

Single-spin asymmetry $A_{UT}^{\sin\phi_{S_h}}$ of proton and Λ production in SIDIS at subleading twist

Wenjuan Mao

School of Physics and Telecommunication Engineering
Zhoukou Normal University

Email: wjmao@seu.edu.cn

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OUTLINE



1. Formalism

2. Calculation on the structure functions

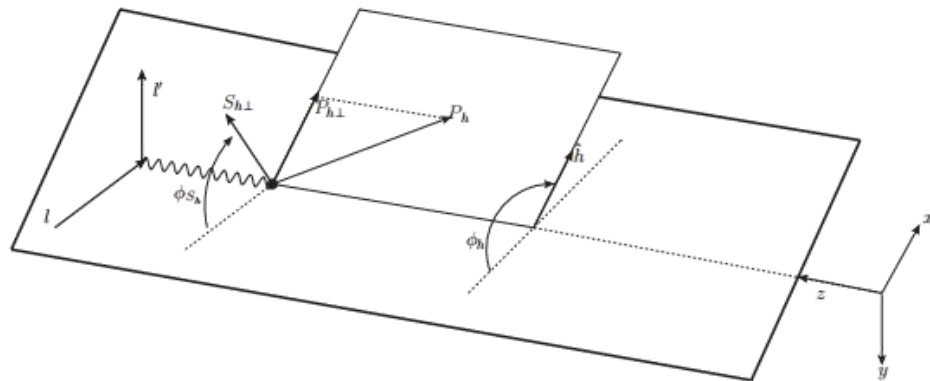
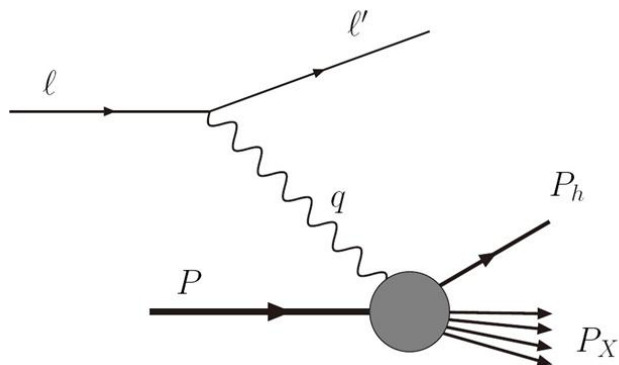
3. Numerical Estimate at JLab12 and COMPASS

4. Summary

Based on arXiv:1808:10565(2018)

◆ Transversely polarized hadron production in semi-inclusive DIS by scattering off the unpolarized target

$$l(l) + p(P) \rightarrow l(l') + h^{\uparrow}(P_h) + X(P_X)$$



◆ The invariants defined as

$$x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx}{Q}$$

$$Q^2 = -q^2, \quad s = (P + \ell)^2, \quad W^2 = (P + q)^2,$$

- ◆ The general form for the differential cross section with an unpolarized nucleon target in SIDIS ([D.Boer and P.J. Mulders, Phys. Rev. D57,5780\(1998\)](#))

$$\frac{d\sigma}{dx dy dz d\phi d\psi dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UUU} \right. \\ \left. + |S_{hT}| \sin \phi_{S_h} \sqrt{2\varepsilon(1+\varepsilon)} F_{UUT}^{\sin \phi_{S_h}} + \dots \right\}$$

the differential cross section

$$A_{UT}^{\sin \phi_{S_h}} \equiv \frac{F_{UUT}^{\sin \phi_{S_h}}}{F_{UUU}}$$

the single spin asymmetry

- ◆ The ratio of the longitudinal and transverse photon flux ([Bacchetta et al., JHEP0702, 093 \(2007\)](#))

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}.$$

- ◆ The $P_{h\perp}$ dependent SSA $A_{UT}^{\sin\phi_{S_h}}$ has the form

$$A_{UT}^{\sin\phi_{S_h}}(P_{h\perp}) = \frac{\int dx \int dy \int dz C_{UT} F_{UT}^{\sin\phi_{S_h}}}{\int dx \int dy \int dz C_{UU} F_{UU}},$$

- ◆ The kinematical factors C_{UU} and C_{UT} are defined as

$$C_{UU} = \frac{1}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right),$$
$$C_{UT} = \frac{1}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \sqrt{2\varepsilon(1+\varepsilon)},$$

- ◆ Sizable SSAs or DSAs cannot be explained by pQCD ([Ahmed & Gehrmann, PLB465, 297 \(1999\)](#)).
- ◆ In the (assumed) TMD factorization ([Bacchetta, Mulders, Pijlman PLB595, 309 \(2004\)](#); [Bacchetta et al., JHEP0702, 093 \(2007\)](#))

- ◆ Introduce the convolution integral

$$\mathcal{I}[\omega f D] = x \sum_q e_q^2 \int d^2 \mathbf{p}_T \int d^2 \mathbf{k}_T \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T) w(\mathbf{p}_T, \mathbf{k}_T) f^q(x, \mathbf{p}_T^2) D^q(z, \mathbf{k}_T^2).$$

- ◆ The spin-averaged structure function F_{UUU} can in the be given as

$$F_{UUU} = \mathcal{I}[f_1 D_1],$$

- ◆ The structure function in the numerator F_{UUT} are given as

$$F_{UUT}^{\sin \phi_{S_h}} = \frac{2M}{Q} \mathcal{I} \left\{ \left(\frac{M_h}{M} f_1 \frac{\tilde{D}_T}{z} - x h H_1 \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2M M_h} \left[\left(\frac{M_h}{M} h_1^\perp \tilde{H}_T^\perp - x f_1 D_{1T}^\perp \right) - \left(\frac{M_h}{M} h_1^\perp \frac{\tilde{H}_T}{z} + x g^\perp G_{1T} \right) \right] \right\}.$$

- ◆ Using the WW (Wandzura-Wilczek) approximation to ignore the contributions from the twist-3 TMD fragmentation functions

$$F_{UUT}^{\sin \phi_{S_h}} \approx \frac{2M}{Q} \mathcal{I} [-x h H_1 + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2M M_h} (-x f^\perp D_{1T}^\perp - x g^\perp G_{1T})]. \quad \text{Need further studies!}$$

- ◆ Nonzero contributions with the collinear twist-3 factorization (Z.-B. Kang, F. Yuan and J. Zhou, Phys. Lett. B 691, 243(2010); A. Metz and D. Pitonyak, Phys. Lett. B 723, 365 (2013); K. Kanazawa, Y. Koike, A. Metz, and D. Pitonyak, PRD 89, 111501(R) (2014)).
- ◆ The contribution from the quark-gluon-quark correlators may still be important, the twist-3 FFs might not be necessarily negligible (R. Kundu and A. Metz, Phys. Rev. D 65 014009(2002). A. Accardi, A. Bacchetta, W. Melnitchouk and M. Schlegel, JHEP 0911, 093(2009). B. Pasquini and S. Rodini, arXiv:1806.10932 [hep-ph].)
- ◆ Focus on the contribution from the twist-3 TMD distribution functions and twist-2 fragmentation functions

$$F_{UUT}^{\sin \phi_{S_h}} \approx \frac{2M}{Q} \mathcal{I}[-xhH_1 + \frac{k_T \cdot p_T}{2MM_h}(-xf^\perp D_{1T}^\perp - xg^\perp G_{1T})].$$

The TMD twist-3 distribution functions

◆ The quark-quark corrector

$$\Phi(x, \mathbf{p}_T; S) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ip\xi} \langle P, S | \bar{\psi}(0) \mathcal{L}[0, \xi] \psi(\xi) | P, S \rangle \Big|_{\xi^+ = 0},$$

$$\frac{1}{4} \text{Tr}[(\Phi(x, \mathbf{p}_T; S) + \Phi(x, \mathbf{p}_T; -S)) i\sigma^{\alpha\beta} \gamma_5] = -\frac{M}{P^+} \epsilon_T^{\alpha\beta} h, \quad \text{T-odd: } h \text{ and } g^\perp$$

$$\frac{1}{4} \text{Tr}[(\Phi(x, \mathbf{p}_T; S) + \Phi(x, \mathbf{p}_T; -S)) \gamma^\alpha] = \frac{p_T^\alpha}{P^+} f^\perp, \quad \text{T-even: } f^\perp$$

$$\frac{1}{4} \text{Tr}[(\Phi(x, \mathbf{p}_T; S) + \Phi(x, \mathbf{p}_T; -S)) \gamma^\alpha \gamma_5] = -\frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{P^+} g^\perp.$$

- ◆ Using the similar spectator model calculation in the work ([Bacchetta, Conti, Radici, PRD78, 074010\(2008\)](#)) and considering the following sum for axial-vector diquark propagator ([Brodsky et al., NPB593, 311\(2011\)](#))

$$d_{\mu\nu}(P-k) = -g_{\mu\nu} + \frac{(P-k)_\mu n_{-\nu} + (P-k)_\nu n_{-\mu}}{(P-k) \cdot n_-} - \frac{M_v^2}{[(P-k) \cdot n_-]^2} n_{-\mu} n_{-\nu}$$

The TMD twist-3 distribution functions

- ◆ The distributions contributed by the scalar diquark

$$h(x, \mathbf{p}_T^2)_s = - \frac{N_s^2 (1-x)^3}{16\pi^3 M} \frac{e_s e_q}{4\pi} \frac{(m + xM)(L_s^2 - \mathbf{p}_T^2)}{L_s^2 (\mathbf{p}_T^2 + L_s^2)^3},$$

$$f^\perp(x, \mathbf{p}_T^2)_s = - \frac{N_s^2 (1-x)^2}{16\pi^3} \frac{(\mathbf{p}_T^2 - 2mM(1-x) - (1-x^2)M^2 + m_s^2)}{(\mathbf{p}_T^2 + L_s^2)^4}.$$

- ◆ The distributions contributed by the axial-vector diquark

$$h(x, \mathbf{p}_T^2)_a = 0,$$

$$f^\perp(x, \mathbf{p}_T^2)_a = \frac{N_a^2 (1-x)}{16\pi^3} \frac{(x\mathbf{p}_T^2 + 2mM(1-x)^2 + (x-1)m^2 + (x^3 - 2x^2 + 1)M^2 - m_a^2)}{L_a^2 (\mathbf{p}_T^2 + L_a^2)^3}.$$

- ◆ Model results for the twist-3 T-odd distribution function g^\perp have been given in [Wenjuan Mao and Zhun Lu, Phys. Rev. D87.014012](#).

- ◆ Parameters are taken from [Bacchetta, Conti, Radici, PRD78, 074010\(2008\)](#)

$$f^u = c_s^2 f^s + c_a^2 f^a, \quad f^d = c_{a'}^2 f^{a'}$$

The TMD twist-3 distribution functions



◆ Model results for the distribution functions f^\perp

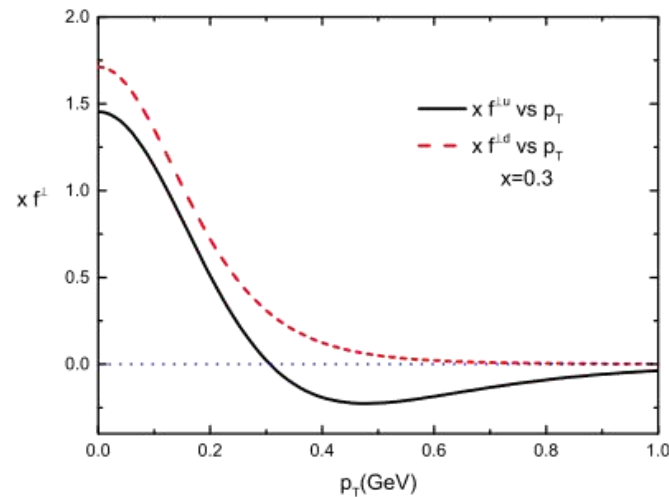
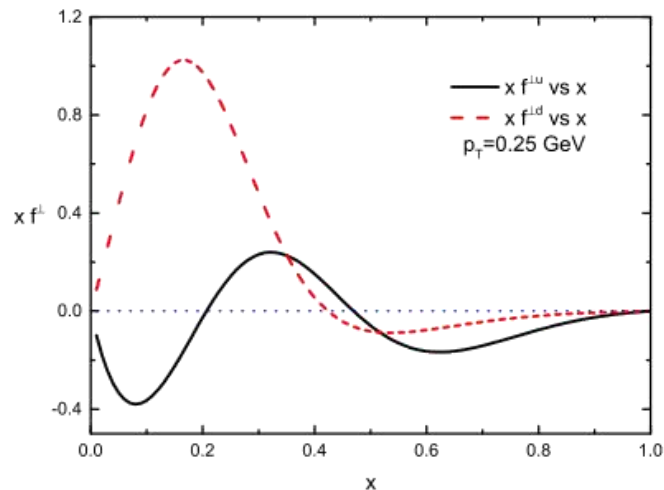


FIG. 1: Left panel: model results for $x f^{\perp u}(x, \mathbf{p}_T^2)$ (solid line) and $x f^{\perp d}(x, \mathbf{p}_T^2)$ (dashed line) as functions of x at $p_T = 0.3$ GeV. Right panel: model results for $x f^{\perp u}$ (solid line) and $x f^{\perp d}$ (dashed line) as functions of p_T at $x = 0.3$.

The TMD twist-3 distribution functions



◆ Model results for the distribution function h

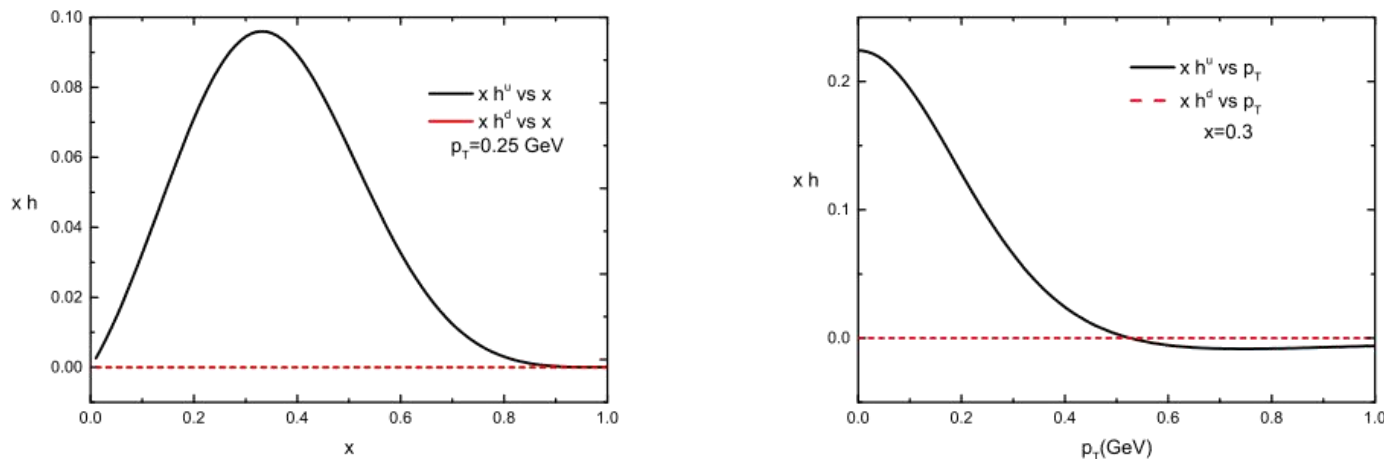


FIG. 2: Similar to Fig. 1, but for the model results of $xh^u(x, \mathbf{p}_T^2)$ (solid line) and $xh^d(x, \mathbf{p}_T^2)$ (dashed line).

- ◆ For the twist-2 distribution function f_1 , we adopt the model results in Bacchetta, Conti, Radici, Phys. Rev. D78, 074010(2008).

The TMD twist-2 fragmentation functions

- ◆ To estimate the asymmetry, we also need to know the twist-2 fragmentation functions $H_1, G_{1T}, D_{1T}^\perp$

$$\begin{aligned}\frac{\epsilon_T^{\alpha\beta} k_{T\alpha} S_{hT\beta}}{M} D_{1T}^\perp(z, \mathbf{k}_T^2) &= \frac{1}{2} \text{Tr}[(\Delta(z, \mathbf{k}_T; \mathbf{S}_{hT}) - \Delta(z, \mathbf{k}_T; -\mathbf{S}_{hT})) \gamma^-], \\ S_{hL} G_{1L}(z, \mathbf{k}_T^2) + \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} G_{1T}(z, \mathbf{k}_T^2) &= \frac{1}{4} \text{Tr}[(\Delta(z, \mathbf{k}_T; \mathbf{S}_{hT}) - \Delta(z, \mathbf{k}_T; -\mathbf{S}_{hT})) \gamma^- \gamma_5], \\ S_T^\alpha H_1(z, \mathbf{k}_T^2) &= \frac{1}{4} \text{Tr}[(\Delta(z, \mathbf{k}_T; \mathbf{S}_{hT}) - \Delta(z, \mathbf{k}_T; -\mathbf{S}_{hT})) i\sigma^{\alpha-} \gamma_5],\end{aligned}$$

with the fragmentation correlation function

$$\Delta(z, \mathbf{k}_T; \mathbf{S}_{hT}) = \frac{1}{2z} \sum_X \int \frac{d\xi^+ d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 | \mathcal{U}_{(+\infty, \xi)}^{n+} \psi(\xi) | P_h, S_h; X \rangle \langle P_h, S_h; X | \bar{\psi}(0) \mathcal{U}_{(0, +\infty)}^{n+} | 0 \rangle \Big|_{\xi^- = 0},$$

and the spin vector of the outgoing hadron

$$S_h^\mu = S_{hL} \frac{(P_h \cdot n_+) n_-^\mu - (P_h \cdot n_-) n_+^\mu}{M_h} + S_{hT}^\mu.$$

The TMD twist-2 fragmentation functions

- ◆ For the fragmentation function $D_1^{(s)}(z, k_T^2)$, we can obtain its expression as

$$D_1^{(s)}(z, k_T^2) = D_1^v(z, k_T^2) = \frac{g_D^2}{2(2\pi)^3} \frac{1}{z^2} e^{\frac{-2k^2}{\Lambda^2}} \frac{(1-z)[z^2 k_T^2 + (M + zm)^2]}{z^4(k_T^2 + L^2)^2},$$

- ◆ Assuming the SU(6) spin-flavor symmetry for the final state hadron, the relation between quark flavors and diquark types for proton and Λ we can write as

$$D^{u \rightarrow p} = \frac{3}{2}D^{(s)} + \frac{1}{2}D^{(v)}, \quad D^{d \rightarrow p} = D^{(v)}, \quad D^{s \rightarrow p} = 0$$

$$D^{u \rightarrow \Lambda} = D^{d \rightarrow \Lambda} = \frac{1}{4}D^{(s)} + \frac{3}{4}D^{(v)}, \quad D^{s \rightarrow \Lambda} = D^{(s)},$$

- ◆ For $D_1^p(z)$, we fit it to the HKNS LO parametrization at the initial scale $\mu_{LO}^2 = 1\text{GeV}^2$

$$g_D = 1.588_{-0.096}^{+0.1}, \quad m_D = 0.849_{-0.0376}^{+0.04} \text{GeV},$$

$$\lambda = 10.192_{-1.11}^{+1.34} \text{GeV}, \quad \alpha = 0.5(\text{fixed}), \quad \beta = 0(\text{fixed}).$$

The TMD twist-2 fragmentation functions



- ◆ Model result of the unpolarized fragmentation function $D_1(z)$

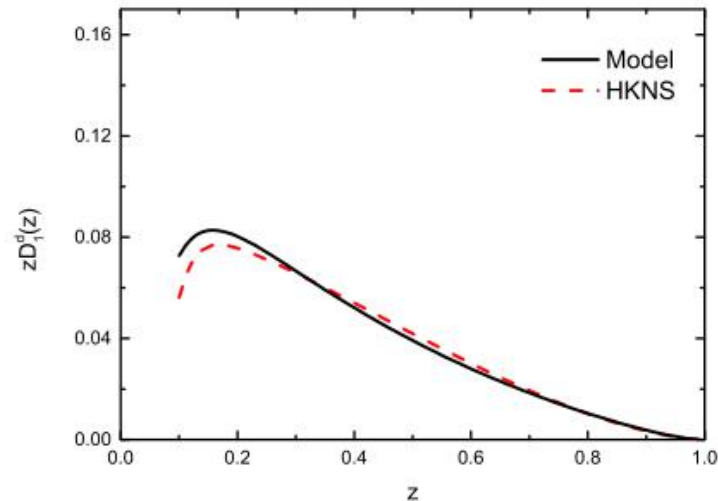
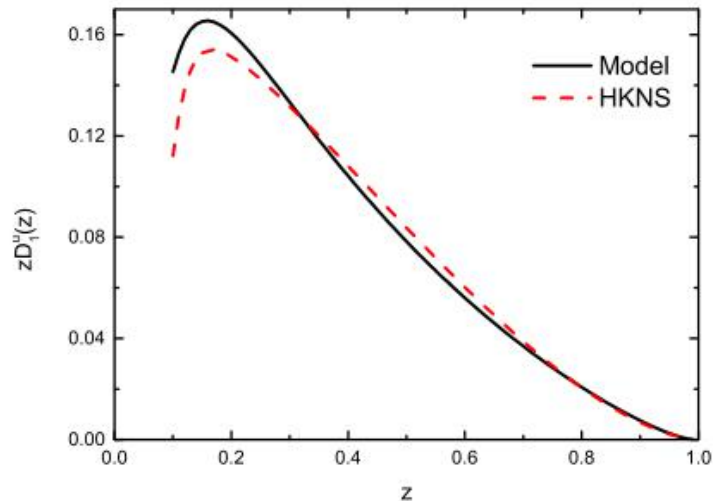


FIG. 3: Unpolarized fragmentation functions $zD_1^u(z)$ (left panel) and $zD_1^d(z)$ (right panel) of the proton (solid lines) compared with the HKNS parametrization (dashed lines), respectively.

The TMD twist-2 fragmentation functions

- ◆ For the fragmentation function $D_{1T}^\perp(z, k_T^2)$, we adopt the form in Y. Yang, Z. LU and I. Schmidt PRD96, 034010(2017)
- ◆ For the fragmentation function $H_1(z, k_T^2)$ and $G_{1T}(z, k_T^2)$, we obtain the results

$$G_{1T}^R(z, \mathbf{k}_T^2) = a_R \frac{g_D^2}{(2\pi)^3} \frac{1}{z^2} e^{\frac{-2k^2}{\Lambda^2}} \frac{M_h(zm + M_h)(1 - z)}{z^3(\mathbf{k}_T^2 + L_f^2)^2},$$

$$H_1^R(z, \mathbf{k}_T^2) = a_R \frac{g_D^2}{2(2\pi)^3} \frac{1}{z^2} e^{\frac{-2k^2}{\Lambda^2}} \frac{(1 - z)[(zm + M_h)^2]}{z^4(\mathbf{k}_T^2 + L_f^2)^2},$$

with
$$L_f^2 = \frac{1 - z}{z^2} M_h^2 + m^2 + \frac{m_D^2 - m^2}{z}.$$

The TMD twist-2 fragmentation functions



◆ Model result of the fragmentation functions $zD_{1T}^{\perp u}$, $zH_1^u(z)$ and $zG_{1T}^u(z)$

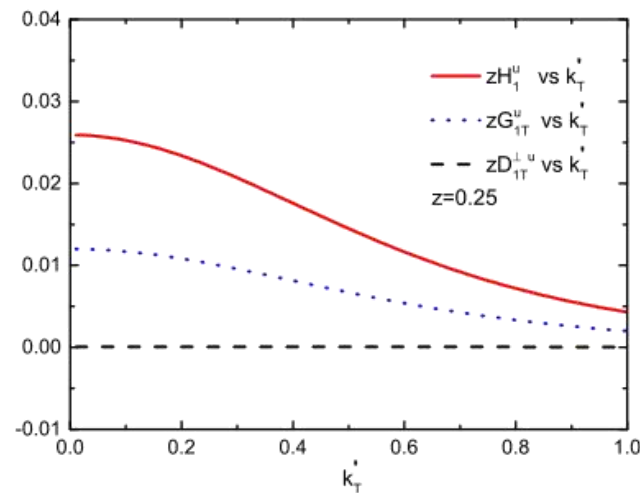
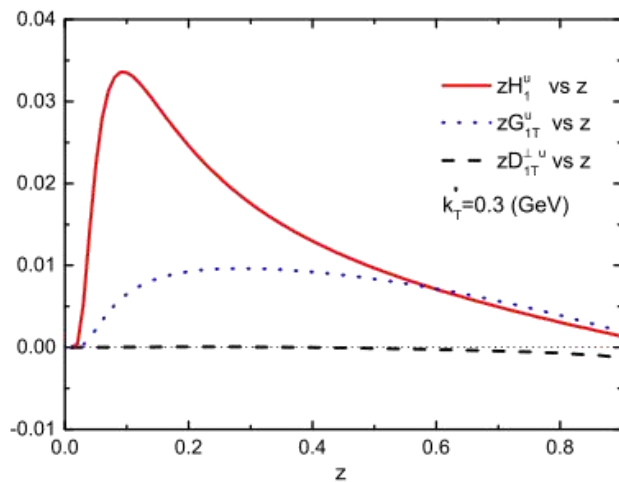


FIG. 4: Left panel: the model results of zH_1^u (solid line), zG_{1T}^u (dotted line) and $zD_{1T}^{\perp u}$ (dashed line) for the proton as functions of z at $k_T = 0.3$ GeV; Right panel: the model results of zH_1^u (solid line), zG_{1T}^u (dotted line) and $zD_{1T}^{\perp u}$ (dashed line) for the proton as functions of k_T at $z = 0.25$.

The TMD twist-2 fragmentation functions



- ◆ Model result of the fragmentation functions $zD_{1T}^{\perp d}$, $zH_1^d(z)$ and $zG_{1T}^d(z)$

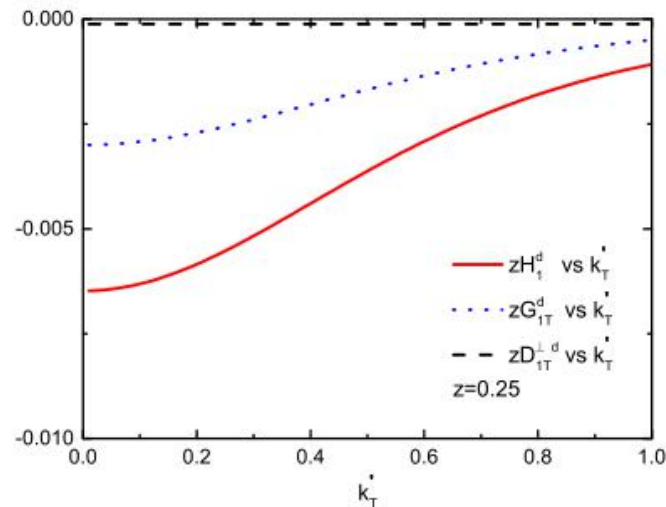
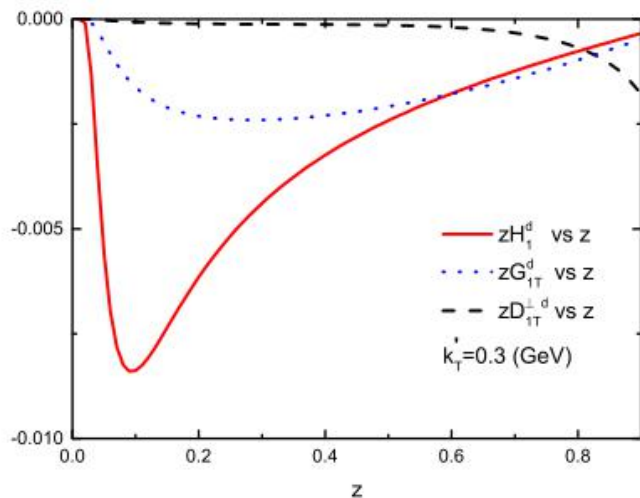


FIG. 5: Similar to Fig. 4, but for the model results of zH_1^d (solid line), zG_{1T}^d (dotted line) and $zD_{1T}^{\perp d}$ (dotted line).

Numerical estimate at JLab 12GeV

- ◆ Constraints for quark transverse momentum p_T^2 (Boglione, Melis, Prokudin, *Phy. Rev. D*84, 034033(2011)) :

$$\begin{cases} p_T^2 \leq (2-x)(1-x)Q^2, & \text{for } 0 < x < 1; \\ p_T^2 \leq \frac{x(1-x)}{(1-2x)^2} Q^2, & \text{for } x < 0.5. \end{cases}$$

- ◆ At JLab 12GeV, the kinematics are as follows

$$0.1 < x < 0.6, \quad 0.3 < z < 0.7, \quad Q^2 > 1\text{GeV}^2, \\ W^2 > 4\text{GeV}^2, P_{h\perp} > 0.05\text{GeV}.$$

- ◆ The invariant mass of the hadron final state

$$W^2 = (P + q)^2 \approx \frac{1-x}{x} Q^2.$$

Numerical estimate at JLab 12GeV

◆ The single-spin asymmetry of proton production in SIDIS at JLab 12GeV.

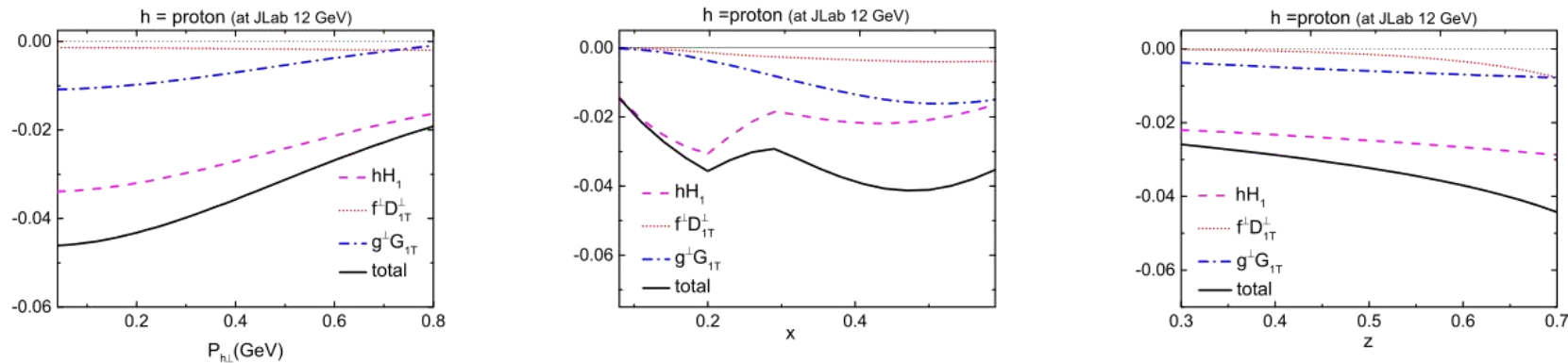


FIG. 7: Predictions on the transverse SSA $A_{UT}^{\sin \phi_{S_h}}$ for proton production in SIDIS at JLab 12 GeV. The dashed, dotted and dash-dotted curves represent the asymmetries from the hH_1 , $f^\perp D_{1T}^\perp$ and $g^\perp G_{1T}$ terms, respectively. The solid curves correspond to the total contribution.

Numerical estimate at JLab 12GeV

- ◆ The single-spin asymmetry of Λ production in SIDIS at JLab 12GeV.

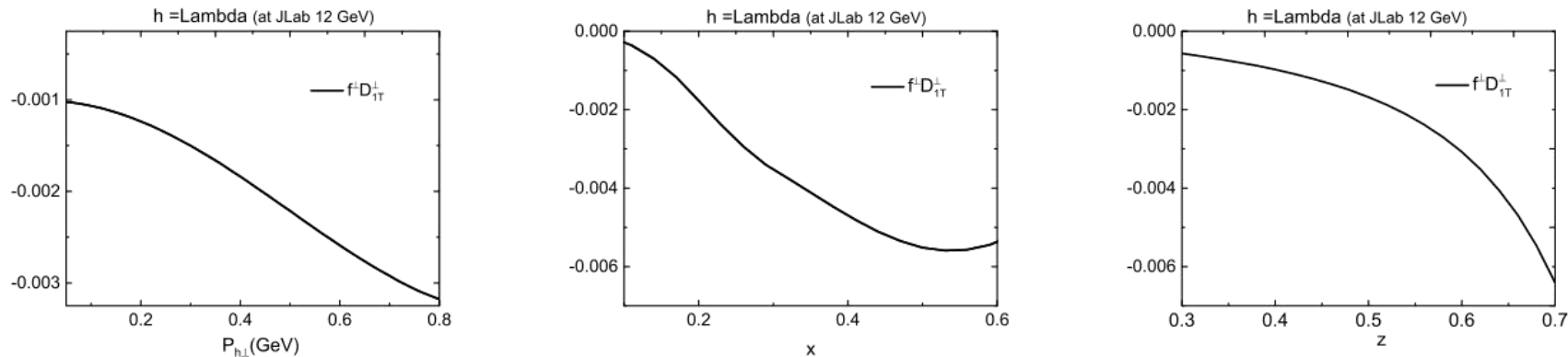


FIG. 8: Predictions on the transverse SSA $A_{\text{UT}}^{\sin \phi_{S_h}}$ for Λ production in SIDIS at JLab. The solid curves correspond to the total asymmetry (it only receives contribution from the $f^\perp D_{1T}^\perp$ term).



Numerical estimate at COMPASS

- ◆ At COMPASS, the kinematical cuts are as follows (M. G. Alekseev et al. [COMPASS Collaboration], Phys. Lett. B 692, 240 (2010) [arXiv:1005.5609 [hep-ex]])

$$0.004 < x < 0.7, \quad 0.1 < y < 0.9, \quad z > 0.2,$$

$$P_{h\perp} > 0.1 \text{ GeV}, \quad Q^2 > 1 \text{ GeV}^2,$$

$$W > 5 \text{ GeV}, \quad E_h > 1.5 \text{ GeV}.$$

- ◆ At the twist-3 level, the effect will be suppressed by $1/Q$, since the averaged Q value at COMPASS is much higher than that at JLab 12GeV

$$E_{beam} = 160 \text{ GeV}$$

Suppressed by $1/Q$!

Numerical estimate at COMPASS

- ◆ The single-spin asymmetry of proton production in SIDIS at COMPASS.

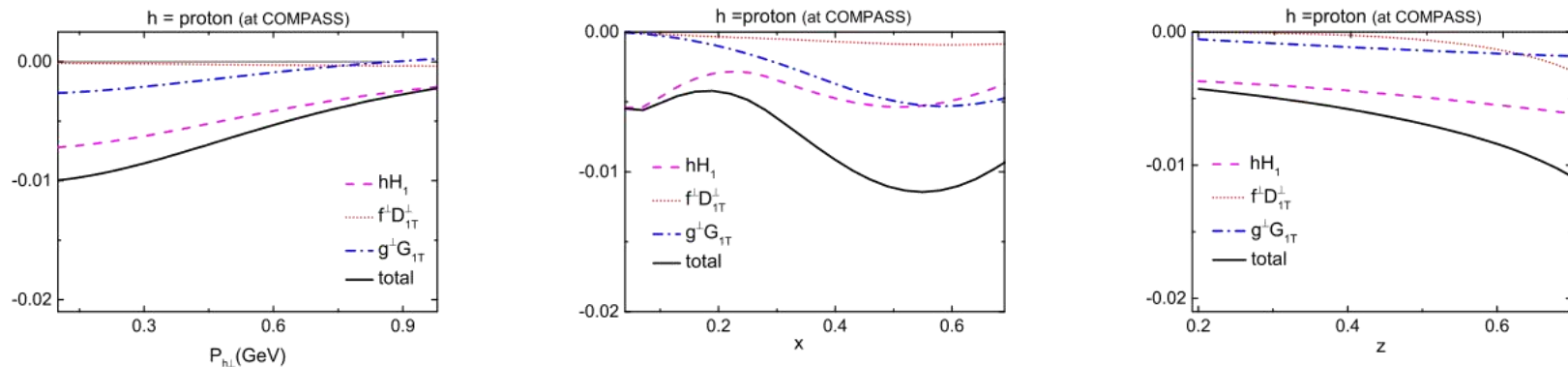


FIG. 9: Predictions on the transverse SSA $A_{UU}^{\sin \phi_{S_h}}$ for the proton production in SIDIS at COMPASS, contributed by the hH_1 (dashed lines), $f^\perp D_{1T}^\perp$ (dotted lines) and $g^\perp G_{1T}$ (dashed-dotted) terms, respectively. The solid curves correspond to the total contribution.

Numerical estimate at COMPASS

- ◆ The single-spin asymmetry of Λ production in SIDIS at COMPASS.

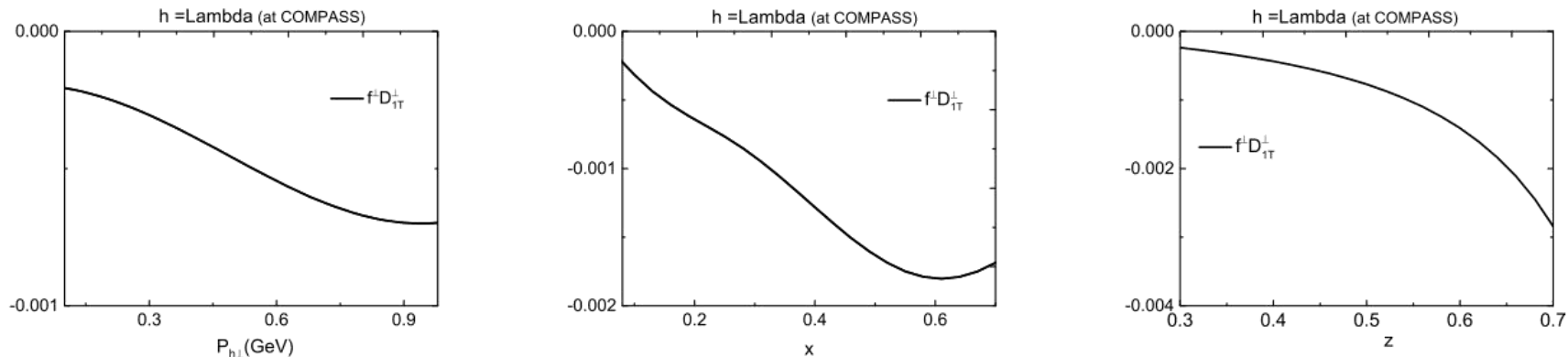


FIG. 10: Predictions on the transverse SSA $A_{\text{UUT}}^{\sin \phi S_h}$ for the Λ production in SIDIS at COMPASS, contributed by the $f^\perp D_{1T}^\perp$ term (The asymmetry only receives contribution from $f^\perp D_{1T}^\perp$ term in our model).

SUMMARY



- ◆ We predict the single-spin asymmetry $A_{UT}^{\sin\phi_{Sh}}$ of proton and Λ production in SIDIS at JLab 12GeV and COMPASS with a muon beam of 160GeV.
- ◆ We consider the contributions from both the twist-3 TMD distribution functions h, f^\perp and g^\perp and the twist-2 fragmentation functions H_1, D_{1T}^\perp and G_{1T} .
- ◆ The asymmetry of proton at JLab 12GeV is sizable, around 4 percent, while at COMPASS it is about 1 percent.
- ◆ The asymmetry of Λ is much smaller, since only the $f^\perp D_{1T}^\perp$ can survive in our model calculation.



THANK YOU!