α_s extraction from N^{3}LO hadronic W (and Z) decays

α_s(2019) workshop

ECT*, 14th Feb. 2019

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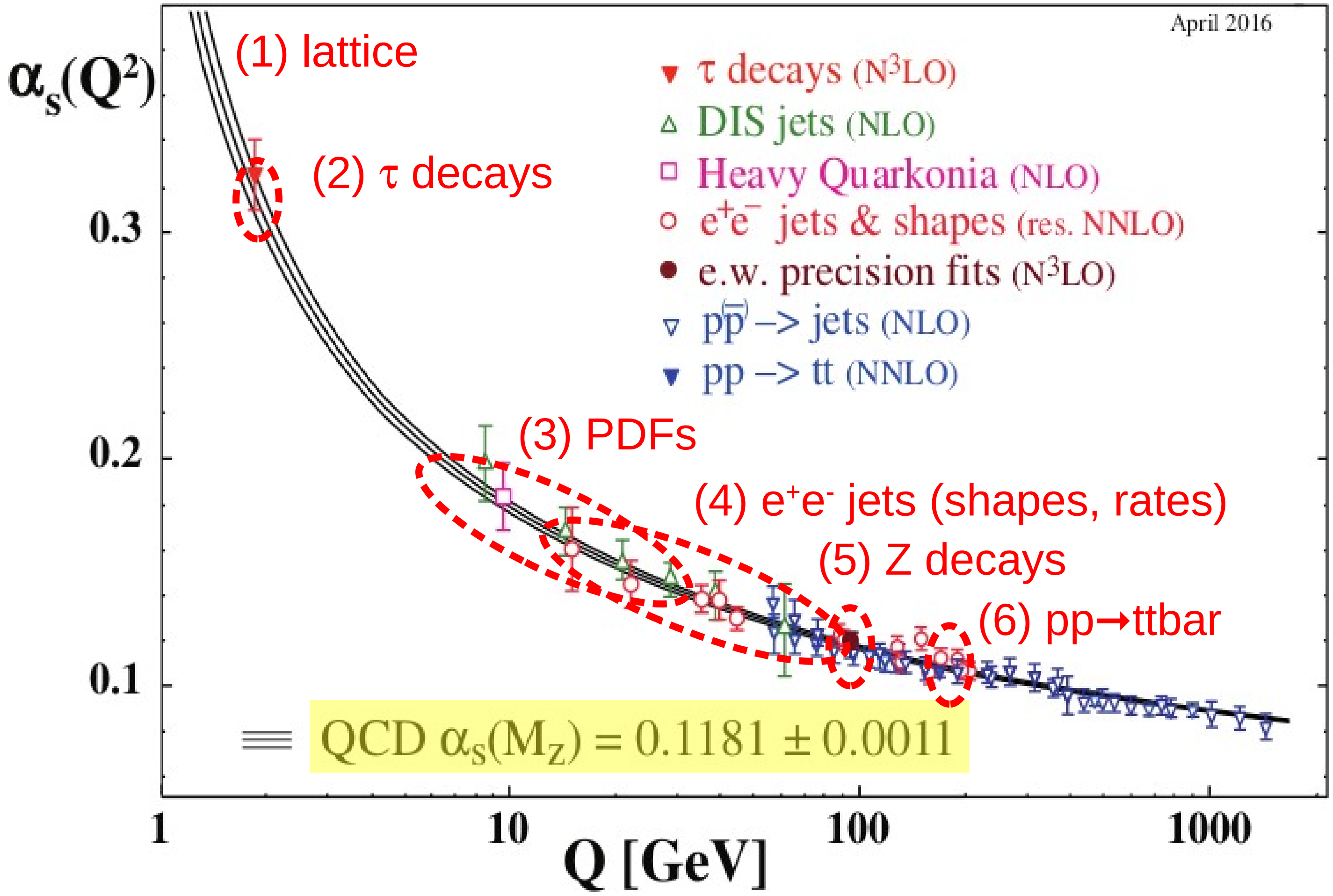
Mostly based on: *D. d'Enterria, M. Srebre, "α_s and V_{cs} determination, and CKM unitarity test, from W decays at NNLO" PLB763 (2016) 465.*
Plus FCC-ee CDR updates: http://inspirehep.net/record/1713706

World α_s determination

- Determined today by comparing 6 experimental observables to pQCD NNLO,N^3LO predictions, plus global average at the Z pole scale:

[Bethke/Dissertori/Salam]



α_s from hadronic Z decays: W? future?

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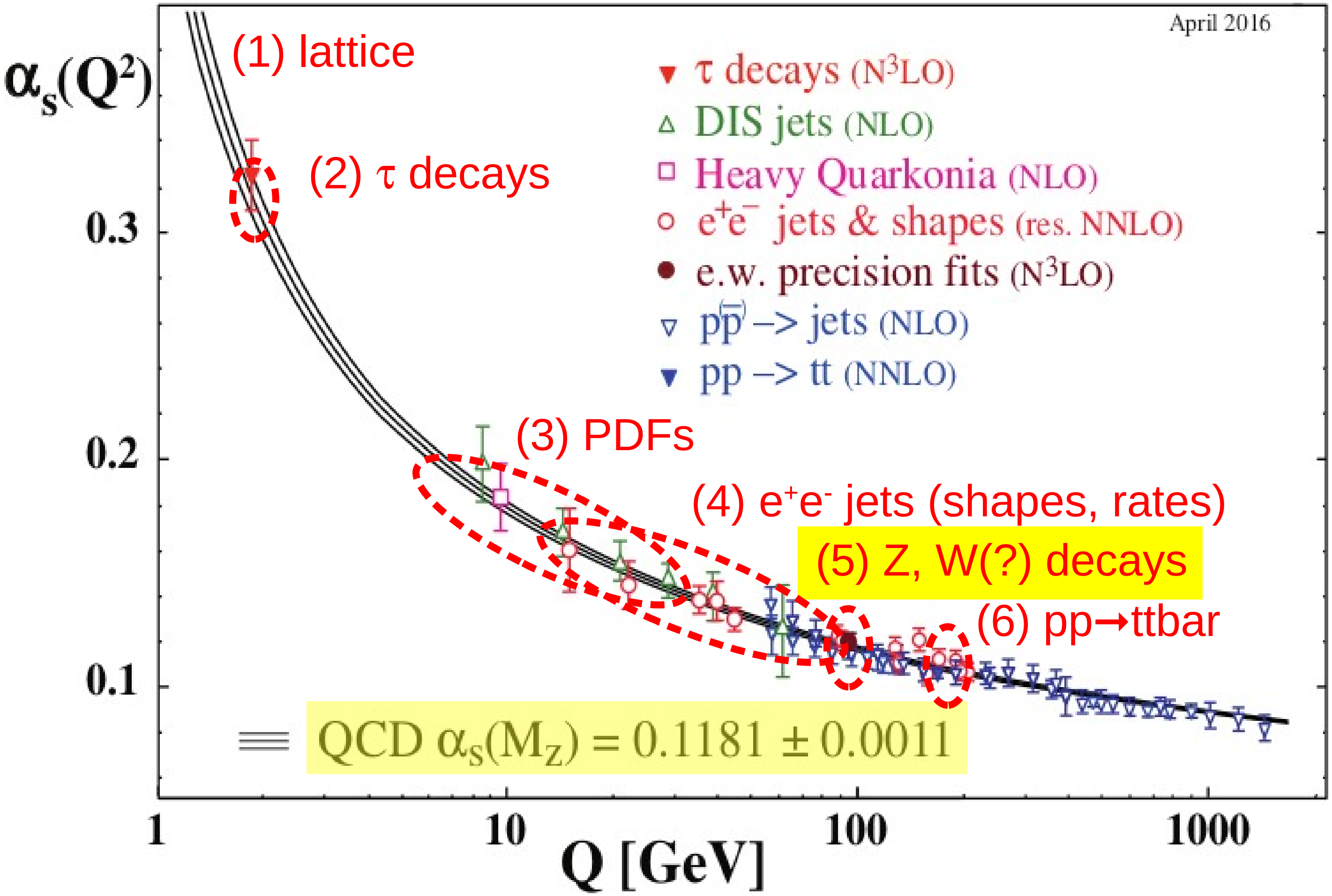


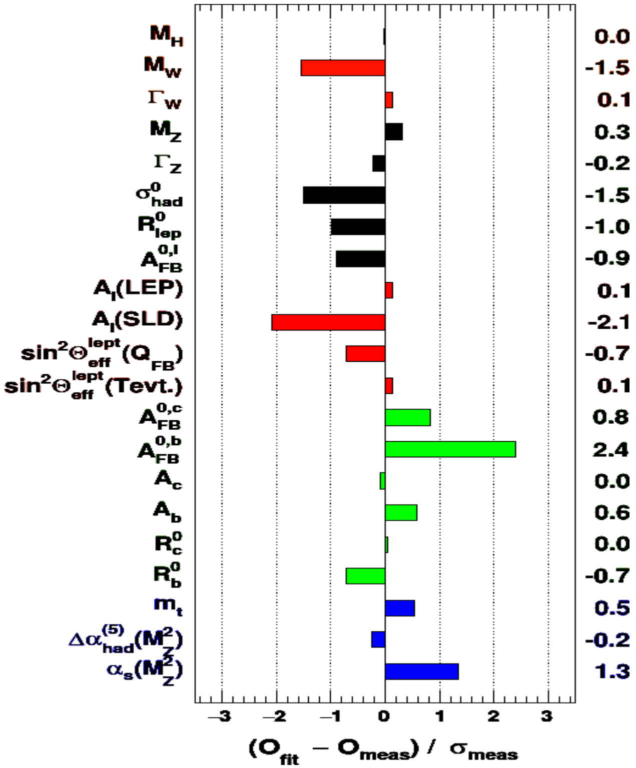
α_s from hadronic Z decays

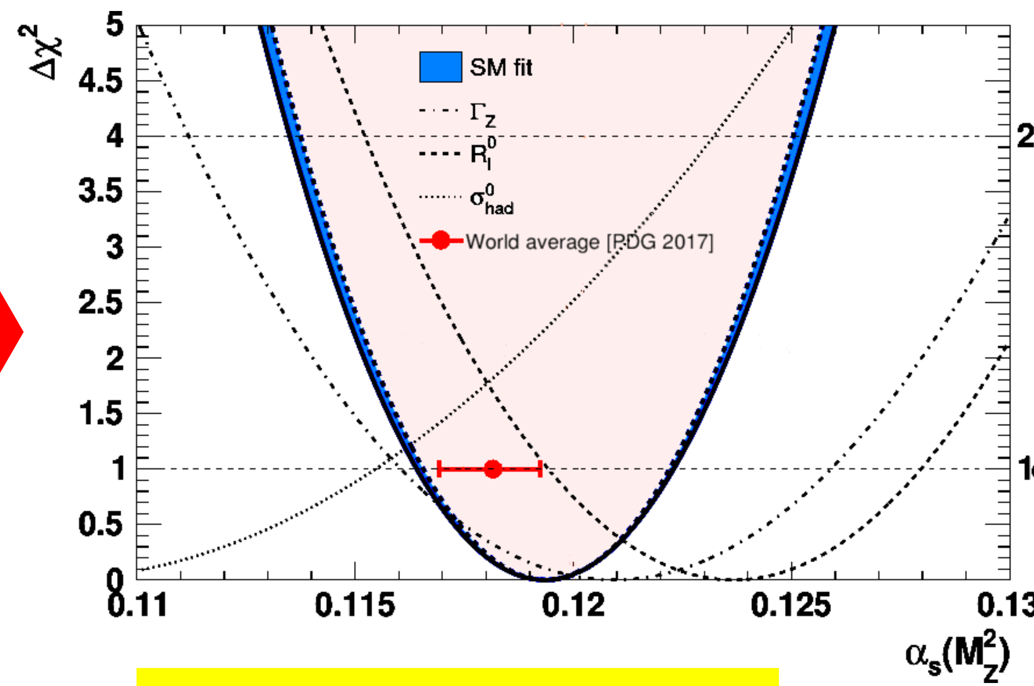
- Computed at N³LO: $R_l^0 \equiv \frac{\Gamma(Z \to \mathrm{h})}{\Gamma(Z \to l)} = R_Z^{\mathrm{EW}} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\mathrm{m}} + \delta_{\mathrm{np}})$
- Extraction from fits to all LEP data (mostly 3 Z-peak pseudo-observables):

Γ_Z = 2.4952±0.0023 GeV (±0.1%) $R_\ell^0 = \frac{\Gamma_{\mathrm{had}}}{\Gamma_\ell}, \ \sigma_{\mathrm{had}}^0 = \frac{12\pi}{m_Z}\frac{\Gamma_e \Gamma_{\mathrm{had}}}{\Gamma_Z^2}, \ \sigma_\ell^0 = \frac{12\pi}{m_Z}\frac{\Gamma_\ell^2}{\Gamma_Z^2}$

α_s (M_Z) = 0.1221 ± 0.0031 (±2.5%, PDG, LEP alone)

After Higgs discovery, α_s can be also directly determined from full fit of SM:



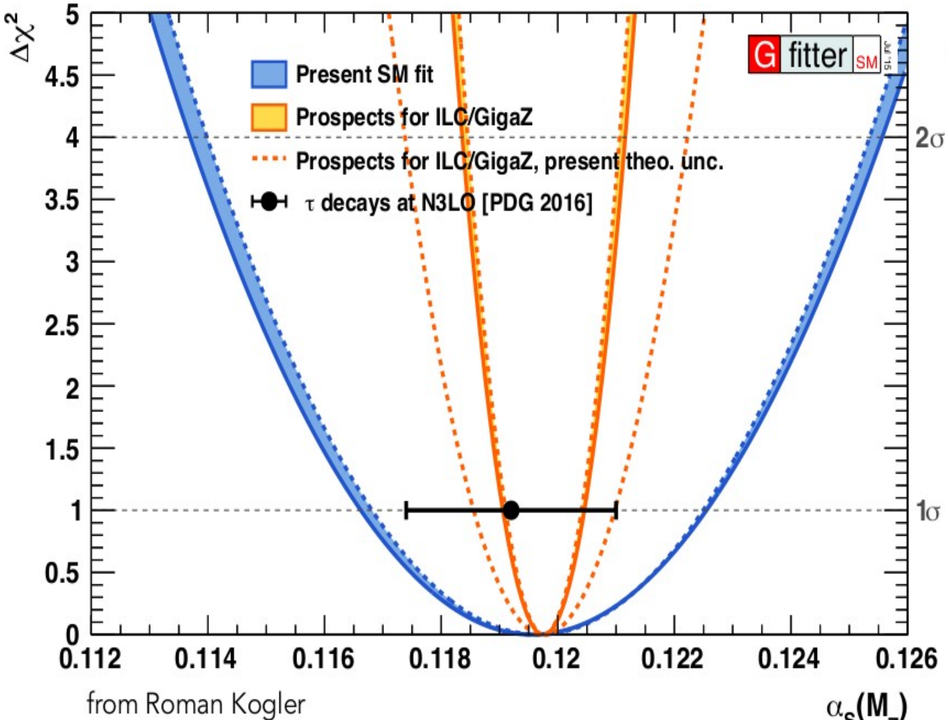


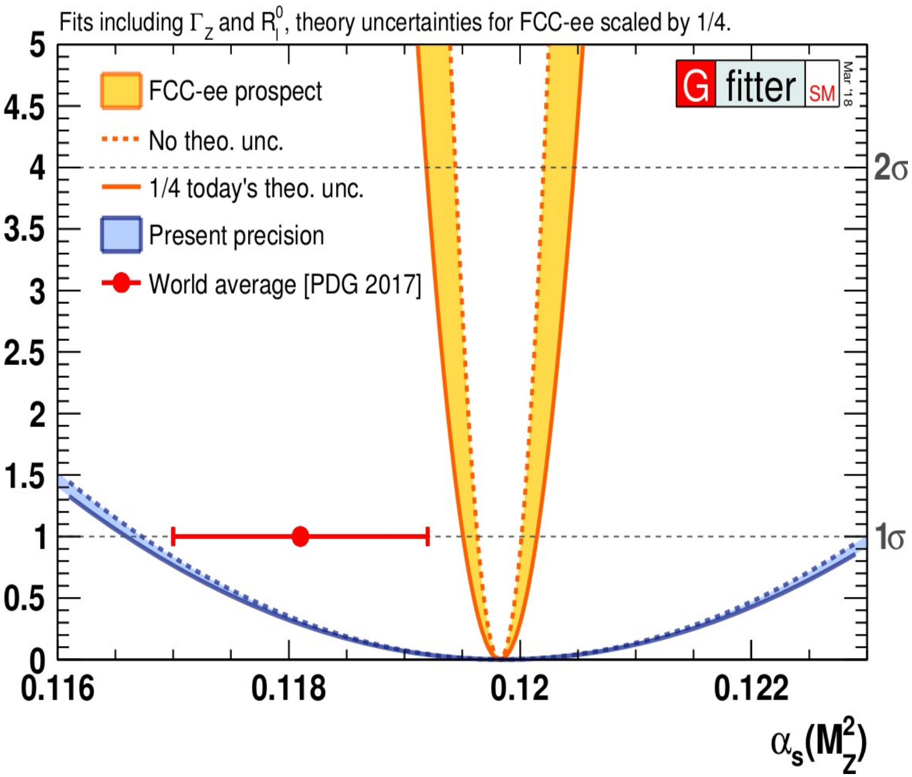
α_s (M_Z) = 0.1196 ± 0.0030 (±2.5%)

α_s from hadronic Z decays (future)

- Extraction from N^{3}LO fits to 3 $e^+e^- \to Z$ pseudo-observables:

Giga-Z at LHC (10^9 Z's) **FCC-ee** (10^{12} Z's)





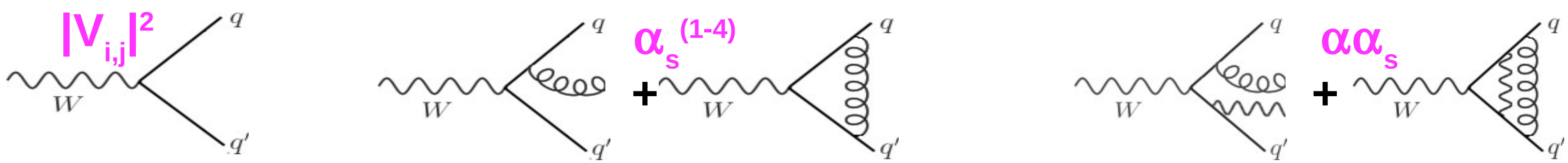
– Huge FCC-ee Z stats (×10^5 LEP) will lead to: **$\delta\alpha_s/\alpha_s$ < 0.2%**

– Compare to full SM-fit extraction (BSM?): Parallel reduction of parametric uncertainties ($\sin^2\theta_{eff}$, m_W, m_{top}) will be achieved too.

α_s from hadronic W width

- Width known at N[3]LO. Small sensitivity to α_s (only beyond Born):

$$\Gamma_{\rm W,had} = \frac{\sqrt{2}}{4\pi} G_{\rm F} m_{\rm W}^3 \sum_{\rm quarks\ i,j} |V_{\rm i,j}|^2 \left[1 + \sum_{\rm k=1}^{4} \left(\frac{\alpha_{\rm s}}{\pi}\right)^k + \delta_{\rm electroweak}(\alpha) + \delta_{\rm mixed}(\alpha\alpha_{\rm s}) \right]$$



- The hadronic W-boson decay width has not been used so far for α_s extraction because:
  - a complete N^{3}LO/NNLO formula with all computed corrections [1] was not available until recently [2] (albeit with a few approximations),
  - the 2% relative experimental uncertainty on Γ_W(hadronic) was significantly large compared to 0.1% of Γ_Z(hadronic).

- We recalculated Γ_W(hadronic) through the $\mathcal{O}(\alpha_s^4)$ or N^{3}LO formula using [2]:

$$\Gamma_{\rm W}(\text{hadronic}) = \frac{\sqrt{2} G_F m_W^3}{4\pi} \left[\sum_{i,j} |V_{i,j}|^2 \right] \left(1 + \sum_{k=1}^{4} \left(\frac{\alpha_s}{\pi}\right)^k + \delta_{\rm EW} + \delta_{\rm Mixed} \right) = \sum_{i=0}^{4} \Gamma^{(i)}_{\rm QCD} + \Gamma^{(1)}_{\rm EW} + \Gamma^{(2)}_{\rm Mixed}$$

where

- $\Gamma^{(k)}_{\rm QCD}$ is the leading order decay width and QCD corrections of order $\mathcal{O}(\alpha_s^k)$ and $k = 1, \ldots, 4$,
- $\Gamma^{(1)}_{\rm EW}$ electroweak corrections of order $\mathcal{O}(\alpha)$,
- $\Gamma^{(2)}_{\rm Mixed}$ mixed corrections of order $\mathcal{O}(\alpha\alpha_s)$.

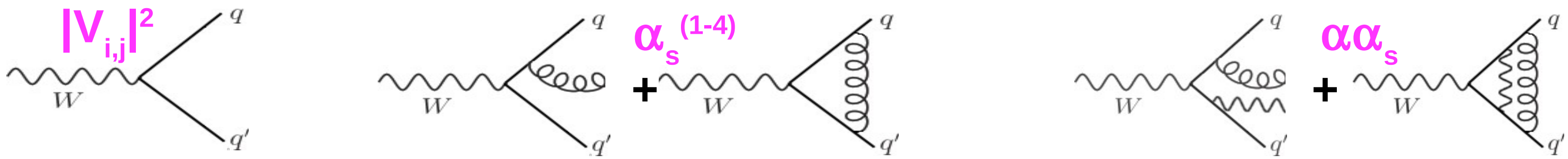
[1] - A. Denner, B. Kniehl, J. Kühn, K. Chetyrkin, ...

[2] - D. Kara, Nucl. Phys. B 877, 3 (2013)

Hadronic W width at N^{3}LO

- Width known at N^{3}LO. Small sensitivity to α_s (only beyond Born):

$$\Gamma_{\mathrm{W,had}} = \frac{\sqrt{2}}{4\pi} G_{\mathrm{F}} m_{\mathrm{W}}^3 \sum_{\text{quarks i,j}} |V_{\mathrm{i,j}}|^2 \left[1 + \sum_{k=1}^{4} \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^k + \delta_{\mathrm{electroweak}}(\alpha) + \delta_{\mathrm{mixed}}(\alpha\alpha_{\mathrm{s}})\right]$$



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- In our calculations we carry out the following improvements compared to previous works:
  1. implement finite quark masses in the dominant Γ_W(hadronic) terms: Born and first-order QCD corrections
  2. use N^{3}LO α_s running instead of LO (between m_W and m_Z)
  3. use current PDG world average values for parameters of the Standard Model (α_{QED}, G_{F}, m_{q}, m_ℓ, m_{W}, m_{Z}, m_{H}, *CKM matrix elements* $|V_{i,j}|$)
  4. determination of associated theoretical and parametric uncertainties.

Hadronic W width at N^{3}LO

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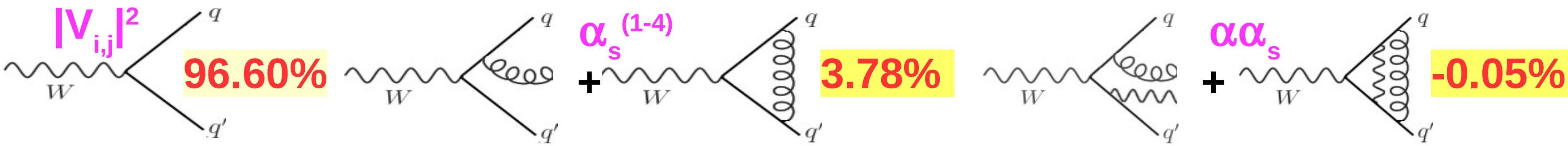
[EWK: -0.35%]



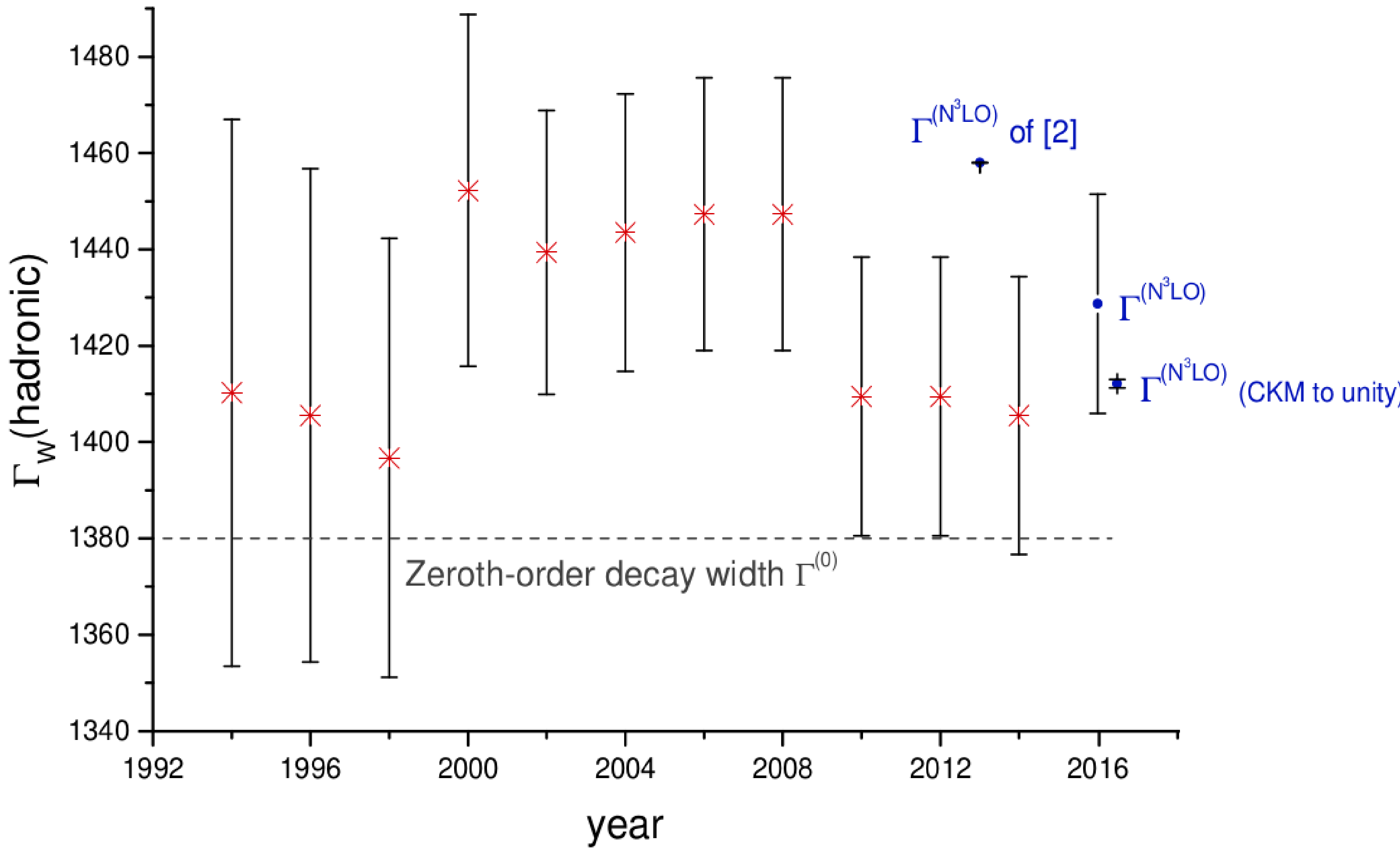
- Derivation of partial and total hadronic W widths:

Partial widths (MeV)	$\Gamma^{(0)}$	$\Gamma^{(1)}_{\mathrm{QCD}}$	$\Gamma^{(2)}_{\mathrm{QCD}}$	$\Gamma^{(3)}_{\mathrm{QCD}}$	$\Gamma^{(4)}_{\mathrm{QCD}}$	Γ_{ewk}	Γ_{mixed}	$\Gamma^{\mathrm{W}}_{\mathrm{had}}$
$W \to qq'$ (exp. V_{ij})	1379.851	52.931	2.857	−0.992	−0.238	−5.002	−0.755	$1428.65 \pm 22.40_{\mathrm{par}} \pm 0.04_{\mathrm{th}}$
$W \to qq'$ ($V_{ij}V_{jk} = \delta_{ik}$)	1363.197	52.291	2.822	−0.980	−0.235	−4.942	−0.746	$1411.40 \pm 0.96_{\mathrm{par}} \pm 0.04_{\mathrm{th}}$
$W \to qq'$ (exp. V_{ij}) [5]	1408.980	54.087	2.927	−1.018	−0.245	−5.132	−0.779	$1458.820 \pm 0.006_{\mathrm{th}}$
$W \to qq'$ ($V_{ij}V_{jk} = \delta_{ik}$) [5]	1363.640	52.346	2.833	−0.985	−0.237	−4.940	−0.748	$1411.910 \pm 0.006_{\mathrm{th}}$

- Careful evaluation of parametric ($V_{i,j}$, m_W) & theoretical uncertainties:
  – Parametric uncertainty: ±22.40 MeV (dominated by V_{cs})
  ±0.96 MeV (CKM unitarity, dominated by m_W)
  – Higher-order QCD corrections: ±0.019 MeV (diffs. N^{3}LO vs N^{4}LO for Γ_Z)
  – Higher-order EWK, mixed corrections: ±0.035 MeV (from D.Kara NPB877(2013)683)
  – Others (non-pQCD $(\Lambda_{QCD}/m_W)^4$, finite q masses above LO, ren. scheme,…): negligible

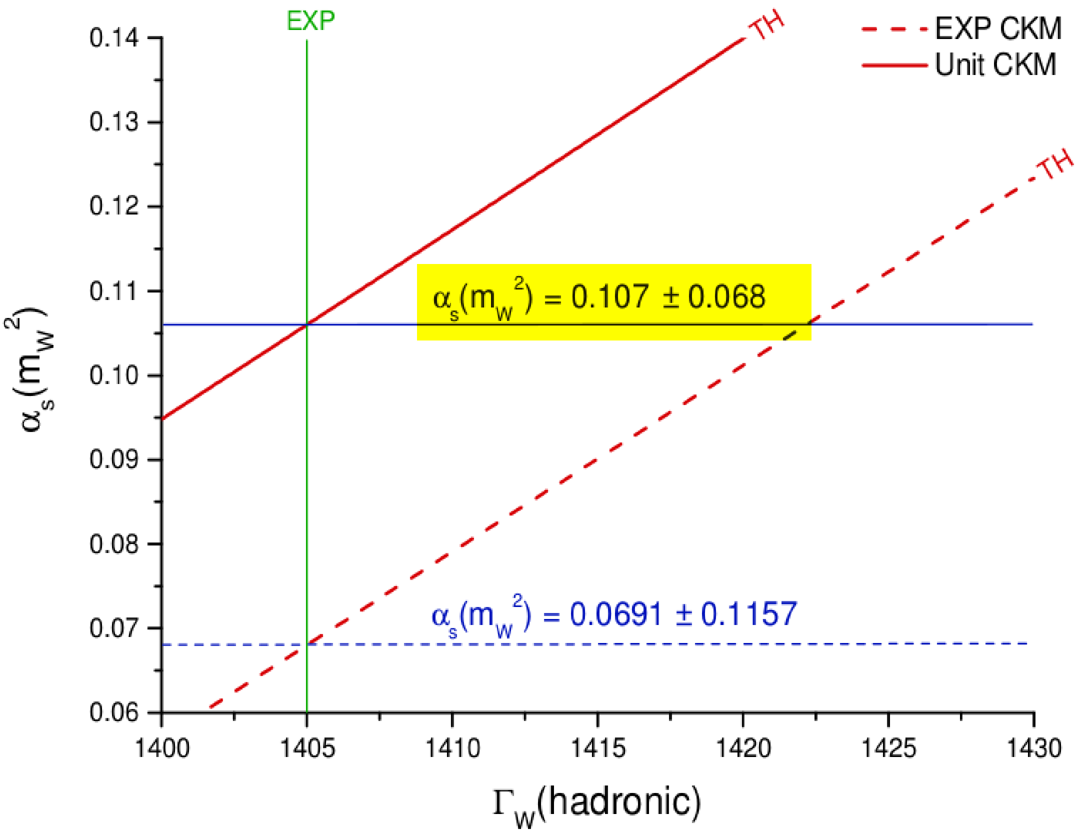
Hadronic W width: Data vs. N³LO

- Evolution of the PDG world average value of Γ_W^{EXP}(hadronic) (current value is (1405 ± 29) MeV) by year compared to theoretically predicted decay widths.



α_s from hadronic W width at N³LO

- Using the $\mathcal{O}(\alpha_s^4)$ W-boson decay width formula we can extract α_s by comparing it to the experimental value Γ_W^{EXP}(hadronic) $= (1405 \pm 29)$ MeV.



$\Rightarrow$ Current large parametric ($\pm$ 23 MeV) and experimental ($\pm$29 MeV) uncertainties on Γ_W(hadronic) propagate into a huge α_s uncertainty $\sim$ 60%.

Experimental priorities should be:

- measure $|V_{cs}|$ with better precision (current 1.6%),
- significantly reduce uncertainty of Γ_W(hadronic) measurement to a few MeV,
- reduce m_W uncertainty (now it propagates to $\pm$0.8 MeV on Γ_W(hadronic)).

α_s extraction method	$\alpha_s(m_W^2)$	$\alpha_s(m_Z^2)$
Γ_{had}^W (experimental CKM)	$0.069 \pm 0.065_{exp} \pm 0.050_{par}$	$0.068 \pm 0.064_{exp} \pm 0.050_{par}$
Γ_{had}^W (CKM unitarity)	$0.107 \pm 0.066_{exp} \pm 0.002_{par} \pm 0.001_{th}$	$0.105 \pm 0.065_{exp} \pm 0.002_{par} \pm 0.001_{th}$

Hadronic W branching ratio at N³LO

- Γ_W(hadronic) has a 2% experimental uncertainty, BR_W(hadronic) has an uncertainty of 0.4%.

  $\Rightarrow$ We try $\text{BR}_W(\text{hadronic}) = \frac{\Gamma_W(\text{hadronic})}{\Gamma_W(\text{total})}$, $R_W \equiv \text{BR}_W(\text{had})/\text{BR}_W(\text{lep}) = \text{BR}_W(\text{had})/(1 - \text{BR}_W(\text{had}))$

- For the total decay width Γ_W(total) we use the ZFitter NNLO (includes up to $\mathcal{O}(\alpha_s^3)$ QCD, $\mathcal{O}(\alpha)$ electroweak and $\mathcal{O}(\alpha\alpha_s)$ mixed corrections) fitted result by [3] parametrized as

  $$\Gamma_W(\text{total}) = G_W^0 m_W^3 \quad \text{and} \quad G_W^0 = 4.0279 \times 10^{-6}(1 + 0.00095 x_H - 0.0024 x_H^2 + 0.0016 x_H^3 + 000065 x_s)\ \text{GeV}^{-2}$$

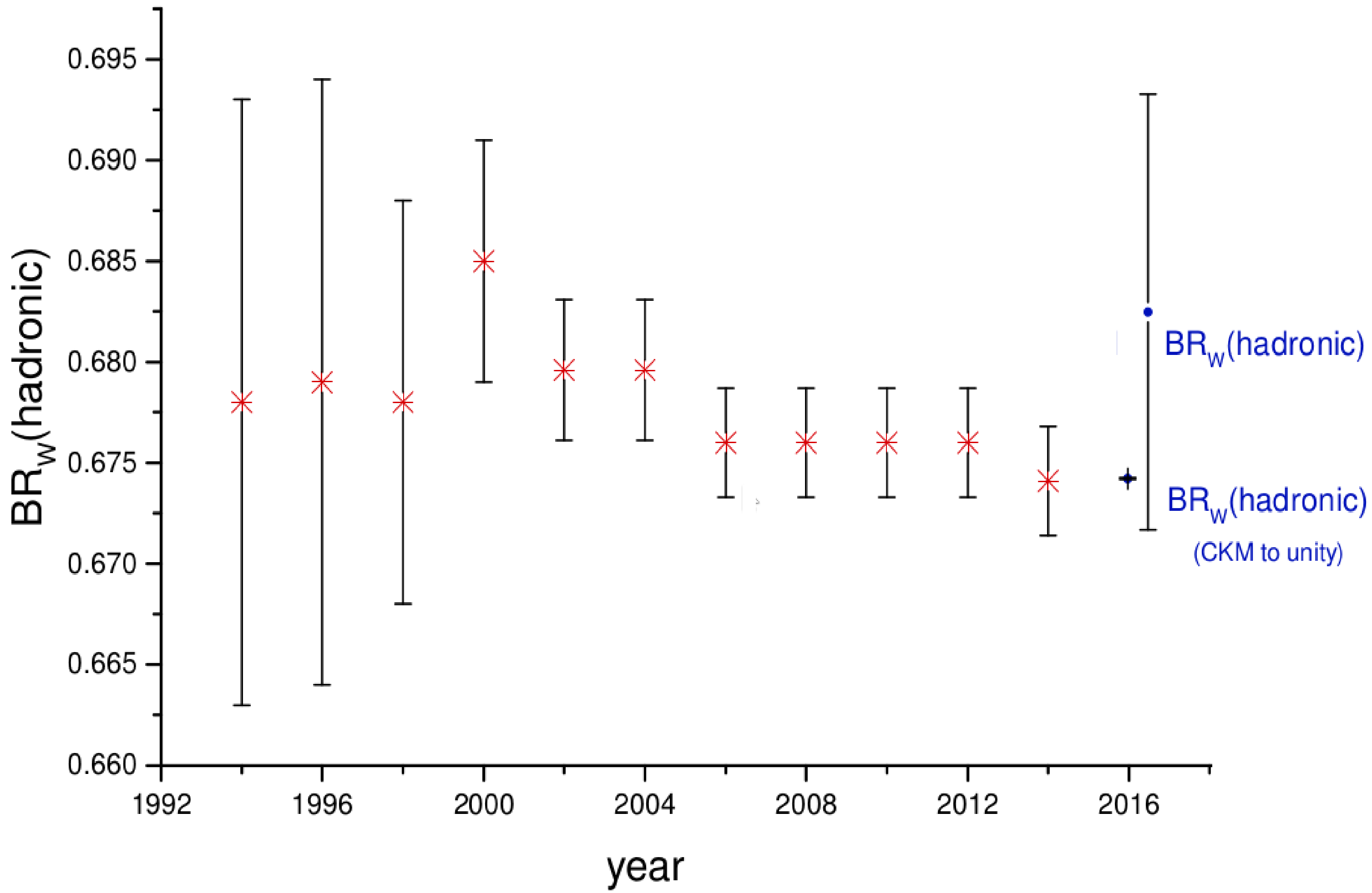
  where $x_s = f(\alpha_s)$, $x_H = f(m_H)$. [3] - G. C. Cho, K. Hagiwara, Y. Matsumoto, D. Nomura, JHEP 111, 068 (2011)

- We also computed the associated parametric uncertainties as done for Γ_W(hadronic).

Observable	(Full calculation)	$(V_{ij}V_{jk} = \delta_{ik})$	Experimental value
Γ^W_{had} (MeV)	$1428.65 \pm 22.40_{par} \pm 0.04_{th}$	$1411.40 \pm 0.96_{par} \pm 0.04_{th}$	1405 ± 29
Γ^W_{tot} (MeV)	$2093.4 \pm 1.2_{par} \pm 0.8_{th}$	–	2085 ± 42
$\mathcal{B}^W_{had}$	$0.682 \pm 0.011_{par}$ $(\pm 0.0002_{th})$	$0.6742 \pm 0.0002_{th} \pm 0.0001_{par}$	0.6741 ± 0.0027
R_W	$2.15 \pm 0.11_{par}$ $(\pm 0.002_{th})$	$2.069 \pm 0.002_{th} \pm 0.001_{par}$	2.068 ± 0.025

Hadronic W branching ratio: Data vs. N^3LO

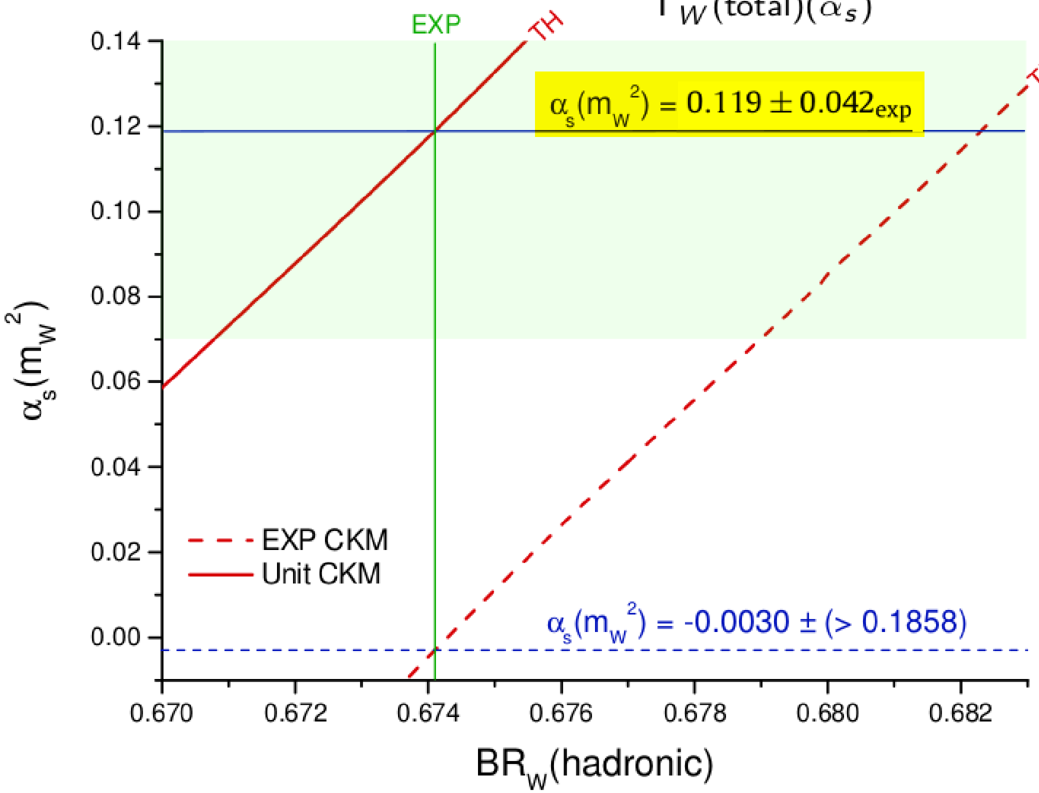
- Evolution of the PDG world average value of BR_W(hadronic) (current value is 0.6741 ± 0.0027) by year compared to theoretically predicted decay widths.



α_s from hadronic W branching ratio at N³LO

➧ Extract α_s by comparing the theoretical hadronic branching ratio formula to the experimental world average value

$$\frac{\Gamma_W(\text{hadronic})(\alpha_s)}{\Gamma_W(\text{total})(\alpha_s)} = \text{BR}_W(\text{hadronic})^{\text{EXP}}.$$



⇒ Setting the CKM matrix to unit matrix instead of using experimental values, we can extract α_s with 35% uncertainy.

To extract α_s with a higher precision:

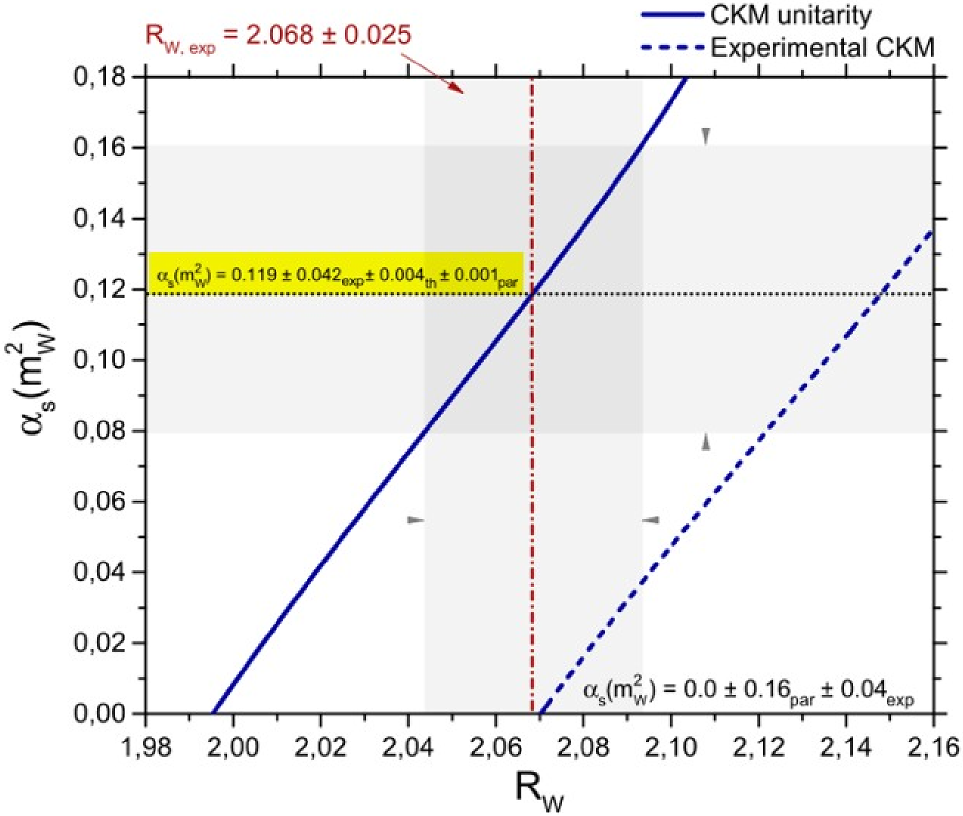
- reduce the uncertainty of $|V_{cs}|$ as mentioned earlier,
- reduce the uncertainty of m_W (measured to 0.02%) which becomes dominant once the $|V_{cs}|$ uncertainty is reduced below 0.05%,
- measure BR_W(hadronic) with a better precision than today (0.4% now).

α_s extraction method	$\alpha_s(m_W^2)$	$\alpha_s(m_Z^2)$
$\mathcal{B}^W_{had}$ (experimental CKM)	$0.0 \pm 0.04_{exp} \pm 0.16_{par}$	$0.0 \pm 0.04_{exp} \pm 0.16_{par}$
$\mathcal{B}^W_{had}$ (CKM unitarity)	$0.119 \pm 0.042_{exp} \pm 0.004_{th} \pm 0.001_{par}$	$0.117 \pm 0.042_{exp} \pm 0.004_{th} \pm 0.001_{par}$

α_s from hadronic W had./lep. BR at N^{3}LO

- Extract α_s by comparing the theoretical hadronic branching ratio formula to the experimental world average value

$$R_W \equiv \mathcal{B}^W_{had}/\mathcal{B}^W_{lep} = \mathcal{B}^W_{had}/(1 - \mathcal{B}^W_{had})$$



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- measure BR_W(hadronic) with a better precision than today (0.4% now).

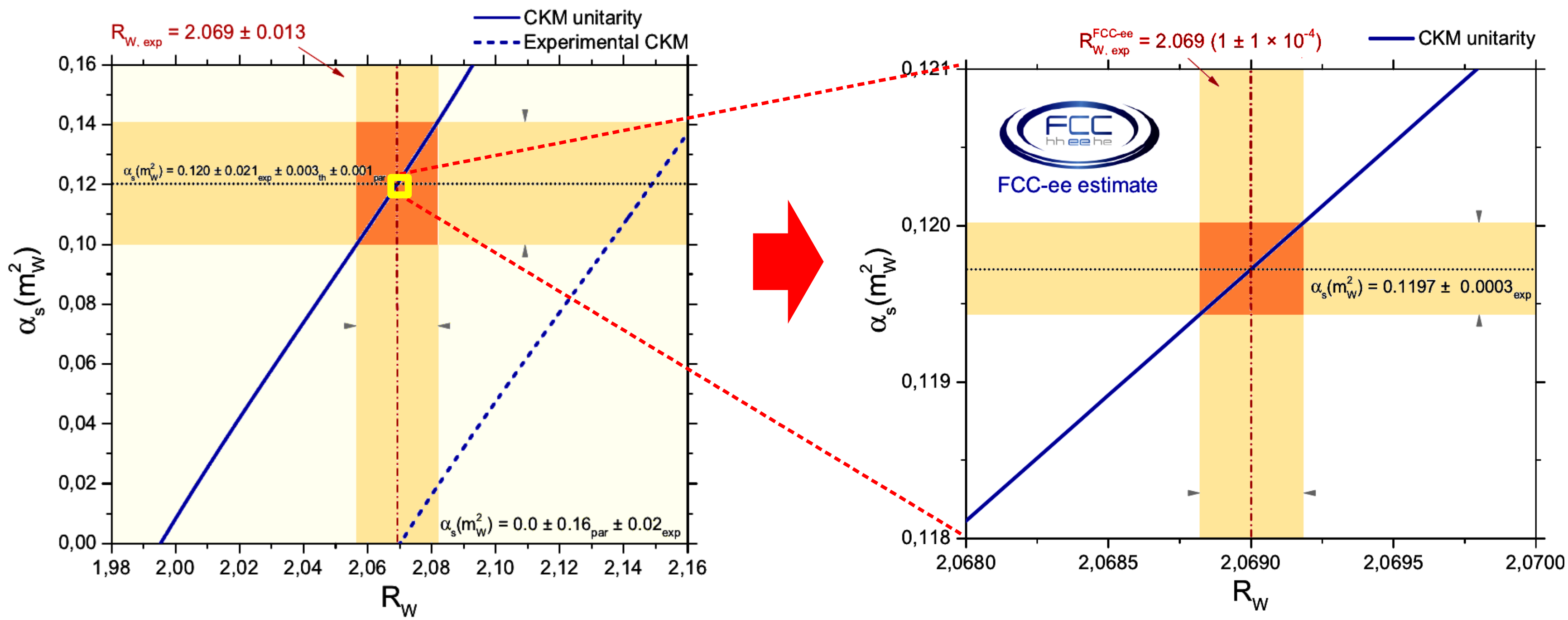
α_s extraction method	$\alpha_s(m^2_W)$	$\alpha_s(m^2_Z)$
R_W (experimental CKM)	$0.0 \pm 0.04_{exp} \pm 0.16_{par}$	$0.0 \pm 0.04_{exp} \pm 0.16_{par}$
R_W (CKM unitarity)	$0.119 \pm 0.042_{exp} \pm 0.004_{th} \pm 0.001_{par}$	$0.117 \pm 0.042_{exp} \pm 0.004_{th} \pm 0.001_{par}$

Summary: α_s from hadronic W decays

- Computed at N^3LO: $\Gamma_{\rm W,had} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\rm quarks\ i,j} |V_{i,j}|^2 \left[1 + \sum_{k=1}^{4} \left(\frac{\alpha_s}{\pi}\right)^k + \delta_{\rm electroweak}(\alpha) + \delta_{\rm mixed}(\alpha\alpha_s)\right]$
- LEP: Γ_W = 1405±29 MeV (±2%), BR_W = 0.6741±0.0027 (±0.4%)

  Extraction with large exp. & parametric ($CKM\ V_{cs}$) uncertainties today:

$\alpha_s(M_z)$ = 0.117 ± 0.040 (±35%)



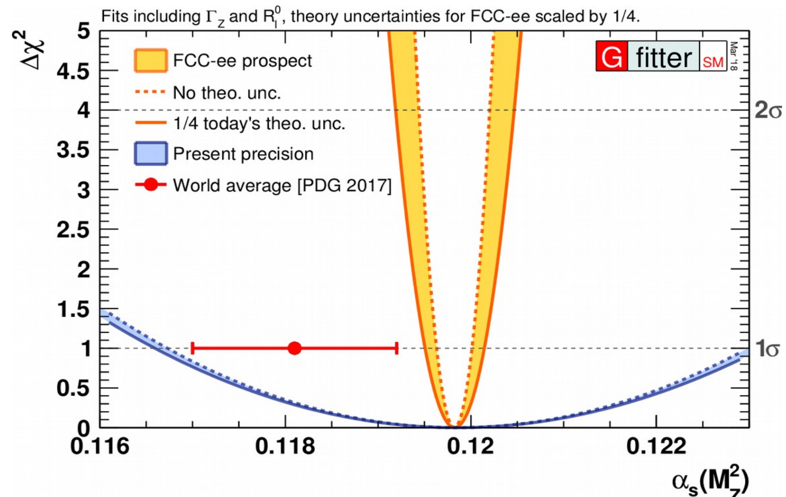
- FCC-ee: – Huge W stats (×10^4 LEP) will lead to: **$\delta\alpha_s/\alpha_s$ < 0.3%**

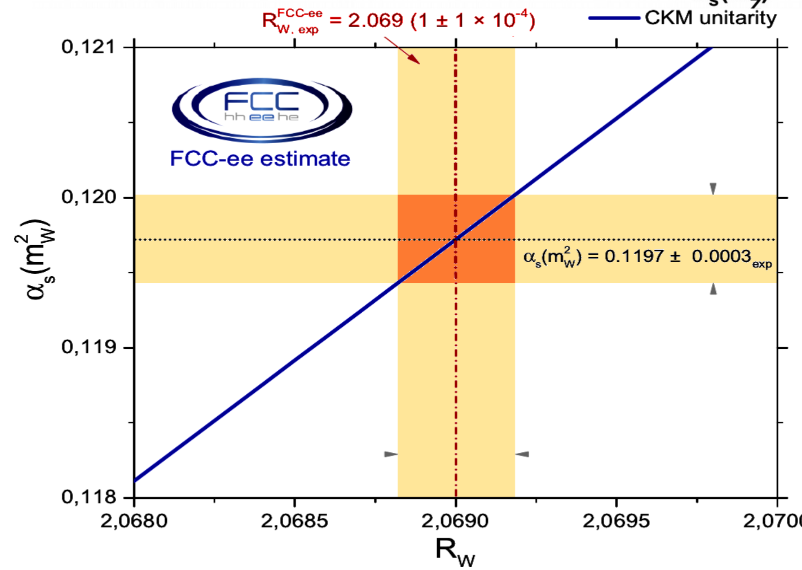
  – TH (param.) uncertainty: $|\delta V_{cs}|$ to be significantly improved (10^{-4})

Summary: Future α_s with permille precision

- World-average QCD coupling at N[2,3]LO:
  - *Determined today from 6 observables with ~1% uncertainty (least well-known coupling).*
  - *Impacts all LHC (& FCC-hh) QCD x-sections & decays.*
  - *Role beyond SM: GUT, EWK vacuum stability New colored sectors?*
- New NNLO extractions/updates: PDF fits, e-p jets, pp→ttbar, jets
- Reduction of hadronization & resummation uncertainties:
  - New TH developments needed
  - New precise e^+e^- data needed
- Other NLO(*) extraction methods proposed. Need TH work towards NNLO accuracy.

- ~0.1% uncertainty, combining Z,W hadronic decays ONLY possible with machine like FCC-ee with $10^{12,8}$ Z,W:





Backup slides

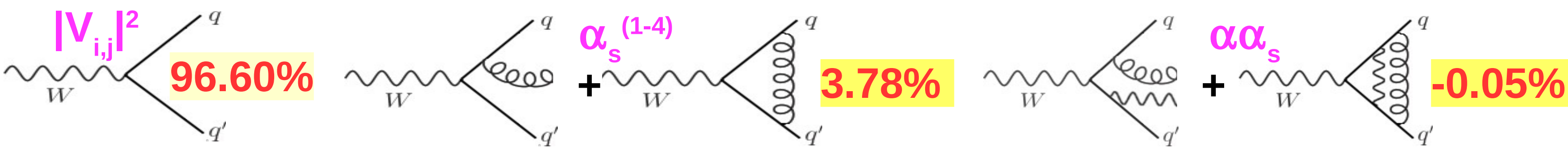
α_s from hadronic W decays

[D.d'E, M.Srebre, arXiv:1603.06501]

- Width (BR) known at N^{3}LO (NNLO). Small sensitivity to α_s (beyond Born)

$$\Gamma_{\rm W,had} = \frac{\sqrt{2}}{4\pi} G_{\rm F} m^3_{\rm W} \sum_{\rm quarks\ i,j} |V_{\rm i,j}|^2 \left[1 + \sum_{k=1}^{4} \left(\frac{\alpha_{\rm s}}{\pi}\right)^k + \delta_{\rm electroweak}(\alpha) + \delta_{\rm mixed}(\alpha\alpha_{\rm s}) \right]$$

[EWK: -0.35%]



- TH improvements: finite quark-mass effects included (LO), updated PDG parameters, careful evaluation of parametric ($V_{i,j}$, m_W) & theoretical uncert.

- Calculation dominated by ±1.5% parametric (mostly V_{cs}) uncertainty:

Γ_W(MeV) = $1428.67 \pm 22.40_{\rm (par)} \pm 0.04_{\rm (th)}$ (exp. CKM)

$1411.40 \pm 0.96_{\rm (par)} \pm 0.04_{\rm (th)}$ (CKM=1)

$BR_W = \Gamma_W/\Gamma_{tot}$ = $0.6820 \pm 0.0110_{\rm (par)} \pm 0.0002_{\rm (th)}$ (exp. CKM)

$0.6742 \pm 0.0001_{\rm (par)} \pm 0.0002_{\rm (th)}$ (CKM=1)

- TH uncertainty (missing α_s^5 terms, non-pQCD $(\Lambda_{QCD}/m_W)^4$ power corrs., finite quark masses beyond LO, CKM matrix renorm. scheme): ±0.03%