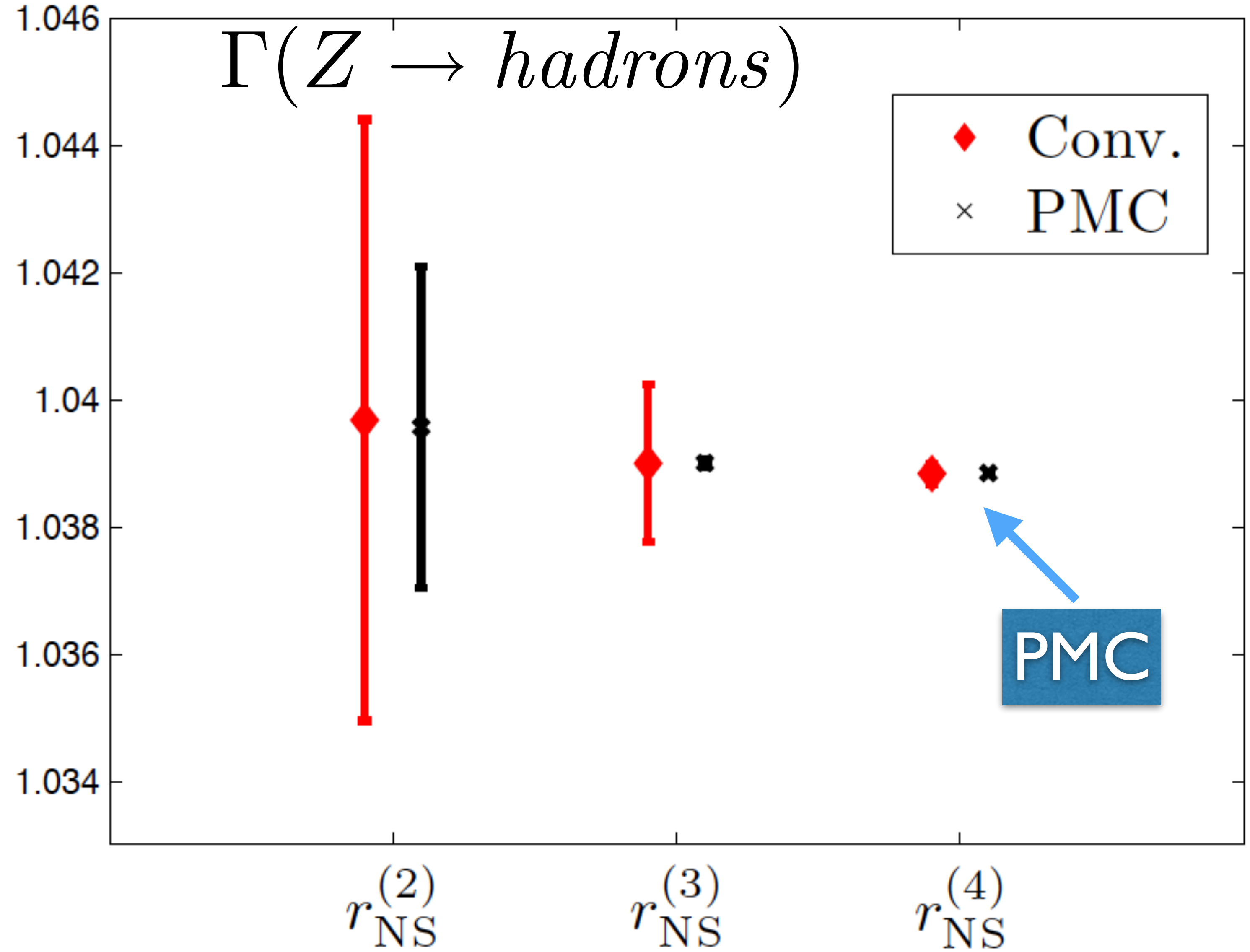
Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

A robot can compute the PMC scales

Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\rm NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\rm NS} a_s^i$ and their errors $\pm|C_n^{\rm NS} a_s^n|_{\rm MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\rm init} = M_Z$.

Since ρ is a physical observable, it must be independent of the arbitrary renormalization scheme and scale. That is,

initial

$$\frac{\partial \rho_\delta}{\partial \mu_\delta} = 0 \, , \qquad \frac{\partial \rho_\delta}{\partial \delta} = 0 \, , \tag{16}$$

Generalization: use δ_n at n-loops.

$$\begin{aligned} \rho_\delta(Q^2) =& r_0 + r_1 a_1(Q) + (r_2 - \beta_0 r_1 \underline{\delta_1}) a_2(Q)^2 \\ & + [r_3 - \beta_1 r_1 \underline{\delta_1} - 2\beta_0 r_2 \underline{\delta_2} + \beta_0^2 r_1 \underline{\delta_1^2}] a_3(Q)^3 \\ & + [r_4 - \beta_2 r_1 \underline{\delta_1} - 2\beta_1 r_2 \underline{\delta_2} - 3\beta_0 r_3 \underline{\delta_3} + 3\beta_0^2 r_2 \underline{\delta_2^2} \\ & - \beta_0^3 r_1 \underline{\delta_1^3} + \frac{5}{2}\beta_1 \beta_0 r_1 \underline{\delta_1^2}] a(Q)^4 + \mathcal{O}(a^5) \end{aligned} \tag{20}$$

Shows the general way nonconformal terms enter an observable and the scheme dependence

Special Degeneracy in PQCD

There is nothing special about a particular value for δ, thus for any δ

General pattern of pQCD

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \underline{\cdots) r_{2,1}}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \underline{\cdots) r_{3,2}} + (\beta_0^3 + \cdots) r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \underline{\cdots) r_{3,1}}$$
$$+ \cdots$$

PMC Scales Q_1 Q_2

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a) r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial \beta}{\partial a} r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}} r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a) r_{3,1} + \cdots$$

M. Mojaza, Xing-Gang Wu, sjb

General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5) \qquad (19)\end{aligned}$$

PMC scales thus satisfy

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1}$$

$$r_{3,0}a(Q_3)^3 = r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a)r_{4,1}$$

$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a)r_{k+1,1}$$

number of flavors n_f depends on Q_k

Features of BLM/PMC

- *Predictions are scheme-independent at every order*
- *Matches conformal series*
- *No n! Renormalon growth of pQCD series*
- *New scale appears at each order; n_F determined at each order - matches virtuality of quark loops!*
- *Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)*
- *Rigorous: Satisfies all Renormalization Group Principles*
- *Realistic Estimate of Higher-Order Terms*
- *Reduces to standard QED scale* $N_C \to 0$
- *GUT: Must use the same scale setting procedure for QED, QCD*
- *Eliminates unnecessary theory error*
- *Maximal sensitivity to new physics*
- *Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)*
- *PMC Reduces to BLM at NLO: Example: BFKL intercept* (Fadin, Kim, Lipatov, Pivovarov, sjb)

standard procedures for PMC

To set the BLM scales up to NNLO, the starting point

$$\rho = r_0 \Big[a_s^n(Q) + (A_1 + A_2 n_f) a_s^{n+1}(Q) + (B_1 + B_2 n_f + B_3 n_f^2) a_s^{n+2}(Q) + (C_1 + C_2 n_f + C_3 n_f^2 + C_4 n_f^3) a_s^{n+3}(Q) + \cdots \Big]$$

free of $a_s = \left(\frac{\alpha_s}{\pi}\right)$

first

to set the effective scale Q^* at LO

$$\rho = r_0 \Big[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^*) + (\widetilde{B}_1 + \widetilde{B}_2 n_f) a_s^{n+2}(Q^*) + (\widetilde{C}_1 + \widetilde{C}_2 n_f + \widetilde{C}_3 n_f^2) a_s^{n+3}(Q^*) + \cdots \Big] . \quad (11)$$

second

The second step is to set the effective scale Q^{**} at NLO

$$\rho = r_0 \Big[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^{**}) + \widetilde{\widetilde{B}}_1 a_s^{n+2}(Q^{**}) + (\widetilde{\widetilde{C}}_1 + \widetilde{\widetilde{C}}_2 n_f) a_s^{n+3}(Q^{**}) + \cdots \Big] , \quad (12)$$

standard procedures for PMC

LO NLO NNLO

final

and the final step is to set the effective scale Q^{***} at NNLO

$$\rho = r_0 \Big[a_s^n(Q^*) + \widetilde{A}_1 a_s^{n+1}(Q^{**}) + \widetilde{\widetilde{B}}_1 a_s^{n+2}(Q^{***}) + \widetilde{\widetilde{\widetilde{C}}}_1 a_s^{n+3}(Q^{***}) + \cdots \Big] . \quad (13)$$

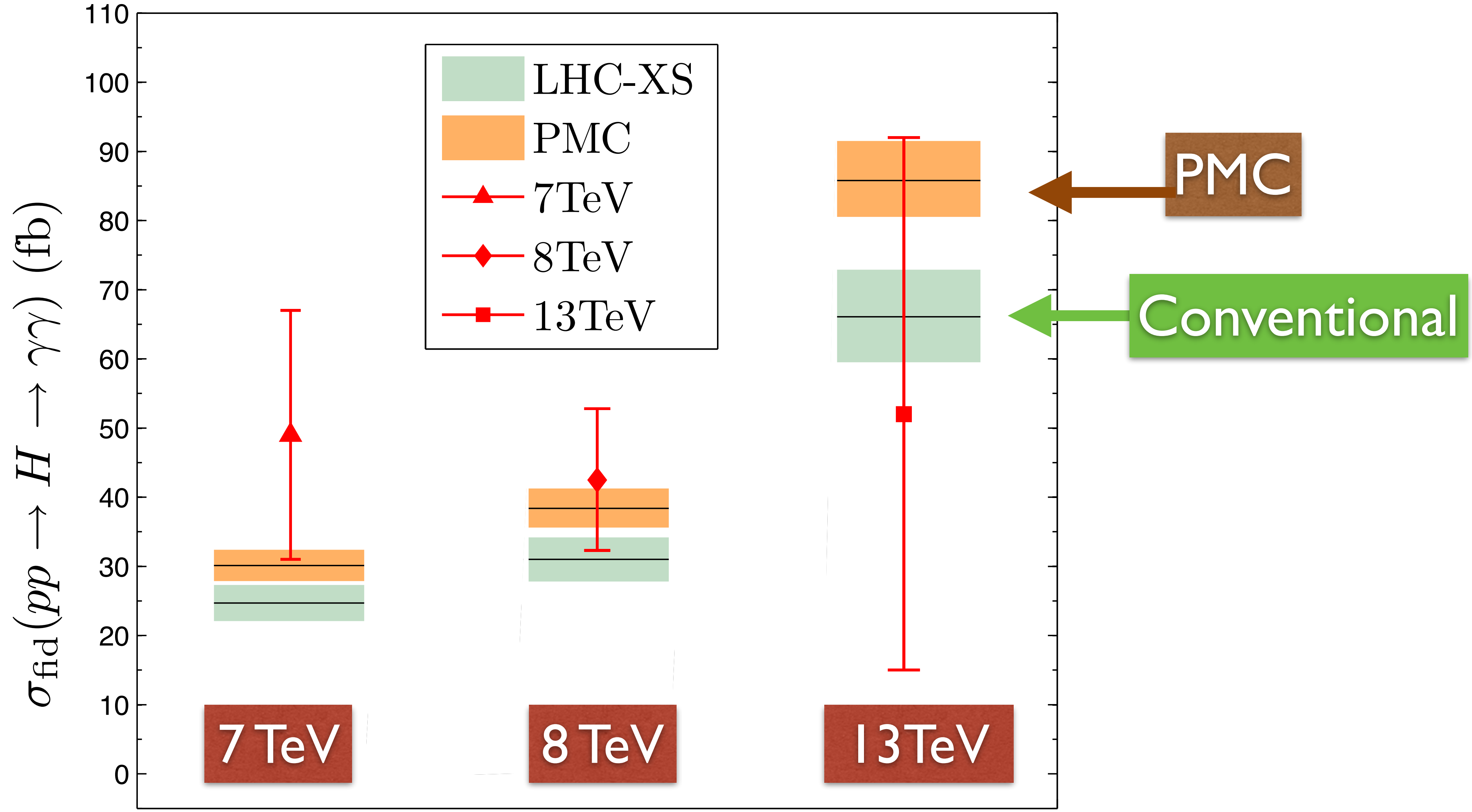
Application of the Principle of Maximum Conformality to the Hadroproduction of the Higgs Boson at the LHC

Sheng-Quan Wang[1],* Xing-Gang Wu[2],† Stanley J. Brodsky[3],‡ and Matin Mojaza[4]§

We present improved pQCD predictions for Higgs boson hadroproduction at the Large Hadronic Collider (LHC) by applying the Principle of Maximum Conformality (PMC), a procedure which resums the pQCD series using the renormalization group (RG), thereby eliminating the dependence of the predictions on the choice of the renormalization scheme while minimizing sensitivity to the initial choice of the renormalization scale. In previous pQCD predictions for Higgs boson hadroproduction, it has been conventional to assume that the renormalization scale μ_r of the QCD coupling $\alpha_s(\mu_r)$ is the Higgs mass, and then to vary this choice over the range $1/2m_H < \mu_r < 2m_H$ in order to estimate the theory uncertainty. However, this error estimate is only sensitive to the non-conformal β terms in the pQCD series, and thus it fails to correctly estimate the theory uncertainty in cases where pQCD series has large higher order contributions, as is the case for Higgs boson hadroproduction. Furthermore, this *ad hoc* choice of scale and range gives pQCD predictions which depend on the renormalization scheme being used, in contradiction to basic RG principles. In contrast, after applying the PMC, we obtain next-to-next-to-leading order RG resummed pQCD predictions for Higgs boson hadroproduction which are renormalization-scheme independent and have minimal sensitivity to the choice of the initial renormalization scale. Taking $m_H = 125$ GeV, the PMC predictions for the $pp \to HX$ Higgs inclusive hadroproduction cross-sections for various LHC center-of-mass energies are: $\sigma_{\rm Incl}|_{7\,{\rm TeV}} = 21.21^{+1.36}_{-1.32}$ pb, $\sigma_{\rm Incl}|_{8\,{\rm TeV}} = 27.37^{+1.65}_{-1.59}$ pb, and $\sigma_{\rm Incl}|_{13\,{\rm TeV}} = 65.72^{+3.46}_{-3.01}$ pb, respectively. We also predict the fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$: $\sigma_{\rm fid}|_{7\,{\rm TeV}} = 30.1^{+2.3}_{-2.2}$ fb, $\sigma_{\rm fid}|_{8\,{\rm TeV}} = 38.3^{+2.9}_{-2.8}$ fb, and $\sigma_{\rm fid}|_{13\,{\rm TeV}} = 85.8^{+5.7}_{-5.3}$ fb. The error limits in these predictions include the small residual high-order renormalization-scale dependence, plus the uncertainty from the factorization-scale. The PMC predictions show better agreement with the ATLAS measurements than the LHC-XS predictions which are based on conventional renormalization scale-setting.

S-Q Wang, X-G Wu, sjb

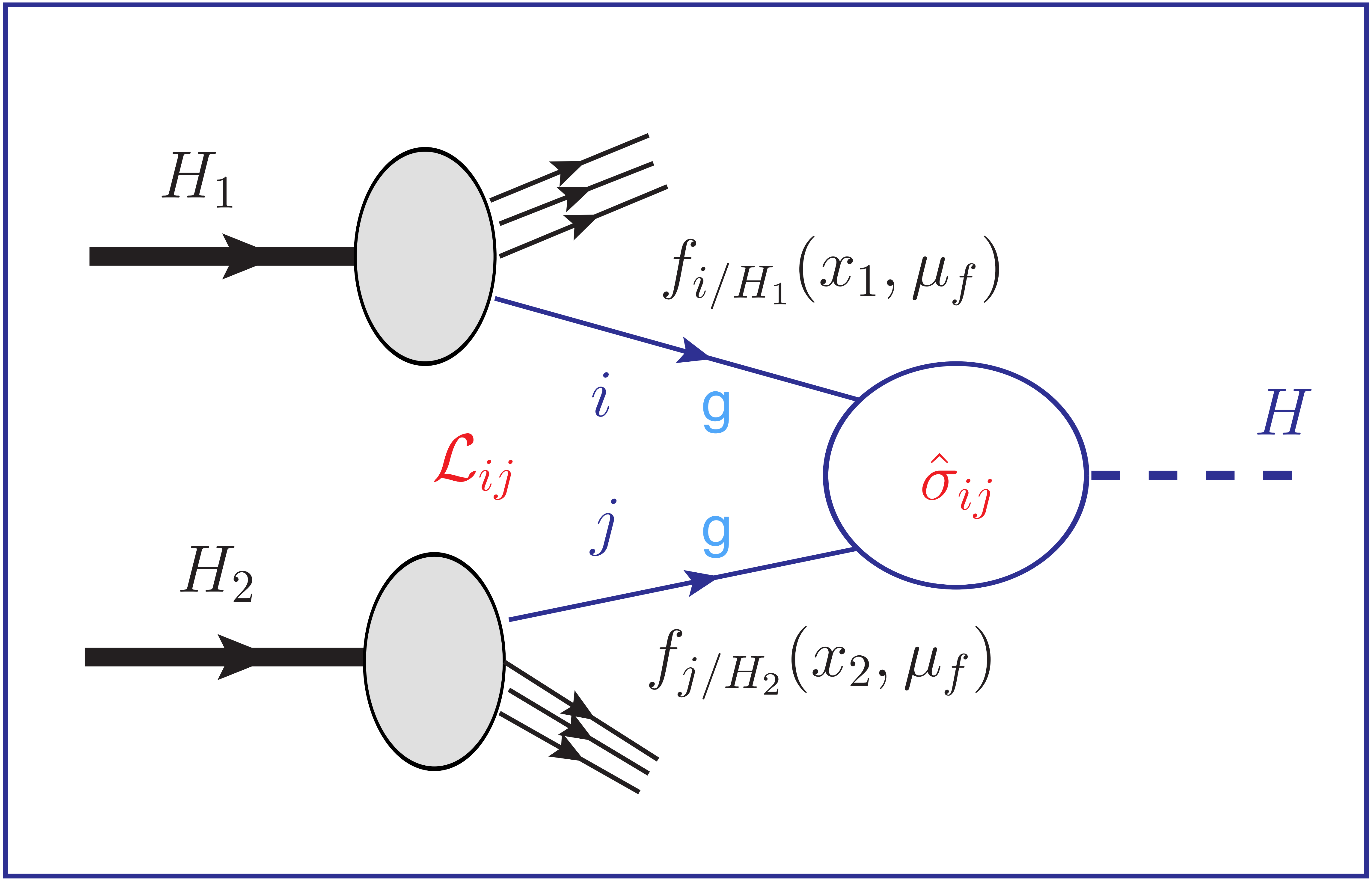
$$\sigma(pp \to HX \to \gamma\gamma X)$$



Comparison of the PMC predictions for the fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ with the ATLAS measurements at various collision energies. The LHC-XS predictions are presented as a comparison.

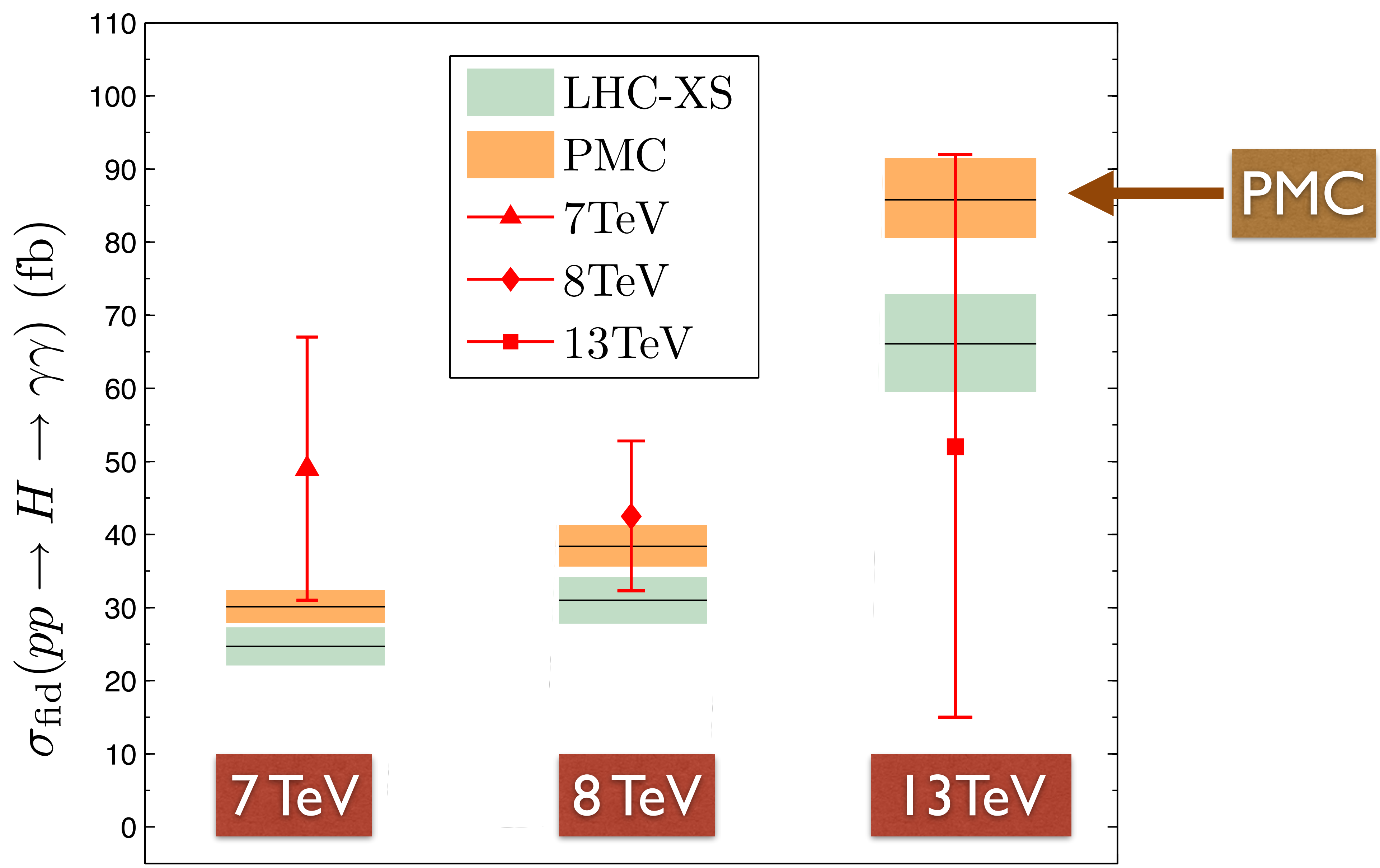
$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [48]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.4^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

$\sigma^{gg}(pp \to HX)$



S-Q Wang, X-G Wu, sjb

$$\sigma(pp \to HX \to \gamma\gamma X)$$

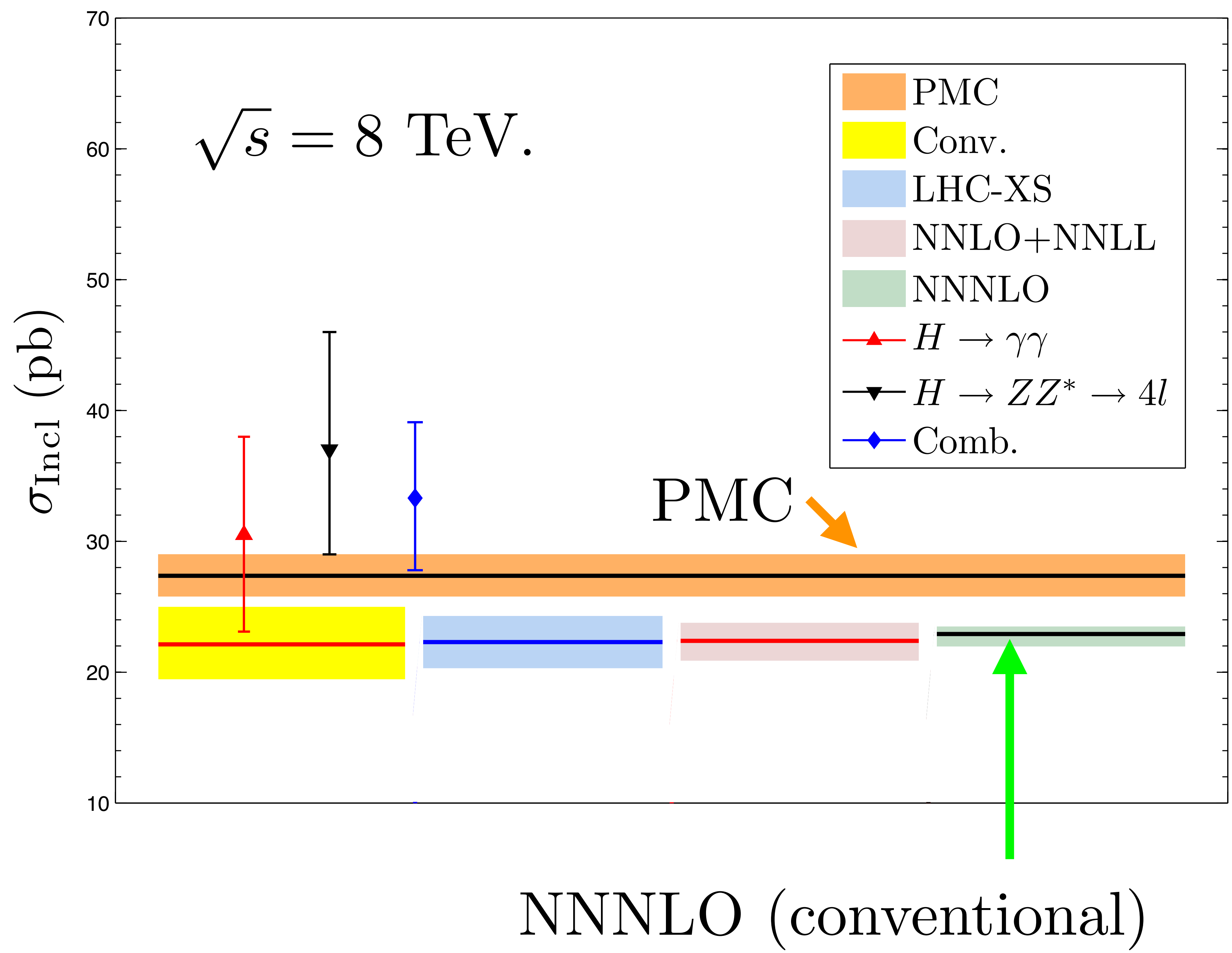


Comparison of the PMC predictions for the fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ with the ATLAS measurements at various collision energies. The LHC-XS predictions are presented as a comparison.

$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [48]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.4^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

S-Q Wang, X-G Wu, sjb

$$\sigma^{gg}(pp \to HX)$$

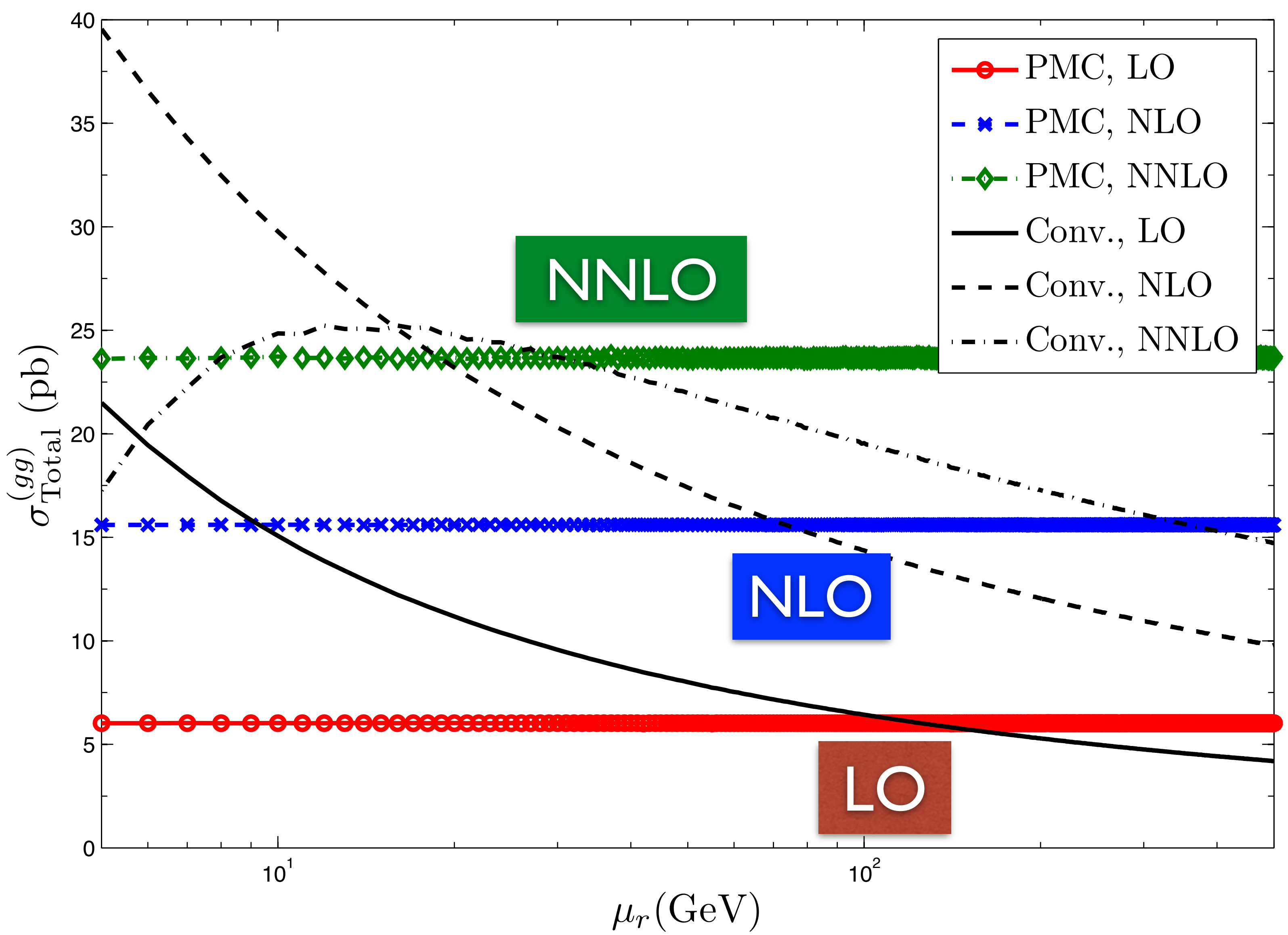


S-Q Wang, X-G Wu, sjb

PMC insensitive to initial scale choice

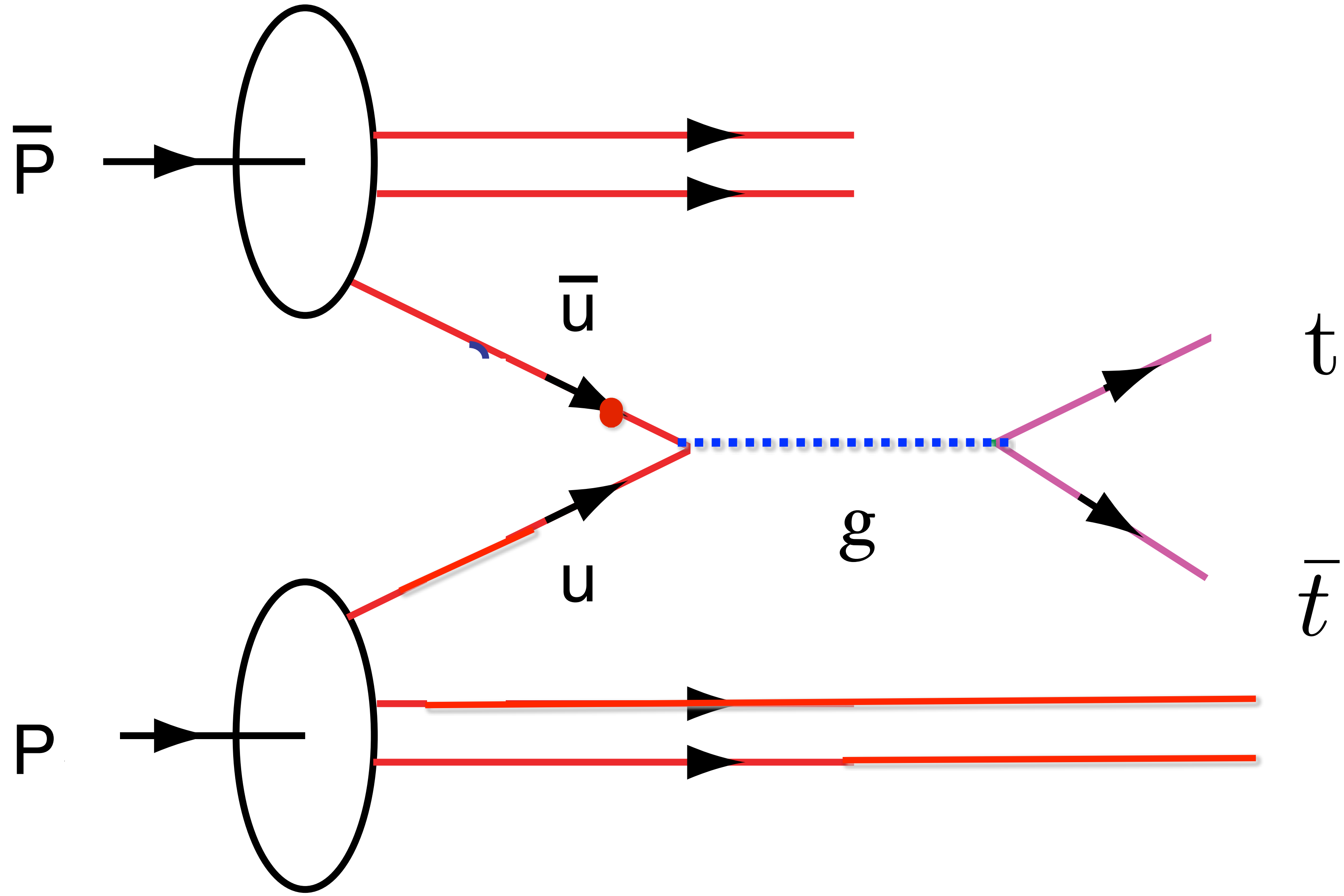
Different PMC scales at each order!

$$\sigma^{gg}(pp \to HX)$$



The gluon-fusion total cross-sections $\sigma^{(gg)}_{\rm Total}$ up to LO, NLO and NNLO levels versus the initial scale μ_r under conventional (Conv.) and PMC scale-settings with the collision energy $\sqrt{S} = 8$ TeV.

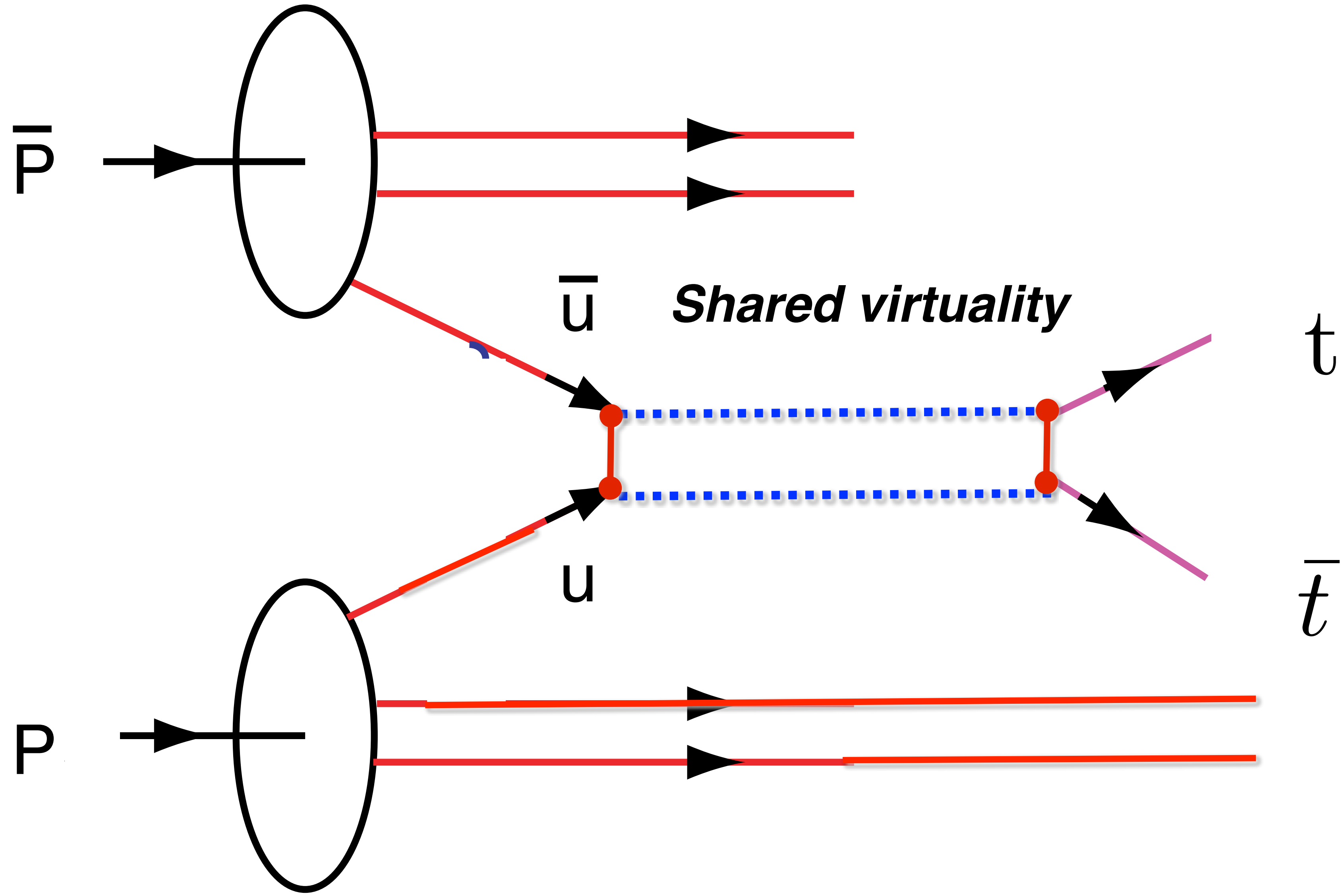
Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron



Born term.

Xing-Gang Wu, sjb

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron

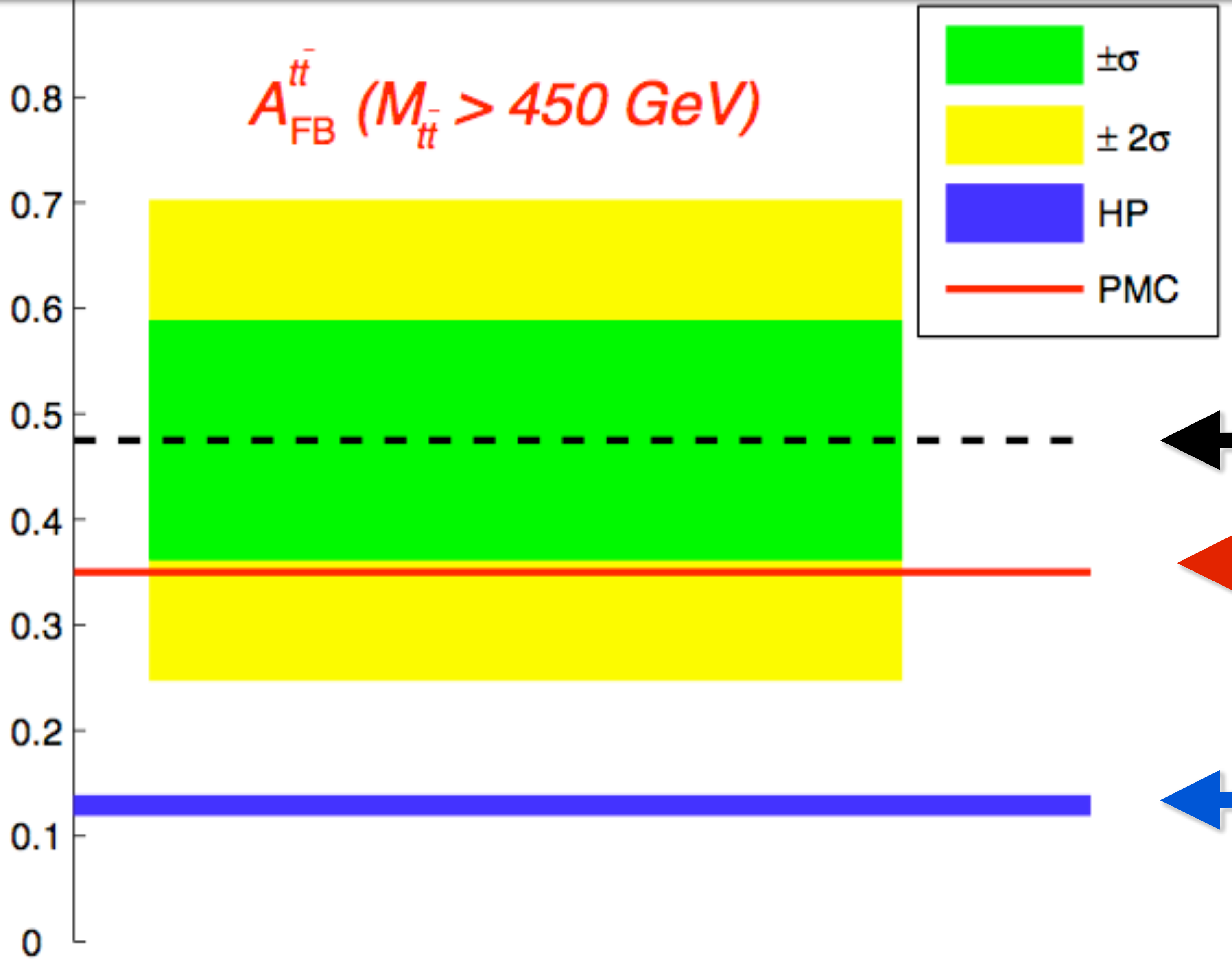


Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED!!

Xing-Gang Wu, sjb

The Renormalization Scale Ambiguity for Top-Pair Production Asymmetry at the Tevatron is Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
S.-Q. Wang
Z.-G. Shi
SJB

Experimental asymmetry

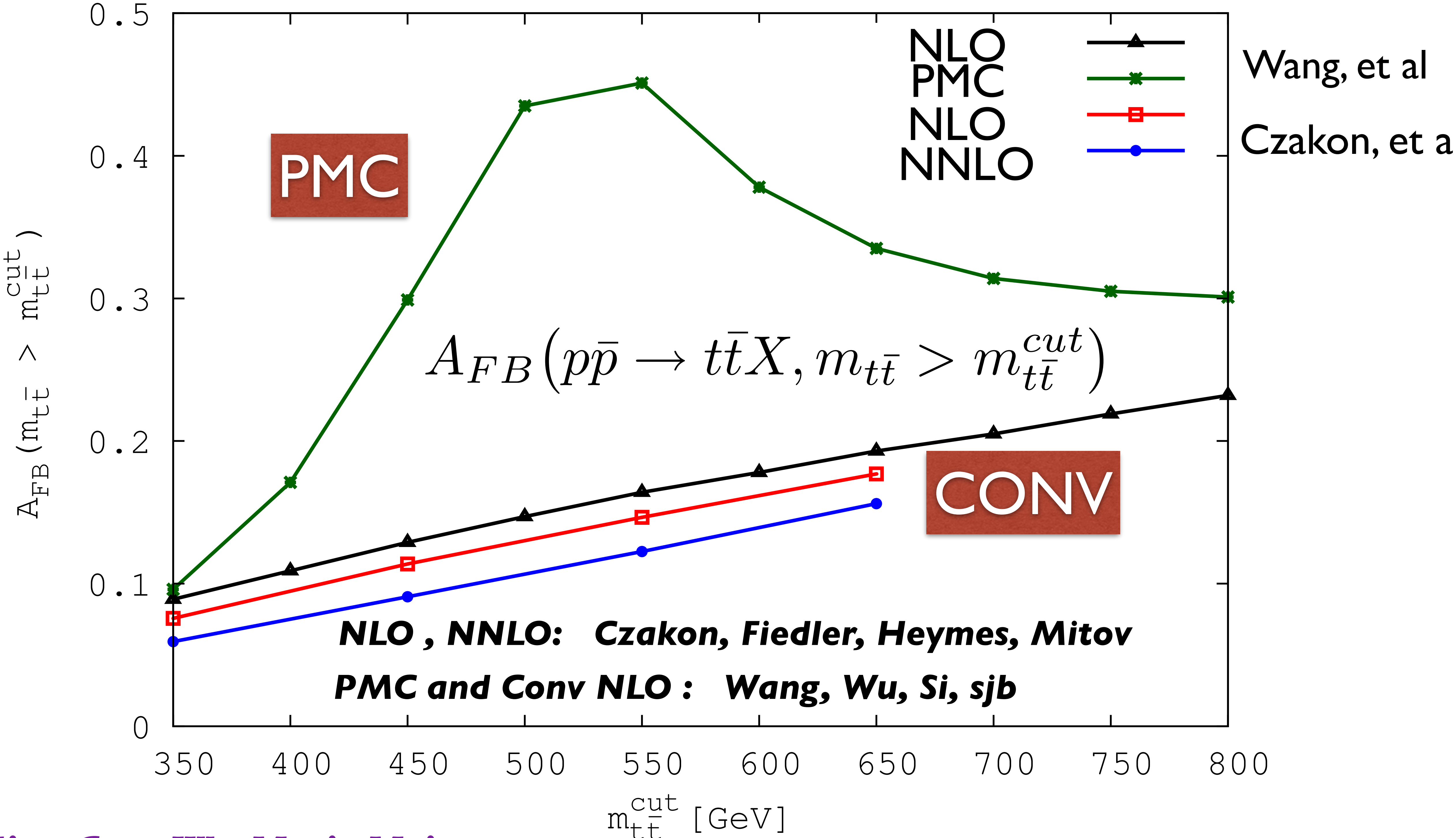
PMC Prediction

Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

NNLO QCD predictions for fully-differential top-quark pair production at the Tevatron arXiv:1601.05375

Michał Czakon,[a] Paul Fiedler,[a] David Heymes[b] and Alexander Mitov[b]

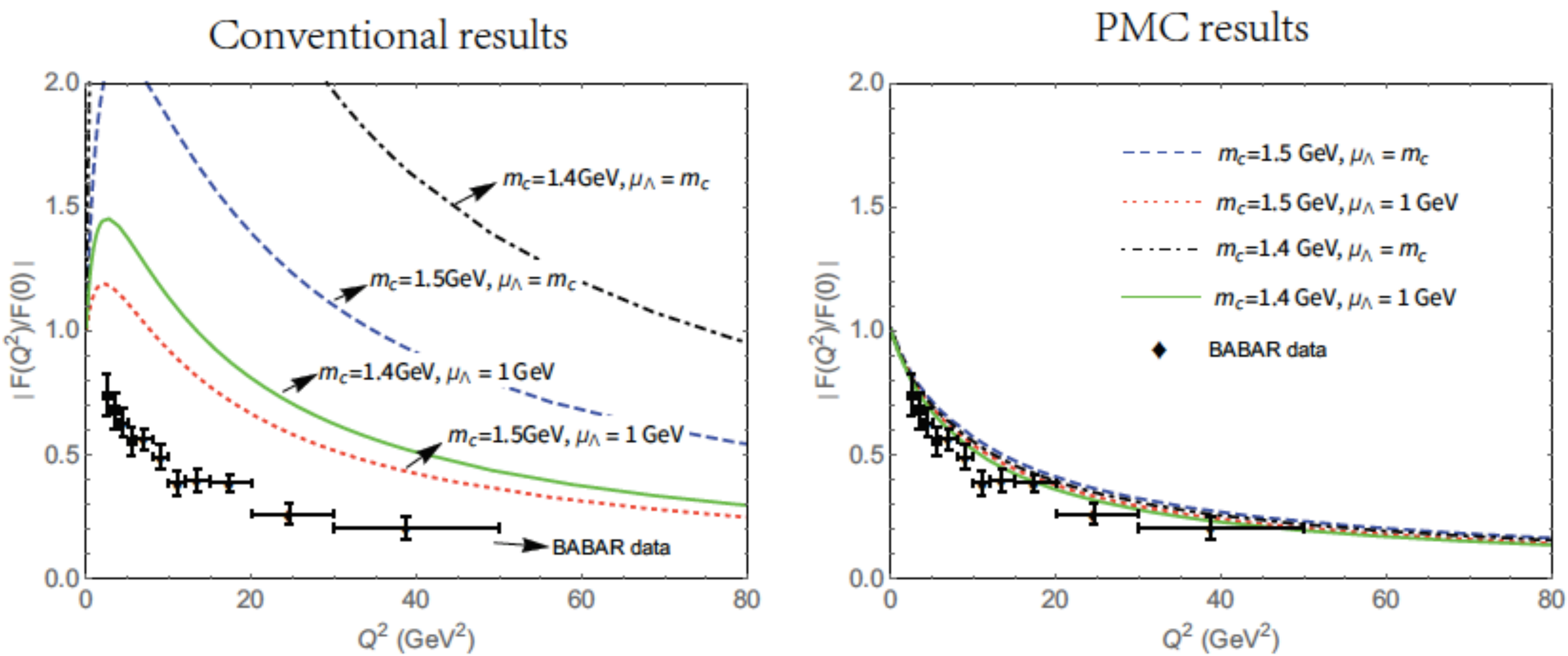


Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

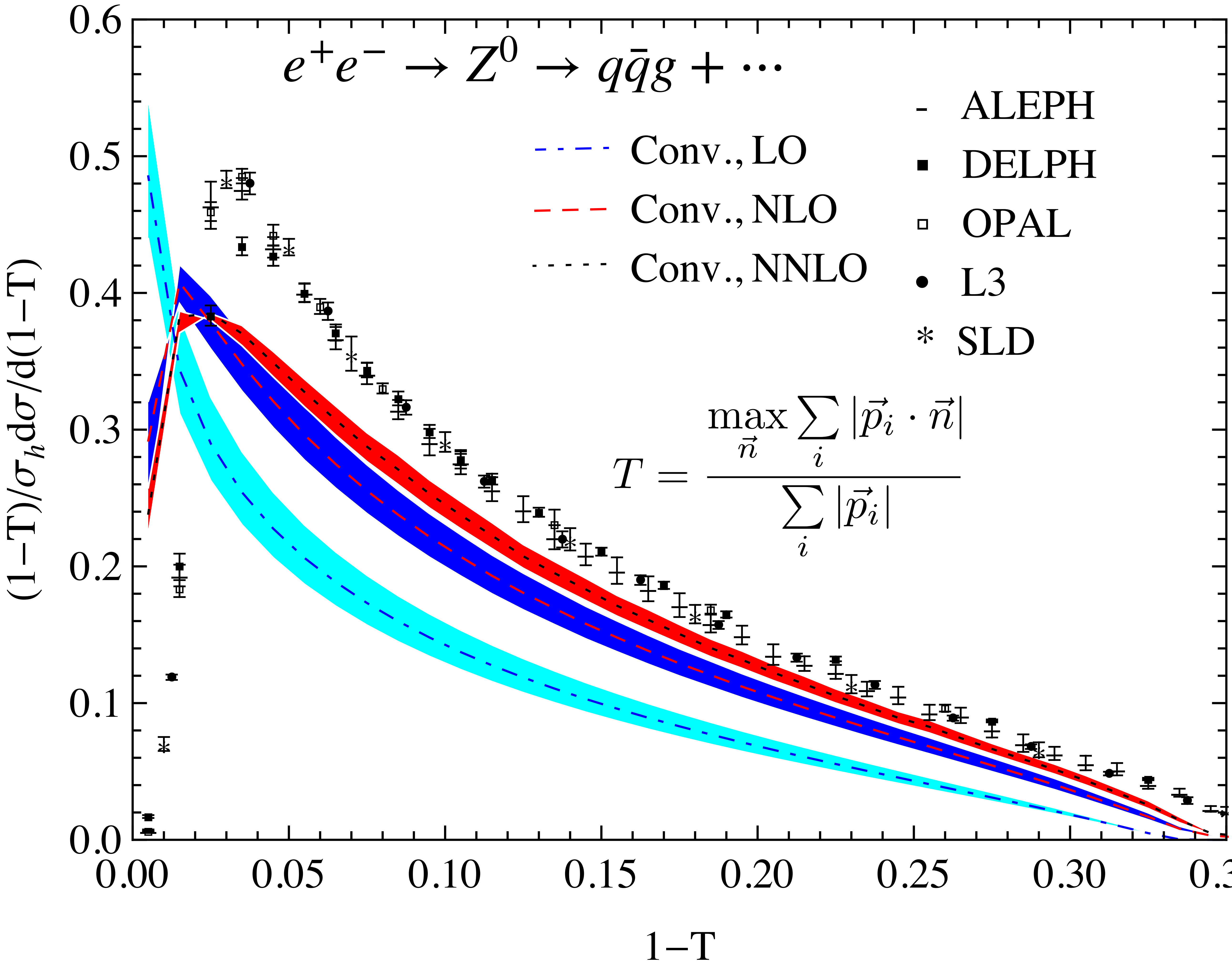
Predictions for the cumulative front-back asymmetry.

A solution to the $\gamma\gamma^* \to \eta_c$ puzzle using the Principle of Maximum Conformality

Sheng-Quan Wang[1,2,*] Xing-Gang Wu[2,†] Wen-Long Sang[3,4,‡] and Stanley J. Brodsky[5§]



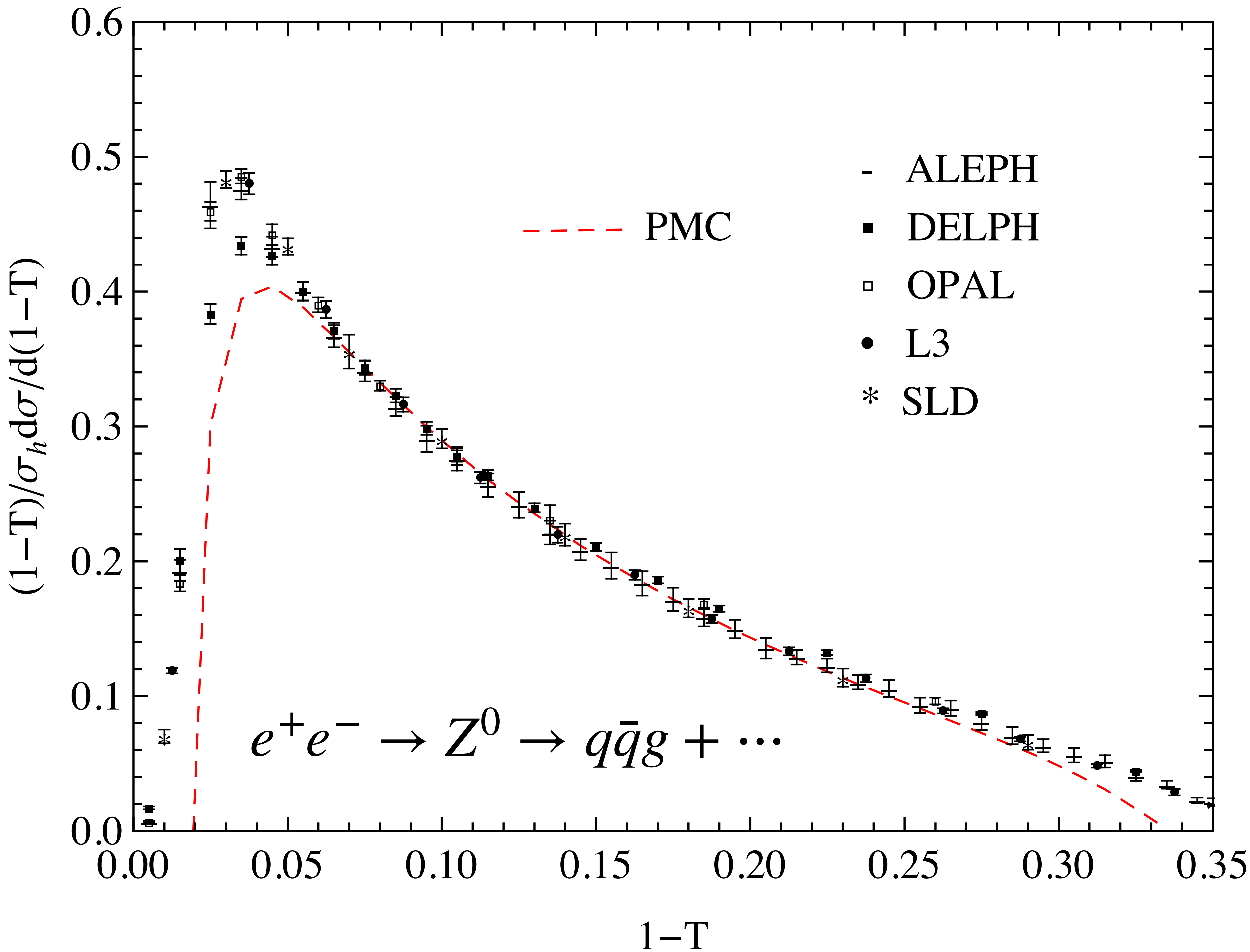
The transition form factor ratio $|F(Q^2)/F(0)|$ versus the momentum transfer squared Q^2 under conventional (Up) [3] and PMC (Down) scale setting. $m_c = 1.5$, 1.4 GeV.



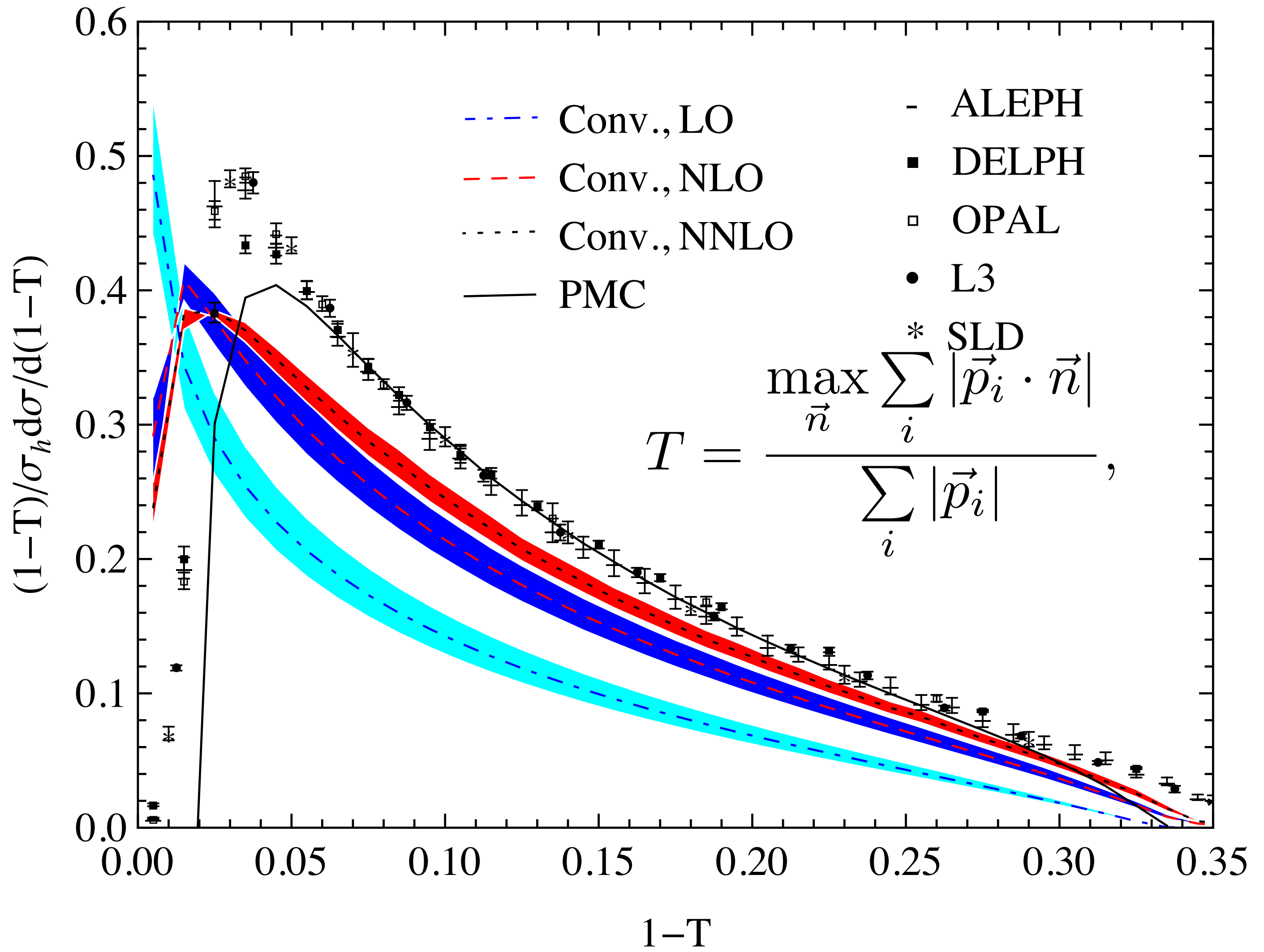
$e^+e^- \to Z^0 \to q\bar{q}g + \cdots$
Conv., LO
Conv., NLO
Conv., NNLO
ALEPH
DELPH
OPAL
L3
SLD
$T = \frac{\max_{\vec{n}} \sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$
(1−T)/σ_hdσ/d(1−T)
1−T
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.00
0.05
0.10
0.15
0.20
0.25
0.30

Thrust Distribution in Electron-Positron Annihilation using the Principle of Maximum Conformality

S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB

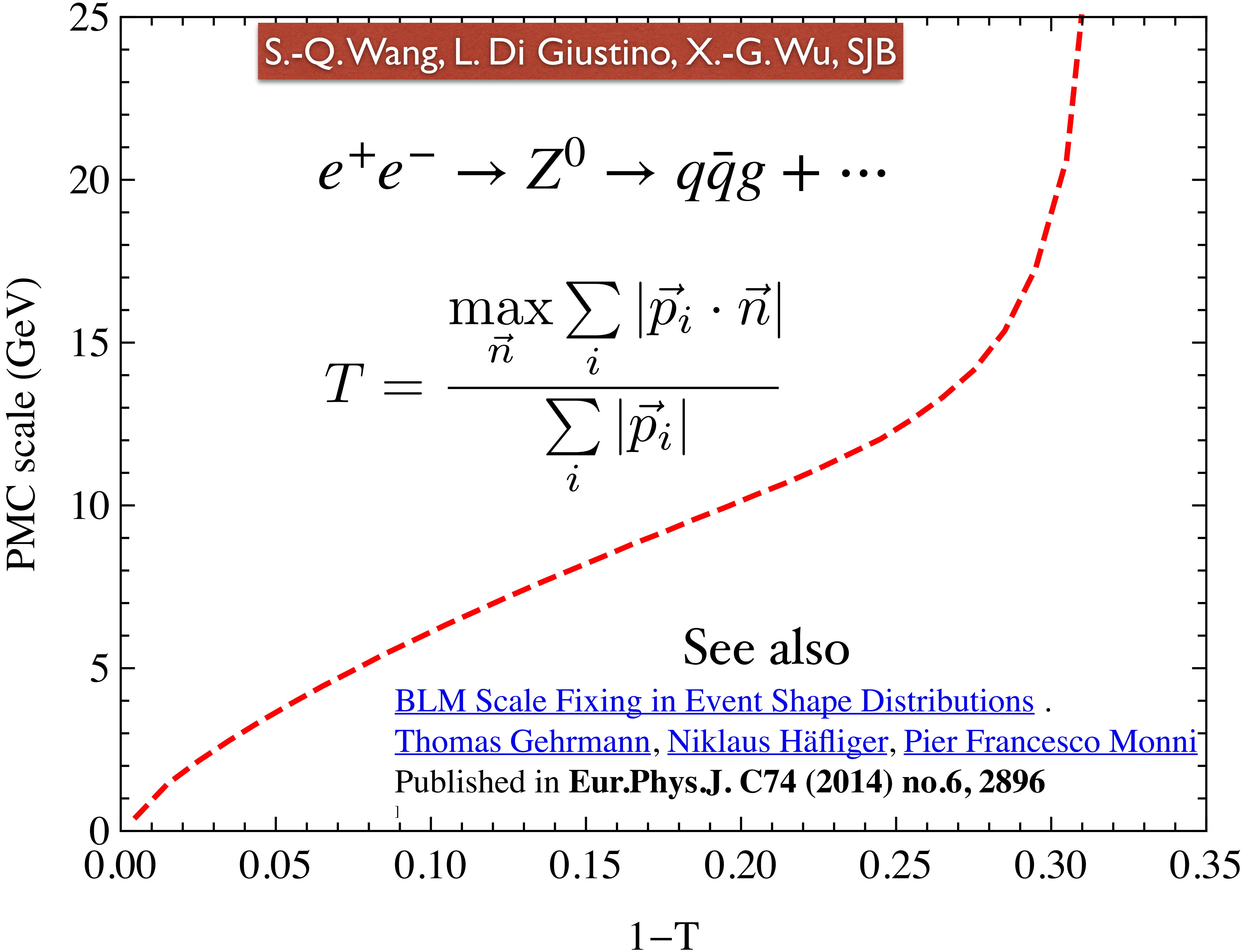


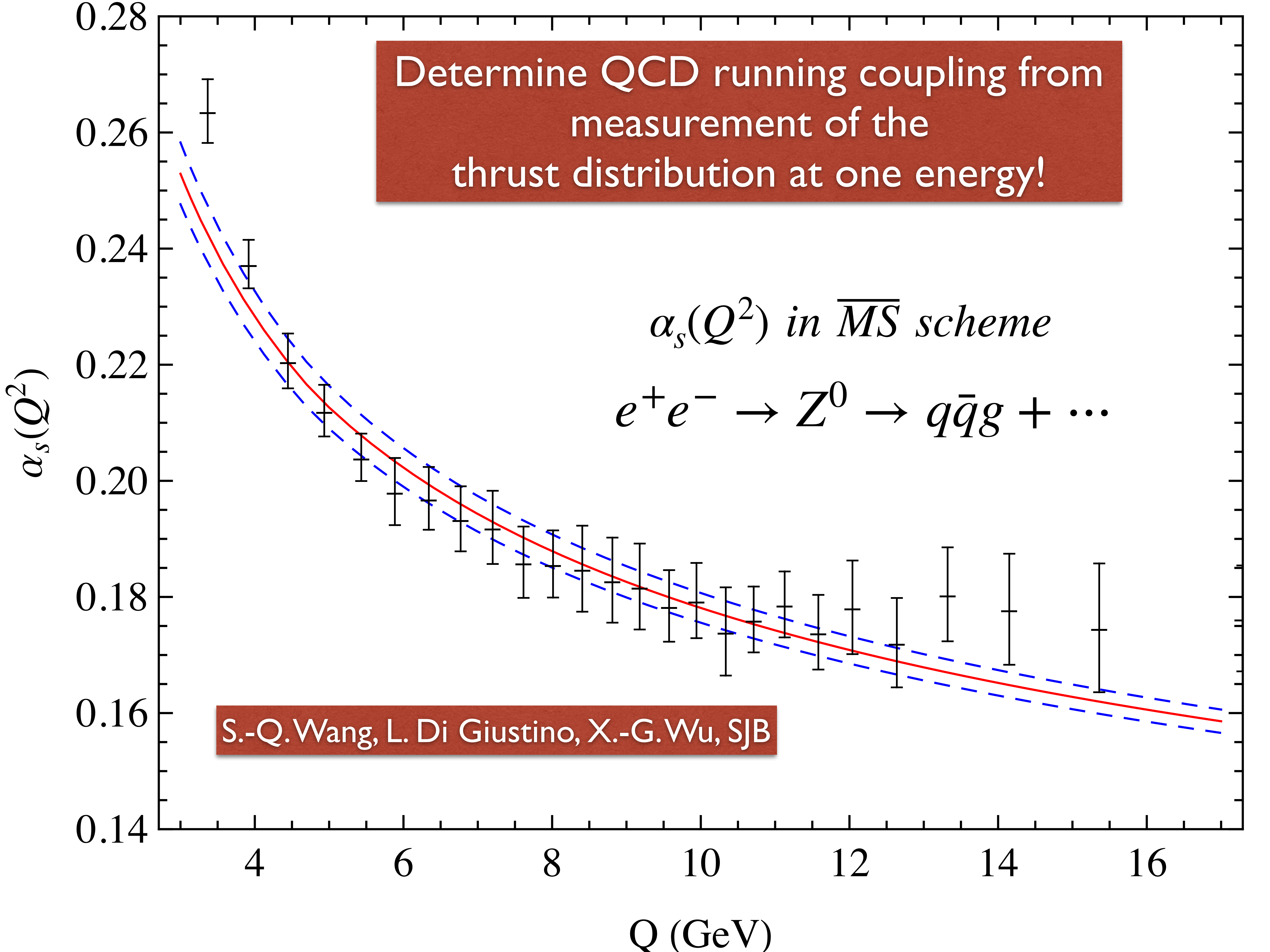
S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB



The thrust $(1-T)$ differential distributions using the conventional (Conv.) and PMC scale settings. The dot-dashed, dashed and dotted lines are the conventional results at LO, NLO and NNLO, respectively. The solid line is the PMC result. The bands for the theoretical predictions are obtained by varying $\mu_r \in [M_Z/2, 2M_Z]$. The PMC prediction eliminates the scale μ_r uncertainty. The experimental data points are taken from the ALEPH [2], DELPH [3], OPAL [4], L3 [5] and SLD [38] experiments.

Renormalization scale depends on thrust T!





Determine QCD running coupling from measurement of the thrust distribution at one energy!
$\alpha_s(Q^2)$ in $\overline{MS}$ scheme
$e^+e^- \to Z^0 \to q\bar{q}g + \cdots$
S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB
$\alpha_s(Q^2)$
Q (GeV)
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
4
6
8
10
12
14
16

Problems with traditional scale setting

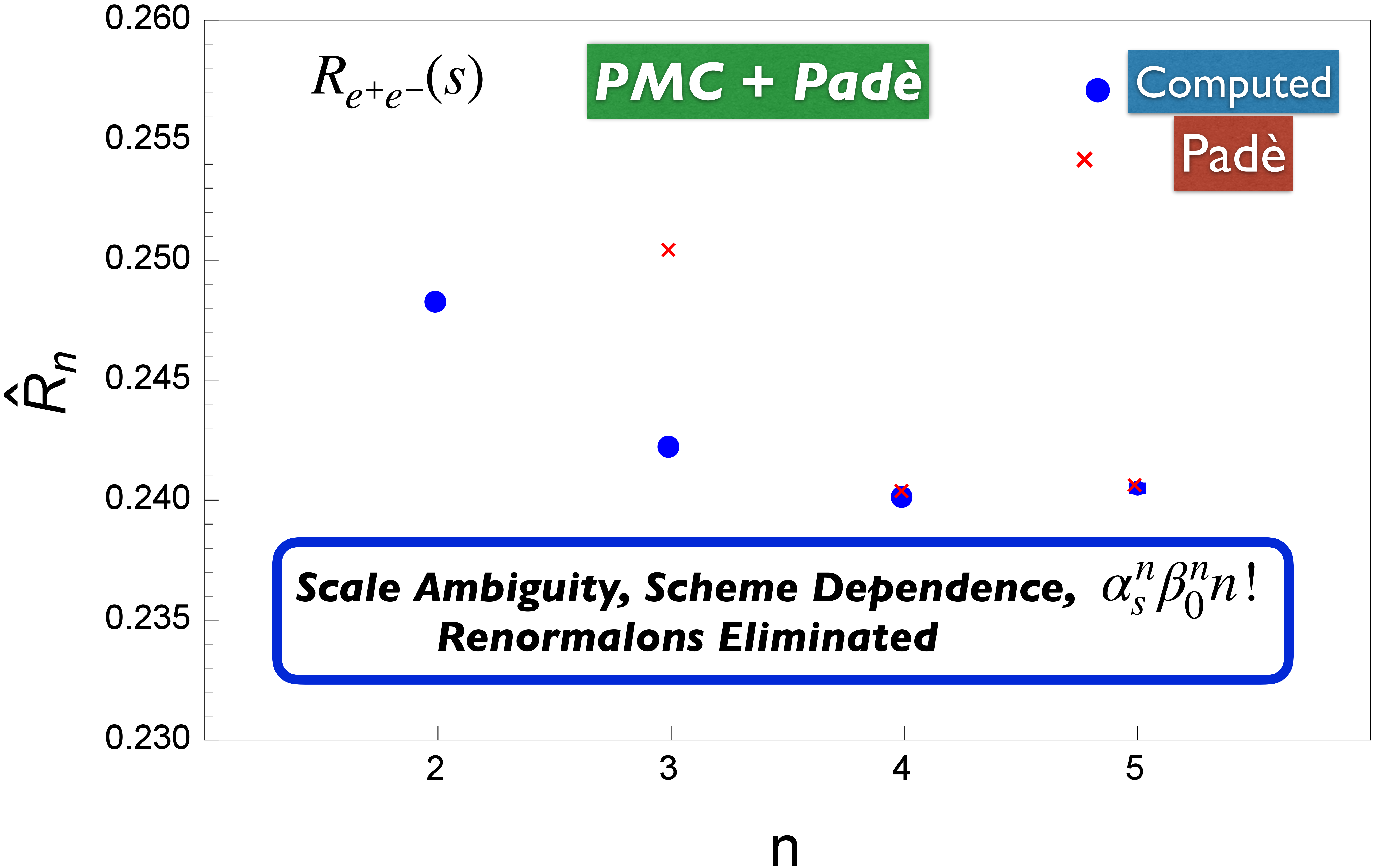
- *Predictions are scheme-dependent! At every order! This fundamental flaw does not get repaired at high orders*
- *Fails to satisfy Renormalization Group Principles*
- *Guessing the renormalization scale and its range is heuristic*
- *Gives wrong predictions for QED*
- *GUT: Must use the same scale-setting procedure for QED, QCD*
- *n! Renormalon growth — no convergence of pQCD*
- *Uses the same scale at each order.*
- *guessed value for n_f does not correctly reflect quark loop virtuality*
- *Multiple Physical Scales cannot be Incorporated*
- *Unrealistic Estimate of Higher-Order Terms: Only β-terms exposed by scale variation*
- *Introduces an unnecessary theory error!*
- *Can give wrong predictions for pQCD observables*
- *Obscures sensitivity to new physics*

Features of BLM/PMC

- ***Predictions are scheme-independent***
- ***Matches conformal series***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation***
- ***No n! Renormalon growth***
- ***New scale at each order; n_F determined at each order***
- ***Multiple Physical Scales Incorporated***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Abelian Limit: Gell-Mann-Low pQED***
- ***Realistic Estimate of Higher-Order Terms***

Essential Points

- ***Physical Results cannot depend on choice of Scheme***
- ***Different PMC scales at each order***
- ***No scale ambiguity!***
- ***Series identical to conformal theory***
- ***Relation between observables scheme independent, transitive***
- ***Choice of initial scale irrelevant even at finite order***
- ***Identify β terms using R_δ method***



Extending the Predictive Power of pQCD

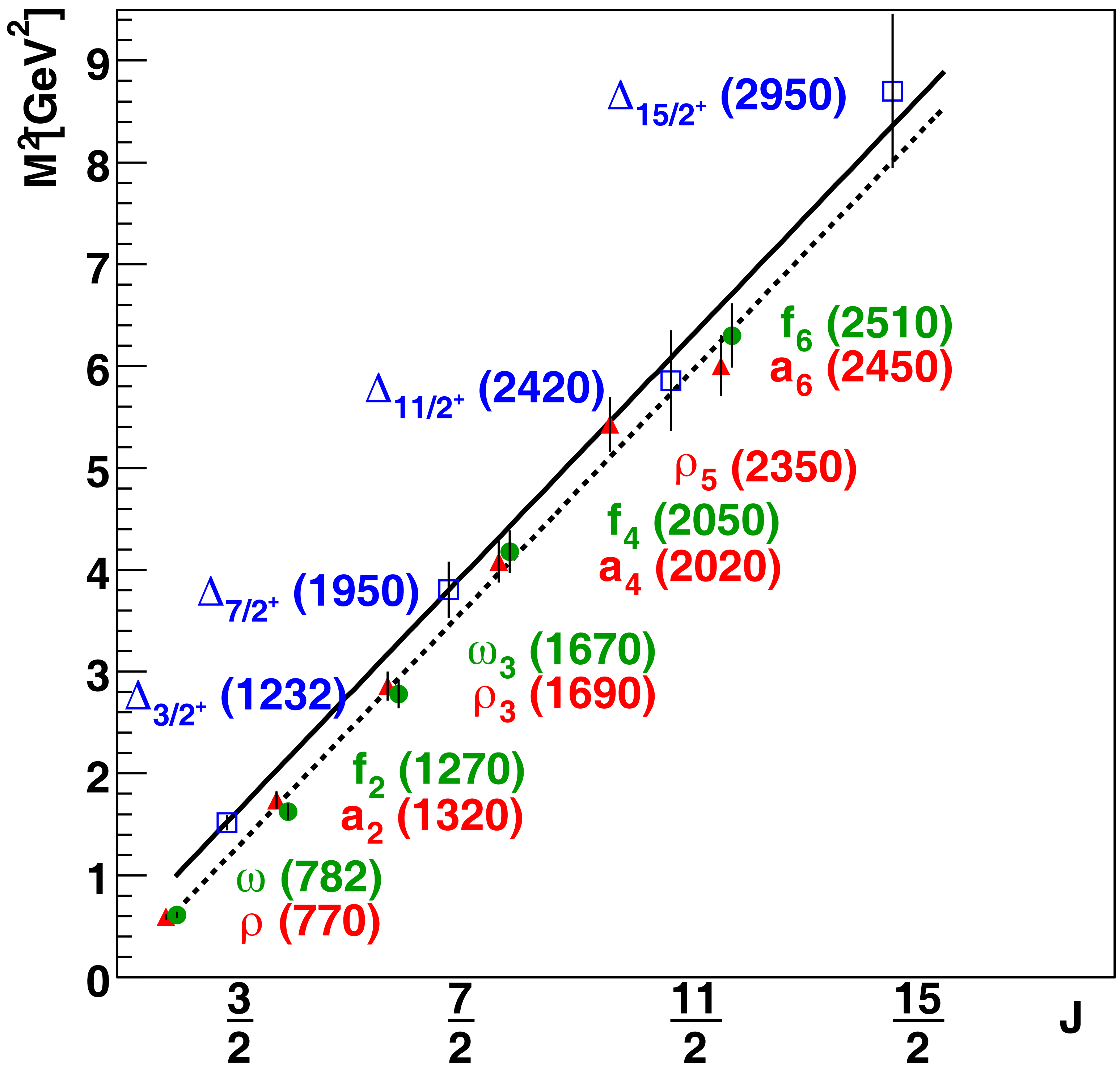
B-L Dun, X-G. Wu, J.M, Shen, sjb

Profound Questions for Hadron Physics

- *Color Confinement*
- *Origin of QCD Mass Scale*
- *Spectroscopy: Tetraquarks, Pentaquarks, Gluonium, Exotic States*
- *Universal Regge Slopes: n, L, both Mesons and Baryons*
- *Massless Pion: Bound State*
- *Dynamics and Spectroscopy*
- *QCD Coupling at all Scales*
- *QCD Vacuum —Do Condensates Exist?*

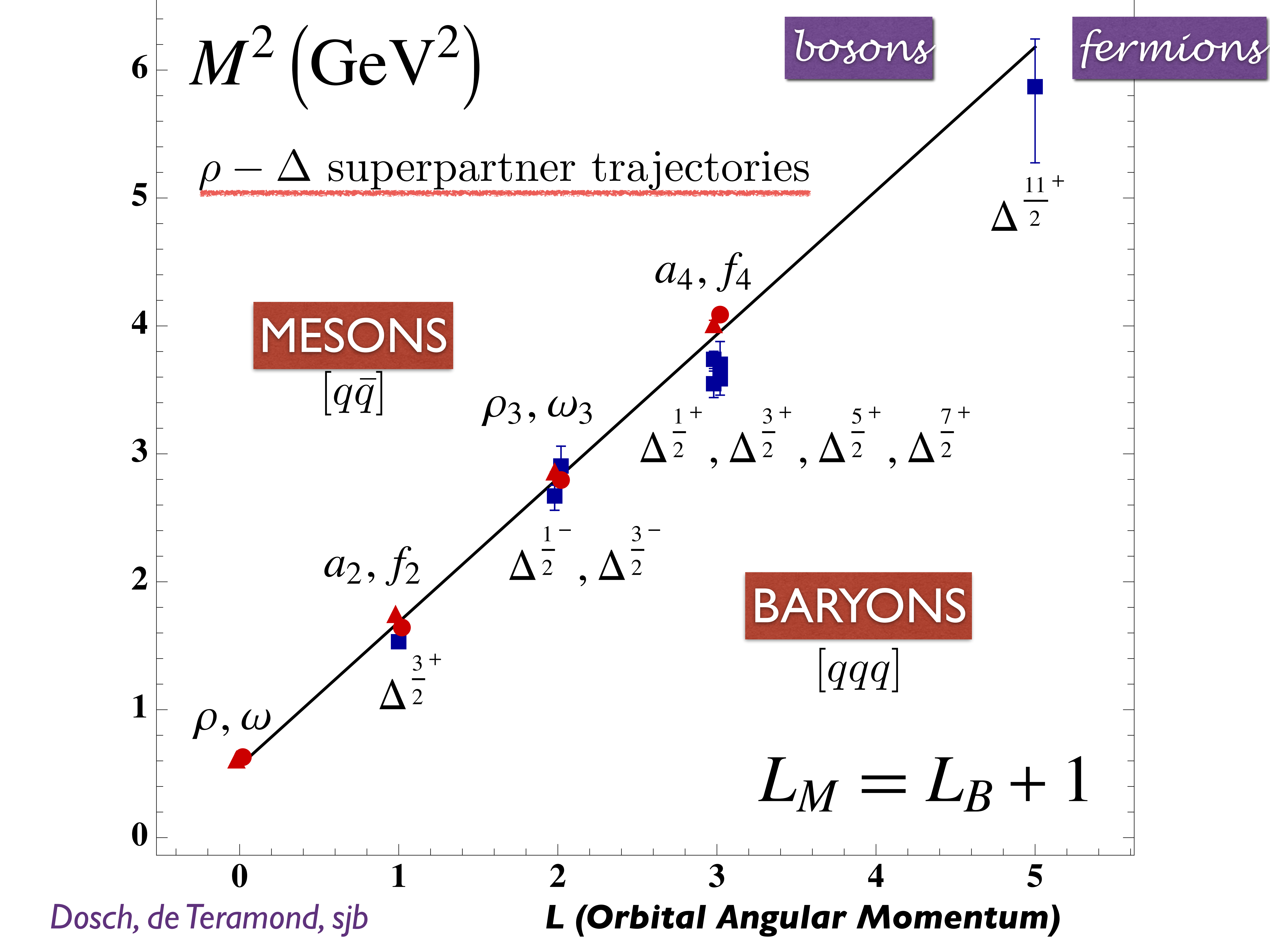
Mesons and Baryons: Same Regge Slope $M^2 \propto J$!

M²[GeV²]



The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.

E. Klempt and B. Ch. Metsch

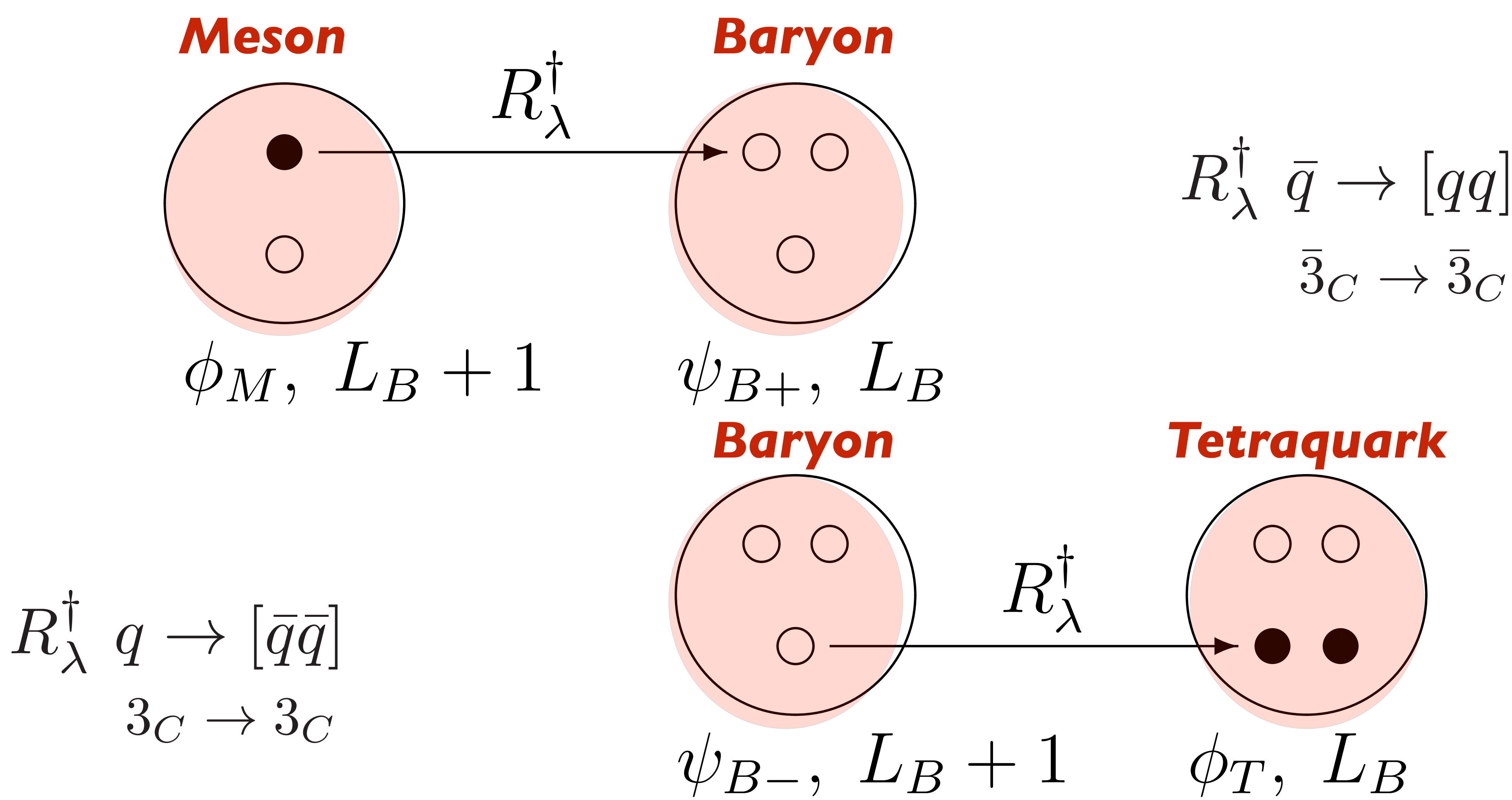


M^2 (GeV^2)
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta^{\frac{3}{2}^+}$
$\Delta^{\frac{1}{2}^-}, \Delta^{\frac{3}{2}^-}$
$\Delta^{\frac{1}{2}^+}, \Delta^{\frac{3}{2}^+}, \Delta^{\frac{5}{2}^+}, \Delta^{\frac{7}{2}^+}$
$\Delta^{\frac{11}{2}^+}$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

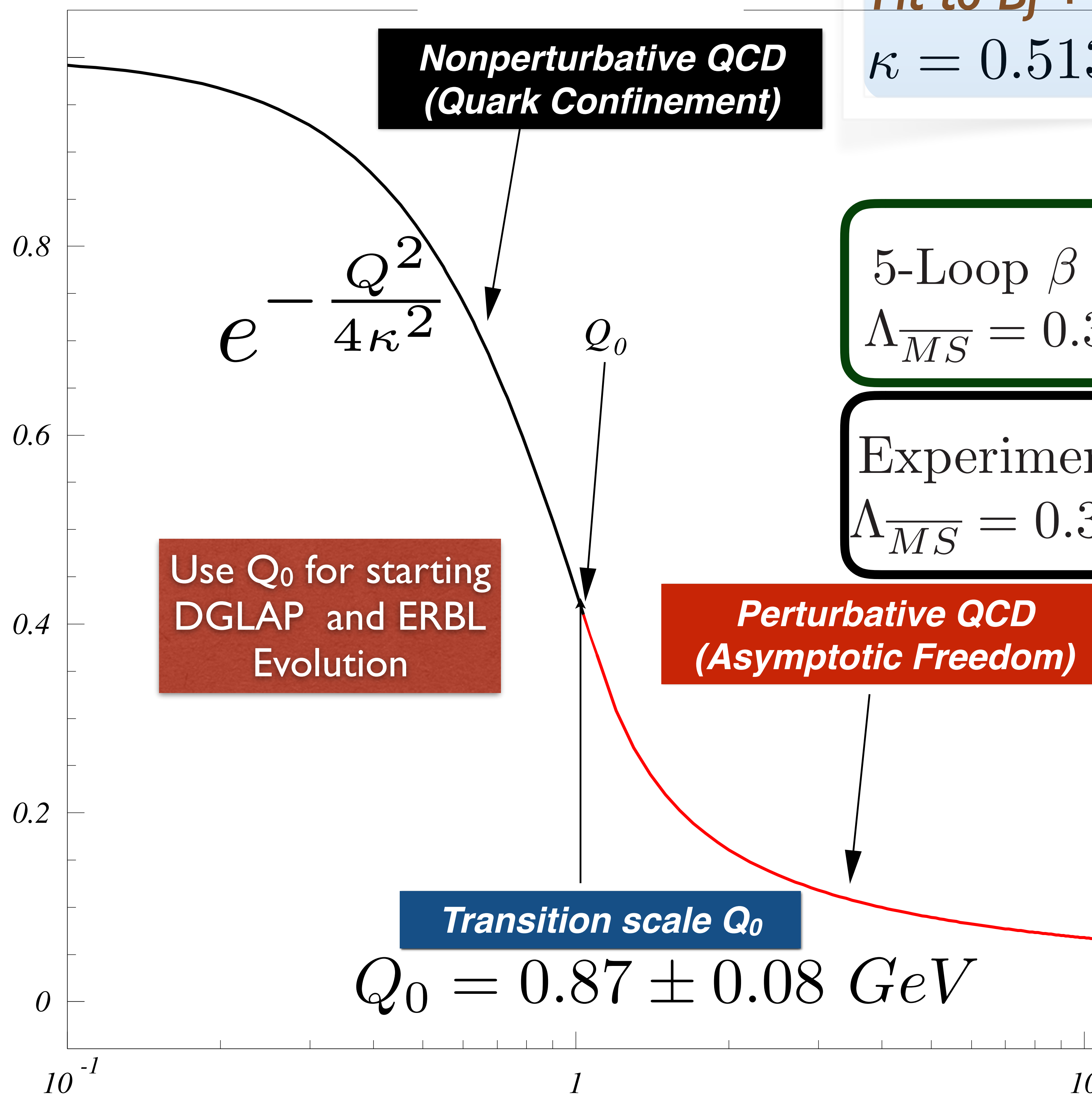
All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:

$\kappa = 0.513 \pm 0.007\ GeV$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



5-Loop β Prediction:

$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

Experiment:

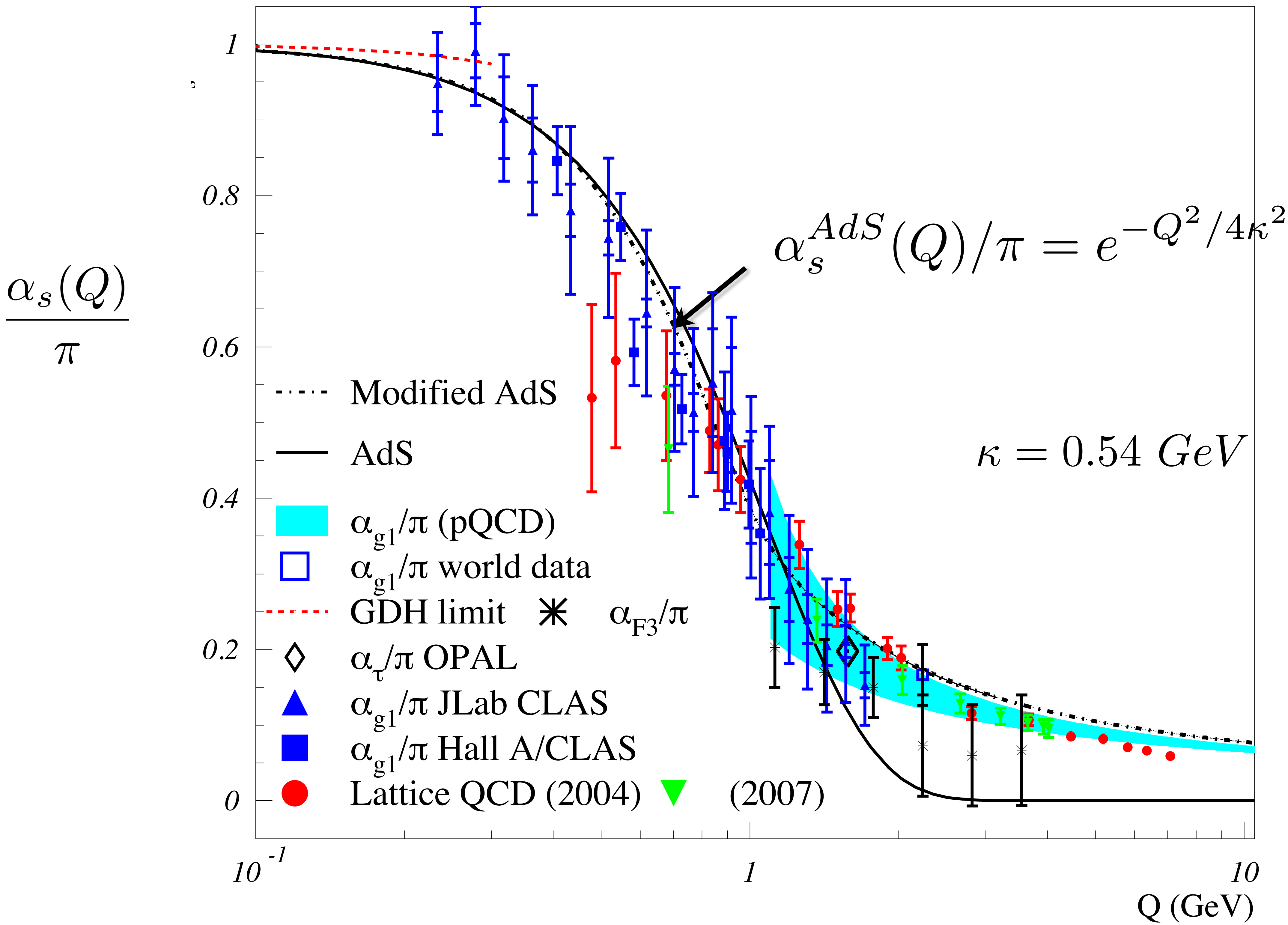
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

$\lambda \equiv \kappa^2$

$\overline{MS}$ *scheme*

Reverse Dimensional Transmutation!

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Supersymmetry in QCD

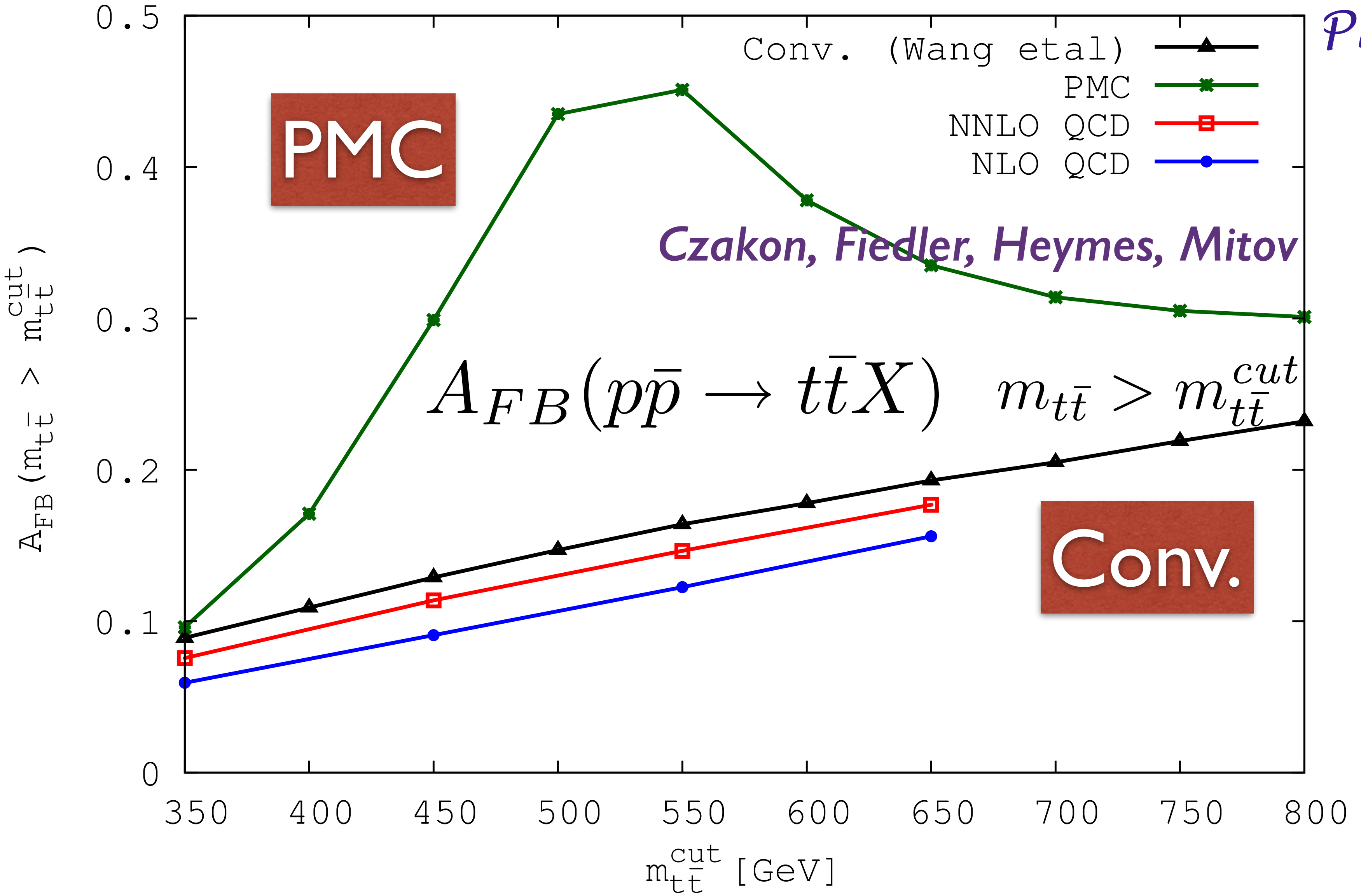
- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincarè Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests
- Systematically improvable: Basis LF Quantization (BLFQ)

The QCD coupling at all scales and the elimination of renormalization scale uncertainties

The Principle of Maximum Conformality (PMC)



BLM: G. Peter Lepage
Paul Mackenzie

PMC: Leonardo di Giustino,
Xing-Gang Wu
Matin Mojaza

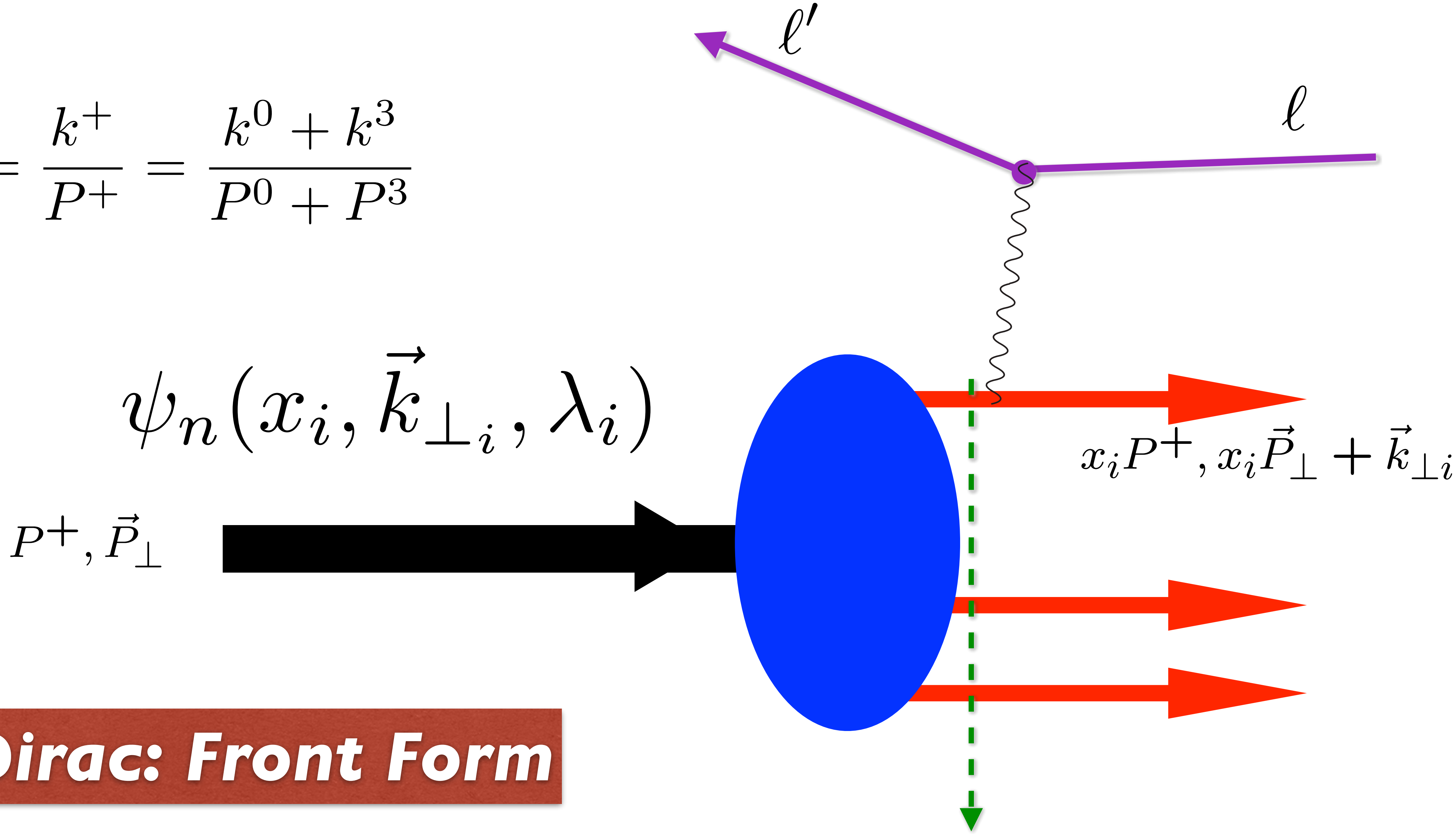
Stan Brodsky
SLAC

ECT*
February 12, 2018

α_s Workshop

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

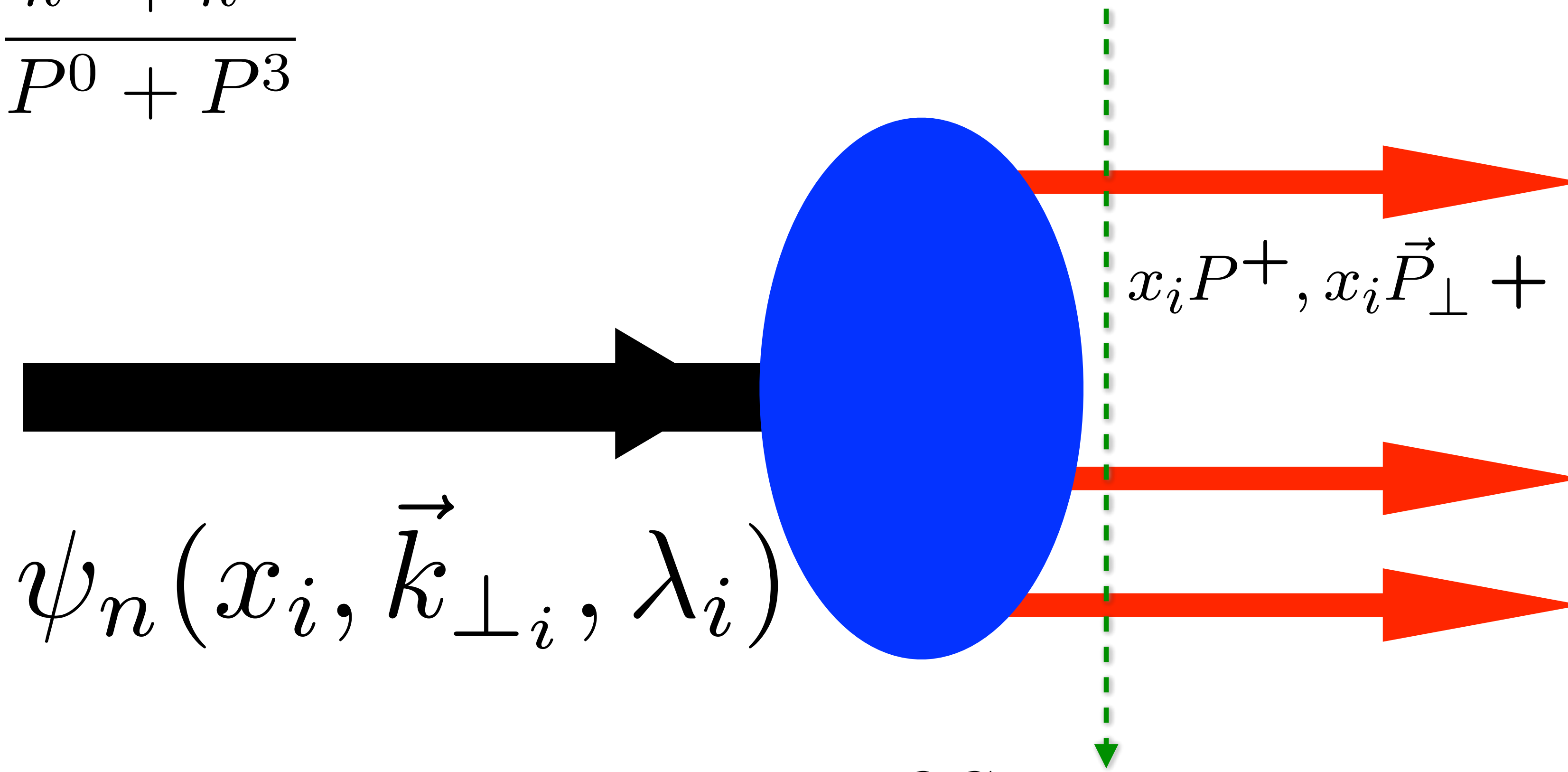
$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

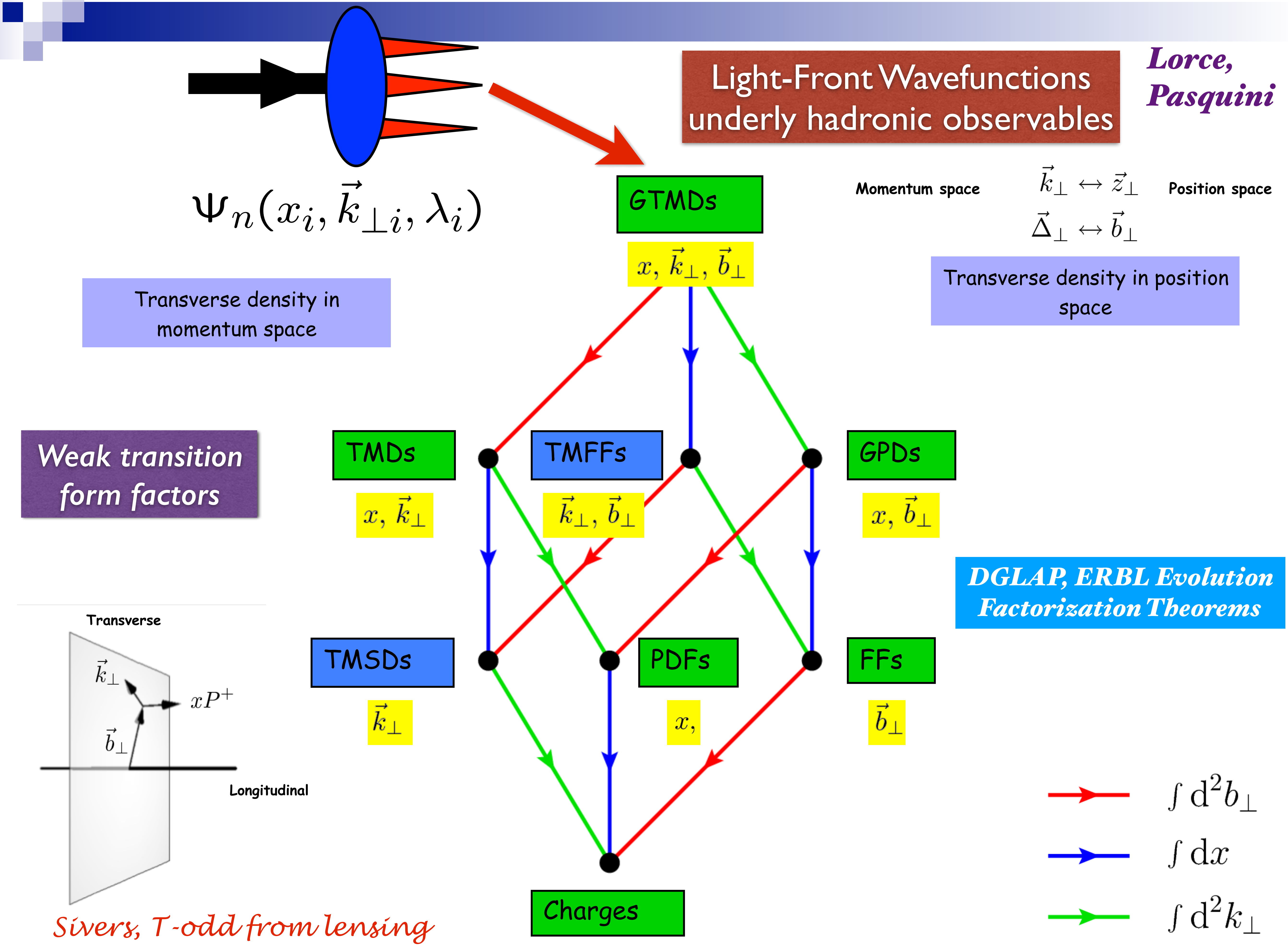
Eigenstate of LF Hamiltonian

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS



Light-Front Wavefunctions underly hadronic observables
Lorce, Pasquini
$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
GTMDs
$x, \vec{k}_\perp, \vec{b}_\perp$
Momentum space
$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$
Position space
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
Transverse density in momentum space
Transverse density in position space
Weak transition form factors
TMDs
$x, \vec{k}_\perp$
TMFFs
$\vec{k}_\perp, \vec{b}_\perp$
GPDs
$x, \vec{b}_\perp$
DGLAP, ERBL Evolution Factorization Theorems
Transverse
$\vec{k}_\perp$
xP^+
$\vec{b}_\perp$
Longitudinal
TMSDs
$\vec{k}_\perp$
PDFs
$x,$
FFs
$\vec{b}_\perp$
Charges
$\int d^2 b_\perp$
$\int dx$
$\int d^2 k_\perp$
Sivers, T-odd from lensing

Advantages of the Dirac's Front Form for Hadron Physics
Poincare' Invariant

Physics Independent of Observer's Motion

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!

Penrose, Terrell, Weisskopf

- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- Jz Conservation, bounds on ΔLz *Chiu, sjb*
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no vacuum condensates!

Roberts, Shrock, Tandy, sjb

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

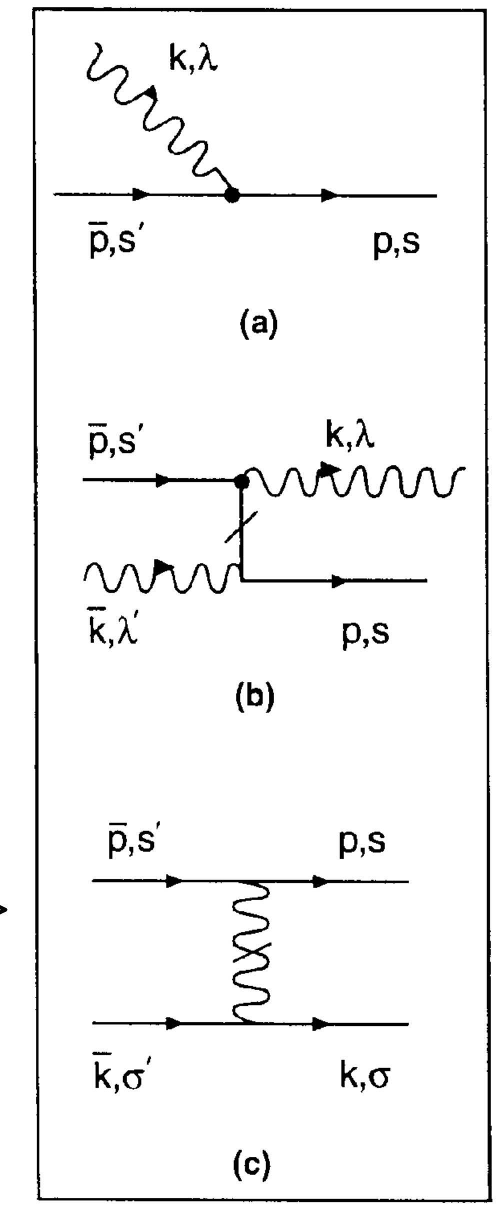
H_{LF}^{int}: Matrix in Fock Space

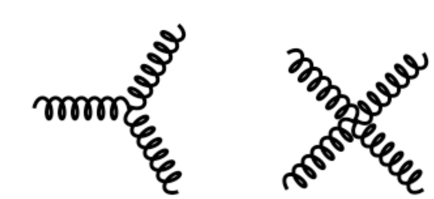
$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





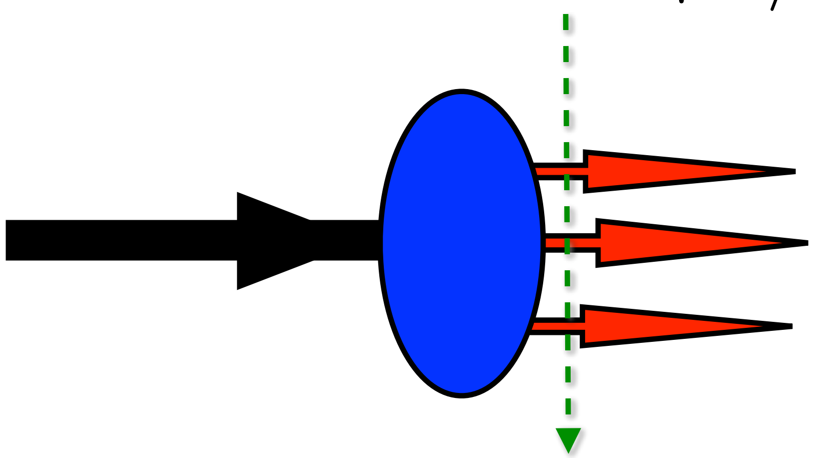
$$H_{LF}^{int}$$

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

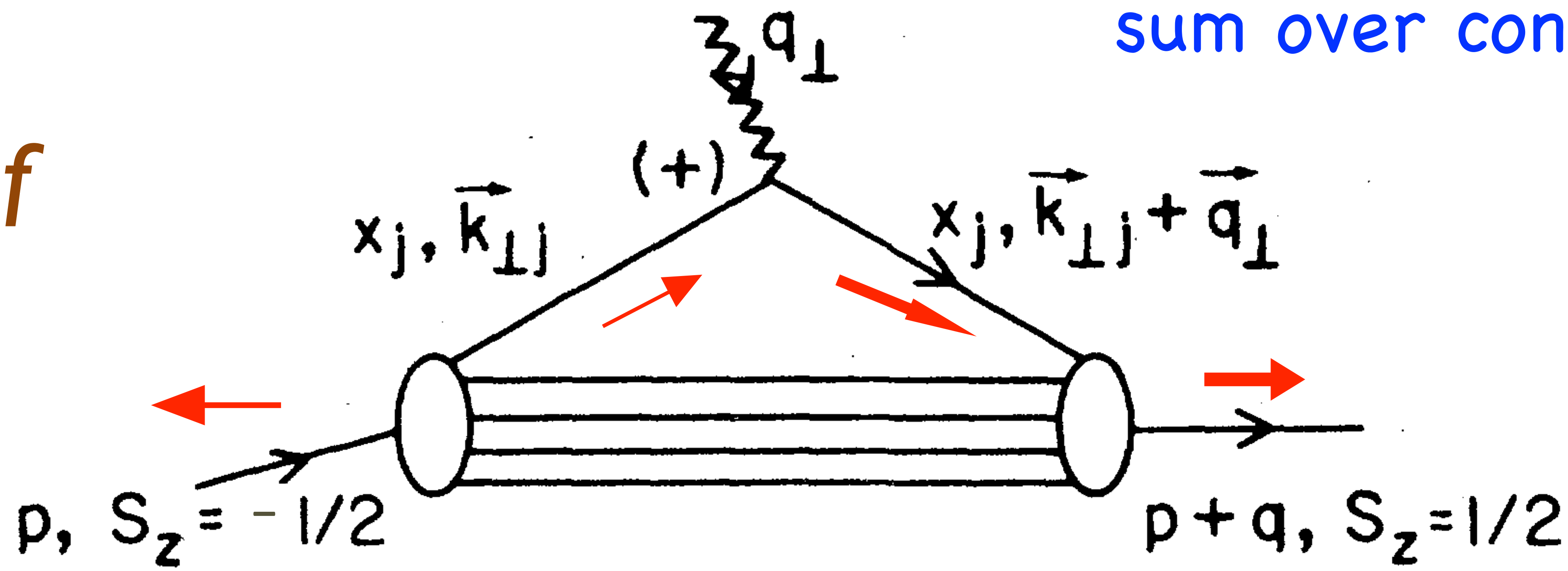
P. Lowdon, K. Chiu, Dae Sung Hwang, Bo-Qiang Ma, Ivan Schmidt, sjb

Terayev, Okun: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents

LF Proof



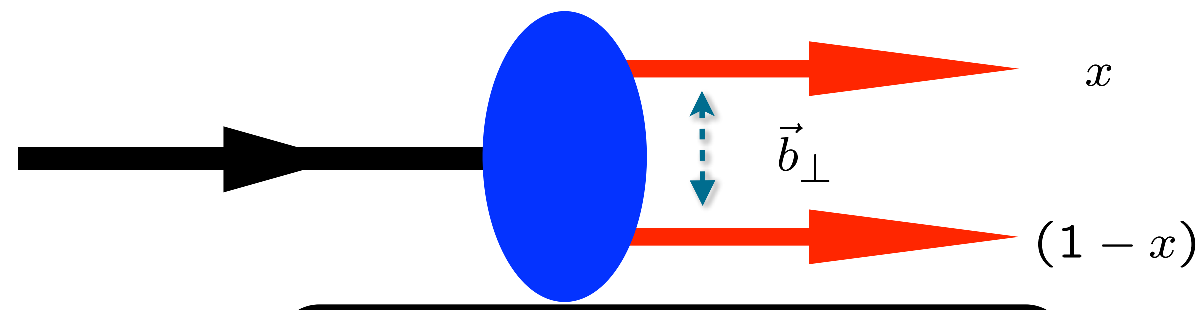
B(0) = 0

Each Fock State

Vanishing Anomalous gravitomagnetic moment B(0)

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\text{eff}}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$$m_q = 0$$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

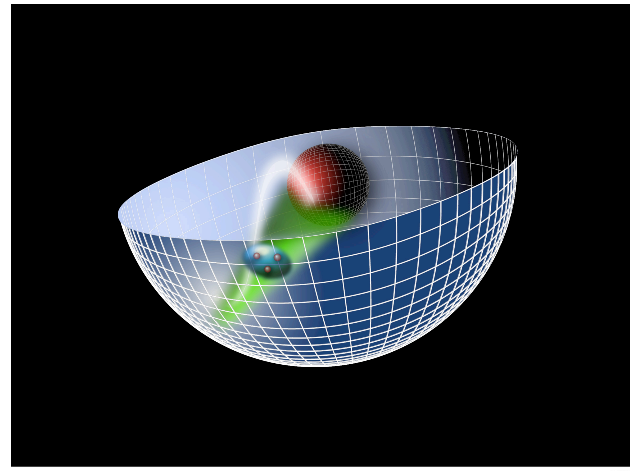
Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD *de Tèramond, Dosch, sjb*

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

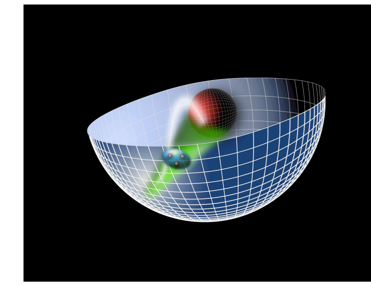
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

GeV units external to QCD: Only Ratios of Masses Determined

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

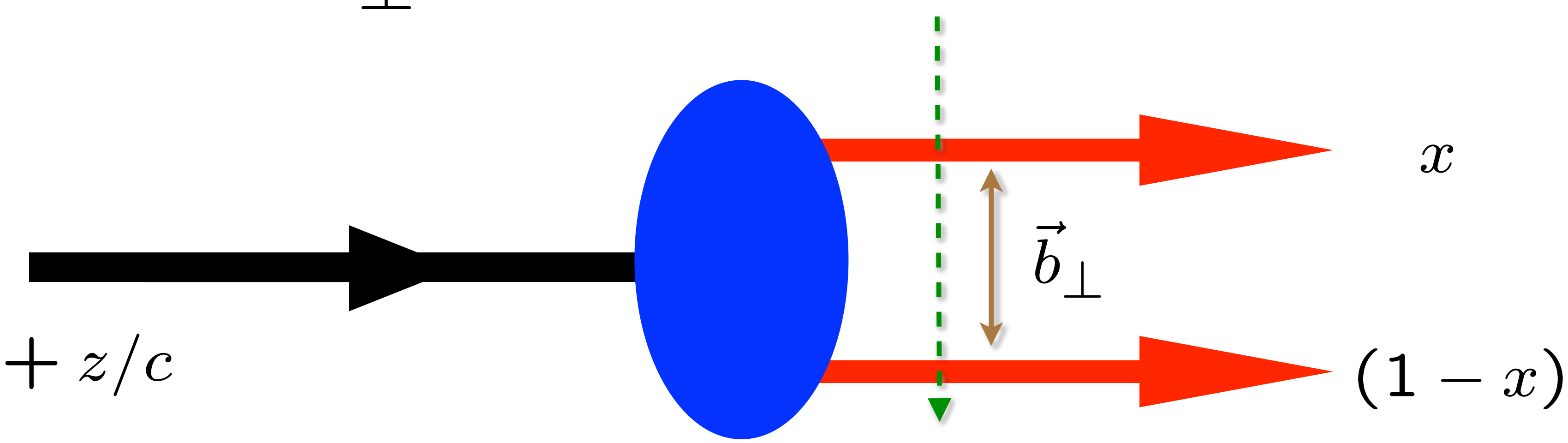
de Teramond, sjb

LF(3+1) ↔ AdS_5

Light-Front Holographic Dictionary

$\psi(x, \vec{b}_\perp)$ ↔ $\phi(z)$

$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ ↔ z



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Massless pion!

Meson Spectrum in Soft Wall Model

$m_\pi = 0 \text{ if } m_q = 0$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

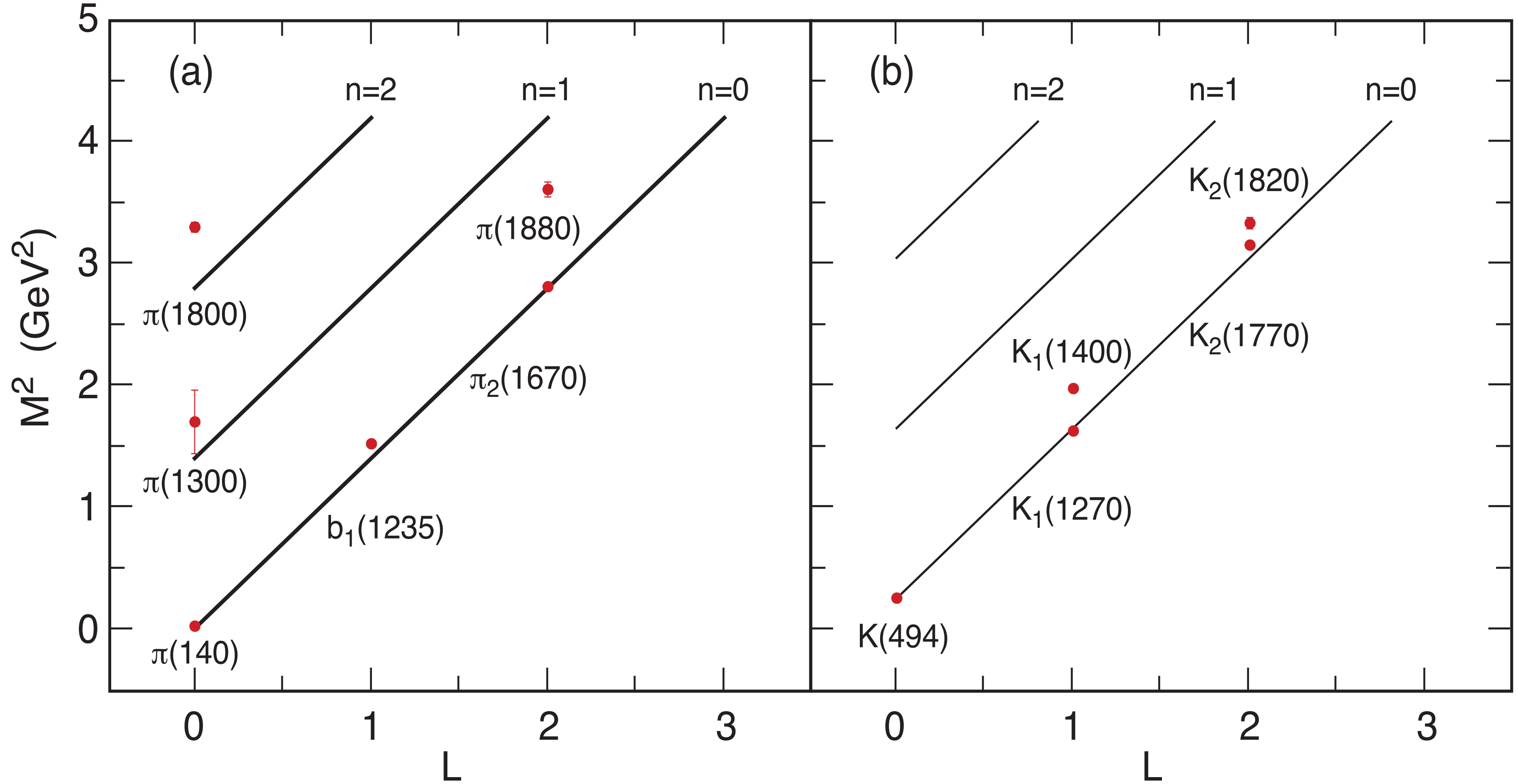
$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

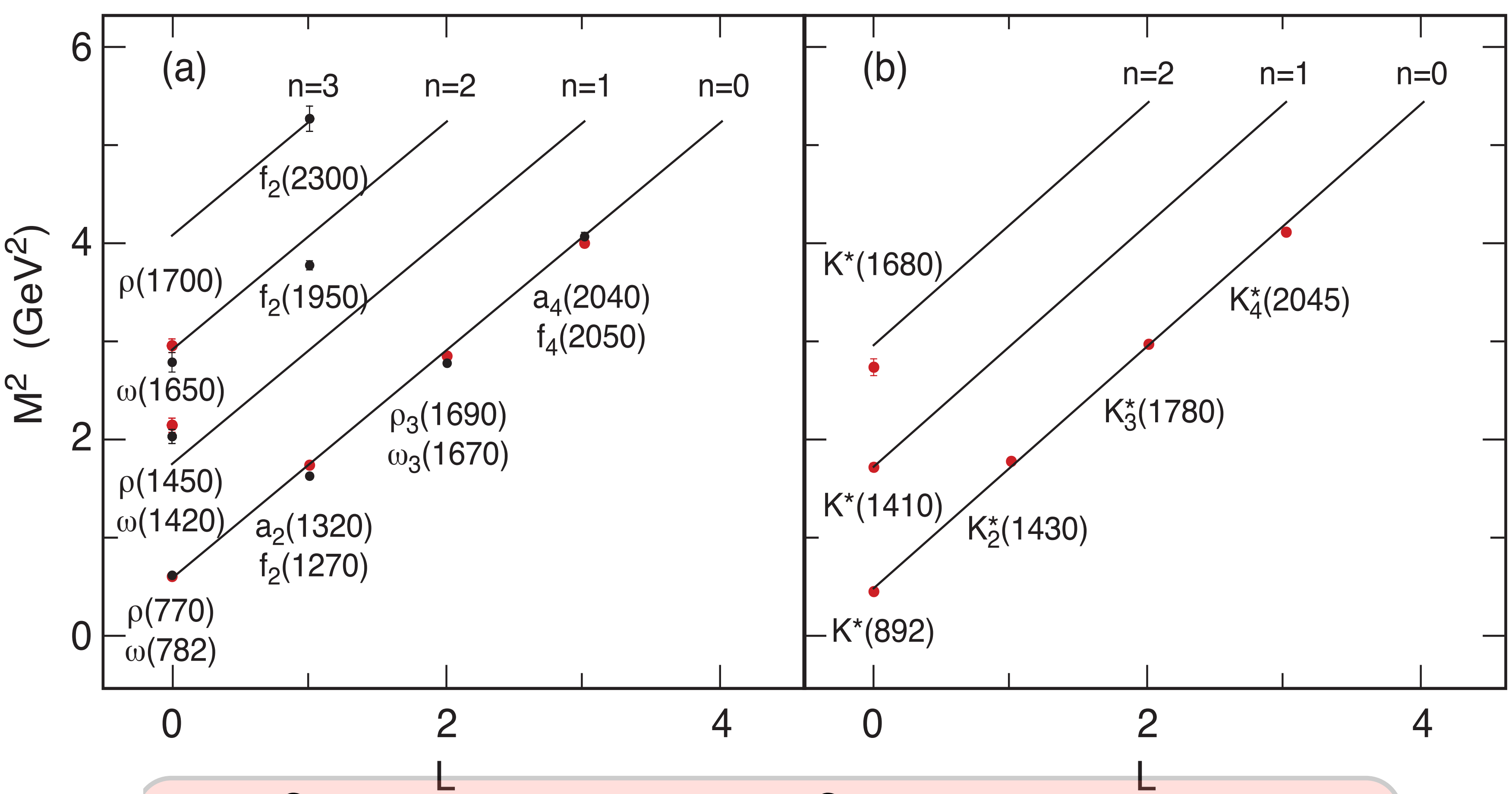
- Eigenvalues

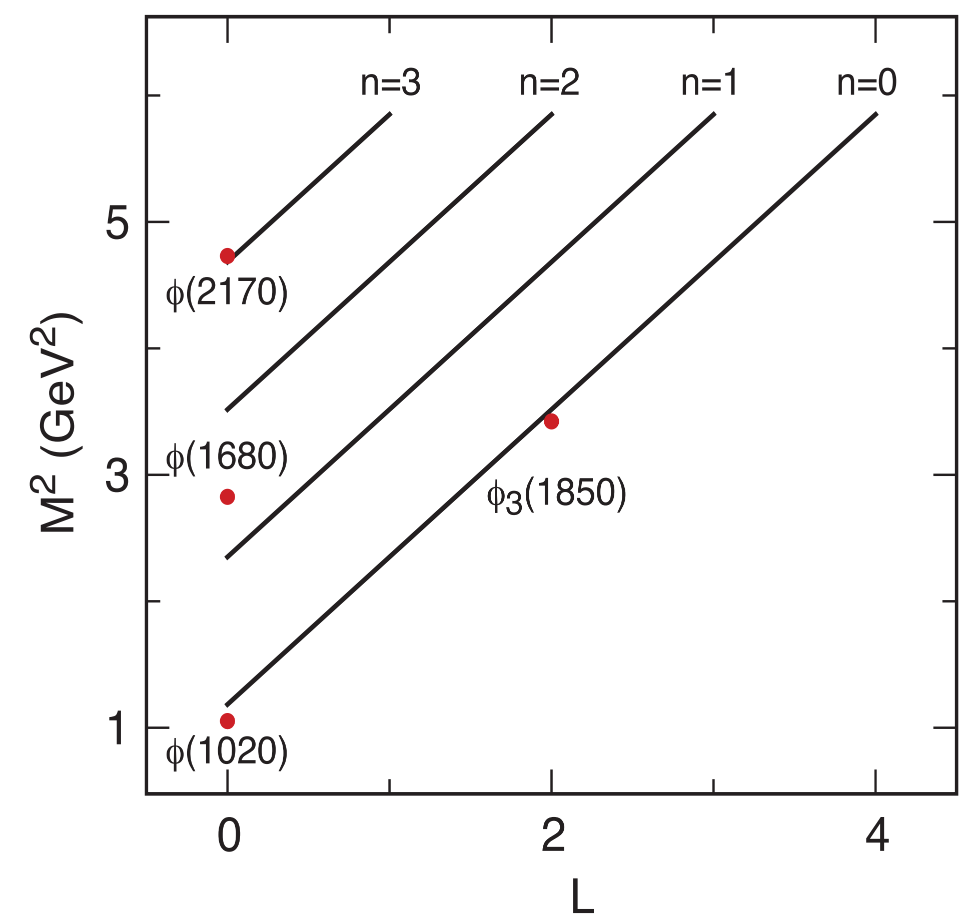
$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$

G. de Teramond, H. G. Dosch, sjb







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

Quark separation increases with L

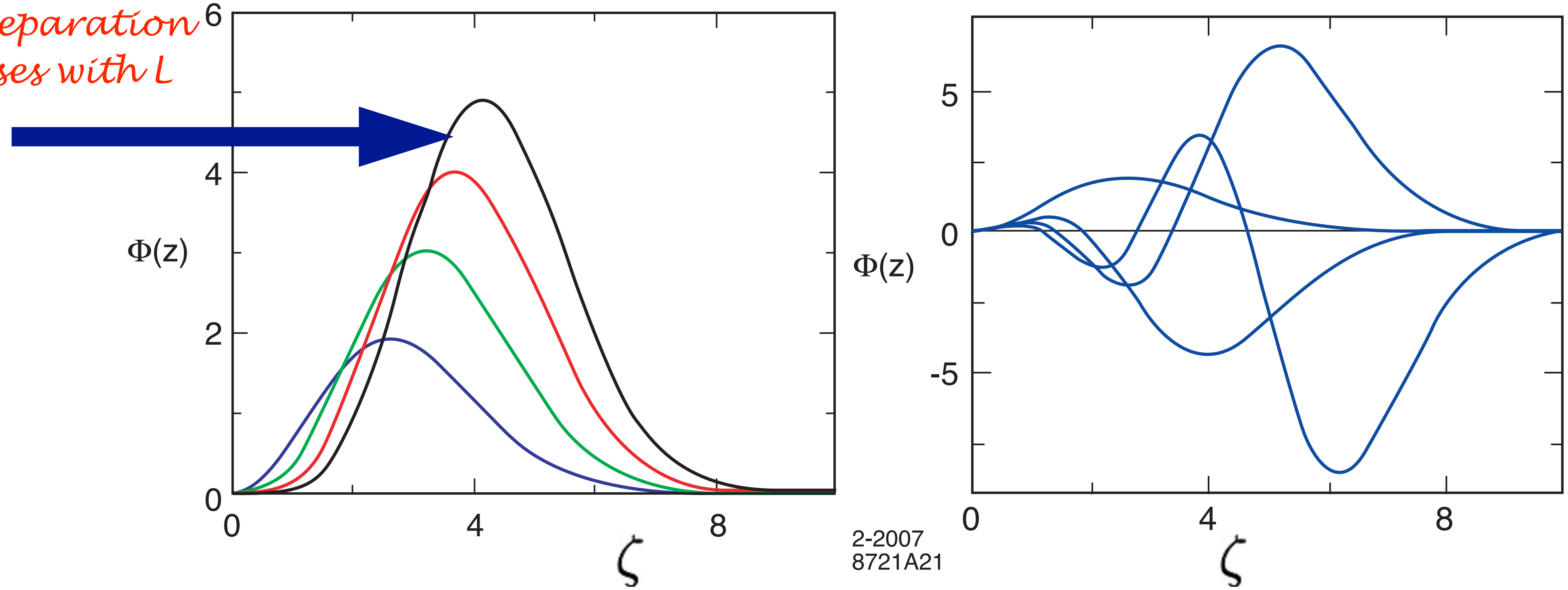
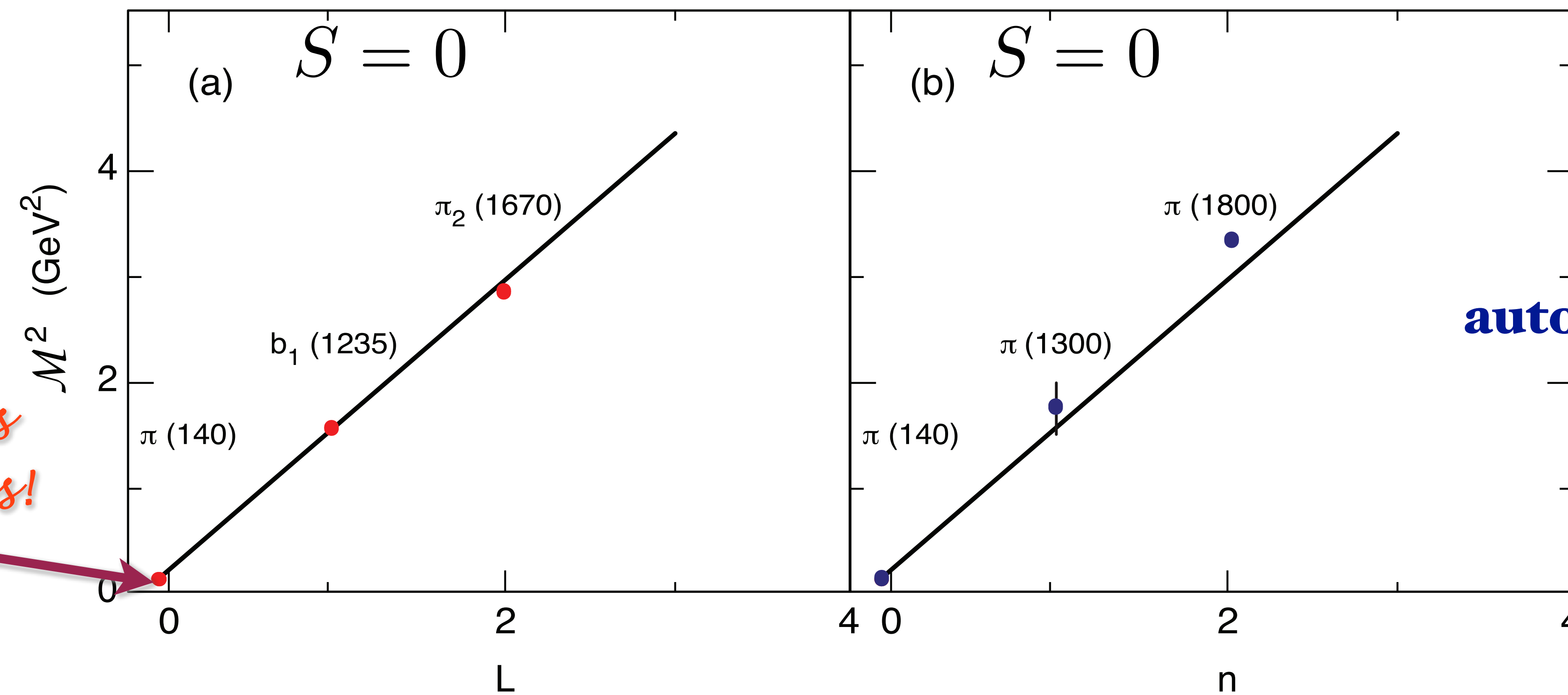


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

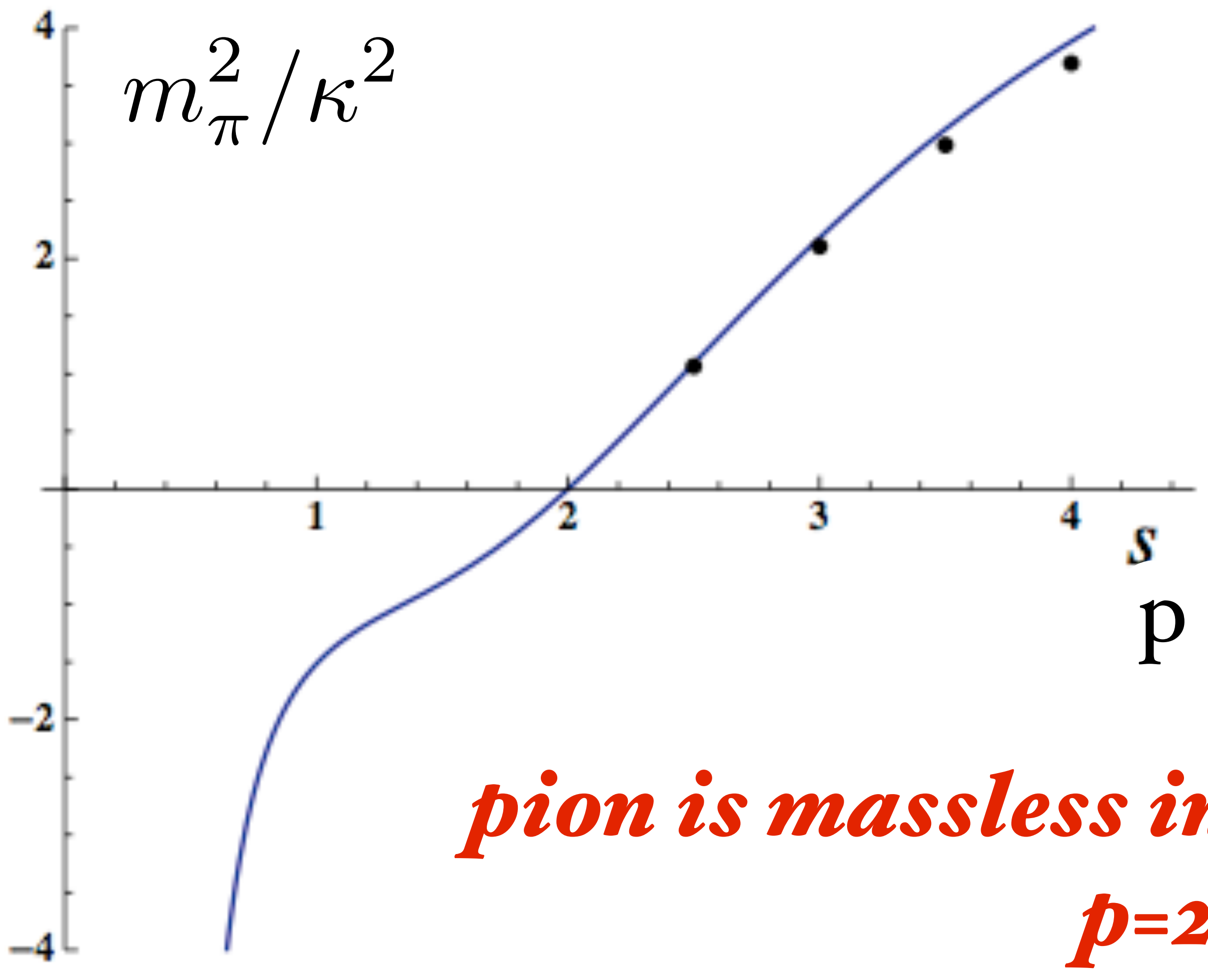
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

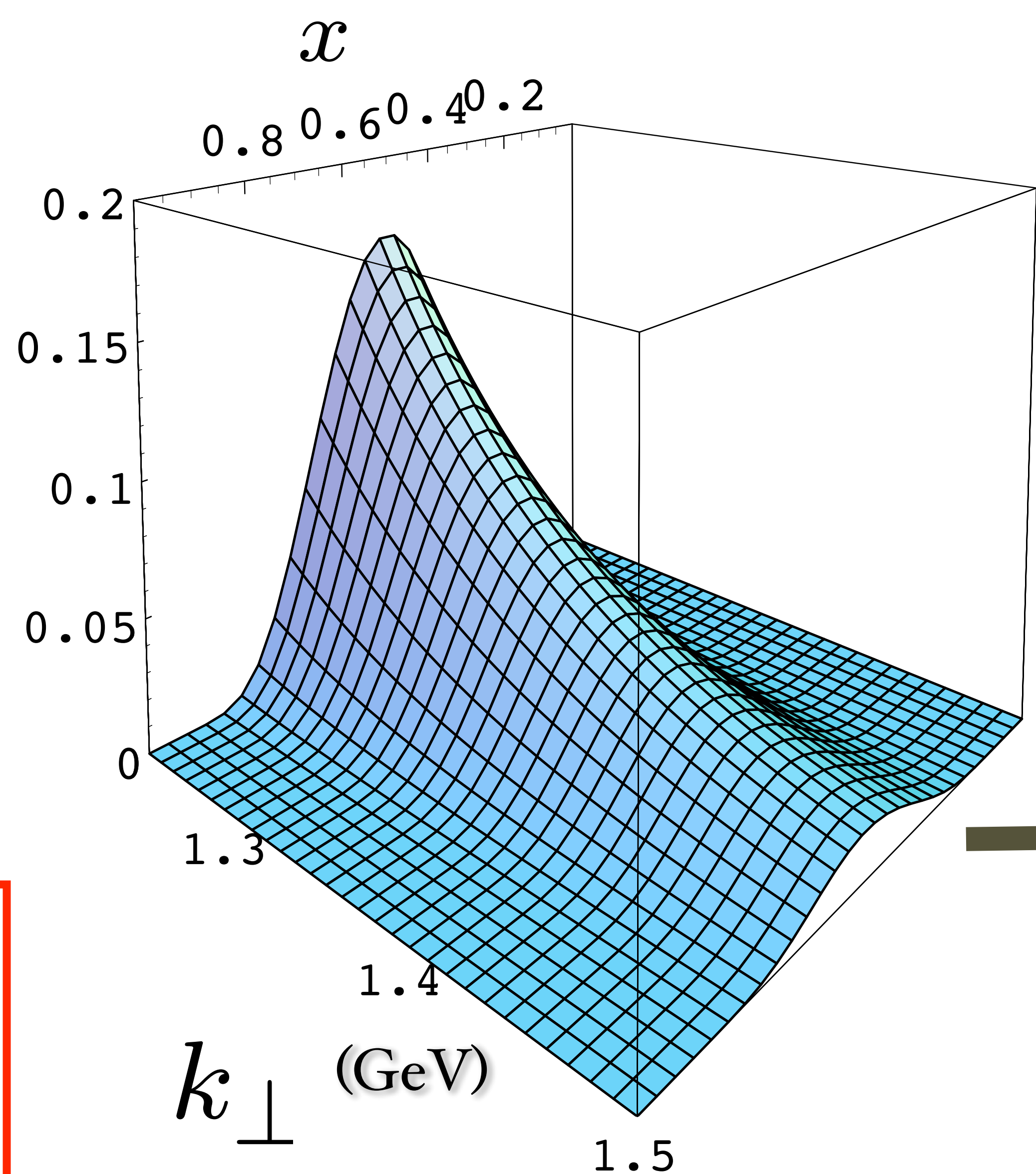
$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

Prediction from AdS/QCD: Meson LFWF

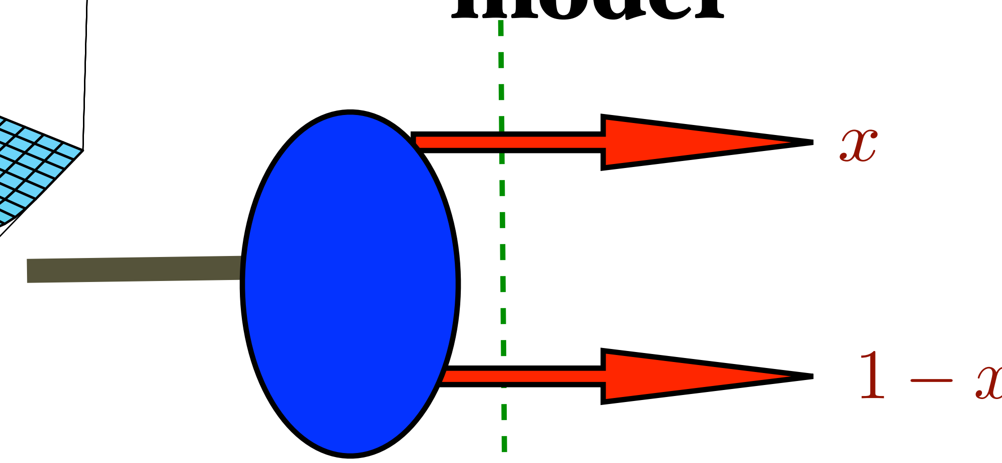
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

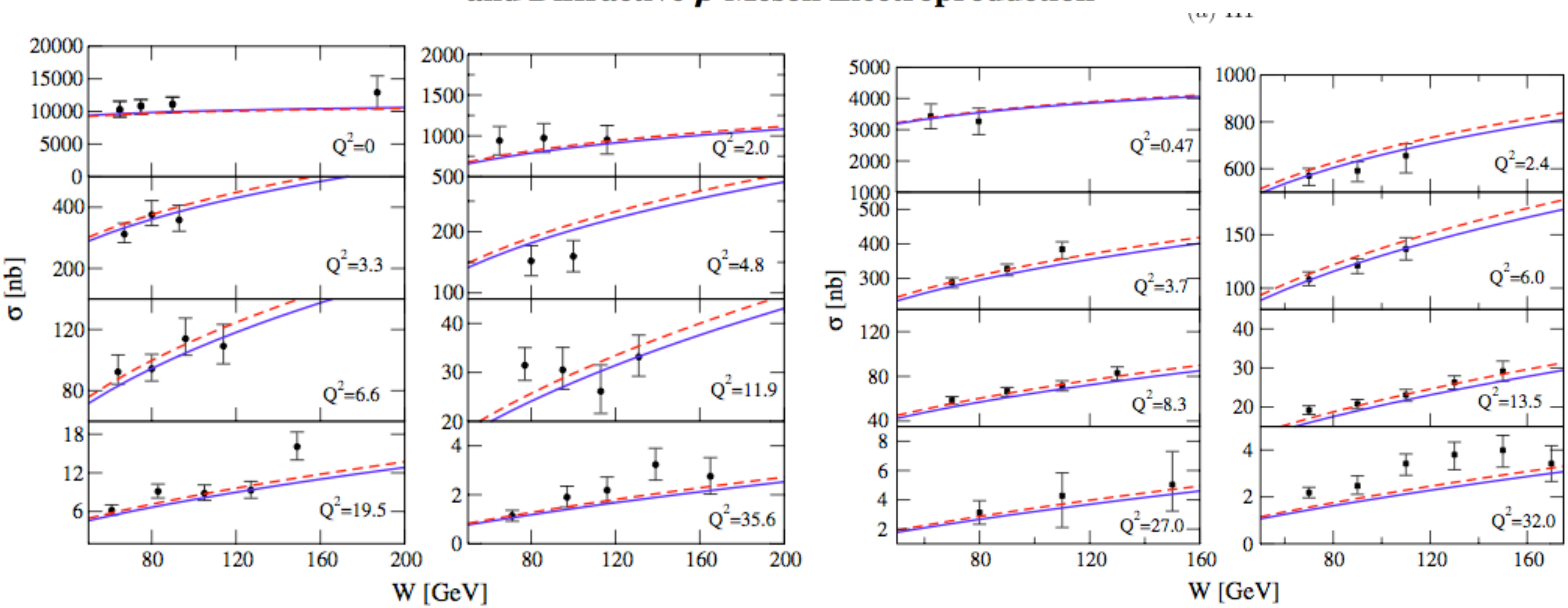
$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

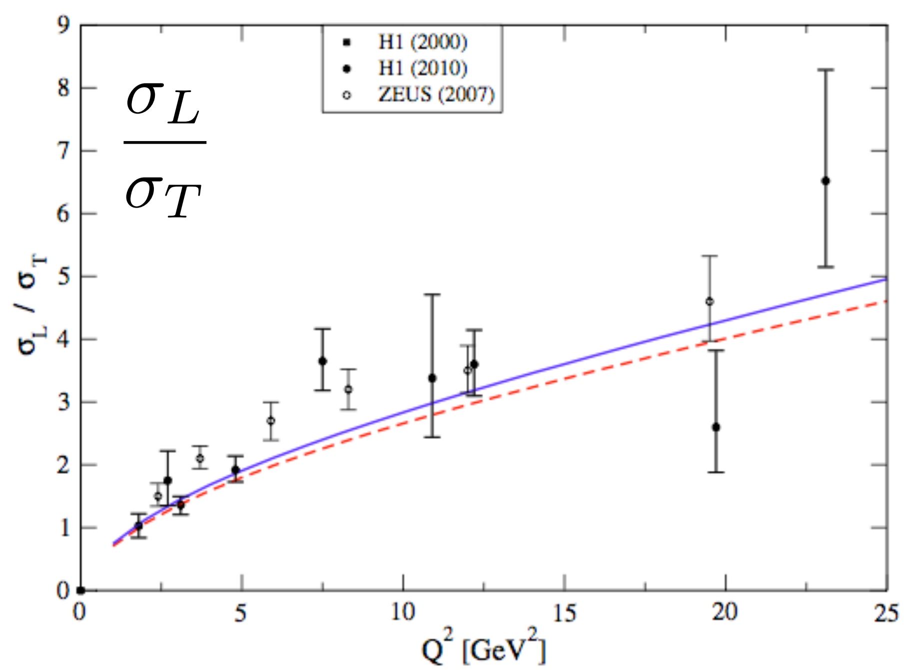
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H1 (b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\frac{\sigma_L}{\sigma_T}$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Light-Front Perturbation Theory for pQCD

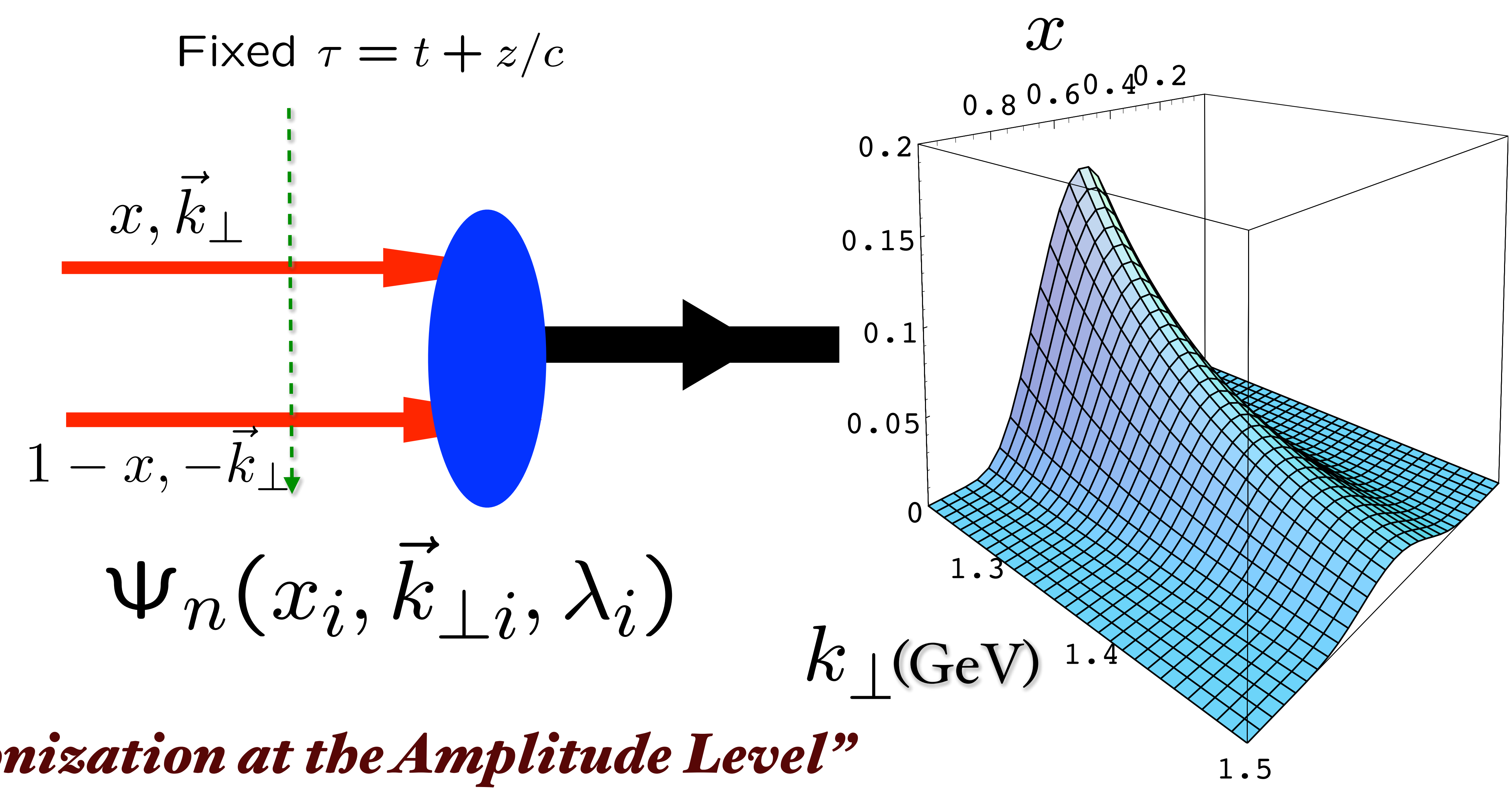
$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + cdots$$

- "History" : Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!
- Wick Theorem applies, but few amplitudes since all $k^+ > 0$.
- J_z Conservation at every vertex $| \sum_{initial} S^z - \sum_{final} S_z | \leq n$ at order g^n
- Unitarity is explicit
- Loop Integrals are 3-dimensional $\int_0^1 dx \int d^2 k_\perp$
- hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

K. Chiu, sjb

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

Connection to the Linear Instant-Form Potential

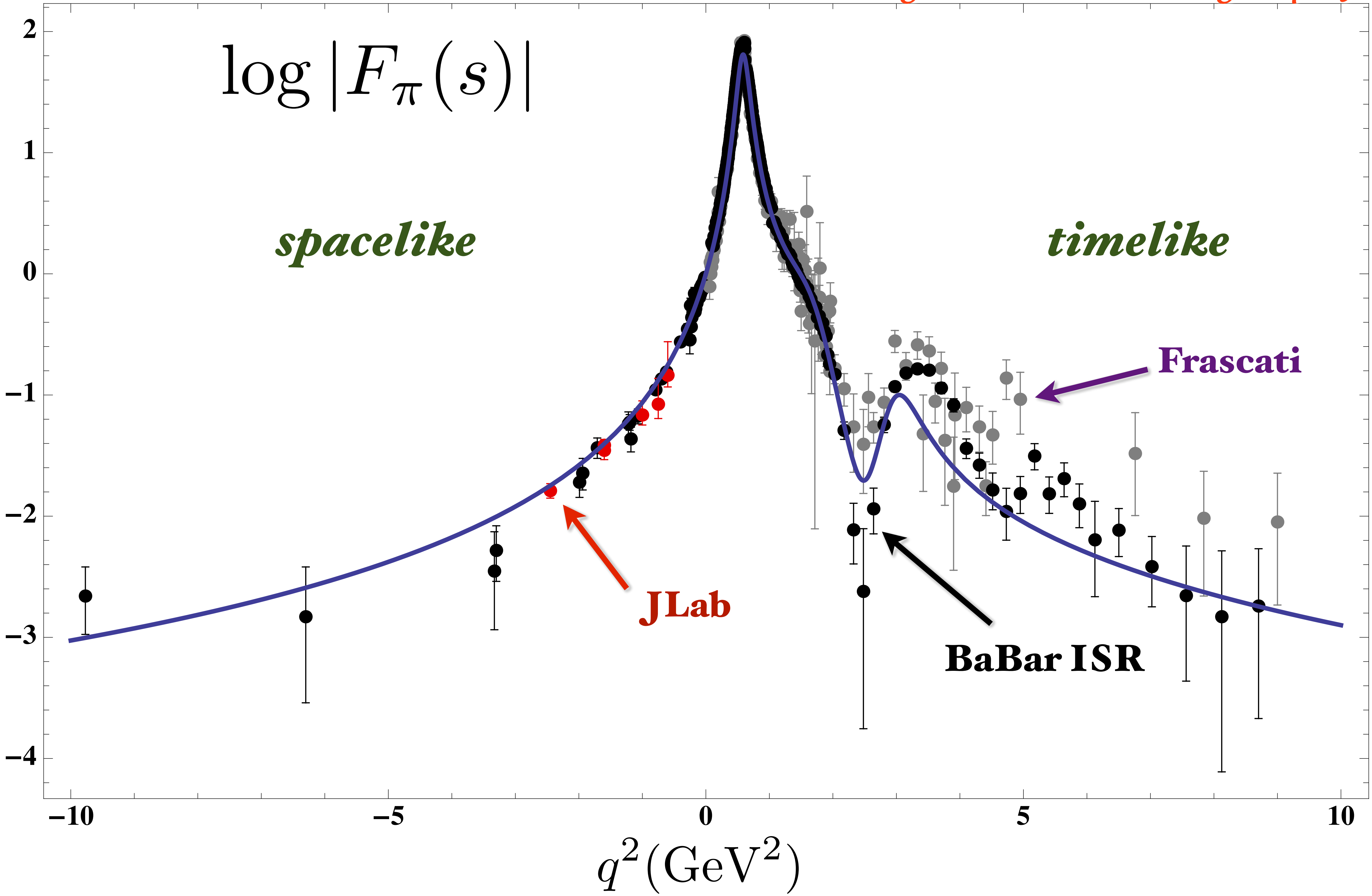
Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Pion Form Factor from AdS/QCD and Light-Front Holography

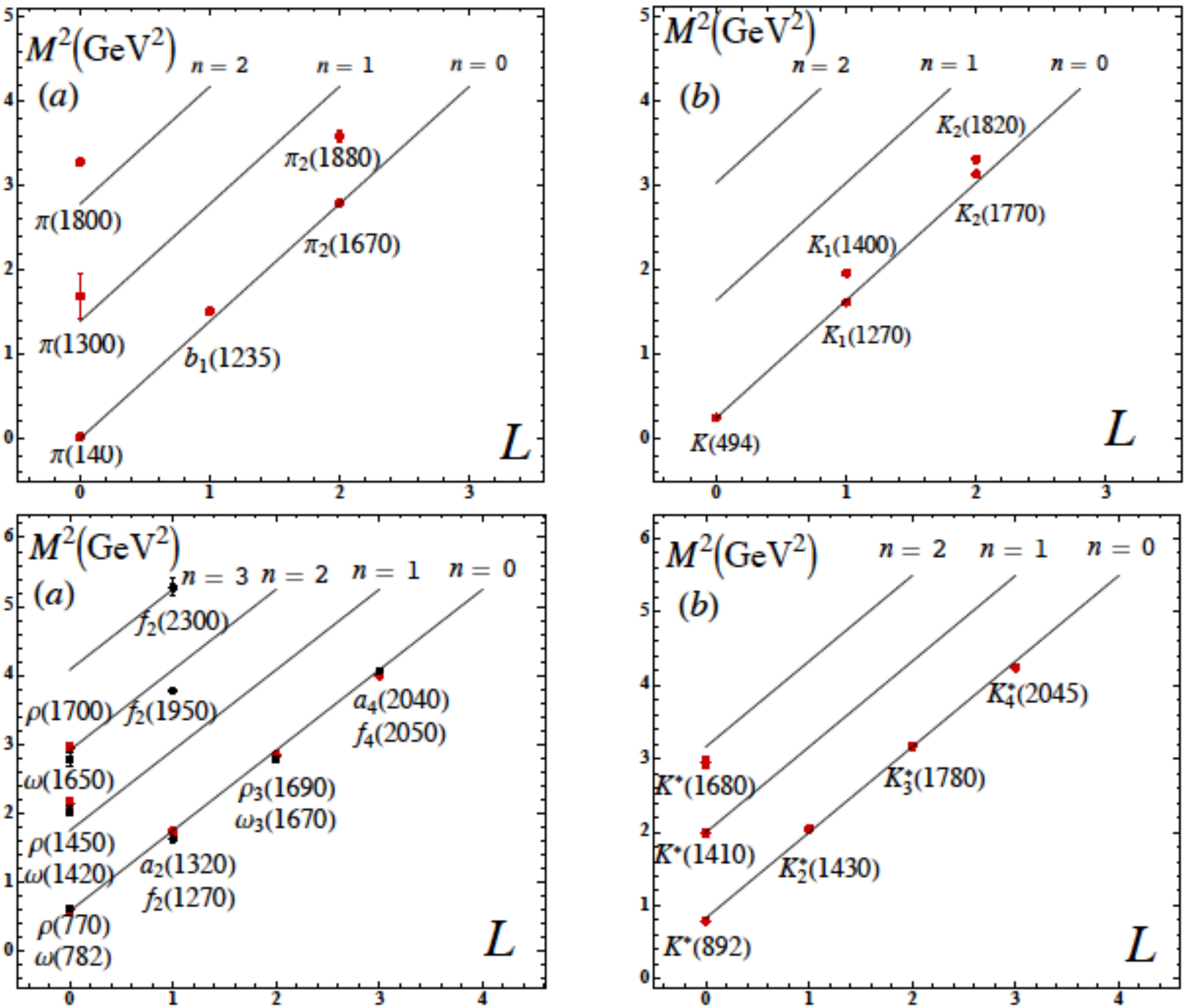


De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



Effective mass from $m(p^2)$

Tandy, Roberts, et al

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- *Relativistic, frame-independent*
- *QCD scale appears - unique LF potential*
- *Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter*
- *Zero-mass pion for zero mass quarks!*
- *Regge slope same for n and L -- not usual HO*
- *Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry*
- *Phenomenology: LFWFs, Form factors, electroproduction*
- *Extension to heavy quarks*

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**

- **Finite range**

- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[H,D] = i\,H, \quad [H, K] = 2\,i\,D, \quad [K, D] = -\,i\,K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action *Fubini and Rabinovici*

New Extended Hamiltonian G is diagonal:

$$G_{11} = \Big(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \Big)$$

$$G_{22} = \Big(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \Big)$$

Identify $f - \frac{1}{2} = L_B \;, \quad w = \kappa^2$ $\qquad \lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ *!*

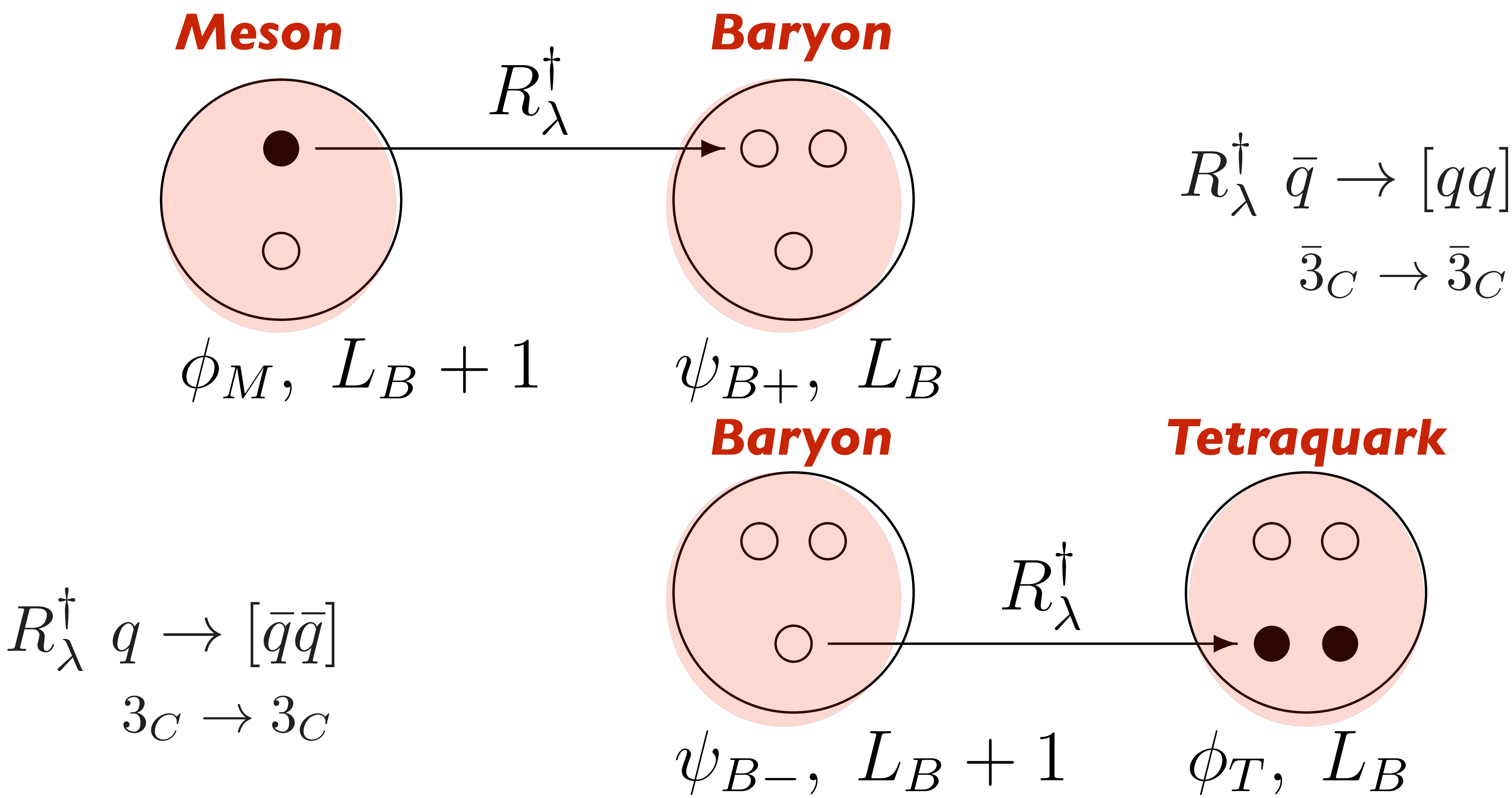
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

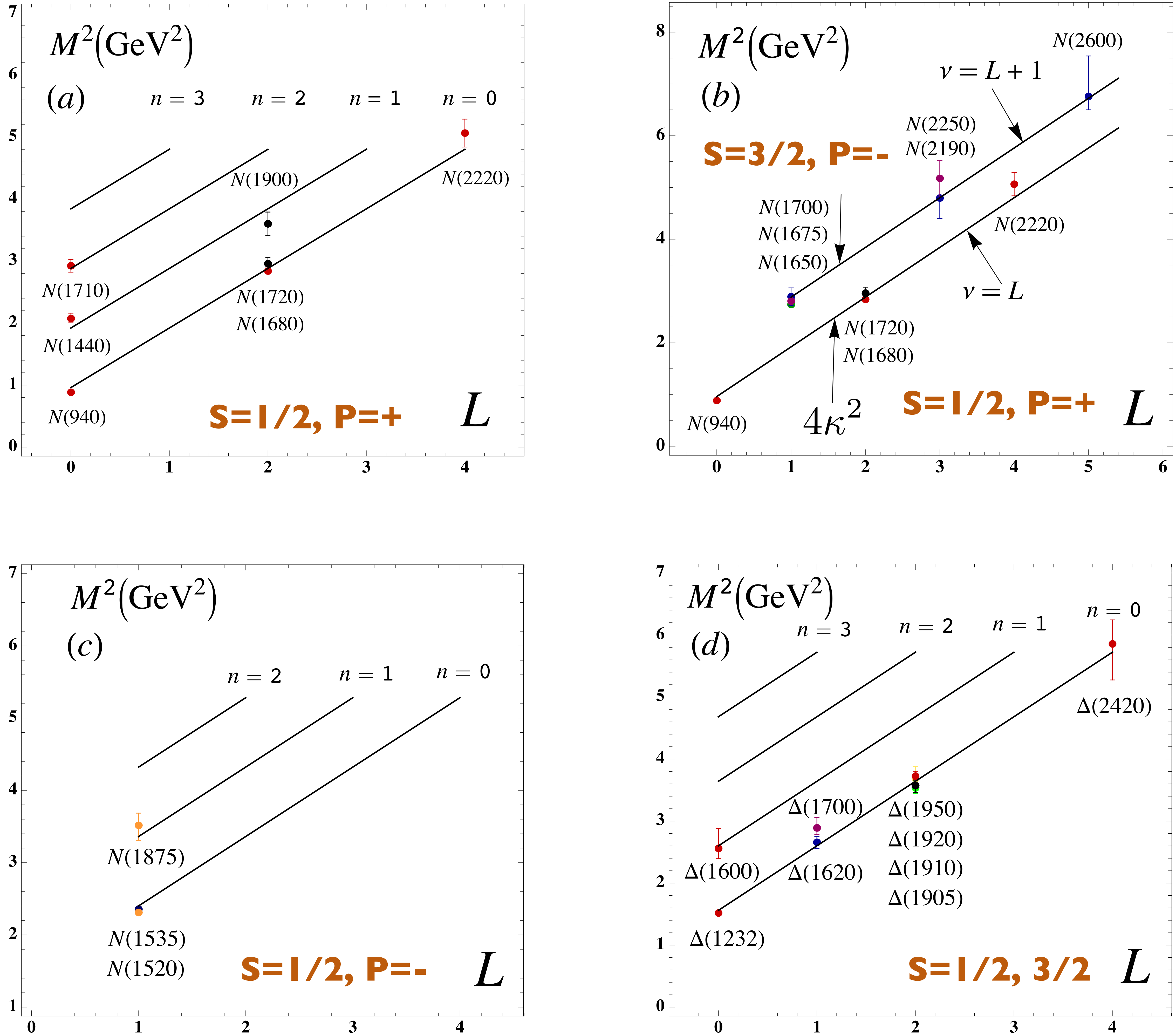
Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1



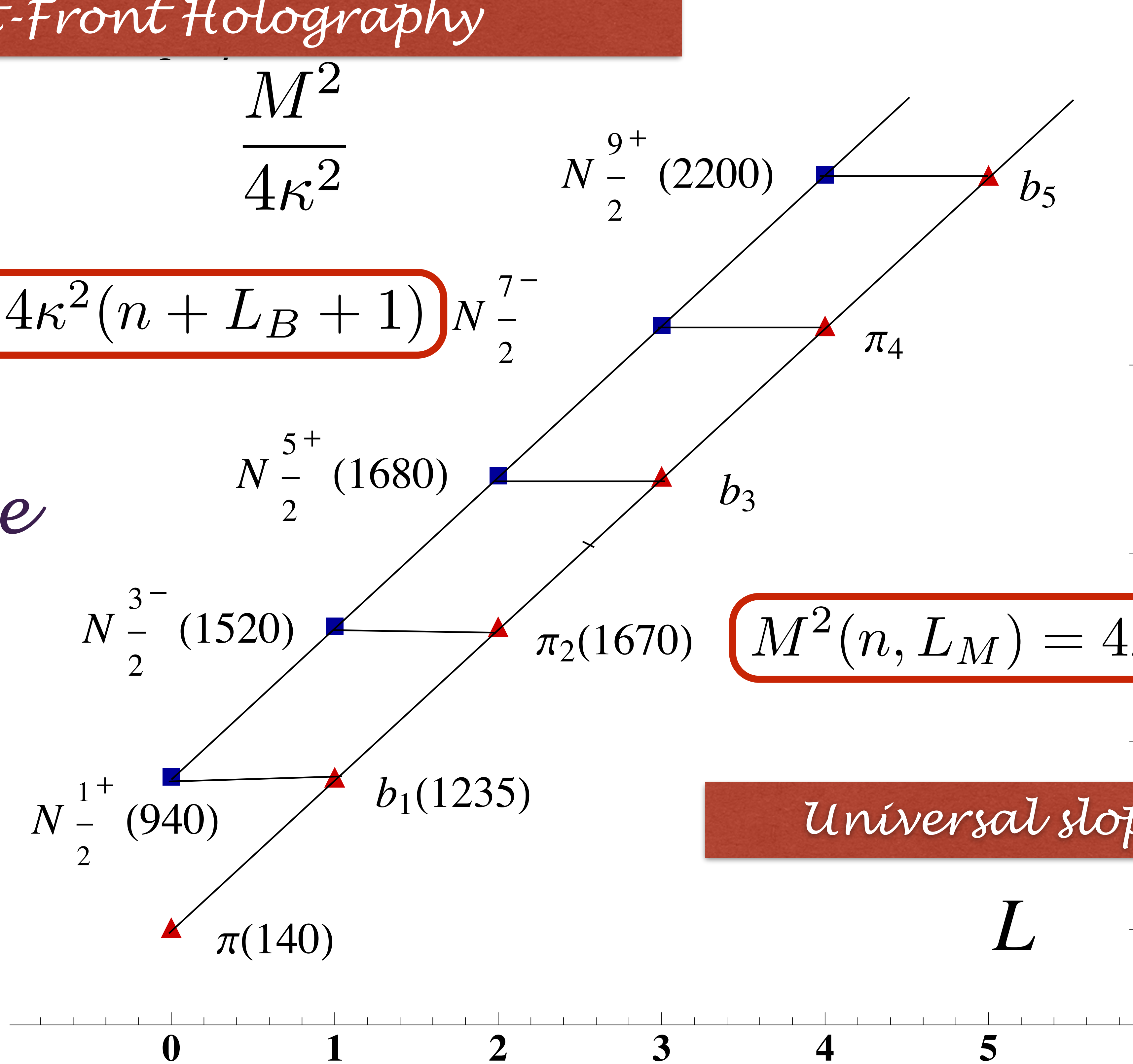
$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L + 1$
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Superconformal Quantum Mechanics
Light-Front Holography

de Tèramond, Dosch, Lorcè, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

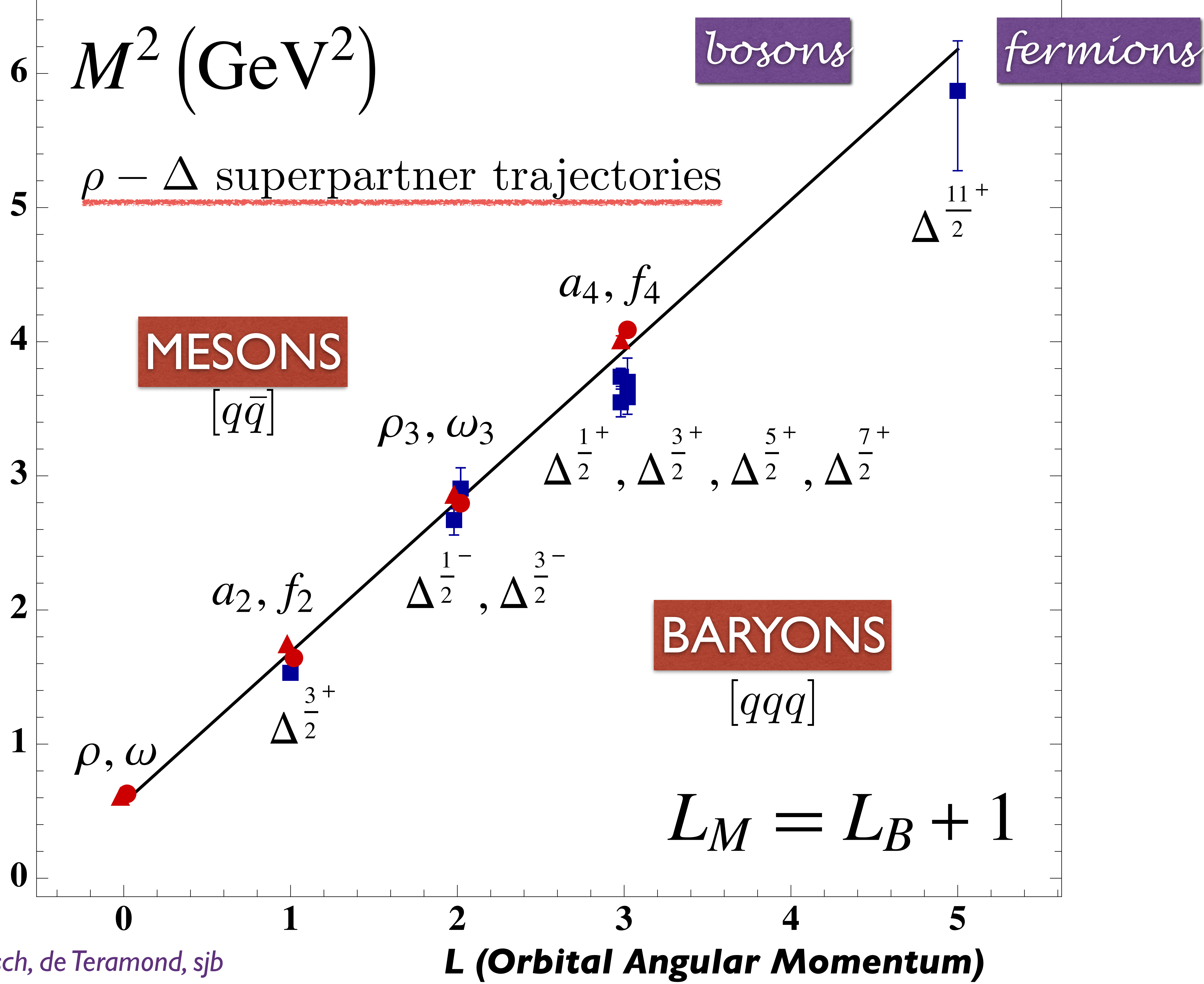


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



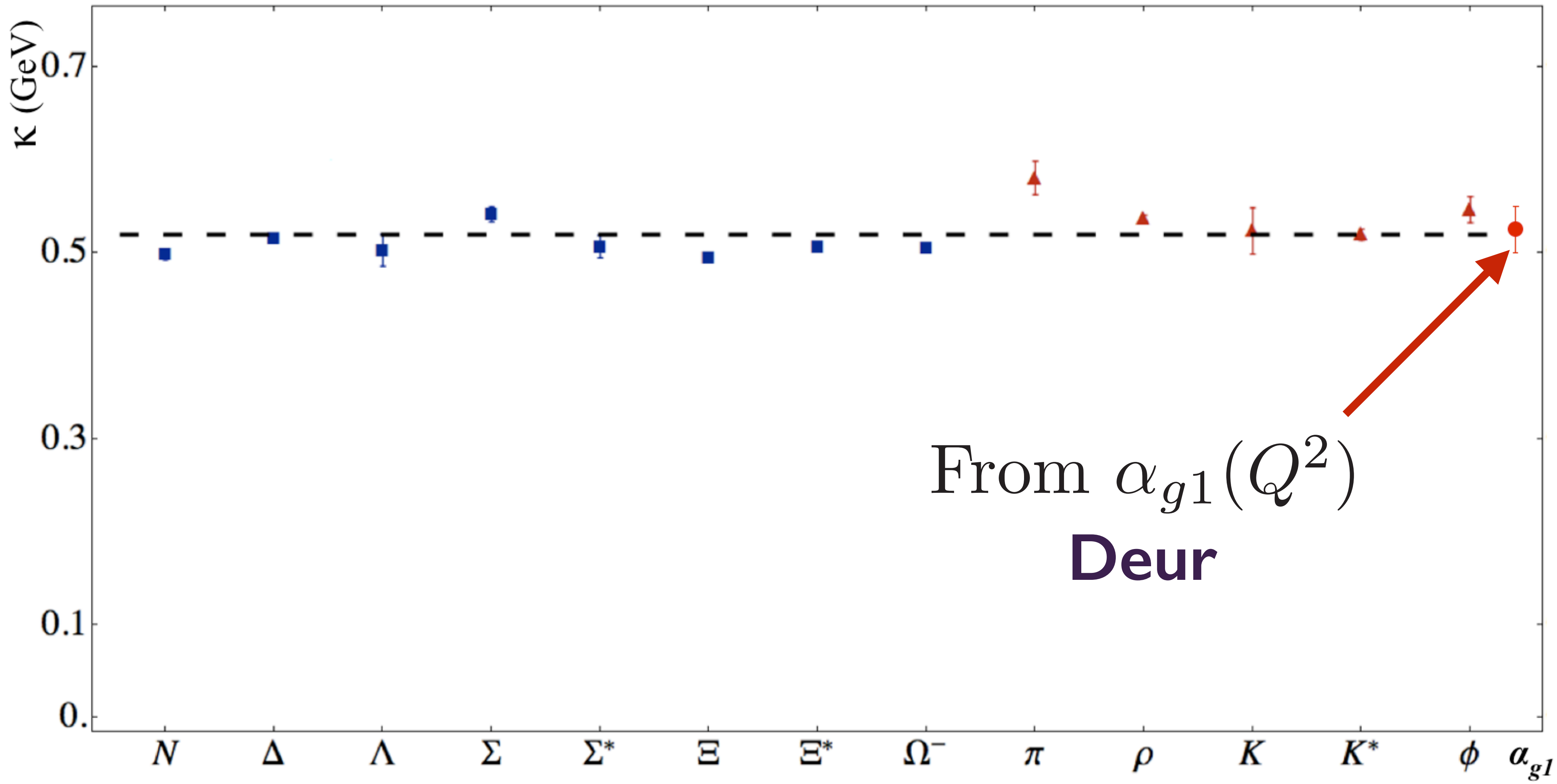
M^2 (GeV^2)
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

$\lambda = \kappa^2$

de Tèramond, Dosch, Lorce', sjb

$$m_u = m_d = 46 \text{ MeV}, \ m_s = 357 \text{ MeV}$$

κ



Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

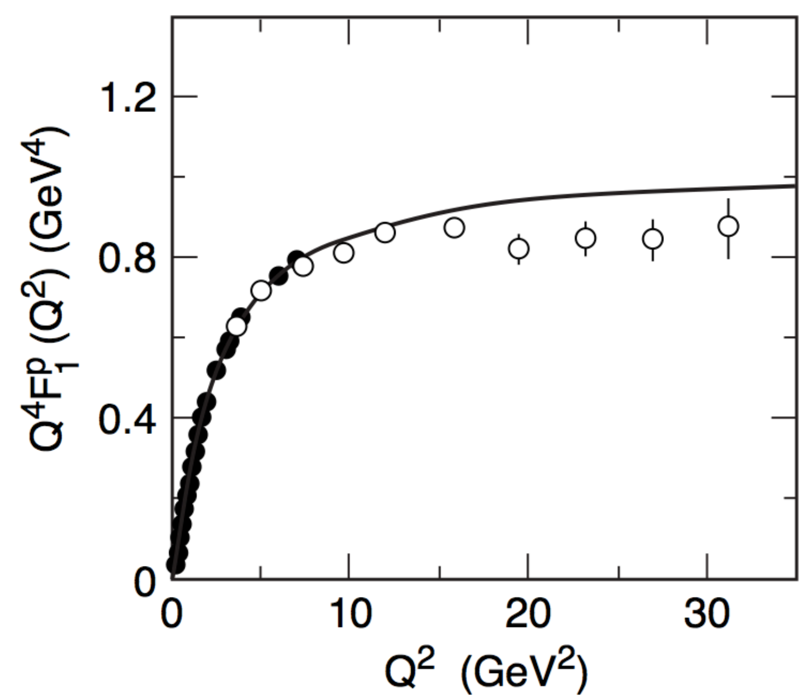
$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$



- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n + 1/2)$

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

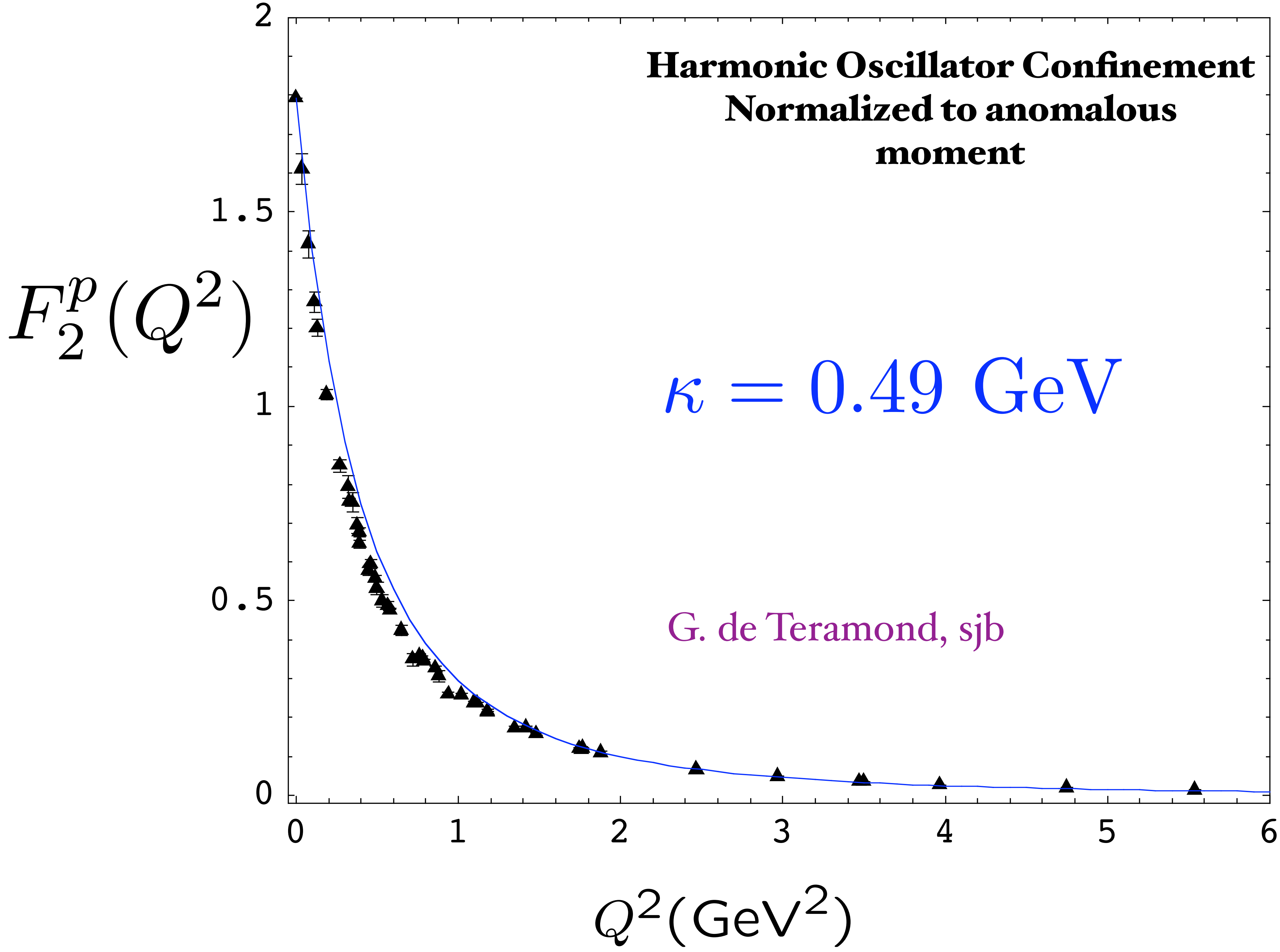
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

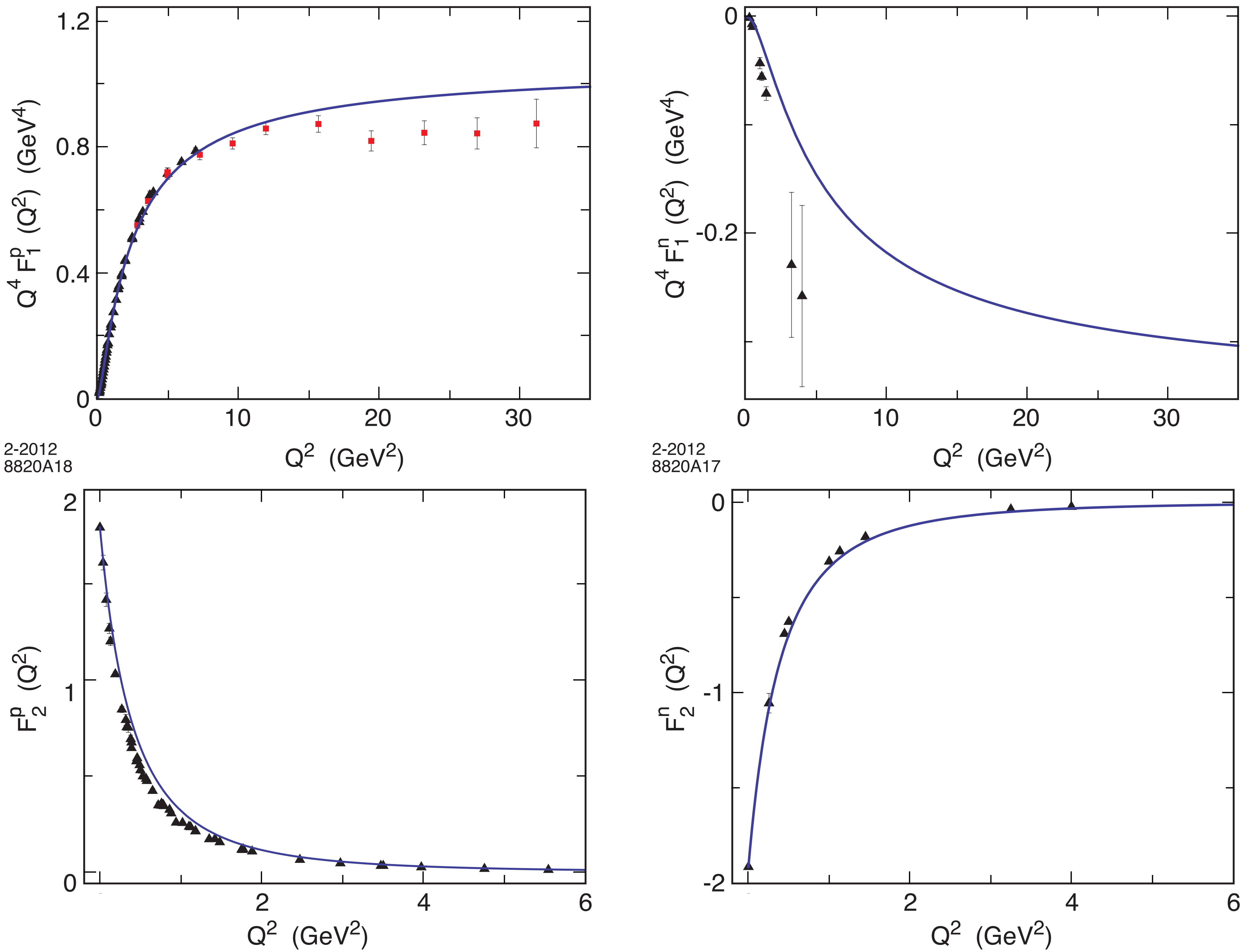
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Using $SU(6)$ flavor symmetry and normalization to static quantities



Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

All-Scale QCD Coupling

Deur, de Tèramond, sjb

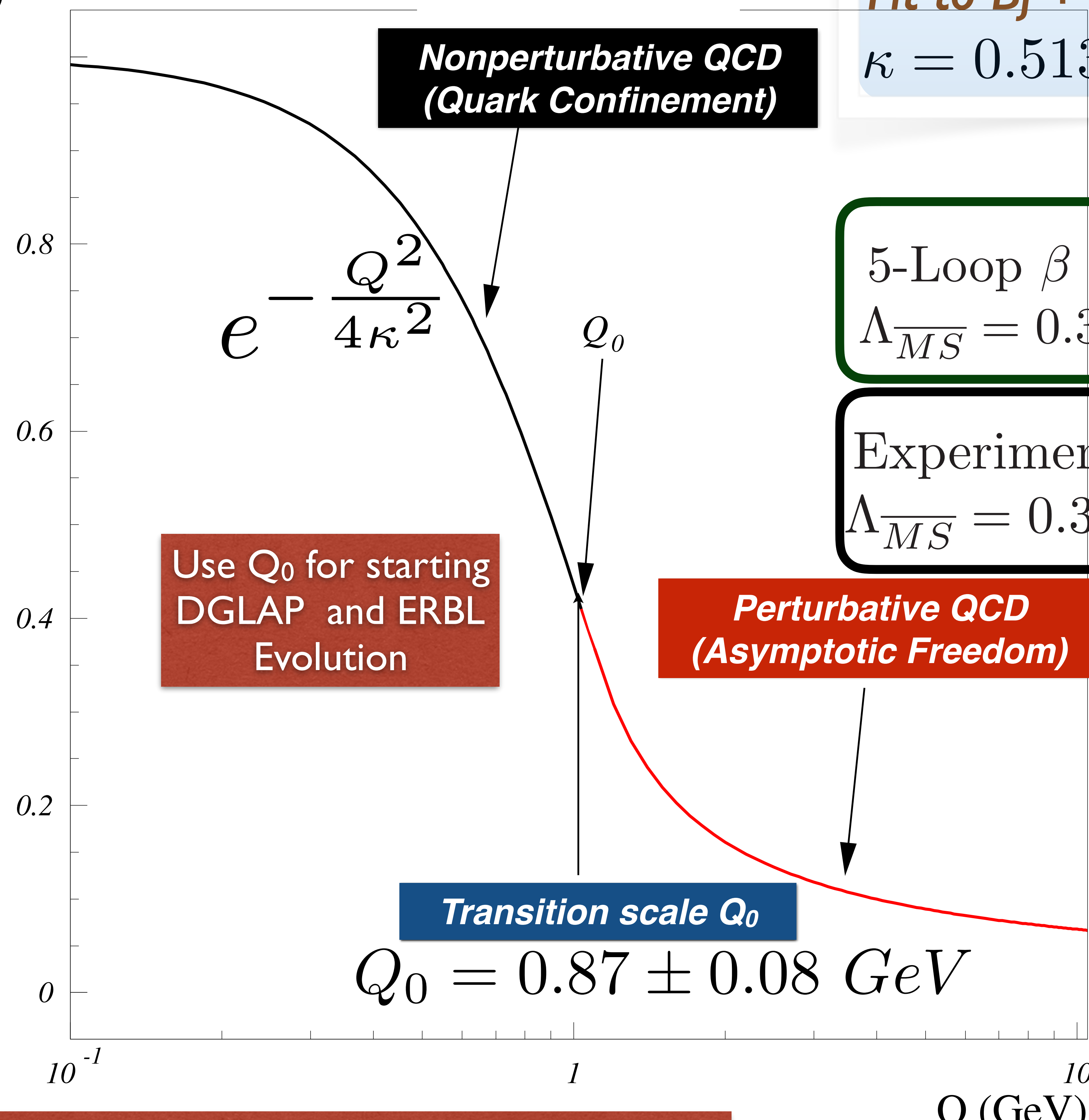
$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:
$$\kappa = 0.513 \pm 0.007 \; GeV$$

5-Loop β Prediction:
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \; GeV$$

Experiment:
$$\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \; GeV$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$

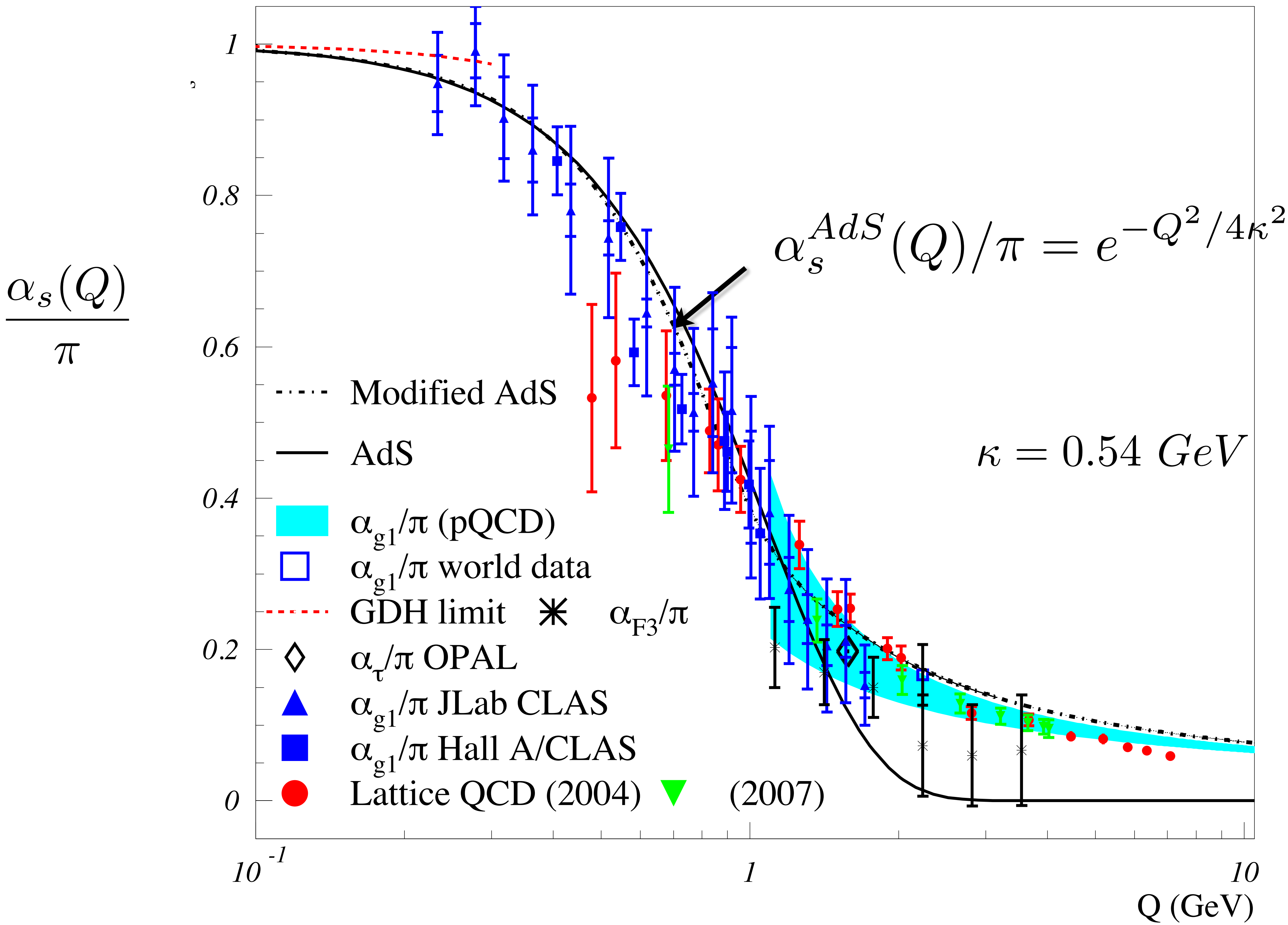


$$\lambda \equiv \kappa^2$$

Reverse Dimensional Transmutation!

$\overline{MS}$ *scheme*

Analytic, defined at all scales, IR Fixed Point

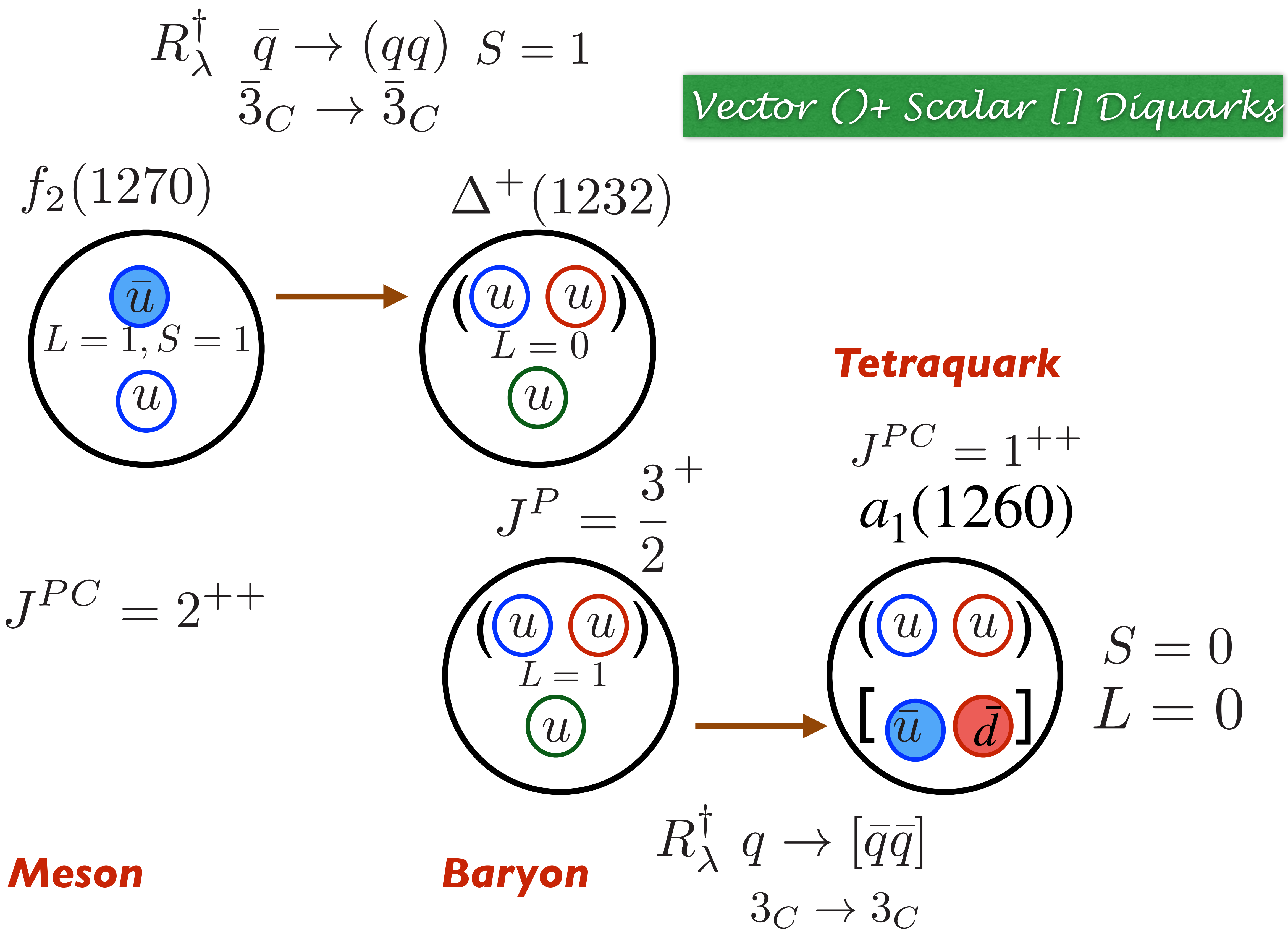


AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Superconformal Algebra 4-Plet



Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$h_1(1170)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$\sigma(500)$
$\bar{q}q$	2^{-+}	$\eta_2(1645)$	$[ud]q$	$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	—
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$(qq)q$	$(3/2)^+$	$\Delta(1232)$	$(qq)[\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$(qq)q$	$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$	$(qq)[\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1600)$
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$(qq)q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$(qq)[\bar{u}\bar{d}]$	—	—
$\bar{q}s$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	1^+	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	0^+	$K_0^*(1430)$
$\bar{q}s$	2^-	$K_2(1770)$	$[ud]s$	$(3/2)^-$	$\Lambda(1520)$	$[ud][\bar{s}\bar{q}]$	1^-	—
$\bar{s}q$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	1^+	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	1^-	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	2^+	$K_2^*(1430)$	$(sq)q$	$(3/2)^+$	$\Sigma(1385)$	$(sq)[\bar{u}\bar{d}]$	1^+	$K_1(1400)$
$\bar{s}q$	3^-	$K_3^*(1780)$	$(sq)q$	$(3/2)^-$	$\Sigma(1670)$	$(sq)[\bar{u}\bar{d}]$	2^-	$K_2(1820)$
$\bar{s}q$	4^+	$K_4^*(2045)$	$(sq)q$	$(7/2)^+$	$\Sigma(2030)$	$(sq)[\bar{u}\bar{d}]$	—	—
$\bar{s}s$	0^{-+}	$\eta'(958)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1380)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1870)$	$[sq]s$	$(3/2)^-$	$\Xi(1620)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	—
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$(sq)s$	$(3/2)^+$	$\Xi^*(1530)$	$(sq)[\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
								$a_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$(sq)s$	$(3/2)^-$	$\Xi(1820)$	$(sq)[\bar{s}\bar{q}]$	—	—
$\bar{s}s$	2^{++}	$f_2(1640)$	$(ss)s$	$(3/2)^+$	$\Omega(1672)$	$(ss)[\bar{s}\bar{q}]$	1^+	$K_1(1650)$

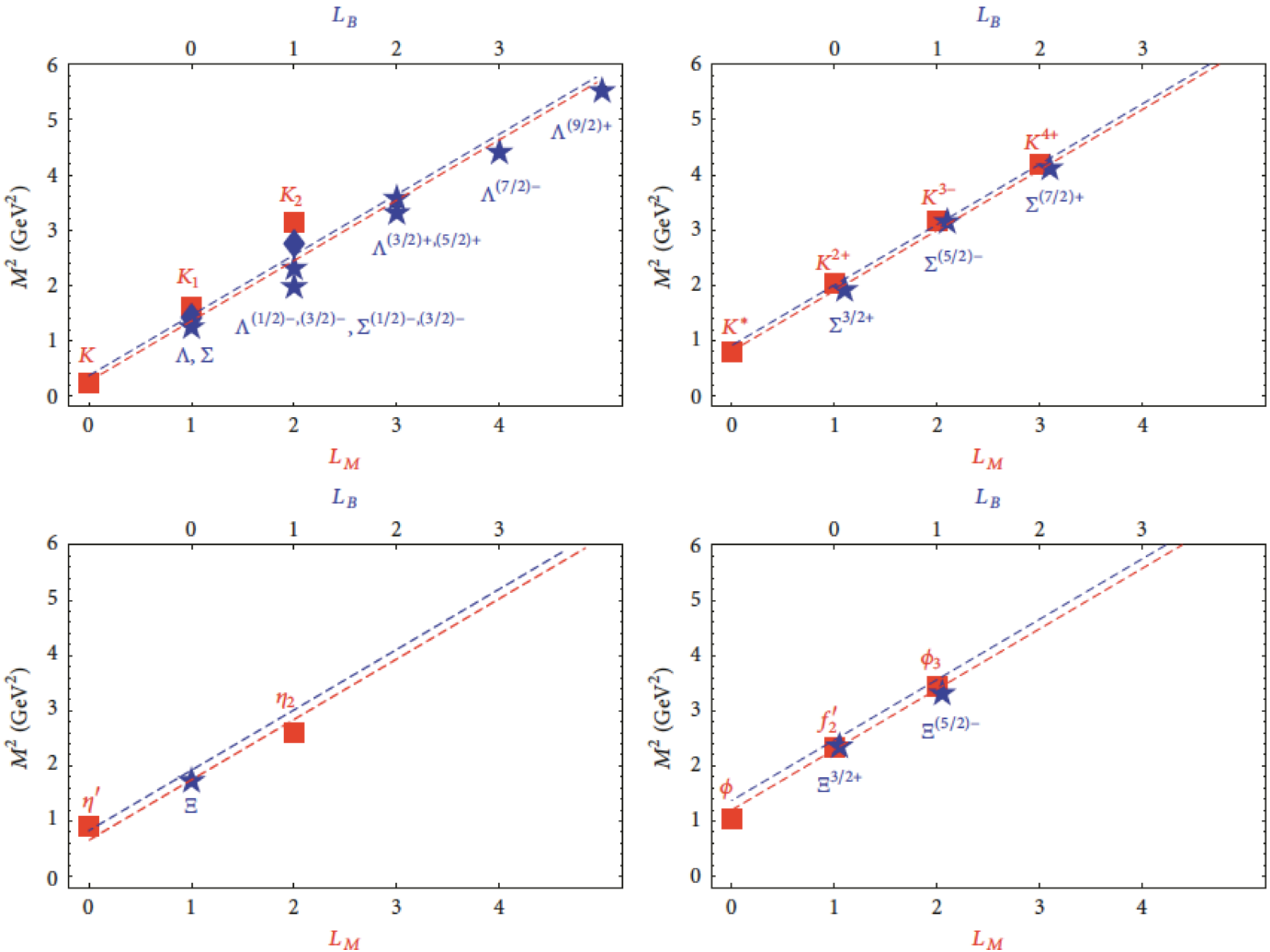
Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

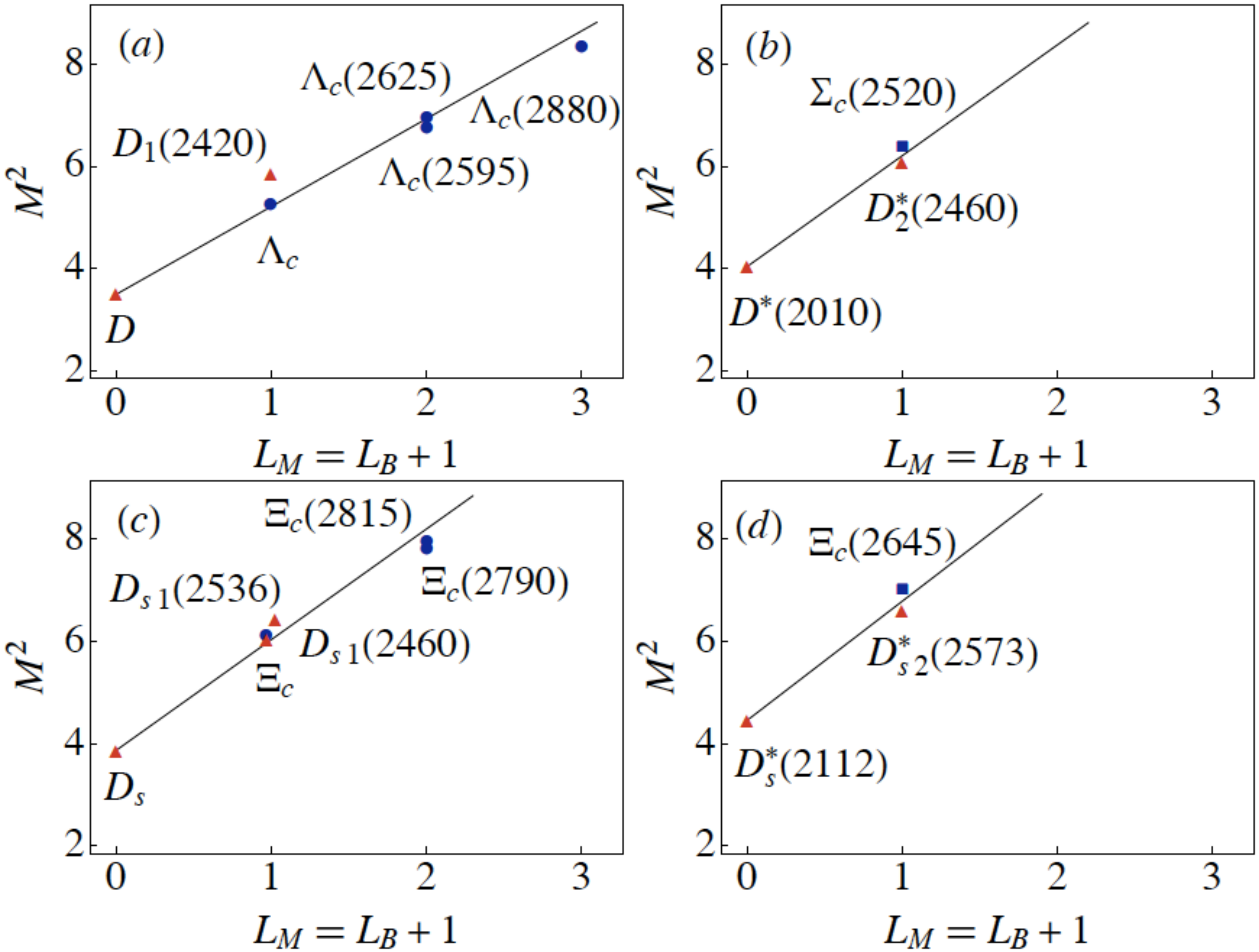
Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

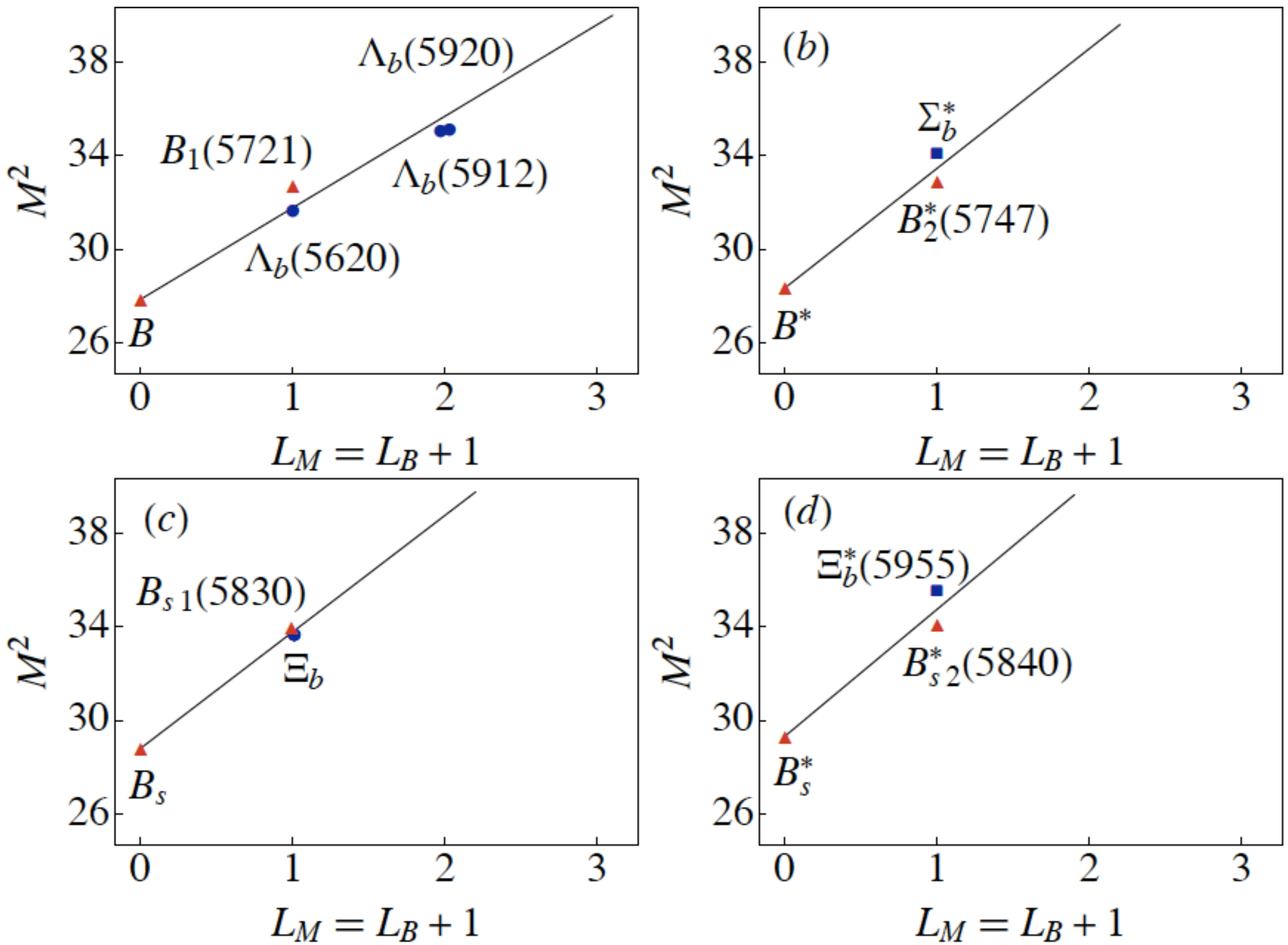
predictions

beautiful agreement!

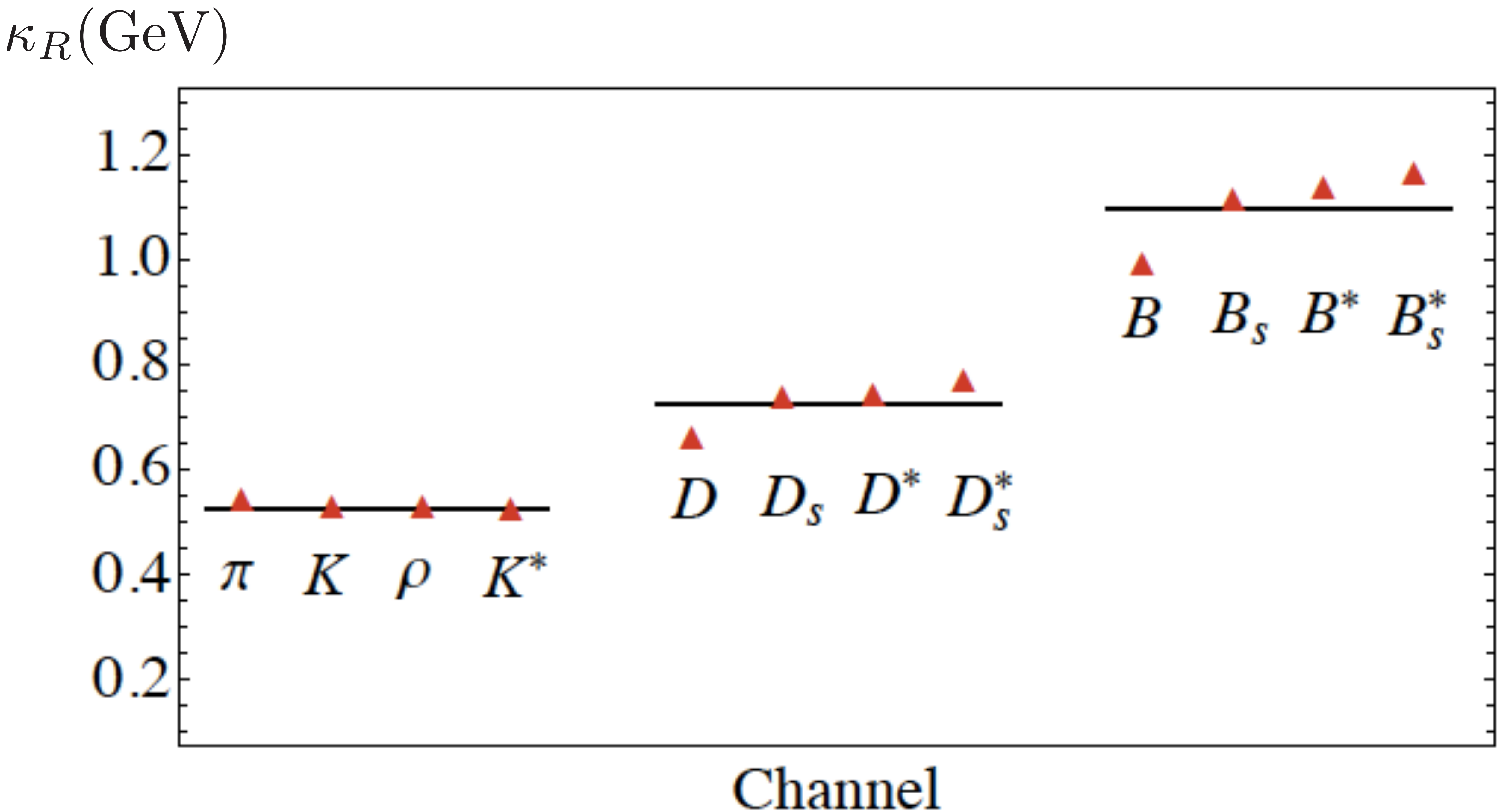
M. Nielsen, sjb

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum

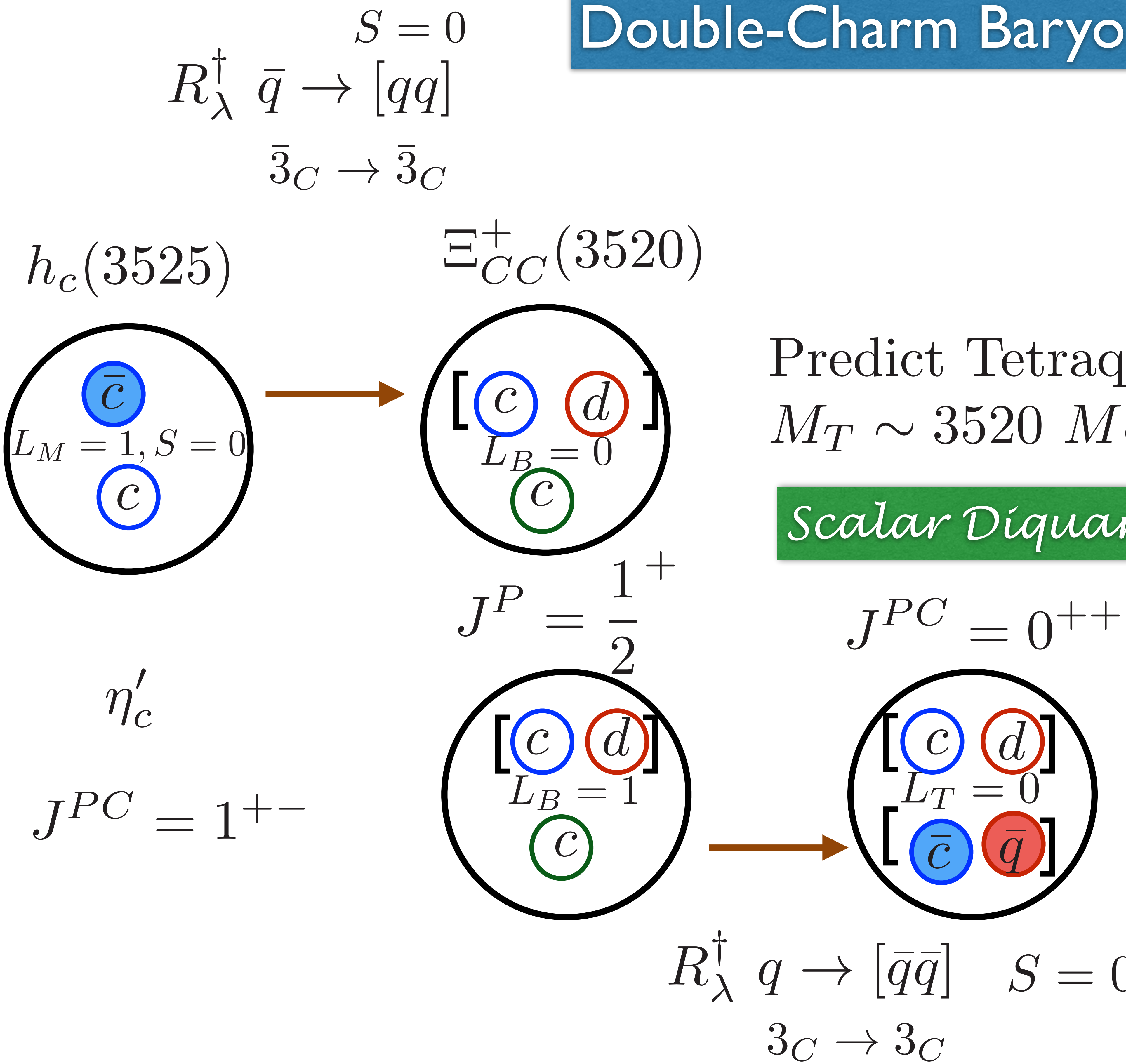


Heavy bottom quark mass does not break supersymmetry



Regge slope for heavy-light mesons, baryons: increases with heavy quark mass

Double-Charm Baryon (SELEX)



Predict Tetraquark $T_{c\bar{c}q\bar{q}}$

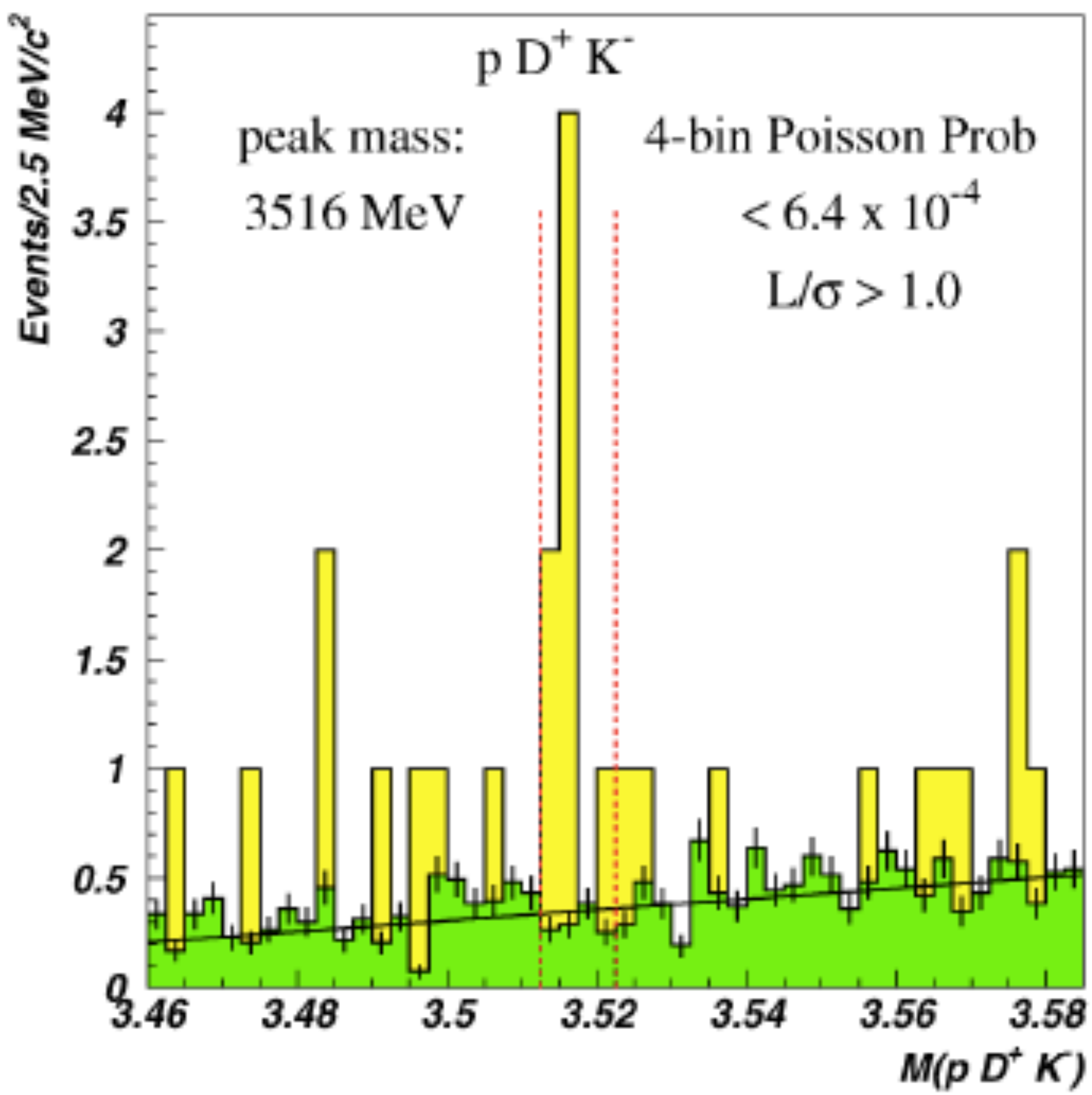
$M_T \sim 3520 \ MeV$

Scalar Diquarks

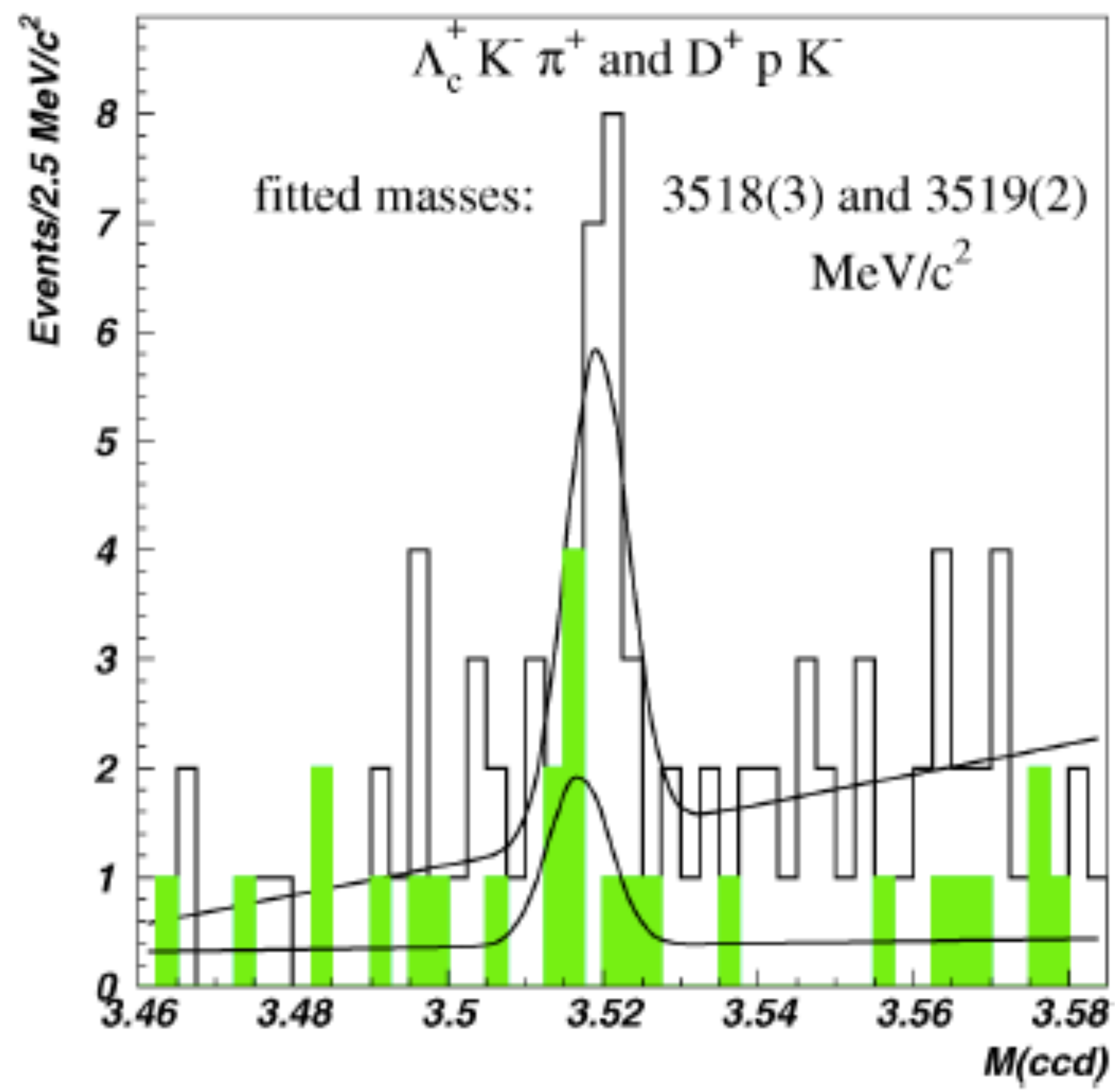
SELEX $(3520 \pm 1\ MeV)$ $J^P = \frac{1}{2}^-$ $|[cd]c>$

Two decay channels: $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+, pD^+K^-$

SELEX Collaboration / Physics Letters B 628 (2005) 18–24

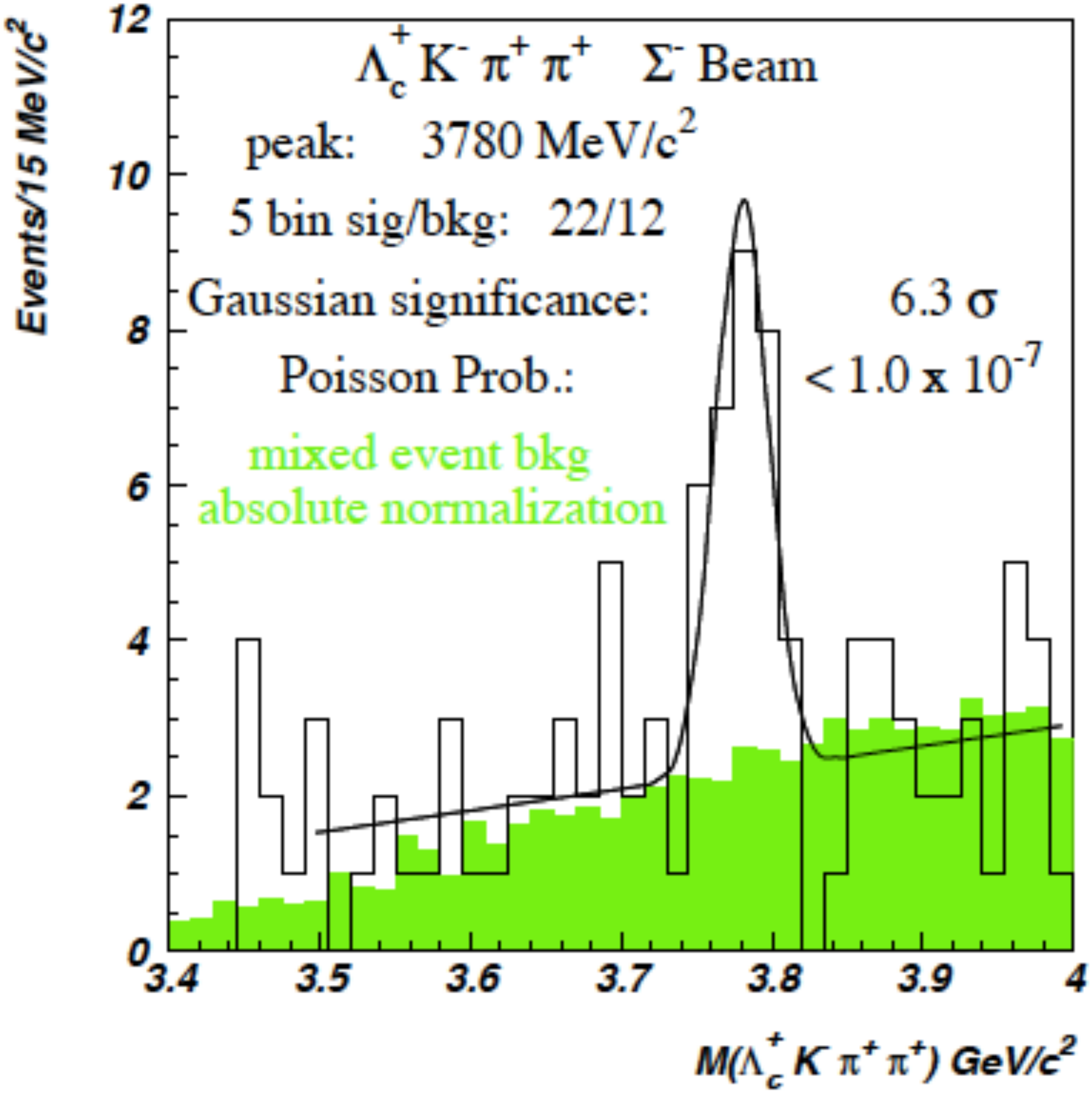


$\Xi_{cc}^+ \to pD^+K^-$ mass distribution from Fig. 2(a) with high-statistics measurement of random combinatoric background computed from event-mixing.

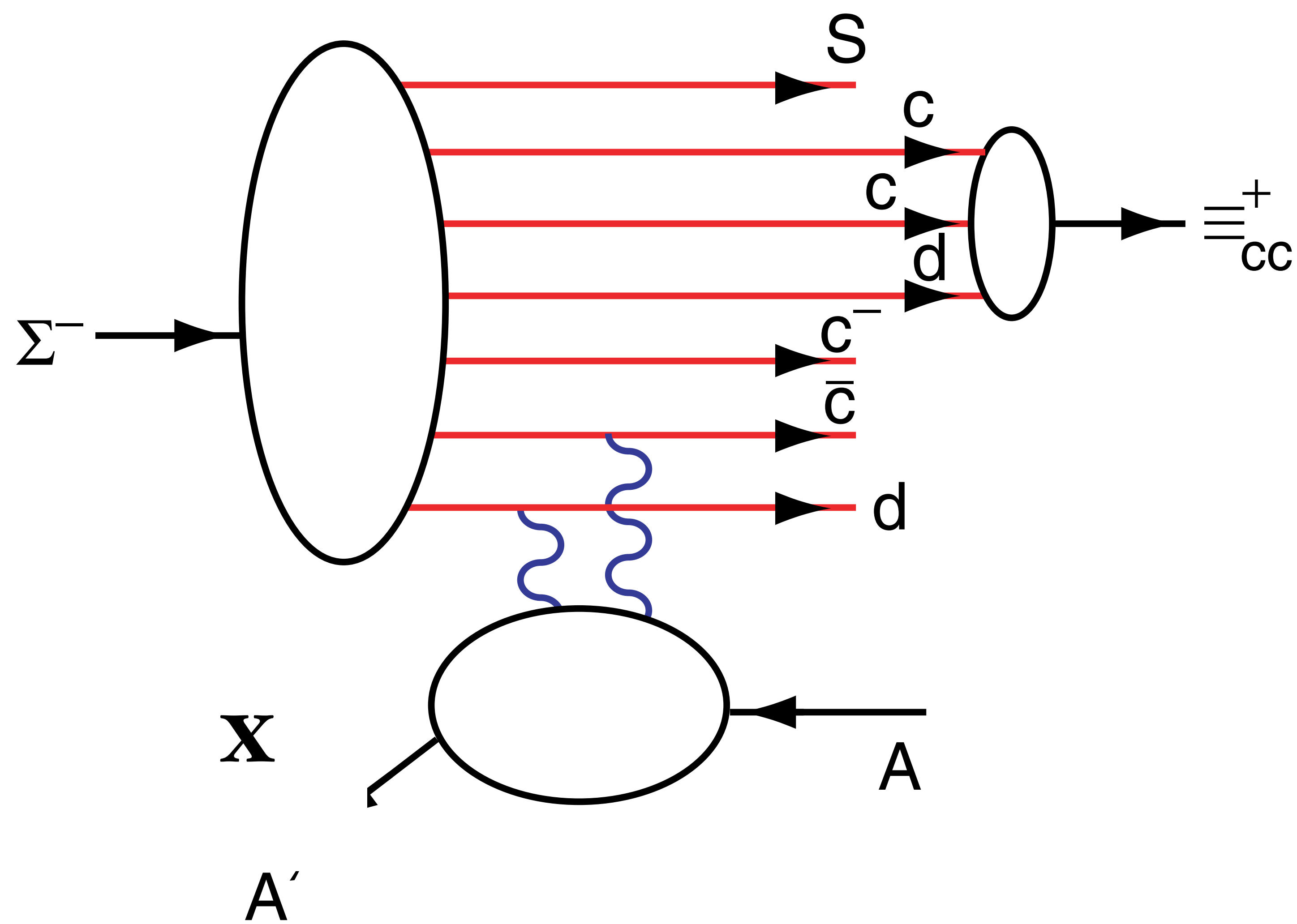


Gaussian fits for $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+ \to pD^+K^-$ (shaded data) on same plot.

SELEX: Recent Progress in the Analysis of Charm-Strange and Double-Charm Baryons



The $\Lambda_c^+ K^- \pi^+ \pi^+$ invariant mass distribution, for Σ^- beam only.



Production of a Double-Charm Baryon

SELEX high x_F $<x_F>= 0.33$

SELEX $(3520 \pm 1\ MeV)$ $J^P = \frac{1}{2}^-$ $|[cd]c>$

Two decay channels: $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+, pD^+ K^-$

LHCb $(3621 \pm 1\ MeV)$ $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$ $|(cu)c>$

$$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$$

Groote, Koshkarev, sjb: SELEX& LHCb could both be correct!

Very different production kinematics:

LHCb (central region)

SELEX (Forward, High x_F) where Λ_c , Λ_b were discovered

NA3: Double J/ψ Hadroproduction measured at High x_F

Radiative Decay:

LHCb(3621) → SELEX(3520) + γ

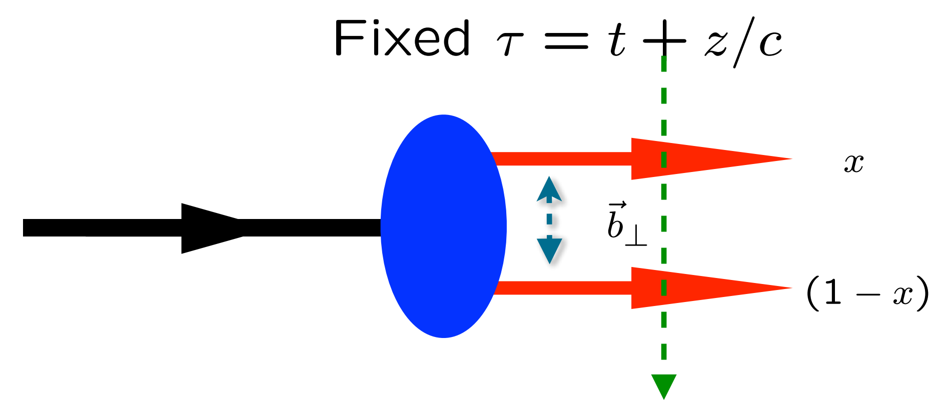
strongly suppressed: $[\frac{100\ MeV}{M_c}]^7$

Also: Different diquark structure possible for LHCb: $|(cc)u>$

Karliner and Rosner

Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame*
- *Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon*
- *Light-Front Holography: AdS_5 = LF (3+1)*



$$\zeta^2 = x(1-x)b_\perp^2$$

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

- *Single fundamental hadronic mass scale κ: but retains the Conformal Invariance of the Action (dAFF)!*

$$U(\zeta^2) = \kappa^4 \zeta^2$$

- *Unique dilaton and color-confining LF Potential!*

$$e^{+\kappa^2 z^2}$$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincarè Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests
- Systematically improvable: Basis LF Quantization (BLFQ)

Invariance Principles of Quantum Field Theory

- *Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- *Causality: Information within causal horizon: Front Form*
- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

Stan Brodsky

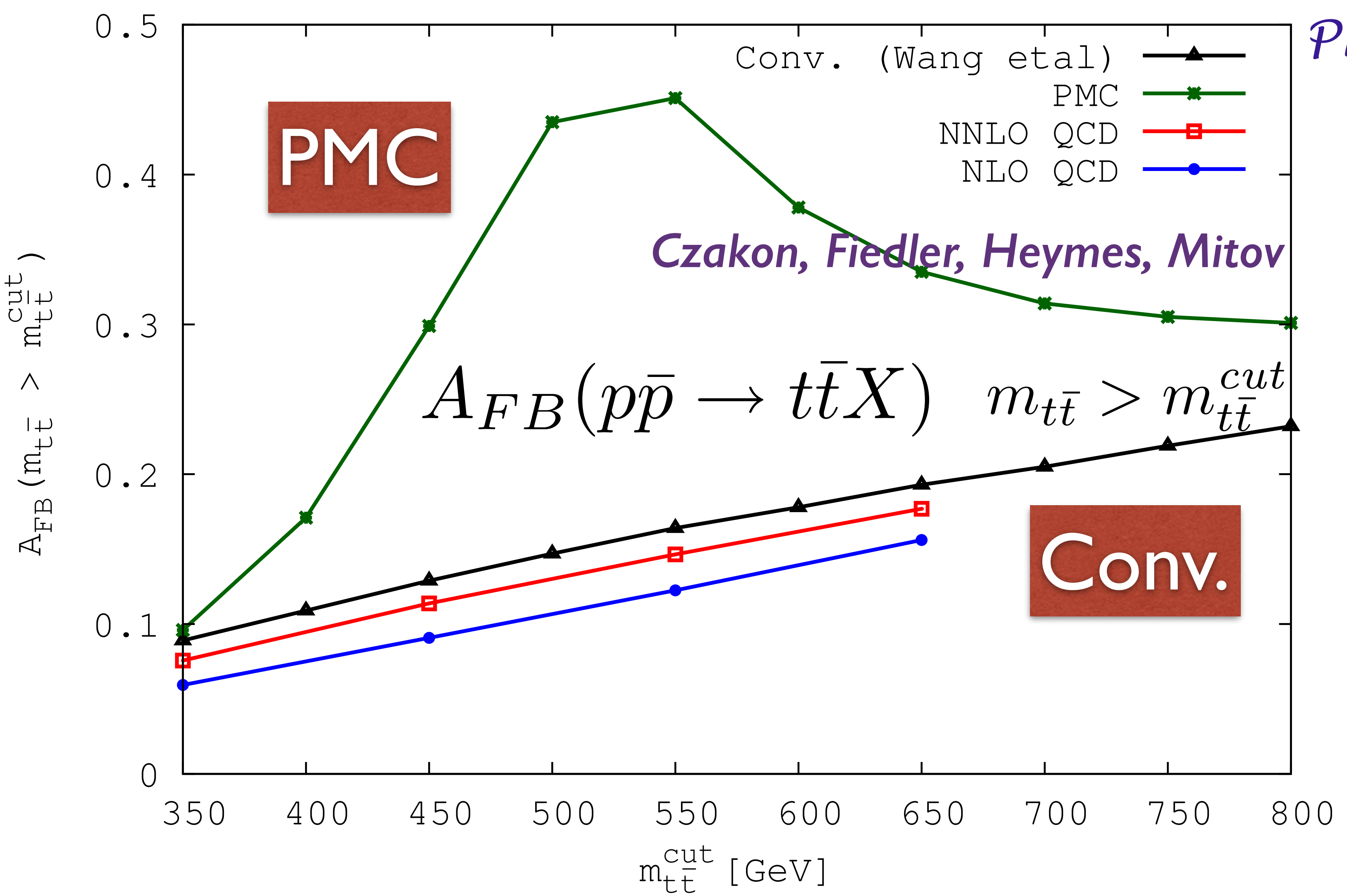
SLAC NATIONAL ACCELERATOR LABORATORY

The QCD coupling at all scales and the elimination of renormalization scale uncertainties

The Principle of Maximum Conformality (PMC)

BLM: G. Peter Lepage
Paul Mackenzie

PMC: Leonardo di Giustino,
Xing-Gang Wu
Matin Mojaza



Stan Brodsky
SLAC

*ECT**
February 12, 2018

α_s Workshop