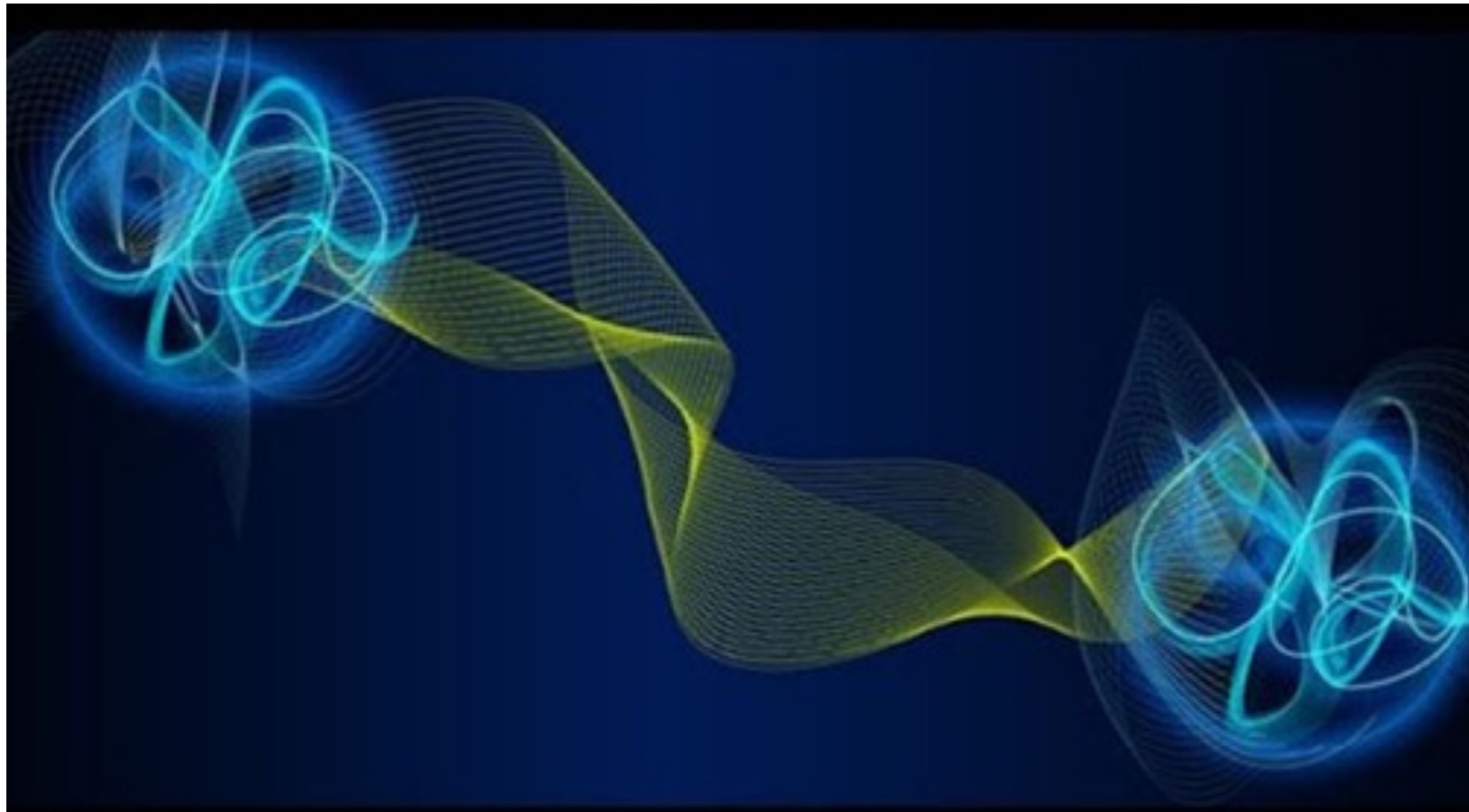


Entanglement & Symmetry



S. Beane, DBK, N. Klco, M. Savage, PRL 122, 102001 (2019), arXiv: 1812.03138

Symmetry is the theorist's friend:

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- Gauge symmetries
- (Approximate) global symmetries
- Anomalous symmetries

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- (Approximate) global symmetries
- Anomalous symmetries
- UV more symmetric than IR
 - ◆ Spontaneously broken symmetries
 - ◆ UV fixed point
- IR more symmetric than UV
 - ◆ IR fixed point
 - ◆ Accidental symmetries

...but never enough approximate symmetry!

- hierarchy
- mass textures
- suppression of FCNC

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Are there other ways that approximate symmetries can emerge?

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This talk:

- unexpected approximate symmetries can arise
- correlated with minimization of entanglement in scattering.

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- hierarchy
- mass textures
- suppression of FCNC

Are there other ways that approximate symmetries can emerge?

This talk:

- unexpected approximate symmetries can arise
- correlated with minimization of entanglement in scattering.

Examples from low energy hadronic physics:

- “Schrödinger symmetry” (nonrelativistic conformal symmetry)
- spin-flavor symmetries

Emergent symmetries seen in the baryons:

- i. $SU(4)$, $SU(6)$ spin-flavor symmetry
- ii. $SU(4)$ Wigner symmetry
- iii. Schrödinger (conformal) symmetry
- iv. $SU(16)$ (?!) in baryon octet

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explanation

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Large- N_c
explanation

No
explanation

Approximate SU(4), SU(6) spin-flavor symmetry (1960s)

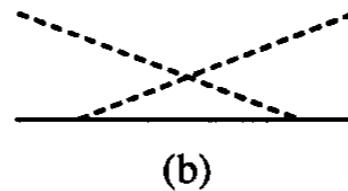
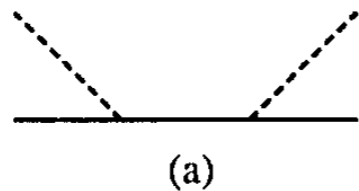
$$\text{SU(4):} \quad 4 = \begin{pmatrix} u \uparrow \\ u \downarrow \\ d \uparrow \\ d \downarrow \end{pmatrix}$$

$$\text{SU(6):} \quad 6 = \begin{pmatrix} u \uparrow \\ u \downarrow \\ d \uparrow \\ d \downarrow \\ s \uparrow \\ s \downarrow \end{pmatrix}$$

- Symmetry of non-relativistic quark model
- Approximate symmetry apparent in nature:
 - masses
 - magnetic moments & transitions
 - semi-leptonic currents
 - meson-baryon couplings
 - NN scattering

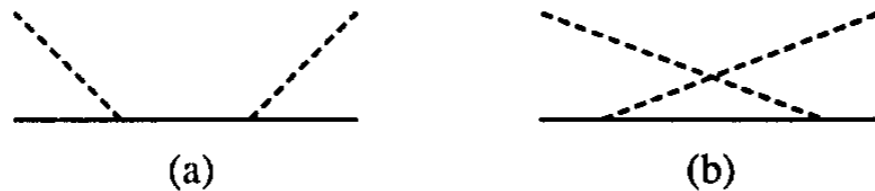
SU(4), SU(6) spin-flavor symmetry

- Cannot be a symmetry of relativistic QFT (Coleman-Mandula)
- For **baryon-meson** couplings, does follow from QCD in large- N_c limit
Gervais, Sakita (1984); Dashen, Manohar (1993), Dashen, Jenkins, Manohar (1994)



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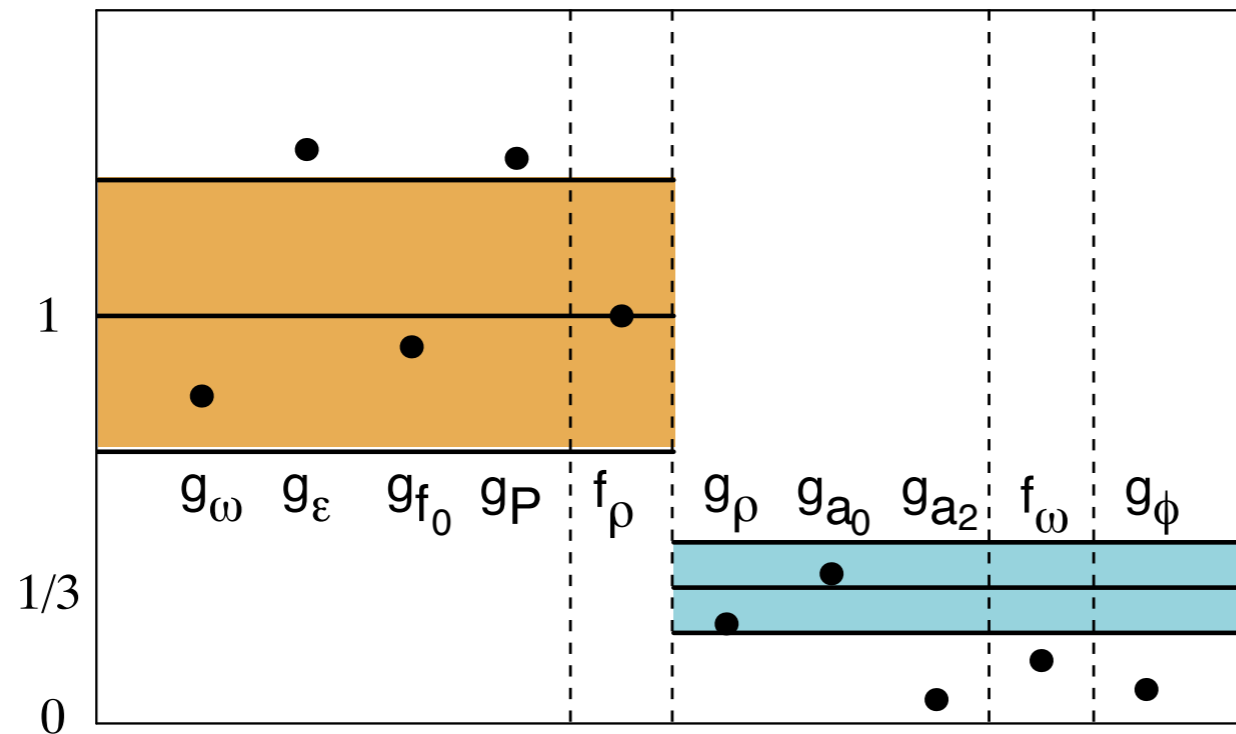
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But what about **baryon-baryon** interactions?

Is large- N is evident in nuclear physics? Yes!

- Large- N_c seems to also work in nuclear physics:



DBK, A. Manohar (1996)

Comparison of phenomenological models vs large- N predictions

- ...and large- N implies spin-flavor symmetries in low energy baryon-baryon interactions... DBK, M.J. Savage (1995)

SU(2N_f) spin-flavor symmetries in low energy baryon-baryon interactions also follows from large-N_c

- N_f = 2: nucleons & Δ in 20 dim irrep of SU(4)

$$\mathcal{L}_6 = -\frac{1}{f_\pi^2} \left[a(\Psi_{\mu\nu\rho}^\dagger \Psi^{\mu\nu\rho})^2 + b\Psi_{\mu\nu\sigma}^\dagger \Psi^{\mu\nu\tau} \Psi_{\rho\delta\tau}^\dagger \Psi^{\rho\delta\sigma} \right]$$

$$\Psi^{(\alpha i)(\beta j)(\gamma k)} = \Delta_{\alpha\beta\gamma}^{ijk} + \frac{1}{\sqrt{18}} \left(N_\alpha^i \epsilon^{jk} \epsilon_{\beta\gamma} + N_\beta^j \epsilon^{ik} \epsilon_{\alpha\gamma} + N_\gamma^k \epsilon^{ij} \epsilon_{\alpha\beta} \right)$$

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- N_f = 2 (restricted to nucleons)

$$\mathcal{L}_6 = -\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger \vec{\sigma} N)^2 \quad \Rightarrow \quad \text{general Weinberg (1990)}$$

$$C_S = \frac{2(a - b/27)}{f_\pi^2}, \quad C_T = 0 \quad \Rightarrow \quad \text{SU(4) prediction}$$

Does this work?

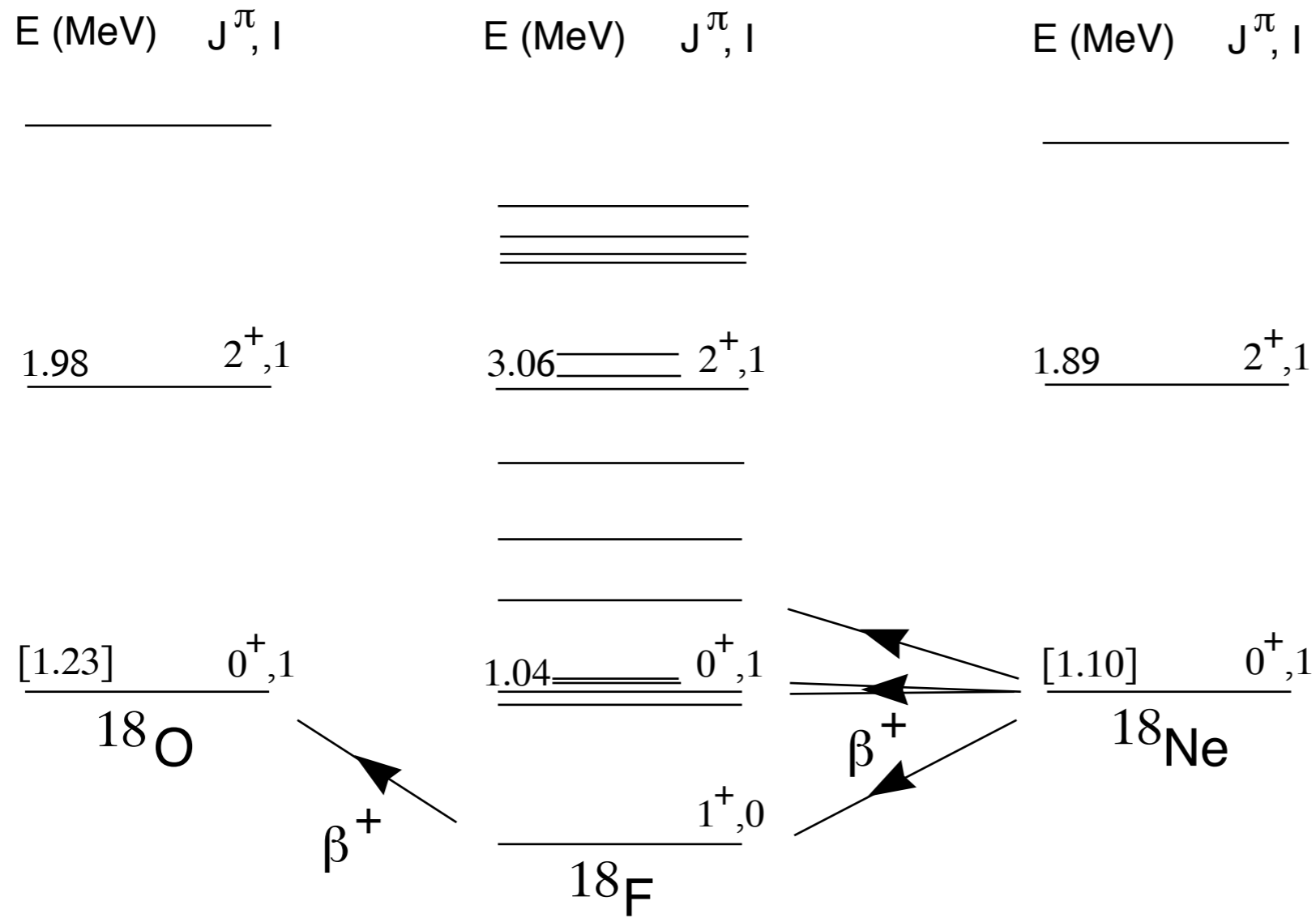
Diagnostic: accidental $SU(4)_{\text{Wigner}}$ symmetry

$$\mathcal{L}_6 = -\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(\cancel{N^\dagger \vec{\sigma} N})^2$$

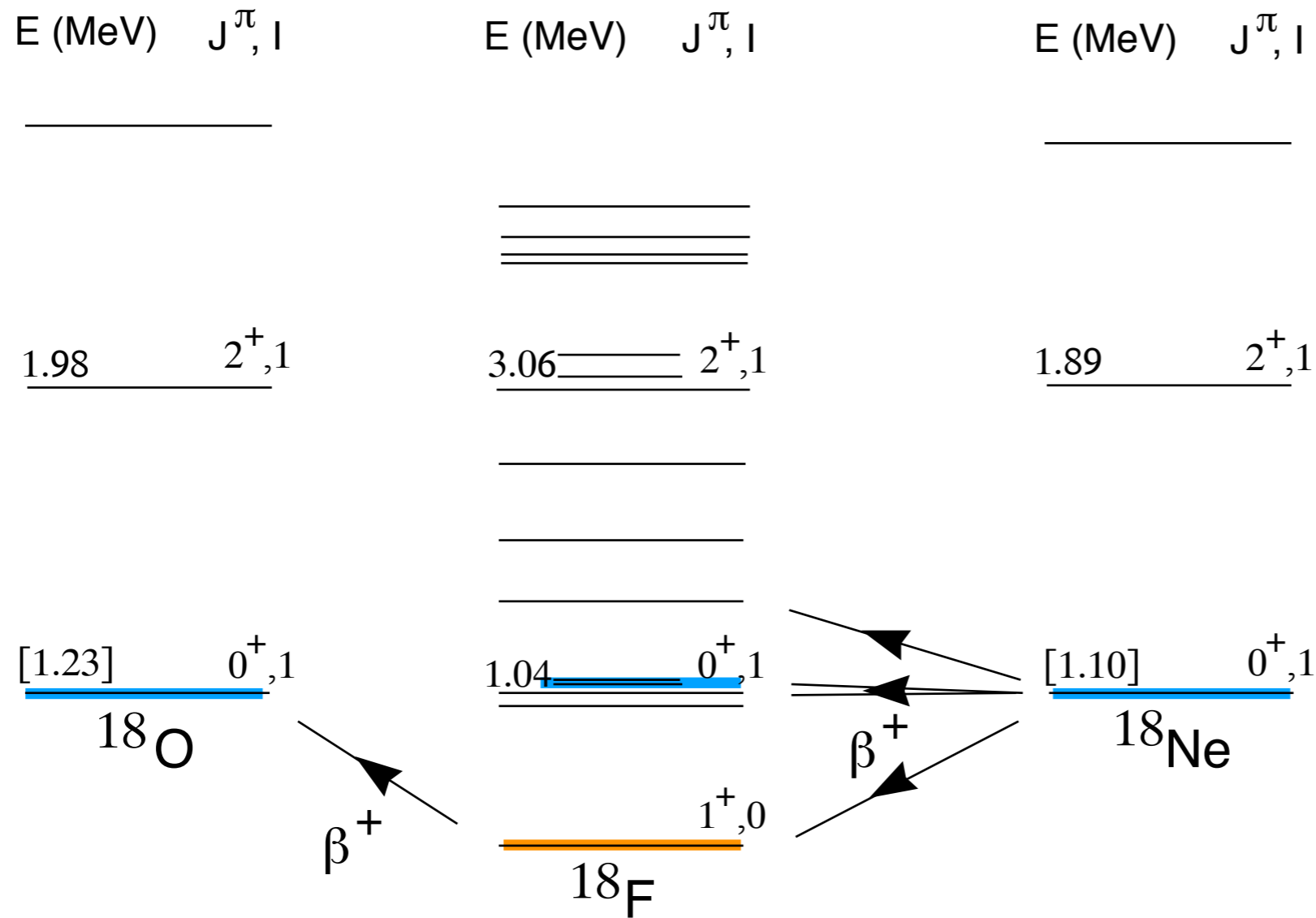
$$N = \begin{pmatrix} p \uparrow \\ p \downarrow \\ n \uparrow \\ n \downarrow \end{pmatrix}$$

Is $SU(4)_{\text{Wigner}}$ symmetry seen in nuclear physics? Yes!

Example of evidence for $SU(4)_{\text{Wigner}}$: β -decay in $A=18$ isobars

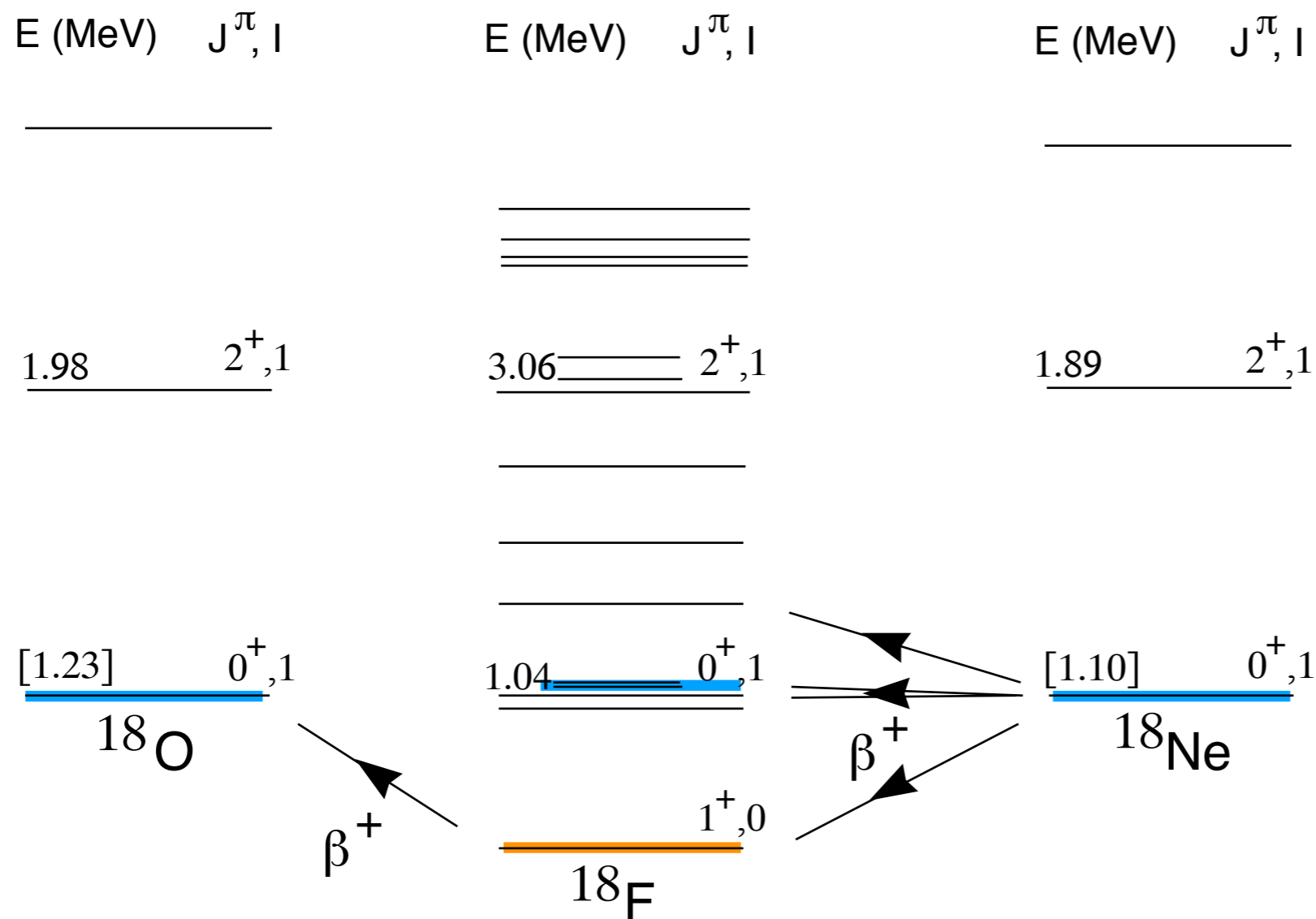


Example of evidence for $SU(4)_{\text{Wigner}}$: β -decay in $A=18$ isobars



$$(1,0) + (0,1) = 6 \text{ of } SU(4)$$

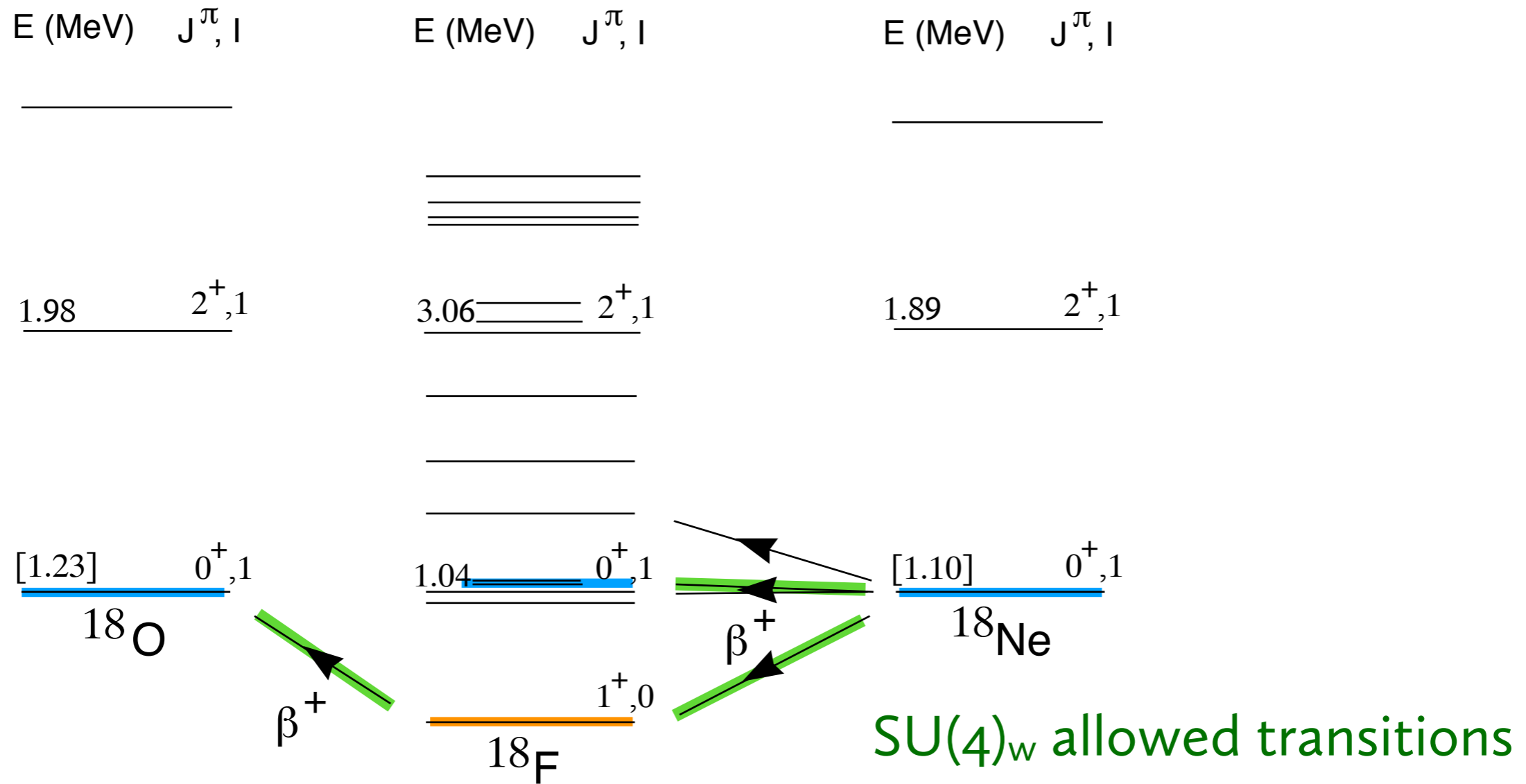
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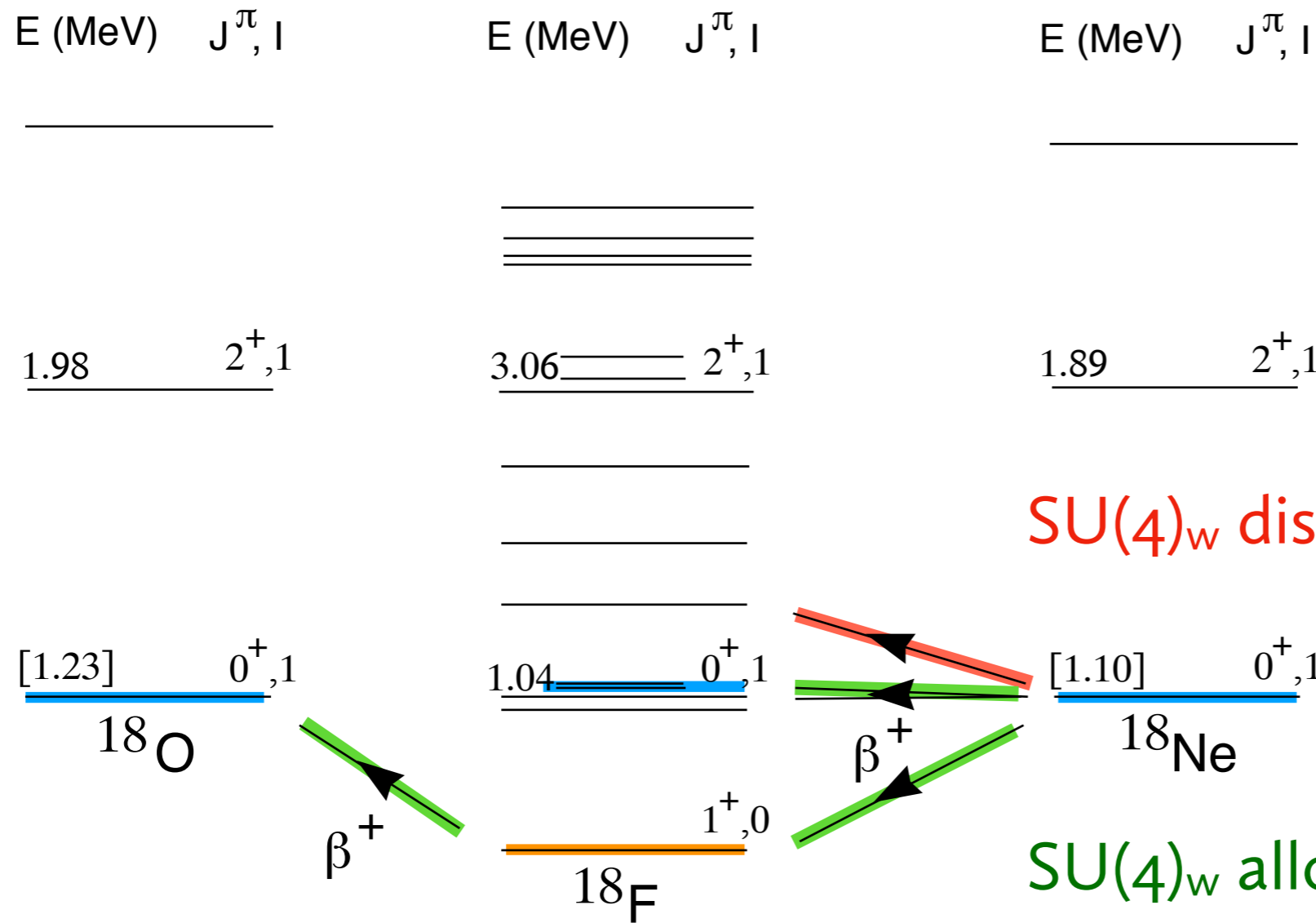
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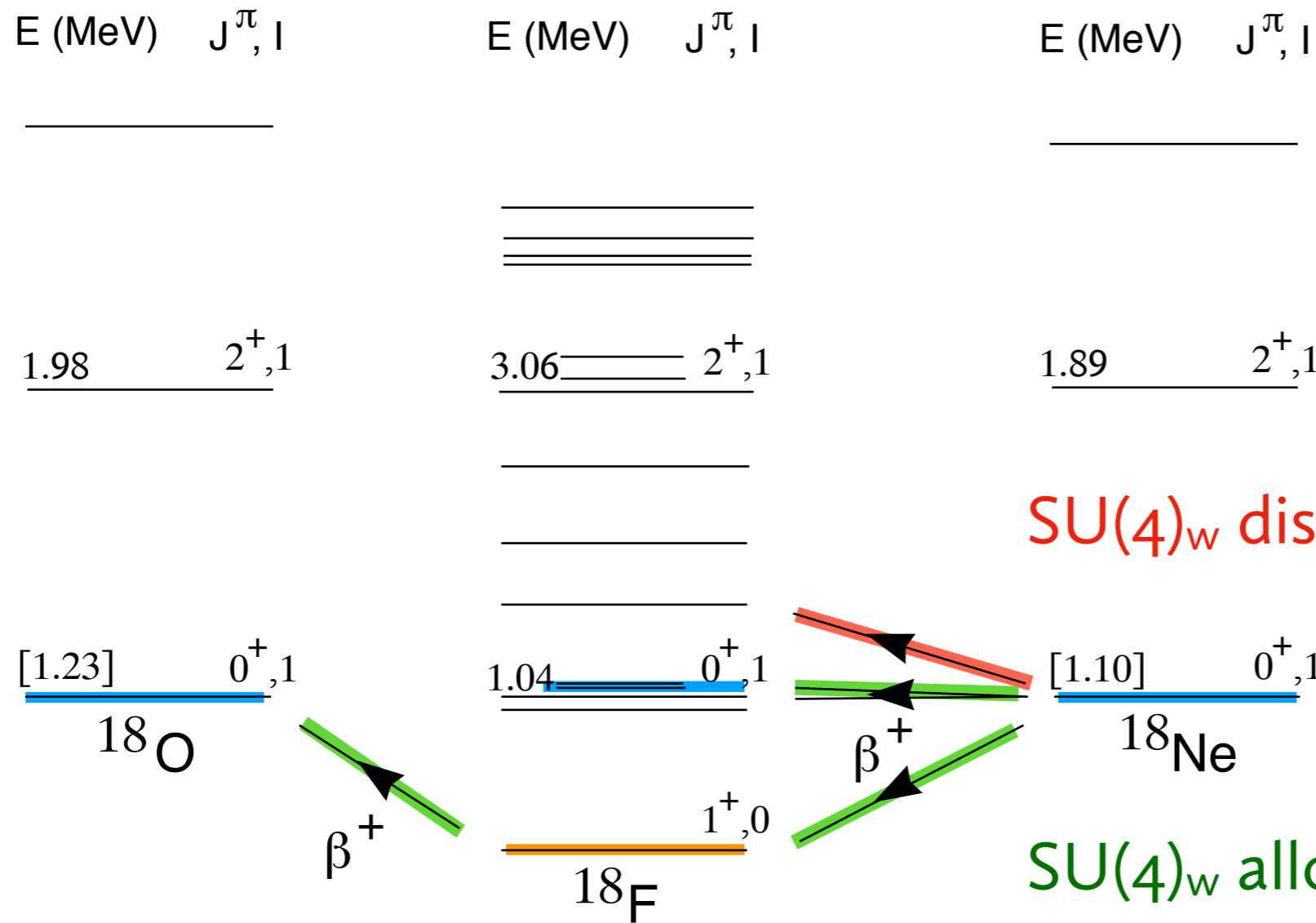
$SU(4)_w$ disallowed transitions

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$SU(4)_w$ allowed matrix elements ~ 10 x greater than $SU(4)_w$ disallowed

So far: no surprises? — emergent spin-flavor symmetries can be explained by large- N_c

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First surprise: unnaturally large scattering lengths in NN scattering give approximate Schrödinger symmetry (nonrelativistic conformal symmetry)

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First surprise: unnaturally large scattering lengths in NN scattering give approximate Schrödinger symmetry (nonrelativistic conformal symmetry)

1S_0 scattering length = -23.7 fm $\sim 1/8$ MeV

3S_1 scattering length = $+5.4$ fm $\sim 1/35$ MeV

$$A \simeq \frac{4\pi}{M} \frac{1}{\left(-\frac{1}{a} + i\sqrt{ME}\right)}$$

$1/a$ is very small for both

Who ordered that??

Second surprise arises for $N_f=3$:

Baryon-baryon interactions $N_f=3$;

SU(6) spin-flavor symmetry predicted by large- N_c

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 baryon decuplet

 baryon octet

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baryon decuplet
baryon octet

Low-energy EFT for just the octet:

M.J. Savage, M.B. Wise (1995)

$$\mathcal{L} = -c_1 \text{Tr} B_i^\dagger B_i B_j^\dagger B_j - c_2 \text{Tr} B_i^\dagger B_j B_j^\dagger B_i - c_3 \text{Tr} B_i^\dagger B_j^\dagger B_i B_j$$

$$-c_4 \text{Tr} B_i^\dagger B_j^\dagger B_j B_i - c_5 \text{Tr} B_i^\dagger B_i \text{Tr} B_j^\dagger B_j - c_6 \text{Tr} B_i^\dagger B_j \text{Tr} B_j^\dagger B_i$$

$$B_i = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}_i$$

$i = \uparrow, \downarrow$

$$\begin{aligned} \mathcal{L} = & -c_1 \text{Tr} B_i^\dagger B_i B_j^\dagger B_j - c_2 \text{Tr} B_i^\dagger B_j B_j^\dagger B_i - c_3 \text{Tr} B_i^\dagger B_j^\dagger B_i B_j \\ & -c_4 \text{Tr} B_i^\dagger B_j^\dagger B_j B_i - c_5 \text{Tr} B_i^\dagger B_i \text{Tr} B_j^\dagger B_j - c_6 \text{Tr} B_i^\dagger B_j \text{Tr} B_j^\dagger B_i \end{aligned}$$

SU(6) prediction:

$$\begin{aligned} c_1 &= -\frac{7}{27}b, & c_2 &= \frac{1}{9}b, & c_3 &= \frac{10}{81}b, \\ c_4 &= -\frac{14}{81}b, & c_5 &= a + \frac{2}{9}b, & c_6 &= -\frac{1}{9}b. \end{aligned}$$

Does this work? Look at lattice data

- NPLQCD collaboration, 2015
- equal quark masses
- $m_\pi = 806 \text{ MeV}$

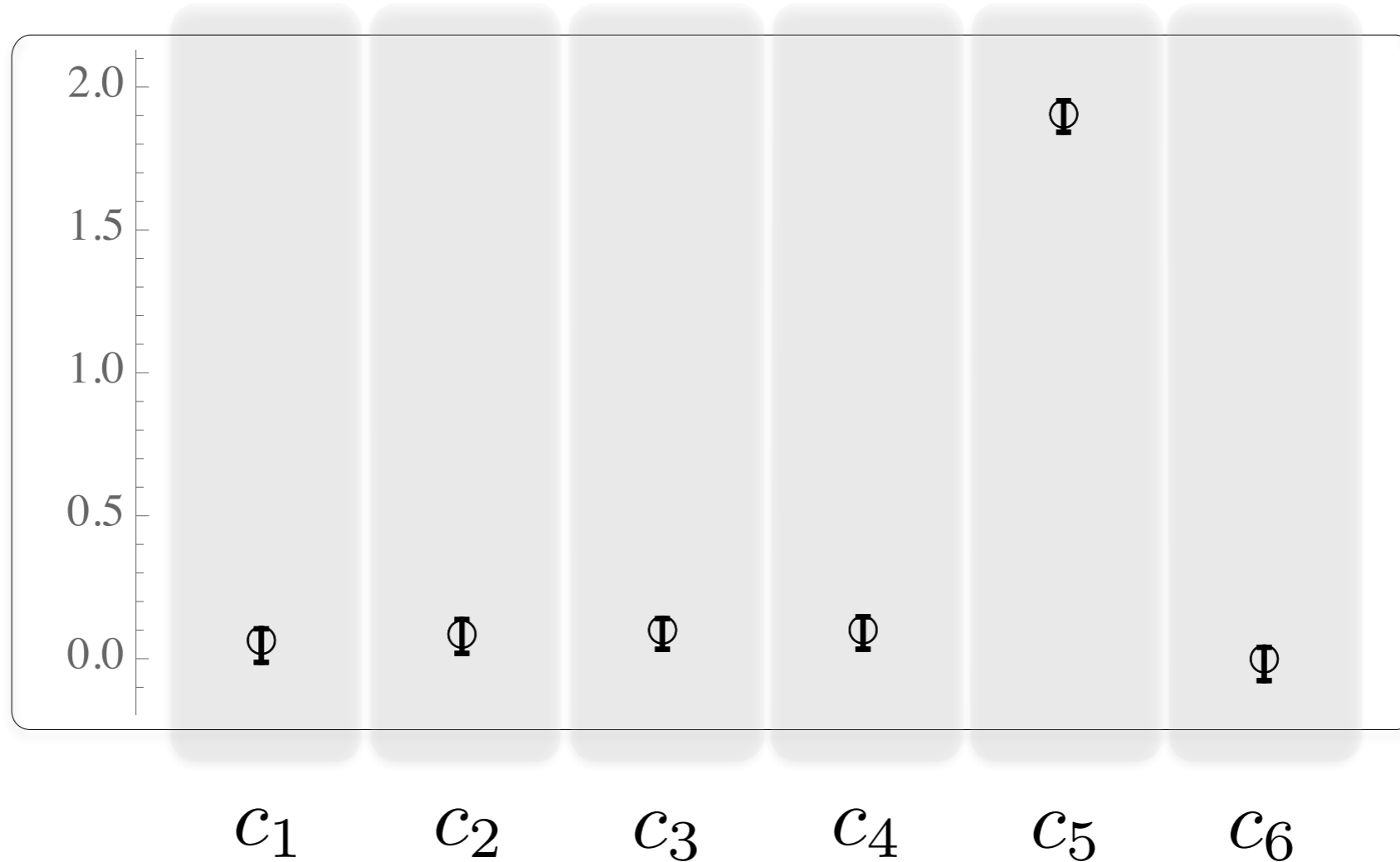
Baryon-baryon interactions and spin-flavor symmetry from lattice quantum chromodynamics

Michael L. Wagman,^{1,2} Frank Winter,³ Emmanuel Chang,² Zohreh Davoudi,⁴ William Detmold,⁴
Kostas Orginos,^{5,3} Martin J. Savage,^{1,2} and Phiala E. Shanahan⁴

(NPLQCD Collaboration)



$$m_\pi \approx 806 \text{ MeV}$$
$$\mu = m_\pi$$



$$\mathcal{L} = -c_1 \text{Tr} B_i^\dagger B_i B_j^\dagger B_j - c_2 \text{Tr} B_j^\dagger B_j B_i^\dagger B_i - c_3 \text{Tr} B_i^\dagger B_j^\dagger B_i B_j - c_4 \text{Tr} B_i^\dagger B_j^\dagger B_j B_i - c_5 \text{Tr} B_i^\dagger B_i \text{Tr} B_j^\dagger B_j - c_6 \text{Tr} B_i^\dagger B_j^\dagger \text{Tr} B_j^\dagger B_i$$

NPLQCD results:

- Only $c_5 \neq 0$
- Near critical value for large scattering lengths

Similar to $N_f=2$ large- N_c result:

$$\mathcal{L}_6 = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2$$

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But:

- EFT possesses **SU(16)** analog of $SU(4)_{\text{Wigner}}$
- + near critical value for large scattering lengths — *conformal symmetry*

NOT large- N_c predictions

low energy symmetries of spin 1/2 baryons

$N_f=2$

$N_f=3$

$SU(4)$ Wigner

$SU(16)$ NPLQCD

~conformal

~conformal

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$N_f=2$

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low energy symmetries of spin 1/2 baryons

$N_f=2$

$N_f=3$

$SU(4)$ Wigner

$SU(16)$ NPLQCD

~conformal

~conformal

No known reason for these symmetries

...but correlated with low entanglement



MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

DISCUSSION OF PROBABILITY RELATIONS BETWEEN
SEPARATED SYSTEMS

By E. SCHRÖDINGER

[Communicated by Mr M. BORN]

[Received 14 August, read 28 October 1935]



When two systems, of which we know representatives, enter into temporary interaction through known forces between them, and when afterwards they separate again and no longer influence the systems separate again, each of them is, as far as the laws of quantum mechanics are concerned, described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or wave-functions) have become **entangled**.

How to quantify entanglement?

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Quantum entropy:

$$S = - \text{Tr } \rho \ln \rho, \quad \rho = \text{density matrix}$$

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E.g. pure state: $|\psi\rangle = |\uparrow_x \downarrow_y\rangle$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\rho = \begin{pmatrix} 1 & 0 & \dots \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad \blacktriangleright \text{rank } 1$$

$S = 0$

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$S = 0$

E.g. mixed state:

$$\rho = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \blacktriangleright \text{rank } 2$$

$S = \ln 2$

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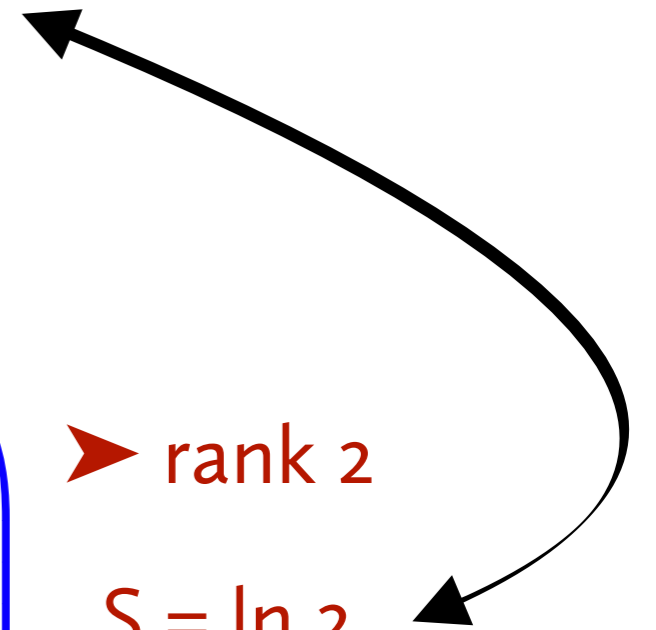
$$\rho = \begin{pmatrix} 1 & 0 & \dots \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad \blacktriangleright \text{rank 1}$$

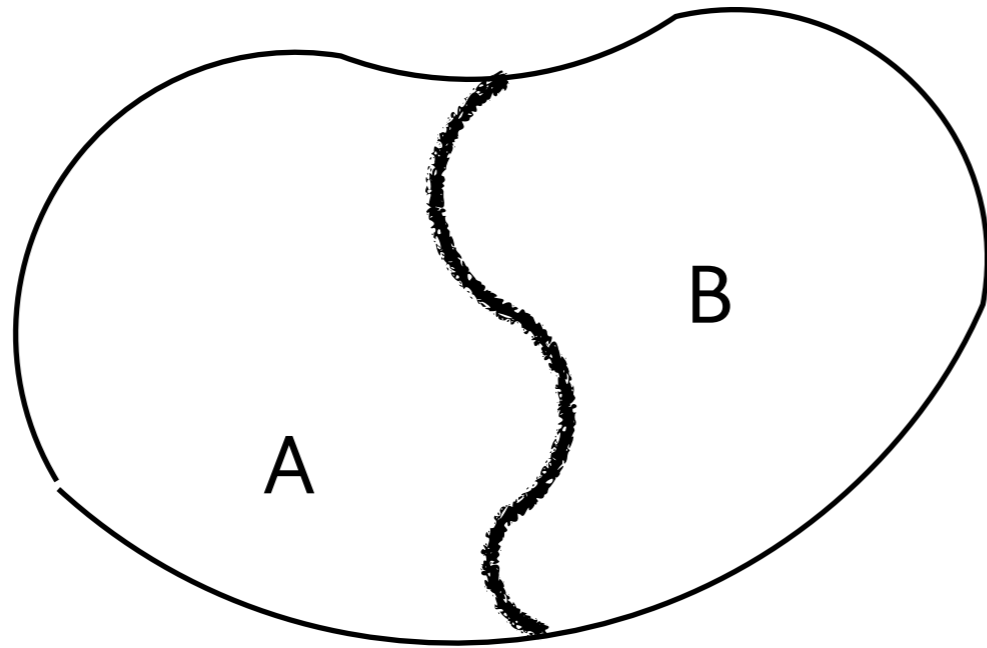
$S = 0$

E.g. mixed state:

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$S = \ln 2$





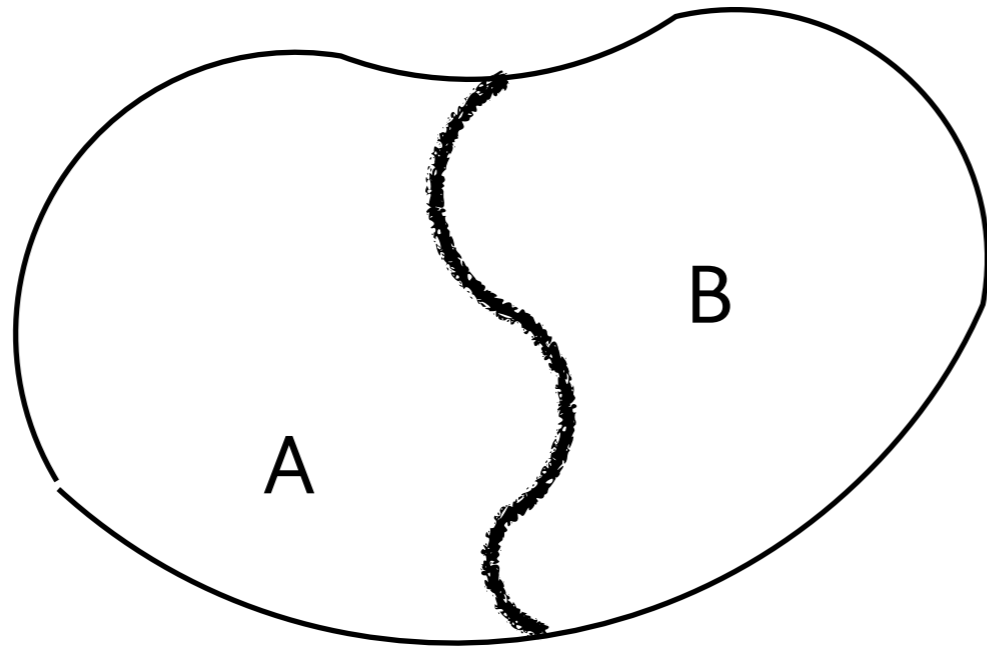
Factorizable Hilbert space:

$$\mathcal{H} = \mathcal{H}_A \times \mathcal{H}_B$$

Reduced density matrix:

$$\rho_A = \text{Tr}_B \rho$$

$$\rho_B = \text{Tr}_A \rho$$



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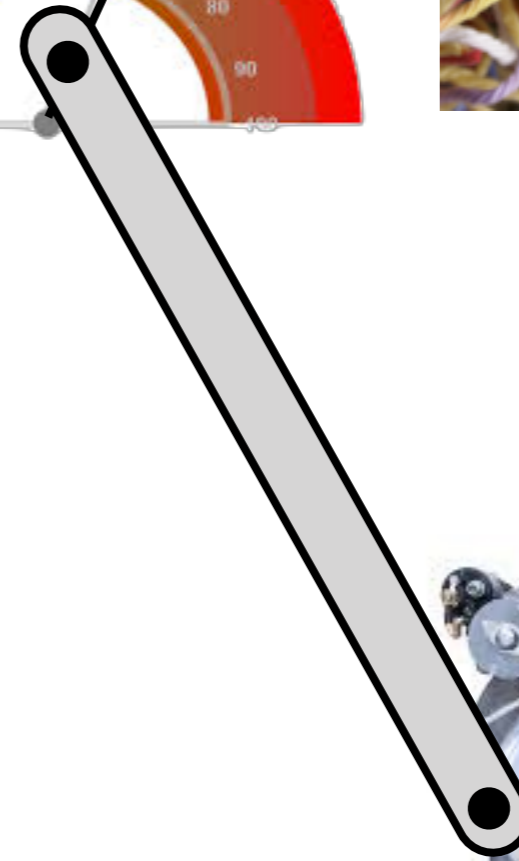
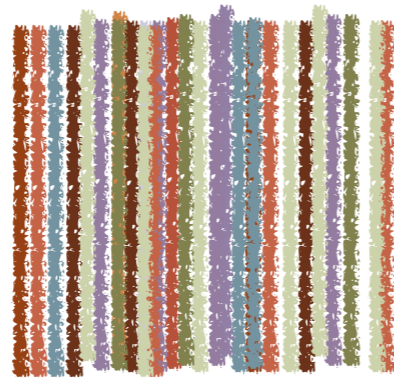
$$\rho_B = \text{Tr}_A \rho$$

Pure state on \mathcal{H} — typically ρ_A, ρ_B will represent mixed states, reflected in entropy:

$$S = 0, \quad S_A = S_B \neq 0$$

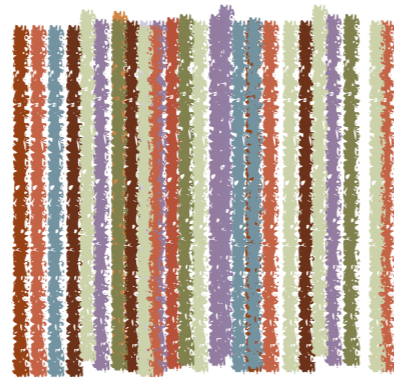
Shows that systems A and B are **entangled**

Is there a connection between entanglement and dynamics?



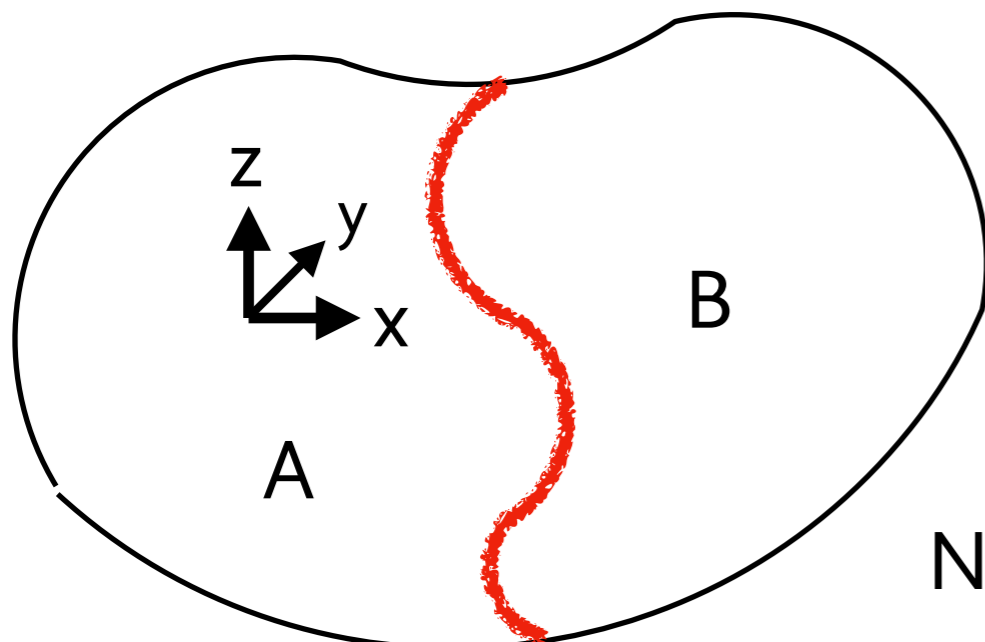
DYNAMICS

Is there a connection between entanglement and dynamics?



Observed that ground states seem to obey **area-law** entanglement

$$S_A = S_B \propto \text{area of shared boundary}$$



DYNAMICS

Not a general feature of wave functions

Entanglement seems to know about dynamics

In a strongly coupled system with composite particles (eg, QCD) can entanglement help determine their wave functions and interactions (and hence their symmetries)?

Quantify the amount of entanglement in the S-matrix

How to quantify entanglement of a N-N scattering process?

One way: PRL 122, 102001 (2019), arXiv: 1812.03138;

A simpler way: in preparation;

Rough description:

- Define entanglement for pure 2-particle state as $[1 - \text{Tr}(\rho_1)^2]$
- Compute entanglement power of the S-matrix as difference in entanglement between $|\psi_{\text{in}}\rangle$ and $|\psi_{\text{out}}\rangle$
- average over initial spin-flavor orientations

Entanglement power in s-wave nucleon-nucleon scattering:

$$\hat{\mathbf{S}} = \frac{1}{4} (3e^{i2\delta_1} + e^{i2\delta_0}) \hat{\mathbf{1}} + \frac{1}{4} (e^{i2\delta_1} - e^{i2\delta_0}) \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\sigma}}$$

$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6} \sin^2 (2(\delta_1 - \delta_0))$$

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$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6} \sin^2 (2(\delta_1 - \delta_0))$$

- Vanishes when:
1. $\delta_0 = \delta_1 \iff \text{SU}(4) \text{ symmetry}$
- or:
2. phase shifts = 0 or $\pi/2 \iff \text{conformal symmetry}$

Entanglement power in s-wave nucleon-nucleon scattering:

$$\hat{\mathbf{S}} = \frac{1}{4} (3e^{i2\delta_1} + e^{i2\delta_0}) \hat{\mathbf{1}} + \frac{1}{4} (e^{i2\delta_1} - e^{i2\delta_0}) \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\sigma}}$$

$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6} \sin^2 (2(\delta_1 - \delta_0))$$

Vanishes when: 1. $\delta_0 = \delta_1 \iff \text{SU}(4) \text{ symmetry}$
or: 2. phase shifts = 0 or $\pi/2 \iff \text{conformal symmetry}$

Look at the low energy EFTs for $p_{\text{cm}} < m_\pi/2$:

$$\mathcal{L}_6 = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2$$

$${}^1S_0 : \quad \bar{C}_0 = (C_S - 3C_T)$$

$${}^3S_1 : \quad \bar{C}_1 = (C_S + C_T)$$

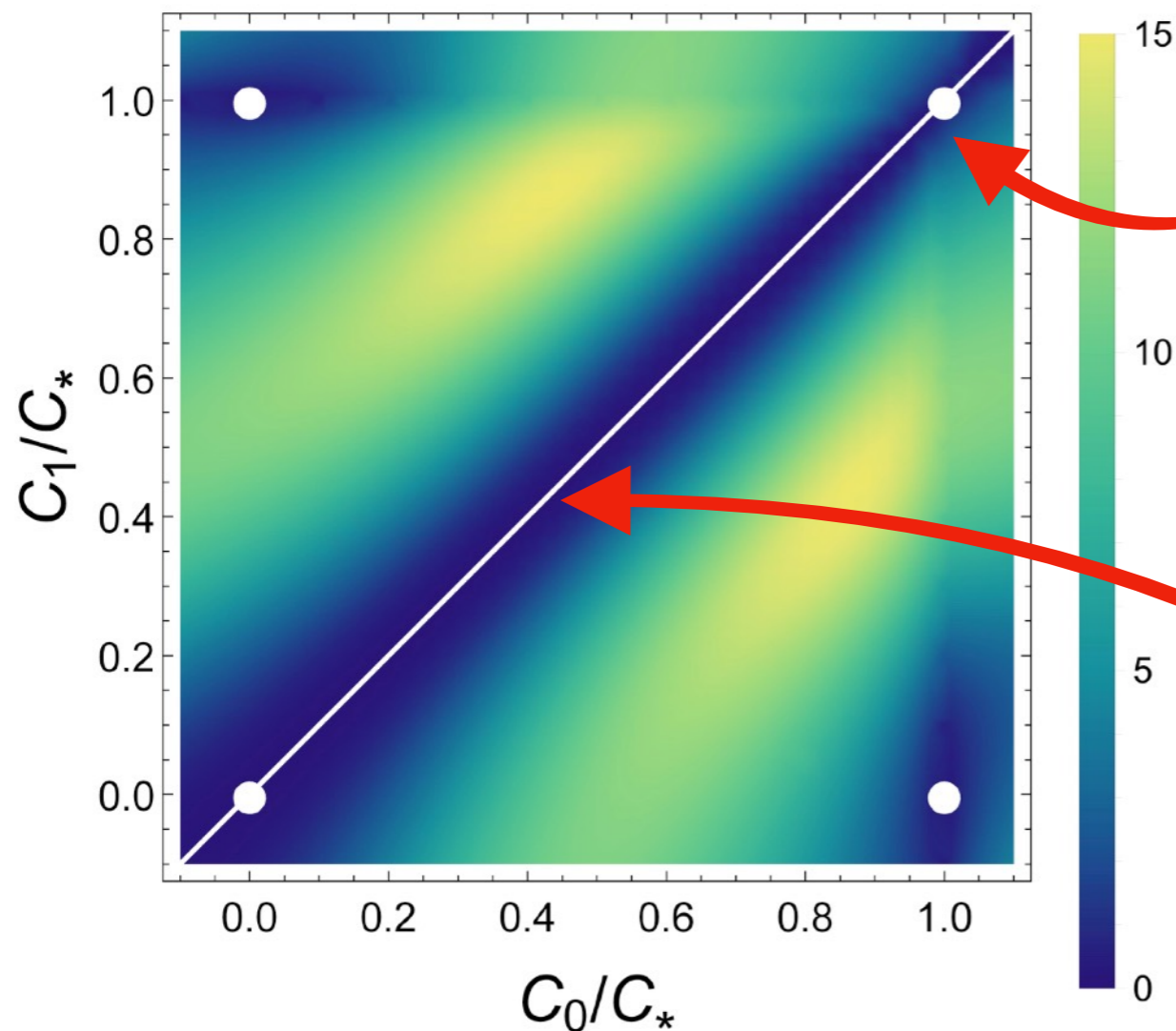
Fit C_0, C_1 to
scattering lengths

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$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6} \sin^2 (2(\delta_1 - \delta_0))$$



Conformal fixed points
(zero or infinite scattering
lengths, $\delta_i = 0, \pi/2$)

$SU(4)_{\text{Wigner}}$ symmetry line
($\delta_0 = \delta_1$)

Real world: Fit C_0, C_1 to scattering lengths

⇒ $C_T/C_S = 0.08 \dots \sim \text{SU}_4$ symmetric

⇒ $C_0 = .94 C_\star,$
⇒ $C_1 = 1.35 C_\star \dots \sim$ pretty close to conformal

OK, what about $N_f=3$?

Find entanglement power of S-matrix is minimized for

- $SU(16)$ symmetry
- Conformal symmetry, for $N_f=2,3$

...exactly the results found by LQCD, with no known QCD explanation

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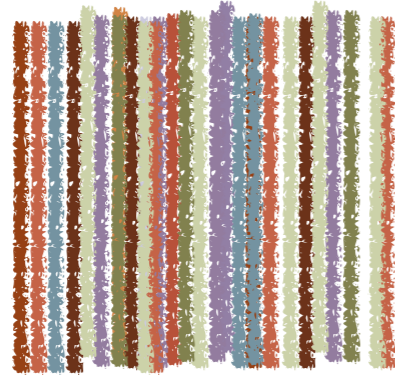
- $SU(16)$ symmetry
- Conformal symmetry, for $N_f=2,3$

...exactly the results found by LQCD, with no known QCD explanation

Could entanglement phobia be a property of strong interactions?
If so, this will in general give rise to enhanced symmetries.

?

LOW ENTANGLEMENT

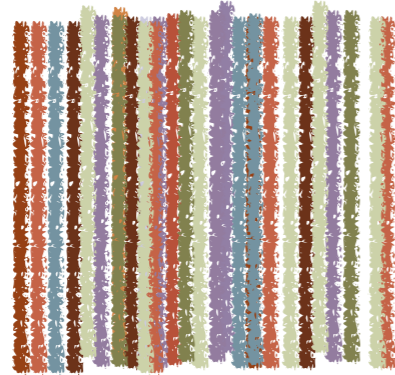


HIGH ENTANGLEMENT

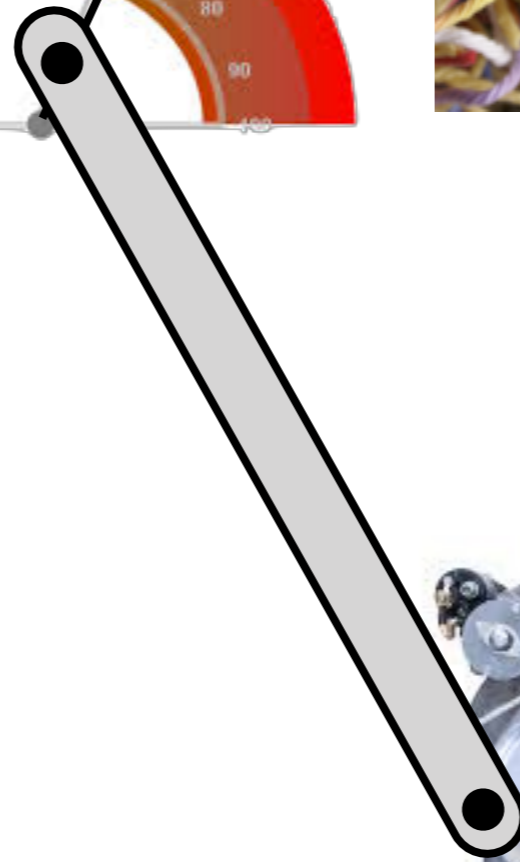


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LOW ENTANGLEMENT



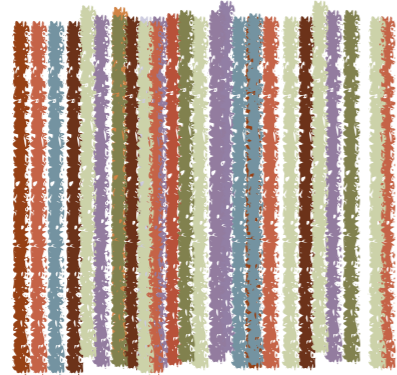
HIGH ENTANGLEMENT



DYNAMICS

?

LOW ENTANGLEMENT



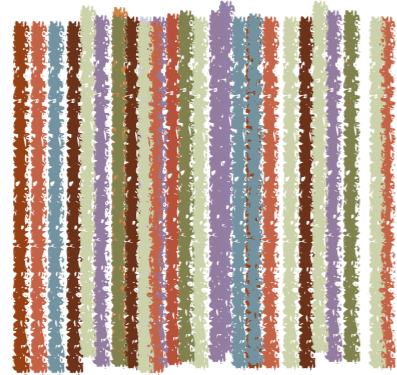
HIGH ENTANGLEMENT



DYNAMICS

?

LOW ENTANGLEMENT



HIGH SYMMETRY



HIGH ENTANGLEMENT



LOW SYMMETRY



DYNAMICS

Conclusions:

In pursuit of a new paradigm...

Empirical approximate symmetries w/o explanation in the strong interactions:

- non-quark spin-flavor symmetries
- NR conformal (Schrödinger) symmetries

Entanglement is minimized for flavors & spin diagonal interactions, as well as for conformal fixed points

Can some symmetries be explained by dynamical systems “wanting” to minimize entanglement?

Need to find more examples; models; perhaps gravity duals?