Relaxion dark matter & tabletop exp.

Gilad Perez

Weizmann Inst.



In Pursuit of New Particles and Paradigms

Aspen, Winter 2019

Outline

Intro.: Relaxion & the naturalness-log-crisis.

Coherent relaxion DM \w dynamical misalignment =>
 time-variation of constant of nature.

Probing heavyish-light-scalar/relaxion DM \w clocks.

Probing scalar-stars \w clocks (earth & space).

Conclusions.

The relaxion mechanism & the hier' problem

Graham, Kaplan & Rajendran (15)

(i) Add a scalar (relaxion) Higgs dependent mass:
$$(\Lambda^2 - g^2 \phi^2) H^\dagger H$$
.

- (ii) Introduce a "rolling" potential for the relaxion.
- (iii) Add (a "backreacion" moguls (wiggly potential) for the relaxion.





The relaxion mechanism & the hier' problem

PHYSICS TODAY

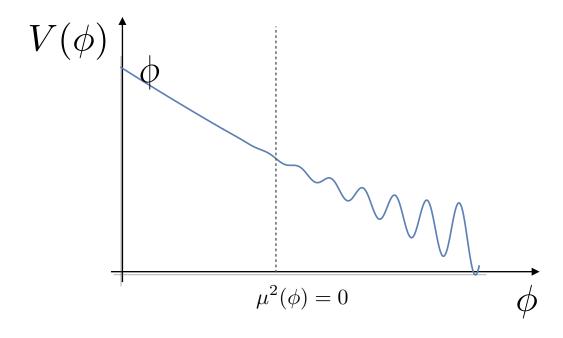


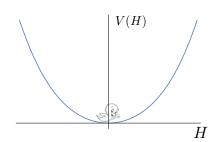


(iv) The moguls height depend on Higgs VEV, they grow till minima is created, which stops the relaxion when $\langle H \rangle \sim 10^2 \, \text{GeV} \ll \Lambda$.

Graham, Kaplan & Rajendran (15)

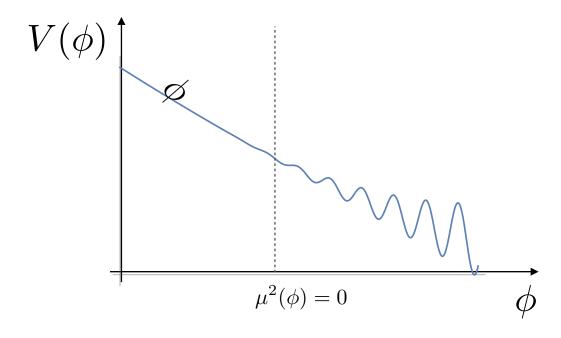
(i) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$

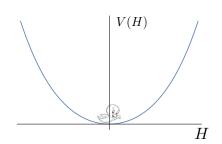




Graham, Kaplan & Rajendran (15)

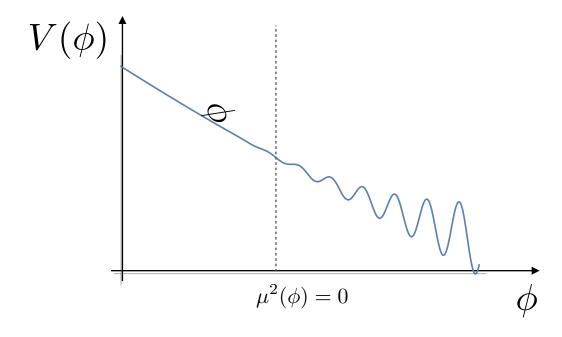
(i) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$

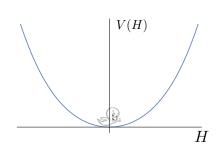




Graham, Kaplan & Rajendran (15)

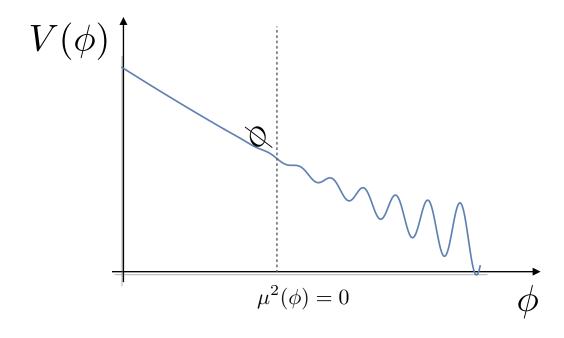
(i) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$

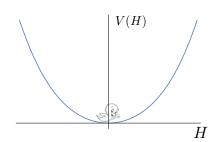




Graham, Kaplan & Rajendran (15)

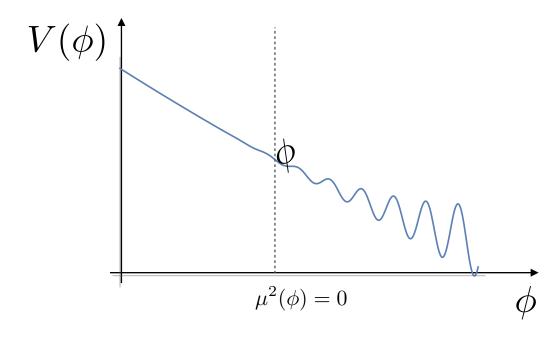
(i) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$

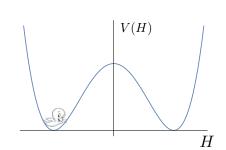




Graham, Kaplan & Rajendran (15)

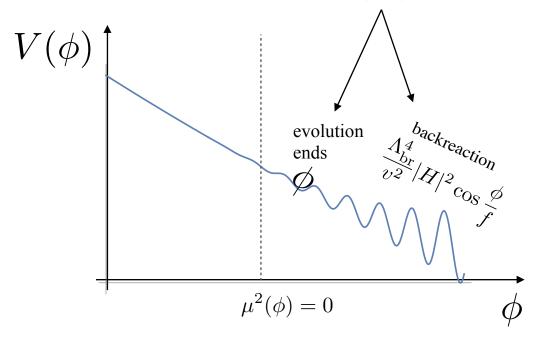
(i) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$

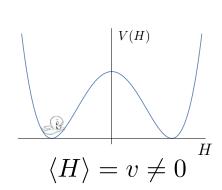




Graham, Kaplan & Rajendran (15)

(i) Add a scalar (relaxion) Higgs dependent mass: $(\Lambda^2 - g^2 \phi^2) H^{\dagger} H$





Summary relaxion physics

♦ A dynamical solution/amelioration of the Higgs fine-tuning problem.

♦ Focus shifts from TeV Higgs dynamics to relaxion, which is a weird axion, naturally light & weakly coupled ...

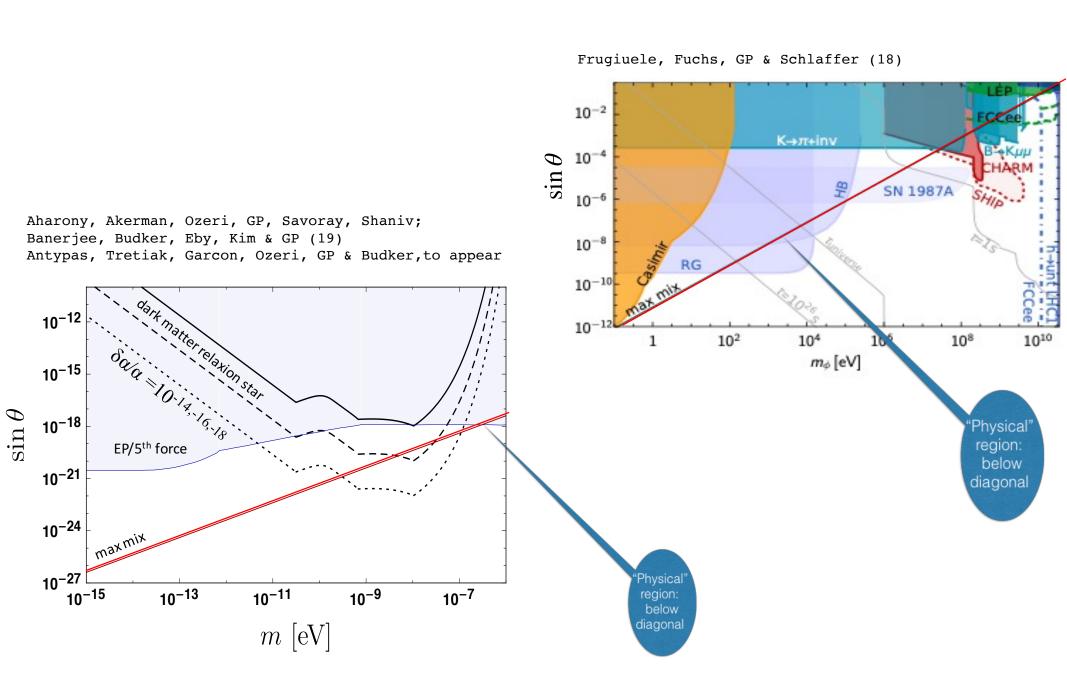
♦ As the minimum of the backreaction potential is not at zero => spontaneous CP violation is induced => QCD CP problem can be solved a la Nelson-Barr (or by giving-up on classical evolution ...)

Davidi, Gupta, GP, Redigolo & Shalit; Gupta; Nelson & Prescod-Weinstein (17)

◆ CP violation => relaxion-Higgs mixing => rich pheno'.

Flacke, Frugiuele, Fuchs, Gupta & GP; Choi & Im (16)

Hunting the relaxion & naturalness log-crisis

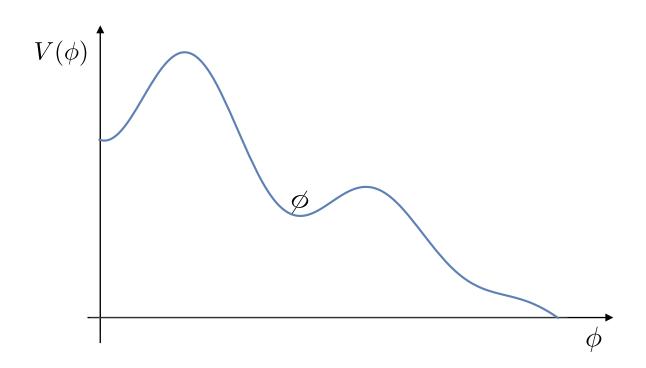


Relaxion/scalar light dark matter

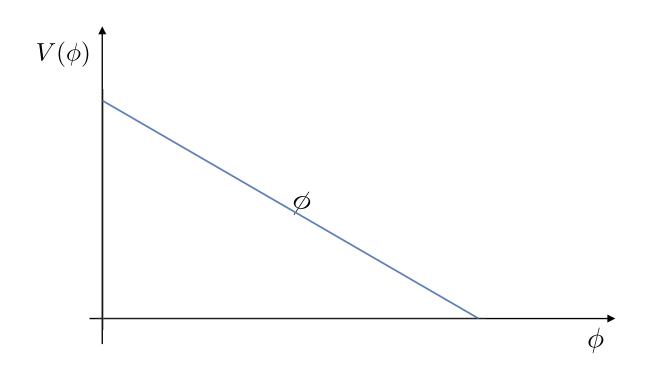
Banerjee, Kim & GP (18)

Banerjee, Kim & GP (18)

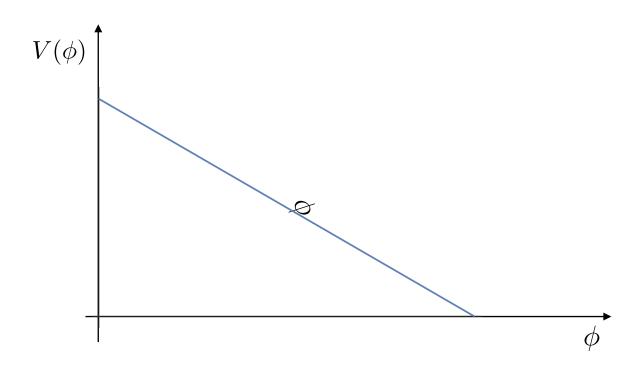
Basic idea is similar to axion DM (but avoiding missalignment problem):



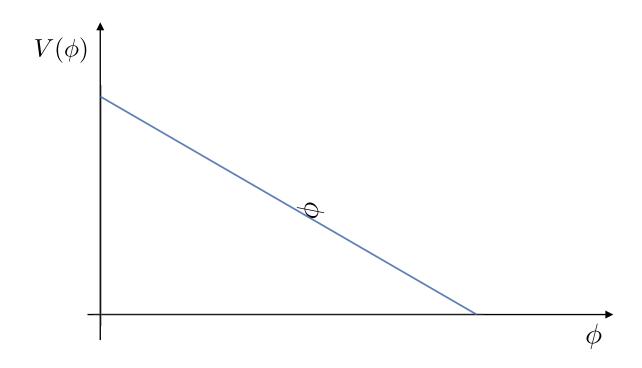
Basic idea is similar to axion DM (but avoiding missalignment problem):
 After reheating the wiggles disappear (sym' restoration):



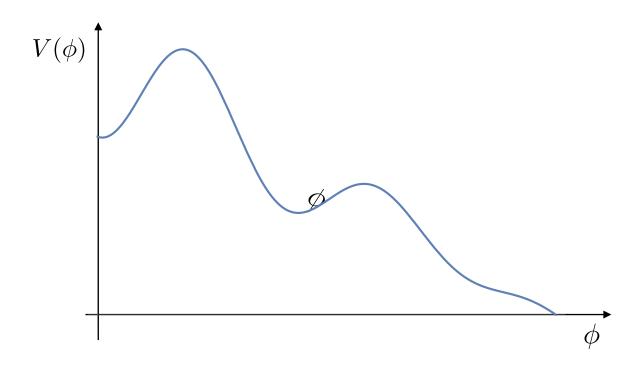
Basic idea is similar to axion DM (but avoiding missalignment problem):
 After reheating the wiggles disappear: and the relaxion roles a bit.



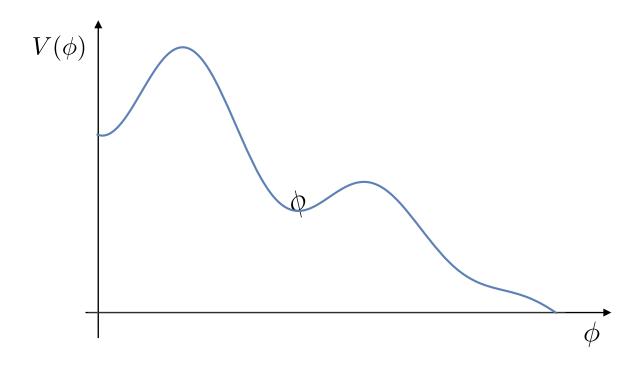
Basic idea is similar to axion DM (but avoiding missalignment problem):
 After reheating the wiggles disappear: and the relaxion roles a bit.



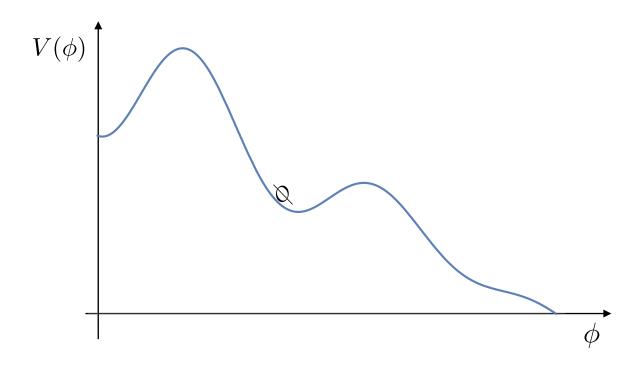
♦ Basic idea is similar to axion DM (but avoiding missalignment problem):



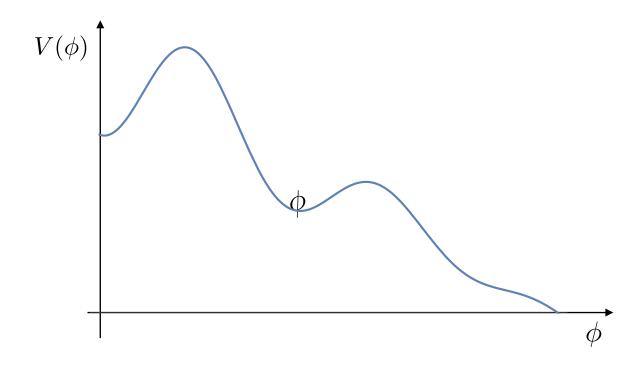
♦ Basic idea is similar to axion DM (but avoiding missalignment problem):



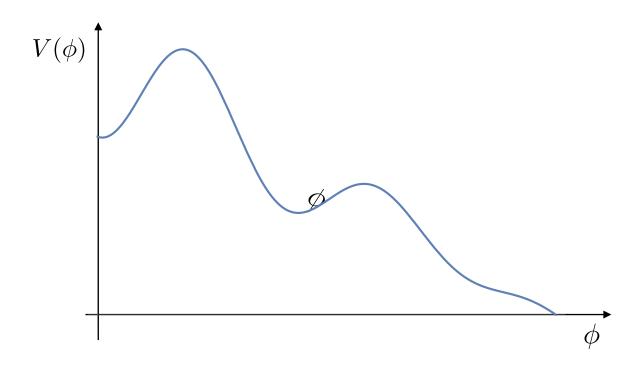
♦ Basic idea is similar to axion DM (but avoiding missalignment problem):



♦ Basic idea is similar to axion DM (but avoiding missalignment problem):



♦ Basic idea is similar to axion DM (but avoiding missalignment problem):



Coherent relaxion DM relic density

Basic idea is similar to axion DM (but avoiding missalignment problem):

Now the relaxion not at the min' and start to oscillates = DM.

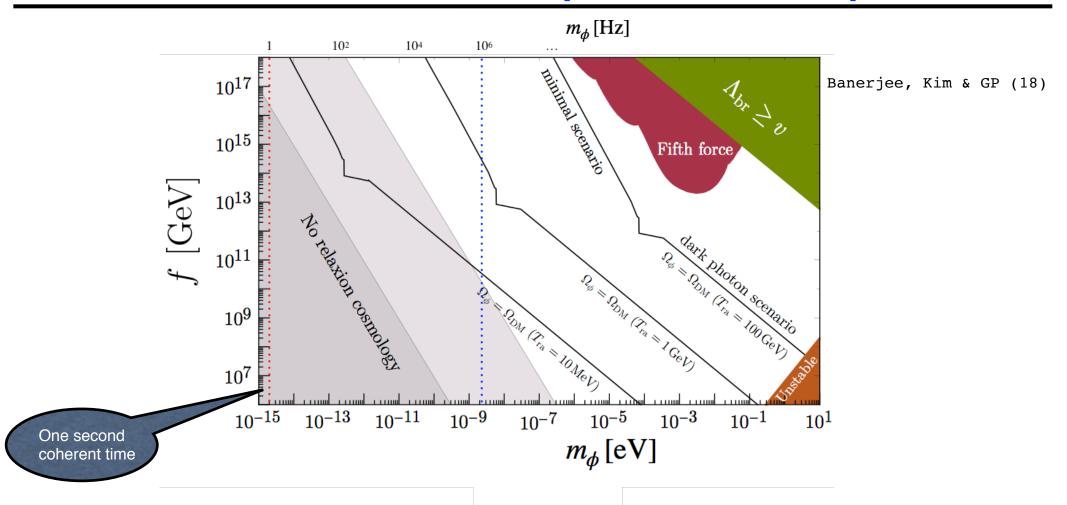
Light-coherent DM abundance: $\rho_{\rm DM}^{\rm cos} \sim m^2 \Delta \phi^2$

For
$$m_{\phi} \gtrsim H(T_{\rm ra})$$
: $\rho_{\rm DM}^{\cos} \sim \Omega_{\phi} h^2 \approx 3 \times (\Delta \theta)_{T=T_{\rm os}}^2 \left(\frac{\Lambda_{\rm br}}{1\,{\rm GeV}}\right)^4 \left(\frac{100\,{\rm GeV}}{T_{\rm os}}\right)^3$

where the observed DM abundance is $\Omega_{\rm DM} h^2 \simeq 0.12$

For $m_{\phi} < H(T_{\rm ra})$: extra suppression is obtained as oscilation starts when $H(T_{\rm osc}) \sim m_{\phi}$.

Relaxion dark matter, parameter space



♦ The relaxion oscillates & mixes with the Higgs, Banerjee, Kim & GP (18) therefore all constants of nature + masses vary with time.

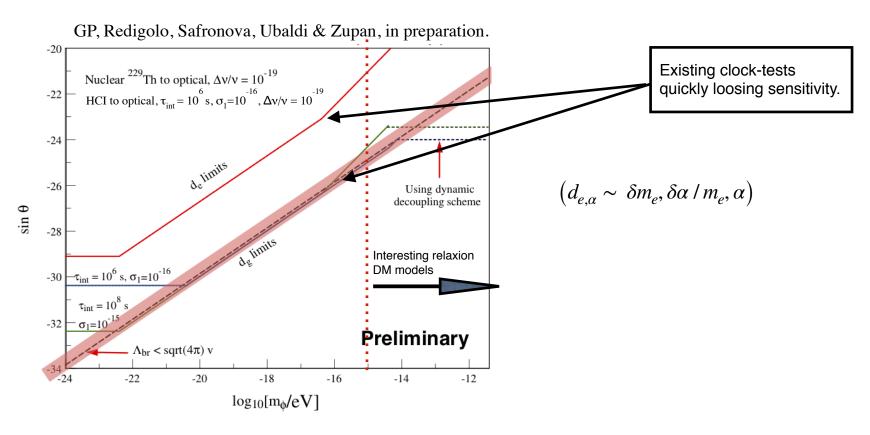
$$\frac{\delta m_e}{m_e} \lesssim y_e \sin_{\phi h} \frac{\sqrt{\rho_{\rm DM}}}{m_e \, m_\phi} \, \sin \left(m_\phi t \right) \, \, _{\rm Arvanitaki, \; Huang \; \& \; Van \; Tilburg \; (15)}$$

Two relevant questions

- (i) Notice that relevant models have osc. freq. I 1014 Hz. Can we probe these?
- (ii) Is the amplitude large enough to probe meaningful models?

The challenging super-Hz-DM mass

Graham, Kaplan, Mardon, Rajendran & Terrano; Arvanitaki, Dimopoulos & Van Tilburg; Van Tilburg, Leefer, Bougas & Budker (15)



 d_e stands for the time dependent component of the fine coupling constant, the bound on d_g (the coefficient of the time dependent component of α_s , the strong coupling) assumes a working ²²⁹Th nuclear clock with a 1: 10^{19} precision, τ_{int} stands for the total assumed integration time and σ_1 stands for the corresponding stability. The dashed-red line on the diagonal corresponds to the maximal mixing allowed in this scenario, Λ_{br} corresponds to a coupling in the relaxion model.

Back to the two questions

(i) Notice that relevant models have osc. freq. I - I0¹⁴ Hz. Can we probe these?

Yes see 2 new exp'that probe this region.



(ii) Is the amplitude large enough to probe meaningful models?



However, gravity can help: dark matter might form "relaxion-planets" that might be trapped around earth-gravitational field.

Banerjee, Budker, Eby, Kim & GP (19)

(similar to axion-stars requiring stability and assuming capturing & coherence)

Method I: Super Hz DM mass \w dynamical decoupling (DD)

Ravid Shaniy & Roee Ozeri

Quantum lock-in force sensing using optical clock Doppler velocimetry

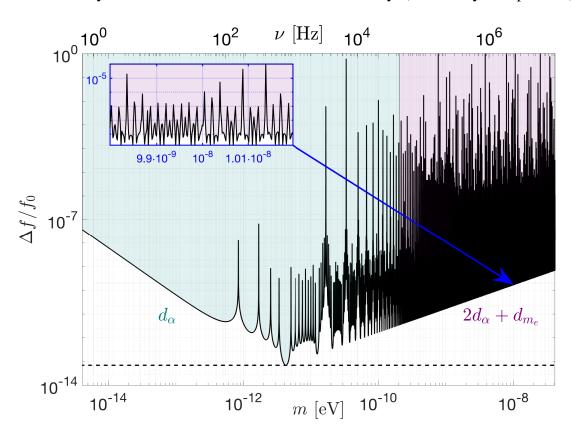
Nature Communications volume 8, Article number: 14157 (2017)

An efficient method of measuring an oscillating signal by a quantum probe, in the presence of noise, is the Quantum lock-in technique. Here, a probe superposition is time modulated by a Hamiltonian term, that does not commute with the noise and signal operators. The resulting *dynamics decouples* the probe phase evolution from any frequency component that does not spectrally overlap with the probe modulation and enables a significant prolongation of the probe coherence. **On the other hand,** frequency components that match some part of the modulation spectrum yield a phase that accumulates as the probe evolves with time. If the desired signal spectrally overlaps with the probe modulation, it imprints a coherently accumulated phase signal that can be subsequently measured. This laser frequency matched the clock dipole-transition $5S_{1/2} \rightarrow 4D_{5/2}$ of a single 88Sr+ ion, on which the DD sequence was applied. Here, V = 1013 Hz was chosen.

Effective 2-state system:
$$\psi(t) = \frac{1}{\sqrt{2}} \left(\left| S, -\frac{1}{2} \right\rangle + \mathrm{e}^{i\phi_{\mathrm{clk}}(t)} \left| D, \frac{1}{2} \right\rangle \right),$$

Beyond I Hz DM mass \w dynamical decoupling (DD)

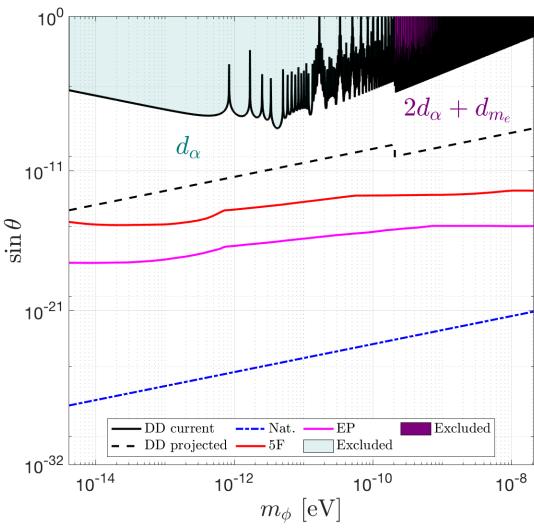
Aharony, Akerman, Ozeri, GP, Shaniv & Savoray, (ion-cavity comparison) 2019



Current bound on the relative modulation of the transition frequency from a DD experiment, placed at 95% CL. The dashed line marks the current sensitivity reach, corresponding to scanning over ν_m . The inset is a magnified view of $m \sim 10^{-8} {\rm eV}$.

Beyond I Hz DM mass \w dynamical decoupling

Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, 19 (ion-cavity comparison)

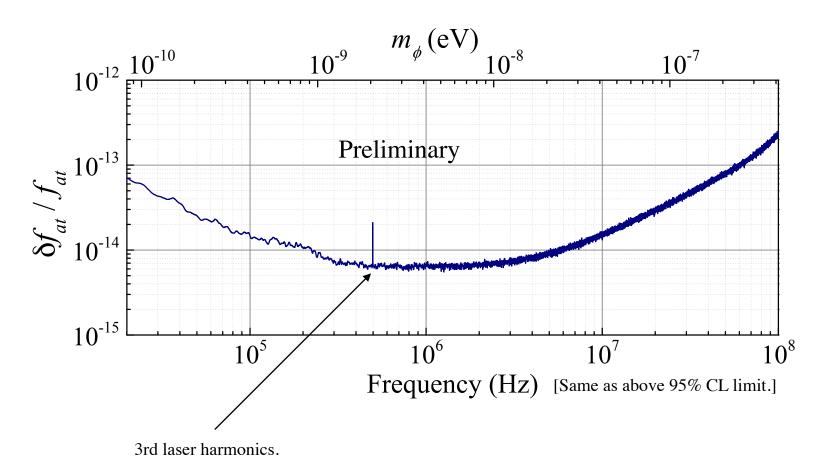


The bounds on the mixing angle of a relaxion DM: Black – current and projected bounds from DD experiments at 95% CL. Red – Bounds from fifth force experiments. Magenta – EP-tests bounds. Dash-dotted – Bounds from Naturalness.

Beyond IHz DM mass \w polarization spectroscopy

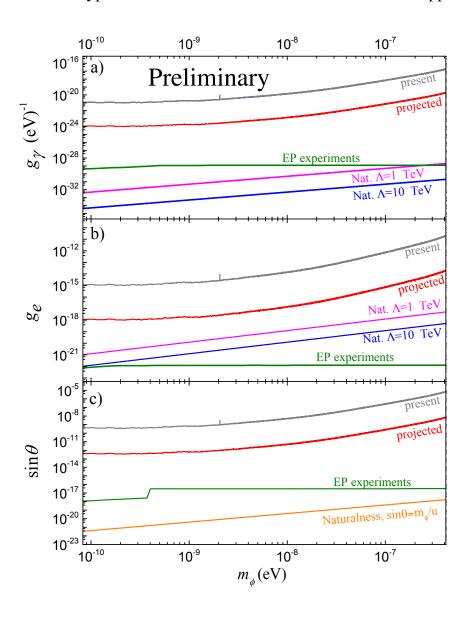
Antypas, Tretiak, Garcon, Ozeri, GP & Budker, to appear

Cs $6S_{1/2} \rightarrow 6P_{3/2}$ transition frequency (10 GHz)



Beyond IHz DM mass \w polarization spectroscopy

Antypas, Tretiak, Garcon, Ozeri, GP & Budker, to appear



Relaxion's Earth & Solar halos vs tabletops

Banerjee, Budker, Eby, Kim & GP (19)

Searching for a relaxion DM planet around us

Assume small DM density & large radius => mass-radii relation:

$$R_{
m star} pprox rac{M_{
m Pl}^2}{m_\phi^2} rac{1}{M_{
m Earth}} \qquad (M_* \ll M_{
m Earth}) \, .$$

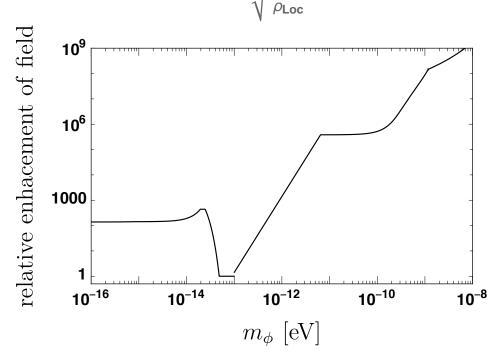
Eby, Leembruggen, Street, Suranyi & Wijewardhana (18);

Banerjee, Budker, Eby, Kim & GP (19)

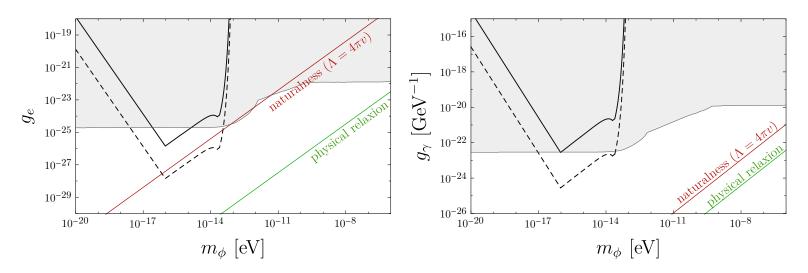
Can obtain large density enhancement:

$$r \equiv \frac{\rho_{\rm star}}{\rho_{\rm loc-DM}} \sim \xi \frac{M_{\rm Earth}^4 \, m_\phi^6}{M_{\rm Pl}^6 \, \rho_{\rm loc-DM}} \sim \xi \times 10^{28} \times \left(\frac{m_\phi}{10^{-10}}\right)^6$$

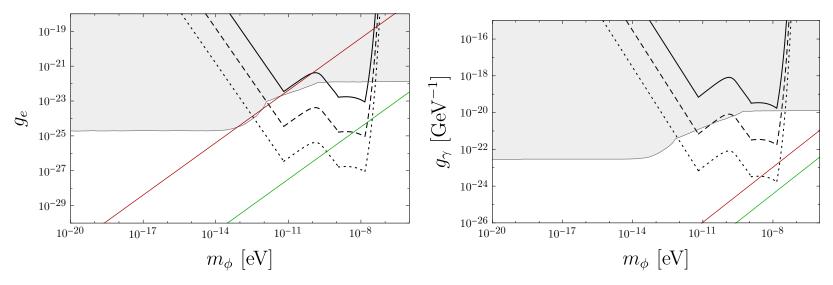
$$\xi \equiv M_{\rm star}/M_{\rm Earth}$$



Other constraints max' allowed mass



Solar halo: projected constraints on for a relaxion Solar halo. Experimental sensitivities = 10^{16} , 10^{18} (solid and dashed lines, respectively). The gray shaded region is excluded by EP. The red line is the naturalness limit, cutoff = 3 TeV, green line is an upper limit on coupling constants which can be obtained from physical relaxion models. The halo mass is taken as $M_2 = \min[(M/2)(R_2/R)^3, (M_2)_{max}]$



Earth halo: as above sensitivities = 10^{14} , 10^{16} , 10^{18} .

Conclusions

- ♦ Null-LHC + new paradigms + incredible sensitivity => new era!
- Relaxion-benchmarking allows to compare sensitivities.
- ♦ Relaxion-DM: dynamic decoupling -> strong bounds but cannot compete \w 5th force & can't probe physical region.

◆ Relaxion-DM-stars: table-top probe physical region, stronger than 5th force & can/should compare \w space.

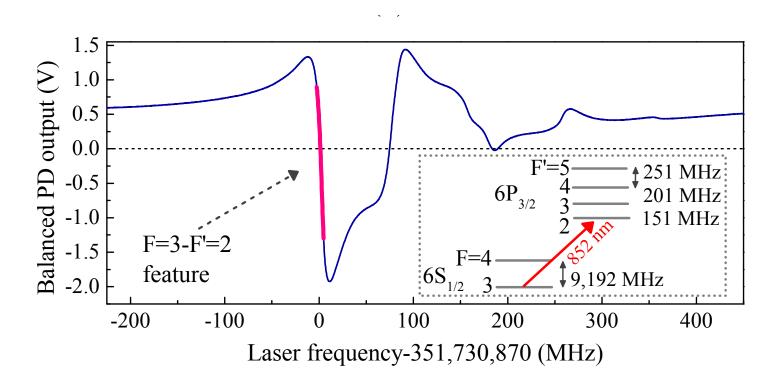
Interesting implications for ALP searches (GNOME & others).

Backups

Beyond IHz DM mass \w polarization spectroscopy

Antypas, Tretiak, Garcon, Ozeri, GP & Budker, to appear

Cs $6S_{1/2} \rightarrow 6P_{3/2}$ transition frequency (10 GHz)



Other constraints max' allowed mass

Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, in prep. (ion-cavity comparison); Banerjee, Budker, Eby, Kim, GP, in Prep.

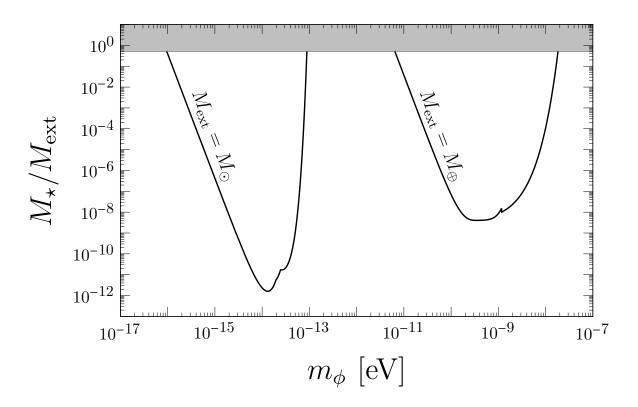


FIG. 2: The upper bound $(M_{\star})_{\text{max}}$ on the relaxion halo mass M_{\star} as a function of scalar particle mass m_{ϕ} ; the regions above the black lines are excluded by either (right side, assuming an Earth halo) lunar laser ranging [39], or (left side, assuming a Solar halo) planetary ephemerides [40]. We also require $M_{\star} \leq M_{\text{ext}}/2$ (boundary of gray shaded region), as explained in the Supplementary Material S2.

Very interesting implications to GNOME

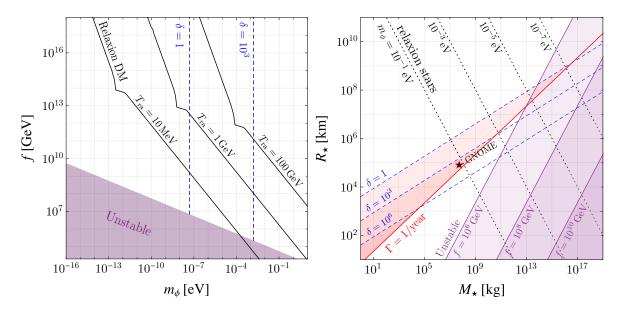
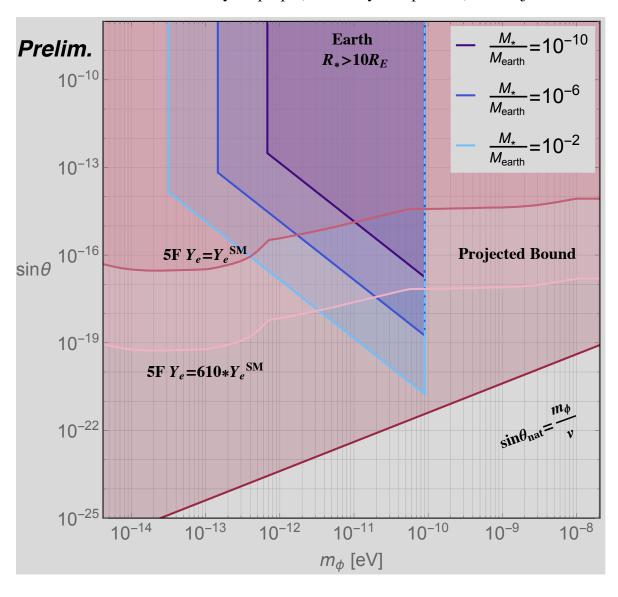


FIG. 1: The relevant parameter space for transient DM boson stars encountering the Earth. In both panels, the dashed blue lines are contours of constant overdensity δ , and the purple shaded regions indicate instability through self-interactions. Left: parameter space in scalar mass m_{ϕ} and decay constant f allowing for gravitationally stable objects, assuming $\Gamma = 1/\text{year}$; black lines denote the relaxion DM model of [9] for different choices of T_{ra} , the cosmological temperature at which the relaxion backreaction potential reappears. Right: M_{\star} and R_{\star} are treated as independent parameters; the black dotted lines denote stable configurations formed from scalars of mass m_{ϕ} , and the red shaded region represents $\Gamma > 1/\text{year}$ and $\delta > 1$. The black star represents the benchmark point used by the GNOME collaboration [34].

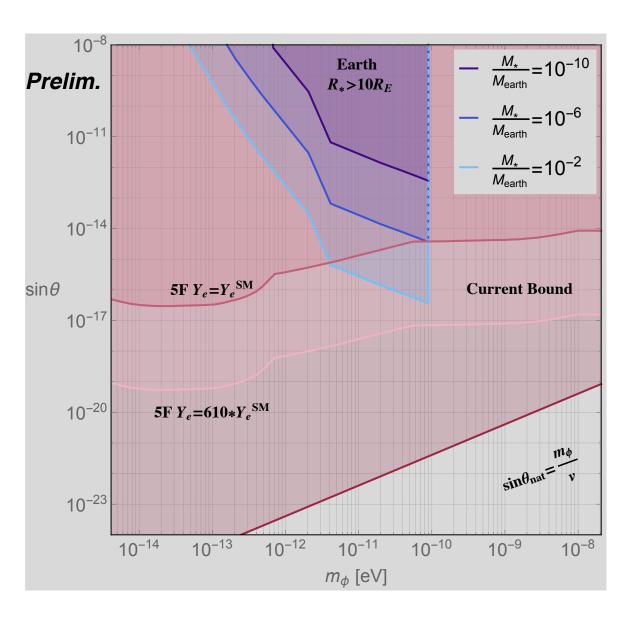
Large star DM density => visible effect

Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, in prep. (ion-cavity comparison); Banerjee, Budker, Eby, Kim, GP, in Prep.

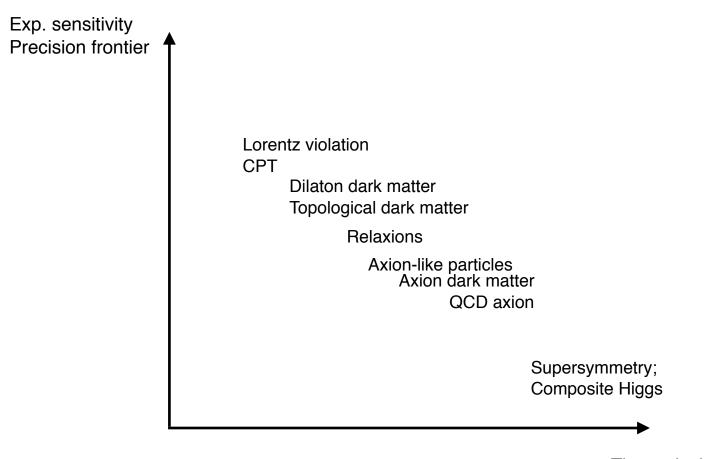


Large star DM density => visible effect

Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, in prep. (ion-cavity comparison); Banerjee, Budker, Eby, Kim, GP, in Prep.



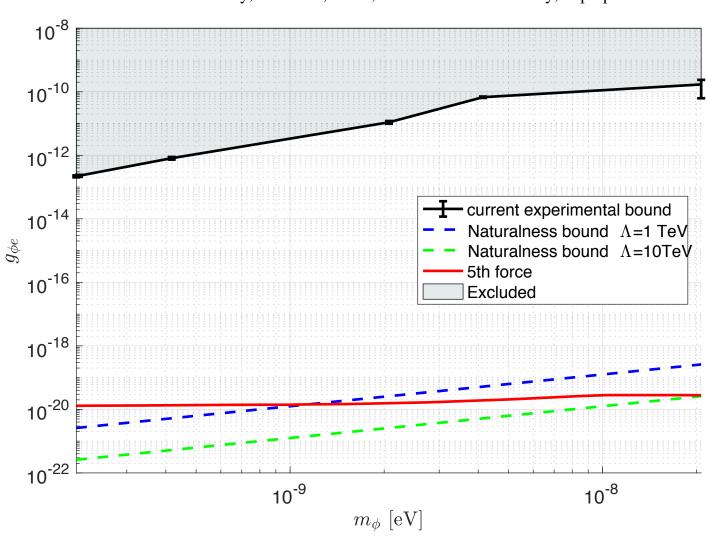
Subjective particle physicist perspective



Theoretical motivation

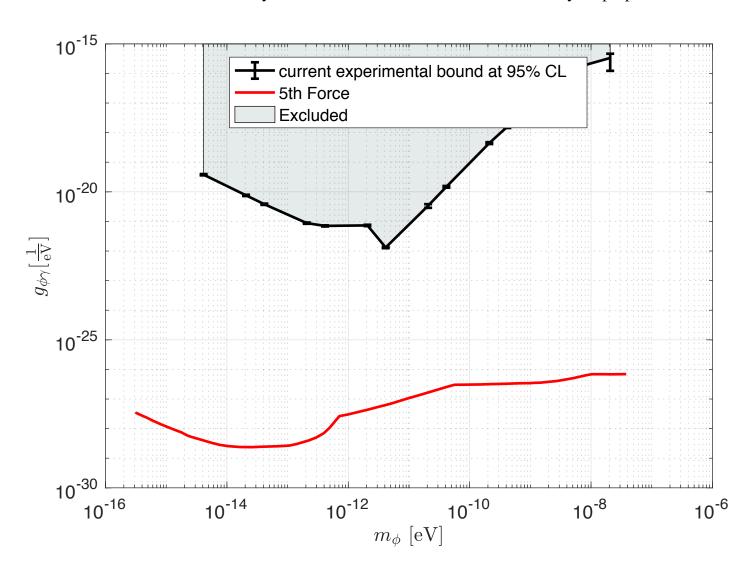
Beyond I Hz DM mass \w dynamical decoupling





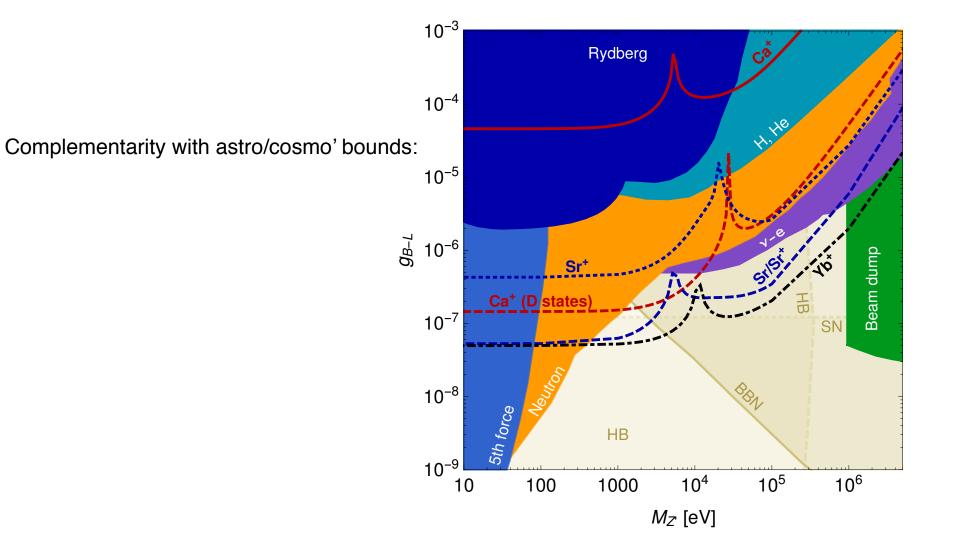
Beyond I Hz DM mass \w dynamical decoupling

Aharony, Akerman, Ozeri, GP & Shaniv & Savoray, in prep.





Frugiuele, Fuchs, GP & Schlaffer (16)



King comparison

• Level of linearity can be quantified by comparing area of triangle to that of a cube: $NL/|\overrightarrow{m}\overrightarrow{\nu}_2||\overrightarrow{m}\overrightarrow{\nu}_1|\ll 1$.

$$\overrightarrow{m}\overrightarrow{\mu}\equiv (1,1,1)\ .$$

$$m\nu_2 \qquad \qquad \text{NL} = \frac{1}{2} \left| (\overrightarrow{m}\overrightarrow{\nu}_1 \times \overrightarrow{m}\overrightarrow{\nu}_2) \cdot \overrightarrow{m}\overrightarrow{\mu} \right|\ .$$

$$m\nu_2^{AA'_2} \qquad \qquad \text{Or volume of prallelepiped:}$$

$$m\nu_2^{AA'_2} \qquad \qquad NL \qquad \qquad AA'_3 \qquad \qquad \overrightarrow{m}\overrightarrow{\nu}_1$$

$$m\nu_2^{AA'_1} \qquad m\nu_1^{AA'_1} \qquad m\nu_1^{AA'_2} \qquad m\nu_1^{AA'_3} \qquad m\nu_1$$

King linearity implications

• Linearity implies that $\overrightarrow{m}\overrightarrow{\nu}_2 \& \overrightarrow{m}\overrightarrow{\nu}_1$ must be linearly dependent:

$$\overrightarrow{m\nu}_{2} = K_{2} \overrightarrow{m}_{\mu} + F_{2} \overrightarrow{v} + \mathcal{O} \left(10^{-4}\right)$$

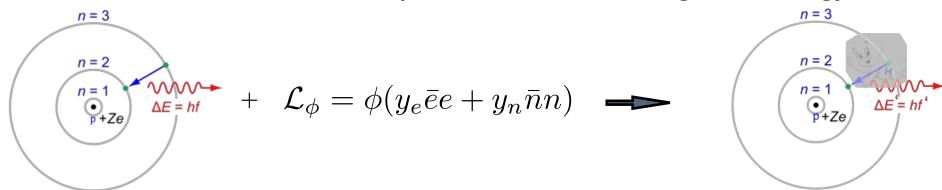
$$\overrightarrow{m\nu}_{1} = K_{1} \overrightarrow{m}_{\mu} + F_{1} \overrightarrow{v} + \mathcal{O} \left(10^{-4}\right)$$

$$\overrightarrow{m\nu}_{2} \cong K_{21} \overrightarrow{m}_{\mu} + F_{21} \overrightarrow{m}_{\nu} + F_{21} \overrightarrow{m}_{\nu}$$
with $F_{21} \equiv F_{2}/F_{1}$ and $K_{21} \equiv K_{2} - F_{21}K_{1}$.

 $F_i \& \vec{v}$ are unknown but $F_{21} \& K_{21}$ can be measured precisely.

Adding light new physics (NP)

New forces acts on electron & quarks leads to change of energy levels.



New physics part known, precisely calculated:

CI+MBPT: Dzuba, Flambaum & Kozlov (96) Berengut, Flambaum & Kozlov (06);

GRASP2K: Jonsson, Gaigalas, Biero, Fischer & Grant (2013)

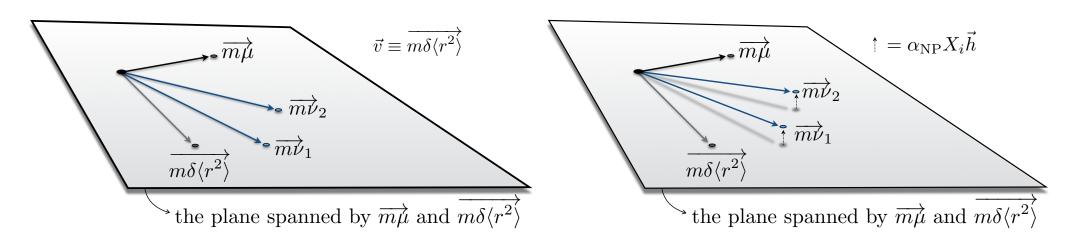
(Combination of the many-body perturbation theory with the configuration-interaction method)

$$\overrightarrow{m}\overrightarrow{\nu}_i = K_i \, \overrightarrow{m}_\mu + F_i \, \overrightarrow{v} + y_e y_n X_i \overrightarrow{h},$$
Delaunay, Ozeri, GP & Soreq (16)

$$\overrightarrow{m\nu}_2 = K_{21}\overrightarrow{m\mu} + F_{21}\overrightarrow{m\nu}_1 + \alpha_{NP}\overrightarrow{h}X_1 (X_{21} - F_{21}),$$

and $X_{21} \equiv X_2/X_1.$

Illustration: adding light new physics (NP)

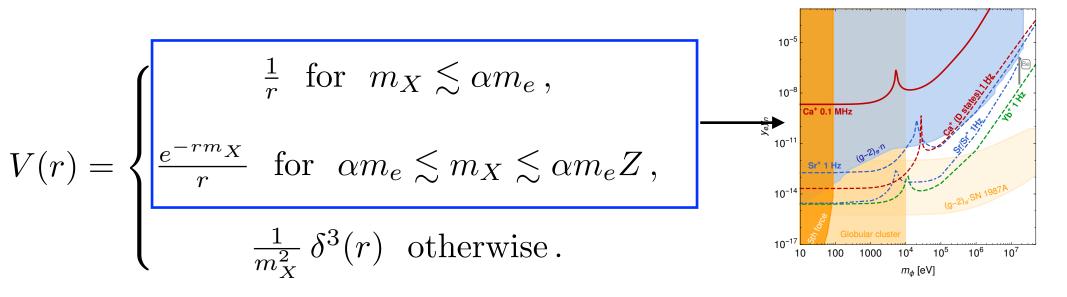


Light mediators

If mediator's mass, m_X , is smaller than inverse of outer electrons than the potential is Coulombic.

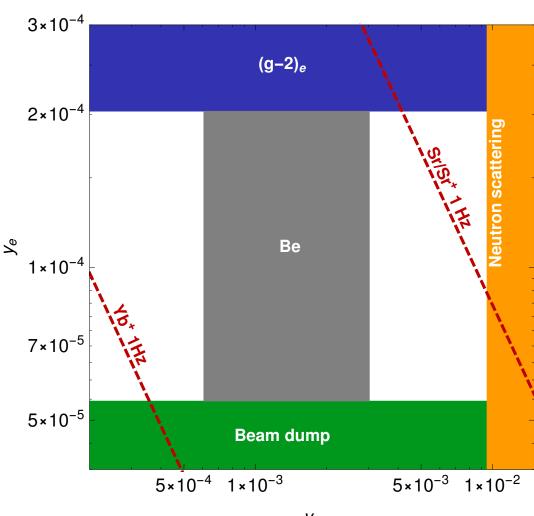
If mediator's mass is smaller than inverse distance of most inner electron from the nucleus then the full Yukawa potential is required.

Otherwise the potential is described via a delta function.

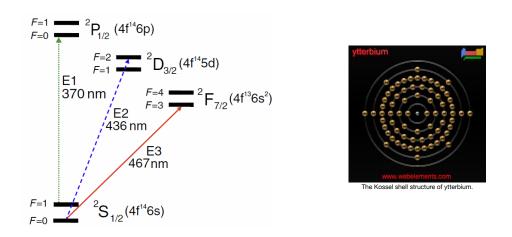


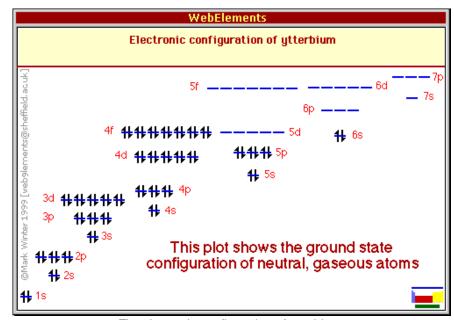
Be 17 MeV anomaly

Frugiuele, Fuchs, GP & Schlaffer v2 (16)



Ex.: Yb+ with Z=70, n=6 and A=168(4)-174(6).





The electronic configuration of ytterbium.

nuclide symbol	Z(p)	N(n)	isotopic mass (u)	half-life	decay mode(s) ^{[2][n 1]}	daughter isotope(s) ^[n 2]	nuclear spin	representative isotopic composition	range of natura
symbol	-	excitation energy			mode(s)	isotope(s).	spin	(mole fraction)	(mole fraction)
148 _{Yb}	70	78	147.96742(64)#	250# ms	β*	148 _{Tm}	0+		
⁴⁹ Yb	70	79	148.96404(54)#	0.7(2) s	B ⁺	¹⁴⁹ Tm	(1/2+.3/2+)		
150 _{Yb}	70	80	149.95842(43)#	700# ms [>200 ns]	β+	150Tm	0+		
					β*	¹⁵¹ Tm	-		
¹⁵¹ Yb	70	81	150.95540(32)	1.6(5) s	β ⁺ , p (rare)	150Er	(1/2+)		
					β+	¹⁵¹ Tm			
^{151m1} Yb	7	50(10	D)# keV	1.6(5) s	β ⁺ , p (rare)	150Er	(11/2-)		
151m2 _{Yb}	1	790(5	00)# keV	2.6(7) µs	P / P (=)		19/2-#		
151m3 _{Yb}			00)# keV	20(1) µs			27/2-#		
150					β*	¹⁵² Tm			
¹⁵² Yb	70	70 82	151.95029(22)	3.04(6) s	β ⁺ , p (rare)	¹⁵¹ Er	0+		
¹⁵³ Yb			83 152.94948(21)#	4.2(2) s	a (50%)	¹⁴⁹ Er	7/2-#		
	70	83			β ⁺ (50%)	¹⁵³ Tm			
					β*, p (.008%)	¹⁵² Er			
153mYb	2	700(1	00) keV	15(1) µs	,		(27/2-)		
		·			a (92.8%)	¹⁵⁰ Er			
154Yb	70	84	4 153.946394(19)	0.409(2) s	β+ (7.119%)	¹⁵⁴ Tm	0+		
155 _{Yb}			6 155.942818(12)	1.793(19) s 26.1(7) s	a (89%)	¹⁵¹ Er			
199Yb	70	85			β+ (11%)	¹⁵⁵ Tm	(7/2-)		
					β* (90%)	¹⁵⁶ Tm			
156 _{Yb}	70	86			a (10%)	¹⁵² Er	0+		
					β* (99.5%)	¹⁵⁷ Tm			
¹⁵⁷ Yb	70	87	156.942628(11)	38.6(10) s	a (.5%)	153Er	7/2-		
					β* (99.99%)	158 _{Tm}	0+		
¹⁵⁸ Yb	70	88	157.939866(9)	1.49(13) min	a (.0021%)	154Er			
159 _{Yb}	70	89	158.94005(2)	1.67(9) min	β*	159Tm	5/2(-)		
160 _{Yb}	70	90	159.937552(18)	4.8(2) min	β+	¹⁶⁰ Tm	0+		
161 _{Yb}	70	91	160.937902(17)	4.2(2) min	β*	¹⁶¹ Tm	3/2-		
¹⁶² Yb	70	92	161.935768(17)	18.87(19) min	β+	¹⁶² Tm	0+		
163 _{Yb}	70	93	162.936334(17)	11.05(25) min	β*	163 _{Tm}	3/2-		
¹⁶⁴ Yb	70	94	163.934489(17)	75.8(17) min	EC	¹⁶⁴ Tm	0+		
165 _{Vh}	70	95	164.93528(3)	9.9(3) min	β*	165 _{Tm}	5/2-		
166 _{Yb}	70	96	165.933882(9)	56.7(1) h	EC	¹⁶⁶ Tm	0+		
167 _{Yb}	70	97	166.934950(5)	17.5(2) min	β+	¹⁶⁷ Tm	5/2-		
168 _{Yb}	70	98	167.933897(5)		rationally Stable		0+	0.0013(1)	
169Yb	70		168.935190(5)	32.026(5) d	EC C	¹⁶⁹ Tm	7/2+	0.0010(1)	
169mYb			3) keV	46(2) s	IT	169 _{Yb}	1/2-		
170Yb	70	_	169.9347618(26)		rationally Stable	[n 4]	0+	0.0304(15)	
170mYb			5(14) keV	370(15) ns			4-	,	
171Yb			170.9363258(26)		rationally Stable	[n 5]	1/2-	0.1428(57)	
171m1Yb			2) keV	5.25(24) ms	IT STABLE	¹⁷¹ Yb	7/2+		
171m2Yb			5(2) keV	265(20) ns			5/2-		
172Yb	70	_	171.9363815(26)		rationally Stable	[n 6]	0+	0.2183(67)	
173 _{Yb}	70		172.9382108(26)		rationally Stable		5/2-	0.1613(27)	
173mYb		98.9(5		2.9(1) µs	,		1/2-	, , ,	
174Yb	70		173.9388621(26)		rationally Stable	[n 8]	0+	0.3183(92)	
175 _{Yb}	70		174.9412765(26)	4.185(1) d	β"	175Lu	7/2-		
			5(4) keV	68.2(3) ms	-		1/2-		
175myb	70		175.9425717(28)		rationally Stable	[n 9]	0+	0.1276(41)	
175mYb 176Yb		.00					(8)-	-//-/	
176 _{Yb}		050.0/	3) keV						
¹⁷⁶ Yb ^{176m} Yb		107	.,	11.4(3) s 1.911(3) h	β"	¹⁷⁷ Lu	(9/2+)		
176 _{Yb}	70		176.9452608(28)	1.911(3) h 6.41(2) s	β" IT	¹⁷⁷ Lu ¹⁷⁷ Yb ¹⁷⁸ Lu	(9/2+) (1/2-)		

Precision mass measurements: 10⁻¹⁰



Contents lists available at ScienceDirect

International Journal of Mass Spectrometry

journal homepage: www.elsevier.com/locate/ijms



The most precise atomic mass measurements in Penning traps

Edmund G. Myers*

Florida State University, Department of Physics, Tallahassee, FL 32306-4350, USA

Table 10Atomic masses of the most abundant isotopes of strontium and ytterbium measured at FSU [109].

Atom	FSU mass (u)	σ_m/m (ppt)
⁸⁶ Sr	85.909 260 730 9(91)	105
⁸⁷ Sг	86.908 877 497 0(91)	105
⁸⁸ Sr	87.905 612 257 1(97)	110
¹⁷⁰ Yb	169.934 767 241(18)	105
¹⁷¹ Yb	170.936 331 514(19)	110
¹⁷² Yb	171.936 386 655(18)	105
¹⁷³ Yb	172.938 216 213(18)	105
¹⁷⁴ Yb	173.938 867 539(18)	105
¹⁷⁶ Yb	175.942 574 702(22)	125

Partial solution, comparing different isotope shift, searching of nonlinearity in "King plot"

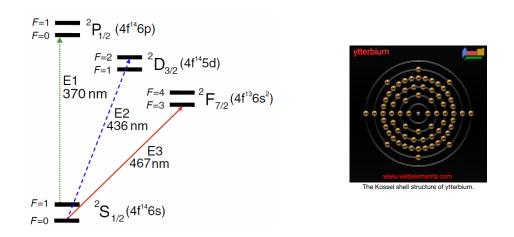
King's factorisation formula (King, 1963): only depend on e-transition $\delta \nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'} = K_i \, \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'},$ $(\mu_{AA'} \equiv 1/m_A - 1/m_{A'} = (A' - A)/(AA') \, \text{amu}^{-1}, \, \text{where amu} \approx 0.931 \, \text{GeV})$ only depend on nucleus

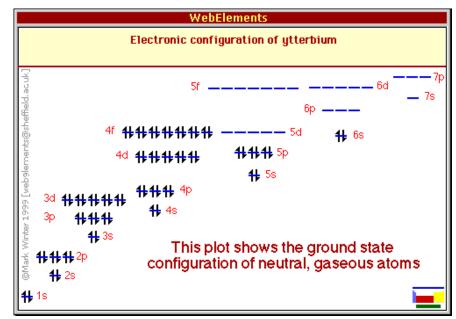
We can solve for $\delta \langle r^2 \rangle_{AA'}$ to get a linear relation:

$$m\delta\nu_{AA'}^2 = F_{21}m\delta\nu_{AA'}^1 + K_{21}$$
,

(with $K_{21} \equiv (K_2 - F_{21}K_1)$ and $F_{21} \equiv F_2/F_1$ and $m\delta\nu_{AA'}^i \equiv \delta\nu_{AA'}^i/\mu_{AA'}$.)

Ex.: Yb+ with Z=70, n=6 and A=168(4)-174(6).





The electronic configuration of ytterbium.

nuclide symbol	Z(p)	N(n)	isotopic mass (u)	half-life	decay	daughter	nuclear	representative	range of natura
symbol	\vdash	excitation energy			mode(s) ^{[2][n 1]}	isotope(s) ^[n 2]	spin	composition (mole fraction)	(mole fraction)
148 _{Yb}	70		147,96742(64)#	250# ms	β*	¹⁴⁸ Tm	0+	(more maction)	
149 _{Yb}	70	79	148.96404(54)#	0.7(2) s	β*	¹⁴⁹ Tm	(1/2+,3/2+)		
150Yb	70	80	149.95842(43)#	700# ms [>200 ns]	β*	150Tm	0+		
		00	143.33042(43)#	100# 1118 [*200 118]	β*	¹⁵¹ Tm	0.		
¹⁵¹ Yb	70	81	150.95540(32)	1.6(5) s	β ⁺ , p (rare)	¹⁵⁰ Er	(1/2+)		
					β ⁺	¹⁵¹ Tm			
^{151m1} Yb	7	50(100	D)# keV	1.6(5) s	β ⁺ , p (rare)	150Er	(11/2-)		
151m2 _{Yb}	1	790(50	00)# keV	2.6(7) µs			19/2-#		
151m3 _{Yb}	2	450(50	00)# keV	20(1) µs			27/2-#		
152 _{Yb}					β*	¹⁵² Tm			
YD	70	82	151.95029(22)	3.04(6) s	β ⁺ , p (rare)	¹⁵¹ Er	0+		
			152.94948(21)#		a (50%)	¹⁴⁹ Er	7/2-#		
153 _{Yb}	70	83		4.2(2) s	β+ (50%)	¹⁵³ Tm			
					β*, p (.008%)	¹⁵² Er			
153mYb	2	700(10	00) keV	15(1) µs			(27/2-)		
154 _{Yb}	70	84	153.946394(19)	0.409(2) s	a (92.8%)	¹⁵⁰ Er	0+		
10	70	04		0.409(Z) \$	β+ (7.119%)	¹⁵⁴ Tm			
155 _{Yb}	70	85	154.945782(18)	1,793(19) s	a (89%)	¹⁵¹ Er	(7/2-)		
10	70	65	134.543762(10)	1.793(19)5	β+ (11%)	¹⁵⁵ Tm			
156 _{Yb}	70	86	155.942818(12)	26.1(7) s	β* (90%)	¹⁵⁶ Tm	0+		
10	70	00			a (10%)	¹⁵² Er			
157 _{Yb}	70	87	7 156.942628(11)	38.6(10) s	β* (99.5%)	¹⁵⁷ Tm	7/2-		
10	70	01			a (.5%)	¹⁵³ Er			
158 _{Yb}	70	88	157.939866(9)	1.49(13) min	β* (99.99%)	¹⁵⁸ Tm	0+		
	10	00	137.333000(3)	1.40(15)11111	a (.0021%)	¹⁵⁴ Er	0.		
¹⁵⁹ Yb	70	89	158.94005(2)	1.67(9) min	β*	¹⁵⁹ Tm	5/2(-)		
¹⁶⁰ Yb	70	90	159.937552(18)	4.8(2) min	β+	¹⁶⁰ Tm	0+		
¹⁶¹ Yb	70	91	160.937902(17)	4.2(2) min	β*	¹⁶¹ Tm	3/2-		
¹⁶² Yb	70	92	161.935768(17)	18.87(19) min	β+	¹⁶² Tm	0+		
¹⁶³ Yb	70	93	162.936334(17)	11.05(25) min	β*	¹⁶³ Tm	3/2-		
¹⁶⁴ Yb	70		163.934489(17)	75.8(17) min	EC	¹⁶⁴ Tm	0+		
				9.9(3) min	β*	¹⁶⁵ Tm	5/2-		
¹⁶⁵ Yb	70		164.93528(3)						
¹⁶⁶ Yb	70	96	165.933882(9)	56.7(1) h	EC	¹⁶⁶ Tm	0+		
¹⁶⁶ Yb ¹⁶⁷ Yb	70 70	96 97	165.933882(9) 166.934950(5)	56.7(1) h 17.5(2) min	EC β ⁺	¹⁶⁷ Tm	5/2-		
¹⁶⁶ Yb ¹⁶⁷ Yb ¹⁶⁸ Yb	70 70 70	96 97 98	165.933882(9) 166.934950(5) 167.933897(5)	56.7(1) h 17.5(2) min Observ	EC β* rationally Stable	¹⁶⁷ Tm [n 3]	5/2- 0+	0.0013(1)	
¹⁶⁶ Yb ¹⁶⁷ Yb ¹⁶⁸ Yb	70 70 70 70	96 97 98 99	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5)	56.7(1) h 17.5(2) min Observ 32.026(5) d	EC β* vationally Stable	¹⁶⁷ Tm ^[n 3]	5/2- 0+ 7/2+	0.0013(1)	
165Yb 167Yb 168Yb 169Yb 169mYb	70 70 70 70	96 97 98 99	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s	EC β ⁺ rationally Stable EC	¹⁶⁷ Tm ^[n 3] 169Tm 169 _{Yb}	5/2- 0+ 7/2+ 1/2-		
166Yb 167Yb 168Yb 169Yb 169mYb	70 70 70 70 70 2	96 97 98 99 4.199(165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s	EC β* vationally Stable	¹⁶⁷ Tm ^[n 3] 169Tm 169 _{Yb}	5/2- 0+ 7/2+ 1/2- 0+	0.0013(1)	
166Yb 167Yb 168Yb 169Yb 169mYb 1700Yb 170mYb	70 70 70 70 70 2 70	96 97 98 99 4.199(100 258.46	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 5(14) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns	EC β* rationally Stable EC IT	167Tm [n 3] 169 Tm 169 Y b	5/2- 0+ 7/2+ 1/2- 0+ 4-	0.0304(15)	
166Yb 167Yb 168Yb 169Yb 169mYb 170Yb 170mYb	70 70 70 70 70 2 70 1	96 97 98 99 4.199(100 258.46	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 3(14) keV 170.9363258(26)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ	EC β* rationally Stable EC IT rationally Stable rationally Stable	167 Tm [n 3] 169 Tm 169 Yb [n 4]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2-		
166Yb 167Yb 168Yb 169Yb 169mYb 170mYb 171Tyb 171m1Yb	70 70 70 70 2 70 1 70	96 97 98 99 4.199(100 258.46 101	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 3(14) keV 170.9363258(26) 2) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms	EC β* rationally Stable EC IT	167Tm [n 3] 169 Tm 169 Y b	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+	0.0304(15)	
166Yb 167Yb 168Yb 169Wyb 169myb 170Yb 170myb 171Yb 171m1Yb 171m1Yb	70 70 70 70 70 2 70 1 70 9	96 97 98 99 4.199(100 258.46 101 95.282(22.416	165.933882(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 5(14) keV 170.9363258(26) 2) keV 5(2) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns	EC β* ationally Stable EC IT ationally Stable IT	167Tm In 3] 169Tm 169Yb In 4] In 5]	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2-	0.0304(15)	
166Yb 167Yb 168Yb 169Yb 169MYb 170MYb 170MYb 171M1Yb 171m1Yb 171m2Yb	70 70 70 70 2 70 1 70 9 1	96 97 98 99 4.199(100 258.46 101 5.282(22.416	165 933882(9) 166 934950(5) 167 933897(5) 168 935190(5) 3) keV 169 9347618(26) 5(14) keV 170 9363258(26) 2) keV 171 9363815(26)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ	EC β* Autionally Stable EC IT	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 5] 171 Yb	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+	0.0304(15) 0.1428(57) 0.2183(67)	
166Yb 167Yb 168Yb 169Yb 169Wyb 169Wyb 170Yb 170Wyb 171m1Yb 171m2Yb 172Yb	70 70 70 70 2 70 1 70 9 1 70	96 97 98 99 4.199(100 258.46 101 5.282(22.416 102 103	165.933682(9) 166.934950(5) 167.933697(5) 167.933697(5) 3) keV 169.9347618(26) 5(14) keV 170.9963258(26) 2) keV 171.9363815(26) 172.9382108(26)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ	EC β* ationally Stable EC IT ationally Stable IT	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 5] 171 Yb	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2-	0.0304(15)	
1664yb 167yb 168yb 168myb 169myb 170myb 171myb 171m1yb 171m2yb 172yb 173myb 173myb	70 70 70 70 2 70 1 70 9 1 70 70	96 97 98 99 4.199(100 258.46 101 5.282(22.416 102 103 98.9(5	165.933862(9) 166.934950(5) 167.933897(5) 168.935190(5) 3) keV 169.9347618(26) 5(14) keV 170.9363258(26) 2) keV 5(2) keV 171.9363815(26) 172.9382108(26)) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ 2.9(1) µs	EC p* artionally Stable EC rationally Stable rationally Stable iT rationally Stable it rationally Stable	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 5] 171 Yb	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166Yb 167Yb 168Yb 169Yb 169MYb 170MYb 170MYb 171M1Yb 171M2Yb 172Yb 173MYb 173MYb 173MYb	70 70 70 70 2 70 1 70 9 1 70 70	96 97 98 99 14.199(100 258.46 101 15.282(22.416 102 103 198.9(5 104	165 933862(9) 166 934950(5) 168 934950(5) 168 935190(5) 3) keV 169 9347618(26) 5(14) keV 170 9363258(26) 2) keV 171 9363815(26) 172 9362108(26)) keV 173 9388621(26)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ 2.9(1) µs Observ	EC p* attionally Stable EC rationally Stable rationally Stable iT rationally Stable rationally Stable rationally Stable	167 Tm n 3 169 Tm 169 Yb n 4 n 5 171 Yb n 6 n 7	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0+	0.0304(15) 0.1428(57) 0.2183(67)	
166Yb 167Yb 168Yb 169Yb 169MYb 170MYb 170MYb 171M1Yb 171M2Yb 172Yb 173MYb 173MYb 174Yb 174Wb 175Yb	70 70 70 70 70 1 1 70 9 1 1 70 70 70	96 97 98 99 99 100 258.46 101 15.282(22.416 102 103 103 198.9(5 104 105	165 933862(9) 166 934950(5) 167 933897(5) 167 933897(5) 33 keV 169 9347618(26) 31 keV 170 9363258(26) 22 keV 171 9363258(26) 172 9362108(26) 174 9412765(26)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ 2.9(1) µs Observ 4.185(1) d	EC p* artionally Stable EC rationally Stable rationally Stable iT rationally Stable it rationally Stable	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 5] 171 Yb	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0+ 7/2-	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166Yb 167Yb 168Yb 168Yb 170Yb 170Yb 171Yb 171m2Yb 172Yb 173mYb 173mYb 174Yb 175mYb 175mYb 175mYb	70 70 70 70 70 1 1 70 9 1 1 70 70 70 70 70	96 97 98 99 100 258.46 101 155.282(22.416 102 103 198.9(5) 104 105 114.865	165 933862(9) 166 934950(5) 167 933897(5) 167 933897(5) 30 keV 169 9347618(26) \$(14) keV 170 9363258(26) 20 keV 171 9363258(26) 21 keV 171 9363815(26) 172 9382108(26) 0 keV 173 9388621(26) 174 9412765(26) 5(4) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Quality Observ 4.185(1) d 68.2(3) ms	EC β* Tationally Stable EC IT Tationally Stable Attainally Stable Attainally Stable Attainally Stable	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 6] 171 Yb [n 6] [n 7] [n 8] 175 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27) 0.3183(92)	
1664yb 167yb 168yb 169yb 169yb 170yb 170yb 171yb 171m1yb 172yb 173myb 173myb 174yb 175myb 175myb 175myb 175myb	70 70 70 70 1 70 9 1 70 70 70 70 70	96 97 98 99 94.199(100 258.46 101 105.282(22.416 102 103 104 105 104 105 106	165 933682(9) 166 934950(5) 167 933697(5) 167 933697(5) 169 9347618(26) (14) keV 169 9347618(26) (14) keV 170 9963258(26) 2) keV 171 9363815(26) 172 9382108(26)) keV 173 9386621(26) 174 9412765(26) (44) keV 175 9425717(28)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ 4.185(1) d 68.2(3) ms Observ	EC p* attionally Stable EC rationally Stable rationally Stable iT rationally Stable rationally Stable rationally Stable	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 6] 171 Yb [n 6] [n 7] [n 8] 175 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 1/2- 0- 1/2- 1/2- 0- 1/2- 1/	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27)	
166Yb 167Yb 168Yb 168Wb 169Wyb 170Yb 171mYb 171m2Yb 171m2Yb 1773Wb 173mYb 175Yb 175Yb 175Yb 175Yb	70 70 70 70 70 1 1 70 9 1 1 70 70 70 70 5 70	96 97 98 99 94.199(100 258.46 101 15.282(22.416 102 103 198.9(5 104 105 106 0050.0(165 933862(9) 166 934950(5) 166 934950(5) 167 933897(5) 168 935190(5) 3) keV 169 9347618(26) 5(14) keV 170 9363258(26) 2) keV 171 9363815(26) 172 9382108(26) 3) keV 173 9388621(26) 174 9412765(26) 5(4) keV	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ 4.185(1) d 68.2(3) ms Observ 11.4(3) s	$\begin{array}{c} EC \\ \beta^* \\ pt^* \\ pt^* \\ EC \\ iT \\ attionally Stable \\ pt^* \\ $	167 Tm	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 1/2- 0- 0- 1/2- 0- 0- 1/2- 0- 1/2- 0- 0- 1/2- 0- 0- 1/2- 0- 0- 1/2- 0- 0- 0- 1/2- 0- 0- 1/2- 0- 0- 0- 0- 0- 1/2- 0- 0- 0- 0- 0- 0- 0- 0- 0- 0	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27) 0.3183(92)	
166Yb 167Yb 168Yb 168Yb 168Yb 170Yb 177Yb 1771Yb 1771Yb 1771Yb 173YYb 173YYb 173YYb 175YYb 175YYb 175YYb	70 70 70 70 70 11 70 9 11 70 70 70 5 70 11 70	96 97 98 99 94.199(100 258.46 101 15.282(22.416 102 103 198.9(5 104 105 106 0050.0(165 933862(9) 166 934950(5) 168 934950(5) 168 935190(5) 3) keV 169 9347618(26) 8(14) keV 170.9363258(26) 2) keV 171.9363215(26) 172.9382108(26)) keV 173.9389621(26) 174.9412765(26) (4) keV 175.9425717(28) 3) keV 176.9452608(28)	56.7(1) h 17.5(2) min Observ 32.026(5) d 46(2) s Observ 370(15) ns Observ 5.25(24) ms 265(20) ns Observ Observ 4.185(1) d 68.2(3) ms Observ	EC β* Tationally Stable EC IT Tationally Stable Attainally Stable Attainally Stable Attainally Stable	167 Tm [n 3] 169 Tm 169 Yb [n 4] [n 6] 171 Yb [n 6] [n 7] [n 8] 175 Lu	5/2- 0+ 7/2+ 1/2- 0+ 4- 1/2- 7/2+ 5/2- 0+ 5/2- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 0- 1/2- 1/2- 0- 1/2- 1/2- 0- 1/2- 1/	0.0304(15) 0.1428(57) 0.2183(67) 0.1613(27) 0.3183(92)	

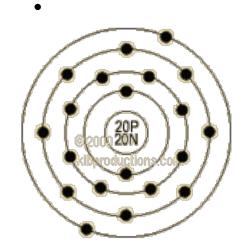
Ex.: $Sr^{(+)}$ with Z=38, n=5 and A=84-88 (90).

- Electron Configuration:1s² 2s²p⁶ 3s²p⁶d¹⁰ 4s²p⁶ 5s²(1)
- Electrons per **Energy Level**: 2,8,18,8,2(1)

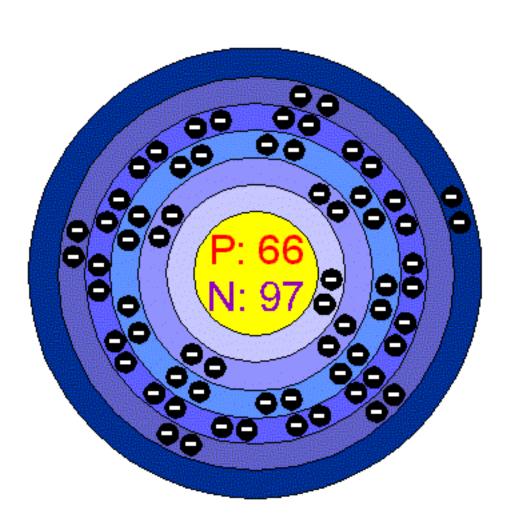


Ex.: $Ca^{(+)}$ with Z=20, n=4 and A=40-48.

- Electron Configuration: 1s² 2s²p⁶ 3s²p⁶ 4s¹
- Electrons per **Energy Level**: 2,8,8,2(1)



Ex.: Dy with Z=66, n=6 and A=158-164.



Number of Energy Levels: 6

First Energy Level: 2 Second Energy Level: 8 Third Energy Level: 18 Fourth Energy Level: 28 Fifth Energy Level: 8

Sixth Energy Level: 2

The observables

• We have 3 isotope shifts $(AA'_{1,2,3})$ for 2 transitions (i=1,2):

$$\overrightarrow{m\nu}_i \equiv \left(m\nu_i^{AA_1'}, m\nu_i^{AA_2'}, m\nu_i^{AA_3'}\right)$$

$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'}. \qquad m\nu_i^{AA'} \equiv \nu_i^{AA'}/\mu_{AA'}$$

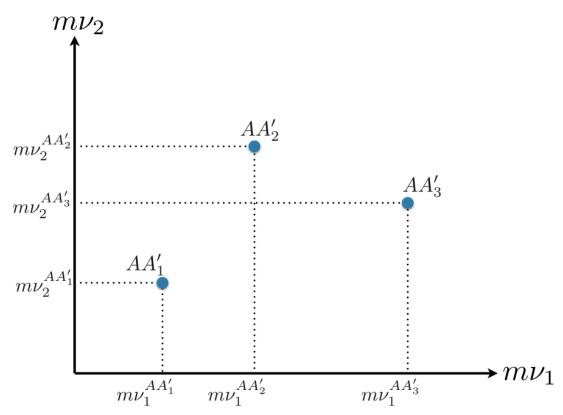
$$\mu_{AA'} \equiv m_A^{-1} - m_{A'}^{-1}$$

Target accuracy: $\Delta m \nu_i^{AA'}/m \nu_i^{AA'} \lesssim 10^{-6}$. (currently: 10^{-4} , projected $< 10^{-9}$)

The observable: King comparison (1964)

• What would be the generic form of $\overrightarrow{m\nu}_2$ vs. $\overrightarrow{m\nu}_1$?

• 3 ISs -
$$m\nu_2 = am\nu_1^2 + bm\nu_1 + c$$
:



What about existing data?

Limitation of method

$$\alpha_{\rm NP} = \frac{(\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu}}{(\overrightarrow{m\mu} \times \overrightarrow{h}) \cdot (X_1 \, \overrightarrow{m\nu}_2 - X_2 \, \overrightarrow{m\nu}_1)}$$

Berengut, Budker, Delaunay, Flambaum, Frugiuele, Fuchs, Grojean, Harnik, Ozeri, GP & Soreq (17)

- Only useful to bound new physics (barring cancellation).
- Short range NP: $X_i \propto F_i \Rightarrow \vec{v}$ is redefined to absorb NP; requires extra carefulness when approaching this limit.
- As long as linearity holds bounds are limited by exp' accuracy:

$$\alpha_{\rm NP} \lesssim \sigma_{\alpha_{\rm NP}} = \sqrt{\Sigma_k (\partial \alpha_{\rm NP}/\partial O_k)^2 \sigma_k^2},$$

$$(O_K \text{ various exp' observables.})$$

Once non-linearity observed bound will be set by observation.