On the $\sin\phi_R$ azimuthal asymmetry single longitudinal-spin asymmetry in dihadron production in SIDIS

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Outline

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- The model calculation of $\tilde{G}_{ot}(z, M_h^2)$
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Understanding the parton structure of the nucleon and the fragmentation mechanism of hadrons are the main tasks in QCD and hadronic physics.

The azimuthal asymmetries in SIDIS process have been recognized as useful tools.

In recent years, the study of dihadron production in SIDIS has received a lot of attention. S. Gliske, A. Bacchetta, and M. Radici, Phys. Rev. D 90, 114027 (2014). A. Bacchetta and M. Radici, Phys. Rev. D 69, 074026 (2004).

Recently, some results on the azimuthal spin asymmetries in dihadron production were obtained by the COMPASS, HERMES collaboration and so on. A. Airapetian et al. [HERMES Collaboration], J. High Energy Phys 0806, 017 (2008). S. Sirtl, in 22nd International Symposium on Spin Physics (SPIN 2016) Urbana, IL, USA

The spectator model has been applied to calculate the dihadron fragmentation function (DiFF) of pion pair. A. Bacchetta and M. Radici, Phys. Rev. D 74, 114007 (2006).
The process under study the dihadron production in SIDIS off a longitudinally polarized proton target as follow

\[ \mu(\ell) + p^{\rightarrow}(P) \rightarrow \mu(\ell') + h^{+}(P_1) + h^{-}(P_2) + X, \]
Introduction

- We adopt the following kinematical variables

\[
\begin{align*}
x &= \frac{k^+}{P^+}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z_i = \frac{P_i^-}{k^-}, \\
z &= \frac{P_h^-}{k^-} = z_1 + z_2, \quad Q^2 = -q^2, \quad s = (P + l)^2, \\
P_h &= P_1 + P_2, \quad R = (P_1 - P_2)/2, \quad M_h = \sqrt{P_h^2}.
\end{align*}
\]

- The momenta \( P_h^\mu, k^\mu \) and \( R^\mu \) thus can be written as

\[
\begin{align*}
P_h^\mu &= \begin{bmatrix} P_h^- & \frac{M_h^2}{2P_h^-} & 0 \end{bmatrix}, \\
k^\mu &= \begin{bmatrix} P_h^- & z(k^2 + k_T^2) & \frac{z}{2P_h^-} \end{bmatrix}, \\
R^\mu &= \begin{bmatrix} |\vec{R}| P_h^- \cos \theta - \frac{|\vec{R}| M_h}{2P_h^-} \cos \theta \cos \phi_R, |\vec{R}| \sin \theta \cos \phi_R \end{bmatrix} \\
&= \begin{bmatrix} |\vec{R}| P_h^- \cos \theta - \frac{|\vec{R}| M_h}{2P_h^-} \cos \theta, \vec{R}_T \end{bmatrix},
\end{align*}
\]
There are several useful expression of the scalar products as follows

\[ P_h \cdot R = 0, \]
\[ P_h \cdot k = \frac{M_h^2}{2z} + z \frac{k^2 + |k_T|^2}{2}, \]
\[ R \cdot k = \left( \frac{M_h}{2z} - z \frac{k^2 + |k_T|^2}{2M_h} \right) |\vec{R}| \cos \theta - k_T \cdot \vec{R}_T. \]

The differential cross section for an unpolarized target and a longitudinally polarized target can be cast to

\[
\frac{d^6\sigma_{UU}}{d\cos \theta \, dM_h^2 \, d\phi_R \, dz \, dx \, dy} = \frac{\alpha^2}{Q^2 y} \left( 1 - y + \frac{y^2}{2} \right) \sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2, \cos \theta),
\]
\[
\frac{d^6\sigma_{UL}}{d\cos \theta \, dM_h^2 \, d\phi_R \, dz \, dx \, dy} = -\frac{\alpha^2}{Q^2 y} S_L \frac{2(1-y)}{\sqrt{2-y}} \sum_a e_a^2 \frac{M |\vec{R}|}{Q M_h} \sin \theta \sin \phi_R \times \left[ xh_L^a(x)H_1^{<,a}(z, M_h^2, \cos \theta) + \frac{M_h}{M_z} g_1(x) \tilde{G}_<^{<,a}(z, M_h^2, \cos \theta) \right].
\]

Model calculation

The twist-3 DiFF $\tilde{G}^\alpha$ arises from by the quark-gluon-quark correlator

$$
\tilde{\Delta}_A^\alpha(z, k_T, R) = \frac{1}{2z} \sum_X \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3} e^{ik\cdot\xi} \langle 0 | \int_{\pm\infty^+} d\eta^+ \mathcal{U}_{(\infty^+,\xi^+)}^{\xi_T} \\
\times gF_{\perp}^{-\alpha} \mathcal{U}_{(\eta^+,\xi^+)}^{\xi_T} \psi(\xi) | P_h, R; X \rangle \langle P_h, R; X | \bar{\psi}(0) \mathcal{U}_{(0^+,\infty^+)}^{0_T} \mathcal{U}_{(0_T,\xi_T)}^{\infty^+} | 0 \rangle | n^+=\xi^+=0, \eta_T=\xi_T \rangle.
$$

After integrating out $\vec{k}_T$, one obtains

$$
\tilde{\Delta}_A^\alpha(z, \cos \theta, M_h^2, \phi_R) = \frac{z^2|\vec{R}|}{8M_h} \int d^2\vec{k}_T \tilde{\Delta}_A^\alpha(z, k_T, R).
$$

The dihadron fragmentation function $\tilde{G}^\alpha$ is obtained from

$$
\frac{\epsilon_T^{\alpha\beta} R_T^{\beta}}{z} \tilde{G}^\alpha(z, \cos \theta, M_h^2) = 4\pi \text{Tr} [\tilde{\Delta}_A^\alpha(z, \cos \theta, M_h^2, \phi_R) \gamma^- \gamma_5].
$$
Model calculation

The function can be expanded in the relative partial waves of the pion pair system. Up to the p-wave level we obtain

\[ \tilde{G}^\Delta(z, \cos \theta, M_h^2) = \tilde{G}_\text{ot}^\Delta(z, M_h^2) + \tilde{G}_\text{lt}^\Delta(z, M_h^2) \cos \theta. \]

For simplicity, we will not consider the \( \cos \theta \)-dependent terms in the expansion of DiFFs.

The quark-gluon-quark correlator for dihardon fragmentation in the spectator model can be written as

\[
\tilde{\Delta}_A^\alpha(k, P_h, R) = i \frac{C_F \alpha_s}{2(2\pi)^2(1-z)P_h^-} \frac{1}{k^2 - m^2} \int \frac{d^4l}{(2\pi)^4} (l^- g_{\mu\nu}^{\alpha\beta} - l_{T\mu}^{\alpha} g_{T\nu}^{\beta}) (l' - l + m)(F^s e^{-\frac{k^2}{\Lambda^2}} + F_p e^{-\frac{k^2}{\Lambda_p^2}} R)(l' - P_h - l + m_s) \gamma_\mu (l' - P_h + m_s)(F^s e^{-\frac{k^2}{\Lambda^2}} + F_p e^{-\frac{k^2}{\Lambda_p^2}} R) (l' + m) \frac{(-l^- \pm i\epsilon)((k - l)^2 - m^2 - i\epsilon)(((k - P_h - l)^2 - m_s^2 - i\epsilon)(l^2 - i\epsilon)}{(-l^- \pm i\epsilon)((k - l)^2 - m^2 - i\epsilon)(((k - P_h - l)^2 - m_s^2 - i\epsilon)(l^2 - i\epsilon)} ,
\]
Model calculation

The diagrammatic representation of the correlation function $\widetilde{\Delta}_A^\alpha$ in the spectator model

Where the vertices $F^s$, $F^p$ have following forms

\[
F^s = f_s,
\]
\[
F^p = f_\rho \frac{(M_h^2 - M_\rho^2) - i\Gamma_\rho M_\rho}{(M_h^2 - M_\rho^2) + \Gamma_\rho^2 M_\rho^2} + f_\omega \frac{(M_h^2 - M_\omega^2) - i\Gamma_\omega M_\omega}{(M_h^2 - M_\omega^2) + \Gamma_\omega^2 M_\omega^2}
\]
\[
- if'_\omega \frac{\sqrt{\tilde{\lambda}(M_\omega^2, M_h^2, m_\pi^2)}\Theta(M_\omega - m_\pi - M_h)}{4\pi \Gamma_\omega^2 [4M_\omega^2 m_\pi^2 + \tilde{\lambda}(M_\omega^2, M_h^2, m_\pi^2)]^{1/4}}.
\]
Model calculation

We choose $\Lambda$-cutoffs having the form

$$\Lambda_{s,p} = \alpha_{s,p} z^{\beta_{s,p}} (1 - z)^{\gamma_{s,p}},$$

The on-shell condition of the spectator gives the relation between $k^2$ and the transverse momentum $\vec{k}_T$

$$k^2 = \frac{z}{1 - z} |\vec{k}_T|^2 + \frac{M^2_s}{1 - z} + \frac{M^2_h}{z}.$$

Then, we can obtain the final result

$$\tilde{G}_{ot}^{\alpha}(z, M^2_h) = \frac{\alpha_s C_F z^2 |\vec{R}|}{8(2\pi)^4 (1 - z) M_h} \frac{1}{k^2 - m^2} \int d|\vec{k}_T|^2 e^{-\frac{2k^2}{\Lambda^2_{s,p}}} \left\{ \text{Im}(F^* F^p) C \right.$$ 

$$\left. + \text{Re}(F^* F^p)(k^2 - m^2) m_s [(A + zB) - I_2] \right\}. $$
Model calculation

Where the coefficients A and B come from the decomposition of the integral

\[ \int d^4 l \frac{l^\mu \delta(l^2) \delta((k - l)^2 - m^2)}{(k - P_h - l)^2 - m_s^2} = Ak^\mu + BP_h^\mu, \]

And have the expressions

\[ A = \frac{I_1}{\lambda(M_h, m_s)} \left( 2k^2 (k^2 - m_s^2 - M_h^2) \frac{I_2}{\pi} + (k^2 + M_h^2 - m_s^2) \right), \]

\[ B = -\frac{2k^2}{\lambda(M_h, m_s)} I_1 \left( 1 + \frac{k^2 + m_s^2 - M_h^2}{\pi} \right). \]

\[ I_1 = \int d^4 l \delta(l^2) \delta((k - l)^2 - m^2) = \frac{\pi}{2k^2} (k^2 - m^2), \]

\[ I_2 = \int d^4 l \frac{\delta(l^2) \delta((k - l)^2 - m^2)}{(k - P_h - l)^2 - m_s^2} = \frac{\pi}{2\sqrt{\lambda(M_h, m_s)}} \ln \left( 1 - \frac{2\sqrt{\lambda(M_h, m_s)}}{k^2 - M_h^2 + m_s^2 + \sqrt{\lambda(M_h, m_s)}} \right), \]
Model calculation

\[ \lambda(M_h, m_s) = (k^2 - (M_h + m_s)^2)(k^2 - (M_h - m_s)^2). \]

and

\[ C = m \int_0^1 dx \int_0^{1-x} dy \frac{-2 [(x + y)k \cdot P_h - yM_h^2] + (k^2 - m^2)}{x(1-x)k^2 + 2k \cdot (k - P_h)xy + xm^2 + y^2m_s^2}, \]

The coefficient C has the form
The numerical result for $\tilde{G}_{ot}(z, M_h^2)$ dependent on $z$ and $M_h$ as follows,

The region $0.2 < z < 0.9$ and $0.3\text{GeV} < M_h < 1.6\text{GeV}$.

The fitting parameters results are

$$
\begin{align*}
\alpha_s &= 2.60\text{GeV}, & \beta_s &= -0.751, & \gamma_s &= -0.193, \\
\alpha_p &= 7.07\text{GeV}, & \beta_p &= -0.038, & \gamma_p &= -0.085, \\
f_s &= 1197\text{GeV}^{-1}, & f_\rho &= 93.5, & f_\omega &= 0.63, \\
f'_\omega &= 75.2, & M_s &= 2.97M_h, & m &= 0.0\text{GeV(fixed)}. 
\end{align*}
$$
Numerical estimate

The $A_{UL}^{\sin \phi R}$ asymmetry of dihadron production in the single longitudinally polarized SIDIS expressed as

$$A_{UL}^{\sin \phi R}(x, z, M_h^2) = -\frac{\sum_a e_a^2 \frac{\vec{R}_a}{Q}}{\sum_a e_a^2 f_1^a(x) D_{1,oo}^a(z, M_h^2)} \left[ \frac{|M|}{M_h} x h_L^u(x) H_{1,ot}^q(z, M_h^2) + \frac{1}{z} g_1(x) \tilde{G}_{ot}^q(z, M_h^2) \right].$$

Following the COMPASS convention, the depolarization factors are not included in the numerator and denominator.

So we can obtain the expressions of the $x$-dependent, $z$-dependent and $M_h$-dependent asymmetry as follows

$$A_{UL}^{\sin \phi R}(x) = -\frac{\int dz \int dM_h 2M_h \frac{|\vec{R}_a|}{Q} \left[ \frac{|M|}{M_h} (4h_L^u(x) + h_L^d(x)) x H_{1,ot}^q(z, M_h^2) + \frac{1}{z} (4g_1^u(x) + g_1^d(x)) \tilde{G}_{ot}^q(z, M_h^2) \right]}{\int dz \int dM_h 2M_h (4f_1^u(x) + f_1^d(x)) D_{1,oo}(z, M_h^2)},$$

$$A_{UL}^{\sin \phi R}(z) = -\frac{\int dx \int dM_h 2M_h \frac{|\vec{R}_a|}{Q} \left[ \frac{|M|}{M_h} (4h_L^u(x) + h_L^d(x)) x H_{1,ot}^q(z, M_h^2) + \frac{1}{z} (4g_1^u(x) + g_1^d(x)) \tilde{G}_{ot}^q(z, M_h^2) \right]}{\int dx \int dM_h 2M_h (4f_1^u(x) + f_1^d(x)) D_{1,oo}(z, M_h^2)},$$

$$A_{UL}^{\sin \phi R}(M_h) = -\frac{\int dx \int dz \frac{|\vec{R}_a|}{Q} \left[ \frac{|M|}{M_h} (4h_L^u(x) + h_L^d(x)) x H_{1,ot}^q(z, M_h^2) + \frac{1}{z} (4g_1^u(x) + g_1^d(x)) \tilde{G}_{ot}^q(z, M_h^2) \right]}{\int dx \int dz (4f_1^u(x) + f_1^d(x)) D_{1,oo}(z, M_h^2)}.$$
Numerical estimate

Plot the $A_{UL}^{\sin \phi_R}$ asymmetry at the kinematics of COMPASS

The full circles are the COMPASS preliminary data. The dashed curves denote the contribution from the $h_L H_{1,\eta}^\alpha$ term, the dashed-dotted curves represent the contribution from the $g_1 \tilde{G}_{1}^\alpha$ term, and the solid lines display the sum of two contributions.

Where the kinematical cuts as follow

\[ 0.003 < x < 0.4, \quad 0.1 < y < 0.9, \quad 0.2 < z < 0.9, \]
\[ 0.3 \text{GeV} < M_h < 1.6 \text{GeV}, \quad Q^2 > 1 \text{GeV}^2, \quad W > 5 \text{GeV}. \]
Numerical estimate

Plot the $A_{UL}^{\sin \phi_R}$ asymmetry at the future EIC

Where the kinematical cuts as follow

$$\sqrt{s} = 45\text{GeV}, \quad 0.001 < x < 0.4, \quad 0.01 < y < 0.95, \quad 0.2 < z < 0.8, \quad 0.3\text{GeV} < M_h < 1.6\text{GeV}, \quad Q^2 > 1\text{GeV}^2, \quad W > 5\text{GeV}.$$
Summary

In this work, we have studied the single longitudinal-spin asymmetry with a \( \sin \phi_R \) modulation of dihadron production in SIDIS.

- We considered the contributions from the coupling of the twist-3 distributions \( h_L \) and the DiFF \( H_{1,ot}^{\alpha} \), as well as the coupling of the helicity distribution \( g_1 \) and the twist-3 DiFF \( \tilde{G}^{\alpha} \).

- We calculated the twist-3 T-odd DiFF \( \tilde{G}_{ot}^{\alpha} \) and found that the contribution to come from the interference of the s and p waves.

- Using the numerical results of the DiFFs, we estimated the \( A_{UL}^{\sin \phi_R} \) asymmetry and compared it with the COMPASS measurement.
In addition, we also made a prediction $A_{UL}^{\sin \phi_R}$ at the typical kinematics of a future EIC.

Our study shows that the twist-3 DiFF $\tilde{G}_{ot}^{\alpha}$ should be considered in phenomenological analysis in order to provide a better understanding of the $\sin \phi_R$ asymmetry in dihadron production in SIDIS.
Thanks for your attention!