# Spectrally novel FF sum rules and dynamical generation of mass


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*Based on:* *Accardi, Signori, arXiv:1903.04458*

*Accardi, Bacchetta, PLB 773 (2017) 632*

*+ in progress w/ Bacchetta, Radici, Signori*

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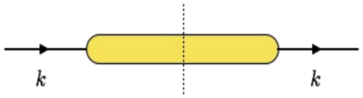
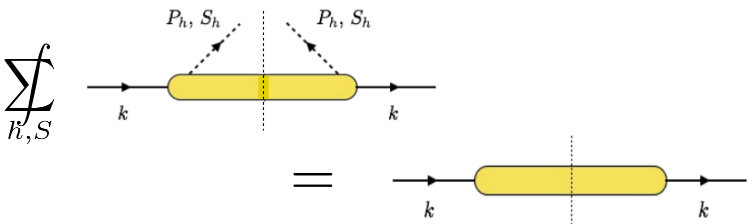
# Overview

- **"Inclusive jet" correlator**
  - Quarks are not asymptotic states




- **Gauge invariant spectral represntation**
  - jet/dressed quark mass

- **New FF sum rules**
  - Jet/ dressed quark mass is eperimentally observable!




- **New phenomenology**

- **Conclusions**

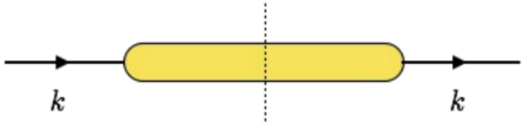
# Inclusive jet correlator

*AA, Signori, 1903.04458*
*Sterman, NPB 281 ('87)*

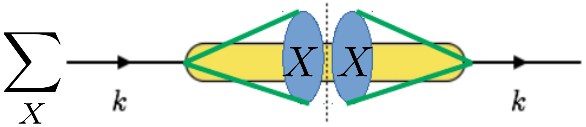
Inclusive $q \to X$ "inclusive jet" correlator

$$\Xi_{ij}(k;n_+) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \frac{\text{Tr}_c}{N_c} \langle\Omega|\mathcal{T}\, W^{n_+}_{(\infty,\xi)}\, \psi_i(\xi)\overline{\psi}_j(0)\, W^{n_+}_{(0,\infty)}|\Omega\rangle$$

- Partonic picture: **gauge invariant dressed quark correlator**
  - Quarks are not asymptotic states
  - Note color averaging




- Hadronic picture: **"inclusive jet" correlator**
  - Hadronization products pass the cut
  - Interpret as a gauge invariant quark-to-jet amplitude squared
  - No measured hadrons $\longrightarrow$ no jet cone / energy




- Can study **fragmentation w/o fragments**
  - In particular, dynamical mass generation & χ–symmetry breaking

# Gauge invariant spectral representation

*AA, Signori, 1903.04458*

- First: convolution representation

$$\Xi_{ij}(k) = \text{Disc} \int d^4p \, \frac{\text{Tr}_c}{N_c} \langle \Omega | \widetilde{S}_{ij}(p) \widetilde{W}(k-p) | \Omega \rangle \; ,$$

where

$$\widetilde{S}_{ij}(p) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{\mathbf{i}\xi\cdot p} \, \mathcal{T} \, \psi_i(\xi) \overline{\psi}_j(0) \, ,$$

$$\widetilde{W}(k-p) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{\mathbf{i}\xi\cdot(k-p)} \, \mathcal{T} \, W(0,\xi) \, .$$

- Invariant decomposition of quark's bilinear operator:

$$\widetilde{S}_{ij}(p) = \hat{s}_3(p^2) \not{p}_{ij} + \sqrt{p^2} \, \hat{s}_1(p^2) \mathbb{I}_{ij} \; + g.f. \not{n}$$

"Spectral operators"

gauge fixing term (for axial gauges)

# Gauge invariant spectral representation

*AA, Signori, 1903.04458*

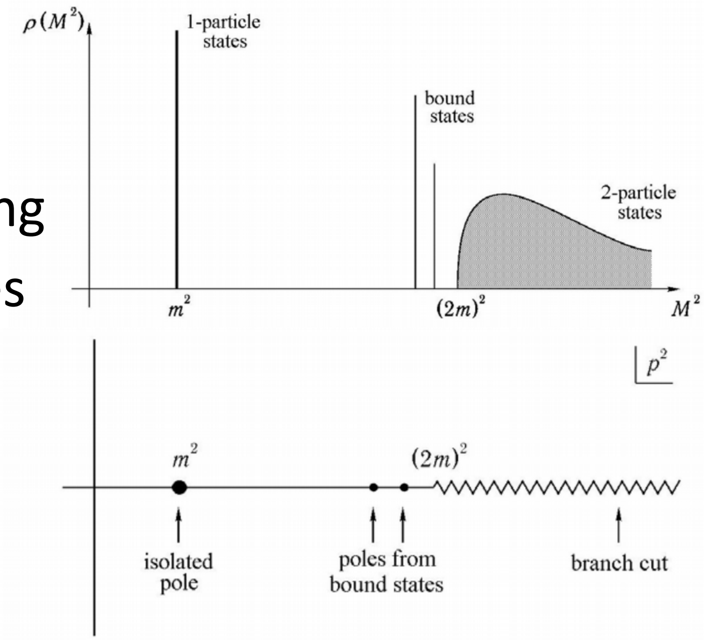
- Kallen-Lehman representation for Feynman propagator

$$\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|\widetilde{S}(p)|\Omega\rangle = \int_{-\infty}^{+\infty}\frac{d\mu^2}{(2\pi)^4}\frac{\mathbf{i}}{p^2-\mu^2+i\epsilon}\Big\{\not{p}\,\rho_3(\mu^2)+\sqrt{\mu^2}\,\rho_1(\mu^2)\Big\}\theta(\mu^2)$$

**$\rho_{1,3}$ are spectral functions:**

⟶ strength of quark-to-multihadron coupling

⟶ color averaging: only colorless final states



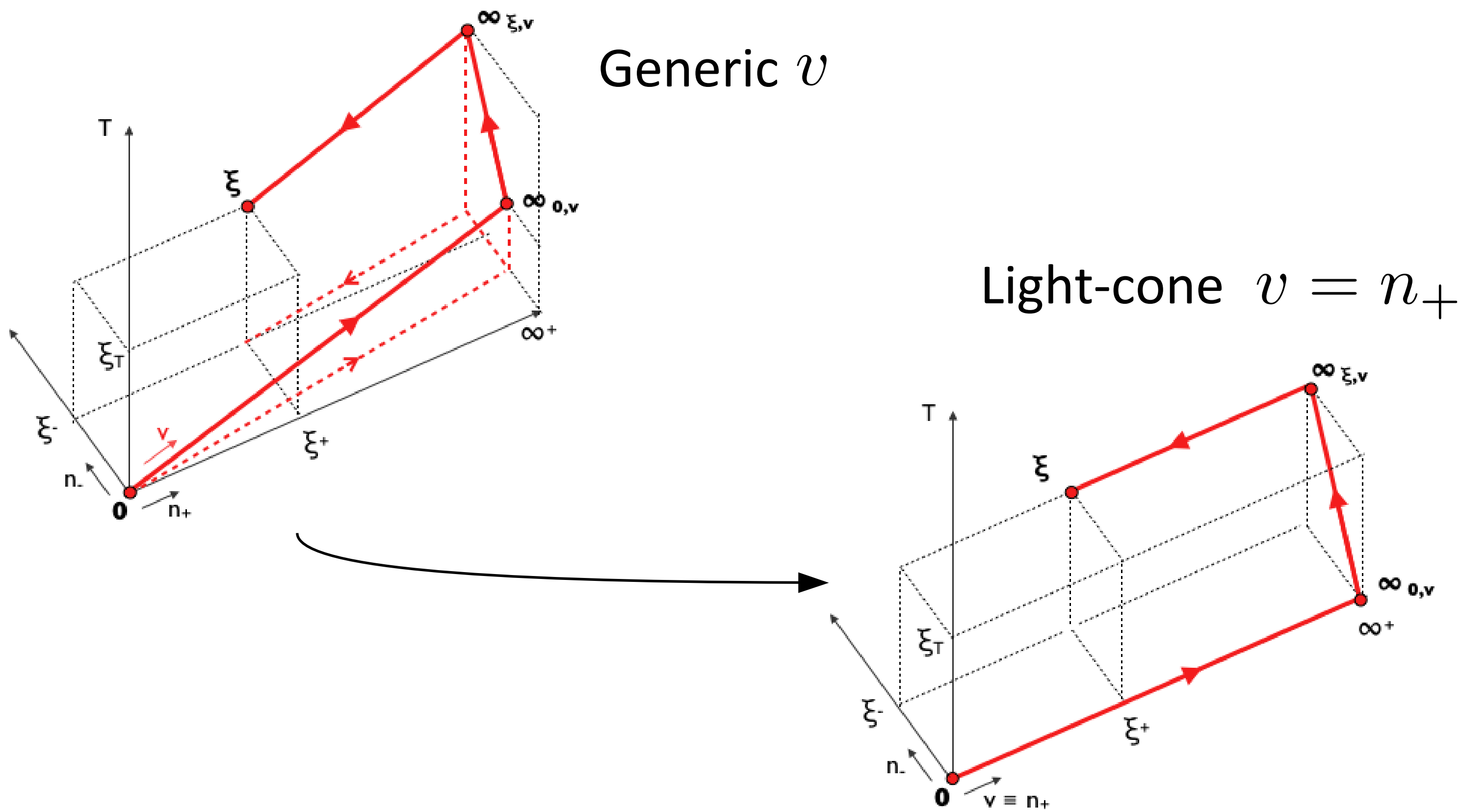

- In terms of **spectral propagators**:

$$(2\pi)^3\,\mathrm{Disc}\,\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|\hat{s}_{1,3}(p^2)|\Omega\rangle = \rho_{1,3}(p^2)\,\theta(p^2)\,\theta(p^-)$$

# Wilson line structure

*AA, Signori, 1903.04458*

- Focus on (l.c.) staple-like Wilson lines
  - But spectral convolution method is general



# TMD jet correlator

*AA, Signori, 1903.04458*

- Boost the quark at large light-cone momentum
  - (e.g., as it happens in large-*Q* DIS)

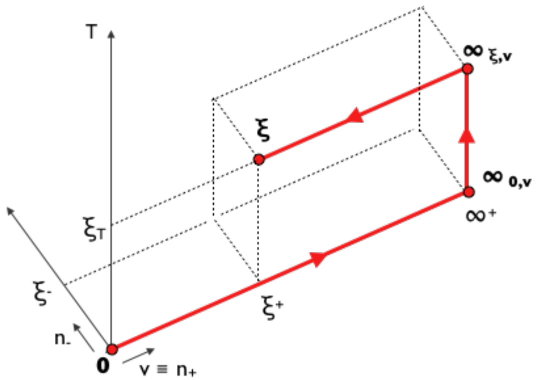
$$k^- \gg |\mathbf{k_T}| \gg k^+$$

- Integrate out the small momentum component:

TMD Inclusive jet correlator

$$J_{ij}(k^-, \boldsymbol{k}_T) \equiv \frac{1}{2} \int dk^+ \, \Xi_{ij}(k) \, ,$$

obtain standard staple,



# TMD jet correlator in full glory

*AA, Signori, 1903.04458*
*+ work in progress*

- Expand in Dirac structures, order in powers of $1/k^-$

$$J(k^-, k_T) = \left\{ \gamma^+ + \frac{M_j}{k^-} + \frac{\not{k}_T}{k^-} + \frac{(K_j^2 + T_j^2 + g.f.) + k_T^2}{2(k^-)^2} \not{n}_- \right\} \theta(k^-)$$

- where, using spectral convolution in light-cone gauge,

$$M_j = \int_0^\infty d\mu^2 \, \mu \, \rho_1(\mu^2)$$

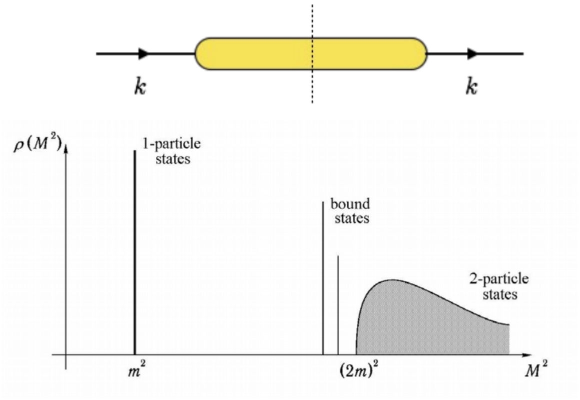
Jet "mass" ~ dressed quark mass ~ O(100 MeV)

$$K_j^2 = \int_0^\infty d\mu^2 \, \mu^2 \, \rho_3(\mu^2)$$

Jet's "virtuality"

$$T_j^2 \sim \langle\langle \mu_T^2 \rangle\rangle$$

Jet's "transverse size"



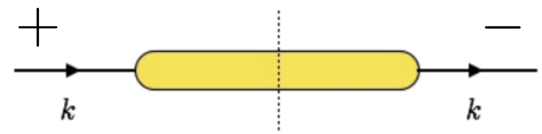

- **NOTE**:
  - Average jet shape (dynamics of hadronization) encoded in TMD jet correlator !!
  - Explicit g.f. contributions pushed to twist-4 in light-cone gauge

# The jet/quark mass

*AA, Signori, 1903.04458*

- Mass associated with **chiral-odd component** of jet amplitude squared:

$$M_{jet} \sim \frac{\mathrm{Tr_c}}{N_c} \int dk^+ \, \mathrm{Tr_D} \Bigg($$




$$\Bigg)$$

  – In light cone gauge, it is interpreted as average mass of the color-neutral QCD d.o.f ("hadrons") through cut

$$M_{jet} = \int_0^\infty d\mu^2 \, \mu \, \rho_1(\mu^2)$$

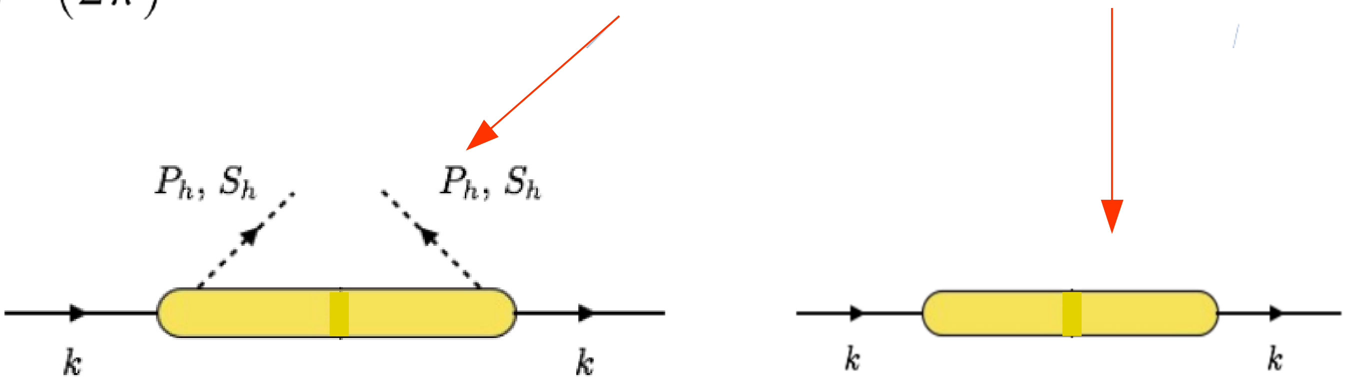
- Provides definition for **mass of a colored-screened dressed quark**, which is:
  – Gauge-invariant
  – Renormalization-scale dependent (grows with jet energy scale $k^-$)
  – Calculable theoretically (through spectral functions)
  – **Most importantly, measurable via a new momentum FF sum rule**

# Momentum sum rule - operator level

*AA, Signori, 1903.04458*
*+ work in progress*

- Extend field-theoretical technique of *Meissner, Metz, Pitonyak, PLB 2010* :

$$\sum_{h,\,S_h} \int \frac{d^4 P_h}{(2\pi)^3}\, \delta(P_h^2 - M_h^2) P_h^\mu \, \Delta^h(k, P_h, S_h) = k^\mu \, \Xi^{uncut}(k)$$



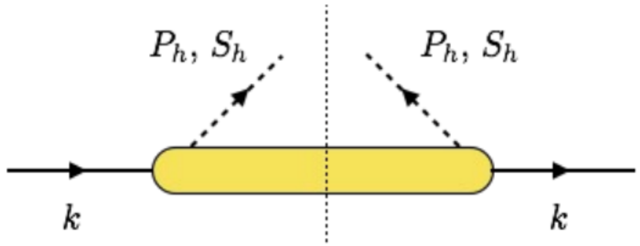

– **Dressed quark propagator as "average" on-shell four momentum produced by hadronization**

– **Dirac projections give momentum sum rules for TMD FFs !**

# Dirac structures

*AA, Signori, 1903.04458*

- TMD Fragmentation Functions




$$\Delta^h(z, P_{h\perp} = \frac{1}{2}\gamma^+ D_1^h + \frac{M_h}{2P_h^-} E^h \mathbb{I} + \frac{\not{P}_{h\perp}}{2zP_h^-} D^{\perp h} \quad + \text{quark polarized terms}$$

- For the inclusive jet correlator


$$J(k^-, k_T) = \frac{1}{2}\gamma^+ + \frac{M_j}{2k^-}\mathbb{I} + \frac{\not{k}_T}{2k^-} \quad + \text{higher-twist terms}$$

# Mass sum rule

*AA, Signori, 1903.04458*

- Projecting the sum rule onto the identity matrix,

$$M_j = \sum_{h,S_h} \int dz M_h E^h(z)$$

jet/quark mass as
**average of produced hadron masses weighted by chiral-odd FFs**

- **Dynamical mass component:**

EOM relations:

$$E = \tilde{E} + z \frac{m_q}{M_h} D_1$$

q-g-q correlations neglecting →

WW approx.

$$M_j = m_q$$

↓

**full QCD**

$$M_j \equiv m_q + m^{corr}$$

**Dynamical mass!**

$$m^{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z)$$

Expect non-zero in χ-limit ⟶ **observable χ-symmetry order parameter!**

# Full set of sum rules

*AA, Signori, PoS(DIS2018)*
*+ in progress*

- Sum rules for quarks into unpolarized hadrons, up to twist-3
  - (only thing missing for twist-4: full FF-TMD analysis)

$$\sum_{h\,S_h}\int dz\, z D_1^h(z) = 1$$

*Collins-Soper*

NEW
$$\sum_{h\,S_h}\int dz M_h E^h(z) = M_j$$

NEW
$$\sum_{h\,S_h}\int dz M_h \tilde{E}^h(z) = M_j - m_{q0} = m_q^{corr}$$

NEW
$$\sum_{h\,S_h}\int dz M_h H^h(z) = 0$$

$$\sum_{h\,S_h}\int dz M_h \tilde{H}^h(z) = 0$$

*Diehl-Sapeta*

$$\sum_{h\,S_h}\int dz\, z M_h H_1^{\perp\,(1)\,h}(z) = 0$$

*Schaefer-Teryaev*

NEW
$$\sum_{h\,S_h}\int dz M_h^2 D^{\perp\,(1)\,h}(z) = 0$$

NEW
$$\sum_{h\,S_h}\int dz M_h^2 \tilde{D}^{\perp\,(1)\,h}(z) = -\frac{1}{2}\langle P_\perp^2/z\rangle$$

NEW
$$\sum_{h\,S_h}\int dz M_h^2 G^{\perp\,(1)\,h}(z) = 0$$

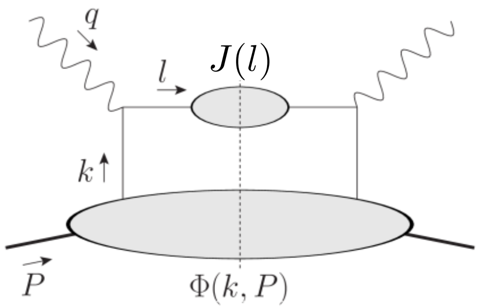
NEW
$$\sum_{h\,S_h}\int dz M_h^2 \tilde{G}^{\perp\,(1)\,h}(z) = 0\,.$$

**fully dynamical quantities**

# χ-odd phenomenology at large x

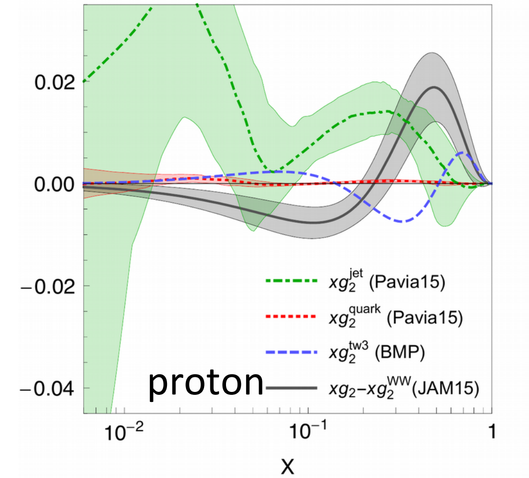
*AA, Bacchetta, PLB 2017*
*AA, Signori, PoS(DIS2018)*

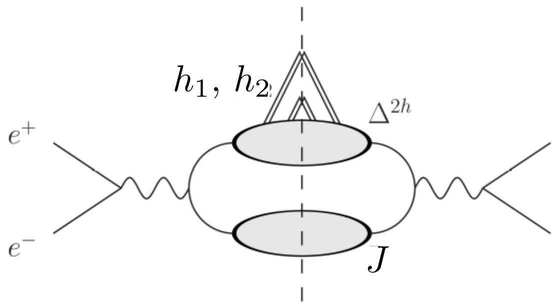
*AA, Bacchetta, Melnitchouk, Schlegel, 2009*

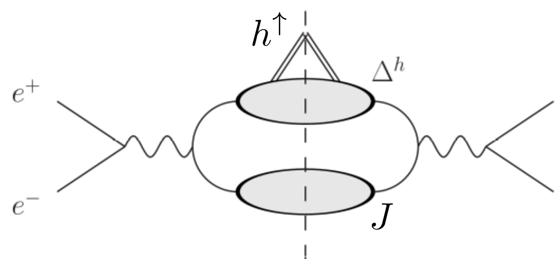



$$g_2(x_B) = \text{“usual”} + \frac{m^{corr}}{M}\frac{h_1^q(x_B)}{x_B}$$

$$\int_0^1 dx\, g_2(x) = m^{corr} \int_0^1 dx\, \frac{h_1(x)}{x} \neq 0$$




*AA, Bacchetta, Radici, Signori, in progress*

*... and more: the door is now open...*

# Conclusions

- We can quantitatively connect quark fragmentation to the dynamical generation of mass
  - **Gauge invariant definition for dressed quark mass, $M_j$**
  - The dynamical component $m^{corr} = M_j - m_q$ is recognized as an **observable order parameter for χ-simmetry breaking**

$$m^{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z)$$

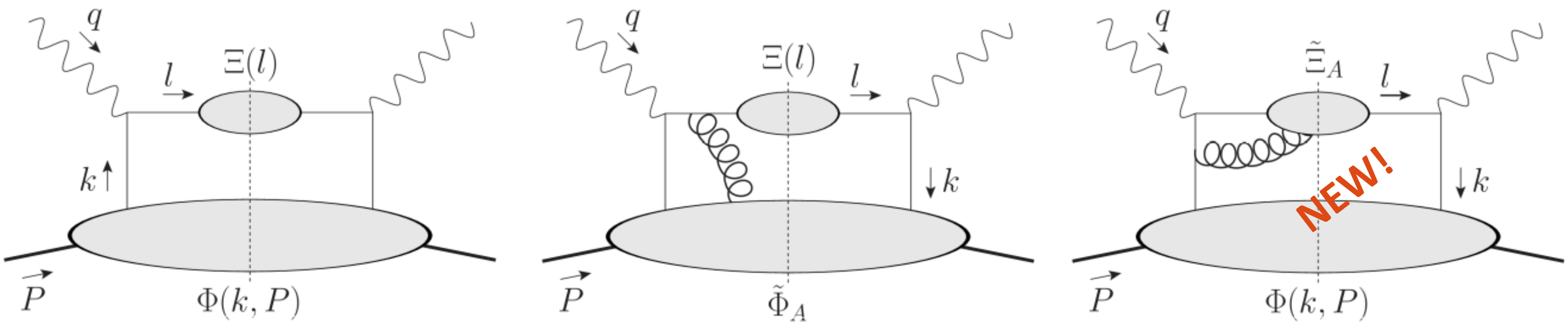
**Practical exp. recipe:**
– measure $\widetilde{E}$, obtain $m^{corr}$
– flavor decomposition, too!

- **Novel phenomenology:**
  - Transversity in g2, same side di-hadrons, …
- **New sum rules:** guidance for future fits
- **New spectral convolution technique** for treating Wilson lines

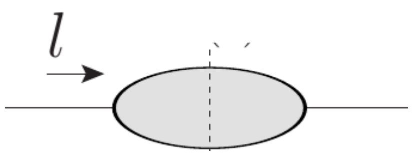
# Backup

# Inclusive DIS with jet correlators

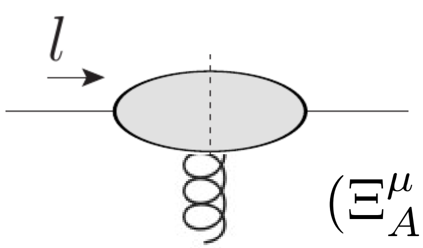
*AA, Bacchetta, PLB 773 ('17) 632*




**Jet correlators:** → non-asymptotic quark states / dressed quarks




$$\Xi_{ij}(l, n_+) = F.T. \langle \Omega |\, W^{n_+}_{(+\infty,\xi)}\, \psi_i(\xi) \bar{\psi}_j(0)\, W^{n_+}_{(0,+\infty)}\, |\Omega\rangle$$

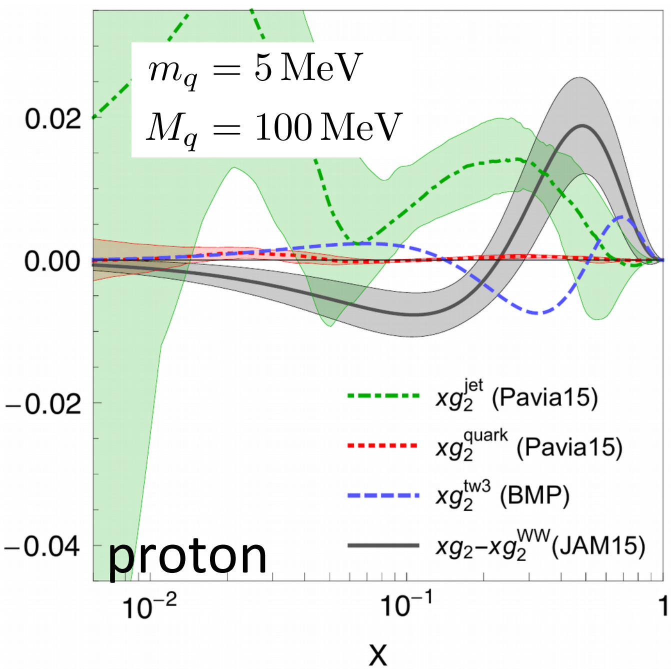



$$(\Xi^\mu_A)_{ij} = F.T. \,\langle \Omega |\, W^{n_+}_{(+\infty,\xi)}\, g A^\mu(\xi)\; \psi_i(\xi)\, \bar{\psi}_j(0)\, W^{n_+}_{(0,+\infty)} |\Omega\rangle$$

# g2 structure function revisited

- Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \Big( g_2^{q,\mathrm{tw3}}(x_B) + \overbrace{\frac{m_q}{M}\Big(\frac{h_1^q}{x}\Big)^{\star}(x_B)}^{\equiv\, g_2^{quark}} + \overbrace{\boxed{\frac{M_j - m_q}{M}\frac{h_1^q(x_B)}{x_B}}}^{\equiv\, g_2^{jet}} \Big)$$




**Consequences:**

- h1 accessible in inclusive DIS

  $\leftrightarrow$ Potentially large signal

- Burkardt-Cottingham sum rule broken

$$\int_0^1 g_2(x) = (M_j - m_q)\int_0^1 dx\, \frac{h_1(x)}{x}$$

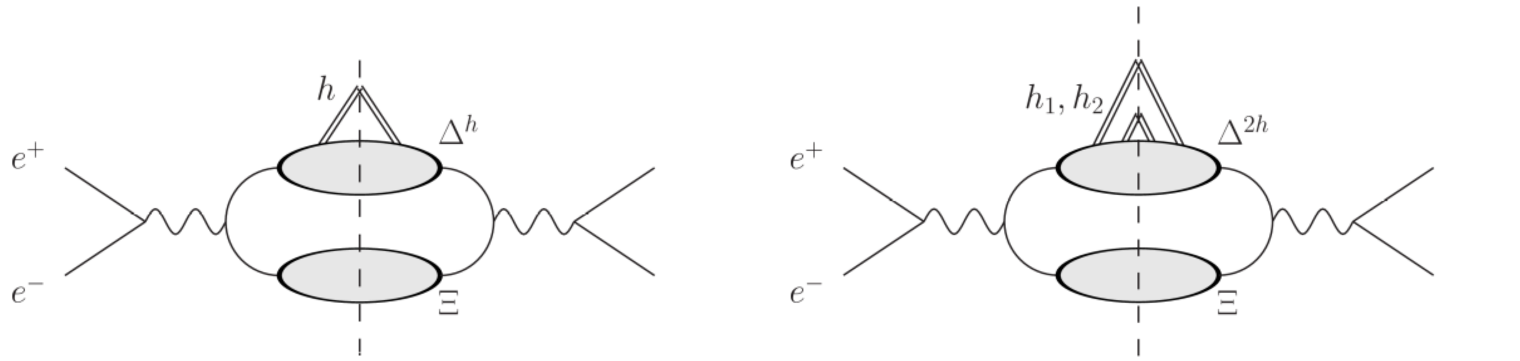
- ETL: novel way to measure tensor charge

$$\int_0^1 x g_2^{q-\bar q}(x) = 2\,(M_j - m_q)\int_0^1 dx\, h_1^{q-\bar q}(x)$$

# Measuring the jet correlator

*Accardi, Bacchetta, Signori, Radici, in progress*

- Jet mass $\mathbf{M_{jet}}$ **can be measured in polarized e⁺ + e⁻ :**




  – Needs **LT asymmetry** in semi-inclusive **Lambda** production

$$\frac{d\sigma^L(e^+e^- \to \text{jet}\, h\, X)}{d\Omega dz}$$

$$= \frac{3\alpha^2}{Q^2} \lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2} \lambda_h G_1 + D(y) |\boldsymbol{S}_T| \cos(\phi_S) \frac{2M_h}{Q} \left( \frac{G_T}{z} + \frac{M_q - m_q}{M_h} H_1 \right) \right\}$$

  – Similarly a **LU asymmetry** in unpolarized **dihadron** production

# Where can we measure jet correlators?

- Can we get a (polarized) **e+ e- collider at JLab / BNL?**
  - At JLab12 ? EIC + positron beam ?
- Are **existing facilities** enough?

| | BEPC | super KEKB | ILC | JLab/BNL |
|---|---|---|---|---|
| **E beam [GeV]** | 1.9 | 4 ($e^-$)<br>7 ($e^-$) | 250 | ? |
| **√s [GeV]** | 3 – 5 | 10 | 500 | ? |
| **polarization** | ? | maybe | 80% $e^-$<br>60% $e^+$ | YES! |

- What else?

# A new “universal” fits

- **Chiral-odd collinear sector across processes:**