Spectrally novel FF sum rules and dynamical generation of mass

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Based on: Accardi, Signori, arXiv:1903.04458
Accardi, Bacchetta, PLB 773 (2017) 632
+ in progress w/ Bacchetta, Radici, Signori
Overview

- “Inclusive jet” correlator
  - Quarks are not asymptotic states

- Gauge invariant spectral representation
  - Jet/dressed quark mass

- New FF sum rules
  - Jet/dressed quark mass is experimentally observable!

- New phenomenology

- Conclusions
Inclusive jet correlator

Inclusive $q \rightarrow X$ “inclusive jet” correlator

\[
\Xi_{ij}(k; n_+) = \text{Disc} \int \frac{d^4 \xi}{(2\pi)^4} e^{ik \cdot \xi} \frac{\text{Tr}_c}{N_c} \langle \Omega | \mathcal{T} \mathcal{W}^{n_+}_{(\infty, \xi)} \psi_i(\xi) \bar{\psi}_j(0) \mathcal{W}^{n_+}_{(0, \infty)} | \Omega \rangle
\]

- **Partonic picture:** gauge invariant dressed quark correlator
  - Quarks are not asymptotic states
  - Note color averaging

- **Hadronic picture:** “inclusive jet” correlator
  - Hadronization products pass the cut
  - Interpret as a gauge invariant quark-to-jet amplitude squared
  - No measured hadrons $\rightarrow$ no jet cone / energy

- Can study **fragmentation w/o fragments**
  - In particular, dynamical mass generation & $\chi$–symmetry breaking

*AA, Signori, 1903.04458
Sterman, NPB 281 ('87)*
Gauge invariant spectral representation

First: convolution representation

\[ \Xi_{ij}(k) = \text{Disc} \int d^4 p \frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}_{ij}(p) \tilde{W}(k - p) | \Omega \rangle , \]

where

\[ \tilde{S}_{ij}(p) = \int \frac{d^4 \xi}{(2\pi)^4} e^{i \xi \cdot p} \mathcal{T} \psi_i(\xi) \bar{\psi}_j(0) , \]

\[ \tilde{W}(k - p) = \int \frac{d^4 \xi}{(2\pi)^4} e^{i \xi \cdot (k - p)} \mathcal{T} W(0, \xi) . \]

Invariant decomposition of quark’s bilinear operator:

\[ \tilde{S}_{ij}(p) = \hat{s}_3(p^2) \phi_{ij} + \sqrt{p^2} \hat{s}_1(p^2) \Pi_{ij} + g \cdot f \cdot \Phi \]

“Spectral operators”

gauge fixing term
(for axial gauges)
Gauge invariant spectral representation

- Kallen-Lehman representation for Feynman propagator

\[
\frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}(p) | \Omega \rangle = \int_{-\infty}^{+\infty} \frac{d\mu^2}{(2\pi)^4} \frac{i}{p^2 - \mu^2 + i\epsilon} \left\{ \phi \rho_3(\mu^2) + \sqrt{\mu^2} \rho_1(\mu^2) \right\} \theta(\mu^2)
\]

\(\rho_{1,3}\) are spectral functions:
- strength of quark-to-multihadron coupling
- color averaging: only colorless final states

- In terms of spectral propagators:

\[(2\pi)^3 \text{Disc} \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_{1,3}(p^2) | \Omega \rangle = \rho_{1,3}(p^2) \theta(p^2) \theta(p^-)\]
Wilson line structure

- Focus on (l.c.) staple-like Wilson lines
  - But spectral convolution method is general

Generic \( \nu \)

Light-cone \( \nu = n_+ \)
TMD jet correlator

- Boost the quark at large light-cone momentum (e.g., as it happens in large-$Q$ DIS)
  \[ k^- \gg |k_T| \gg k^+ \]

- Integrate out the small momentum component:

  TMD Inclusive jet correlator

  \[ J_{ij}(k^-, k_T) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k), \]

  obtain standard staple,
TMD jet correlator in full glory

- Expand in Dirac structures, order in powers of $1/k^-$

$$J(k^-, k_T) = \left\{ \gamma^+ + \frac{M_j}{k^-} + \frac{k_T}{k^-} + \frac{(K_j^2 + T_j^2 + g.f.) + k_T^2}{2(k^-)^2} \theta(k^-) \right\} \theta(k^-)$$

- where, using spectral convolution in light-cone gauge,

$$M_j = \int_0^\infty \mu^2 \rho_1(\mu^2)$$  
Jet “mass” $\sim$ dressed quark mass  
$\sim O(100 \text{ MeV})$

$$K_j^2 = \int_0^\infty \mu^2 \rho_3(\mu^2)$$  
Jet’s “virtuality”

$$T_j^2 \sim \langle \langle \mu_T^2 \rangle \rangle$$  
Jet’s “transverse size”

- NOTE:
  - Average jet shape (dynamics of hadronization) encoded in TMD jet correlator!!
  - Explicit g.f. contributions pushed to twist-4 in light-cone gauge
The jet/quark mass

- Mass associated with **chiral-odd component** of jet amplitude squared:

  \[ M_{\text{jet}} \sim \frac{\text{Tr}_c}{N_c} \int d\k^+ \text{Tr}_D \left( \frac{+}{k} \frac{-}{k} \right) \]

  - In light cone gauge, it is interpreted as average mass of the color-neutral QCD d.o.f ("hadrons") through cut

  \[ M_{\text{jet}} = \int_0^\infty d\mu^2 \mu \rho_1(\mu^2) \]

- Provides definition for **mass of a colored-screened dressed quark**, which is:
  - Gauge-invariant
  - Renormalization-scale dependent (grows with jet energy scale \( k^- \))
  - Calculable theoretically (through spectral functions)
  - **Most importantly, measurable** via a new momentum FF sum rule

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AA, Signori, 1903.04458
Momentum sum rule - operator level

- Extend field-theoretical technique of *Meissner, Metz, Pitonyak, PLB 2010*:

\[
\sum_{h, S_h} \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2) P_h^\mu \Delta^h(k, P_h, S_h) = k^\mu \Xi^{uncut}(k)
\]

- Dressed quark propagator as "average" on-shell four momentum produced by hadronization

- Dirac projections give momentum sum rules for TMD FFs!
Dirac structures

- **TMD Fragmentation Functions**

\[ \Delta^h(z, P_{h\perp}) = \frac{1}{2} \gamma^+ D_1^h + \frac{M_h}{2P^-_h} E^h \mathbb{I} + \frac{\not{P}_{h\perp}}{2zP^-_h} D^{\perp h} + \text{quark polarized terms} \]

- **For the inclusive jet correlator**

\[ J(k^-, k_T) = \frac{1}{2} \gamma^+ + \frac{M_j}{2k^- \mathbb{I}} + \frac{k_T}{2k^-} + \text{higher-twist terms} \]
Mass sum rule

- Projecting the sum rule onto the identity matrix,

\[ M_j = \sum_{h,S_h} \int dz M_h E^h(z) \]

jet/quark mass as average of produced hadron masses weighted by chiral-odd FFs

- Dynamical mass component:

EOM relations:

\[ E = \tilde{E} + z \frac{m_q}{M_h} D_1 \]

q-g-q correlations neglecting

WW approx.

\[ M_j \approx m_q \]

full QCD

Dynamical mass!

\[ m^{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z) \]

\[ M_j \equiv m_q + m^{corr} \]

Expect non-zero in \( \chi \)-limit → observable \( \chi \)-symmetry order parameter!
Full set of sum rules

- Sum rules for quarks into unpolarized hadrons, up to twist-3
  - (only thing missing for twist-4: full FF-TMD analysis)

\[ \sum_{h S_h} \int dzzD_{1}^{h}(z) = 1 \quad \text{(Collins-Soper)} \]
\[ \sum_{h S_h} \int dzM_{h}E_{1}^{h}(z) = M_{J} \]
\[ \sum_{h S_h} \int dzM_{h}H_{1}^{h}(z) = 0 \quad \text{(Schaefer-Teryaev)} \]
\[ \sum_{h S_h} \int dzzM_{h}H_{1}^{h}(z) = 0 \quad \text{(Diehl-Sapeta)} \]
\[ \sum_{h S_h} \int dzM_{h}^{2}D_{1}^{h}(z) = 0 \quad \text{(Teryaev-Teryaev)} \]
\[ \sum_{h S_h} \int dzM_{h}^{2}G_{1}^{h}(z) = 0 \]
\[ \sum_{h S_h} \int dzM_{h}^{2}\tilde{E}_{1}^{h}(z) = M_{J} - m_{q0} = m_{q}^{corr} \]
\[ \sum_{h S_h} \int dzM_{h}\tilde{H}_{1}^{h}(z) = 0 \]
\[ \sum_{h S_h} \int dzM_{h}\tilde{D}_{1}^{h}(z) = -\frac{1}{2}\langle P_{\perp}/z \rangle \]
\[ \sum_{h S_h} \int dzM_{h}\tilde{G}_{1}^{h}(z) = 0 \]
χ-odd phenomenology at large $x$

$g_2(x_B) = \text{“usual”} + \frac{m^{\text{corr}}}{M} \frac{h_1^q(x_B)}{x_B}$

$\int_0^1 dx \, g_2(x) = m^{\text{corr}} \int_0^1 dx \, \frac{h_1(x)}{x} \neq 0$

... and more: the door is now open...
Conclusions

- We can quantitatively connect quark fragmentation to the dynamical generation of mass
  - Gauge invariant definition for dressed quark mass, $M_j$
  - The dynamical component $m^{corr} = M_j - m_q$ is recognized as an observable order parameter for $\chi$-symmetry breaking

\[
m^{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z)
\]

Practical exp. recipe:
- measure $\tilde{E}$, obtain $m^{corr}$
- flavor decomposition, too!

- Novel phenomenology:
  - Transversity in g2, same side di-hadrons, ...

- New sum rules: guidance for future fits

- New spectral convolution technique for treating Wilson lines
Backup
Jet correlators: \[ \Xi_{ij}(l, n_+) = F.T. \langle \Omega | W_{n_+}^{(+\infty, \xi)} \psi_i(\xi) \bar{\psi}_j(0) W_{n_+}^{(+\infty, 0)} | \Omega \rangle \]

\[ (\Xi_A^\mu)_{ij} = F.T. \langle \Omega | W_{n_+}^{(+\infty, \xi)} gA^\mu(\xi) \psi_i(\xi) \bar{\psi}_j(0) W_{n_+}^{(+\infty, 0)} | \Omega \rangle \]
g₂ structure function revisited

- Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

\[ g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2} \sum_a e_a^2 \left( g_2^{q,\text{tw3}}(x_B) + \frac{m_q}{M} \left( \frac{h_1^q}{x} \right)^* (x_B) + \left( \frac{M_j - m_q}{M} \frac{h_1^q(x_B)}{x_B} \right) \right) \]

Consequences:

- h₁ accessible in inclusive DIS
  \( \leftrightarrow \) Potentially large signal
- Burkardt-Cottingham sum rule broken
  \[ \int_0^1 g_2(x) = (M_j - m_q) \int_0^1 dx \frac{h_1(x)}{x} \]
- ETL: novel way to measure tensor charge
  \[ \int_0^1 x g_2^{q-\bar{q}}(x) = 2 \left( M_j - m_q \right) \int_0^1 dx h_1^{q-\bar{q}}(x) \]
Measuring the jet correlator

Jet mass $M_{\text{jet}}$ can be measured in polarized $e^+ + e^-$:

- Needs LT asymmetry in semi-inclusive Lambda production

\[
\frac{d\sigma^L(e^+e^- \rightarrow \text{jet } h X)}{d\Omega dz} = \frac{3\alpha^2}{Q^2} \lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2} \lambda_h G_1 + D(y) |S_T| \cos(\phi_S) \frac{2M_h}{Q} \left( \frac{G_T}{z} + \frac{M_q - m_q}{M_h} H_1 \right) \right\}
\]

- Similarly a LU asymmetry in unpolarized dihadron production
Where can we measure jet correlators?

- Can we get a (polarized) e+ e- collider at JLab / BNL?
  - At JLab12? EIC + positron beam?

- Are existing facilities enough?

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- What else?
A new “universal” fits

Chiral-odd collinear sector across processes:

\[ M_{\text{jet}} \quad H_1^\downarrow \quad H_1^\downarrow \otimes H_1^\downarrow \]

\[(\text{Di})e^+e^- \]

\[ \text{DIS} \quad (\text{Di})\text{SIDIS} \]

\[ M_{\text{jet}} h_1 \quad H_1^\downarrow \otimes h_1 \]