Unfolding the W Momentum: Deep Neural Networks

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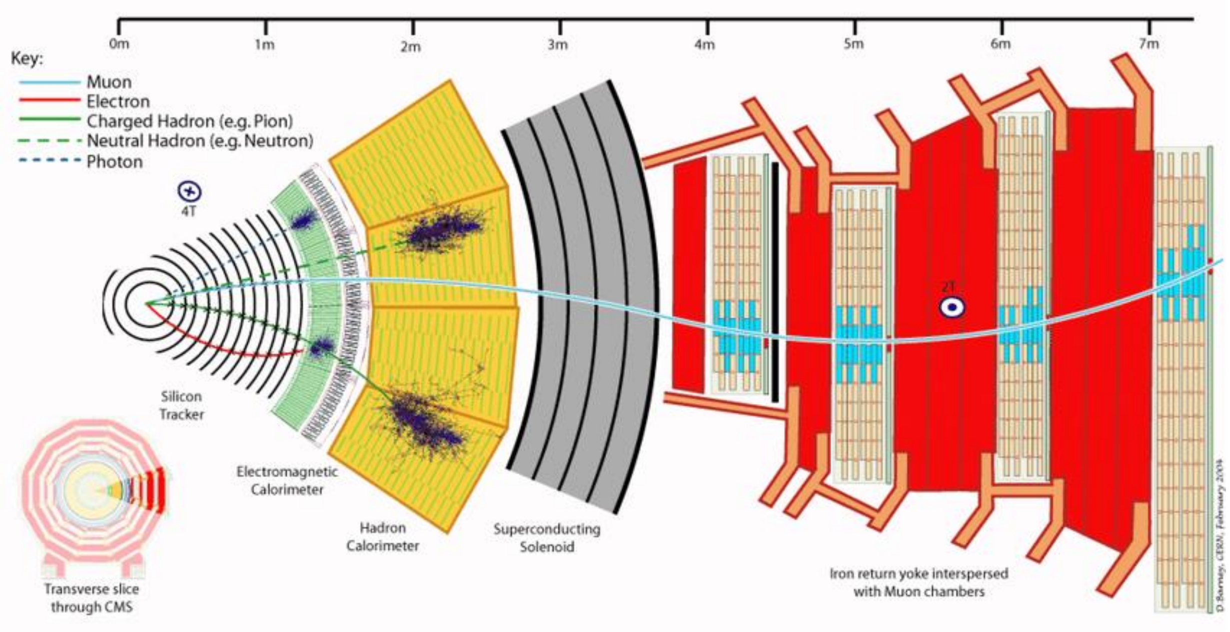
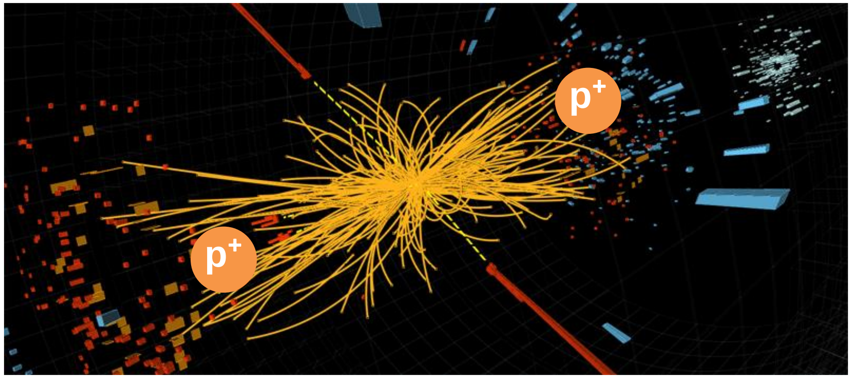


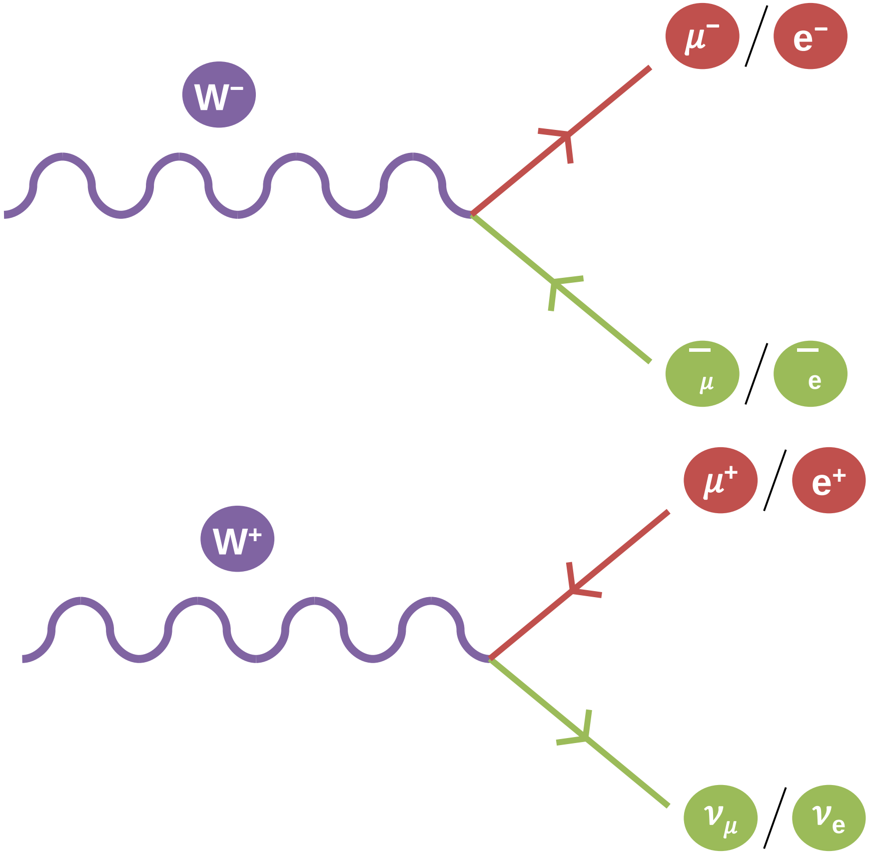
Summer Student Session August 2018

Outline

1. W boson production and detection overview
2. Unfolding:
    a. What is unfolding?
    b. Why do we need to unfold the W momentum?
    c. What are the current unfolding methods?
3. Neural Networks:
    a. How do they work?
    b. How can we use them to unfold the W momentum?
4. Bayes Unfolding:
    a. Priors and D’Agostini Method
    b. How can we apply this to a neural network?
5. Results

W Boson Mass





Unfolding - What is it?

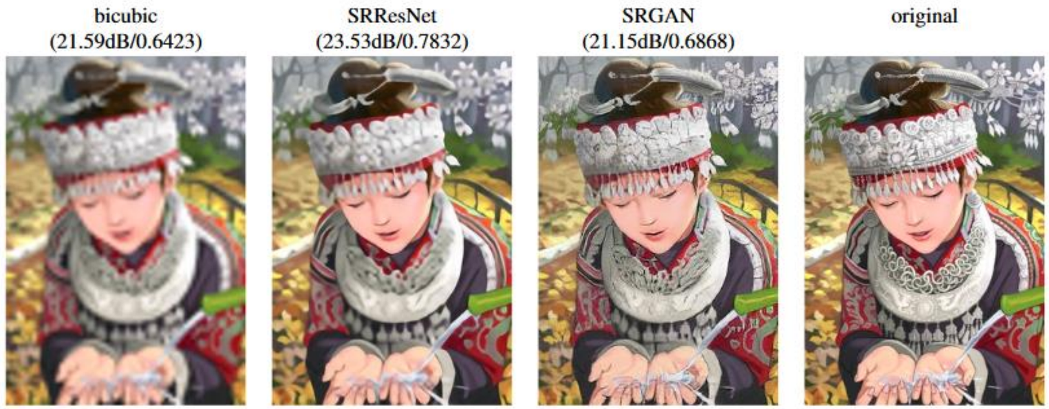
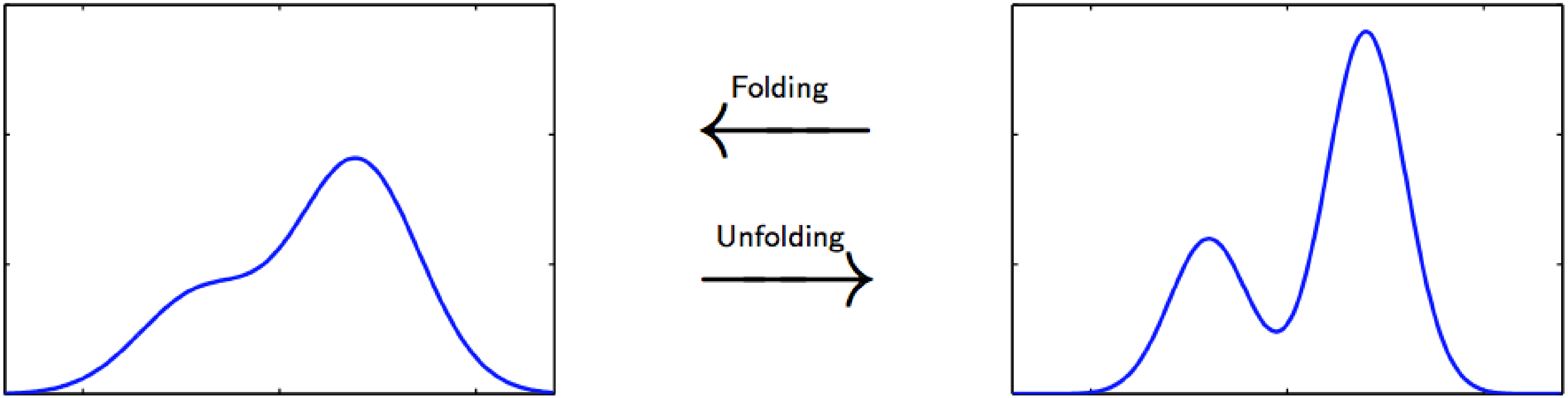


Figure 2: From left to right: bicubic interpolation, deep residual network optimized for MSE, deep residual generative adversarial network optimized for a loss more sensitive to human perception, original HR image. Corresponding PSNR and SSIM are shown in brackets. [4× upscaling]

- Similar to reconstructing a blurred image
- Given a probability distribution, we want to determine the “true” distribution

Unfolding - Why?



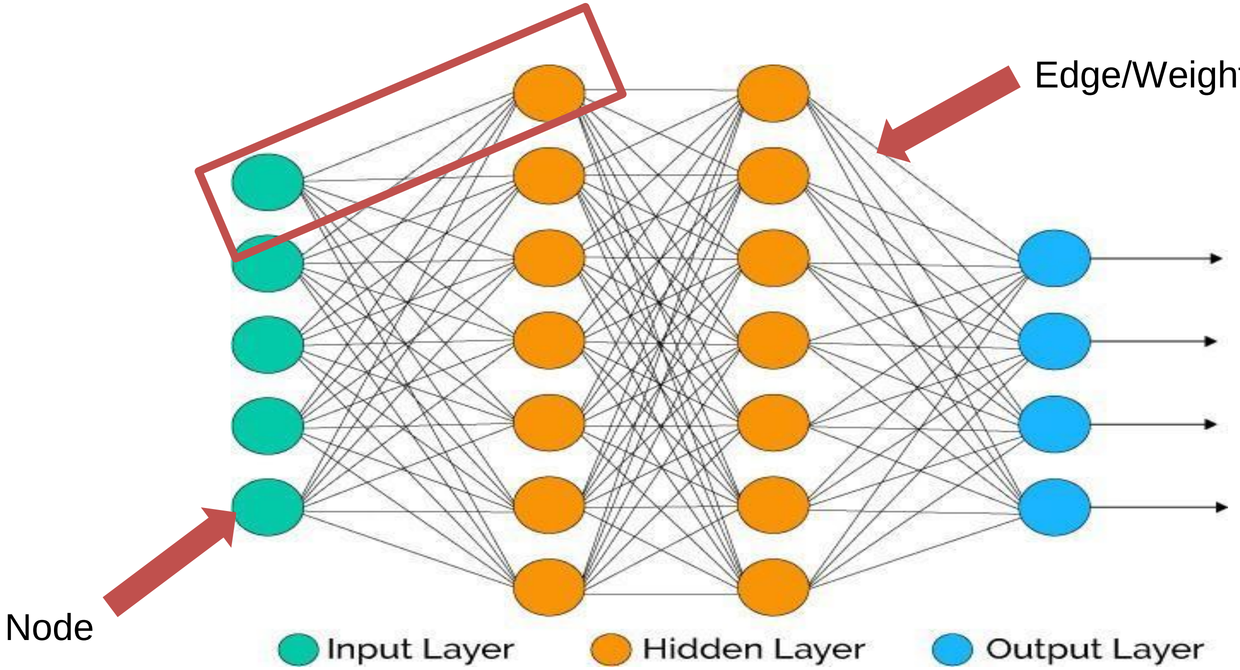
- Momentum measurements collected from the CMS detector have a certain amount of “noise”
- We observe the picture on the left; we want to know the picture on the right

Unfolding - Current Methods

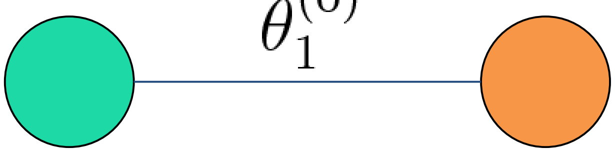
- ❖ Bin-by-bin matrix unfolding
- ❖ Downsides: does not scale well to multidimensional data

$$\begin{bmatrix} 9 & 13 & 5 & 2 \\ 1 & 11 & 7 & 6 \\ 3 & 7 & 4 & 1 \\ 6 & 0 & 7 & 10 \end{bmatrix}$$

Deep Neural Network - What is it?



$a_1^{(0)}$ $\theta_1^{(0)}$ $a_1^{(1)}$



Weight

Layer 0, Node 1

Layer 1, Node 1

$$a_1^{(1)} = \frac{1}{1 + e^{-(\theta_0^{(0)} + \theta_1^{(0)} a_1^{(0)} + \theta_2^{(0)} a_2^{(0)} + \ \dots \ )}}$$

Deep Neural Network - Unfolding

Input Layer: Smeared (folded) data collected from detector

Hidden Layers: 2 hidden layers, 30 nodes each

Output Layer: Binned (unfolded) data, # nodes = # bins

$\{x^{(i)}\}$ Training data, input, Monte Carlo simulated (folded)

$\{z^{(i)}\}$ Training data, output, Monte Carlo simulated (unfolded, true)

$\{y^{(i)}\}$ Real data, input, collected from detector (folded)

Bayesian Unfolding - Bayes' Rule

Probability of smeared distribution given true distribution

$$P(z \mid x) = \frac{P(x \mid z)P(z)}{P(x)}$$

Prior true distribution

Probability of true distribution given smeared distribution

Bayesian Unfolding $P(z \mid x) = \frac{P(x \mid z)P(z)}{P(x)}$

Even under the assumption that the Monte Carlo simulation is perfect, i.e.

$$P(x^{data} \mid z^{data}) = P(x^{MC} \mid z^{MC})$$

the prediction depends on prior true distribution $P(z)$

Since we are trying to measure the W boson momentum, we do not know the exact distribution to initialize the Monte Carlo simulation, so

$$P(z^{data}) \neq P(z^{MC})$$

Therefore:

$$P(z^{data} \mid x^{data}) \neq P(z^{MC} \mid x^{MC})$$

Bayesian Unfolding - D’Agostini

Problem: We are training our neural network on Monte Carlo data, which may result in a prediction biased towards the Monte Carlo prior distribution

Idea to Build on: Giulio D'Agostini developed an iterative algorithm where the prior distribution, $P(z)$, is updated using Bayes’ Theorem:

$$P(z \mid x) = \frac{P(x \mid z)P(z)}{P(x)}$$

Bayesian Unfolding - Reweighting

Our Idea: Apply D'Agostini's unfolding to our neural network by iteratively reweighting the prior distribution in the loss function

$$J = \sum_i \sum_k z_k^{(i)} \log h(x)_k^{(i)} + (1 - z_k^{(i)}) \log (1 - h(x)_k^{(i)})$$

Prior distribution

Prediction

Use the prediction from the neural network to update the prior distribution:

$$P(z) = P(z)^{prev}$$

Putting it all Together

Train neural network using Monte Carlo simulated data

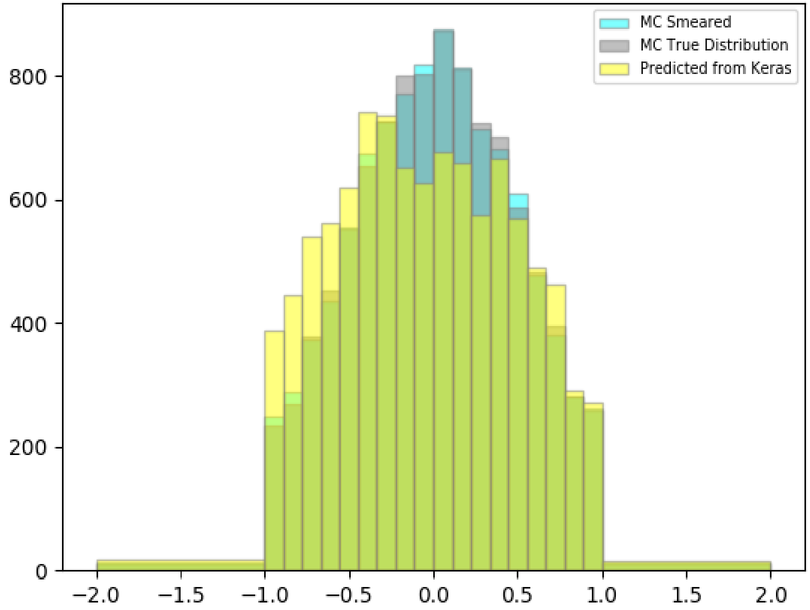
Get a prediction from the real data using the neural network

Update the distribution in the loss function using the prediction

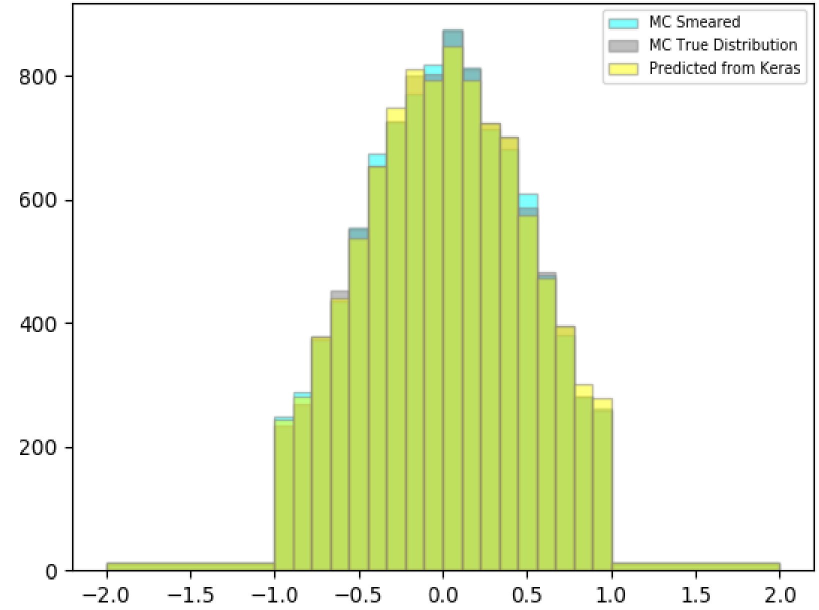
Repeat until convergence

Neural Net Results: Training

Training data after 1000 epochs

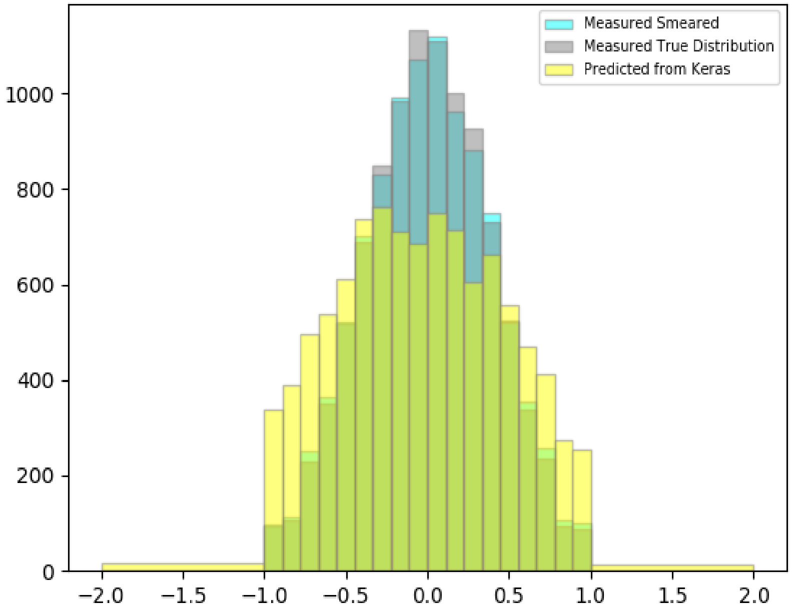


Training data after 15000 epochs

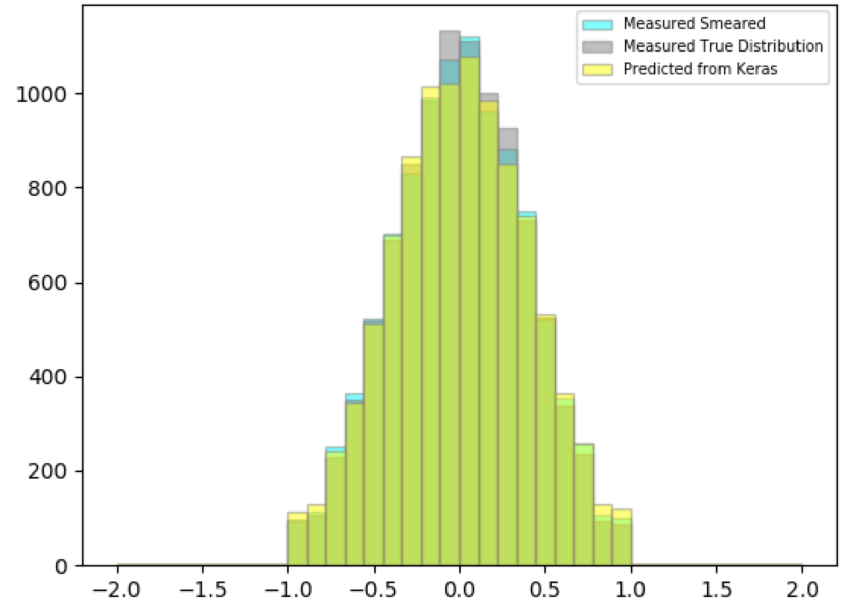


Neural Net Results: Testing

Testing data after 1000 epochs

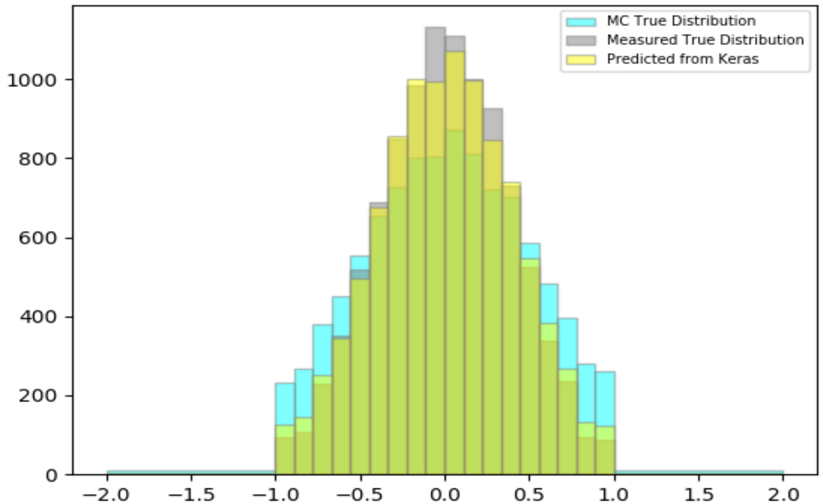


Testing data after 15000 epochs

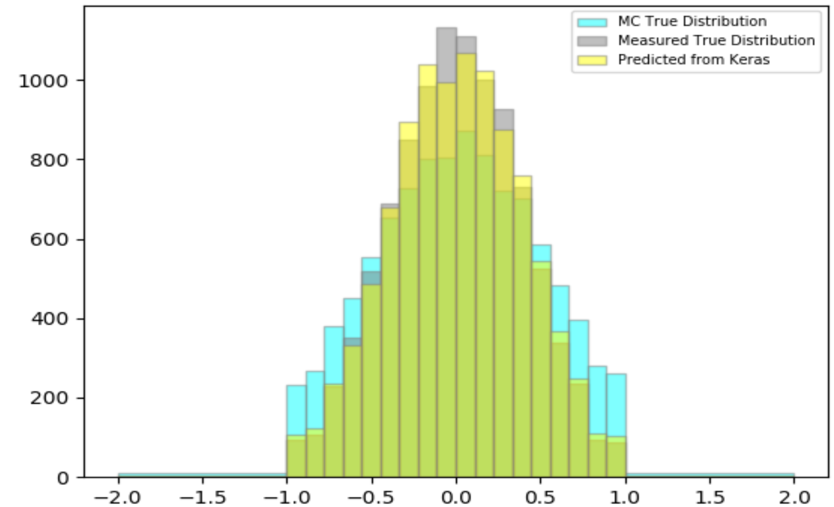


Bayes Reweighting: No Error Term

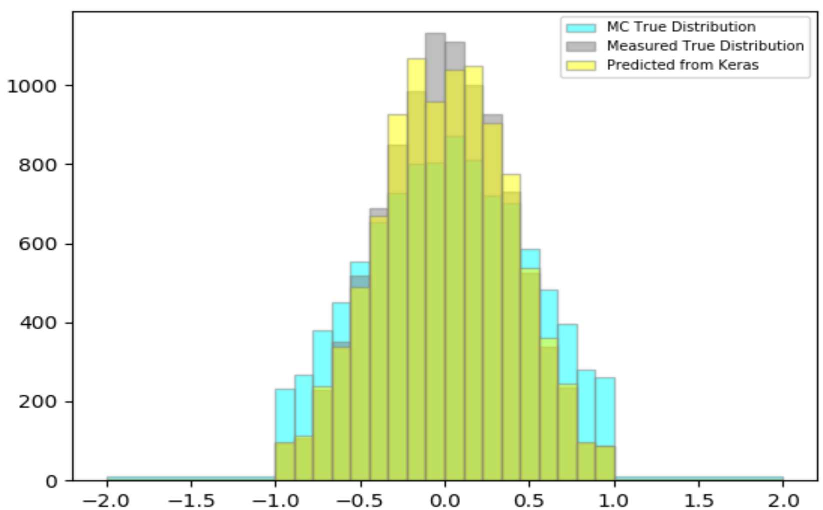
No Reweighting



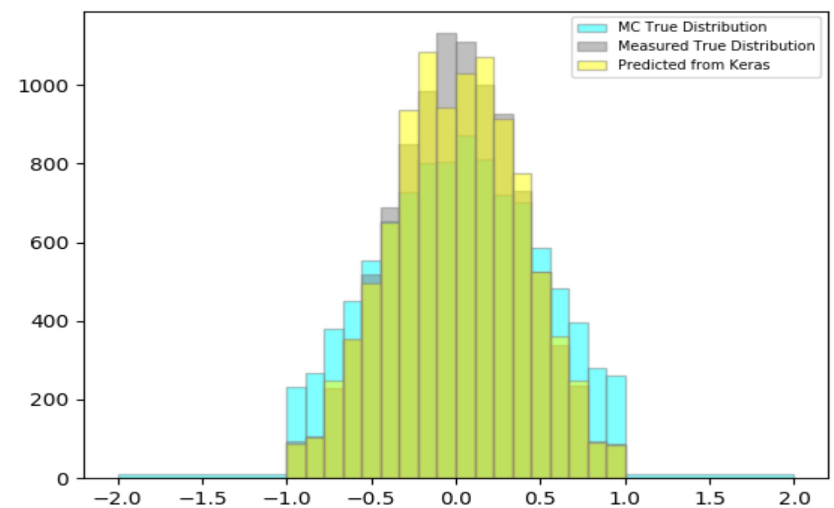
1 Iteration



4 Iterations

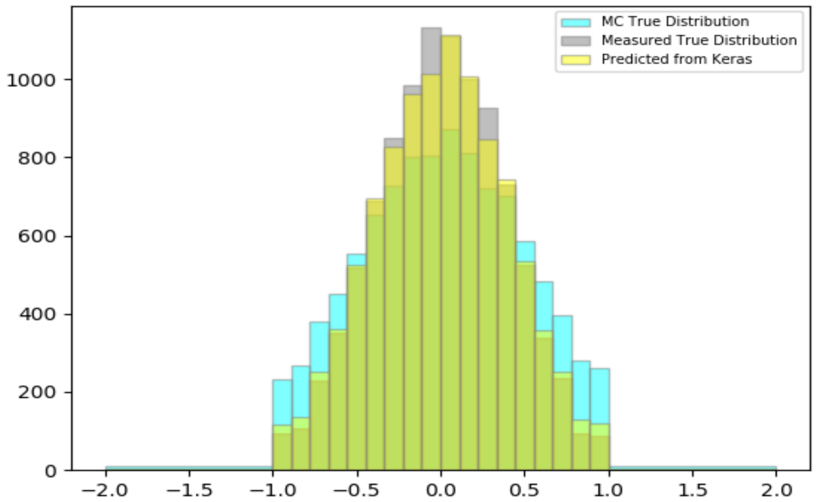


8 Iterations



Bayes Reweighting: Error Term

No Reweighting



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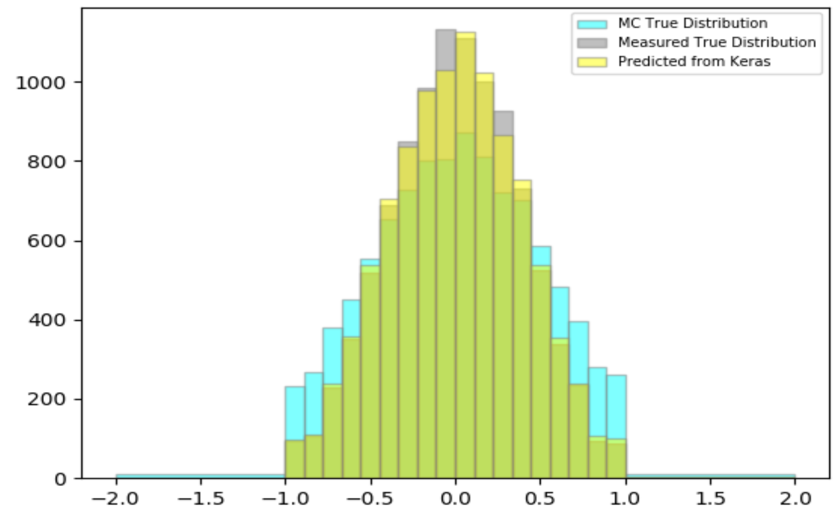
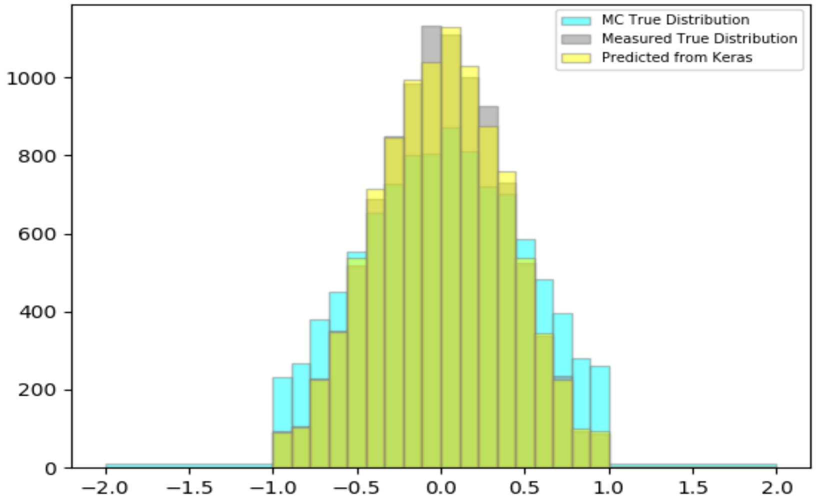


4 Iterations



8 Iterations