

# *Theory uncertainties for inclusive cross-section*

**Sven-Olaf Moch**

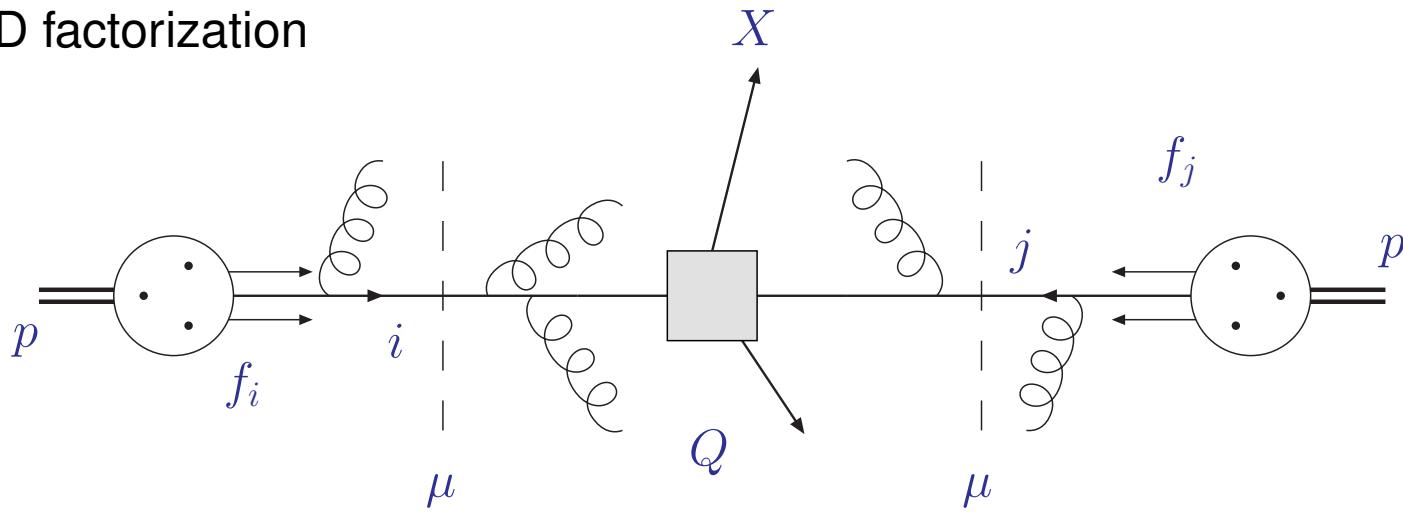
*Universität Hamburg*

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LHC TOP WG meeting, Geneva, Nov 20, 2018

# Top-quark inclusive cross sections

- QCD factorization



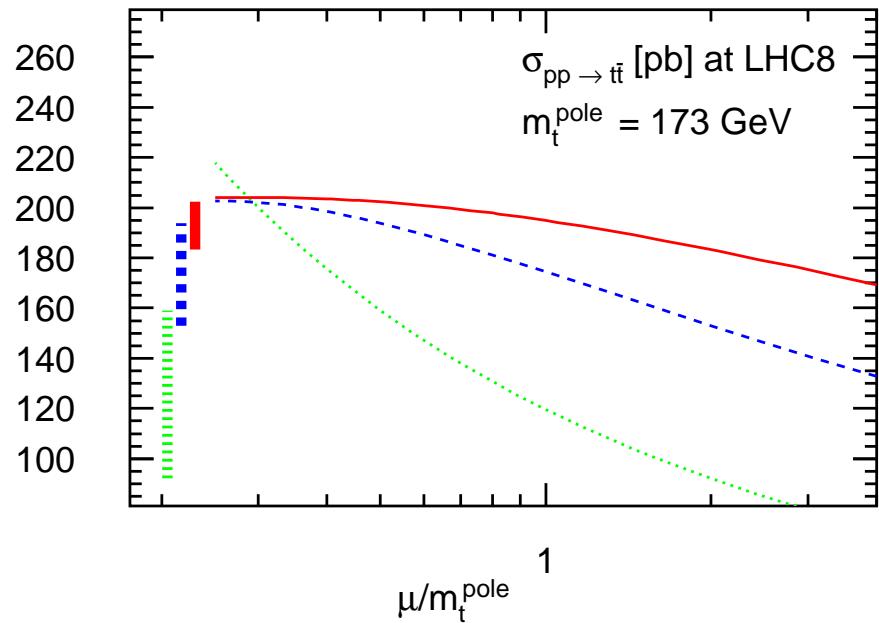
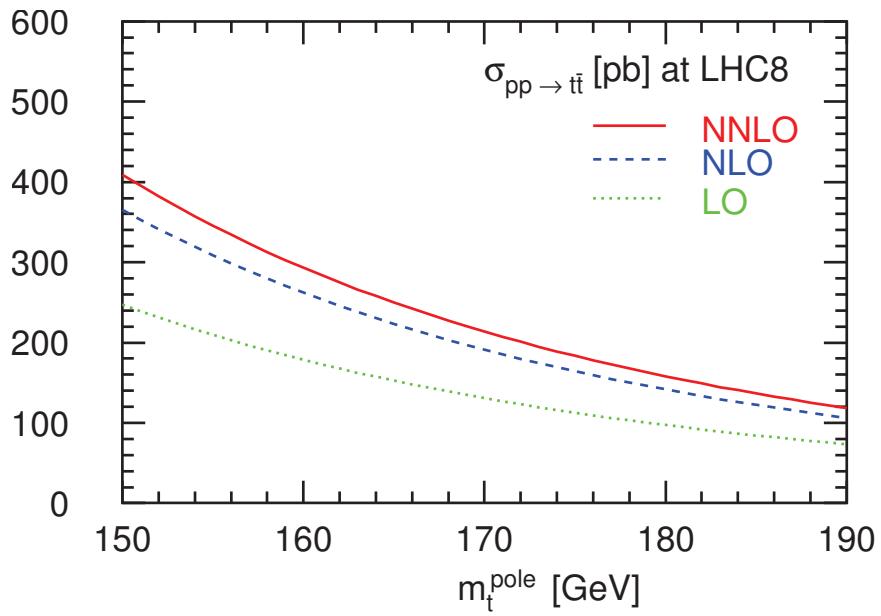
$$\begin{aligned}\sigma_{pp \rightarrow X} &= \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \underbrace{\hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)}_{\hat{\sigma}_{ij \rightarrow X}^{(0)} + \alpha_s \hat{\sigma}_{ij \rightarrow X}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij \rightarrow X}^{(2)} + \dots} \\ &= \hat{\sigma}_{ij \rightarrow X}^{(0)} + \alpha_s \hat{\sigma}_{ij \rightarrow X}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij \rightarrow X}^{(2)} + \dots\end{aligned}$$

- Hard parton cross section  $\hat{\sigma}_{ij \rightarrow X}$  calculable in perturbation theory
  - known to NLO, NNLO, ... ( $\mathcal{O}(\text{few}\%)$  theory uncertainty)
- Non-perturbative parameters: parton distribution functions  $f_i$ , strong coupling  $\alpha_s$ , particle masses  $m_X$ 
  - renormalization scheme for  $\alpha_s$  and particle masses  $m_X$

# Total cross section

## Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13

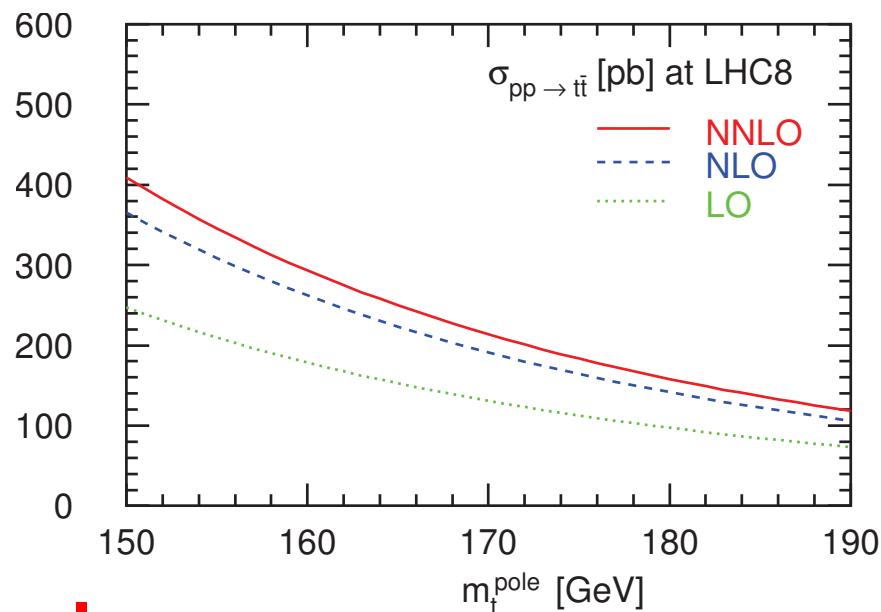


- NNLO perturbative corrections (e.g. at LHC with  $\sqrt{s} = 8$  TeV)
- $\overline{\text{MS}}$  renormalization scheme for  $\alpha_s$ , on-shell scheme for  $m_t$ 
  - $K$ -factors:  $K_{\text{LO} \rightarrow \text{NLO}} = 1.46$  and  $K_{\text{NLO} \rightarrow \text{NNLO}} = 1.12$
  - scale stability at NNLO of  $\mathcal{O}(\pm 5\%)$
  - point of minimal sensitivity at low scales  $\mu \sim \mathcal{O}(m_t/4) \sim \mathcal{O}(45)$  GeV

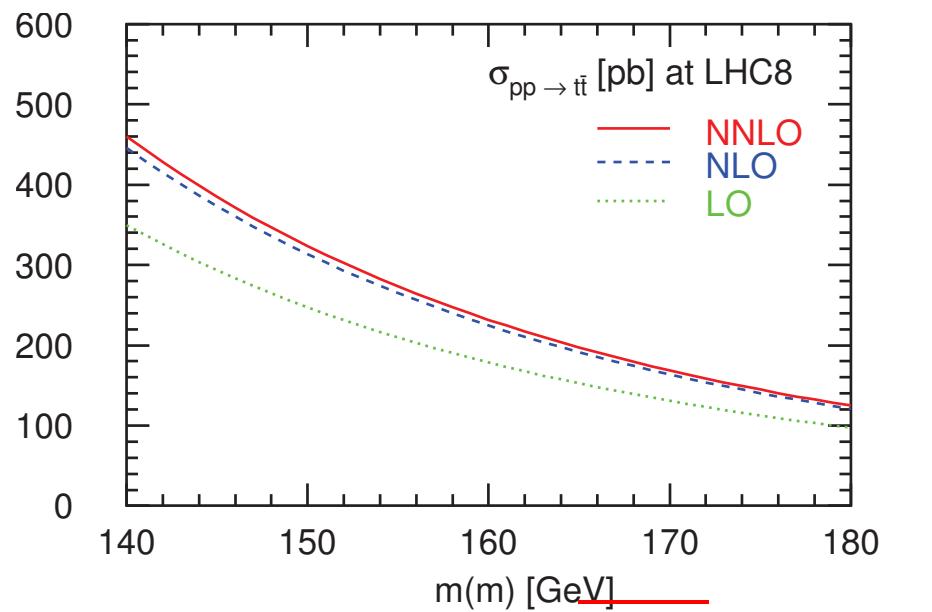
# Total cross section with running mass (I)

## Comparison pole mass vs. $\overline{\text{MS}}$ mass

Dowling, S.M. '13



**pole mass**



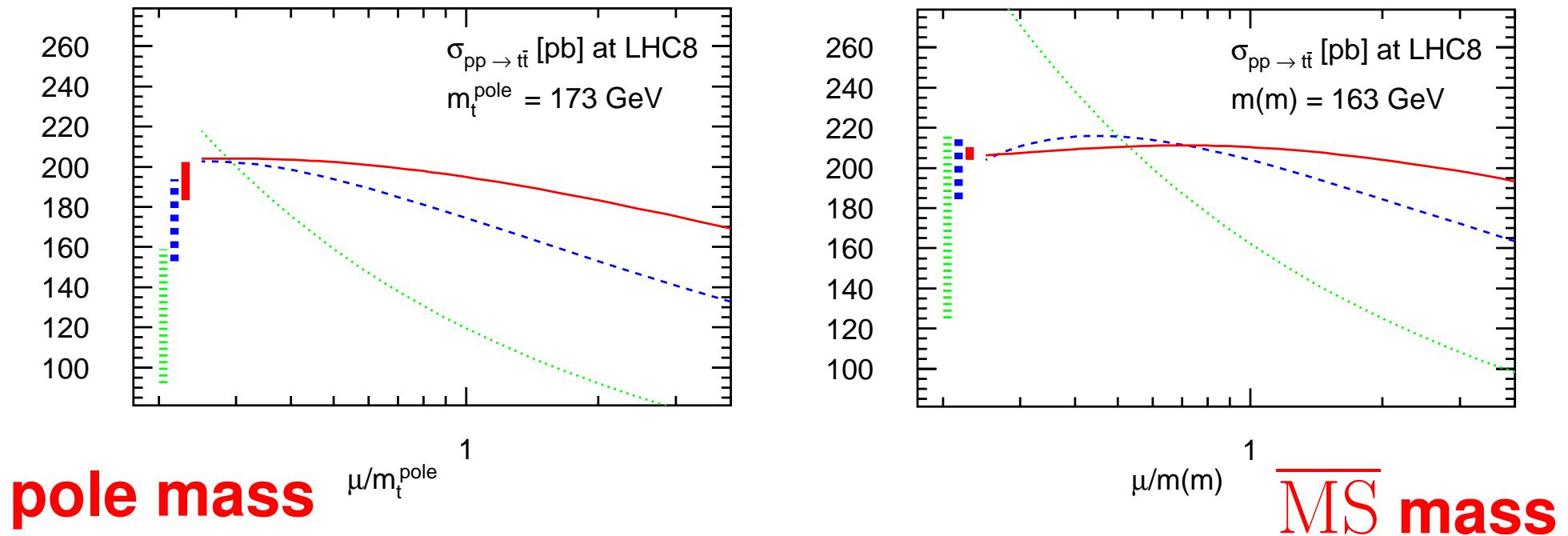
**$\overline{\text{MS}}$  mass**

- NNLO cross section with  $\overline{\text{MS}}$  renormalization scheme for  $\alpha_s$  and  $m_t$ 
  - running mass with better apparent perturbative convergence
  - $K$ -factors:  $K_{\text{LO} \rightarrow \text{NLO}} = 1.26$  and  $K_{\text{NLO} \rightarrow \text{NNLO}} = 1.03$
- Similar observations for other processes with heavy-quark production: hadro-production  $pp \rightarrow c\bar{c}(b\bar{b})$  and DIS  $ep \rightarrow c\bar{c}(b\bar{b})$

# Total cross section with running mass (II)

## Comparison pole mass vs. $\overline{\text{MS}}$ mass

Dowling, S.M. '13

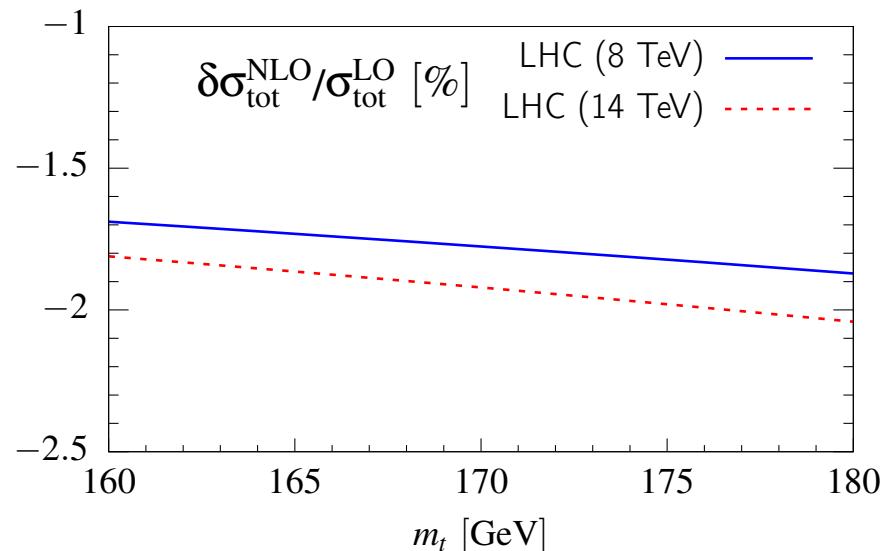
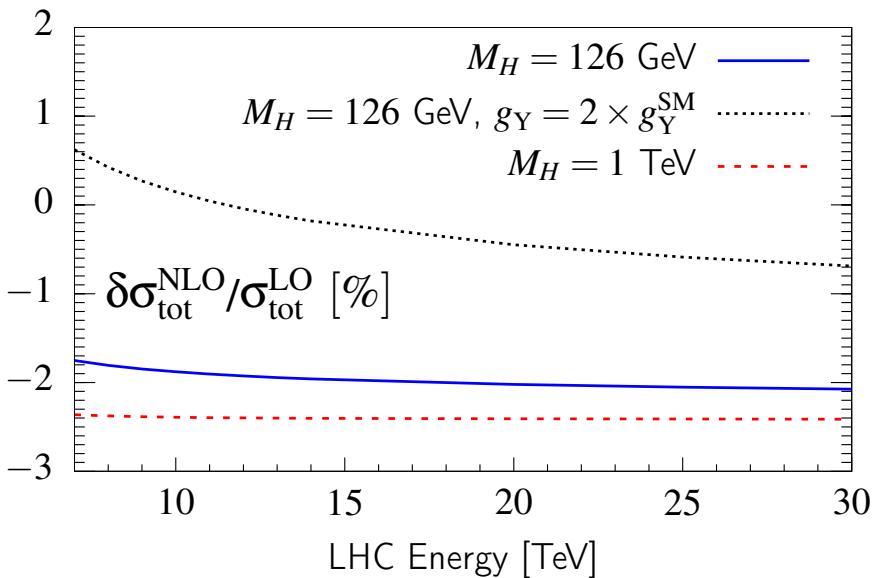


- NNLO cross section with  $\overline{\text{MS}}$  renormalization scheme for  $\alpha_s$  and  $m_t$ 
  - small theoretical uncertainty from scale variation
  - point of minimal sensitivity at natural hard scales  
 $\mu \sim \mathcal{O}(m_t(m_t)) \sim \mathcal{O}(160) \text{ GeV}$

# Electroweak corrections

- Electroweak corrections (ratio of  $\sigma_{\text{EW}}/\sigma_{\text{LO}}$ )

Beenakker, Denner, Hollik, Mertig, Sack, Wackerlo '94; Bernreuther, Fücker '05;  
Kühn, Scharf, Uwer '06



- Left:  $\sigma_{\text{EW}}/\sigma_{\text{LO}}$  as function of total cms energy (effect depends on Higgs mass and Higgs-Yukawa coupling)
- Right:  $\sigma_{\text{EW}}/\sigma_{\text{LO}}$  as function of top-quark mass:  
negative contribution to total cross section;  $\mathcal{O}(2\%)$  with respect to  $\sigma_{\text{LO}}$
- NLO electroweak corrections included in Hathor (v2.1) Kühn, Scharf, Uwer '13

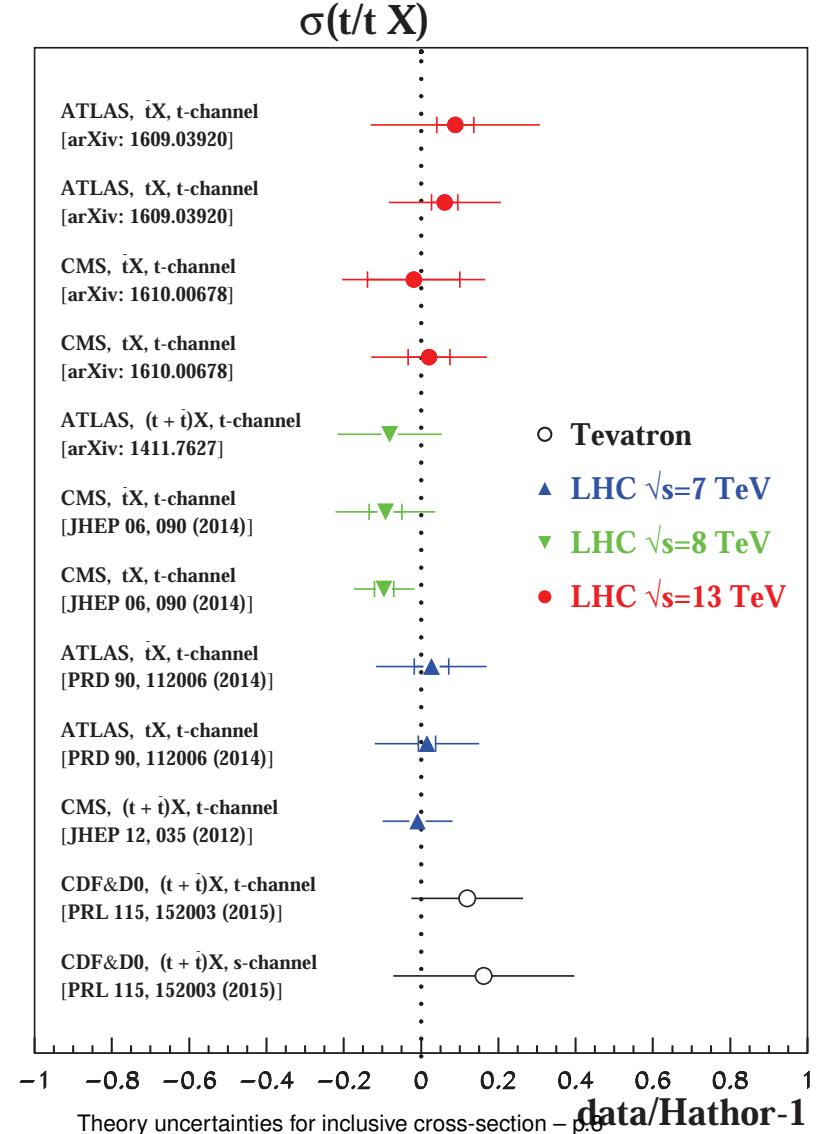
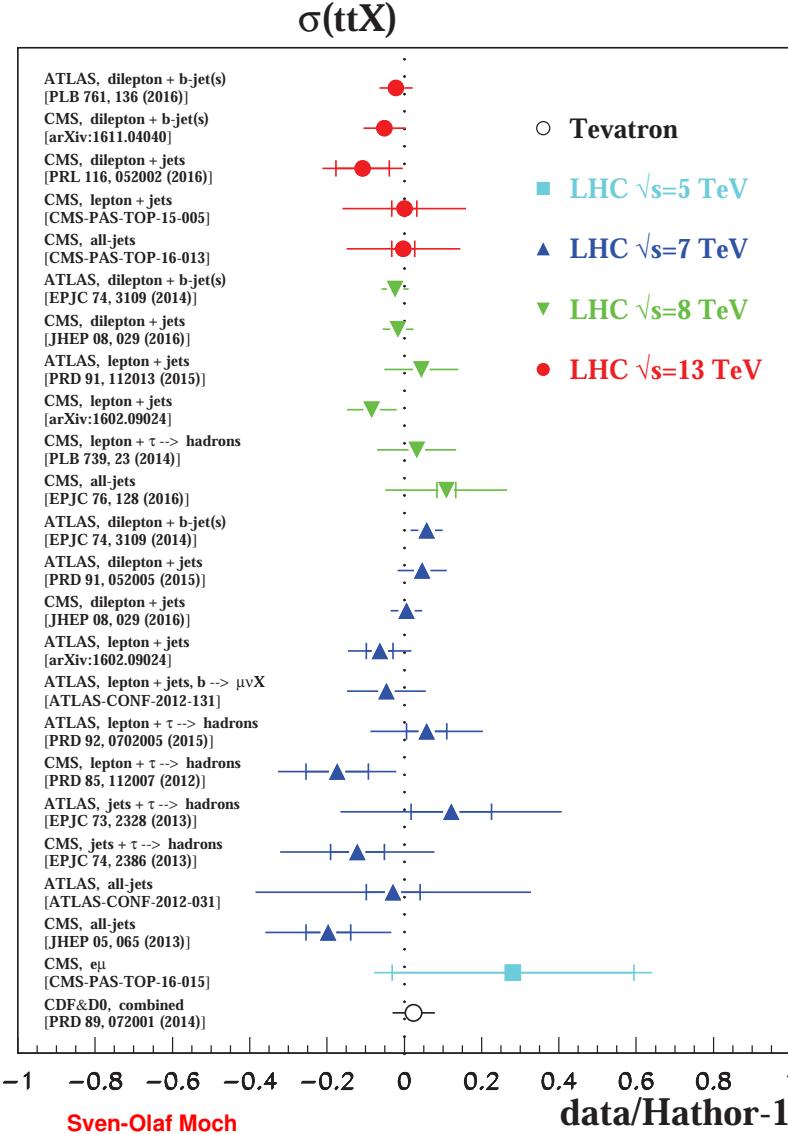
# PDF landscape

- Significant number of active groups ABMP16, CJ15, CT14, HERAPDF2.0, JR14, MMHT14, NNPDF3.1
  - PDFs accurate to NNLO in QCD, except for CJ15 (NLO)
  - different choices of data sets
  - different fitting procedures ( $\Delta\chi^2$  criterium)

PDF sets	$\Delta\chi^2$ criterion	data sets used in analysis
ABMP16 <a href="#">arXiv:1701.05838</a>	1	incl. DIS, DIS charm, DY, $t\bar{t}$ , single $t$
CJ15 <a href="#">arXiv:1602.03154</a>	1	incl. DIS, DY (incl. $p\bar{p} \rightarrow W^\pm X$ ), $p\bar{p}$ jets, $\gamma$ +jet
CT14 <a href="#">arXiv:1506.07443</a>	100	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, $pp$ jets
HERAPDF2.0 <a href="#">arXiv:1506.06042</a>	1	incl. DIS, DIS charm, DIS jets
JR14 <a href="#">arXiv:1403.1852</a>	1	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, DIS jets
MMHT14 <a href="#">arXiv:1510.02332</a>	2.3 ... 42.3 (dynamical)	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, $pp$ jets, $t\bar{t}$
NNPDF3.1 <a href="#">arXiv:1706.00428</a>	n.a.	incl. DIS, DIS charm, DY, $p\bar{p}$ jets, $pp$ jets, $t\bar{t}$ , $W +$ charm, $Zp_T$

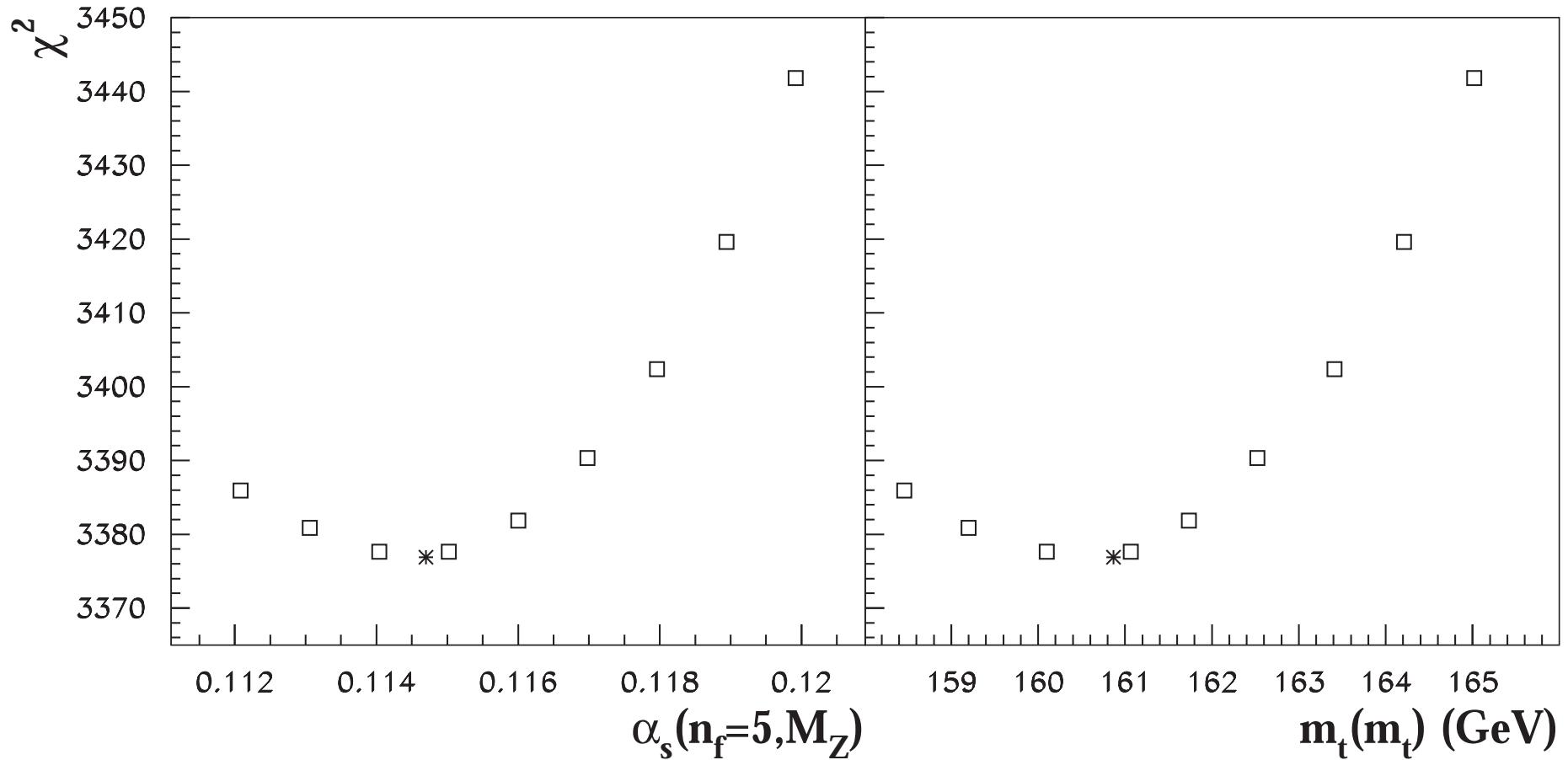
# Top-quark production cross sections

- Inclusive  $t\bar{t}$ - and single- $t$ -data in ABMP16 from CDF&D0, ATLAS, CMS
  - pulls for ABMP16 fit ( $N_{DP} = 36$  for  $t\bar{t}$ - and single- $t$ -data)



# *Quality of fit (I)*

- Goodness-of-fit estimator  $\chi^2$  for extracted  $\alpha_s(M_Z)$  and  $m_t(m_t)$  values
  - $\chi^2$  of global fit ( $NDP = 2834$ )
  - data on  $t\bar{t}$ - and singlet-production CDF&D0, ATLAS, CMS ( $NDP = 36$ )



# Quality of fit (II)

## Statistical tests

- Goodness-of-fit estimator
  - $\chi^2$  values compared to number of data points (typically a few thousand in global fit)

## Covariance matrix

- Positive-definite covariance matrix
  - correlations for fit parameters of ABMP16 PDFs

	$a_u$	$b_u$	$\gamma_{1,u}$	$\gamma_{2,u}$	$\gamma_{3,u}$	$a_d$	$b_d$	$\gamma_{1,d}$	$\gamma_{2,d}$	$\gamma_{3,d}$
$a_u$	1.0	0.7617	0.9372	-0.5078	0.4839	0.4069	0.3591	0.4344	-0.3475	0.0001
$b_u$	0.7617	1.0	0.6124	-0.1533	0.0346	0.3596	0.2958	0.3748	-0.2748	0.0001
$\gamma_{1,u}$	0.9372	0.6124	1.0	-0.7526	0.7154	0.2231	0.2441	0.2812	-0.2606	0.0001
$\gamma_{2,u}$	-0.5078	-0.1533	-0.7526	1.0	-0.9409	0.2779	0.2276	0.2266	-0.1860	0.0
$\gamma_{3,u}$	0.4839	-0.0346	0.7154	-0.9409	1.0	-0.1738	-0.1829	-0.1327	0.1488	0.0
$a_d$	0.4069	0.3596	0.2231	0.2779	-0.1738	1.0	0.7209	0.9697	0.6529	0.0001
$b_d$	0.3591	0.2958	0.2441	0.2276	-0.1829	0.7209	1.0	0.7681	-0.9786	-0.0001
$\gamma_{1,d}$	0.4344	0.3748	0.2812	0.2266	-0.1327	0.9697	0.7681	1.0	-0.7454	0.0002
$\gamma_{2,d}$	-0.3475	-0.2748	-0.2606	-0.1860	0.1488	-0.6529	-0.9786	-0.7454	1.0	-0.0002
$\gamma_{3,d}$	0.0001	0.0001	0.0001	0.0	0.0	0.0001	-0.0001	0.0002	-0.0002	1.0
$a_{us}$	-0.0683	-0.0801	-0.2094	0.3881	-0.3206	0.2266	0.1502	0.2000	-0.1293	0.0
$b_{us}$	-0.3508	-0.3089	-0.3462	0.0906	-0.0537	-0.1045	-0.2000	-0.2241	0.2798	0.0
$\gamma_{1,us}$	0.2296	0.1387	0.3367	-0.4043	0.3474	-0.1171	-0.1127	-0.0810	0.0767	0.0
$\gamma_{1,us}$	-0.4853	-0.4119	-0.3844	-0.0365	0.0064	-0.4380	-0.3592	-0.4957	0.3771	-0.0001
$A_{us}$	0.0506	0.0807	-0.0949	0.3198	-0.2560	0.2527	0.1648	0.2350	-0.1509	0.0
$a_{ds}$	-0.0759	-0.0443	-0.0951	0.0263	-0.0382	-0.2565	-0.2541	-0.2666	0.2380	0.0
$b_{ds}$	0.0452	-0.0197	0.0345	-0.0589	0.0683	-0.2084	0.0190	-0.1841	-0.0522	0.0
$\gamma_{1,ds}$	-0.0492	-0.0809	0.0101	-0.1791	0.1309	-0.5576	-0.2029	-0.4584	0.0946	0.0
$A_{ds}$	-0.1980	-0.1262	-0.2349	0.1526	-0.1428	0.1113	-0.2167	-0.1739	0.2407	0.0
$a_{ss}$	-0.2034	-0.1285	0.2362	0.2328	-0.0280	0.0960	0.1596	0.0661	-0.1054	0.0
$b_{ss}$	-0.1186	-0.0480	0.1532	0.1549	-0.1536	0.0486	0.1508	0.0267	-0.1161	0.0
$A_{ss}$	-0.1013	-0.0411	-0.1458	0.1802	-0.1625	0.1216	0.1678	0.0924	-0.1196	0.0
$a_g$	0.0046	-0.0374	0.1109	-0.1934	0.1653	-0.0288	-0.0122	0.0053	0.0059	0.0
$b_g$	0.2662	0.3141	0.1579	-0.0050	-0.0207	0.0973	0.0870	0.0646	-0.0666	0.0
$\gamma_{1,g}$	0.0008	0.0274	0.0706	0.0876	-0.0835	0.0919	0.0574	0.0493	-0.0364	0.0
$a_s^{(n_j=3)}(\mu_0)$	0.1083	-0.0607	0.0848	-0.0250	0.0765	-0.0763	-0.0306	0.0725	0.0243	0.0
$m_c(m_c)$	-0.0006	0.0170	-0.0104	0.0206	-0.0201	-0.0123	-0.0161	-0.0114	0.0108	0.0
$m_b(m_b)$	0.0661	0.0554	0.0605	-0.0367	0.0287	-0.0116	0.0029	-0.0074	-0.0051	0.0
$m_t(m_t)$	-0.1339	-0.2170	-0.0816	0.0081	0.0250	-0.0616	-0.0813	-0.0491	0.0736	0.0

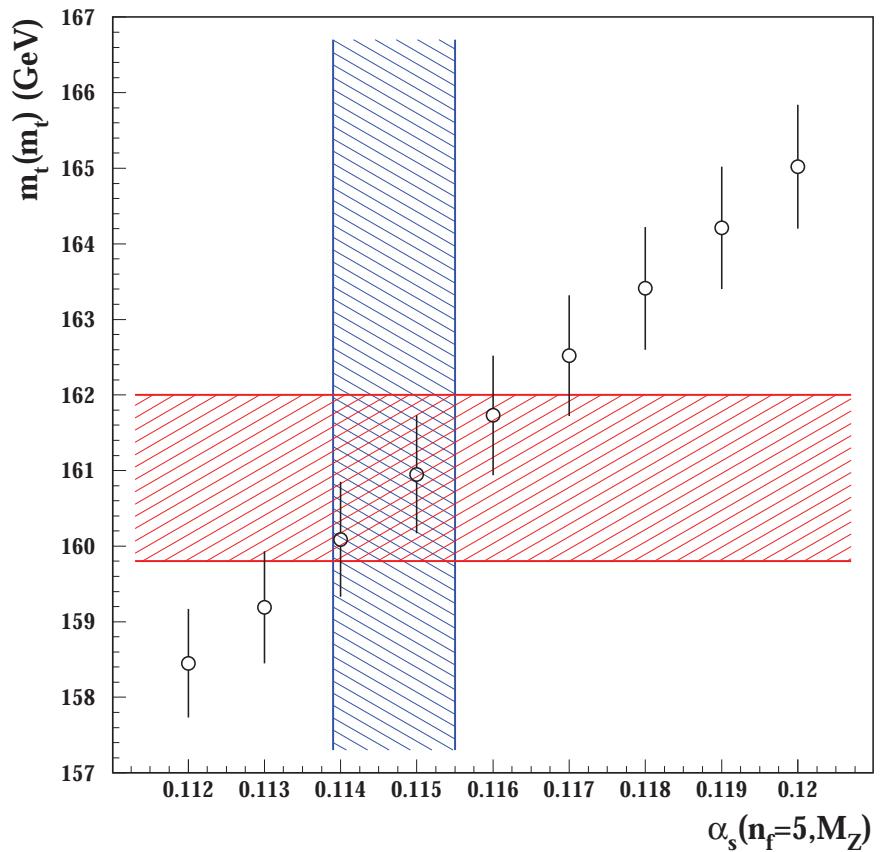
	$a_{us}$	$b_{us}$	$\gamma_{-1,us}$	$\gamma_{1,us}$	$A_{us}$	$a_{ds}$	$b_{ds}$	$\gamma_{1,ds}$	$A_{ds}$	$a_{ss}$
$a_u$	-0.0683	-0.3508	0.2296	0.4853	0.0506	-0.0759	0.0452	-0.0492	-0.1980	-0.2034
$b_u$	-0.0081	-0.3089	0.1387	-0.4119	0.0807	-0.0443	-0.0197	-0.0809	-0.1262	-0.1285
$\gamma_{1,u}$	-0.2094	-0.3462	0.3367	-0.3844	-0.0949	-0.0951	0.0345	0.0101	-0.2349	0.2362
$\gamma_{2,u}$	0.3881	0.0906	-0.4043	0.0365	0.3198	0.0263	-0.0589	-0.1791	0.1526	0.2328
$\gamma_{3,u}$	-0.3206	-0.0537	0.3474	0.0064	-0.2560	-0.0382	0.0683	0.1309	-0.1428	-0.2080
$a_d$	0.2266	-0.1045	-0.1171	0.4380	-0.2527	-0.0265	0.2084	0.5576	-0.1113	0.0960
$b_d$	0.1502	-0.2000	-0.1127	0.3592	0.1648	-0.2541	0.0190	-0.2029	-0.2167	0.1596
$\gamma_{1,d}$	0.2000	-0.2241	-0.0810	-0.4957	0.2350	-0.2666	-0.1841	-0.4584	-0.1739	0.0661
$\gamma_{2,d}$	-0.1293	0.2798	0.0767	0.3771	-0.1509	0.2380	-0.0522	0.0946	0.2407	-0.1054
$\gamma_{3,d}$	0.0	0.0	0.0	-0.0001	0.0	0.0	0.0	0.0	0.0	0.0
$a_{us}$	1.0	-0.3156	-0.8947	-0.5310	0.9719	0.2849	0.0241	-0.0470	0.2983	0.4131
$b_{us}$	-0.3156	1.0	0.1372	0.8258	-0.3995	0.0467	-0.0221	0.1190	0.1856	0.0291
$\gamma_{-1,us}$	-0.8947	0.1372	1.0	0.2611	-0.7829	-0.1695	0.0156	0.0501	-0.2117	0.7191
$\gamma_{1,us}$	-0.5310	0.8258	0.2611	1.0	-0.6479	0.0086	0.0076	0.1460	0.0781	-0.0010
$A_{us}$	0.9719	-0.3995	-0.7829	-0.6479	1.0	0.2983	0.0515	-0.0404	0.3055	0.2811
$a_{ds}$	0.2849	0.0467	-0.1695	0.0086	-0.2983	1.0	-0.1608	0.0719	0.9152	-0.2941
$b_{ds}$	0.0241	-0.0221	0.0156	0.0076	0.0515	-0.1608	1.0	0.7834	-0.3022	-0.0390
$\gamma_{1,ds}$	-0.0470	-0.1190	0.0501	0.1460	-0.0404	0.0719	0.7834	1.0	-0.1838	-0.1373
$A_{ds}$	0.2983	0.1856	-0.2117	0.0781	0.3055	0.9152	-0.3022	-0.1838	1.0	0.1833
$a_{ss}$	0.4131	0.0291	-0.7191	0.0010	-0.2811	-0.2941	-0.0390	-0.1373	-0.1833	1.0
$b_{ss}$	0.2197	0.0643	-0.4479	0.1286	0.1193	-0.1579	-0.0260	0.0169	-0.0896	0.6522
$A_{ss}$	0.3627	0.0261	-0.6319	0.0102	-0.2412	-0.2688	-0.0180	-0.0960	-0.1797	0.9280
$a_g$	-0.2570	0.0001	0.2196	0.0039	-0.2493	-0.2190	-0.0454	-0.1031	-0.2571	0.0626
$b_g$	-0.1419	0.1266	0.0694	0.2648	-0.1715	-0.0515	0.0917	0.2130	-0.0469	0.0092
$\gamma_{1,g}$	-0.0241	0.0332	-0.0226	0.1296	-0.0489	-0.0137	0.0503	0.1409	-0.0022	-0.0279
$a_s^{(n_j=3)}(\mu_0)$	0.0954	-0.2866	-0.0341	0.3493	0.1110	-0.0604	0.1265	-0.1811	-0.1330	-0.0432
$m_c(m_c)$	0.0704	-0.0093	-0.0033	0.0462	0.1182	0.0849	0.0547	0.0413	0.1193	-0.0432
$m_b(m_b)$	-0.0183	-0.0132	0.0044	0.0209	-0.0298	-0.0006	0.0332	0.0695	-0.0432	0.0159
$m_t(m_t)$	0.0641	-0.1841	-0.0408	-0.2635	0.0755	-0.0573	-0.1067	-0.2003	-0.0869	0.0169

	$b_{ss}$	$A_{ss}$	$a_g$	$b_g$	$\gamma_{1,g}$	$a_s^{(n_j=3)}(\mu_0)$	$m_c(m_c)$	$m_b(m_b)$	$m_t(m_t)$
$a_u$	-0.1186	-0.1013	0.0046	0.2662	0.2008	0.1083	0.0006	0.0661	-0.1339
$b_u$	-0.0480	-0.0411	-0.0374	0.3141	0.2274	-0.0607	0.0170	0.0554	-0.2170
$\gamma_{1,u}$	-0.1532	-0.1458	0.1109	0.1579	0.0706	0.0848	-0.0104	0.0605	-0.0816
$\gamma_{2,u}$	0.1549	0.1802	-0.1934	-0.0050	0.0876	-0.0250	0.0206	-0.0367	0.0081
$\gamma_{3,u}$	-0.1536	-0.1625	0.1653	-0.0207	-0.0835	0.0765	0.0201	0.0287	-0.0250
$a_d$	0.0486	0.1216	-0.0288	0.0973	0.0919	0.0763	-0.0123	-0.0161	-0.0116
$b_d$	0.1508	0.1678	-0.0122	0.0870	0.0574	-0.0306	-0.0161	0.0029	0.0813
$\gamma_{1,d}$	0.0267	0.0924	0.0053	0.0646	0.0493	0.0725	-0.0114	-0.0074	-0.0491
$\gamma_{2,d}$	-0.1161	-0.1196	0.0059	-0.0666	-0.0364	0.0243	0.0108	-0.0051	0.0736
$\gamma_{3,d}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$a_{us}$	0.2197	0.3627	-0.2570	-0.1419	0.0241	0.0954	0.0704	-0.0183	0.0641
$b_{us}$	0.0643	0.0261	0.0001	0.1266	0.0332	-0.2866	-0.0093	-0.0132	-0.1841
$\gamma_{-1,us}$	-0.4479	0.6319	0.2197	0.0694	-0.0226	-0.0341	-0.0034	0.0444	0.0408
$\gamma_{1,us}$	0.1286	0.0102	0.0039	0.2648	0.1296	-0.3493	-0.0462	0.0209	-0.2635
$A_{us}$	0.1193	0.2412	-0.2493	-0.1715	0.0489	-0.1110	-0.1330	0.0423	-0.0755
$a_{ds}$	-0.1579	-0.2688	-0.2190	-0.0515	-0.0137	-0.1265	-0.0547	-0.0604	0.0849
$b_{ds}$	-0.0260	-0.0180	-0.0454	0.0917	0.0503	-0.1265	0.0547	0.0332	-0.1067
$\gamma_{1,ds}$	0.0169	-0.0960	-0.1031	0.2130	0.1409	-0.1811	0.0413	0.0695	-0.2003
$A_{ds}$	-0.0896	-0.1797	-0.2571	0.0469	0.0022	-0.1330	0.1193	-0.0432	0.0869
$a_{ss}$	0.6522	0.9280	0.0626	-0.0092	-0.0279	-0.0841	-0.0728	-0.0159	0.0169
$b_{ss}$	1.0	0.6427	-0.0179	0.1967	0.1164	-0.2390	-0.0965	0.0169	-0.1675
$A_{ss}$	0.6427	1.0	-0.0211	0.1403	0.0997	-0.1385	0.0216	0.0072	-0.1109
$a_g$	-0.0179	-0.0211	1.0	-0.5279	-0.8046	0.1838	-0.2829	0.0076	0.3310
$b_g$	0.1967	0.1403	-0.5279	1.0	0.8837	-0.5124	0.1438	0.1255	-0.7275
$\gamma_{1,g}$	0.1164	0.0997	-0.8046						

# Correlations

- Correlations between gluon PDF  $g(x)$ ,  $\alpha_s(M_Z)$  and  $m_t(m_t)$

- PDF sets with fixed values of  $\alpha_s(M_Z)$  display dependence on  $m_t(m_t)$
- PDF fits with fixed values of  $m_t$  and  $\alpha_s(M_Z)$  carry significant bias



# Summary

## Hard scattering cross section

- Precision predictions for inclusive cross section
  - cross section with  $\overline{\text{MS}}$  renormalization scheme for  $\alpha_s$  and  $m_t$  displays clear improvements

## PDFs, $\alpha_s(M_Z)$ and all that

- Precise PDFs are available
  - correlations between gluon PDF  $g(x)$ ,  $\alpha_s(M_Z)$  and  $m_t$

## Outlook

- Improvements in precision of predictions
  - yet higher orders in perturbation theory, resummation, etc
- Improvements in precision of measurements
  - simultaneous fit of data for multiple observables (inclusive cross section, differential distributions, etc)