



On the $\sin\phi_R$ azimuthal asymmetry single longitudinal-spin asymmetry in dihadron production in SIDIS

Wei Yang

School of Physics, Southeast University
Nanjing, China

XXVII International Workshop on Deep-Inelastic
Scattering and Related Subjects

April 8-12, 2019
Turin, Italy



Outline

➤ Introduction

➤ The model calculation of $\tilde{G}_{ot}^{\Delta}(z, M_h^2)$

➤ Numerical estimate the azimuthal
asymmetry $A_{UL}^{\sin \phi_R}$

➤ Summary



Introduction

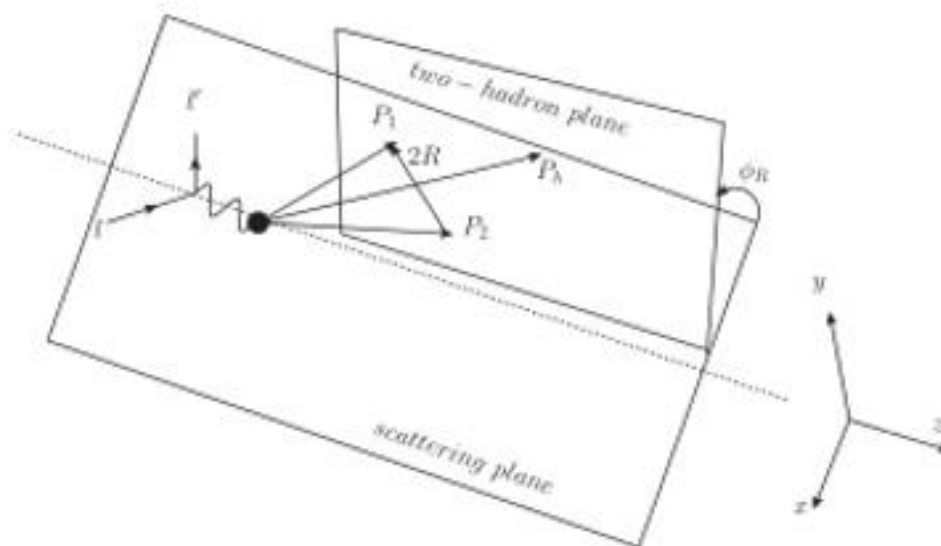
- Understanding the parton structure of the nucleon and the fragmentation mechanism of hadrons are the main tasks in QCD and hadronic physics.
- The azimuthal asymmetries in SIDIS process have been recognized as useful tools.
- In recent years, the study of dihadron production in SIDIS has received a lot of attention.
S. Gliske, A. Bacchetta, and M. Radici, Phys. Rev. D 90, 114027 (2014).
A. Bacchetta and M. Radici, Phys.Rev. D 69, 074026 (2004).
- Recently, some results on the azimuthal spin asymmetries in dihadron production were obtained by the COMPASS, HERMES collaboration and so on.
A. Airapetian et al. [HERMES Collaboration], J. High Energy Phys 0806, 017 (2008).
S. Sirtl, in 22nd International Symposium on Spin Physics (SPIN 2016) Urbana, IL, USA
- The spectator model has been applied to calculate the dihadron fragmentation function (DiFF) of pion pair.
A. Bacchetta and M. Radici, Phys. Rev. D 74, 114007 (2006).



Introduction

- The process under study the dihadron production in SIDIS off a longitudinally polarized proton target as follow

$$\mu(\ell) + p^+(P) \longrightarrow \mu(\ell') + h^+(P_1) + h^-(P_2) + X,$$





Introduction

- We adopt the following kinematical variables

$$\begin{aligned}x &= \frac{k^+}{P^+}, & y &= \frac{P \cdot q}{P \cdot l}, & z_i &= \frac{P_i^-}{k^-}, \\z &= \frac{P_h^-}{k^-} = z_1 + z_2, & Q^2 &= -q^2, & s &= (P + l)^2, \\P_h &= P_1 + P_2, & R &= (P_1 - P_2)/2, & M_h &= \sqrt{P_h^2}.\end{aligned}$$

- The momenta P_h^μ , k^μ and R^μ thus can be written as

$$\begin{aligned}P_h^\mu &= \left[P_h^-, \frac{M_h^2}{2P_h^-}, \vec{0} \right], \\k^\mu &= \left[\frac{P_h^-}{z}, \frac{z(k^2 + \vec{k}_T^2)}{2P_h^-}, \vec{k}_T \right], \\R^\mu &= \left[\frac{|\vec{R}|P_h^-}{M_h} \cos \theta, -\frac{|\vec{R}|M_h}{2P_h^-} \cos \theta, |\vec{R}| \sin \theta \cos \phi_R, |\vec{R}| \sin \theta \sin \phi_R \right] \\&= \left[\frac{|\vec{R}|P_h^-}{M_h} \cos \theta, -\frac{|\vec{R}|M_h}{2P_h^-} \cos \theta, \vec{R}_T^x, \vec{R}_T^y \right],\end{aligned}$$



Introduction

- There are several useful expressions of the scalar products as follows

$$P_h \cdot R = 0,$$

$$P_h \cdot k = \frac{M_h^2}{2z} + z \frac{k^2 + |\vec{k}_T|^2}{2},$$

$$R \cdot k = \left(\frac{M_h}{2z} - z \frac{k^2 + |\vec{k}_T|^2}{2M_h} \right) |\vec{R}| \cos \theta - \vec{k}_T \cdot \vec{R}_T.$$

- The differential cross section for an unpolarized target and a longitudinally polarized target can be cast to

$$\frac{d^6\sigma_{UU}}{d\cos\theta dM_h^2 d\phi_R dz dx dy} = \frac{\alpha^2}{Q^2 y} \left(1 - y + \frac{y^2}{2} \right) \sum_q e_a^2 f_1^q(x) D_1^q(z, M_h^2, \cos\theta),$$

$$\frac{d^6\sigma_{UL}}{d\cos\theta dM_h^2 d\phi_R dz dx dy} = -\frac{\alpha^2}{Q^2 y} S_L 2(1-y)\sqrt{2-y} \sum_a e_a^2 \frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \sin\theta \sin\phi_R$$

$$\times \left[x h_L^a(x) H_1^{\leftarrow, a}(z, M_h^2, \cos\theta) + \frac{M_h}{Mz} g_1(x) \tilde{G}^{\leftarrow, a}(z, M_h^2, \cos\theta) \right].$$

Z. Lu, Phys. Rev. D 90, 014037 (2014).
 A. Bacchetta and M. Radici, Phys. Rev. D 74, 114007 (2006).

A. Bacchetta, F. Conti, and M. Radici,
 Phys. Rev. D 78, 074010 (2008).



Model calculation

The twist-3 DiFF $\tilde{G}^{\triangleleft}$ arises from by the quark-gluon-quark correlator

$$\begin{aligned} \tilde{\Delta}_A^\alpha(z, k_T, R) = & \frac{1}{2z} \sum_X \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 | \int_{\pm\infty^+}^{\xi^+} d\eta^+ \mathcal{U}_{(\infty^+, \xi^+)}^{\xi_T} \\ & \times g F_\perp^{-\alpha} \mathcal{U}_{(\eta^+, \xi^+)}^{\xi_T} \psi(\xi) | P_h, R; X \rangle \langle P_h, R; X | \bar{\psi}(0) \mathcal{U}_{(0^+, \infty^+)}^{0_T} \mathcal{U}_{(0^+, \xi_T)}^{\infty^+} | 0 \rangle |_{\eta^+ = \xi^+ = 0, \eta_T = \xi_T} . \end{aligned}$$

After integrating out \vec{k}_T , one obtains

$$\tilde{\Delta}_A^\alpha(z, \cos\theta, M_h^2, \phi_R) = \frac{z^2 |\vec{R}|}{8M_h} \int d^2\vec{k}_T \tilde{\Delta}_A^\alpha(z, k_T, R).$$

The dihadron fragmentation function $\tilde{G}^{\triangleleft}$ is obtained from

$$\frac{\epsilon_T^{\alpha\beta} R_{T\beta}}{z} \tilde{G}^{\triangleleft}(z, \cos\theta, M_h^2) = 4\pi \text{Tr}[\tilde{\Delta}_A^\alpha(z, \cos\theta, M_h^2, \phi_R) \gamma^- \gamma_5].$$



Model calculation

The function can be expanded in the relative partial waves of the pion pair system. Up to the p-wave level we obtain

$$\tilde{G}^{\triangleleft}(z, \cos \theta, M_h^2) = \tilde{G}_{ot}^{\triangleleft}(z, M_h^2) + \tilde{G}_{lt}^{\triangleleft}(z, M_h^2) \cos \theta.$$

For simplicity, we will not consider the $\cos\theta$ -dependent terms in the expansion of DiFFs

The quark-gluon-quark correlator for dihardon fragmentation in the spectator model can be written as

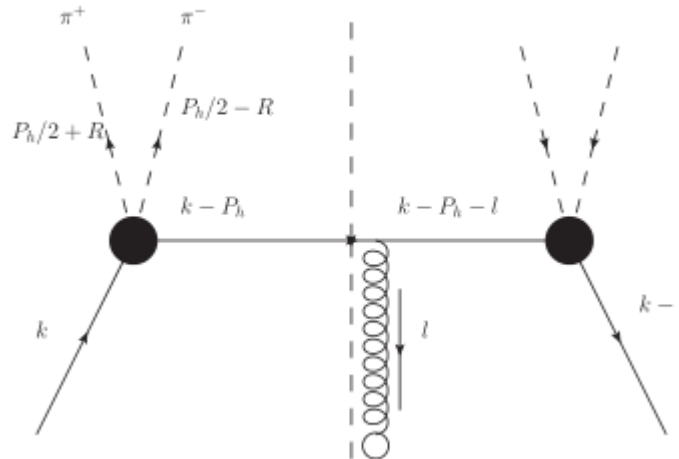
$$\tilde{\Delta}_A^{\alpha}(k, P_h, R) = i \frac{C_F \alpha_s}{2(2\pi)^2(1-z)P_h^-} \frac{1}{k^2 - m^2} \int \frac{d^4 l}{(2\pi)^4} (l^- g_T^{\alpha\mu} - l_T^{\alpha} g^{-\mu})$$

$$\frac{(\not{k} - \not{l} + m)(F^{s*} e^{-\frac{k^2}{\Lambda_s^2}} + F^{p*} e^{-\frac{k^2}{\Lambda_p^2}} \not{R})(\not{k} - \not{P}_h - \not{l} + m_s) \gamma_{\mu} (\not{k} - \not{P}_h + m_s)(F^s e^{-\frac{k^2}{\Lambda_s^2}} + F^p e^{-\frac{k^2}{\Lambda_p^2}} \not{R})(\not{k} + m)}{(-l^- \pm i\epsilon)((k-l)^2 - m^2 - i\epsilon)((k - P_h - l)^2 - m_s^2 - i\epsilon)(l^2 - i\epsilon)},$$



Model calculation

The diagrammatic representation of the correlation function $\tilde{\Delta}_A^\alpha$ in the spectator model



Where the vertices F^s, F^p have following forms

$$F^s = f_s,$$

$$F^p = f_\rho \frac{(M_h^2 - M_\rho^2) - i\Gamma_\rho M_\rho}{(M_h^2 - M_\rho^2) + \Gamma_\rho^2 M_\rho^2} + f_\omega \frac{(M_h^2 - M_\omega^2) - i\Gamma_\omega M_\omega}{(M_h^2 - M_\omega^2) + \Gamma_\omega^2 M_\omega^2}$$

$$- if'_\omega \frac{\sqrt{\hat{\lambda}(M_\omega^2, M_h^2, m_\pi^2)} \Theta(M_\omega - m_\pi - M_h)}{4\pi\Gamma_\omega^2 [4M_\omega^2 m_\pi^2 + \hat{\lambda}(M_\omega^2, M_h^2, m_\pi^2)]^{\frac{1}{4}}}.$$



Model calculation

We choose Λ -cutoffs having the form

$$\Lambda_{s,p} = \alpha_{s,p} z^{\beta_{s,p}} (1-z)^{\gamma_{s,p}},$$

The on-shell condition of the spectator gives the relation between k^2 and the transverse momentum \vec{k}_T

$$k^2 = \frac{z}{1-z} |\vec{k}_T|^2 + \frac{M_s^2}{1-z} + \frac{M_h^2}{z}.$$

Then, we can obtain the final result

$$\begin{aligned} \tilde{G}_{ot}^{\triangleleft}(z, M_h^2) = & \frac{\alpha_s C_F z^2 |\vec{R}|}{8(2\pi)^4 (1-z) M_h} \frac{1}{k^2 - m^2} \int d|\vec{k}_T|^2 e^{-\frac{2k^2}{\Lambda_{sp}^2}} \left\{ \text{Im}(F^{s*} F^p) C \right. \\ & \left. + \text{Re}(F^{s*} F^p) (k^2 - m^2) m_s [(A + zB) - I_2] \right\}. \end{aligned}$$



Model calculation

Where the coefficients A and B come from the decomposition of the integral

$$\int d^4l \frac{l^\mu \delta(l^2) \delta((k-l)^2 - m^2)}{(k - P_h - l)^2 - m_s^2} = Ak^\mu + BP_h^\mu,$$

And have the expressions

$$A = \frac{I_1}{\lambda(M_h, m_s)} \left(2k^2 (k^2 - m_s^2 - M_h^2) \frac{I_2}{\pi} + (k^2 + M_h^2 - m_s^2) \right),$$
$$B = -\frac{2k^2}{\lambda(M_h, m_s)} I_1 \left(1 + \frac{k^2 + m_s^2 - M_h^2}{\pi} I_2 \right).$$

$$I_1 = \int d^4l \delta(l^2) \delta((k-l)^2 - m^2) = \frac{\pi}{2k^2} (k^2 - m^2),$$

$$I_2 = \int d^4l \frac{\delta(l^2) \delta((k-l)^2 - m^2)}{(k - P_h - l)^2 - m_s^2} = \frac{\pi}{2\sqrt{\lambda(M_h, m_s)}} \ln \left(1 - \frac{2\sqrt{\lambda(M_h, m_s)}}{k^2 - M_h^2 + m_s^2 + \sqrt{\lambda(M_h, m_s)}} \right),$$



Model calculation

and

$$\lambda(M_h, m_s) = (k^2 - (M_h + m_s)^2)(k^2 - (M_h - m_s)^2).$$

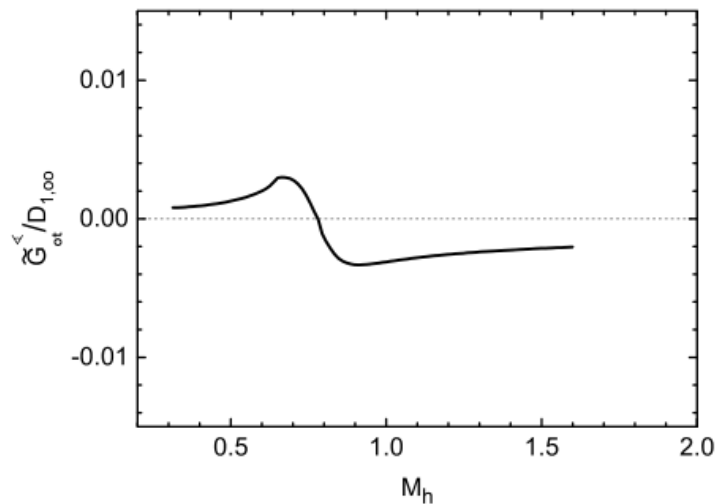
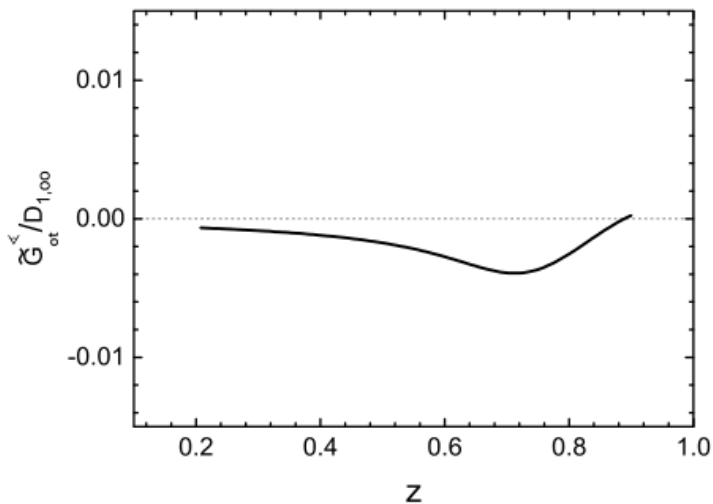
The coefficient C has the form

$$C = m \int_0^1 dx \int_0^{1-x} dy \frac{-2 [(x+y)k \cdot P_h - yM_h^2] + (k^2 - m^2)}{x(1-x)k^2 + 2k \cdot (k - P_h)xy + xm^2 + y^2m_s^2},$$



Numerical estimate

The numerical result for $\tilde{G}_{ot}^{\leq}(z, M_h^2)$ dependent on z and M_h as follows,



The region $0.2 < z < 0.9$ and $0.3\text{GeV} < M_h < 1.6\text{GeV}$.

The fitting parameters results are

$$\begin{aligned} \alpha_s &= 2.60\text{GeV}, & \beta_s &= -0.751, & \gamma_s &= -0.193, \\ \alpha_p &= 7.07\text{GeV}, & \beta_p &= -0.038, & \gamma_p &= -0.085, \\ f_s &= 1197\text{GeV}^{-1}, & f_\rho &= 93.5, & f_\omega &= 0.63, \\ f'_\omega &= 75.2, & M_s &= 2.97M_h, & m &= 0.0\text{GeV}(\text{fixed}). \end{aligned}$$



Numerical estimate

The $A_{UL}^{\sin \phi_R}$ asymmetry of dihadron production in the single longitudinally polarized SIDIS expressed as

$$A_{UL}^{\sin \phi_R}(x, z, M_h^2) = - \frac{\sum_a e_a^2 \frac{|\vec{R}|}{Q} \left[\frac{|M|}{M_h} x h_L^a(x) H_{1,ot}^{\leftarrow, a}(z, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}_{ot}^{\leftarrow}(z, M_h^2) \right]}{\sum_a e_a^2 f_1^a(x) D_{1,oo}^a(z, M_h^2)}.$$

Following the COMPASS convention, the depolarization factors are not included in the numerator and denominator.

So we can obtain the expressions of the x-dependent, z-dependent and M_h -dependent asymmetry as follows

$$A_{UL}^{\sin \phi_R}(x) = - \frac{\int dz \int dM_h 2M_h \frac{|\vec{R}|}{Q} \left[\frac{|M|}{M_h} (4h_L^u(x) + h_L^d(x)) x H_{1,ot}^{\leftarrow}(z, M_h^2) + \frac{1}{z} (4g_1^u(x) + g_1^d(x)) \tilde{G}_{ot}^{\leftarrow}(z, M_h^2) \right]}{\int dz \int dM_h 2M_h (4f_1^u(x) + f_1^d(x)) D_{1,oo}(z, M_h^2)},$$

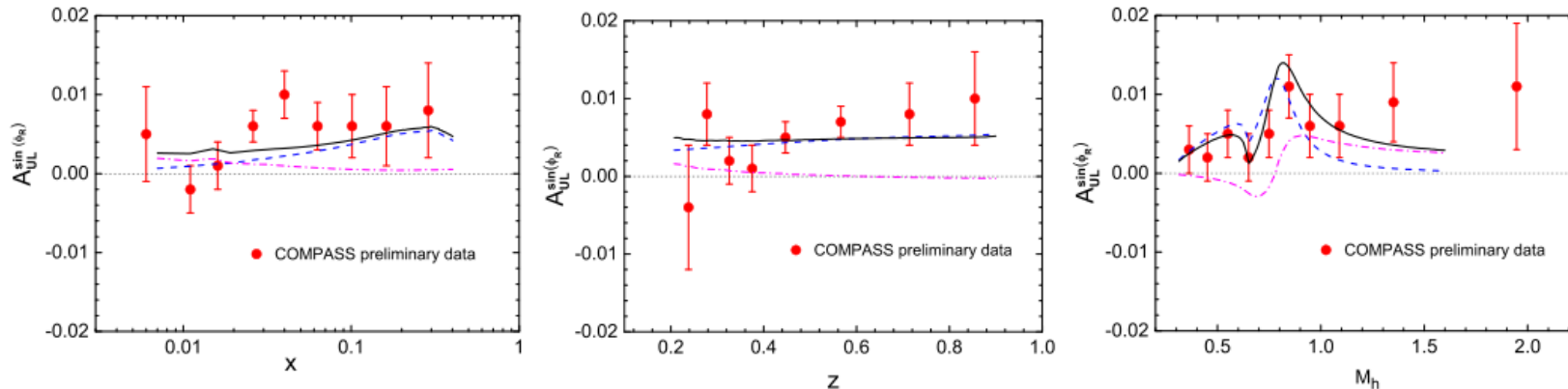
$$A_{UL}^{\sin \phi_R}(z) = - \frac{\int dx \int dM_h 2M_h \frac{|\vec{R}|}{Q} \left[\frac{|M|}{M_h} (4h_L^u(x) + h_L^d(x)) x H_{1,ot}^{\leftarrow}(z, M_h^2) + \frac{1}{z} (4g_1^u(x) + g_1^d(x)) \tilde{G}_{ot}^{\leftarrow}(z, M_h^2) \right]}{\int dx \int dM_h 2M_h (4f_1^u(x) + f_1^d(x)) D_{1,oo}(z, M_h^2)},$$

$$A_{UL}^{\sin \phi_R}(M_h) = - \frac{\int dx \int dz \frac{|\vec{R}|}{Q} \left[\frac{|M|}{M_h} (4h_L^u(x) + h_L^d(x)) x H_{1,ot}^{\leftarrow}(z, M_h^2) + \frac{1}{z} (4g_1^u(x) + g_1^d(x)) \tilde{G}_{ot}^{\leftarrow}(z, M_h^2) \right]}{\int dx \int dz (4f_1^u(x) + f_1^d(x)) D_{1,oo}(z, M_h^2)}.$$



Numerical estimate

Plot the $A_{UL}^{\sin\phi_R}$ asymmetry at the kinematics of COMPASS



The full circles are the COMPASS preliminary data. The dashed curves denote the contribution from the $h_L H_{1,ot}^{\Delta}$ term, the dashed-dotted curves represent the contribution from the $g_1 \tilde{G}^{\Delta}$ term, and the solid lines display the sum of two contributions.

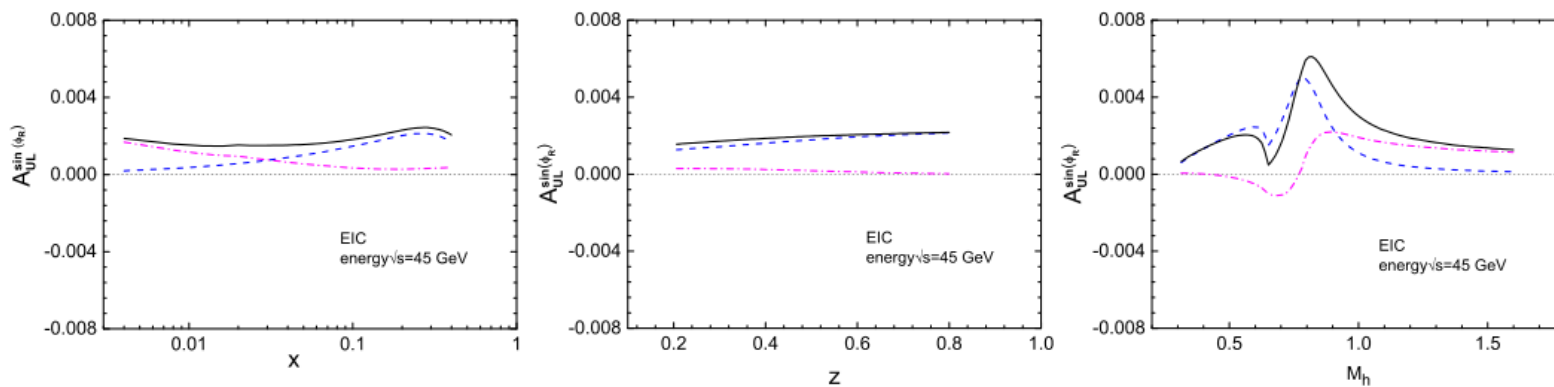
Where the kinematical cuts as follow

$$0.003 < x < 0.4, \quad 0.1 < y < 0.9, \quad 0.2 < z < 0.9,$$
$$0.3\text{GeV} < M_h < 1.6\text{GeV}, \quad Q^2 > 1\text{GeV}^2, \quad W > 5\text{GeV}.$$



Numerical estimate

Plot the $A_{UL}^{\sin\phi_R}$ asymmetry at the future EIC



Where the kinematical cuts as follow

$$\sqrt{s} = 45\text{GeV}, \quad 0.001 < x < 0.4, \quad 0.01 < y < 0.95, \quad 0.2 < z < 0.8, \\ 0.3\text{GeV} < M_h < 1.6\text{GeV}, \quad Q^2 > 1\text{GeV}^2, \quad W > 5\text{GeV}.$$



Summary

In this work, we have studied the single longitudinal-spin asymmetry with a $\sin\phi_R$ modulation of dihadron production in SIDIS.

- We considered the contributions from the coupling of the twist-3 distributions h_L and the DiFF $H_{1,ot}^{\triangleleft}$, as well as the coupling of the helicity distribution g_1 and the twist-3 DiFF $\tilde{G}^{\triangleleft}$
- We calculated the twist-3 T-odd DiFF $\tilde{G}_{ot}^{\triangleleft}$ and found that the contribution to come from the interference of the s and p waves.
- Using the numerical results of the DiFFs, we estimated the asymmetry and compared it with the COMPASS measurement.



Summary

- In addition, we also made a prediction $A_{UL}^{\sin \phi_R}$ at the typical kinematics of a future EIC.
- Our study shows that the twist-3 DiFF $\tilde{G}_{ot}^{\triangleleft}$ should be considered in phenomenological analysis in order to provide a better understanding of the $\sin \phi_R$ asymmetry in dihadron production in SIDIS.



Thanks for your attention!

