Comments on the perturbative and non-perturbative contributions in unpolarized SIDIS

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Outline

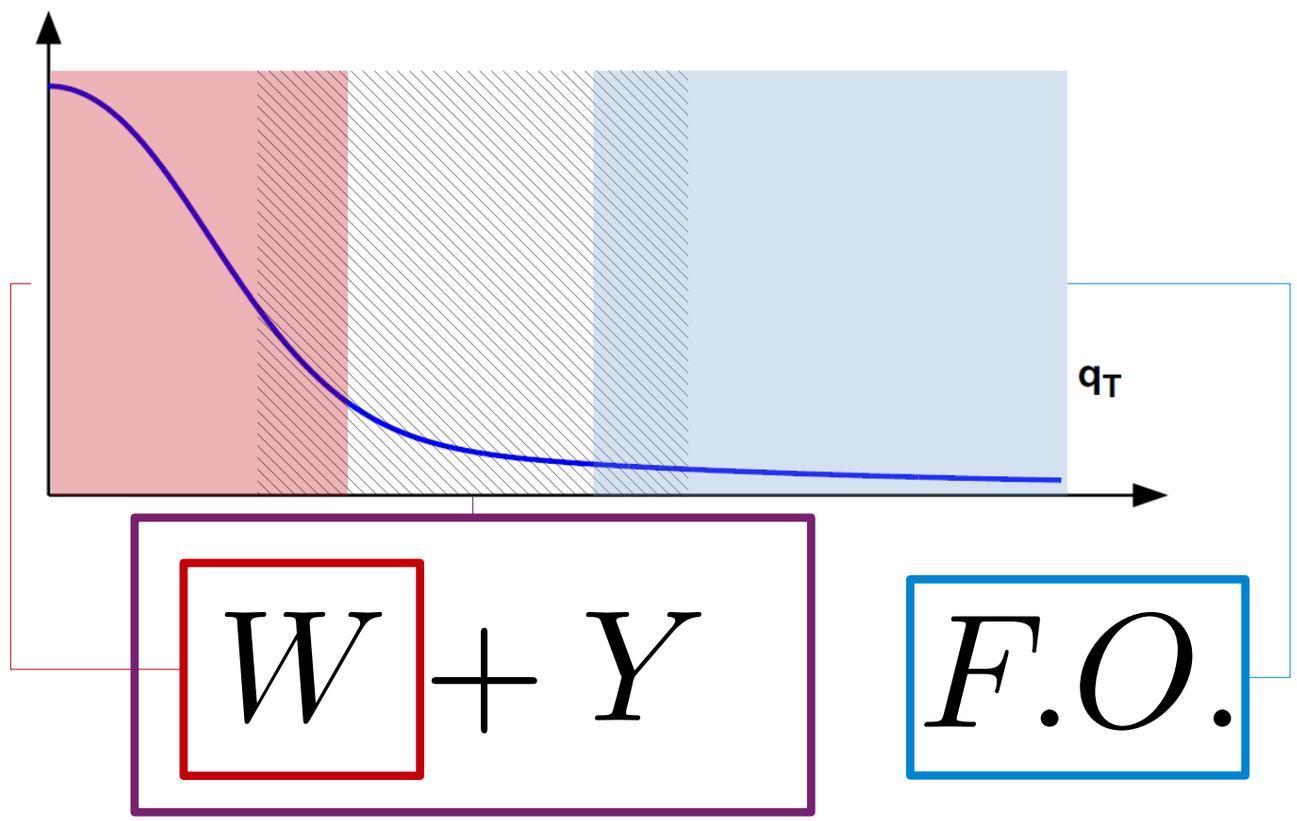
- Describing SIDIS data from small to large qT.

W (TMD small qT)
FO(large qT), Y(Matching)

- Structure of the TMD definition (CSS).

- Towards a LO analysis of the entire qT spectra.

Describing SIDIS data from small to large qT.



SIDIS variables: $Q^2,\ x_{Bj},\ z_h,\ P_T$

Often useful to consider: $q_T = P_T/z_h,\ \ y_h = \frac{1}{2}\log\left(\frac{P_h^+}{P_h^-}\right)$

Structure of the TMD definition (CSS).

W **(TMD region)**

$$\sum_q \mathcal{H}_q \text{ F.T.} \left\{ \tilde{D}_{h/q}(z, z\boldsymbol{b}_\perp; Q) \; \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q) \right\}$$

Fourier Transform of:

$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b} \right)^{K(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}^{in}_{ji}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b)$$
$$\times \exp \left\{ \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left(\gamma_F(\mu; 1) - \ln \left(\frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\}$$
$$\times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\} ,$$

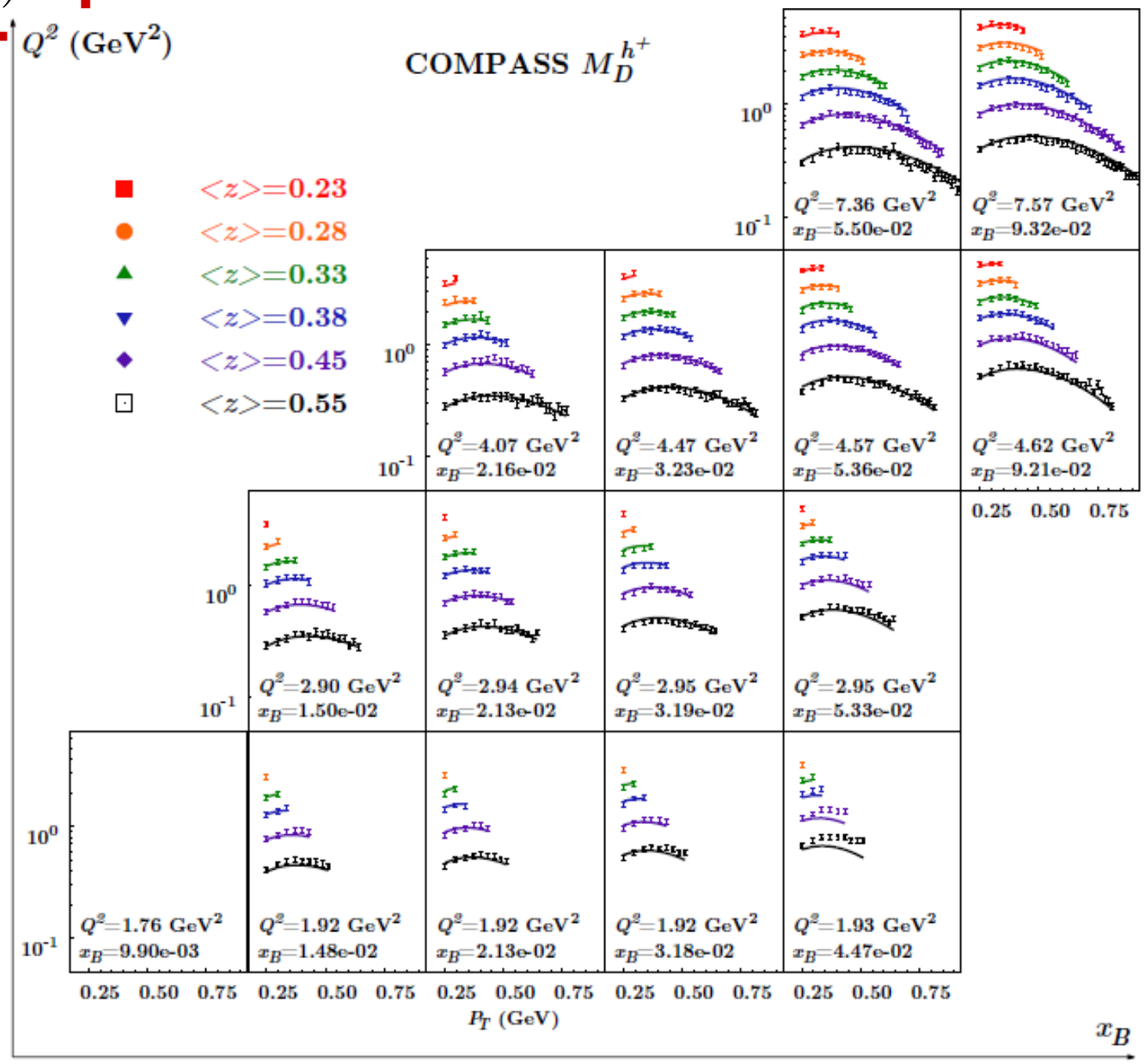
pQCD

Input (extraction from collinear cross section)

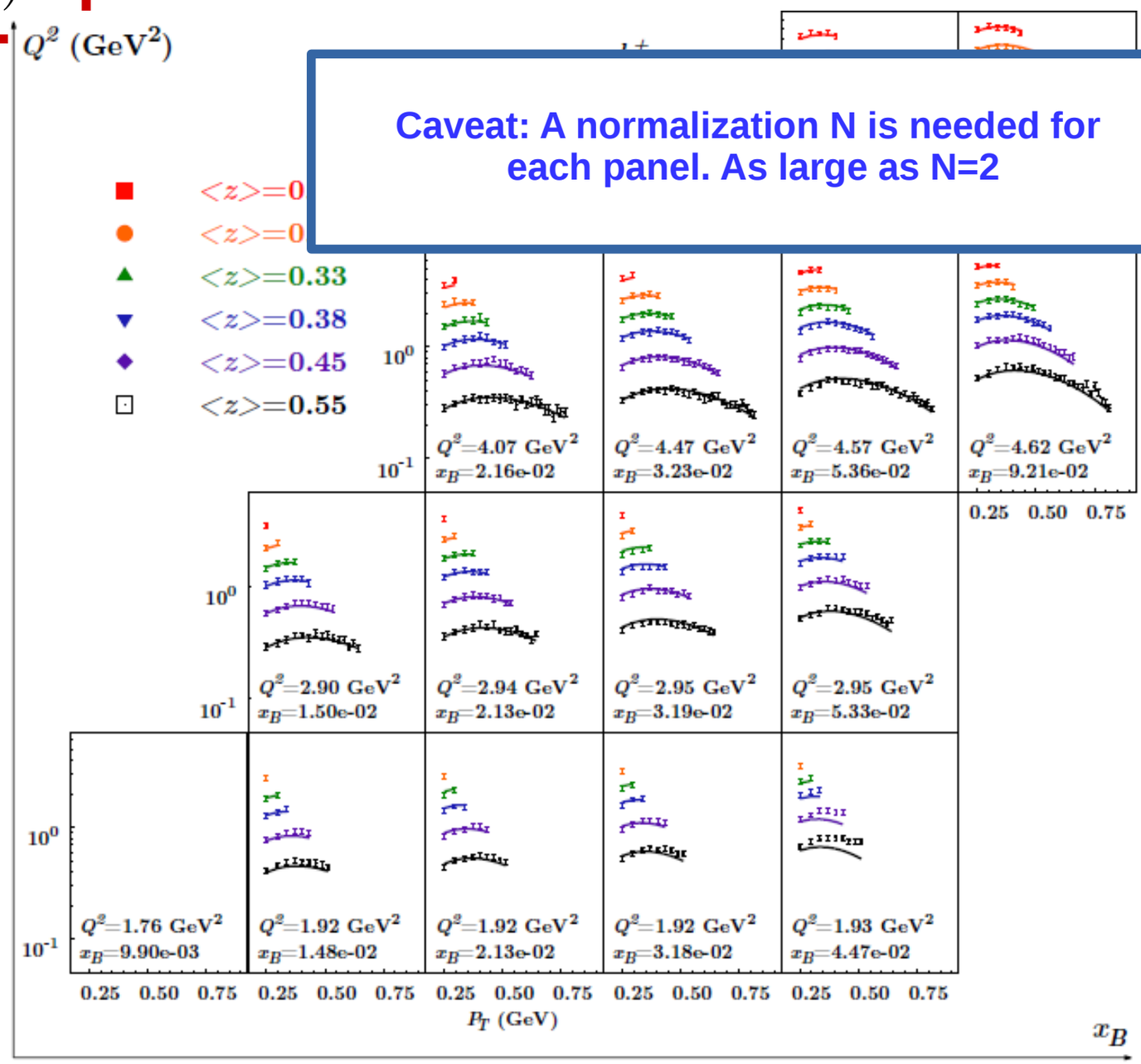
Non-perturbative functions to extract from data.

Towards a LO analysis of the entire qT spectra

W at $\mathcal{O}(\alpha_s)$



W at $\mathcal{O}(\alpha_s)$



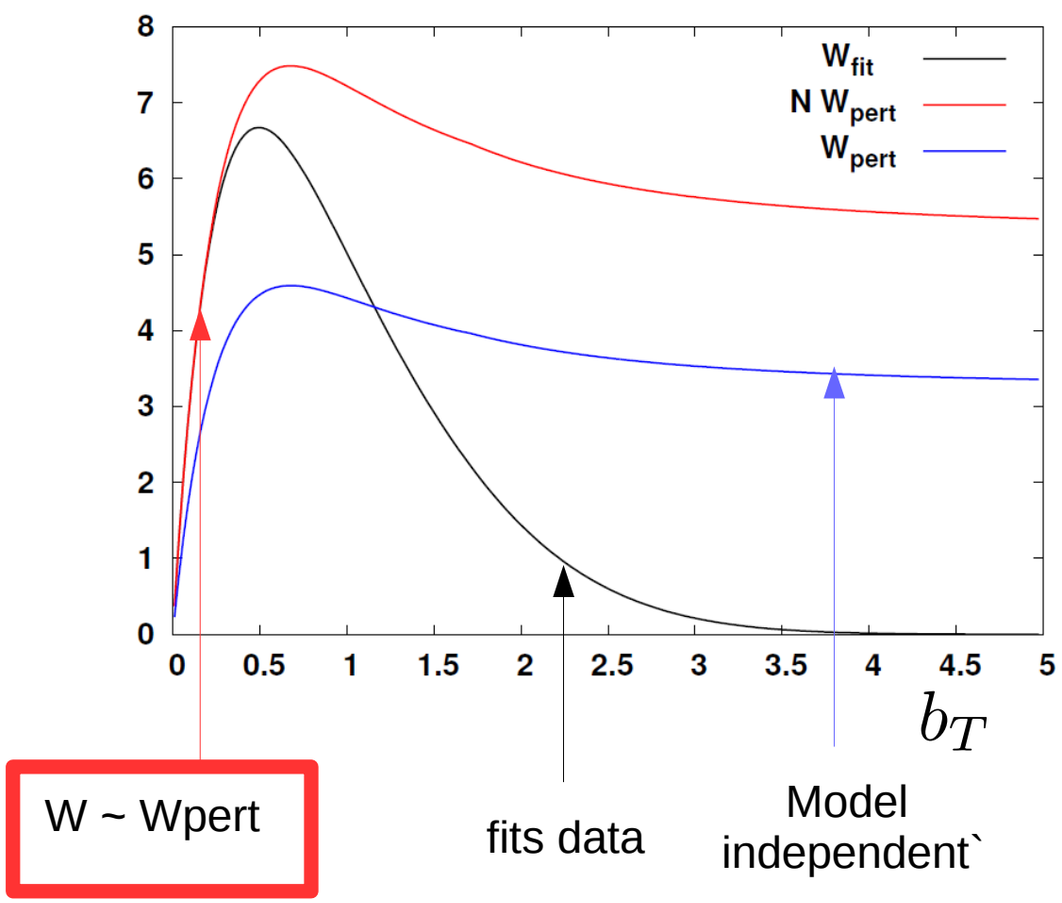
Caveat: A normalization N is needed for each panel. As large as N=2

$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \boxed{\left(\frac{\sqrt{\zeta_F}}{\mu_b}\right)^{\tilde{K}(b_*,\mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}^{\rm in}_{ji}\left(x/\hat{x}, b_*, \mu_b, \mu_b^2\right) f_i(\hat{x}, \mu_b) \times \exp\left\{\int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(\gamma_F(\mu;1) - \ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right)\gamma_K(\mu)\right)\right\}}$$

$$\times \exp\left\{-g_P(x, b_T) - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right)\right\},$$

At small bT, the W term (full) should be equal to the "perturbative" part of the TMD (model independent)

The role of the normalization is to help with <u>this</u>. One should try to avoid this



look at the most recent data
by
COMPASS

Similar issues as before:

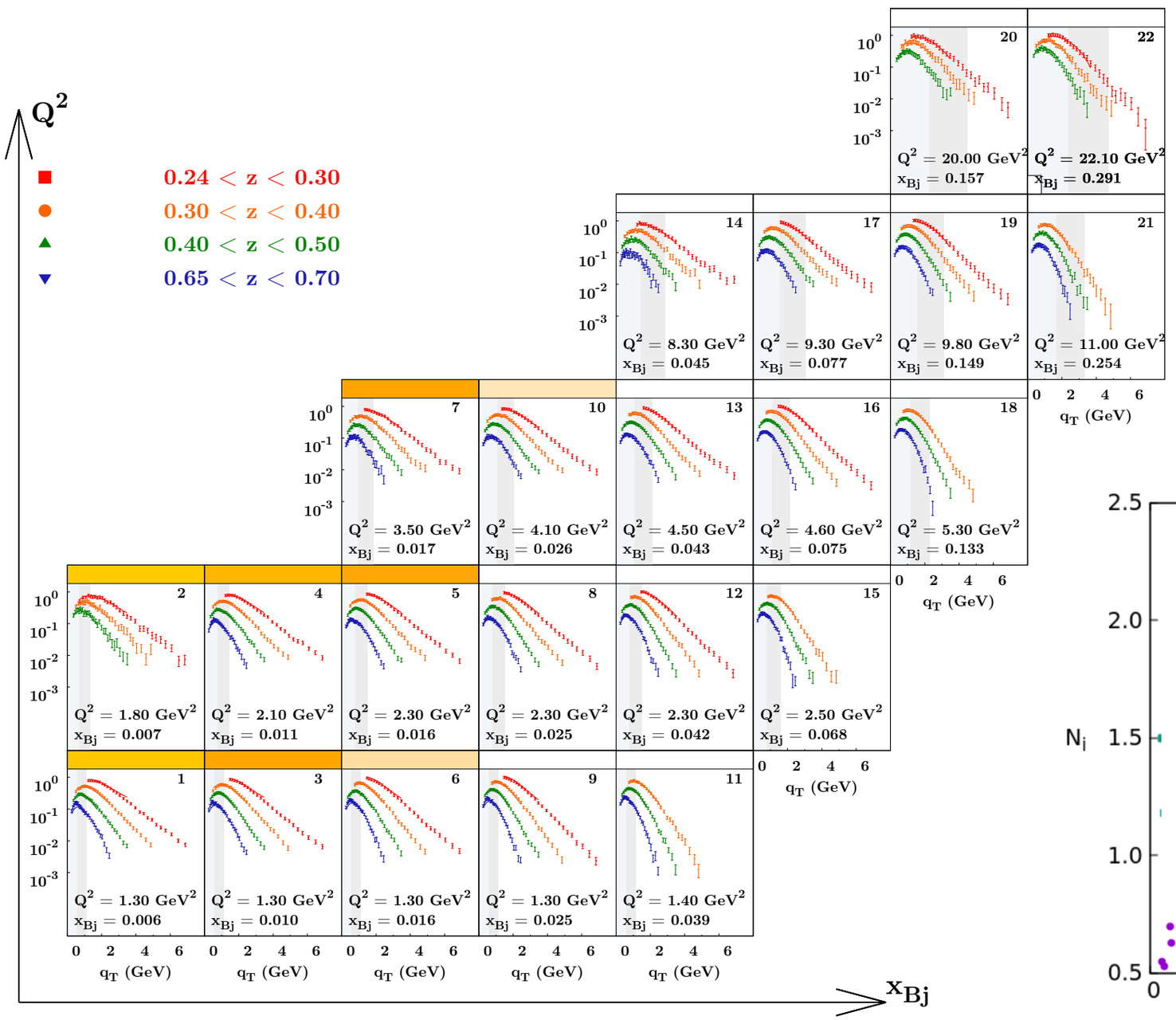
Need to introduce spurious normalizations in the analysis.

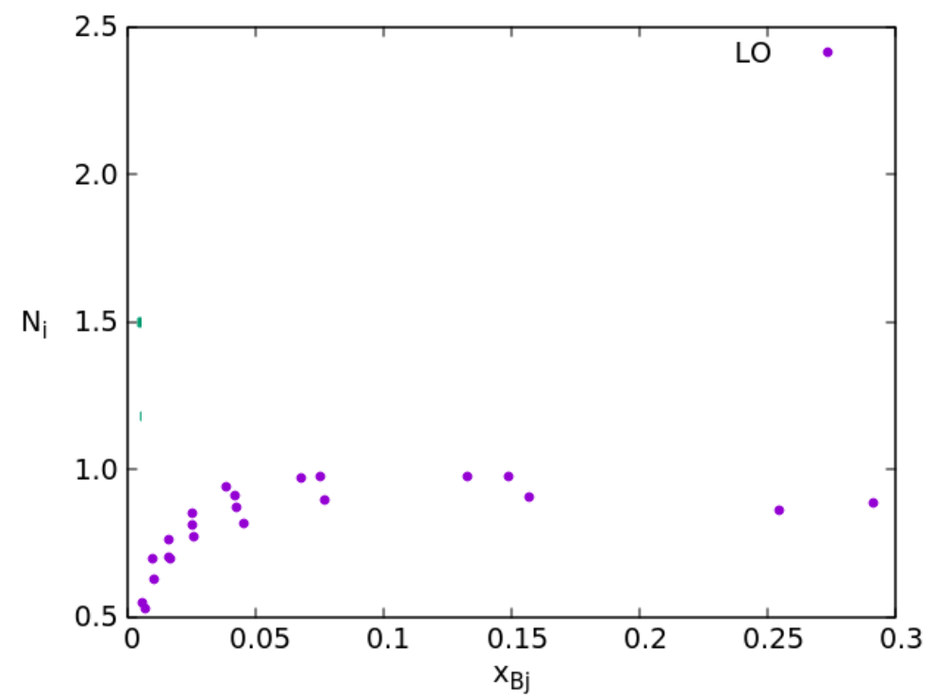
No reported normalization errors in new data sets.

Need to "debug". Diagnosis strategy (not a TMD extraction):

1) choose a range to fit only W term: PT < 0.9 GeV
2) Use "standard freezing prescription" with bmax=1.0 GeV^-1
3) Fit Panel by panel (fixed Q^2 and xbj)
4) Use a normalization **Ni** as a free parameter for each panel
5) Use the values of **Ni** to quantify the problem

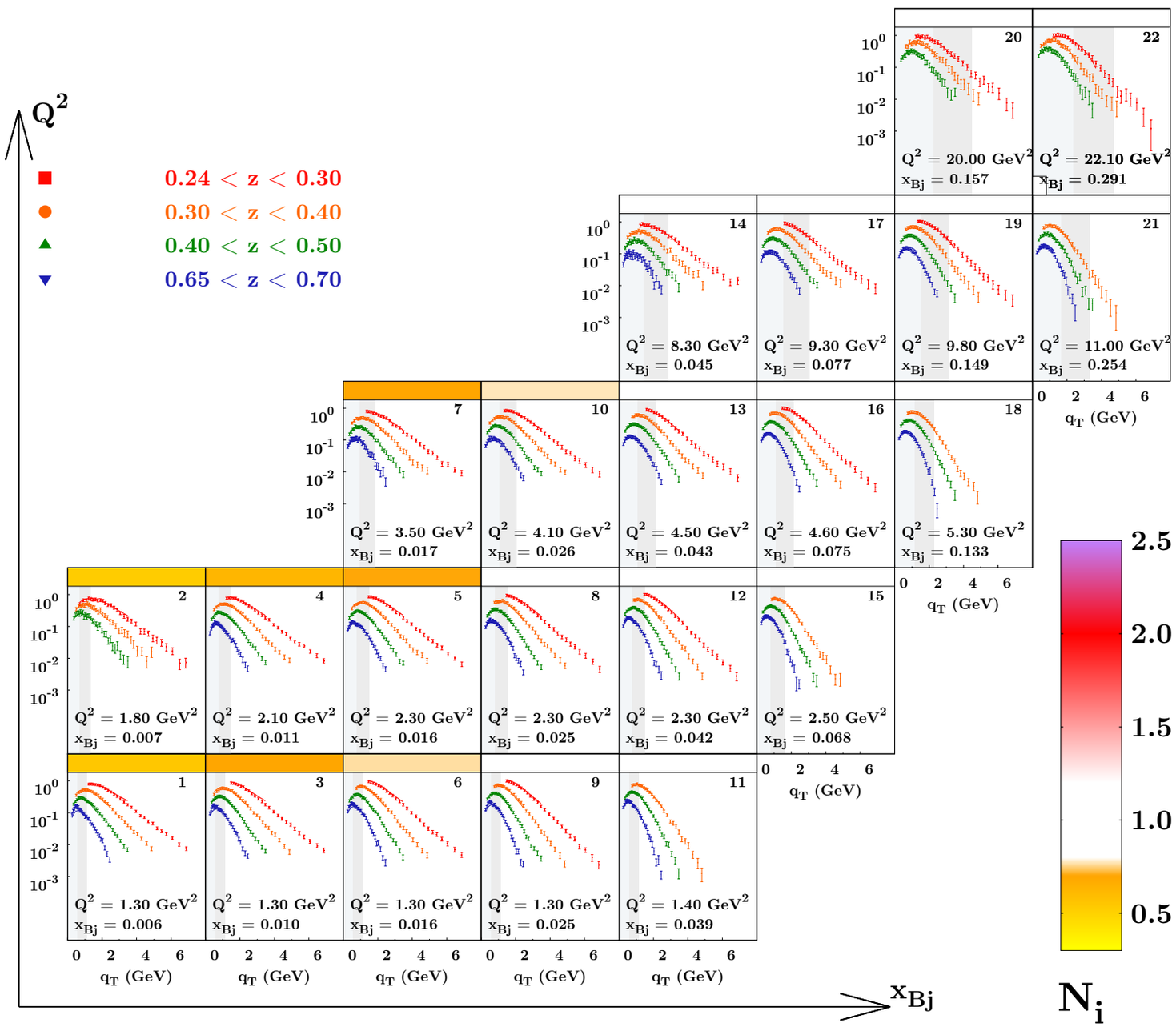
$\mathcal{O}(\alpha_s^0)$

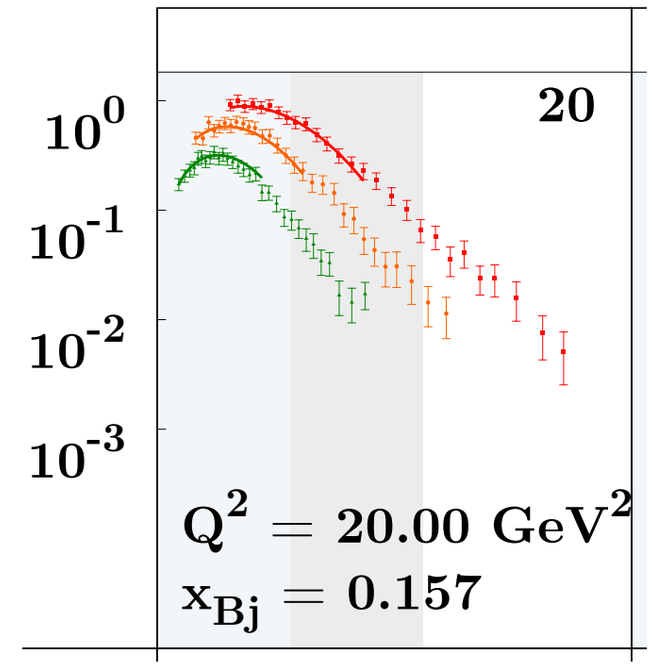


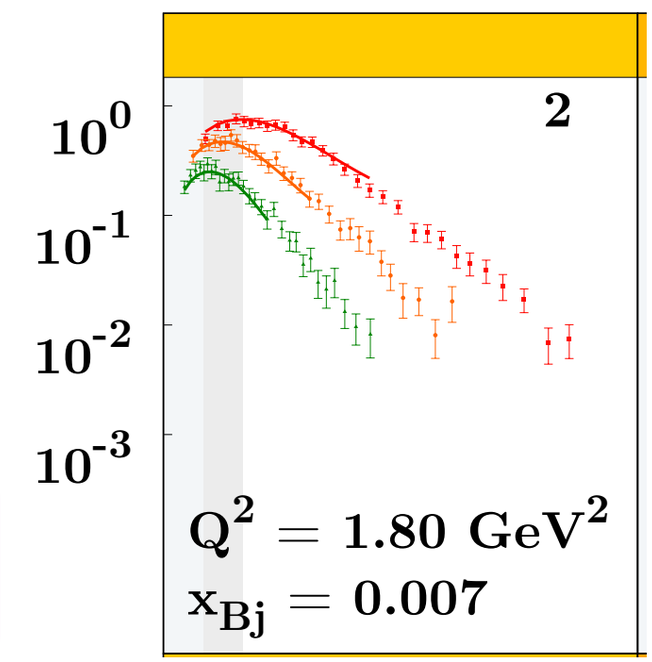


$$\tilde{F}_j = f(x,\mu_b)\exp\left\{g_P(x,b_T) - g_K(b_T)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

$\mathcal{O}(\alpha_s^0)$

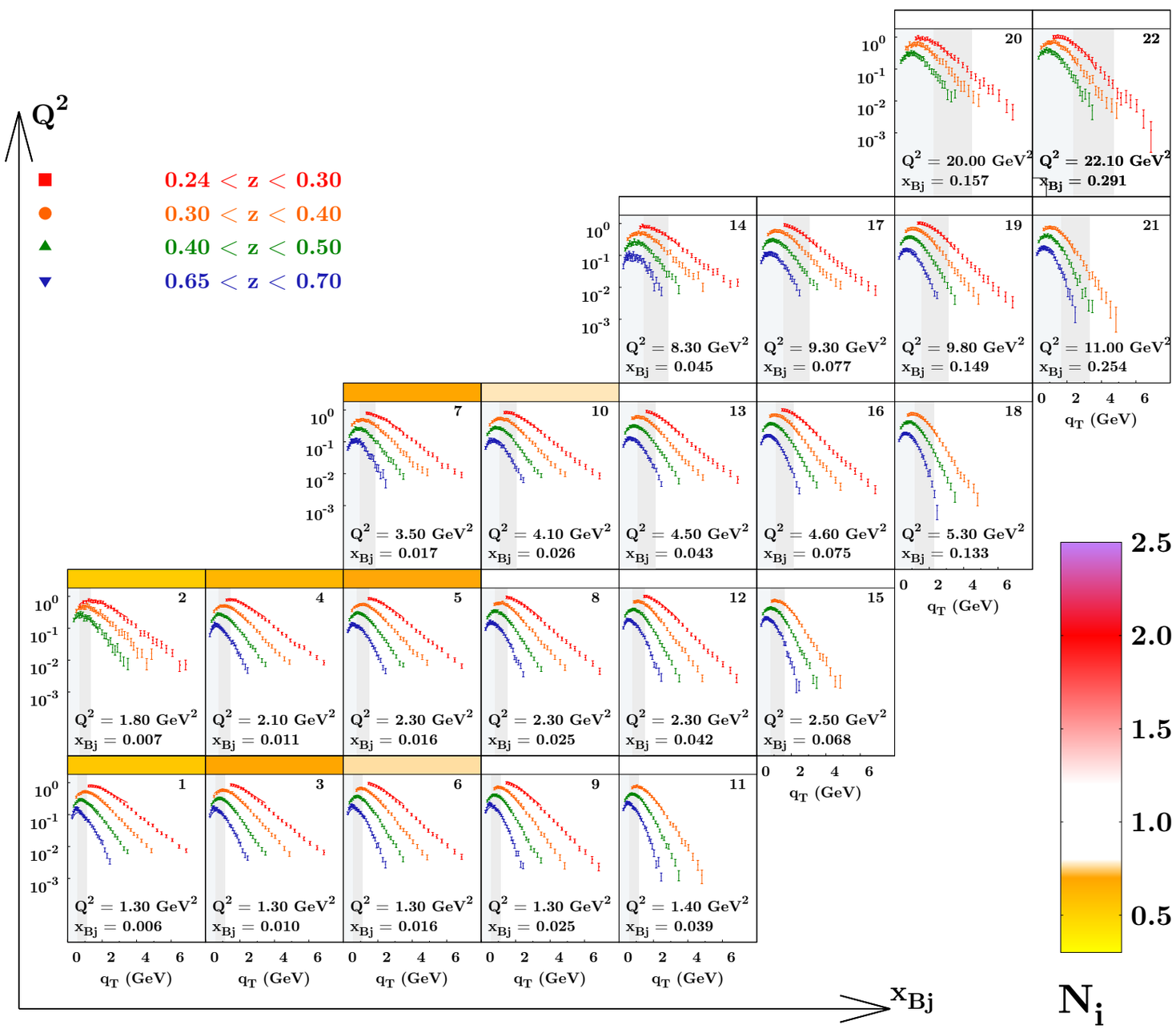






$$\tilde{F}_j = f(x,\mu_b)\exp\left\{g_P(x,b_T) - g_K(b_T)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

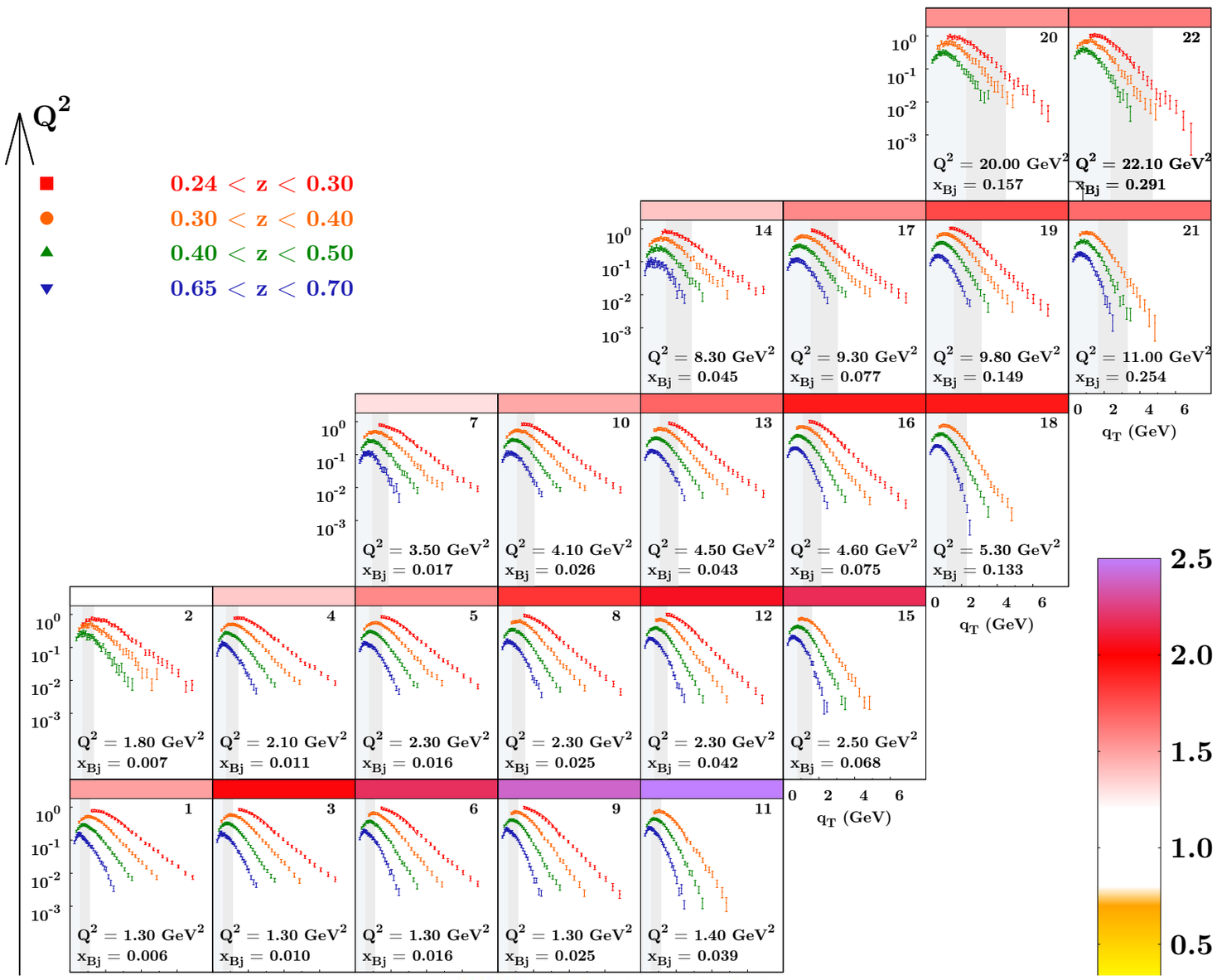
$\mathcal{O}(\alpha_s^0)$



- **First non-vanishing contribution of large qT region appears at order O(α_S)**
- **No matching at this order.**
- **Reduced predictive power.**

$$\tilde{F}_j = f(x, \mu_b) \exp\left\{ g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

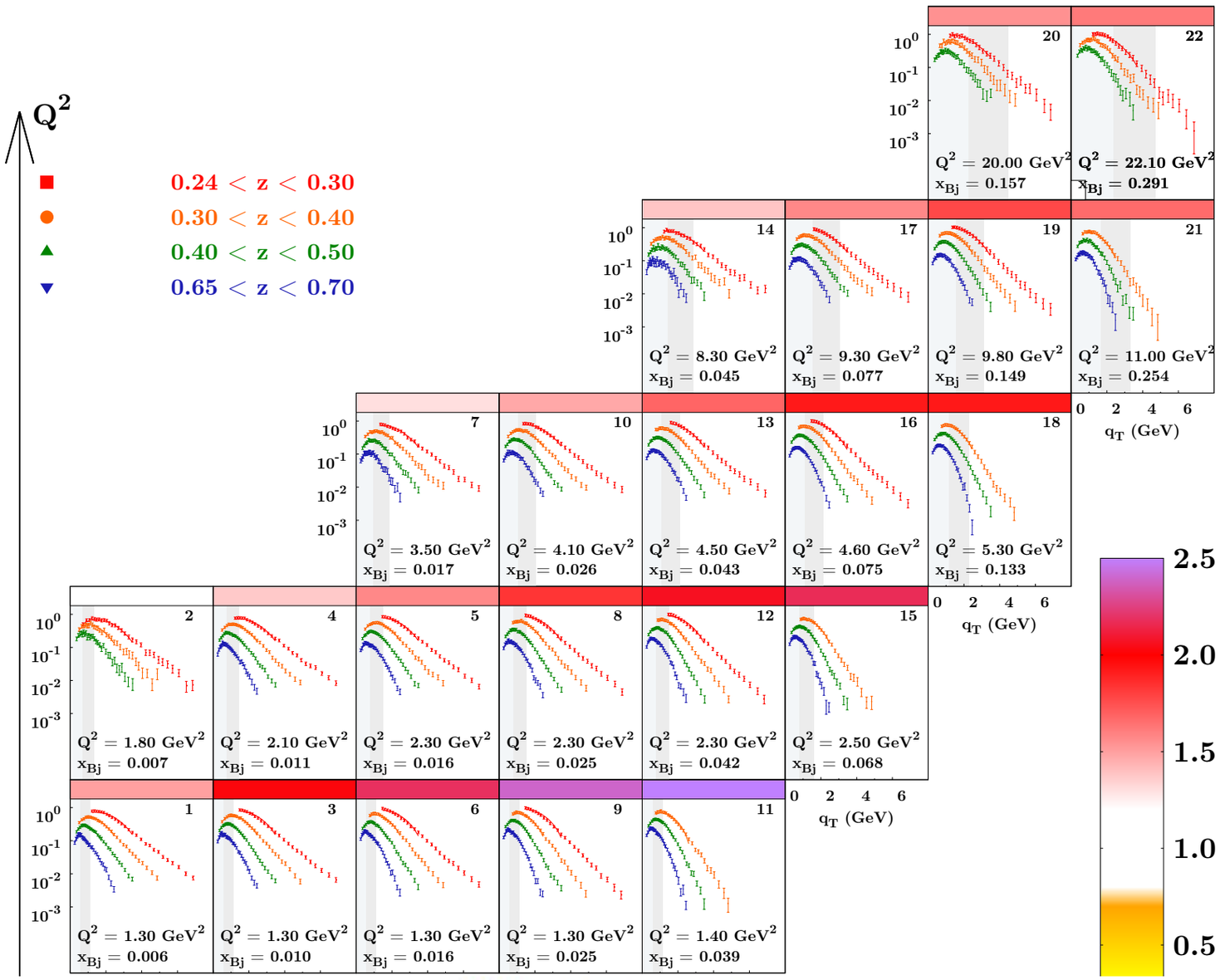
$\mathcal{O}(\alpha_s)$

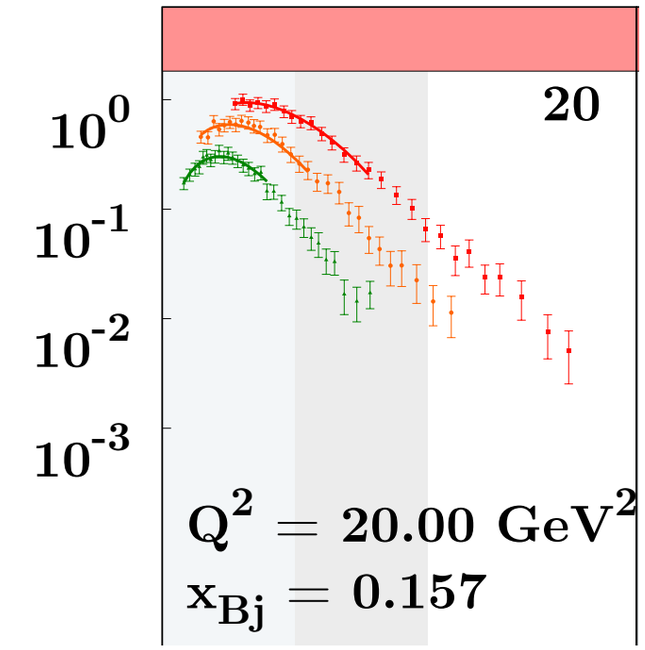


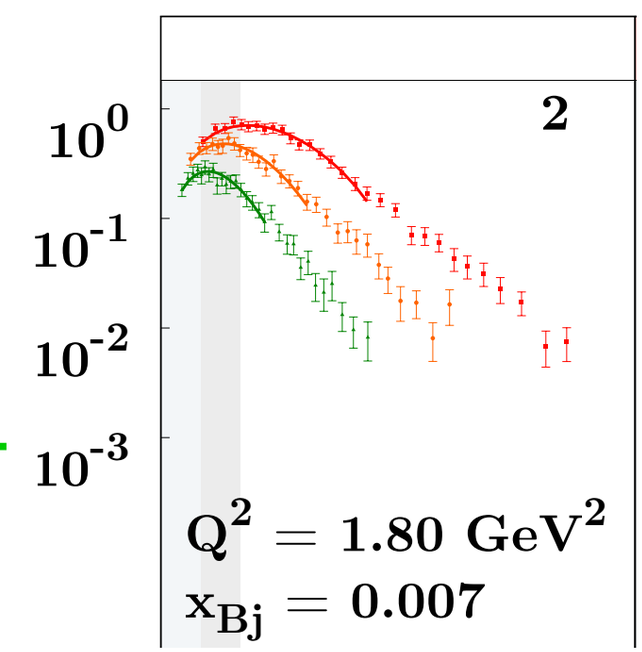
- **First order in which implementation of CSS scheme in full qT region is possible.**
- **More problems to fit data compared to O(α_s^0) case.**

$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b}\right)^{\tilde{K}(b_*,\mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{in}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b)$$
$$\times \exp\left\{\int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(\gamma_F(\mu;1) - \ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right)\gamma_K(\mu)\right)\right\}$$
$$\times \exp\left\{-g_P(x, b_T) - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right)\right\},$$

$\mathcal{O}(\alpha_s)$





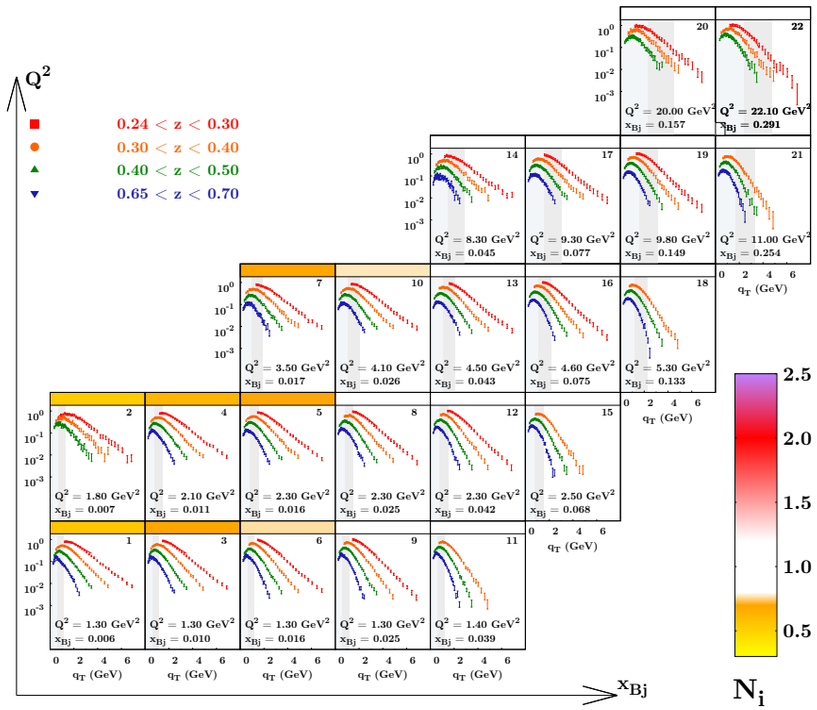


$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b}\right)^{\tilde{K}(b_*,\mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}^{in}_{ji}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b)$$

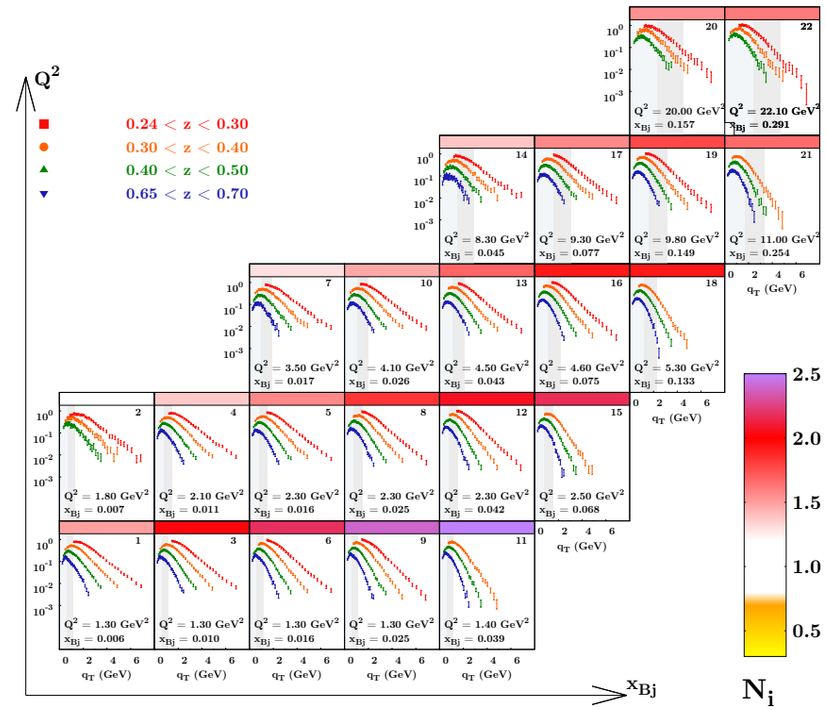
$$\times \exp\left\{\int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(\gamma_F(\mu;1) - \ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right)\gamma_K(\mu)\right)\right\}$$

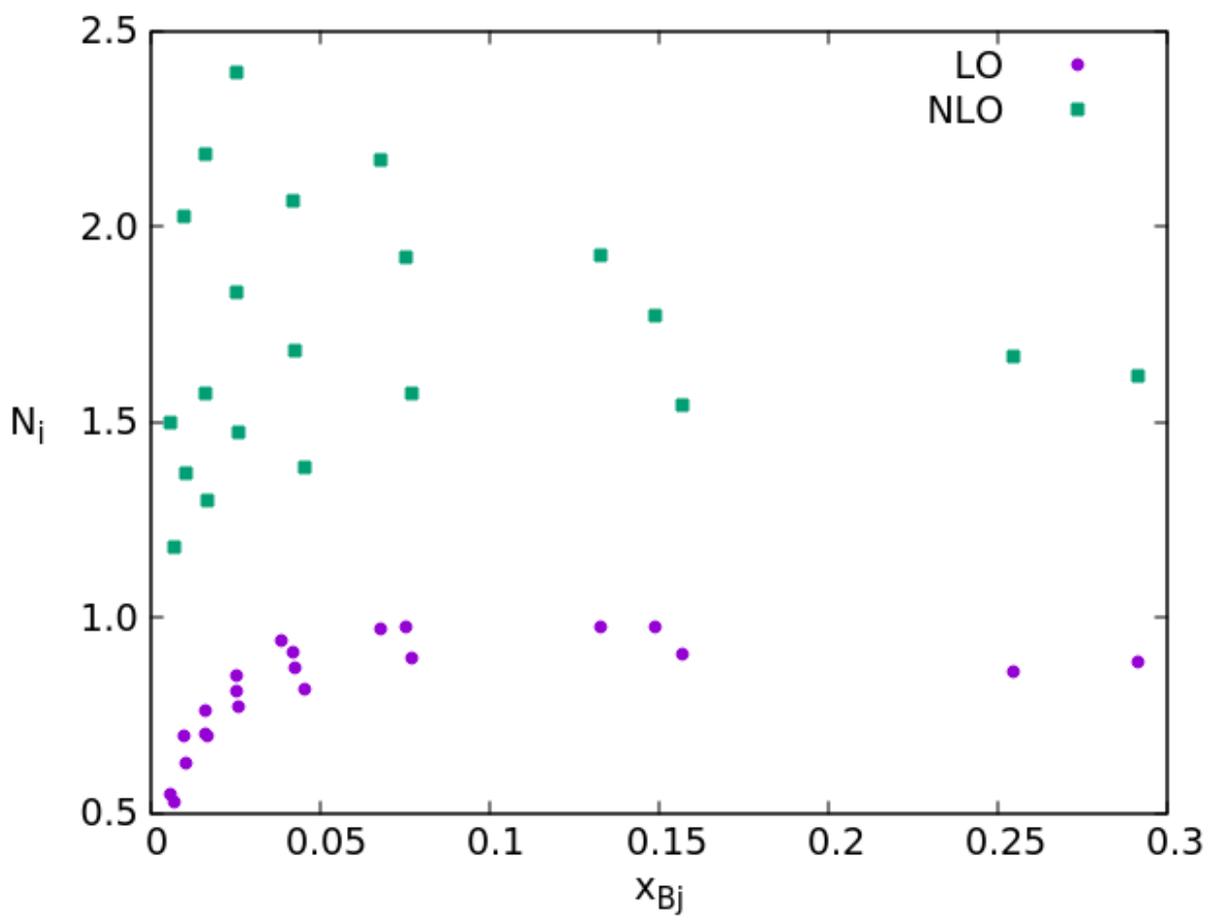
$$\times \exp\left\{-g_P(x, b_T) - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right)\right\},$$

$\mathcal{O}(\alpha_s^0)$

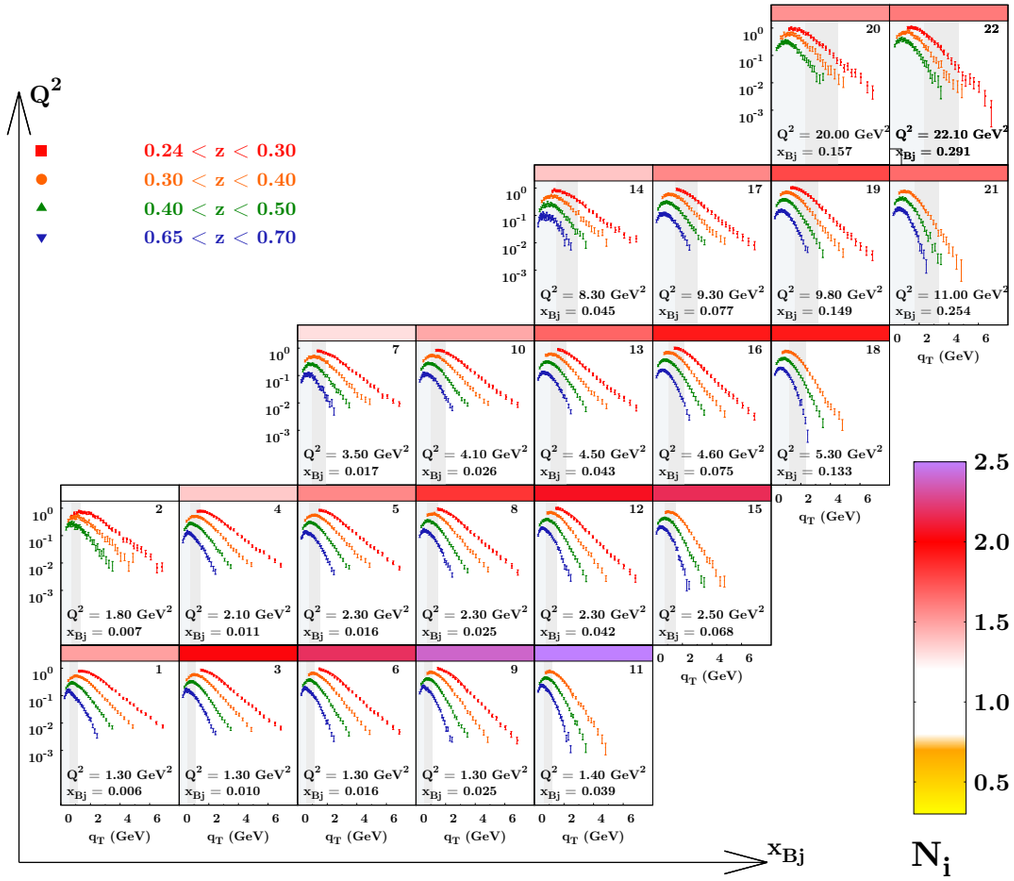


$\mathcal{O}(\alpha_s)$

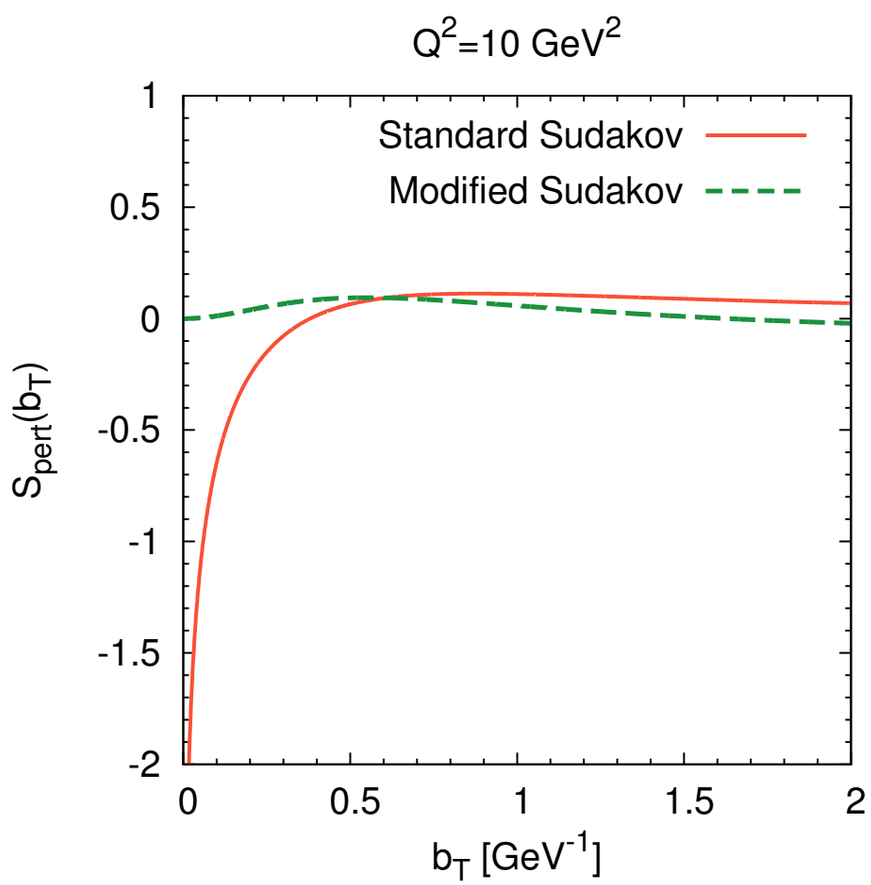




A bit counter-intuitive



$\mathcal{O}(\alpha_s)$

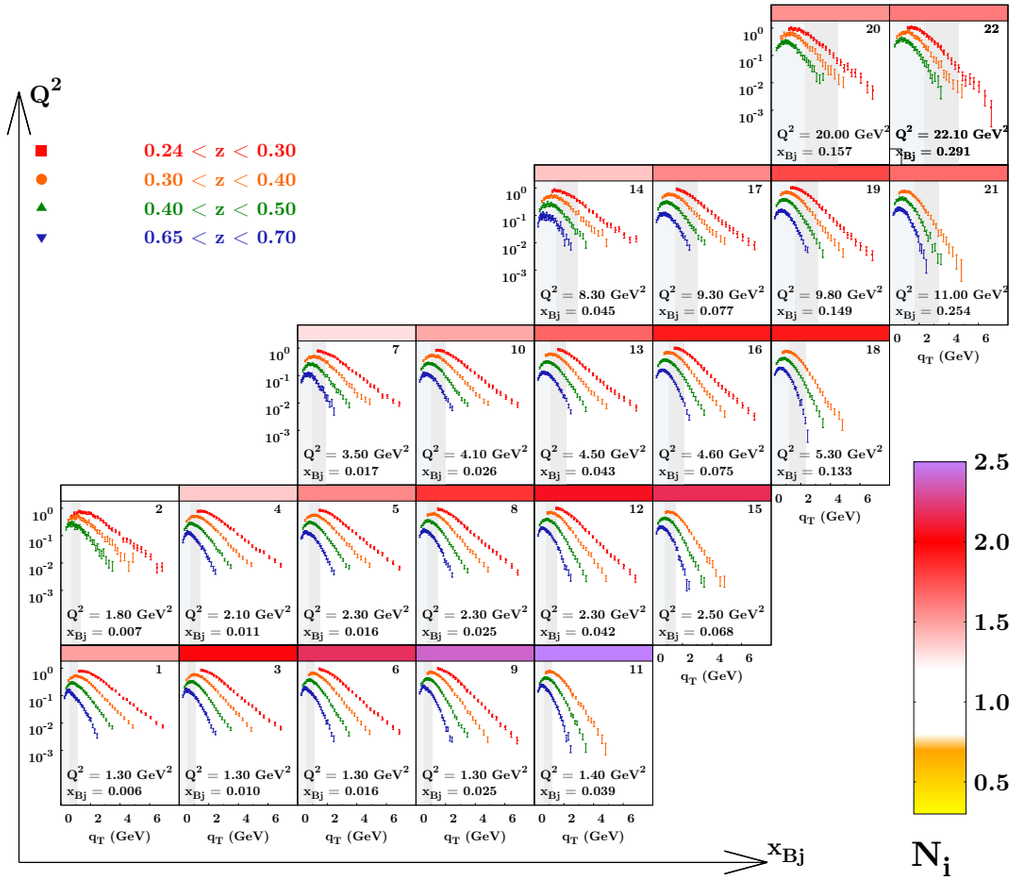


$\mathcal{O}(\alpha_s)$

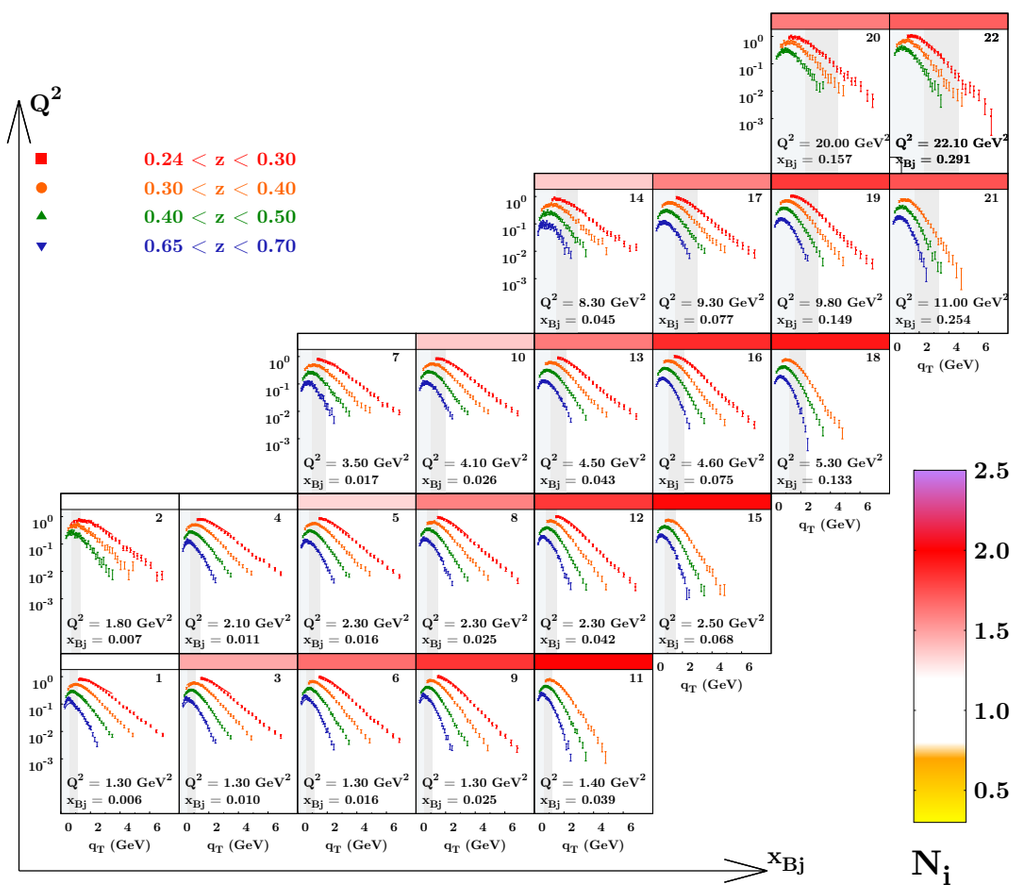
$$\exp\left\{\int_{\mu_b}^{Q}\frac{d\mu}{\mu}\left(\gamma_F(\mu;1)-\ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right)\gamma_K(\mu)\right)\right\}$$

$$\log\left(Q^2/\mu_b^2\right)\to\log\left(1+Q^2/\mu_b^2\right)$$

Modified "Sudakov"
(similar to a cut at low bT)



$\mathcal{O}(\alpha_s)$



$\mathcal{O}(\alpha_s)$

$$\exp\left\{\int_{\mu_b}^{Q}\frac{d\mu}{\mu}\left(\gamma_F(\mu;1)-\ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right)\gamma_K(\mu)\right)\right\}$$

$$\log\left(Q^2/\mu_b^2\right)\to\log\left(1+Q^2/\mu_b^2\right)$$

Modified "Sudakov"
(similar to a cut at low bT)

PDF C coefficients LO

FF C coefficients LO, (otherwise $\mathcal{O}(\alpha_s)$)

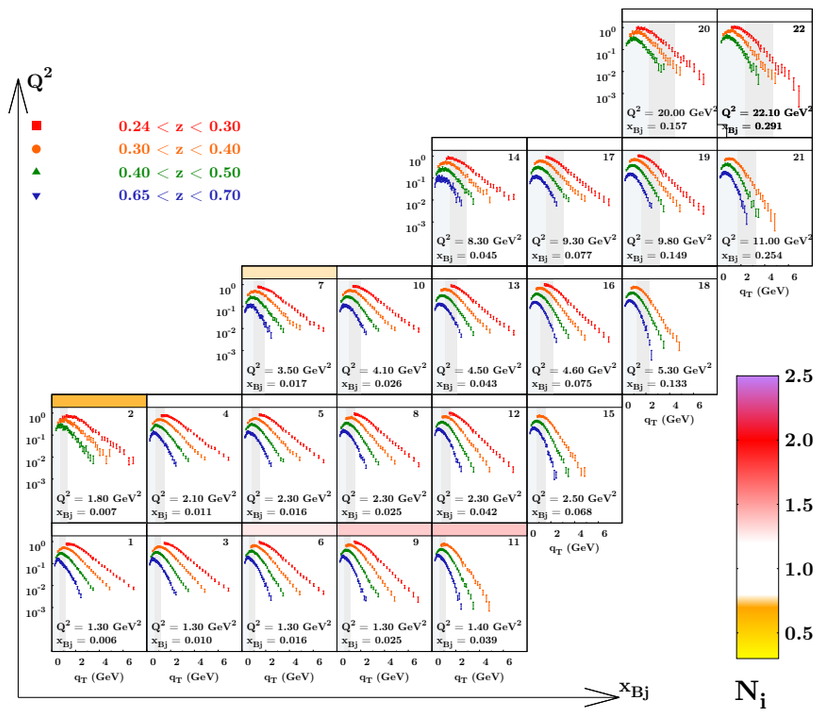
$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b}\right)^{\tilde{K}(b_*,\mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \boxed{\tilde{C}_{ji}^{\text{in}}\left(x/\hat{x}, b_*, \mu_b, \mu_b^2\right)} f_i(\hat{x}, \mu_b)$$
$$\times \exp\left\{\int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(\gamma_F(\mu;1) - \ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right)\gamma_K(\mu)\right)\right\}$$
$$\times \exp\left\{-g_P(x, b_T) - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right)\right\},$$

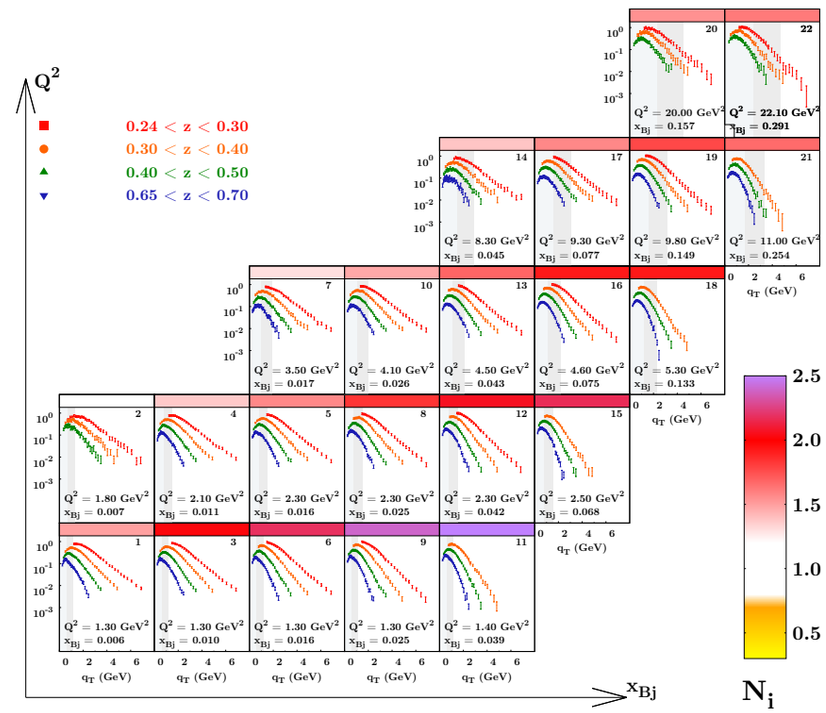
$$\tilde{D}_j(z, b_T, Q, \zeta_D) = \left(\frac{\sqrt{\zeta_D}}{\mu_b}\right)^{\tilde{K}(b_*,\mu_b)} \sum_k \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \boxed{\tilde{C}_{kj}^{\text{out}}\left(z/\hat{z}, b_*, \mu_b, \mu_b^2\right)} D_j(\hat{z}, \mu_b)$$
$$\times \exp\left\{\int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(\gamma_D(\mu;1) - \ln\left(\frac{\sqrt{\zeta_D}}{\mu}\right)\gamma_K(\mu)\right)\right\}$$
$$\times \exp\left\{-g_H(z, b_T) - g_K(b_T)\ln\left(\frac{\sqrt{\zeta_D}}{\sqrt{\zeta_{D0}}}\right)\right\}.$$

PDF C coefficients LO

FF C coefficients LO

$\mathcal{O}(\alpha_s)$





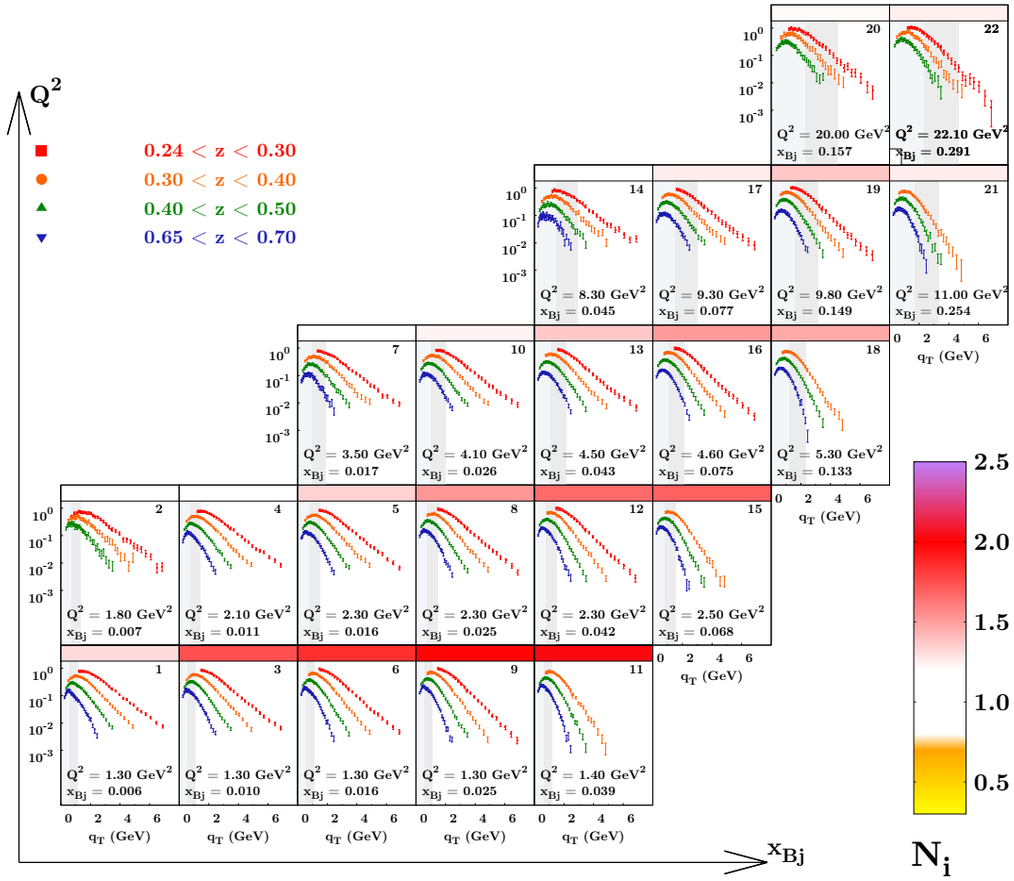
Convolution in TMD definition

$$\tilde{F}_j(x, b_T, Q, \zeta_F) \sim \quad C \otimes f(x) \equiv \int_x^1 \frac{d\hat{x}}{\hat{x}} C\left(\frac{x}{\hat{x}}\right) f(\hat{x})\,.$$

$$C_{qq'}^{(0)\mathrm{in}}(x) = \delta_{qq'}\delta(1-x)$$

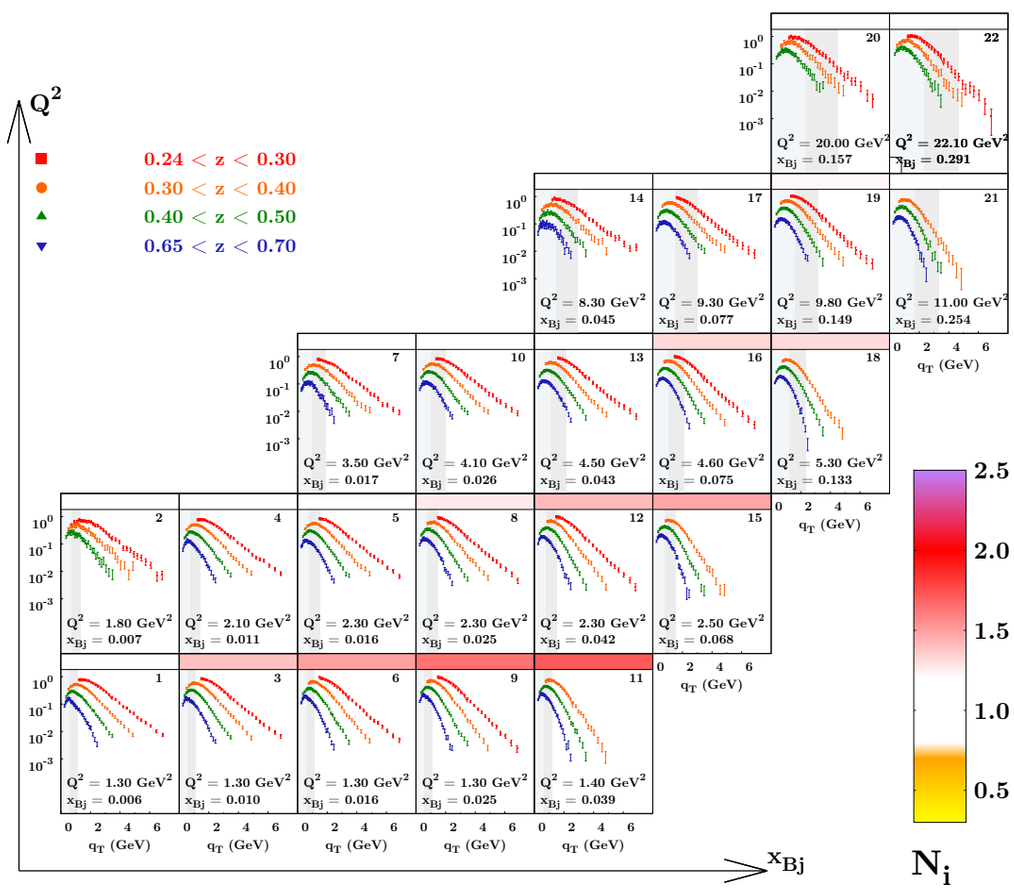
$$C_{qq'}^{(1)\mathrm{in}}(x) = \delta_{qq'}\frac{C_F}{2}\Big\{(1-x) - 4\delta(1-x)\Big\}$$

$$C_{qg}^{(1)\mathrm{in}}(x) = T_F[x(1-x)]$$



PDF C coefficients LO (only)

(otherwise $\mathcal{O}(\alpha_s)$)



FF C coefficients LO (only)

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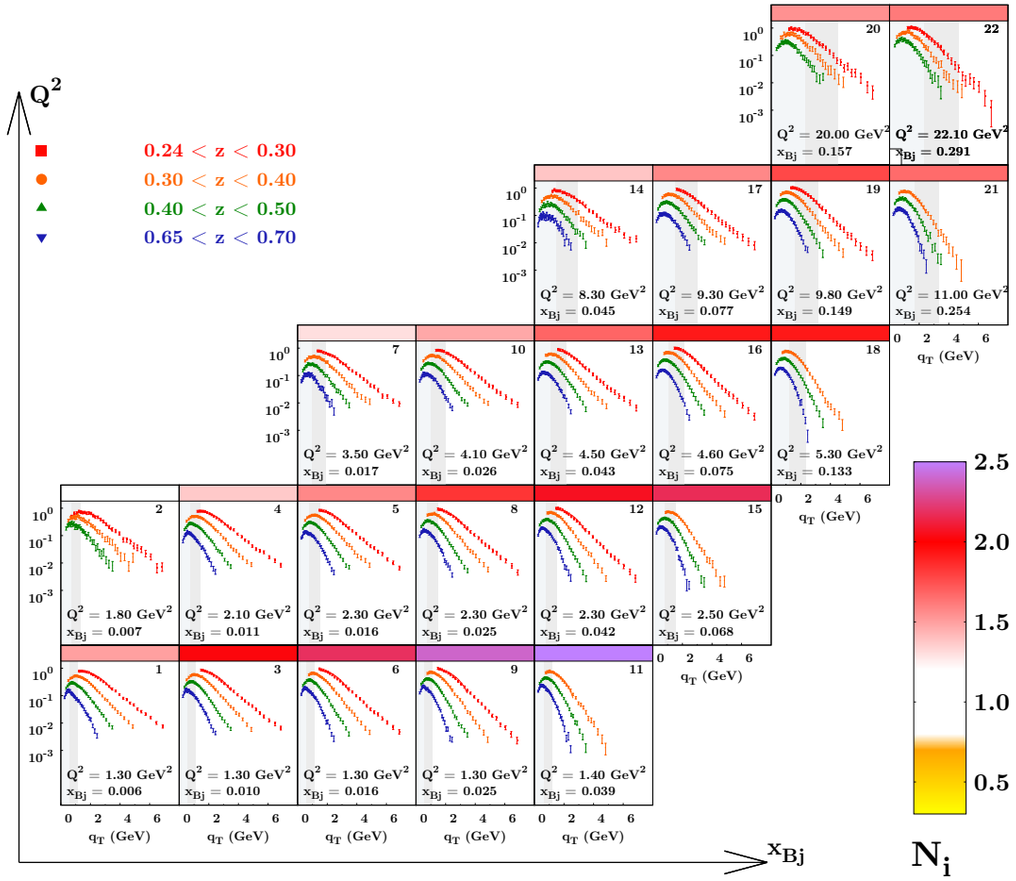
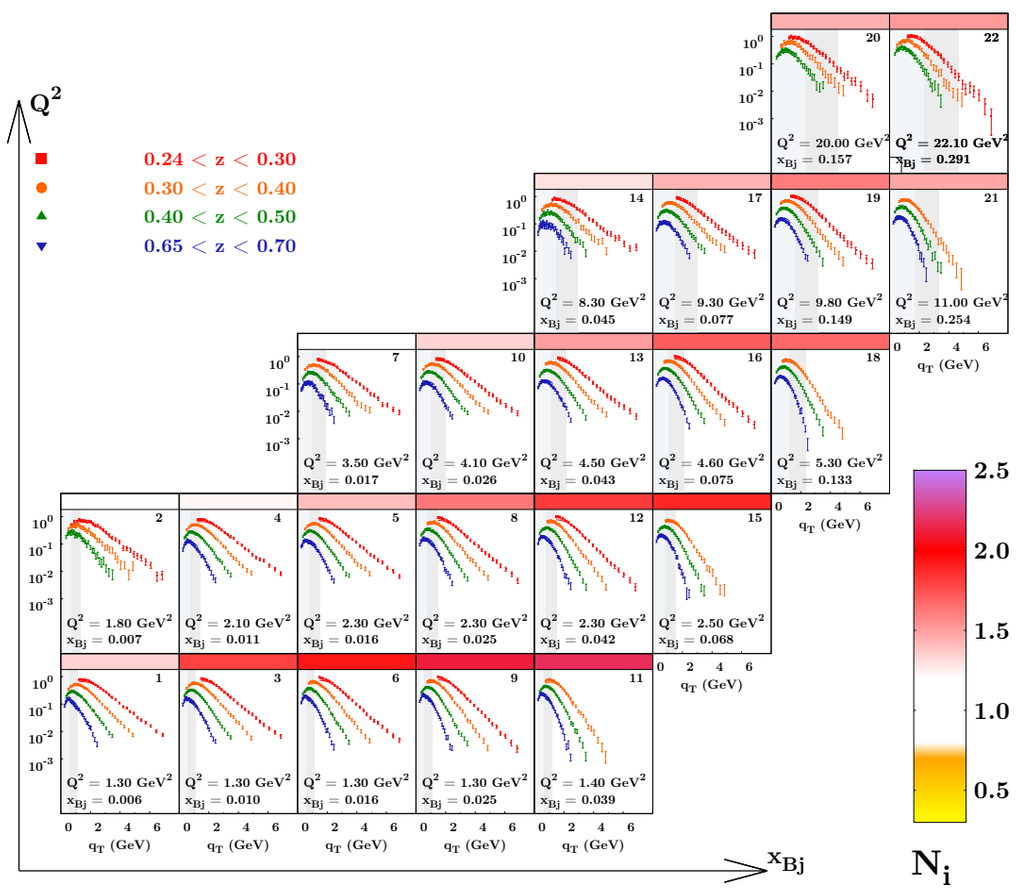


$\mathcal{O}(\alpha_s)$



Changing Fragmentation functions
(here, for no particular reason other than simplicity, I use **DSS LO**)

Final Remarks

- **Describing the full SIDIS qT spectra requires at least an order $\mathcal{O}(\alpha_s)$ analysis.**

- **The major obstacle is the suppression of the cross section due to:**

  terms in perturbative ingredients
  (possibly) inadequacy of collinear functions

- **Other choices like cuts in qT, smaller bmax, model, do not change the picture discussed in this talk.**

- **Things to try:**

  $\mathcal{O}(\alpha_s^2)$ analysis
  Collinear functions tuned for the large qT regime
  (do they improve the picture for the TMD region?)

Thanks