Old and new observables for α_s from e^+e^- to hadrons

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based on work in collaboration with A. Kardos., S. Kluth, Z. Trócsányi, Z. Tulipánt, A. Verbytskyi

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Introduction

Introduction

The strong coupling α_S is a fundamental parameter of the Standard Model and must be measured precisely

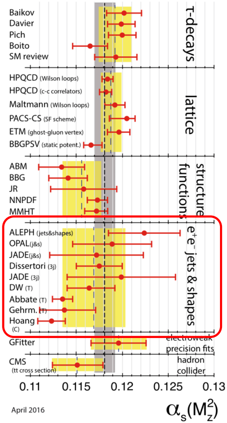
Obtained from fits of theory to measured data

High precision measurements demand highly accurate theoretical predictions

One option: three-jet event shapes in electron-positron annihilation

- extensively measured by multiple experiments
- the Born contribution is already proportional to α_S
- state-of-the-art theory: NNLO fixed order + NNLL resummation (N^3LL for thrust and C-parameter)

α_S world average



[S. Bethke, Nucl. Part. Phys. Proc. 282-284 (2017) 149]

Determinations from e^+e^- annihilation based on

- jet rates (see A. Verbytskyi's talk)
- event shapes describing global event topology (thrust, C-parameter, heavy jet mass, etc.)
- longstanding problem of low $\alpha_S(M_Z)$ from NNLO+N^{3}LL event shapes (using analytic hadronization models)

Can also revisit old event shapes as well as examine new ones

- energy-energy correlation (old)
- groomed event shapes (new)

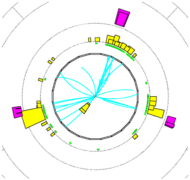
An old observable: energy-energy correlation

Energy-energy correlation

Energy-energy correlation (EEC): energy weighted distribution of angles χ between particles, one of the oldest event shapes [Basham, Brown, Ellis, Love 1978]

$$\frac{1}{\sigma_t}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi} \equiv \frac{1}{\sigma_{\mathrm{tot}}}\int\sum_{i,j}\frac{E_i E_j}{Q^2}\mathrm{d}\sigma_{e^+e^-\to ij+X}\,\delta(\cos\chi-\cos\theta_{ij})$$

- Particles in the same jet:
  forward region, peak near small χ
- Particles in different jets:
  back-to-back region, peak near $\chi\simeq\pi$



Was measured extensively at LEP and predecessors (but no measurements beyond LEP1)

Accurate theory predictions available

- NNLO fixed order from CoLoRFulNNLO [Del Duca, Duhr, Kardos, GS, Trócsányi 2016]
- NNLL resummation in back-to-back region [de Florian, Grazzini 2005]

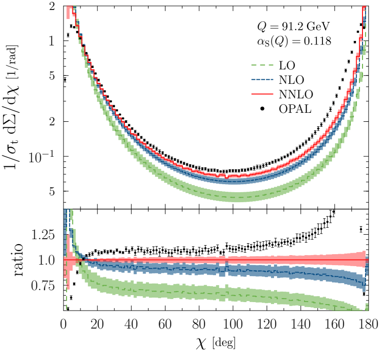
Potential for precision $\alpha_S(M_Z)$ measurement

EEC predictions: fixed-order

The fixed-order prediction to NNLO accuracy reads (note: here $\chi = \pi - \theta_{ij}$)

$$\left[\frac{1}{\sigma_0}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi}\right]_{(\mathrm{f.o.})} = \frac{\alpha_S}{2\pi}\frac{\mathrm{d}A(\chi)}{\mathrm{d}\cos\chi} + \left(\frac{\alpha_S}{2\pi}\right)^2\frac{\mathrm{d}B(\chi)}{\mathrm{d}\cos\chi} + \left(\frac{\alpha_S}{2\pi}\right)^3\frac{\mathrm{d}C(\chi)}{\mathrm{d}\cos\chi} + \mathcal{O}(\alpha_S^4)$$

- NLO correction is large as judged by scale variation ⇒ must go to NNLO
- Higher order predictions improve agreement with data
- Fixed order prediction diverges in the forward and back-to-back regions ⇒ resummation is required
- Sizeable deviations from data even at NNLO ⇒ must take into account hadronization corrections



[Tulipánt, Kardos, GS 2017]

Resummation

Fixed order diverges in the back-to-back limit as $\sim \alpha_S^n \ln^{2n-1} y$ where $y = \cos^2(\chi/2)$, the fixed-order coefficients at n-th order include terms $\{\ln^k y\}_{k=1}^{2n-1}$.

For small y the logarithms become large, $\alpha_S^n \ln^{2n-1} y \sim 1$, invalidating the use of fixed-order perturbation theory.

The logarithmically enhanced terms must be resummed to all orders to obtain a description appropriate in the $y \to 0$ limit.

- resummation can be systematically improved by resumming more towers of logarithms: leading logs (LL), next-to-leading logs (NLL), etc.

$$
\begin{array}{lcccccccccl}
\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} \sim \frac{1}{y}\Big\{ \alpha_S\Big[& \log y & + & 1 & & & & & & \Big] & \text{LO} \\
+\, \alpha_S^2\Big[& \log^3 y & + & \log^2 y & + & \log y & + & 1 & & \Big] & \text{NLO} \\
+\, \alpha_S^3\Big[& \log^5 y & + & \log^4 y & + & \log^3 y & + & \log^2 y & \ldots & \Big]\Big\} & \text{NNLO} \\
& \vdots & & & & & & & & & \\
& \text{LL} & & \text{NLL} & & \text{NNLL} & & & & &
\end{array}
$$

EEC predictions: resummation

Resummation in the back-to-back region known up to NNLL accuracy (and N^3LL is on the way using SCET) [de Florian, Grazzini 2005; Moult, Zhu 2018]

$$\left[\frac{1}{\sigma_t}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi}\right]_{(\mathrm{res.})} = \frac{Q^2}{8}H(\alpha_S)\int_0^\infty db\, J_0(b\,Q\sqrt{y})S(Q,b)$$

The logarithmically enhanced terms are collected in the Sudakov form factor

$$S(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_S(q^2))\right]\right\}$$

The $A(\alpha_S)$, $B(\alpha_S)$ and $H(\alpha_S)$ functions can be computed perturbatively

$$A(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n A^{(n)}, \quad B(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n B^{(n)}, \quad H(\alpha_S) = 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n H^{(n)}$$

Unitarity constraint (vanishing of the distribution at kinematical limit $y = 1$)

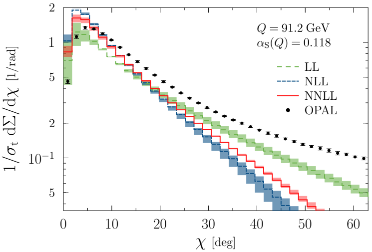
$$A^{(n)} \to A^{(n)}(1-y)^p \quad \text{and} \quad B^{(n)} \to B^{(n)}(1-y)^p, \quad p = 1,2$$

EEC predictions: resummation

Resummation in the back-to-back region known up to NNLL accuracy (and N^3LL is on the way using SCET) [de Florian, Grazzini 2005; Moult, Zhu 2018]

$$\left[\frac{1}{\sigma_t}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi}\right]_{(\mathrm{res.})} = \frac{Q^2}{8}H(\alpha_S)\int_0^\infty db\, J_0(b\,Q\sqrt{y})S(Q,b)$$

- pure resummed results capture the general behaviour of data for small y (note: here $\chi = \pi - \theta_{ij}$ so small y corresponds to small angles)
- Sizeable deviations from data even for moderate angles



[Tulipánt, Kardos, GS 2017]

Combining fixed-order and resummed predictions

Fixed-order and resummed calculations are complementary to each other: they describe data over different kinematical ranges

In order to obtain predictions over a wide kinematical range, the two computations must be combined without double counting ("matching")

Matched predictions at NNLO+NNLL:

- all terms from first three rows (NNLO)
- in addition, first three terms from all rows (NNLL)
- must take care to count the first three terms of the first three rows only once

$$
\begin{array}{lllllllll}
\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} \sim \frac{1}{y}\Big\{ \alpha_S\Big[& \log y & + & 1 & & & & \Big] & \text{LO} \\
\quad + \alpha_S^2\Big[& \log^3 y & + & \log^2 y & + & \log y & + \quad 1 & \Big] & \text{NLO} \\
\quad + \alpha_S^3\Big[& \log^5 y & + & \log^4 y & + & \log^3 y & + \quad \log^2 y \quad \ldots & \Big]\Big\} & \text{NNLO} \\
 & \vdots & & & & & & & \\
 & \text{LL} & & \text{NLL} & & \text{NNLL} & & &
\end{array}
$$

log-R matching

Standard additive matching (naive R matching)

$$\frac{1}{\sigma_t}\frac{d\Sigma}{d\ \cos\chi} \stackrel{?}{=} \left[\frac{1}{\sigma_t}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi}\right]_{(\mathrm{res.})} + \left[\frac{1}{\sigma_t}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi}\right]_{(\mathrm{f.o.})} - \left\{\left[\frac{1}{\sigma_t}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi}\right]_{(\mathrm{res.})}\right\}\Bigg|_{(\mathrm{f.o.})}$$

- cannot be used, because the order of the logarithmic approximation is not high enough (i.e., the fixed order expansion of the resummed expression does not predict all logarithmically enhanced terms correctly at NNLO)
- N^3LL accuracy required

Use log-R matching instead

- in the log-R scheme, matching is performed at the level of the cumulant
- the fixed order expansion of EEC diverges for both small and large angles making the determination of a simple cumulant unreliable
- use a linear combination of moments to suppress the singularity in the forward region

log-R matching

Prescription for log-R matching

- consider the combination of moments

$$\frac{1}{\sigma_t}\tilde{\Sigma}(\chi) \equiv \frac{1}{\sigma_t}\int_0^{\chi} d\chi'(1+\cos\chi')\frac{d\Sigma}{d\chi'}$$

- this has the fixed order expansion

$$\left[\frac{1}{\sigma_t}\tilde{\Sigma}(\chi)\right]_{\text{(f.o.)}} = 1 + \frac{\alpha_S}{2\pi}\bar{\mathcal{A}}(\chi) + \left(\frac{\alpha_S}{2\pi}\right)^2\bar{\mathcal{B}}(\chi) + \left(\frac{\alpha_S}{2\pi}\right)^3\bar{\mathcal{C}}(\chi) + \mathcal{O}(\alpha_S^4)$$

- NNLO+NNLL matched expression

$$\ln\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right] = \ln\left\{\frac{1}{H(\alpha_S)}\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right]_{\text{(res.)}}\right\} - \ln\left\{\frac{1}{H(\alpha_S)}\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right]_{\text{(res.)}}\right\}\Bigg|_{\text{(f.o.)}}$$
$$+\frac{\alpha_S}{2\pi}\bar{\mathcal{A}} + \left(\frac{\alpha_S}{2\pi}\right)^2\left(\bar{\mathcal{B}} - \frac{1}{2}\bar{\mathcal{A}}^2\right) + \left(\frac{\alpha_S}{2\pi}\right)^3\left(\bar{\mathcal{C}} - \bar{\mathcal{A}}\bar{\mathcal{B}} + \frac{1}{3}\bar{\mathcal{A}}^3\right)$$

- original observable

$$\frac{1}{\sigma_t}\Sigma(\chi) = \frac{1}{1+\cos\chi}\frac{d}{d\chi}\left[\frac{1}{\sigma_t}\tilde{\Sigma}(\chi)\right]$$

Finite b-quark mass corrections

At low energies, the assumption of vanishing quark masses is not entirely justified

- include mass effects directly at the level of matched distributions

$$\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} = (1 - r_b(Q))\left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{\text{massless}} + r_b(Q)\left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]^{\text{NNLO}^*}_{\text{massive}}$$

- the complete NNLO correction to the massive distribution is unknown, so supplement the massive NLO prediction with the NNLO coefficient of the massless calculation
- massive predictions computed with Zbb4 [P. Nason, C. Oleari 1997]
- $r_b(Q)$ is the fraction of b-quark events

$$r_b(Q) = \frac{\sigma_{\text{massive}}(e^+e^- \to b\bar{b})}{\sigma_{\text{massive}}(e^+e^- \to \text{hadrons})}$$

- to assess the uncertainty associated to the modeling of b-quark mass corrections, investigated different prescriptions for including the massive corrections (e.g., do not supplement massive NLO prediction with massless NNLO coefficients)

Hadronization corrections

Effects associated with the parton-to-hadron transition cannot be computed in perturbation theory and must be estimated by other means

Analytic modeling

- Non-perturbative, power-behaved corrections in dispersive approach

  [Y.L. Dokshitzer, G. Marchesini, B.R. Webber 1999]

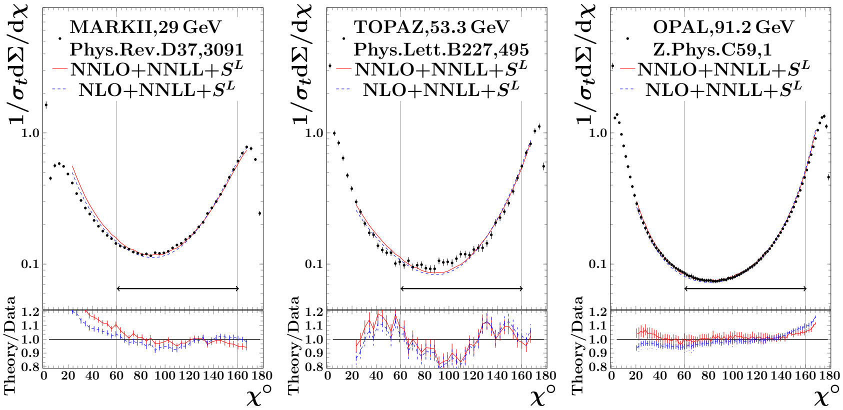
- Multiply Sudakov form factor with non-perturbative correction $S_{\mathrm{NP}} = e^{-\frac{1}{2}a_1 b}(1 - 2a_2 b)$
- a_1 and a_2 are related to moments of α_S at low energy
- Analytic model cannot fully account for hadronization corrections away from back-to-back limit

MC based approach

- Point-by-point multiplicative correction factors using modern MC tools
- Hadronization corrections are ratios of hadron to parton level distributions in the MCs
- Systematics from comparing multiple hadronization models
- See A. Verbytskyi's talk for details

Fits to data: MC based hadronization corrections

Fits to data of NNLO+NNLL and NLO+NNLL predictions



[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

Result

Main result from global fit at NNLO+NNLL

$$\alpha_S(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$$
$$\alpha_S(M_Z) = 0.11750 \pm 0.00287(comb.)$$

See A. Verbytskyi's talk for description of uncertainty

Note using NLO+NNLL only (i.e., no NNLO), we find

$$\alpha_S(M_Z) = 0.12200 \pm 0.00023(exp.) \pm 0.00113(hadr.) \pm 0.00433(ren.) \pm 0.00293(res.)$$
$$\alpha_S(M_Z) = 0.12200 \pm 0.00535(comb.)$$

Inclusion of NNLO corrections crucial in reducing uncertainty: factor of 1/2!

The result is consistent with the world average ($\alpha_S(M_Z) = 0.1175 \pm 0.0029$ vs. 0.1181 ± 0.0011) and competitive with other precision event shapes

New observables: soft-drop event shapes

How to improve precision?

Main source of uncertainty

- renormalization scale uncertainty (truncation of perturbative series)
- hadronization uncertainty

How to improve

- Can we go beyond NNLO for three-jet event shapes? No.[1]
- What can be done with the non-perturbative effects?

Find observables with

- increased perturbative stability (smaller scale uncertainty)
- decreased non-perturbative corrections (a large uncertainty on a small correction is a small overall uncertainty)

One interesting prospect: groomed event shapes, designed to reduce contamination from non-perturbative effects.

Pertinent criticism: grooming results in decreased yield. Are we dropping useful events? Statistics an issue?

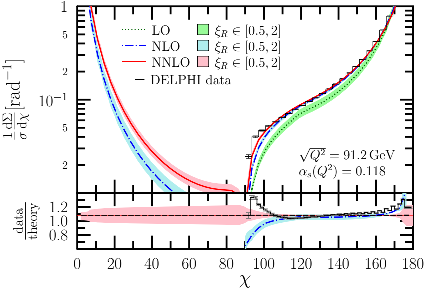
[1] But note that the two jet rate R_2 can be computed to N^3LO, see A. Verbytskyi's talk.

Aside: jet cone energy fraction

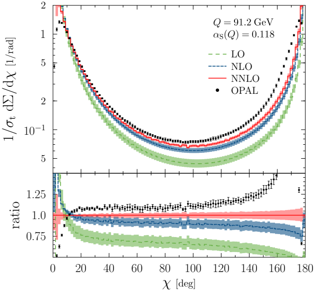
Jet cone energy fraction (JCEF): a "particularly simple and excellent observable for the determination of α_S", since hadronization, detector and perturbative corrections are small

[DELPHI Collaboration Eur. Phys. J. **C14** (2000) 557]

$$\mathrm{JCEF}(\chi) = \frac{1}{\sigma_{\rm had}} \sum_i \int \frac{E_i}{Q} \mathrm{d}\sigma_{e^+e^- \to i+X}\, \delta\left(\cos\chi - \frac{\vec{p}_i \cdot \vec{n}_T}{|\vec{p}_i|}\right)$$



vs.



[Del Duca, Duhr, Kardos, GS, Szőr, Trócsányi, Tulipánt 2016]

Soft-drop grooming

Idea: obtain observables with reduced non-perturbative corrections by removing soft and large-angle radiation from the jet

Soft-drop grooming is defined for Cambridge-Aachen jets of radius R as follows

1. undo the last step of clustering for jet J and split into two subjets
2. check if subjets pass the soft drop condition, for e^+e^- collisions

$$\frac{\min\{E_i, E_j\}}{E_i + E_j} > z_{\mathrm{cut}} \left(\frac{1-\cos\theta_{ij}}{1-\cos R}\right)^{\beta/2} \quad \text{or} \quad z_{\mathrm{cut}}(1-\cos\theta_{ij})^{\beta/2}$$

   for hemisphere jets, where E_i, E_j are subjet energies, θ_{ij} is their angle
3. if the splitting fails this condition, the softer subjet is discarded and the groomer continues to the next step in the clustering
4. if the splitting passes the procedure ends and J is the soft-drop jet

Grooming parameters:

- z_{cut} is an energy threshold, $z_{\mathrm{cut}} \to 0$ corresponds to no grooming
- β controls how strongly wide-angle emissions are discarded, $\beta \to \infty$ corresponds to no grooming

Soft-drop thrust

Soft-drop thrust: first perform a special kind of grooming on the event, then compute event shape from groomed event

[Baron, Marzani, Theeuwes JHEP**08** (2018) 105]

(a) compute the thrust axis, $\vec{n}_T$, divide event into two hemispheres

(b) apply soft-drop grooming to each hemisphere

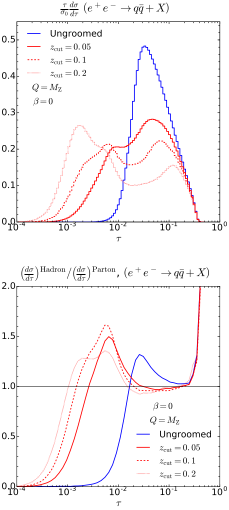
(c') the set of particles left in the two hemispheres after the soft-drop constitute the soft-drop hemispheres $\mathcal{H}^L_{\rm SD}$ and $\mathcal{H}^R_{\rm SD}$, on which the soft-drop thrust $T'_{\rm SD}$ is defined

$$T'_{\rm SD} = \frac{\sum_{i\in\mathcal{H}^L_{\rm SD}} |\vec{n}_L \cdot \vec{p}_i|}{\sum_{i\in\mathcal{E}_{\rm SD}} |\vec{p}_i|} + \frac{\sum_{i\in\mathcal{H}^R_{\rm SD}} |\vec{n}_R \cdot \vec{p}_i|}{\sum_{i\in\mathcal{E}_{\rm SD}} |\vec{p}_i|}$$

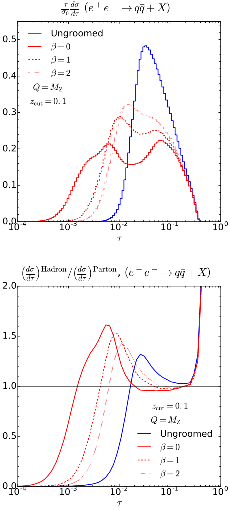
where $\vec{n}_L$ and $\vec{n}_R$ are the jet axes of the left and right hemisphere and $\mathcal{E}_{\rm SD}$ is the soft-drop event, $\mathcal{E}_{\rm SD} = \mathcal{H}^L_{\rm SD} \cup \mathcal{H}^R_{\rm SD}$

Soft-drop event shapes depend on the grooming parameters $z_{\rm cut}$ and β, which can be used to optimize the behavior of the observable (e.g., yield vs. perturbative stability)

$z_{\rm cut}$ dependence



β dependence

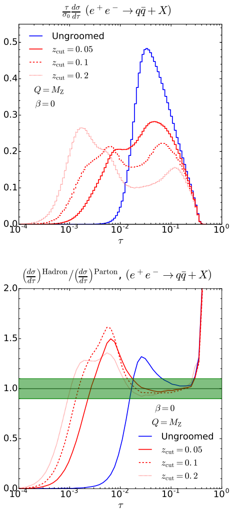


Plots show [Baron, Marzani, Theeuwes JHEP**08** (2018) 105]

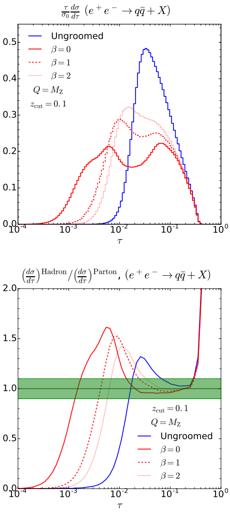
- top: soft-drop thrust distribution in `Pythia`
- bottom: hadron/parton ratios
- left: β fixed, $z_{\rm cut}$ varies
- right: $z_{\rm cut}$ fixed, β varies

Impact of soft-drop on hadronization corrections

z_{cut} dependence



β dependence



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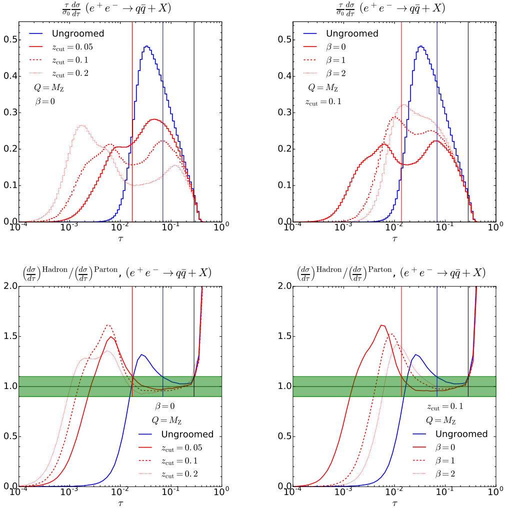
Effects of grooming

- non-perturbative corrections remain moderate over an extended kinematical range
- validity of perturbation theory extended towards smaller τ'_{SD} values ($\tau'_{\mathrm{SD}} = 1 - T'_{\mathrm{SD}}$)

Impact of soft-drop on hadronization corrections

z_{cut} dependence | β dependence



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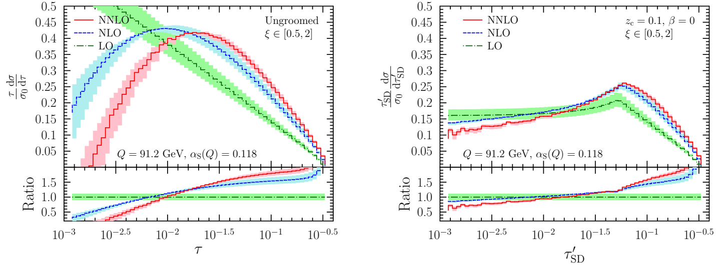
Effects of grooming

- non-perturbative corrections remain moderate over an extended kinematical range
- validity of perturbation theory extended towards smaller τ'_{SD} values ($\tau'_{\mathrm{SD}} = 1 - T'_{\mathrm{SD}}$)
- smaller non-perturbative corrections $\Rightarrow$ better perturbative convergence?

Thrust τ (left) vs. soft-drop thrust τ'_{SD} (right) with $z_{\mathrm{cut}} = 0.1$, $\beta = 0$, bottom panel shows ratio to LO

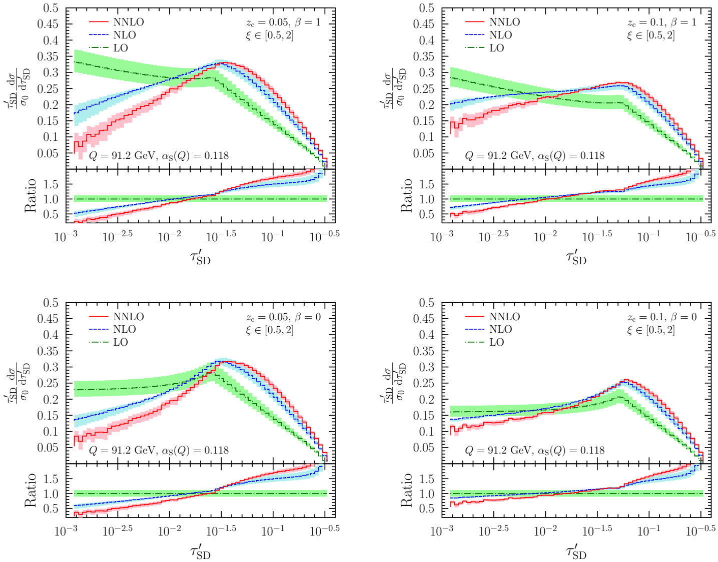


[Kardos, GS, Trócsányi 2018]

- Much improved perturbative stability, as expected
- Smaller dependence on renormalization scale

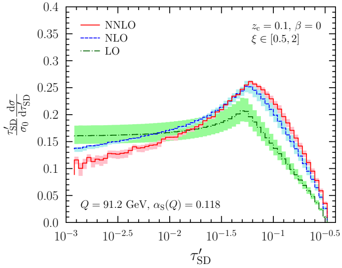
Dependence on grooming parameters

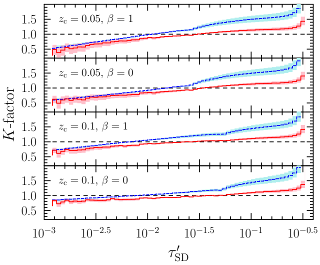
Soft-drop thrust $\tau'_{\rm SD}$ for different $(z_{\rm cut},\,\beta)$ pairs [Kardos, GS, Trócsányi 2018]



K-factors defined as ratios of consecutive orders: "ratio test" for convergence of perturbative series

$$K_{\mathrm{NLO}}(\mu) = \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(\mu)}{\mathrm{d}O} \bigg/ \frac{\mathrm{d}\sigma_{\mathrm{LO}}(Q)}{\mathrm{d}O}\,, \quad K_{\mathrm{NNLO}}(\mu) = \frac{\mathrm{d}\sigma_{\mathrm{NNLO}}(\mu)}{\mathrm{d}O} \bigg/ \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(Q)}{\mathrm{d}O}$$





[Kardos, GS, Trócsányi 2018]

- Perturbatively most stable prediction for strongest grooming: $z_{\mathrm{cut}} = 0.1,\ \beta = 0$

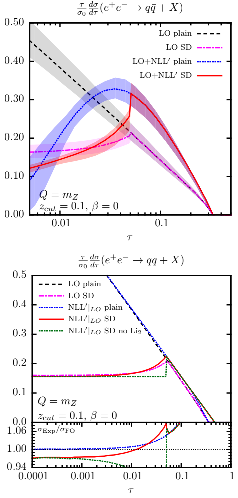
Groomed event shapes: an interesting prospect

Soft-drop

- can help reduce non-perturbative corrections for thrust
- leads to more stable perturbative predictions
- NNLO accuracy in fixed-order
- NNLL accuracy in resummation for small τ'_{SD}

Issues

- grooming reduces yield, statistics an issue?
- all order description of the transition between groomed and ungroomed regime challenging
- detailed understanding of the $\tau'_{\mathrm{SD}} \sim z_{\mathrm{cut}}$ region needed for meaningful matching of fixed order to NNLL resummation



[Baron, Marzani, Theeuwes JHEP**08** (2018) 105]

Summary

New measurement of $\alpha_S(M_Z)$ from global fit of EEC in e^+e^- annihilation to NNLO+NNLL predictions

$$\alpha_S(M_Z) = 0.11750 \pm 0.00287$$

- value consistent with world average ($\alpha_S(M_Z) = 0.1181 \pm 0.0011$)
- uncertainty competitive with other precision event shapes

Impact of NNLO corrections:

- better modelling of the shape of the distribution
- non-negligible shift of extracted $\alpha_S(M_Z)$ towards lower values
- reduced theoretical uncertainties (factor of 1/2)

Latest progress on the theory side:

- analytic computation at NLO [Dixon, Luo, Shtabovenko, Yang, Zhu 2018]
- factorisation formula for N^3LL resummation [Moult, Zhu 2018]

Summary

Groomed event shapes provide an interesting class of new observables

- designed to reduce contamination from non-perturbative effects
- extend the validity of the perturbative description
- better perturbative stability

Example: soft-drop thrust

- can be computed to NNLO accuracy in fixed order
- grooming parameters can be used to optimize the observable
- NNLL resummation known, but transition region must be better understood

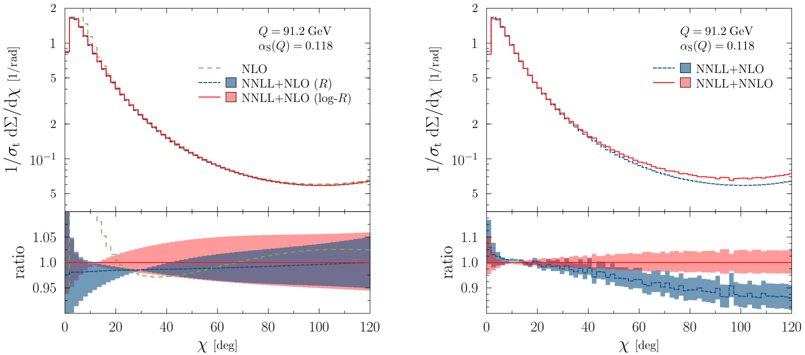
We should exploit these new observables!

Thank you for your attention!

Backup slides

EEC predictions: fixed-order + resummation

Matched predictions at NNLL+NLO and NNLL+NNLO



[Tulipánt, Kardos, GS 2017]

More soft-drop event shapes

Can define other soft-drop event shapes in a similar manner to soft-drop thrust: use the groomed event to compute the hemisphere jet mass $e_2^{(2)}$ and narrow jet mass ρ

Hemisphere jet mass:

- cluster the event into exactly two jets
- soft-drop jets
- look at the largest value of

$$e_2^{(2)} = \frac{m_J^2}{E_J^2}$$

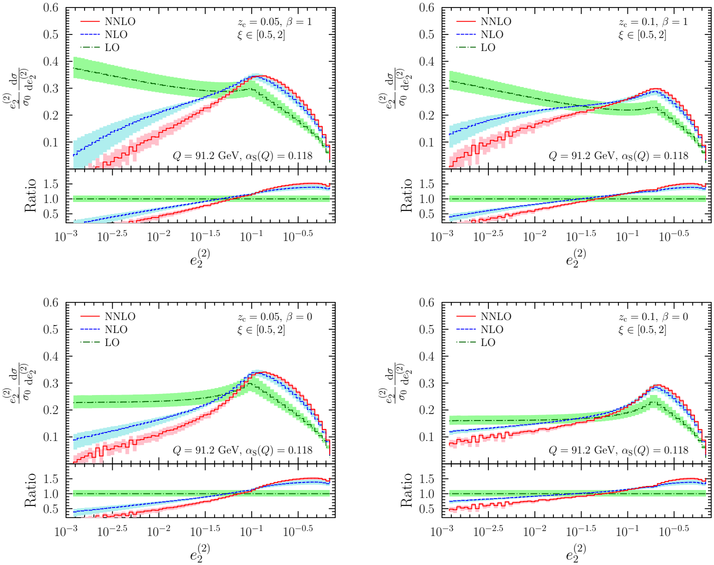
Narrow jet mass:

- use anti-k_t clustering with jet radius R
- soft-drop jets
- consider the observable

$$\rho = \frac{m_J^2}{2E_J^2(1 - \cos R)}$$

$e_2^{(2)}$: dependence on grooming parameters

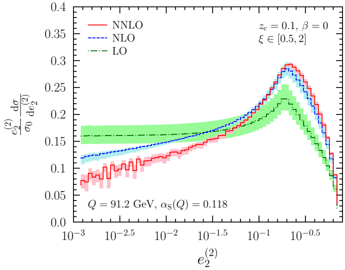
Soft-drop hemisphere jet mass $e_2^{(2)}$ for different $(z_{\rm cut}, \beta)$ pairs [Kardos, GS, Trócsányi 2018]

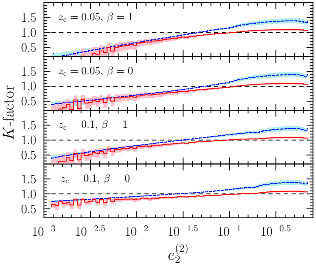


Perturbative stability: K-factors for $e_2^{(2)}$

K-factors defined as ratios of consecutive orders: "ratio test" for convergence of perturbative series

$$K_{\mathrm{NLO}}(\mu) = \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(\mu)}{\mathrm{d}O} \Bigg/ \frac{\mathrm{d}\sigma_{\mathrm{LO}}(Q)}{\mathrm{d}O}\,, \quad K_{\mathrm{NNLO}}(\mu) = \frac{\mathrm{d}\sigma_{\mathrm{NNLO}}(\mu)}{\mathrm{d}O} \Bigg/ \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(Q)}{\mathrm{d}O}$$



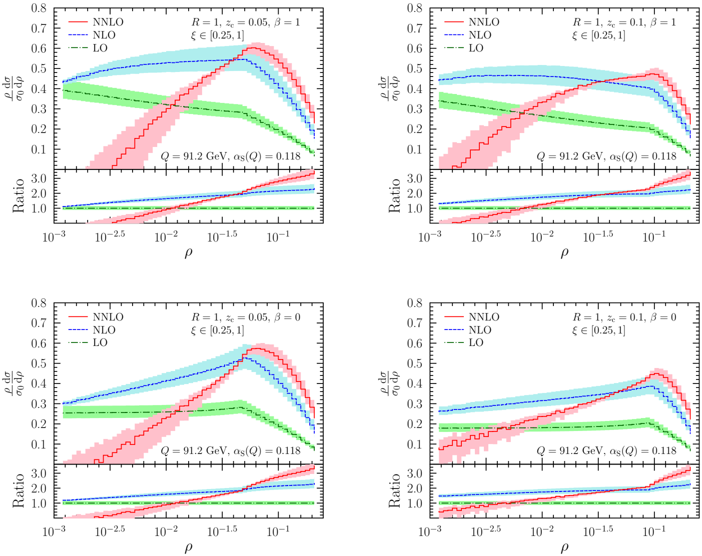


[Kardos, GS, Trócsányi 2018]

- Perturbatively most stable prediction for strongest grooming: $z_{\mathrm{cut}} = 0.1$, $\beta = 0$

ρ: dependence on grooming parameters

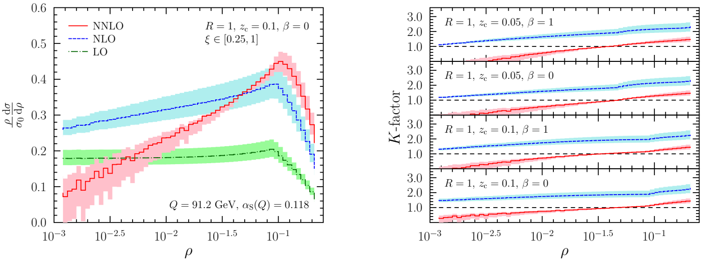
Soft-drop narrow jet mass ρ for different (z_{cut}, β) pairs [Kardos, GS, Trócsányi 2018]



Perturbative stability: K-factors for ρ

K-factors defined as ratios of consecutive orders: "ratio test" for convergence of perturbative series

$$K_{\mathrm{NLO}}(\mu) = \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(\mu)}{\mathrm{d}O} \bigg/ \frac{\mathrm{d}\sigma_{\mathrm{LO}}(Q)}{\mathrm{d}O}\,, \quad K_{\mathrm{NNLO}}(\mu) = \frac{\mathrm{d}\sigma_{\mathrm{NNLO}}(\mu)}{\mathrm{d}O} \bigg/ \frac{\mathrm{d}\sigma_{\mathrm{NLO}}(Q)}{\mathrm{d}O}$$



[Kardos, GS, Trócsányi 2018]

- Stronger grooming improves perturbative convergence from NLO to NNLO, but the NLO K-factor grows with more grooming