QCD coupling: scheme variations. Hadronic tau decays

Ramon Miravitllas Mas
in collaboration with D. Boito and M. Jamin

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Table of contents

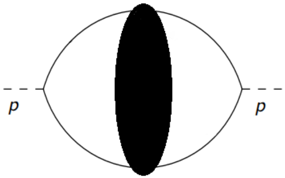
Vector two-point correlator

- Vector two-point correlator: Relevant for hadronic τ decays.

$$\Pi_{\mu\nu}(p) = \int \mathrm{d}^4 x\, e^{ipx} \langle \Omega | J_\mu(x) J_\nu^\dagger(0) | \Omega \rangle$$

where $J_\mu = :\bar{q}\gamma_\mu q':$

- Full QCD:



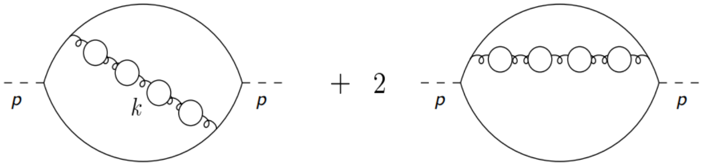
Vector two-point correlator

- Vector two-point correlator: Relevant for hadronic τ decays.

$$\Pi_{\mu\nu}(p) = \int \mathrm{d}^4x\, e^{ipx} \langle \Omega | J_\mu(x) J_\nu^\dagger(0) | \Omega \rangle$$

  where $J_\mu = :\bar{q}\gamma_\mu q':$

- Large-β_0 approximation:



Scheme dependence in the large-β_0 approximation

- Gluon chain:

$$D_{\mu\nu}(k^2) = \frac{-i}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+\Pi_0(k^2)}$$

- Dimensional regularisation:
  $\Pi_0(k^2) = \alpha_s\left[1/\epsilon + \gamma_E - \log(4\pi) + \log(-k^2/\mu^2) - 5/3\right].$

Scheme dependence in the large-β_0 approximation

- Gluon chain:

$$D_{\mu\nu}(k^2) = \frac{-i}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+\Pi_0(k^2)}$$

- Dimensional regularisation:
  $\Pi_0^R(k^2) = \alpha_s^R\left[\log(-k^2/\mu^2) + C\right]$.
- In the $\overline{\text{MS}}$ scheme, $C = -5/3$.

Observation

We are free to choose any scheme, parametrized by the constant C.

Scheme dependence in the large-β_0 approximation

- The Adler function

$$D(Q^2) = -Q^2 \frac{\mathrm{d}\Pi(Q^2)}{\mathrm{d}Q^2}, \quad \Big(\Pi_{\mu\nu}(p) = (p_\mu p_\nu - g_{\mu\nu}p^2)\Pi(p^2)\Big)$$

  is scheme and scale invariant (physical quantity).
- We define

$$a = \alpha_s/\pi\,, \quad (\text{scale } \mu^2 = Q^2).$$

  With this choice, all Q^2 dependence in D comes exclusively from the coupling.
- In the large-β_0 approximation (Beneke 1993; Broadhurst 1993),

$$D(Q^2) = \frac{2}{\beta_1}\int_0^\infty \mathrm{d}u\, e^{-2u/(\beta_1 a)} B[D](u) + \ldots\,,$$

$$B[D](u) = 8C_F \frac{e^{-Cu}}{2-u}\sum_{k\geq 0}\frac{(-1)^k k}{[k^2-(1-u)^2]^2}\,.$$

Scheme dependence in the large-β_0 approximation

- Inside the Laplace integral, we have the combination

$$\exp\left(-\frac{2u}{\beta_1 a} - Cu\right) = \text{independent of } C.$$

- Thus, the C dependence of the coupling has to be

$$\frac{1}{a(C)} = \frac{1}{a(C=0)} - \frac{\beta_1}{2} C$$

in the large-β_0 approximation.

Goal

Define a coupling in full QCD with similar scheme properties to the above large-β_0 coupling.

The C-scheme coupling

- Scale invariant QCD Λ parameter:

$$\Lambda = Q e^{-1/(\beta_1 a)} a^{-\beta_2/\beta_1^2} \exp\left(\int_0^a \frac{\mathrm{d}a}{\tilde{\beta}(a)}\right),$$

where $1/\tilde{\beta}$ is the inverse of the β function, but with subtracted singularities at $a = 0$.

- General scheme transformation (Celmaster, Gonsalves 1979):

$$a' = a + c_1 a^2 + \mathcal{O}(a^3) \quad \longrightarrow \quad \Lambda' = \Lambda e^{c_1/\beta_1}.$$

- We consider the following definition:

The C-scheme coupling

$$\frac{1}{\hat{a}} + \frac{\beta_2}{\beta_1}\log(\hat{a}) = \beta_1 \log\left(\frac{Q}{\Lambda}\right) + \frac{\beta_1}{2} C.$$

- Matching between the two sides ensures a power-like perturbative relation: $\hat{a} = \sum_{n\geq 1} c_n a^n$.

The C-scheme coupling

The C-scheme coupling

$$\frac{1}{\hat{a}} + \frac{\beta_2}{\beta_1}\log(\hat{a}) = \beta_1 \log\left(\frac{Q}{\Lambda}\right) + \frac{\beta_1}{2} C\,.$$

- Simple scale and scheme evolution

$$-Q\frac{\mathrm{d}\hat{a}}{\mathrm{d}Q} = -2\frac{\mathrm{d}\hat{a}}{\mathrm{d}C} = \frac{\beta_1 \hat{a}^2}{1 - \frac{\beta_2}{\beta_1}\hat{a}}\,.$$

- Scheme variations can be compensated by scale variations:

$$\mu_1/\mu_2 = e^{C_1 - C_2}.$$

  They are equivalent transformations.

- β_1 and β_2 are scheme independent $\longrightarrow$ the β function of $\hat{a}$ is also scheme independent.

Borel models

Goal

Determine the large order behaviour of perturbative expansions in order to improve their numerical accuracy.

- Perturbative expansions require exponentially suppressed corrections:

$$D\left(Q^2\right) \sim \sum_{n\geq 0} b_n a^{n+1} \pm i\, b\, e^{-S/a} a^{-\lambda} \sum_{n\geq 0} c_n a^{n+1} + \ldots$$

  Conventionally written as an Operator Product Expansion.
- Connection between the high n behaviour of the b_n and the first few c_n:

$$b_n = b\,\frac{(-1)^{n+1}}{\pi}\frac{\Gamma(n+\lambda)}{(-S)^{n+\lambda}}\left[1 + \frac{-S\,c_1}{n+\lambda-1} + \mathcal{O}\left(\frac{1}{n^2}\right)\right],$$

$$\text{Correction} = \pm i\, b\, e^{-S/a} a^{-\lambda}\left[a + c_1 a^2 + \mathcal{O}\left(a^2\right)\right].$$

Borel models

$$b_n = b \, \frac{(-1)^{n+1}}{\pi} \frac{\Gamma(n+\lambda)}{(-S)^{n+\lambda}} \left[1 + \frac{-S\, c_1}{n+\lambda-1} + \mathcal{O}\left(\frac{1}{n^2}\right) \right],$$

$$\pm\, i\, b\, e^{-S/a} a^{-\lambda} \left[a + c_1 a^2 + \mathcal{O}\left(a^2\right) \right].$$

Structure of the OPE fixes:

- S (position of the singularity in the Borel plane),
- λ (order of the singularity),
- c_1,
- but NOT b (residue of the singularity).

Strategy (Beneke, Jamin 2008)

Use the first few known coefficients b_0, b_1, b_2, $b_3 \ldots$ to fit b.

- This strategy works better if the large n asymptotic behaviour of the b_n sets in for low enough n.

Borel models

- Compute Borel transforms with respect to $\hat{a}$ instead of a.
- The residue b then will depend on the C choice.
- For example, in the large-β_0 approximation:

$$B[D](u) = 8C_F \frac{e^{-Cu}}{2-u} \sum_{k\geq 0} \frac{(-1)^k k}{[k^2-(1-u)^2]^2} \,.$$

$b \sim e^{-CS}$: $C > 0$ enhances negative poles, while $C < 0$ enhances positive poles.

Strategy

Choose C so that the large n asymptotic behaviour of the b_n sets in for low n.

Thanks!