

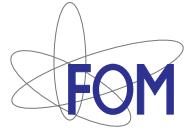
Polarized gluon TMDs at small x

Piet J Mulders and Elena Petreska

p.j.g.mulders@vu.nl



European Research Council



Abstract

■ **Polarized gluon TMDs at small x**

Piet Mulders and Elena Petreska

- Flavor, spin and partonic transverse momenta are important characteristics for parton distribution functions (PDFs), allowing a proliferation of possibilities. This proliferation can provide novel information into the non-perturbative structure of nucleons as well as new probes for high energy processes. Wilson lines are an important ingredient in the operator definitions of transverse momentum dependent PDFs (TMDs). We focus on the small x behavior of unpolarized and linearly polarized gluon TMDs with different gauge link structures for unpolarized and transversely polarized nucleons. For this we employ generalized TMD correlators (GTMDs) involving non-forward matrix elements of Wilson loops. As an example of the richness of GTMDs, we note that the C-odd parts can generate odd harmonics in the two-particle azimuthal correlations in peripheral proton-nucleus collisions.

Gluons at small x

- Starting point: matrix elements of two nonlocal gluon field operators (of course connected via an appropriate gauge link) with light-like separation (collinear PDFs) or light-front separation (TMDs).

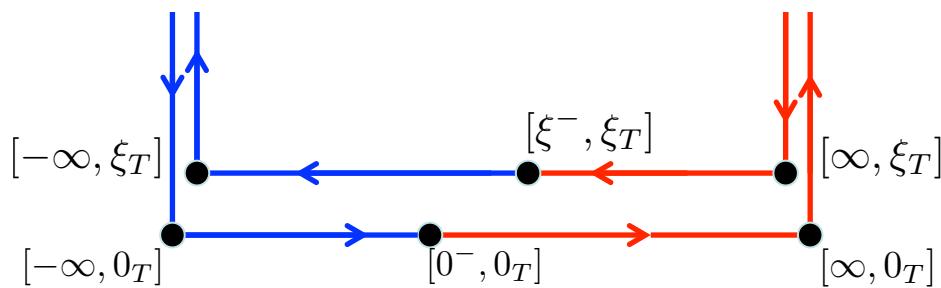
- $\epsilon^\alpha(k)\epsilon^{\beta*}(k) \implies$

$$\Gamma^{[U,U']}{}^{\mu\nu}(x, p_T; n) = \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P | F^{n\mu}(0) U_{[0,\xi]} F^{n\nu}(\xi) U'_{[\xi,0]} | P \rangle|_{\xi \cdot n = 0}$$

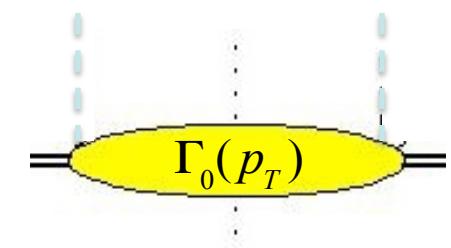
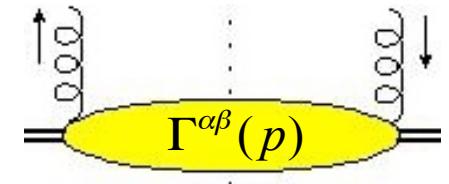
- Compare this with just a Wilson loop with transverse separation, linked to the emergence of transverse structure in QCD

$$\Gamma_0^{[U,U']}(k_T; n) = \int \frac{d^2\xi_T}{(2\pi)^2} e^{ik \cdot \xi} \langle P | U_{[0,\xi]} U'_{[\xi,0]} | P \rangle|_{\xi = \xi_T}$$

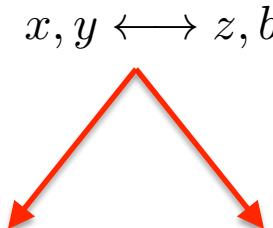
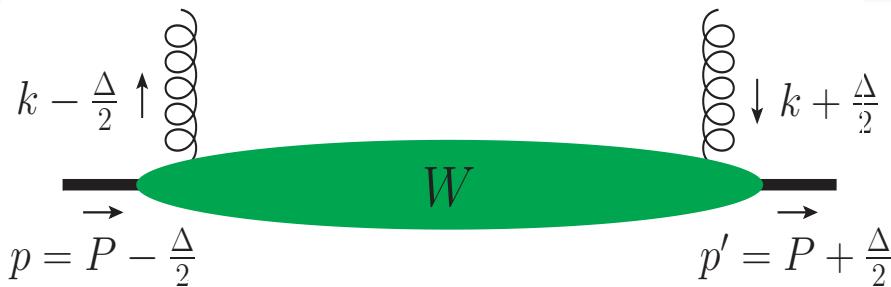
- Gauge links (dipole type $[+,-]$)



$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$



Gluon TMDs and Wilson loops via GTMDs



$$G^{[+,-]\alpha\beta}(x, k_T, \xi, \Delta_T) = 4 \int \frac{d^3z d^3b}{(2\pi)^3} e^{ik \cdot z - i\Delta \cdot b} \frac{\langle p' | F^{n\beta}(x) U_{[x,y]}^{[-]} F^{n\alpha}(y) U_{[y,x]}^{[+]} | p \rangle|_{LF}}{\langle P | P \rangle}$$

$\downarrow x=\xi=0$

$$G^{[+,-]\alpha\beta}(k_T, \Delta_T) = 16 \int \frac{d^2z d^2b}{(2\pi)^3} e^{ik_T \cdot z_T - i\Delta_T \cdot b_T} \frac{\langle p' | G_T^\beta(x) U_{[x,y]}^{[-]} G_T^\alpha(y) U_{[y,x]}^{[+]} | p \rangle|_{LF}}{\langle P | P \rangle}$$

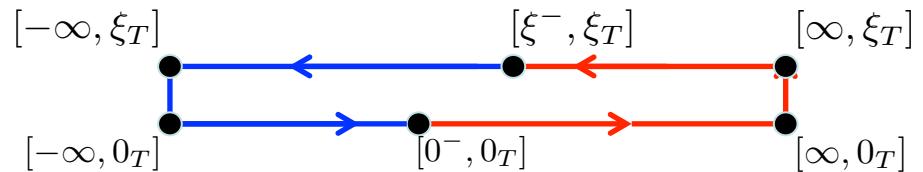
gluonic pole: $G_T^\alpha(x_T) = \int_{-\infty}^{\infty} dz^- U_{[0,x]}^{[-]} F^{n\alpha}(z^-, x_T) U_{[0,x]}^{[+]}$

$$\left[i\partial_x^\alpha, U_{[a,x]}^{[\pm]} \right] = \pm g U_{[a,x]}^{[\pm]} G_T^\alpha(x)$$

■ Basis: Wilson loop GTMD

$$G^{[\square]}(k_T, \Delta_T) = \int \frac{d^2z d^2b}{(2\pi)^4} e^{ik_T \cdot z_T - i\Delta_T \cdot b_T} \frac{\langle p' | \frac{1}{N_c} \text{Tr} (U^{[\square]}(x, y)) | p \rangle|_{LF}}{\langle P | P \rangle}$$

Gluon TMDs and Wilson loops via GTMDs



$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$

- Employ the (off-forward) Wilson loop GTMD,

$$\begin{aligned} G^{[\square]}(k_T, \Delta_T) &= \int \frac{d^2 z d^2 b}{(2\pi)^4} e^{ik_T \cdot z_T - i\Delta_T \cdot b_T} \frac{\langle p' | \frac{1}{N_c} \text{Tr} (U^{[\square]}(x, y)) | p \rangle|_{LF}}{\langle P | P \rangle} \\ &= \frac{\alpha_s}{2N_c M^2} \mathcal{E}(k_T^2, \Delta_T^2, k_T \cdot \Delta_T) \end{aligned}$$

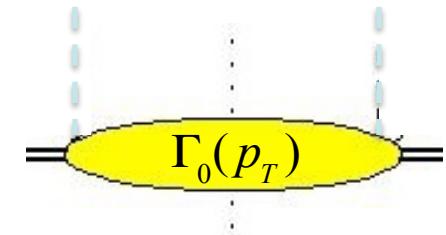
- ... to get the gluon GTMD at $x = 0$ as well as the gluon TMD at $x = 0$

$$\begin{aligned} G^{[+,-]\alpha\beta}(k_T, \Delta_T) &= \left[\frac{k_T^\alpha k_T^\beta}{M^2} - \frac{\Delta_T^\alpha \Delta_T^\beta}{4M^2} - \frac{k_T^{[\alpha} \Delta_T^{\beta]}}{2M^2} \right] \mathcal{E}(k_T^2, \Delta_T^2, k_T \cdot \Delta_T) \\ &\stackrel{\Delta \rightarrow 0}{\Longrightarrow} \frac{k_T^\alpha k_T^\beta}{M^2} e(k_T^2) \end{aligned}$$

Gluon TMDs in unpolarized hadrons

- Wilson loop correlator

$$\Gamma_0^{[+,-]}(k_T) = \frac{1}{2M^2} e^{[+,-]}(k_T^2)$$

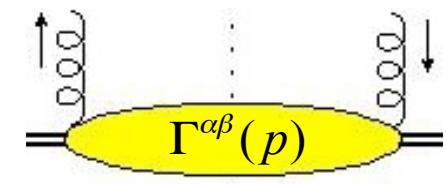


- Gluon (G)TMD at $x=\xi=0$

$$G^{[+,-]\alpha\beta}(k_T, \Delta_T) \xrightarrow{\Delta_T \rightarrow 0} \frac{k_T^\alpha k_T^\beta}{M^2} e^{[+,-]}(k_T^2)$$

- There are several gluon TMDs for unpolarized hadrons:

$$\Gamma^{\alpha\beta[U]}(x, k_T) = \frac{x}{2} \left\{ -g_T^{\alpha\beta} f_1^{[U]}(x, k_T^2) + \frac{k_T^{\alpha\beta}}{M^2} h_1^{\perp[U]}(x, k_T^2) \right\}$$



- Small x behavior of (dipole) gluon TMDs

$$x f_1^{[+,-]}(x, k_T^2) \xrightarrow{x \rightarrow 0} \frac{k_T^2}{2M^2} e^{[+,-]}(k_T^2)$$

$$x h_1^{\perp[+,-]}(x, k_T^2) \xrightarrow{x \rightarrow 0} e^{[+,-]}(k_T^2)$$

- Naively, TMDs behave as $1/x$, diverge for $x \rightarrow 0$ (or less naively: small x behavior x^α with $1 < \alpha < 2$)

Gluon TMDs in polarized nucleon

■ Polarized target (vector polarization)

$$\Gamma_L^{ij[U]}(x, k_T) = \frac{x}{2} \left\{ i\epsilon_T^{ij} S_L g_1^{[U]}(x, k_T^2) + \frac{\epsilon_T^{\{i} k_T^j\}\alpha}{M^2} S_L h_{1L}^\perp [U](x, k_T^2) \right\}$$

$$\begin{aligned} \Gamma_T^{ij[U]}(x, k_T) = & \frac{x}{2} \left\{ \frac{g_T^{ij} \epsilon_T^{kS_T}}{M} f_{1T}^\perp [U](x, k_T^2) - \frac{i\epsilon_T^{ij} k_T \cdot S_T}{M} g_{1T}^{[U]}(x, k_T^2) \right. \\ & \left. - \frac{\epsilon_T^{k\{i} S_T^j\} + \epsilon_T^{S_T\{i} k_T^j\}}{4M} h_1(x, k_T^2) - \frac{\epsilon_T^{\{i} k_T^j\}\alpha S_T}{2M^3} h_{1T}^\perp(x, k_T^2) \right\} \end{aligned}$$

■ Cf. Wilson loop TMDs in polarized nucleon (no TMD for L polarization)

$$\Gamma_0(k_T) = \frac{1}{2M^2} \left\{ e(k_T^2) - \frac{\epsilon^{kS_T}}{M} e_T(k_T^2) \right\}$$

 'pomeron'
  'odderon'

Note on bounds:

$$\frac{|k_T|}{M} |e_T(k_T^2)| \leq e(k_T^2)$$

Dominguez, Xiao, Yuan 2011; Hatta, Xiao, Yuan 2016

D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485

D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

S. Cotogno, T. van Daal, PJM, JHEP 1711 (2017) 185, ArXiv 1709.07827

Small x physics in terms of TMDs

- Dipole gluon TMDs: at **small x** only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

$$x f_1^{[+,-]}(x, k_T^2) \longrightarrow \frac{k_T^2}{2M^2} e^{[+,-]}(k_T^2)$$
$$x h_1^{\perp[+,-]}(x, k_T^2) \longrightarrow e^{[+,-]}(k_T^2)$$

$$x f_{1T}^{\perp[+,-]}(x, k_T^2) \longrightarrow \frac{k_T^2}{2M^2} e_T^{[+,-]}(k_T^2)$$
$$x h_1^{[+,-]}(x, k_T^2) \longrightarrow \frac{k_T^2}{2M^2} e_T^{[+,-]}(k_T^2)$$
$$x h_{1T}^{\perp[+,-]}(x, k_T^2) \longrightarrow e_T^{[+,-]}(k_T^2)$$

- Circularly polarized gluons in transversely polarized nucleons and TMDs in longitudinally polarized nucleons: Naively: TMDs tend to zero for $x \rightarrow 0$ (less naively: small x behavior x^α with $\alpha < 1$)

Nuclear measurements of GTMDs

- Back to unpolarized GTMDs (at $x=\xi=0$)

$$G^{[+,-]\alpha\beta}(k_T, \Delta_T) = \frac{2N_c}{\alpha_s} \left[k_T^\alpha k_T^\beta - \frac{1}{4} \Delta_T^\alpha \Delta_T^\beta - \frac{1}{2} k_T^{[\alpha} \Delta_T^{\beta]} \right] G^{[\square]}(k_T, \Delta_T)$$

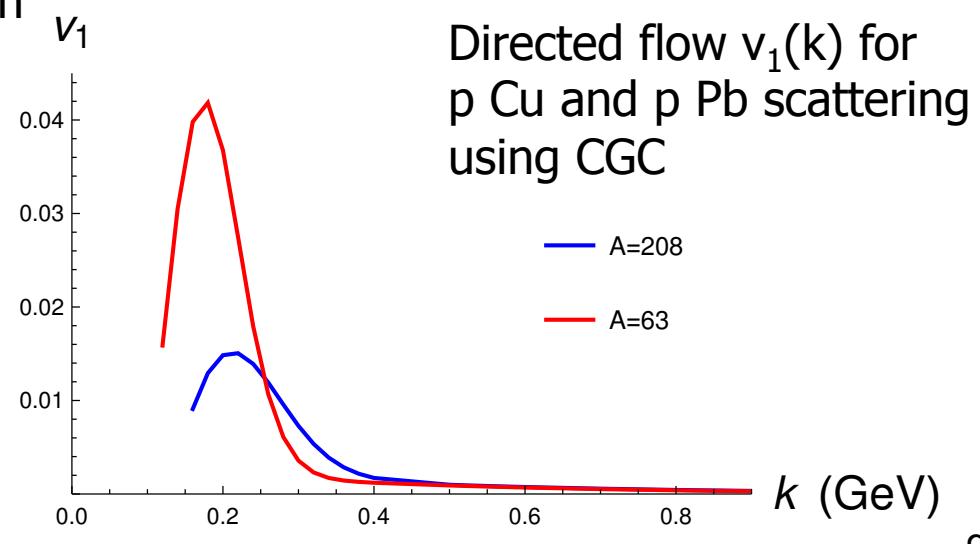
- Can be complex Hermiticity: $G^{[\square]*}(k_T, \Delta_T) = G^{[\square]}(k_T, -\Delta_T)$

Time reversal: $G^{[\square]*}(k_T, \Delta_T) = G^{[\square^\dagger]}(-k_T, -\Delta_T)$

- Odderon (C-odd) contribution in $G^{[\square]}(k_T, \Delta_T)$ only has odd powers in $k_T \cdot \Delta_T$ and vanishes in the forward limit

- As b_T (FT of Δ_T) and z_T (FT of k_T) involve different scales in a nucleus (nuclear for b_T and nucleonic for z_T) the odderon part could show up, e.g. in ultra-peripheral pA scattering via azimuthal asymmetries (directed flow)

- Needs cubic term in CGC action
- Leads to $1/A^{1/3}$ effect



Concluding remarks

- Simplifications of gluon TMDs at small x : linearly polarized gluon distributions in unpolarized and transversely polarized nucleons dominate over circularly polarized distributions.
- Wilson loop matrix elements crucial in low x domain.
- Access to C-odd matrix elements via nuclear density profiles