Off-shell initial state effects and gauge invariance in the Drell-Yan process

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Introduction

The talk is based on [M. A. Nefedov, V. A. Saleev, Phys.Lett. B790 (2019) 551; hep-ph/1810.04061].

Cross-section for the un-polarized Drell-Yan process ($S = (P_1 + P_2)^2$, $Q^2 = q^2 = (k_1 + k_2)^2$):

$$p(P_1) + p(P_2) \to \gamma^\star(q) + X \to l^+(k_1) + l^-(k_2) + X,$$

can be decomposed over *helicity structure functions* (HSFs) $F_{UU}^{(1,\dots)}$ as follows:

$$\begin{aligned} \frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} &= \frac{\alpha^2}{4Q^2}\Big[F_{UU}^{(1)}\cdot\left(1+\cos^2\theta\right) + F_{UU}^{(2)}\cdot\left(1-\cos^2\theta\right) + \\ &+ F_{UU}^{(\cos\phi)}\cdot\sin(2\theta)\cos\phi + F_{UU}^{(\cos 2\phi)}\cdot\sin^2\theta\cos(2\phi)\Big], \end{aligned}$$

where $x_{A,B} = Qe^{\pm Y}/\sqrt{S}$, θ and ϕ are angles defining the direction of l^+-momentum in the c.o.m. frame of l^+l^-. We define them in **Collins-Soper frame**.

TMD-factorization

Can one set-up the TMD-factorization theorem for HSFs?

$$F_{UU}^{(1,\dots)}(x_A, x_B, \mathbf{q}_T) \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1} + \mathbf{q}_{T2} - \mathbf{q}_T) \times$$
$$\times \quad \Phi_q(x_A, \mathbf{q}_{T1}) \Phi_{\bar{q}}(x_B, \mathbf{q}_{T2}) \times f_{q\bar{q}}^{(1,\dots)}(\mathbf{q}_{T1}, \mathbf{q}_{T2})$$

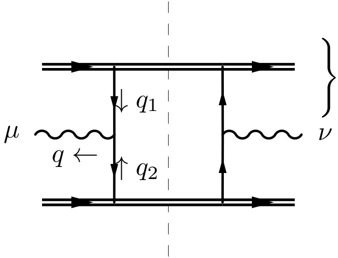
- For $\frac{d\sigma}{dQ^2 d\mathbf{q}_T^2}$ such factorization holds up to power-corrections in $\mathbf{q}_T^2/Q^2$. Is it true for HSFs?
- Possible problem: **QED** gauge-invariance.

TMD Parton-model

Leptonic ($L_{\mu\nu}$) and hadronic ($W_{\mu\nu}$) tensors:

$$d\sigma \sim L^{\mu\nu} W_{\mu\nu},$$

Parton model for hadronic tensor:



Decomposition for quark correlatior (un-polarized protons, $P_1^\mu = \sqrt{S} n_-^\mu / 2$):

$$\Phi_q^{\alpha\beta} = \hat{n}_-^{\alpha\beta} f_1^{(q)} + \frac{q_{T1}^i \epsilon_{ij}}{\Lambda} \left(i\sigma^{-j}\gamma_5\right)^{\alpha\beta} h_1^{(\perp q)},$$

where $f_1^{(q)}(x, \mathbf{q}_T)$ – TMD quark number density, $h_1^{(\perp q)}(x, \mathbf{q}_T)$ – Boer-Mulders function.

$$W_{\mu\nu} \sim \mathrm{tr}\left[\Phi_q \gamma_\mu \Phi_{\bar{q}} \gamma_\nu\right] \Rightarrow \begin{cases} F_{UU}^{(1,2)} \sim f_1^{(q)}(x_A, \mathbf{q}_{T1}) \otimes f_1^{(\bar{q})}(x_B, \mathbf{q}_{T2}) \\ F_{UU}^{(\cos 2\phi)} \sim h_1^{(\perp q)}(x_A, \mathbf{q}_{T1}) \otimes h_1^{(\perp \bar{q})}(x_B, \mathbf{q}_{T2}) \end{cases}$$

But $\mathrm{q}^\mu \mathrm{W}_{\mu\nu} \neq 0$ for $\mathbf{q}_\mathrm{T} \neq 0$!

Curing the Ward-identity

How to restore Ward identity on the level of **partonic tensor** for the **number-density** contribution:

$$w_{\mu\nu} = \frac{1}{4}\mathrm{tr}\left[\hat{\tilde{q}}_2\gamma_\mu\hat{\tilde{q}}_1\gamma_\nu\right],$$

Naively, one can just put momenta $\tilde{q}_{1,2}$ on-shell ($\tilde{q}_{1,2}^2 = 0$), retaining $\tilde{q}_1 + \tilde{q}_2 = q$. This can be done in two ways ("Quasi on-shell schemes"):

1 Without explicit dependence on $\mathbf{q}_{T1,2}$ [Collins, 2011]:

$$(\tilde{q}_1^{(\mathrm{QOS}-1)})^\mu = \frac{1}{4\kappa}\left(q^+(\kappa+1)n_-^\mu + q^-(\kappa-1)n_+^\mu\right) + \frac{q_T^\mu}{2},$$
$$(\tilde{q}_2^{(\mathrm{QOS}-1)})^\mu = \frac{1}{4\kappa}\left(q^+(\kappa-1)n_-^\mu + q^-(\kappa+1)n_+^\mu\right) + \frac{q_T^\mu}{2},$$

where $\kappa = \sqrt{Q_T^2/Q^2}$ and $q^\pm = Q_T e^{\pm Y}$. Then

$$\boxed{f^{(1)}_{\mathrm{QOS}-1} = Q^2,\ f^{(2)}_{\mathrm{QOS}-1} = f^{(\cos\phi)}_{\mathrm{QOS}-1} = f^{(\cos 2\phi)}_{\mathrm{QOS}-1} = 0.}$$

Curing the Ward-identity

2 With explicit dependence on $\mathbf{q}_{T1,2}$:

$$(\tilde{q}_1^{(\mathrm{QOS}-2)})^\mu = \frac{1}{2}\left(q_1^+ n_-^\mu + \frac{\mathbf{q}_{T1}^2}{q_1^+} n_+^\mu\right) + q_{T1}^\mu,$$

$$(\tilde{q}_2^{(\mathrm{QOS}-2)})^\mu = \frac{1}{2}\left(\frac{\mathbf{q}_{T2}^2}{q_2^-} n_-^\mu + q_2^- n_+^\mu\right) + q_{T2}^\mu,$$

where $q_1^+ = (Q_T^2 + t_1 - t_2 + \sqrt{D})/(2q^-)$, $q_2^- = (Q_T^2 - t_1 + t_2 + \sqrt{D})/(2q^+)$ and $D = (Q_T^2 - t_1 - t_2)^2 - 4t_1 t_2$.

Then $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$ get nonzero contribution from **number density**:

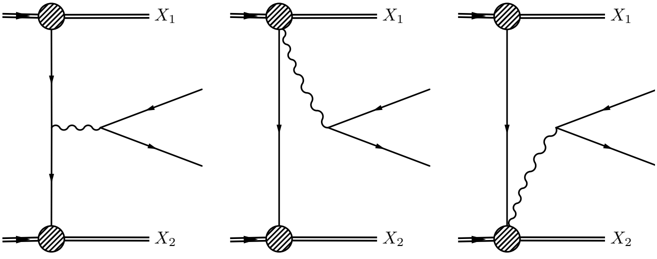
$$f^{(1)}_{\mathrm{QOS}-2} = Q^2 - \frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{2} + \frac{(\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2)^2}{2Q_T^2},\ f^{(2)}_{\mathrm{QOS}-2} = (\mathbf{q}_{T1}-\mathbf{q}_{T2})^2 - \frac{(\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2)^2}{Q_T^2},$$

$$f^{(\cos\phi)}_{\mathrm{QOS}-2} = \sqrt{\frac{Q^2 D}{q_T^2}}\frac{\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2}{Q_T^2},\ f^{(\cos 2\phi)}_{\mathrm{QOS}-2} = -\frac{(\mathbf{q}_{T1}-\mathbf{q}_{T2})^2}{2} + \frac{Q^2+Q_T^2}{2Q_T^2}\frac{(\mathbf{q}_{T1}^2-\mathbf{q}_{T2}^2)^2}{q_T^2}.$$

So answer for HSFs $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$ at **leading power** in q_T^2/Q^2 depends on the way how one restores gauge-invariance of the hadronic tensor. What is the right way?

Full amplitude

In the full theory, not only t-channel ("Parton model") diagram but also diagrams with direct interaction of the photon with the proton and it's remnants are needed to restore gauge-invariance:



(a) (b) (c)

Does that mean that there is no TMD factorization for HSFs $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$?
Could factorization be restored in the High-Energy limit:

$$\boxed{S \gg Q^2 \gg q_T^2} \ ?$$

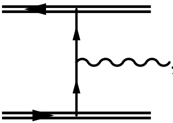
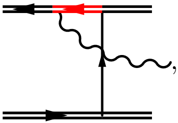
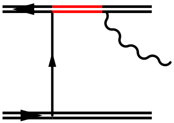
Spectator model

Let's consider the question of factorization in a concrete field-theoretic model, which includes (massless) proton fields, quarks, gluons and *spectator* fields of mass M_s. Protons, quarks and spectators carry $U(1)$ charge.

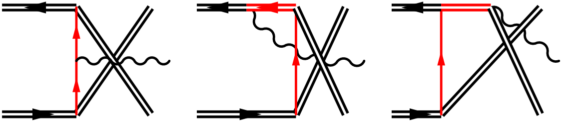
Let's consider the process:

$$\bar{p}(P_1) + p(P_2) \to \gamma^\star(q) + s(P_1') + s(P_2').$$

Most interesting diagrams in High-Energy limit (+ 2 similar diags.):

$\mathcal{D}_1 =$ , $\mathcal{D}_2 =$ , $\mathcal{D}_3 =$ 

Crossed-diagrams are doubly-suppressed:



Fadin-Sherman vertex

In the leading power in $\sqrt{S} = P_1^+ = P_2^-$, diags. 2 and 3 give:

$$\mathcal{D}_2^\mu \supset e_p \bar{v}(P_1)\gamma^\mu \frac{\hat{P}_1 - \hat{q}}{(P_1 - q)^2} \simeq e_p \bar{v}(P_1) \frac{P_1^+ \gamma^\mu \hat{n}_-}{2(-P_1^+ q^-)} = \bar{v}(P_1)\frac{i\hat{q}_1}{q_1^2}\left[ie_p \frac{\hat{q}_1 n_-^\mu}{q_-}\right],$$

$$\mathcal{D}_3^\mu \supset e_s \frac{(2P_1 + 2q_2 - q)^\mu}{(P_1 + q_2)^2}\bar{v}(P_1) \simeq \frac{P_1^+ n_-^\mu}{P_1^+ q^-}\bar{v}(P_1) = \bar{v}(P_1)\frac{i\hat{q}_1}{q_1^2}\left[-ie_s \frac{\hat{q}_1 n_-^\mu}{q_-}\right].$$

Collecting the contributions of all diagrams one obtains

$$\mathcal{M}_\mu \simeq (-\lambda_{spq}^2)\bar{v}(P_1)\frac{i\hat{q}_1}{q_1^2}\left(-i\Gamma_\mu(q_1, q_2)\right)\frac{-i\hat{q}_2}{q_2^2}u(P_2),$$

where **Fadin-Sherman vertex** [Fadin, Sherman, 1976]:

$$\Gamma_\mu(q_1, q_2) = e_q \gamma_\mu - (e_p - e_s)\hat{q}_1 \frac{n_\mu^-}{q_-} - (e_p - e_s)\hat{q}_2 \frac{n_\mu^+}{q_+},$$

depends only on e_q, since $\boxed{e_p - e_s = e_q}$ and it satisfies Ward identity: $\boxed{q^\mu \Gamma_\mu(q_1, q_2) = 0}$.

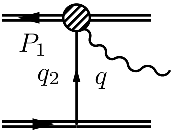
“Dressed” spectator model

Proton is a composite particle and “spectator” also models the soft hadronic system. Interactions $p\gamma p$ and $s\gamma s$ should contain formfactors $F(Q^2) \sim (\Lambda^2/Q^2)^{\#}$ for $Q^2 \gg \Lambda^2$:

$$\gamma^\mu \to F_1^{(p)}(P,q)\gamma_\mu + F_2^{(p)}(P,q)q^\nu\sigma_{\mu\nu} + F_3^{(p)}(P,q)P^\nu\sigma_{\mu\nu} + ...,$$
$$(2P-q)_\mu \to F_1^{(s)}(P,q)(2P-q)_\mu + F_2^{(s)}(P,q)P_\mu + F_3^{(s)}(P,q)q_\mu,$$

However this should be also done consistently with gauge-invariance!

To restore GI of the model, one introduces the $p\gamma qs$-vertex:



And expands it over a basis of 48 possible Lorentz structures:

$$\gamma^\mu,\ \gamma_5\gamma^\mu,\ P_1^\mu,\ q_2^\mu,\ P_1^\mu\gamma_5,\ q_2^\mu\gamma_5,$$
$$P_1^\mu\hat{P}_1, P_1^\mu\hat{q}, P_1^\mu\hat{q}_2, q^\mu\hat{P}_1, ...,$$
$$\gamma^\mu\hat{P}_1,\ \gamma^\mu\hat{q},\ ,...$$
$$P_1^\mu\hat{P}_1\hat{q}_2, ..., \gamma^\mu\hat{P}_1\hat{q}, ...$$

Also we introduce the form-factors to spq-vertex:

$$i\lambda_{spq}(q_{1,2}) \to i\lambda_{spq}(f_1(q_{1,2}) + f_2(q_{1,2})\gamma_5\hat{q}_{1,2}),$$

form-factor f_2 leads to *Boer-Mulders-like* contribution to quark correlator Φ_q.

"Dressed" spectator model

Imposing the Ward-identity for the full amplitude:

$$q_\mu \mathcal{M}^\mu = 0,$$

one expresses some of the coefficients of decomposition of $p\gamma qs$-vertex in terms of formfactors and then goes to the limit $\sqrt{S} \to \infty$. We have found, that Leading-power term of the amplitude is independent on formfactors and is equal to:

$$\lambda^2_{spq} \bar{v}(P_1)(f_1(q_1)+f_2(q_1)\gamma_5 \hat{q}_1)\frac{\hat{q}_1}{q_1^2}\Gamma_\mu(q_1,q_2)\frac{\hat{q}_2}{q_2^2}(f_1(q_2)+f_2(q_2)\gamma_5 \hat{q}_2)u(P_2).$$

This suggests the following **gauge-invariant** factorization for the hadronic tensor of the TMD parton model:

$$\boxed{W_{\mu\nu} \sim \mathrm{tr}\left[\Phi_q \Gamma_\mu(q_1,q_2)\Phi_{\bar{q}}\Gamma_\nu(q_1,q_2)\right],}$$

where Φ_q is decomposed in a standard way over number density and Boer-Mulders TMD PDFs. *Of course, this proposal requires further perturbative tests beyond tree-level in QCD interactions.*

Helicity structure functions in PRA

The partonic tensor for **number-density** contribution in above-proposed *Parton Reggeization Approach* (PRA) reads:

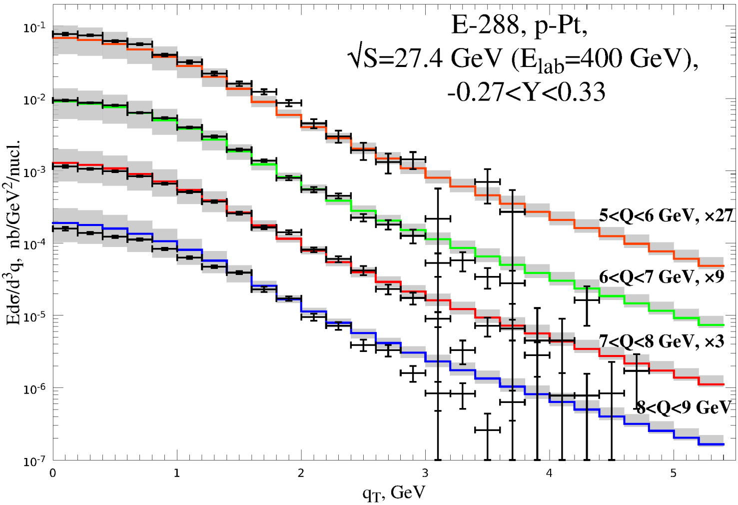
$$w_{\mu\nu}^{\rm PRA} = \frac{1}{4}{\rm tr}\left[\left(\frac{q_2^-}{2}\hat{n}^+\right)\Gamma_\mu(q_1,q_2)\left(\frac{q_1^+}{2}\hat{n}^-\right)\Gamma_\nu(q_1,q_2)\right],$$

and it leads to the following partonic HSFs:

$$f_{\rm PRA}^{(1)} = Q^2 + \frac{q_T^2}{2},\ f_{\rm PRA}^{(2)} = (\mathbf{q}_{T1} - \mathbf{q}_{T2})^2,$$
$$f_{\rm PRA}^{(\cos\phi)} = \sqrt{\frac{Q^2}{q_T^2}}(\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2),\ f_{\rm PRA}^{(\cos 2\phi)} = \frac{q_T^2}{2}.$$

As an **estimate** for number-density TMD PDF, we use the KMR [Kimber-Martin-Ryskin, 2001] formula, which allows one to obtain the unintegrated PDFs from the collinear ones (MSTW-2008 in our case), and resums the $(\alpha_s \log^2 \mathbf{q}_{T1,2}^2/\mu^2)^n$-corrections in Leading Logarithmic Approximation.

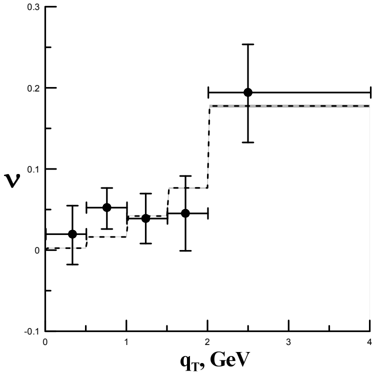
Description of E-288 q_T-spectra

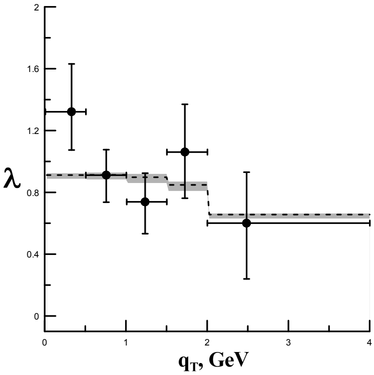


Constant NLO K-factor ~ 1.8 from $\alpha_s \pi^2$-corrections is included.

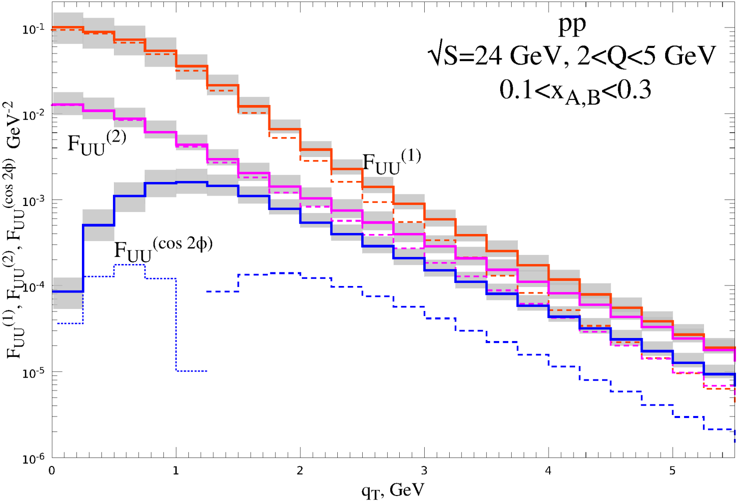
Description of NuSea data ($\sqrt{S} = 39$ GeV) on angular coefficients

See [Nefedov, Nikolaev, Saleev, 2013].





Predictions for HSFs at JINR-NICA ($\sqrt{S} = 24$ GeV)



Solid lines – PRA predictions, dashed lines – $QOS-2$-predictions. At small q_T, the $F_{UU}^{\cos 2\phi}$ HSF in $QOS-2$ is negative.

Conclusions

- Gauge-invariance of hadronic tensor is important for Helicity Structure Functions in Drell-Yan process
- Factorization holds in high-energy limit $S \gg Q^2 \gg q_T^2$, but with modified hard-scattering part $\Rightarrow$ PRA.
- Structure-function $F_{UU}^{\cos 2\phi}$ gets contribution not only from Boer-Mulders TMD PDF but also from number-density TMD PDF. Well-established factorization formula is required to separate them.
- PRA predictions with simple KMR unPDF reproduce existing data rather well, however polarization information is still limited. New experiments are needed, COMPASS, NICA-SPD, RHIC, ...

Thank you for your attention!