Comments on the perturbative and non-perturbative contributions in unpolarized SIDIS

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Outline

- Describing SIDIS data from small to large qT.
  \[ W \) (TMD small qT)
  \[ FO(large \; qT), \; Y(Matching) \]

- Structure of the TMD definition (CSS).

- Towards a LO analysis of the entire qT spectra.
Describing SIDIS data from small to large qT.

SIDIS variables: $Q^2, x_{Bj}, z_h, P_T$

Often useful to consider: $q_T = P_T / z_h$, $y_h = \frac{1}{2} \log \left( \frac{P_h^+}{P_h^-} \right)$
Structure of the TMD definition (CSS).

\[ \mathcal{W} \quad \text{(TMD region)} \]

\[ \sum_q \mathcal{H}_q \ F.T. \left\{ \tilde{D}_{h/q}(z, z b_\perp; Q) \ \tilde{f}_{q/P}(x, b_\perp; Q) \right\} \]

Fourier Transform of:

\[ \tilde{F}_j(x, b_T, Q, \zeta_F) = \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right)^{K(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{\text{in}}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_j(\hat{x}, \mu_b) \]

\[ \times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \]

\[ \times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\} , \]

- **pQCD**
- **Input (extraction from collinear cross section)**
- **Non-perturbative functions to extract from data.**
Towards a LO analysis of the entire qT spectra
$W$ at $\mathcal{O}(\alpha_s)$

$Q^2 (\text{GeV}^2)$

COMPASS $M_D^{h^+}$

- $\langle z \rangle = 0.23$
- $\langle z \rangle = 0.28$
- $\langle z \rangle = 0.33$
- $\langle z \rangle = 0.38$
- $\langle z \rangle = 0.45$
- $\langle z \rangle = 0.55$

$x_B$ vs. $P_T$ (GeV)

$Q^2$ values:
- $Q^2 = 1.76 \text{ GeV}^2$
- $Q^2 = 1.92 \text{ GeV}^2$
- $Q^2 = 3.07 \text{ GeV}^2$
- $Q^2 = 4.07 \text{ GeV}^2$
- $Q^2 = 4.47 \text{ GeV}^2$
- $Q^2 = 4.57 \text{ GeV}^2$
- $Q^2 = 4.62 \text{ GeV}^2$

$x_B$ values:
- $x_B = 5.50 \times 10^{-2}$
- $x_B = 9.32 \times 10^{-2}$
- $x_B = 5.36 \times 10^{-2}$
- $x_B = 6.55 \times 10^{-2}$
- $x_B = 9.21 \times 10^{-2}$
Caveat: A normalization $N$ is needed for each panel. As large as $N=2$. 

$W$ at $\mathcal{O}(\alpha_s)$

$Q^2$ (GeV$^2$)

- $\langle z \rangle = 0$
- $\langle z \rangle = 0$
- $\langle z \rangle = 0.33$
- $\langle z \rangle = 0.38$
- $\langle z \rangle = 0.45$
- $\langle z \rangle = 0.55$

$Q^2 = 4.07$ GeV$^2$
$Q^2 = 4.47$ GeV$^2$
$Q^2 = 4.57$ GeV$^2$
$Q^2 = 4.62$ GeV$^2$

$Q^2 = 2.00$ GeV$^2$
$Q^2 = 2.94$ GeV$^2$
$Q^2 = 2.95$ GeV$^2$
$Q^2 = 2.95$ GeV$^2$

$Q^2 = 1.78$ GeV$^2$
$Q^2 = 1.92$ GeV$^2$
$Q^2 = 1.92$ GeV$^2$
$Q^2 = 1.93$ GeV$^2$

At small $b_T$, the $W$ term (full) should be equal to the "perturbative" part of the TMD (model independent).

The role of the normalization is to help with this. One should try to avoid this.

\[
\tilde{F}_j(x, b_T, Q, \zeta_F) = \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right) K(b_*, \mu_b) \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{\text{in}}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b) \\
\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \\
\times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_F}_0} \right) \right\},
\]
look at the most recent data by COMPASS

Similar issues as before:

Need to introduce spurious normalizations in the analysis.

No reported normalization errors in new data sets.

Need to “debug”. Diagnosis strategy (not a TMD extraction):

1) choose a range to fit only W term: PT < 0.9 GeV
2) Use “standard freezing prescription” with bmax=1.0 GeV^-1
3) Fit Panel by panel (fixed Q^2 and xbj)
4) Use a normalization \( N_i \) as a free parameter for each panel
5) Use the values of \( N_i \) to quantify the problem
\[ \widetilde{F}_j = f(x, \mu_b) \exp \left\{ g_P(x, b_T) - g_K(b_T) \ln \left( \frac{Q}{Q_0} \right) \right\} \]
\[
\tilde{F}_j = f(x, \mu_b) \exp \left\{ g_P(x, b_T) - g_K(b_T) \ln \left( \frac{Q}{Q_0} \right) \right\}
\]
\[ \mathcal{O}(\alpha_s^0) \]

- First non-vanishing contribution of large qT region appears at order \( \mathcal{O}(\alpha_s) \)
- No matching at this order.
- Reduced predictive power.

\[ \tilde{F}_j = f(x, \mu_b) \exp \left\{ g_P(x, b_T) - g_K(b_T) \ln \left( \frac{Q}{Q_0} \right) \right\} \]
• First order in which implementation of CSS scheme in full qT region is possible.

• More problems to fit data compared to $\mathcal{O}(\alpha_s^0)$ case.

$$\mathcal{O}(\alpha_s)$$

$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right) \frac{K(b_*, \mu_b)}{\mu_b} \sum_j \int_x^1 \frac{d\hat x}{\hat x} \tilde C_{ji}^{\text{in}}(x/\hat x, b_*, \mu_b, \mu_b^2) f_i(\hat x, \mu_b)$$

$$\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\}$$

$$\times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\},$$
\[ F_j(x, b_T, Q, \zeta_F) = \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right)^{2 \tilde{K}(b_*, \mu_b)} \sum_j \int_0^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{\text{in}}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b) \]

\[
\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \]

\[
\times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta F_0}} \right) \right\} ,
\]
A bit counter-intuitive
perturbative QCD.

of resummation, or equivalently of the TMD description, to the region of applicability of space, where the direct connection to the conjugate tool. However, its successful implementation is a

4 Conclusions and outlook

be achieved within the corresponding error bands, rather than on individual points of the theoretical errors due, for instance, to the choice of renormalization scale and to the truncation from

allowing for a region of successful matching. However, since section, one also needs to know where to start using

Figure 10

\[ S^{-3.5} \]

Indeed, one should remember that all these contributions are computed within the theoretical prescription is rather limited in this case. The failure of the matching cannot be performed. In fact, the impact of this kinematical configuration the matching cannot be performed. In fact, the impact of this inadequacy of the resummation approaches at such low energies.

almost all the energy configurations (HERA and COMPASS).

We have checked that, even adopting the prescription of eq. (pert)

become large and negative at

The e\[ Q^2 \ll 2 \]

The e\[ Q^2 \gg 2 \]

The e\[ Q^2 \sim 2 \]

\[ \exp\left\{ \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{S_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \]

\[ \log \left( Q^2 / \mu_b^2 \right) \to \log \left( 1 + Q^2 / \mu_b^2 \right) \]

Modified “Sudakov”

(similar to a cut at low bT)
One can see from figure 10 that the perturbative Sudakov exponential \( \exp \left[ -\frac{Q^2}{\mu_b^2} \right] \) is evaluated over the whole range, see the central panels of figures 10–12.\footnote{We refer to \( Q^2 \) and \( \mu_b \) as kinematic variables, without any further notation.} We fix the kinematic range \( 0 < Q^2 < 100 \) GeV, which is the conventional range, see the top-left panel of figure 10. A suggested prescription to avoid this problem, which is also referred to as the bare-\( b \) prescription, is\footnote{An alternative prescription, which is referred to as the \( \mu \)-heavy prescription, is described in ref. \cite{JHEP02(2015)095}.} given by (3.10) and its expansion 1 + \( \alpha_s \) \( \log \left( \frac{Q^2}{\mu_b^2} \right) \rightarrow \log \left( 1 + Q^2/\mu_b^2 \right) \)

\( \mathcal{O}(\alpha_s) \)

\( \mathcal{O}(\alpha_s) \)

\( \exp \left\{ \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{S_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \)

\( \log \left( Q^2/\mu_b^2 \right) \rightarrow \log \left( 1 + Q^2/\mu_b^2 \right) \)

Modified “Sudakov”

(similar to a cut at low \( bT \))
PDF C coefficients LO

FF C coefficients LO, ( otherwise \( \mathcal{O}(\alpha_s) \) )

\[
\tilde{F}_j(x, b_T, Q, \zeta_F) = \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{\text{in}} \left( x/\hat{x}, b_*, \mu_b, \mu_b^2 \right) f_i(\hat{x}, \mu_b) \times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\} \times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\},
\]

\[
\tilde{D}_j(z, b_T, Q, \zeta_D) = \left( \frac{\sqrt{\zeta_D}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_k \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \tilde{C}_{kj}^{\text{out}} \left( z/\hat{z}, b_*, \mu_b, \mu_b^2 \right) D_j(\hat{z}, \mu_b) \times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_D(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_D}}{\mu} \right) \gamma_K(\mu) \right) \right\} \times \exp \left\{ -g_H(z, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_D}}{\sqrt{\zeta_{D0}}} \right) \right\}.
\]
PDF C coefficients LO

FF C coefficients LO

Convolution in TMD definition

\[ \tilde{F}_j(x, b_T, Q, \zeta_F) \sim C \otimes f(x) \equiv \int_x^1 \frac{d\hat{x}}{\hat{x}} C \left( \frac{x}{\hat{x}} \right) f(\hat{x}). \]

\[ C_{qq}^{(0)\text{in}}(x) = \delta_{qq'} \delta(1 - x) \]

\[ C_{qq}^{(1)\text{in}}(x) = \delta_{qq'} \frac{C_F}{2} \left\{ (1 - x) - 4 \delta(1 - x) \right\} \]

\[ C_{qq}^{(1)\text{in}}(x) = T_F [x(1 - x)] \]
PDF C coefficients LO (only) 
(otherwise $\mathcal{O}(\alpha_s)$)

FF C coefficients LO (only) 
(otherwise $\mathcal{O}(\alpha_s)$)
Changing Fragmentation functions

(here, for no particular reason other than simplicity, I use DSS LO)

\( \mathcal{O}(\alpha_s) \)
Final Remarks

- Describing the full SIDIS qT spectra requires at least an order $\mathcal{O}(\alpha_s)$ analysis.

- The major obstacle is the suppression of the cross section due to:
  
  - terms in perturbative ingredients
  - (possibly) inadequacy of collinear functions

- Other choices like cuts in qT, smaller bmax, model, do not change the picture discussed in this talk.

- Things to try:
  
  - $\mathcal{O}(\alpha_s^2)$ analysis
  - Collinear functions tuned for the large qT regime (do they improve the picture for the TMD region?)
Thanks