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Amplitude Parton Showers and Non-Global Logarithms

Simon Plätzer

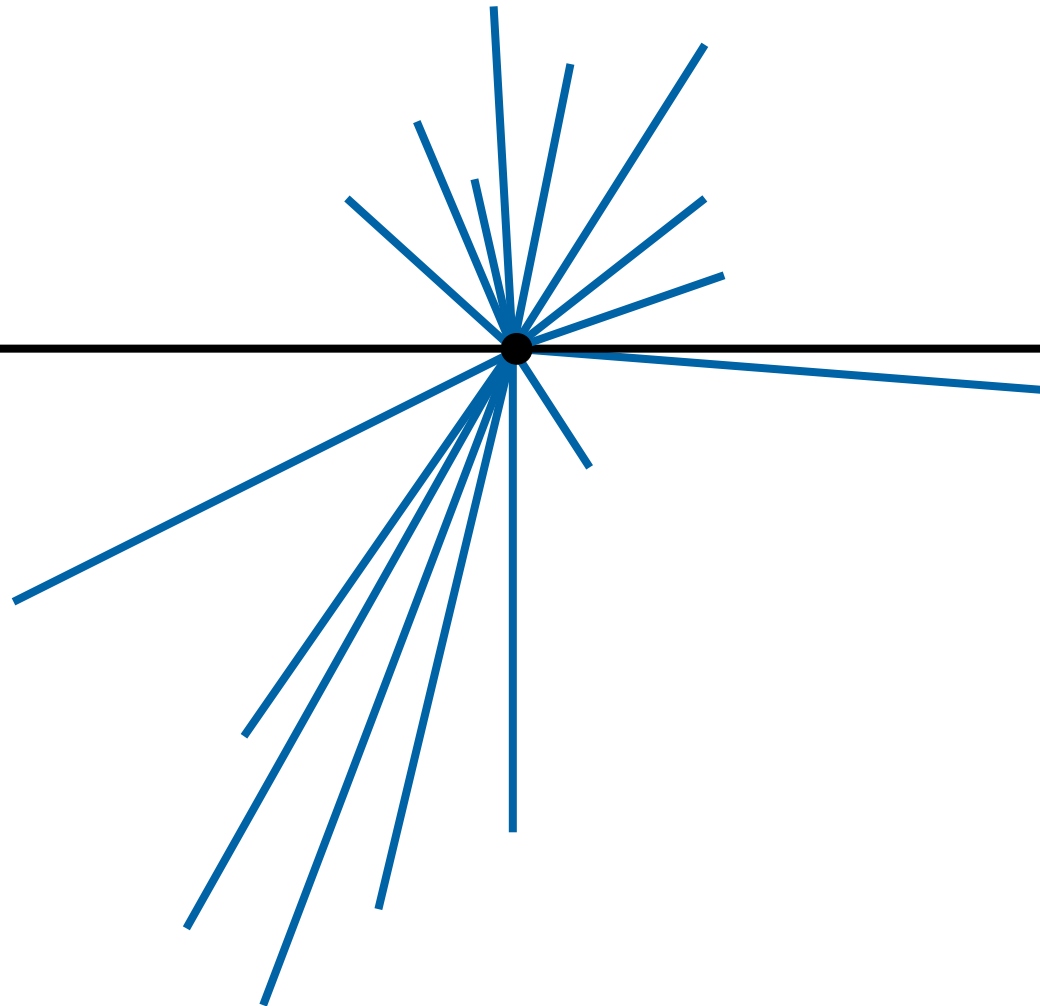
Particle Physics, University of Vienna

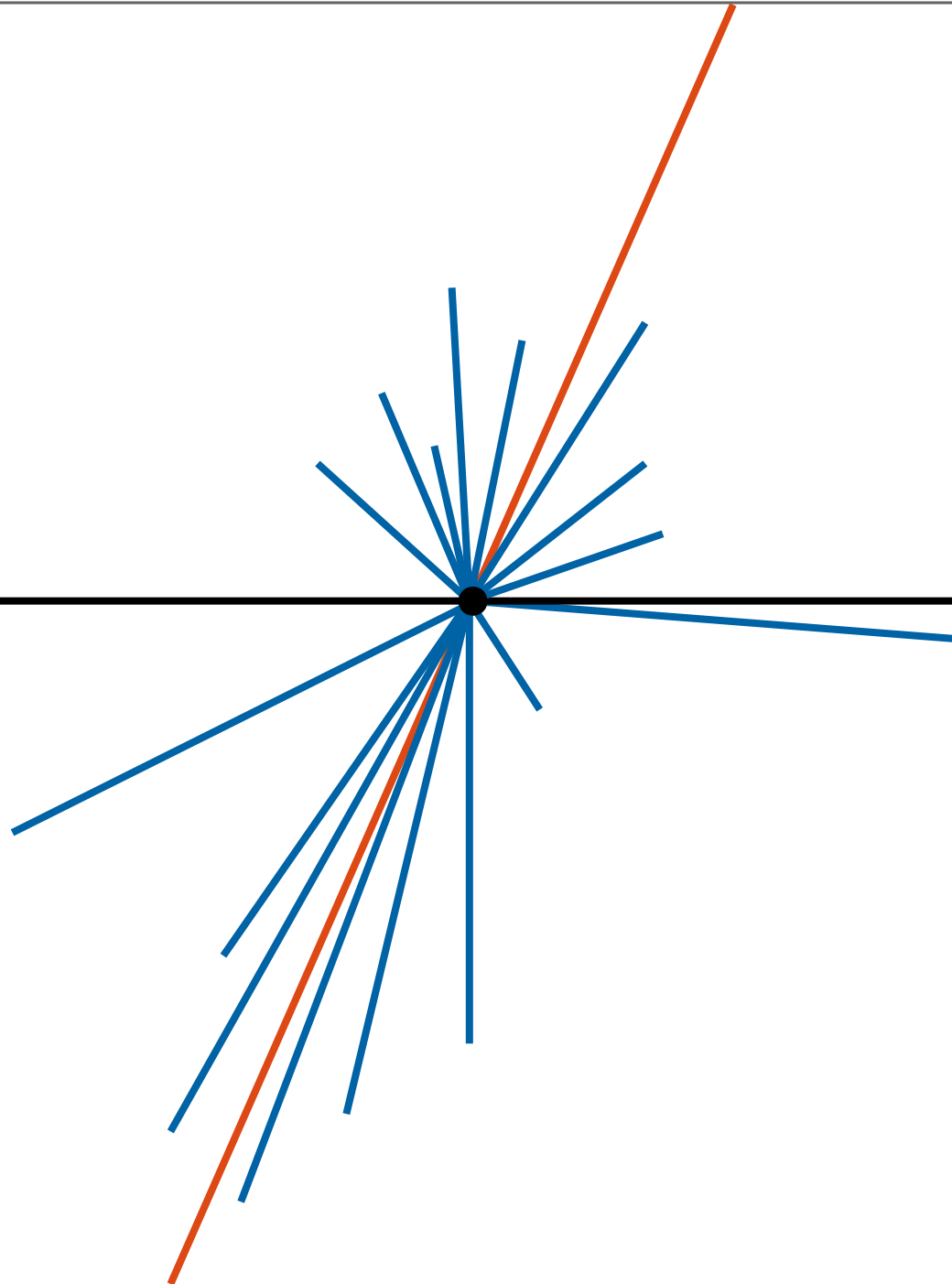
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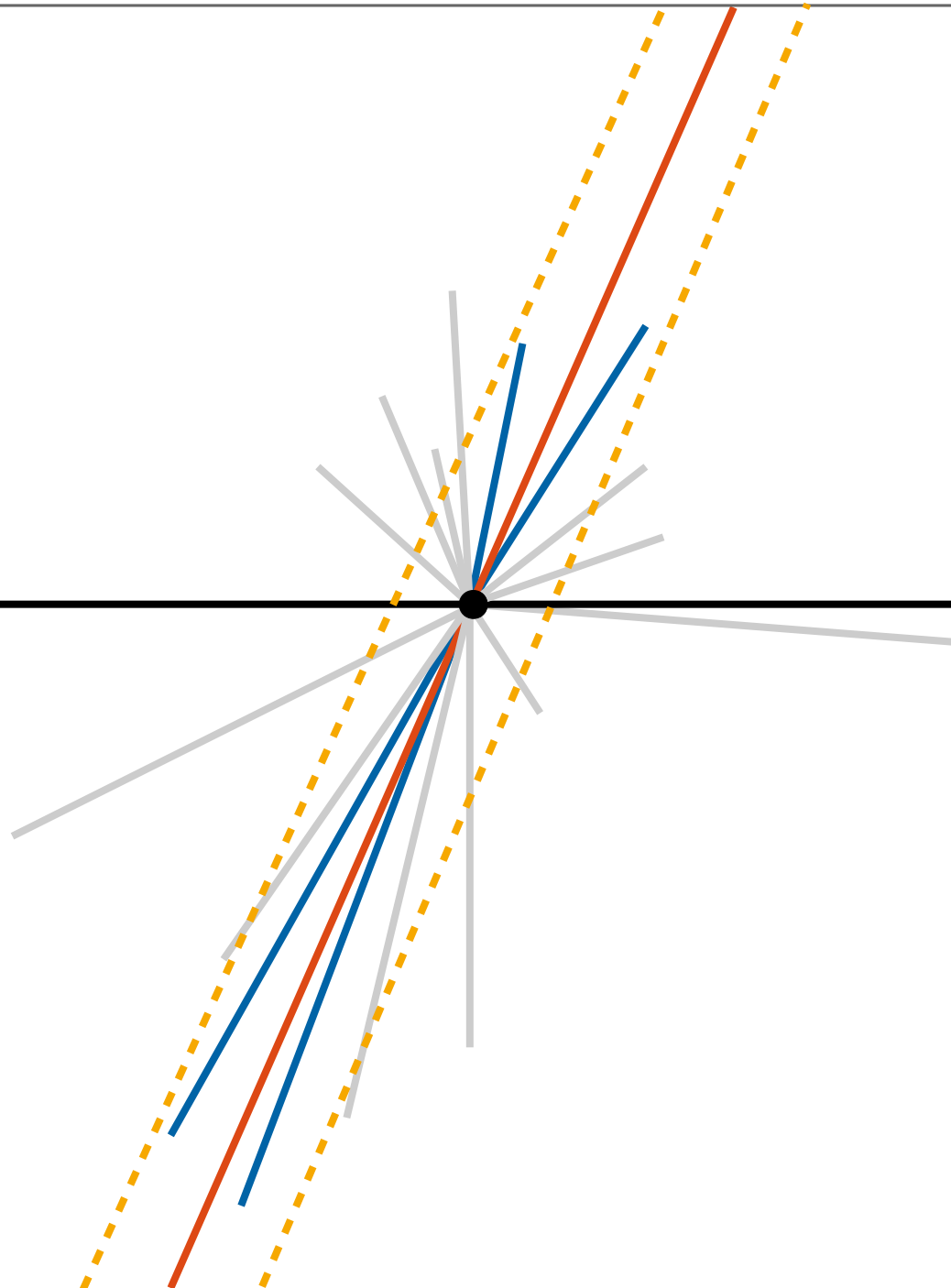
XXVII Workshop on DIS and Related Subjects

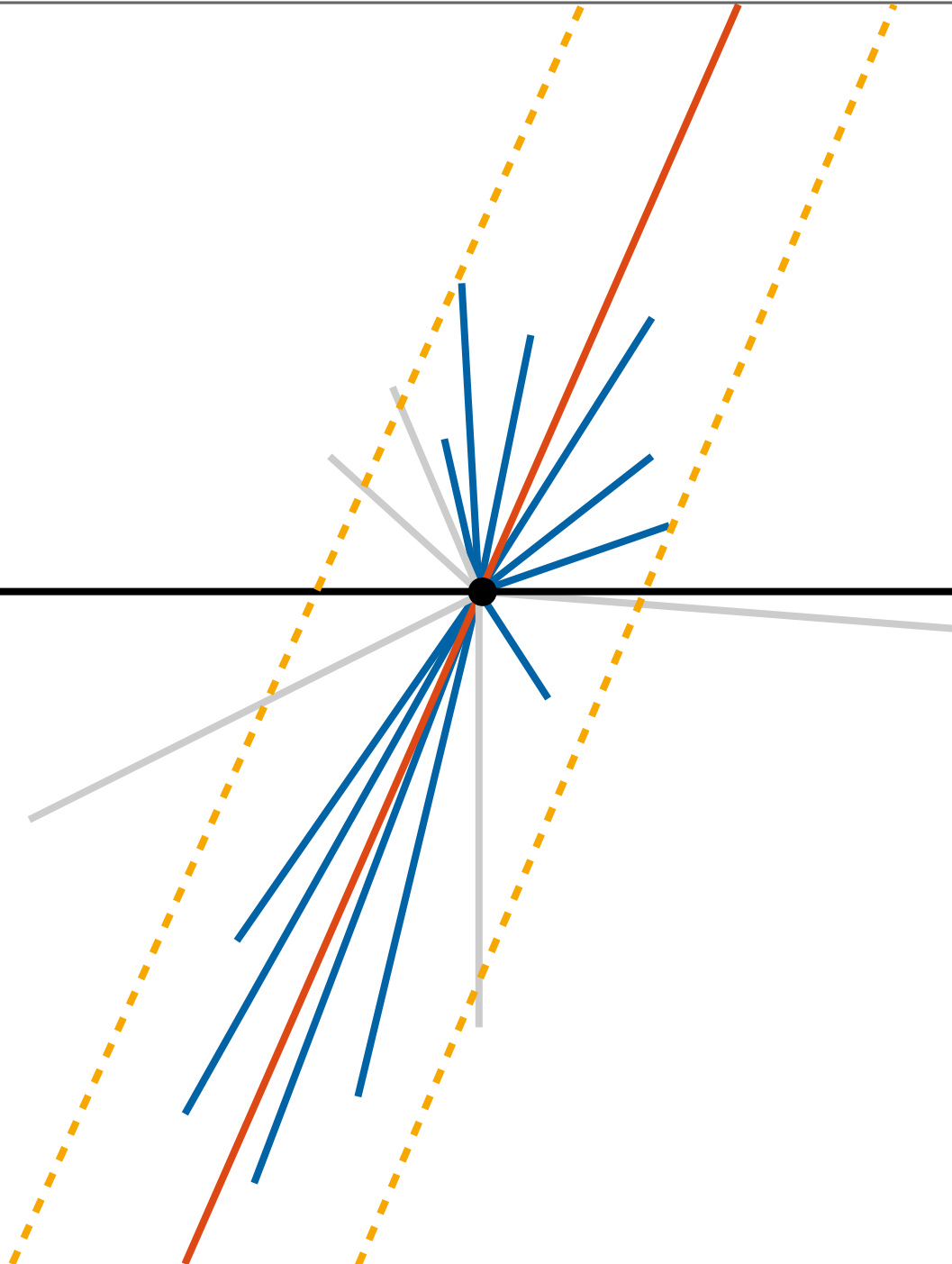
Torino | 8 April 2019







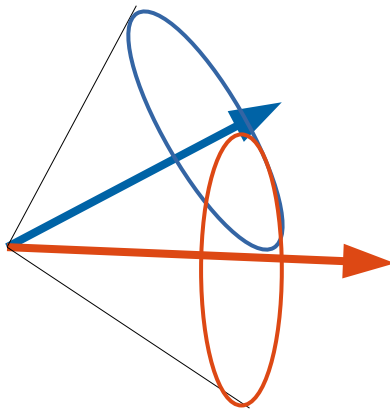




Coherent branching algorithms essential to direct QCD resummation of global event shapes, and to designing parton shower algorithms.

[Catani, Marchesini, Webber]

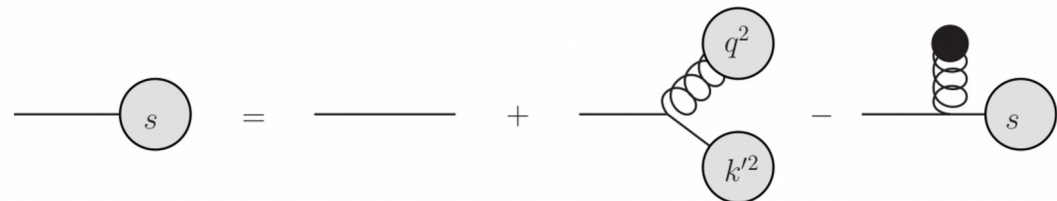
[Gieseke, Stephens, Webber]

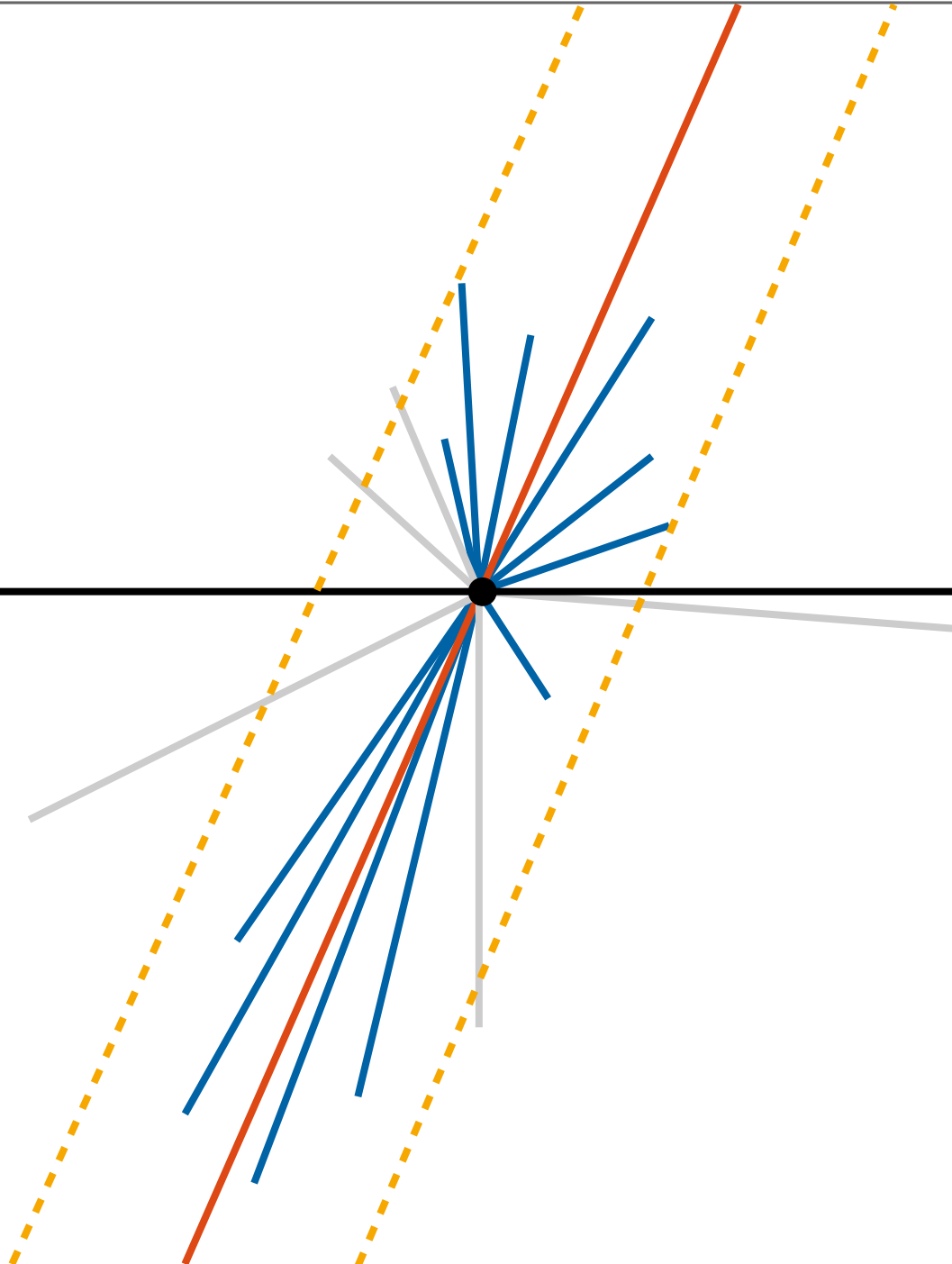


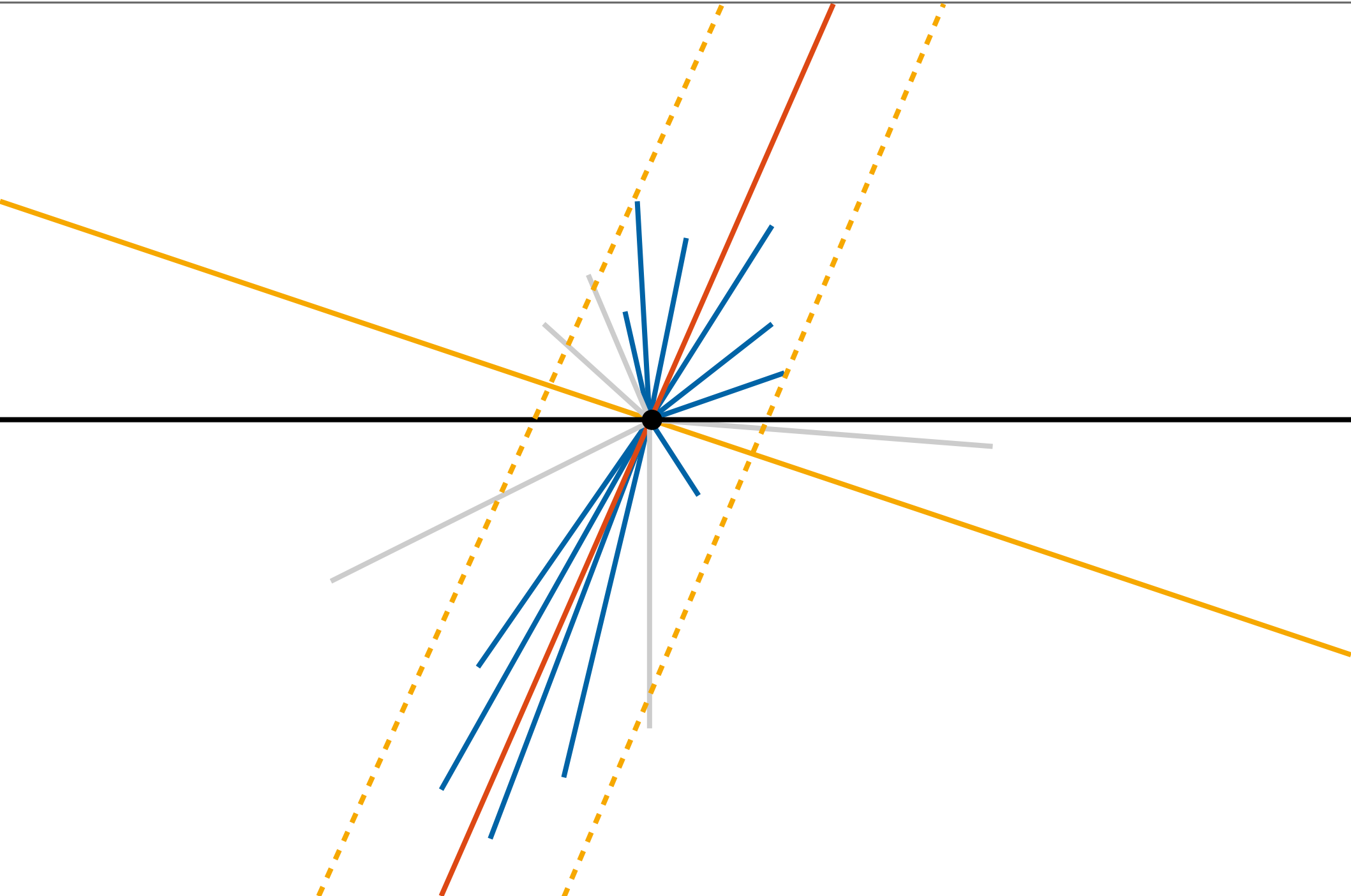
$$\begin{aligned}
 & \text{Diagram with } Q^2 \text{ and } q_1 \rightarrow \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\
 & \xrightarrow{q_1^2 \ll Q^2} \text{Diagram 4} + \mathcal{O}(q_1^2/Q^2) \\
 & \quad \quad \quad \swarrow \quad \searrow \\
 & \quad \quad \text{gauge invariant} \quad \text{decomposition}
 \end{aligned}$$

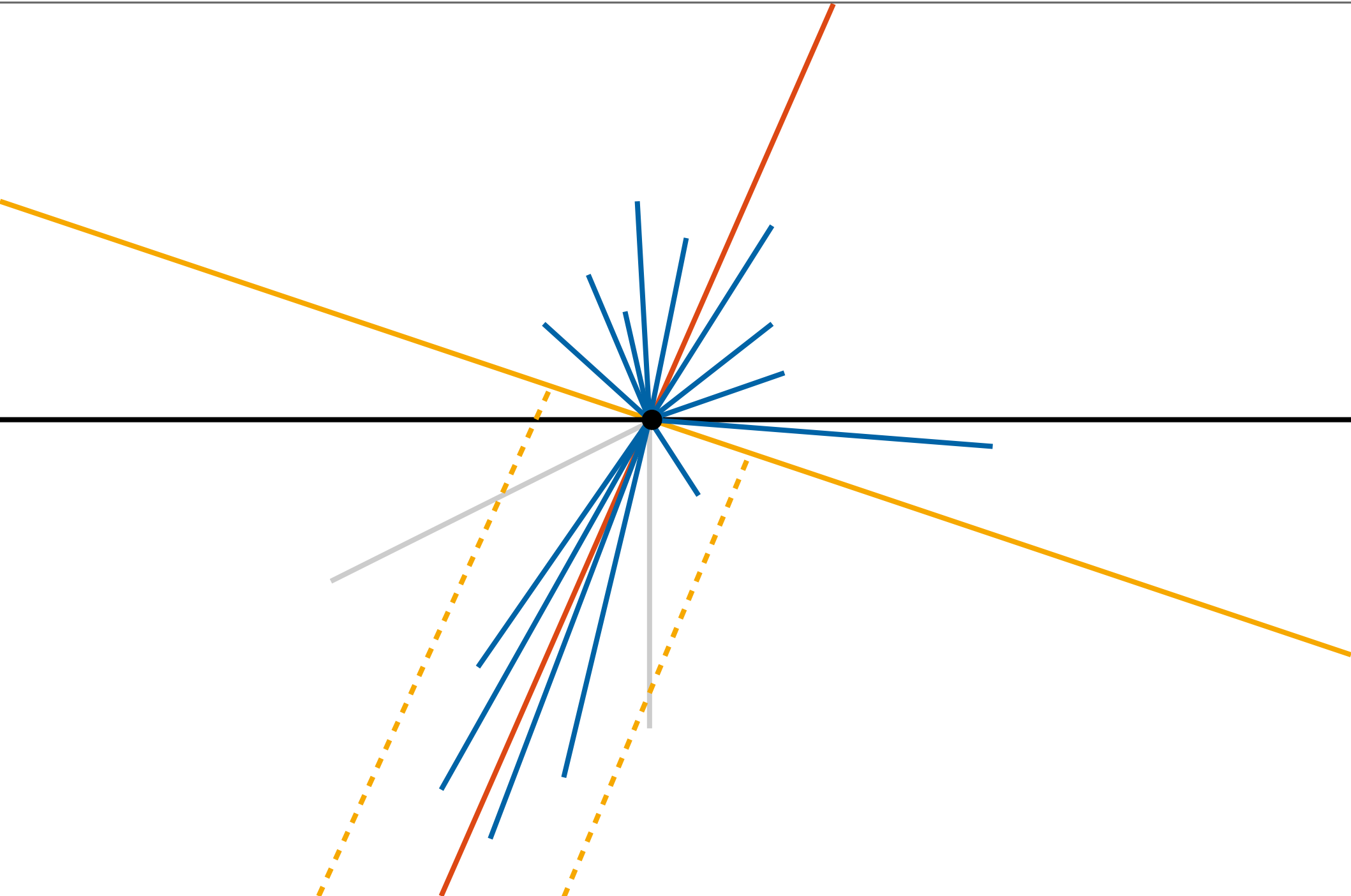
Large-angle soft effects included on average by a clever choice of ordering variable. Multiple emissions collapse to iterating splitting functions: DGLAP-type evolution.

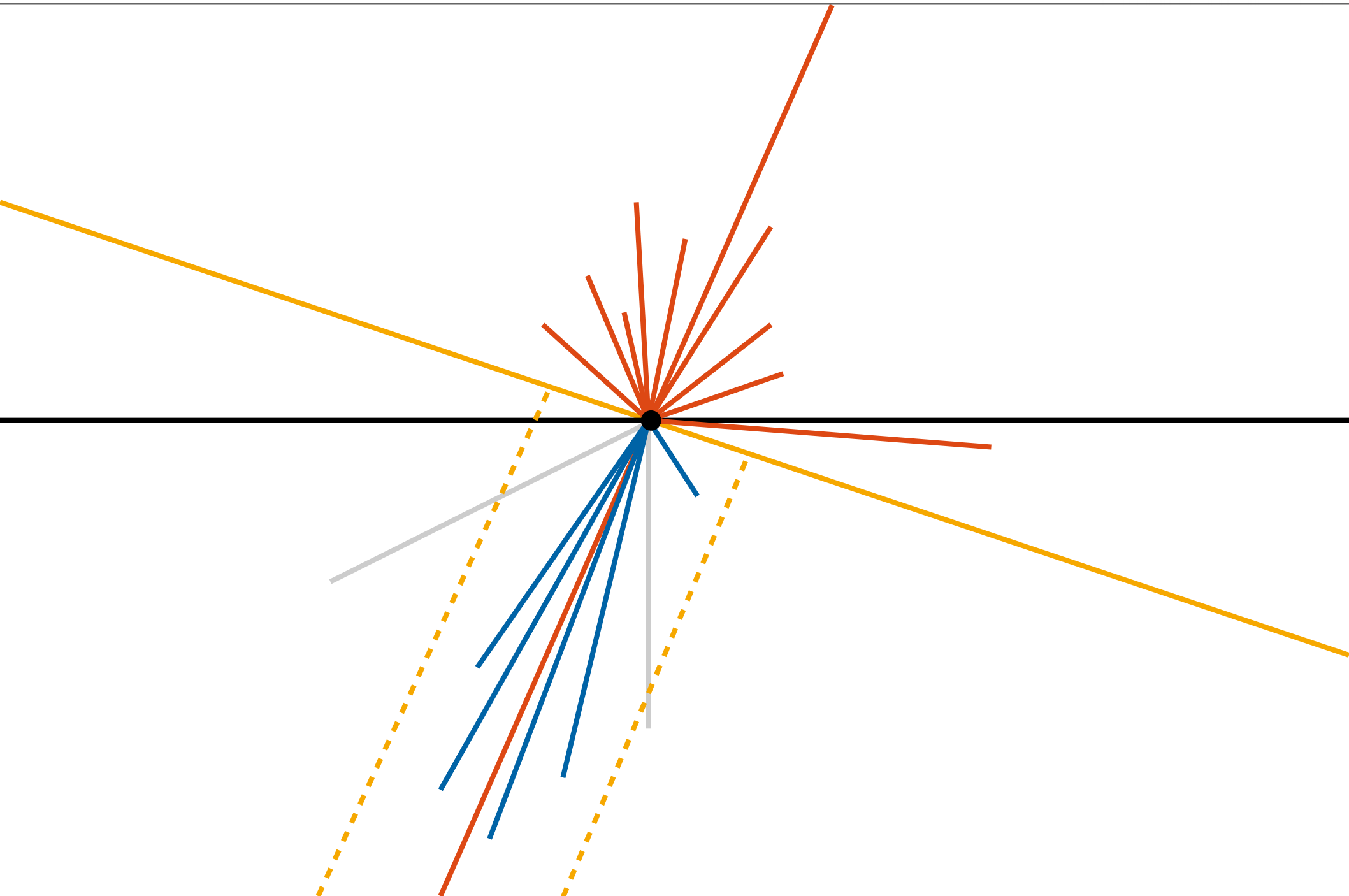
$$\begin{aligned}
 J(s, Q^2) = & \delta(s) + \int_0^{Q^2} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_0^1 dz P_{qq} \left[\alpha_s(z(1-z)\tilde{q}), z \right] \\
 & \times \left[\int_0^\infty dk'^2 \int_0^\infty dq^2 \delta \left(s - \frac{k'^2}{z} - \frac{q^2}{1-z} - z(1-z)\tilde{q}^2 \right) J(k'^2, z^2\tilde{q}^2) J_g(q^2, (1-z)^2\tilde{q}^2) \right. \\
 & \quad \left. - J(s, \tilde{q}^2) \right]
 \end{aligned}$$

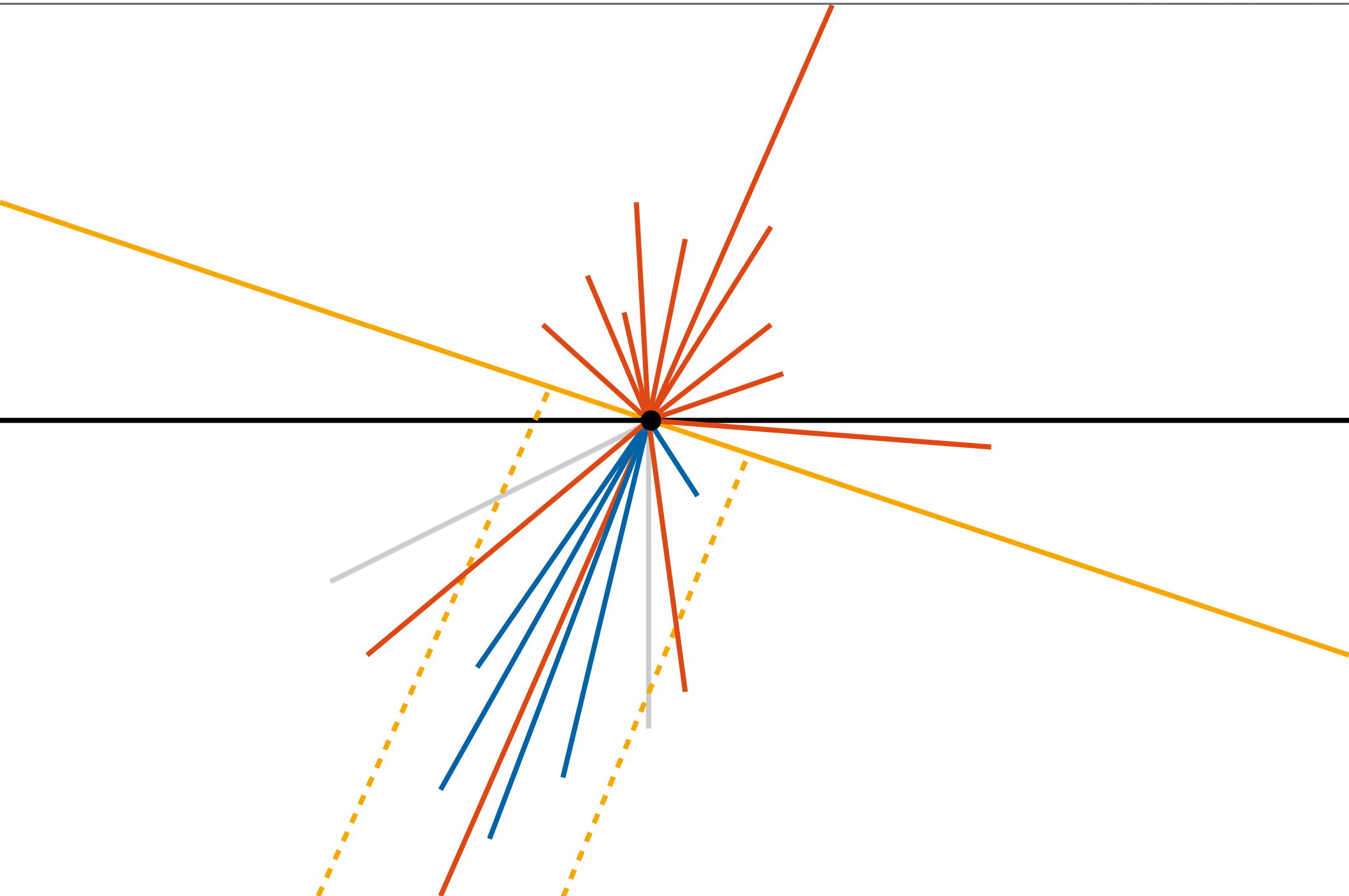












QCD cross sections factorize into hard cross section and emission probability, for **soft (low energy)** and **collinear** parton emission.

For soft emissions **colour correlations** persist, for collinear emissions spin correlations are present.

$$|\mathcal{M}_{n+1}|^2 \sim \sum_{i \neq j} S_{ij} \langle \mathcal{M}_n | \mathbf{T}_i \cdot \mathbf{T}_j | \mathcal{M}_n \rangle + \sum_i C_i \mathbf{T}_i^2 |\mathcal{M}_n|^2$$
$$\sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j = -\mathbf{T}_i^2$$

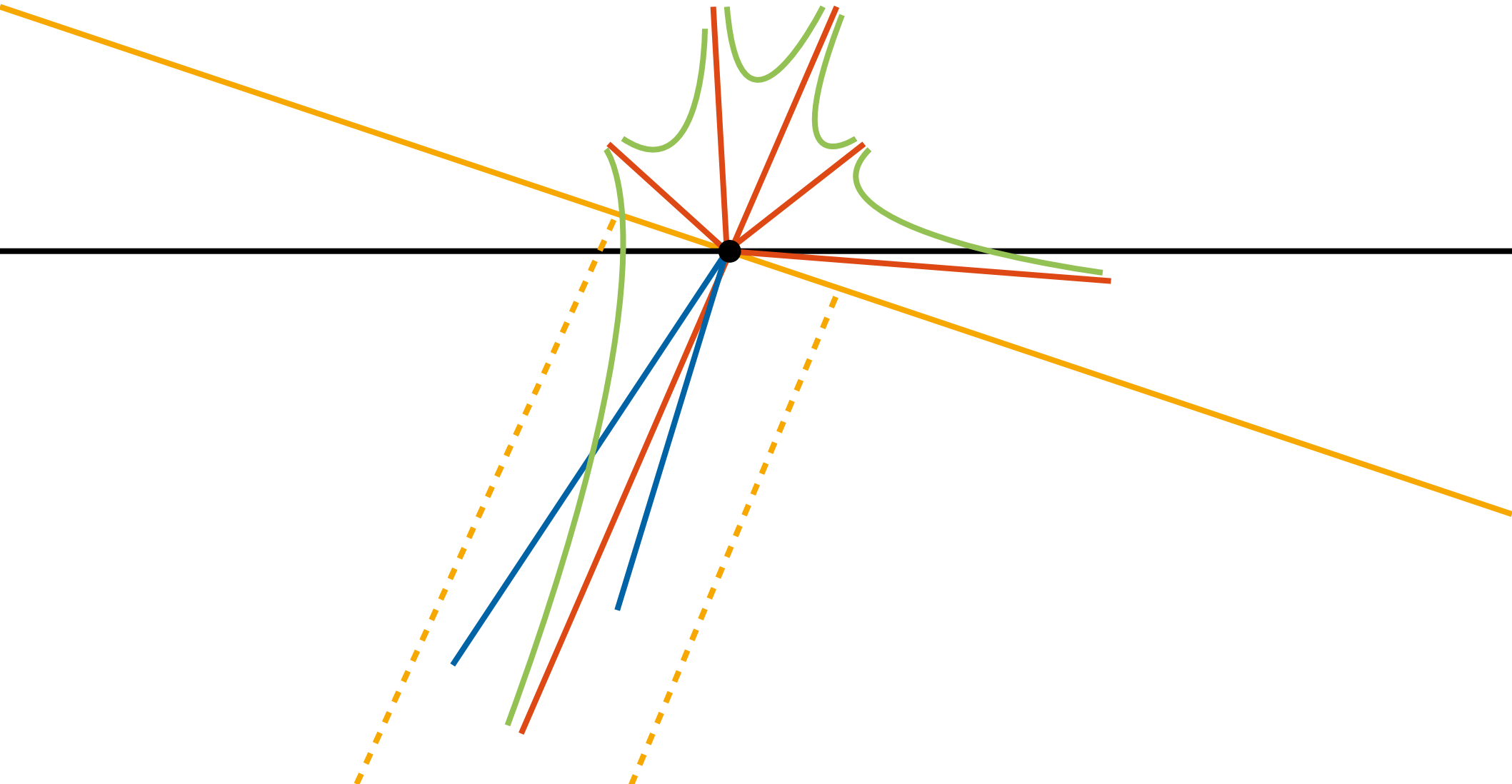
Outside the hemisphere, we have a **coherently radiating sum of colour dipoles**. Any number of emissions in the `out` region can contribute.

$$|\mathcal{M}_n(\mu)\rangle = \mathbf{Z}^{-1}(\mu, \epsilon) |\tilde{\mathcal{M}}_n(\epsilon)\rangle$$

Virtual corrections mix colour in a similar way, factorization at amplitude level.

Resummation of non-global observables is
with dipole cascades in the large- N limit.

[Dasgupta, Salam – Phys.Lett. B512 (2001) 323]
[Balsinger, Becher, Shao – arXiv:1803.07045]

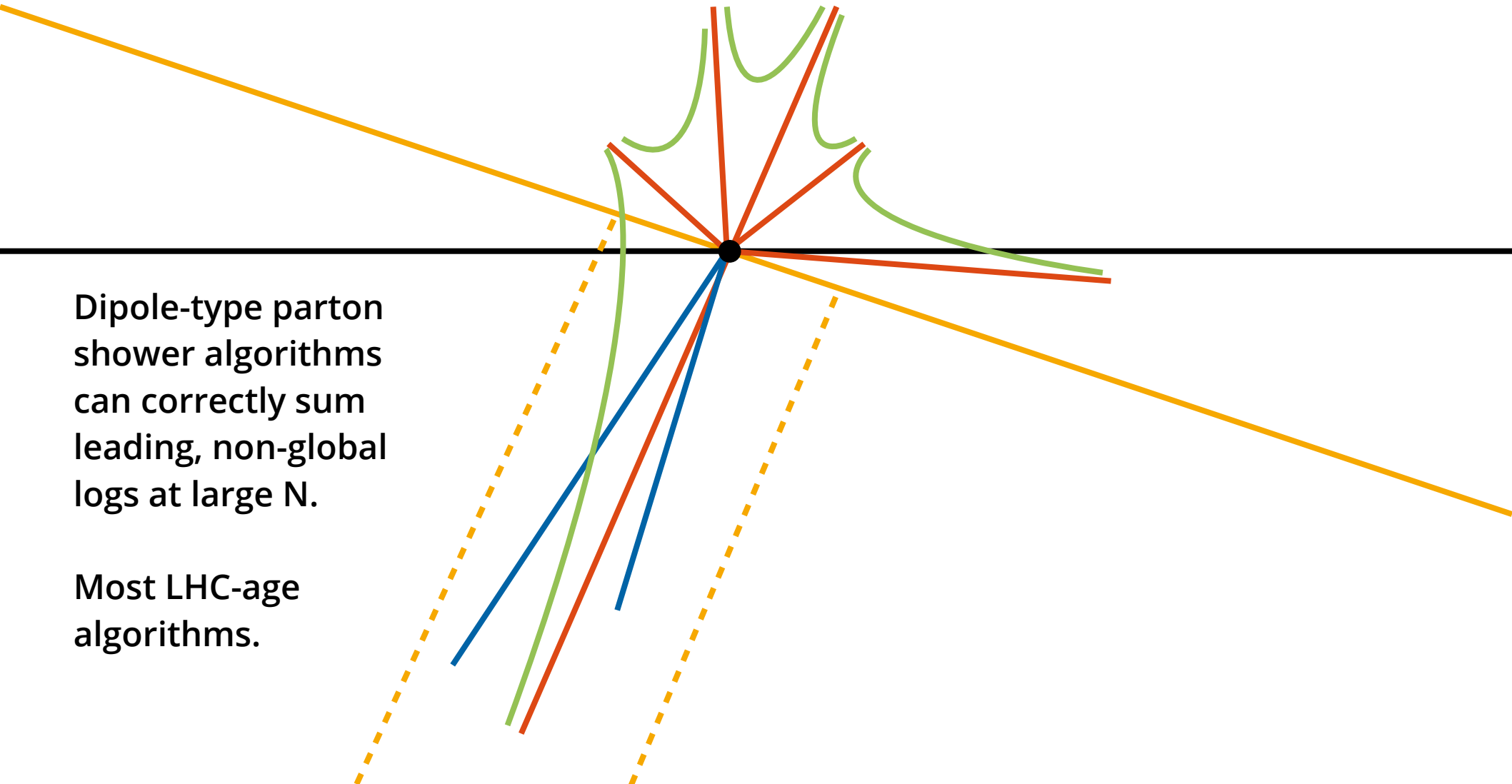


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Dipole-type parton
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Most LHC-age
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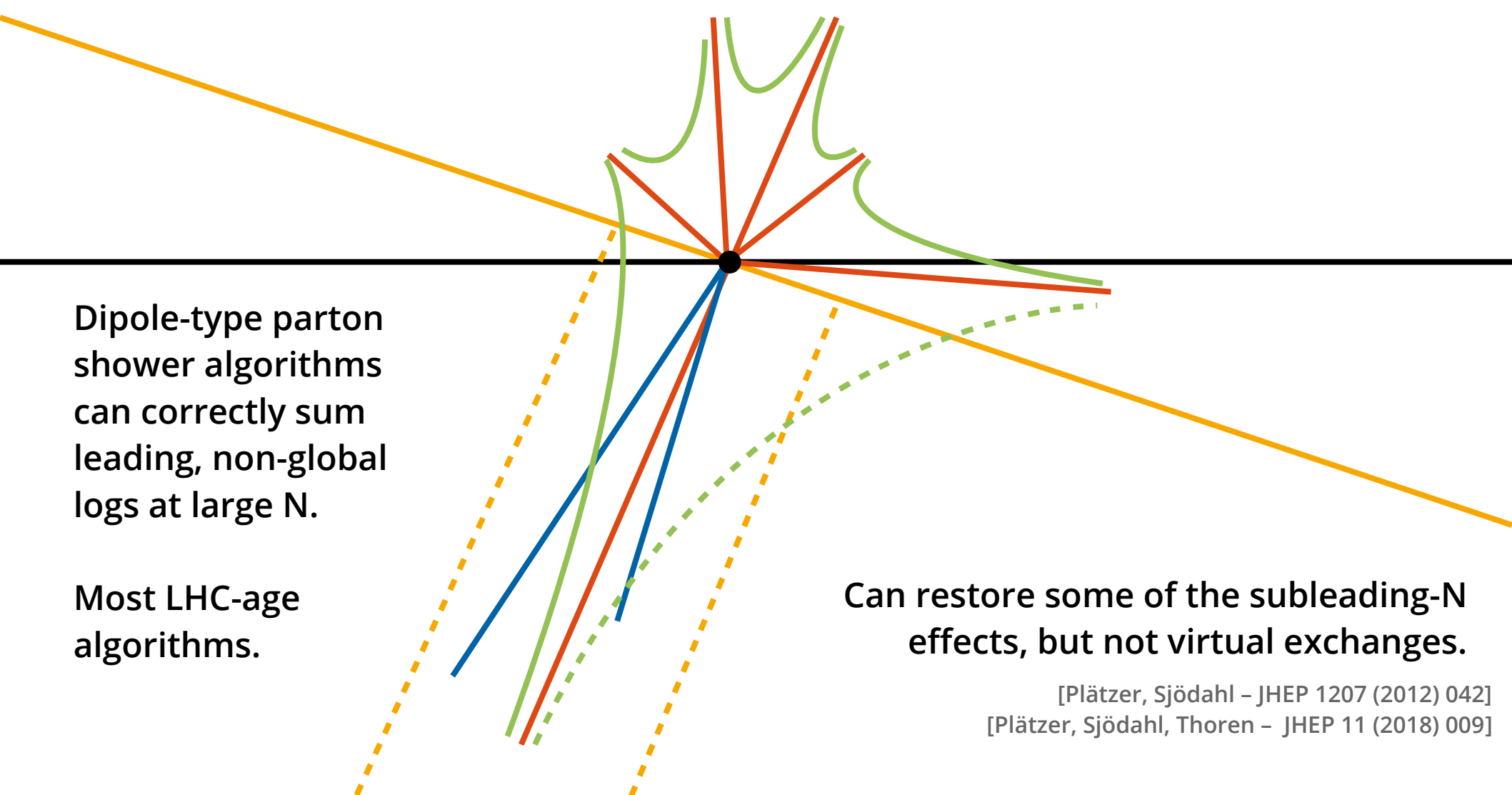
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Most LHC-age
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Can restore some of the subleading- N
effects, but not virtual exchanges.

[Plätzer, Sjödal – JHEP 1207 (2012) 042]
[Plätzer, Sjödal, Thoren – JHEP 11 (2018) 009]



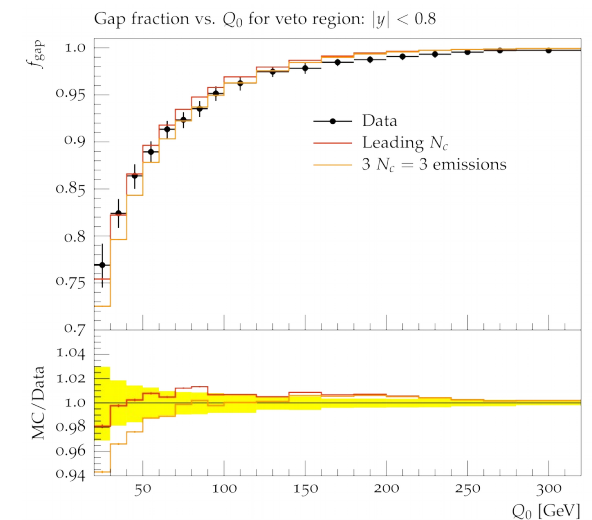
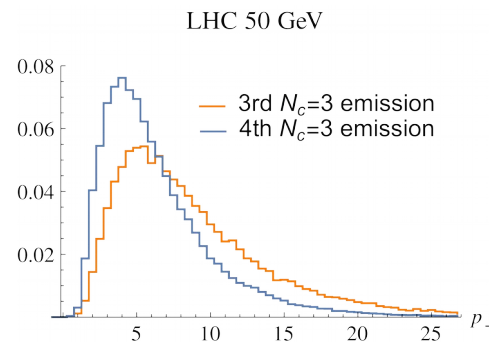
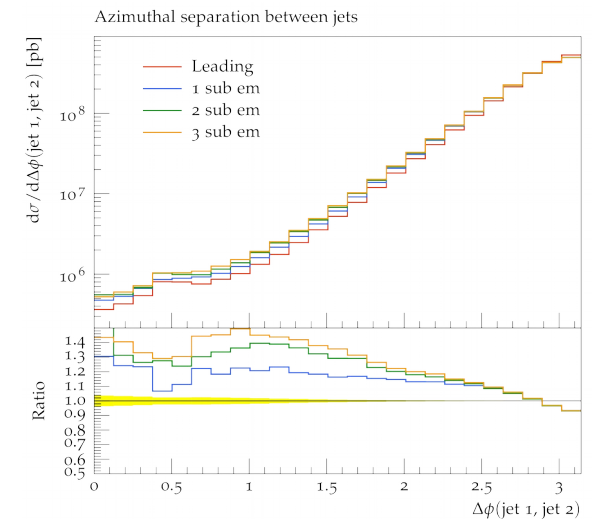
Take into account subleading-N corrections to the radiation pattern, maintain shower unitarity:

Best option we can have within existing probabilistic algorithms, see last talk for approaches beyond.

$$V_{ij,k} \rightarrow - \frac{\langle \mathcal{M} | \mathbf{T}_{ij} \cdot \mathbf{T}_k | \mathcal{M} \rangle}{\mathbf{T}_{ij}^2 |\mathcal{M}|^2} V_{ij,k}$$

Works generic for every process, including hadron colliders and top quarks, weighted Sudakov algorithm technology crucial.

Effects not severe, but need retuning to fully judge.
Probes the relevant soft scales despite limited number of emissions available.



[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Seek a framework to systematically address and improve **a new kind of parton shower algorithm**, not relying on ad-hoc constructions, treating colour exactly as far as possible.

[also see Nagy, Soper]

Non-global observables are a unique playground: At large- N they provide a clean way of deriving a dipole-type parton shower, but the origin of the method used is much more general.

$$\sigma = \sum_n \int \text{Tr} [\mathbf{A}_n(\mu)] u(p_1, \dots, p_n) d\phi_n$$

$$\mathbf{A}_n(\mu) = |\mathcal{M}_n(\mu)\rangle\langle\mathcal{M}_n(\mu)|$$

Evolved **density operator**

Observable

Phase space

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Successive virtual evolution/emission combinations down to an infrared cutoff, which will need to be removed at the end. Observable value itself can act as a cutoff, if fully inclusive for gluon energies below this scale.

$$\mathbf{A}_n(E) = \mathbf{V}(E, E_n) \mathbf{D}_n \mathbf{A}_{n-1}(E_n) \mathbf{D}_n^\dagger \mathbf{V}^\dagger(E, E_n) \theta(E - E_n)$$

$$\mathbf{V}_n(E, Q) = \mathbf{P} \exp \left(- \int_E^Q \frac{dq}{q} \mathbf{\Gamma}_n(q) \right) \quad \mathbf{D}_n = \sum_{i=1}^{n-1} \frac{p_i \cdot \epsilon^*(p_n)}{p_i \cdot p_n} \mathbf{T}_i$$

Non-probabilistic evolution at the amplitude level, keeping full colour structure, virtual corrections encoded in anomalous dimension

$$\mathbf{\Gamma}_n = \frac{\alpha_s}{\pi} \sum_{i < j} \int d\Omega \frac{n_i \cdot n_j}{n_i \cdot n \ n \cdot n_j} (-\mathbf{T}_i \cdot \mathbf{T}_j)$$

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Evolution equations for factorizing observables $u(p_1, \dots, p_n) = u(E_1, \hat{p}_1) \cdots u(E_n, \hat{p}_n)$
integrating over the intermediate scales:

$$E \frac{\partial \mathbf{G}_n(E)}{\partial E} = -\mathbf{\Gamma} \mathbf{G}_n(E) - \mathbf{G}_n(E) \mathbf{\Gamma}^\dagger + \mathbf{D}_n \mathbf{G}_{n-1}(E) \mathbf{D}_n^\dagger u(E, \hat{p}_n)$$

Recursive observables: $u_n(p_1, \dots, p_n) = u(p_n, \{p_1, \dots, p_{n-1}\}) u_{n-1}(p_1, \dots, p_{n-1})$

Genuine non-global case: $u(k, \{q\}) = \Theta_{\text{out}}(k) + \Theta_{\text{in}}(k) u_{\text{in}}(k, \{q\})$

Can identify global and non-global contributions simply by splitting anomalous dimension into ‘out’ and ‘in’ contributions.

Reproduce all available literature.

[Dasgupta, Salam – Phys.Lett. B512 (2001) 323]

[Forshaw, Kyrieleis, Seymour – JHEP 0608 (2006) 059]

[Weigert – Nucl.Phys. B685 (2004) 321]

[Caron-Huot – JHEP 1803 (2018) 036]

[Becher, Neubert, Rothen, Shao – JHEP 1611 (2016) 019]

Includes a **re-derivation of the BMS equation** at leading-N, being able to calculate subleading-N corrections to it.

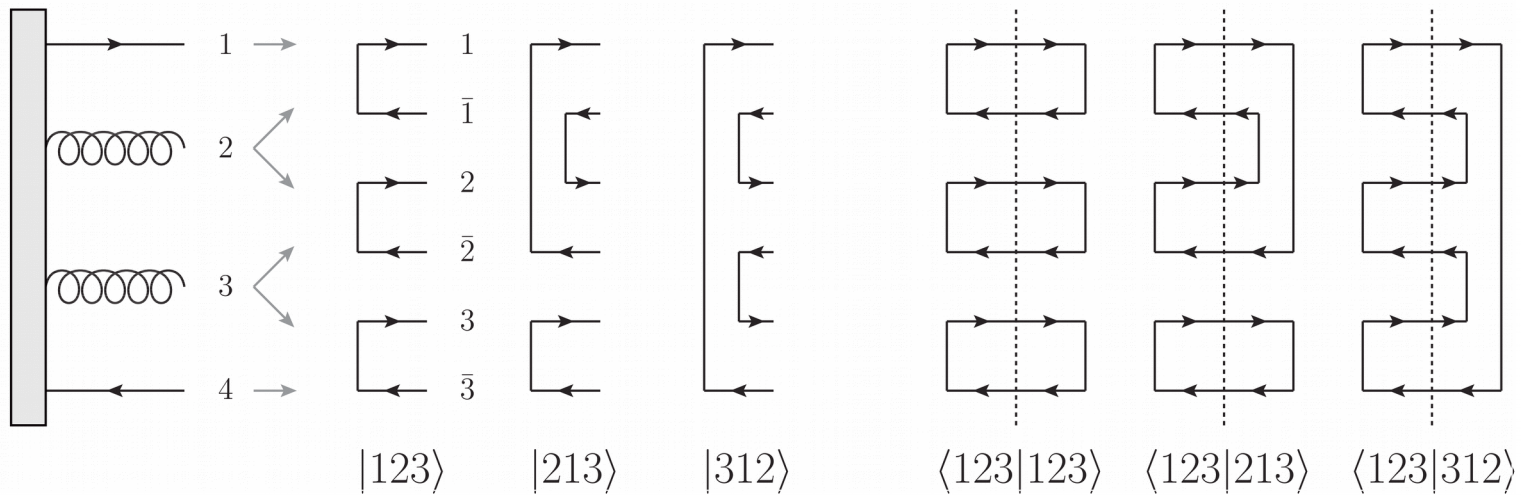
[Banfi, Marchesini, Smye – JHEP 0208 (2002) 006]

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Express amplitudes in combinations of fundamental/anti-fundamental indices:

$$|\mathcal{M}\rangle = \sum_{\sigma} \mathcal{M}_{\sigma} |\sigma\rangle$$

$$|\sigma\rangle = \left| \begin{array}{ccc} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{array} \right\rangle = \delta_{\bar{\alpha}_{\sigma(1)}^{\alpha_1}} \cdots \delta_{\bar{\alpha}_{\sigma(n)}^{\alpha_n}}$$



Non-orthogonal basis:

$$1 = \sum_{\sigma} |\sigma\rangle [\sigma| \quad [\sigma|\tau\rangle = \langle\sigma|\tau\rangle = \delta_{\tau\sigma} \quad \text{Tr}[\mathbf{A}] = \sum_{\tau, \sigma} [\tau|\mathbf{A}|\sigma] \langle\sigma|\tau\rangle$$

Also overcomplete ... but computationally very handy: It's all about permutations.

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Evolution operator in colour flow basis:

$$\begin{aligned}
 [\tau|e^{\Gamma}|\sigma\rangle &= \sum_{k=0}^{\infty} \frac{(-1)^k}{N^k} \sum_{l=0}^{\infty} \frac{(-\rho)^l}{l!} \sum_{\sigma_0, \dots, \sigma_{l-k}} \delta_{\tau\sigma_0} \delta_{\sigma_{l-k}\sigma} \prod_{\alpha=0}^{l-k-1} \Sigma_{\sigma_{\alpha}\sigma_{\alpha+1}} R(\{\sigma_0, \dots, \sigma_{l-k}\}) \\
 &= \delta_{\tau\sigma} \left(e^{-N\Gamma_{\sigma}} + e^{-N\Gamma_{\sigma}} \frac{\rho}{N} \right) - \frac{1}{N} \frac{e^{-N\Gamma_{\tau}} - e^{-N\Gamma_{\sigma}}}{\Gamma_{\tau} - \Gamma_{\sigma}} \Sigma_{\tau\sigma} + \text{NNLC}
 \end{aligned}$$

[Plätzer – EPJ C 74 (2014) 2907]

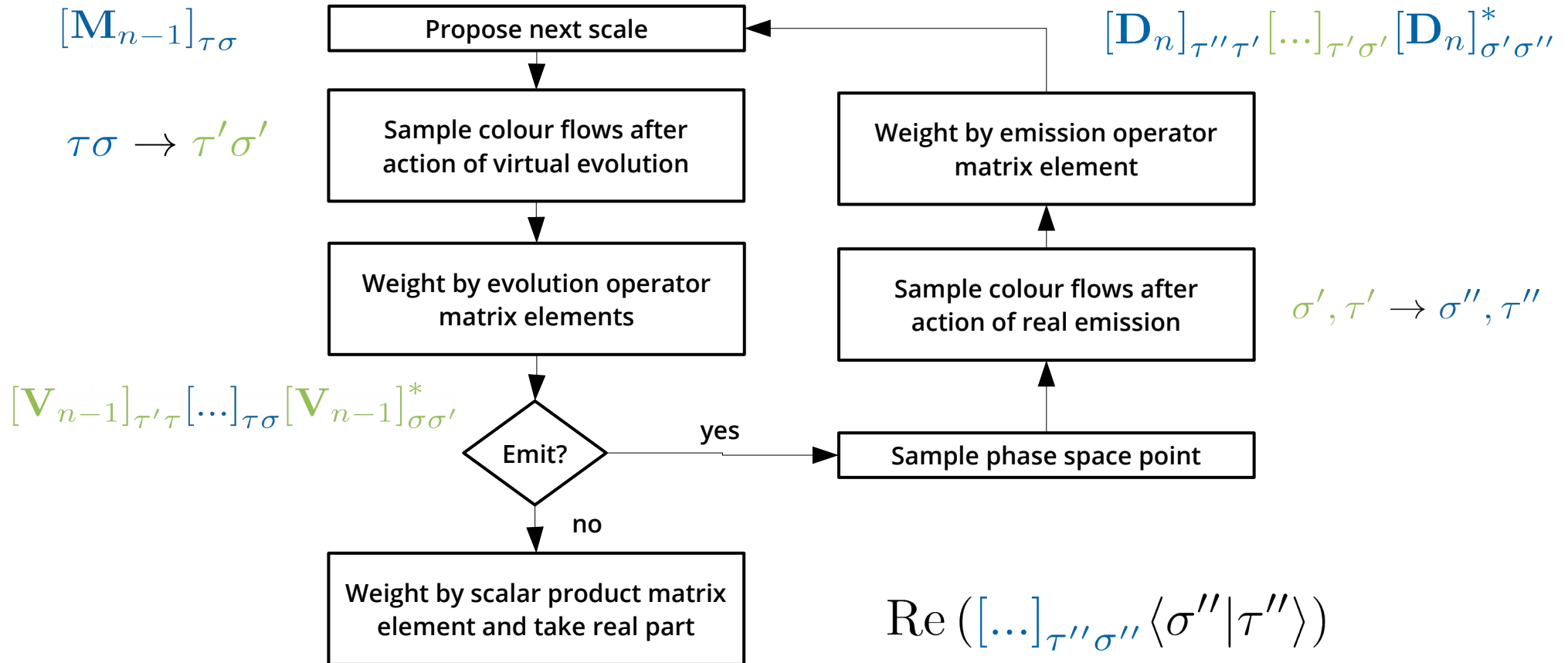
Sum terms enhanced by $\alpha_s N$ to all orders, insert perturbations in $1/N$.

Take into account real emission contributions and the final suppression by the scalar product matrix element.

				virtuals	reals
N^3			Γ^3	$(0 \text{ flips}) \times 1 \times (\alpha_s N)^n$	$(t[\dots]t _{0 \text{ flips}})^{r-1} t[\dots]t _{2 \text{ flips}} \times 1$ $(t[\dots]t _{0 \text{ flips}})^{r-1} t[\dots]s _{1 \text{ flip}} \times N^{-1}$ $(t[\dots]t _{0 \text{ flips}})^{r-1} s[\dots]s _{0 \text{ flips}} \times N^{-2}$
N^2		Γ^2	$\Sigma\Gamma^2$	$(1 \text{ flip}) \times \alpha_s \times (\alpha_s N)^n$	$(t[\dots]t _{0 \text{ flips}})^r$ $(t[\dots]t _{0 \text{ flips}})^{r-1} t[\dots]s _{1 \text{ flip}} \times N^{-1}$
N^1	Γ	$\Sigma\Gamma$	$\rho\Gamma^2$	$(0 \text{ flips}) \times \alpha_s N^{-1} \times (\alpha_s N)^n$	$(t[\dots]t _{0 \text{ flips}})^r$
N^0	1	Σ	$\rho\Sigma\Gamma$	$(0 \text{ flips}) \times \alpha_s^2 \times (\alpha_s N)^n$ $(2 \text{ flips}) \times \alpha_s^3 \times (\alpha_s N)^n$	$(t[\dots]t _{0 \text{ flips}})^r$ $(t[\dots]t _{0 \text{ flips}})^{r-1} t[\dots]t _{2 \text{ flips}}$
N^{-1}		$\rho 1$	$\rho\Sigma$		
N^{-2}			$\rho^2 1$		
N^{-3}					
	α_s^0	α_s^1	α_s^2	α_s^3	

[De Angelis, Forshaw, Plätzer – arXiv:1905.xxxxx, first presented at PSR '18 and others]

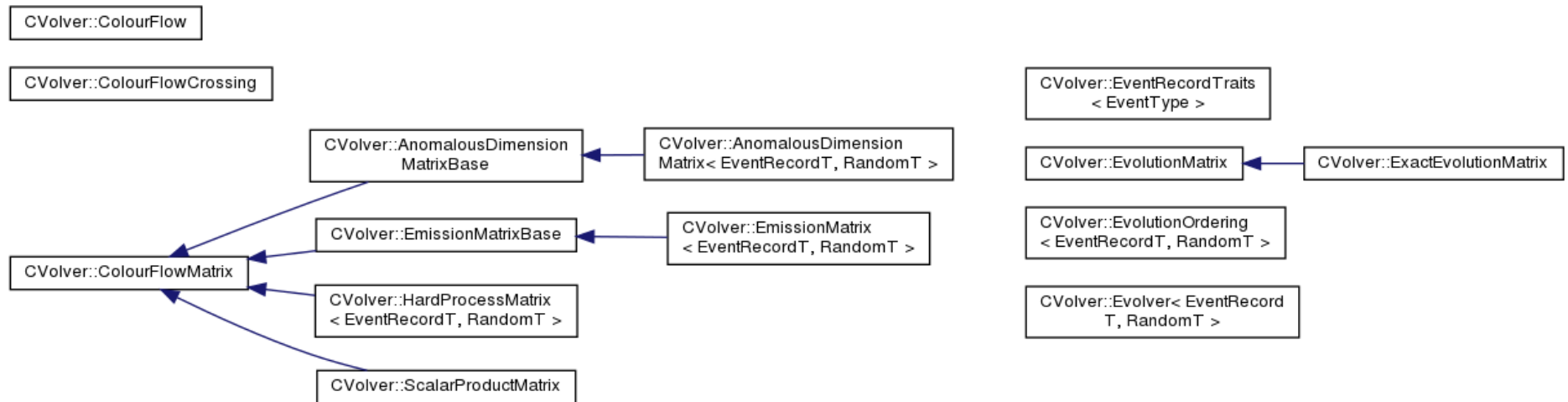
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[De Angelis, Forshaw, Plätzer – arXiv:1905.xxxxx, first presented at PSR '18 and others]

A framework to solve multi-differential evolution equations in colour space.
Concise, simple, and light-weight code structure.

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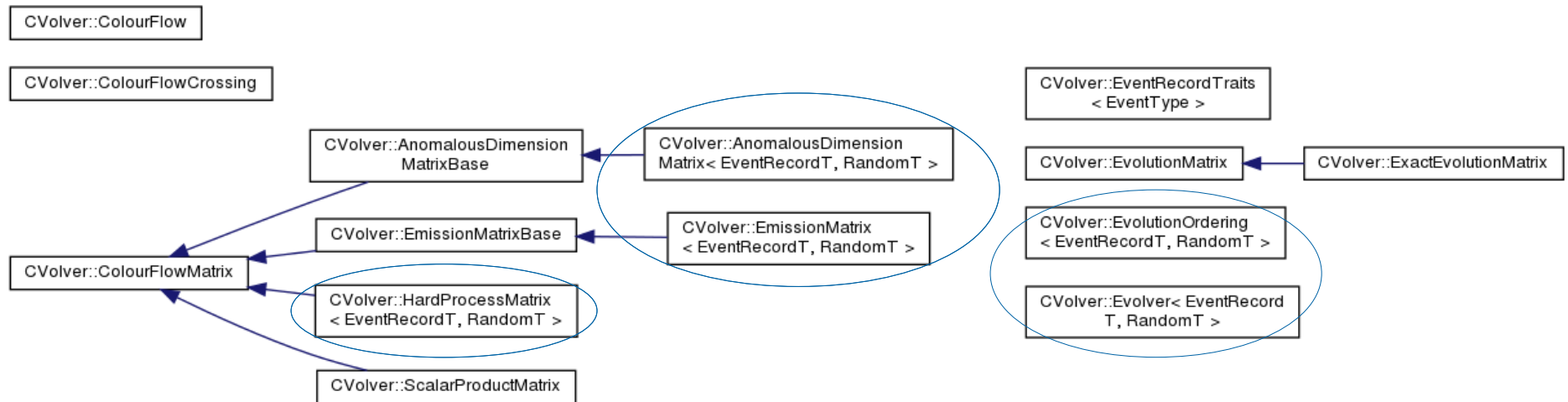
Dedicated Monte Carlo algorithms to sample colour structures.

Plugin approach can accommodate anything from (N)GLs to full parton showers.

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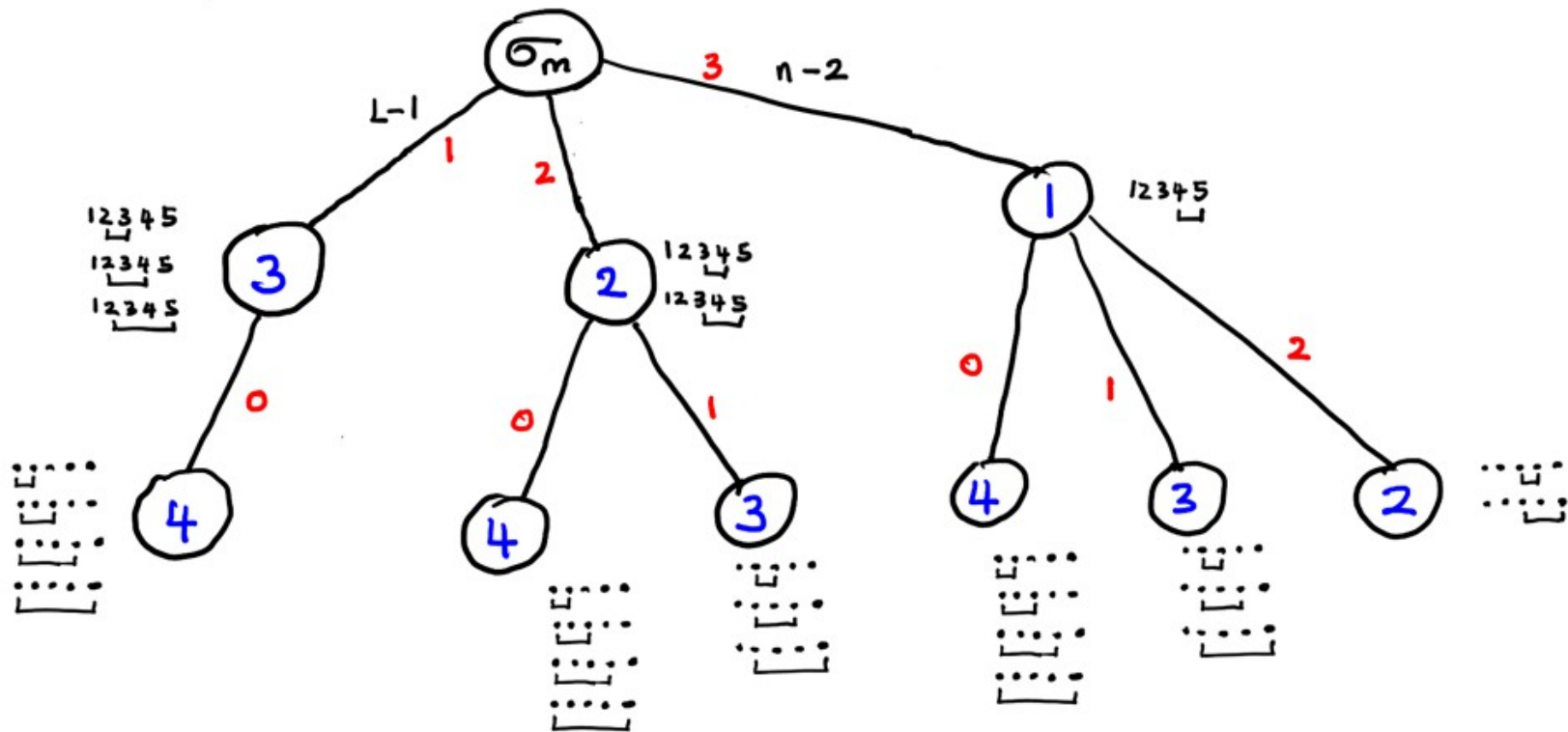
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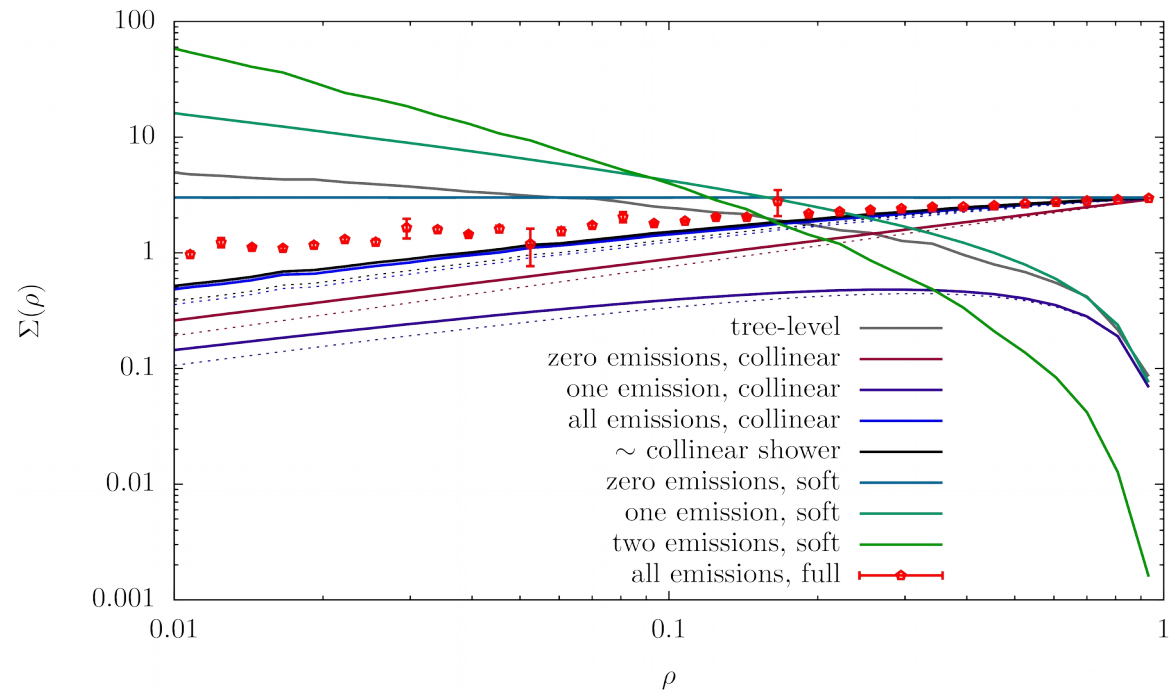
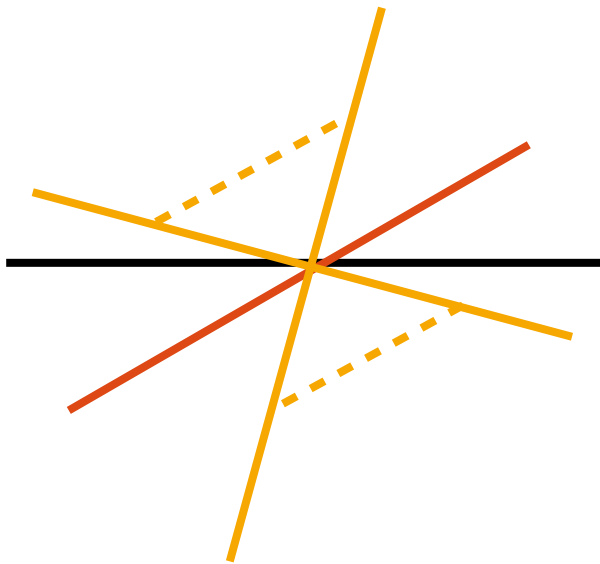
Importance sampling in colour space rules: $\#(\tau, \tau') \sim 1/N^{\#(\tau, \tau')}$

Enumerate and address permutations with fixed cycle length:



[De Angelis, Forshaw, Plätzer – arXiv:1905.xxxxx, first presented at PSR '18 and others]

Code is differential for a large class of (non-global) observables.
Example: Cone-dijet veto cross section.

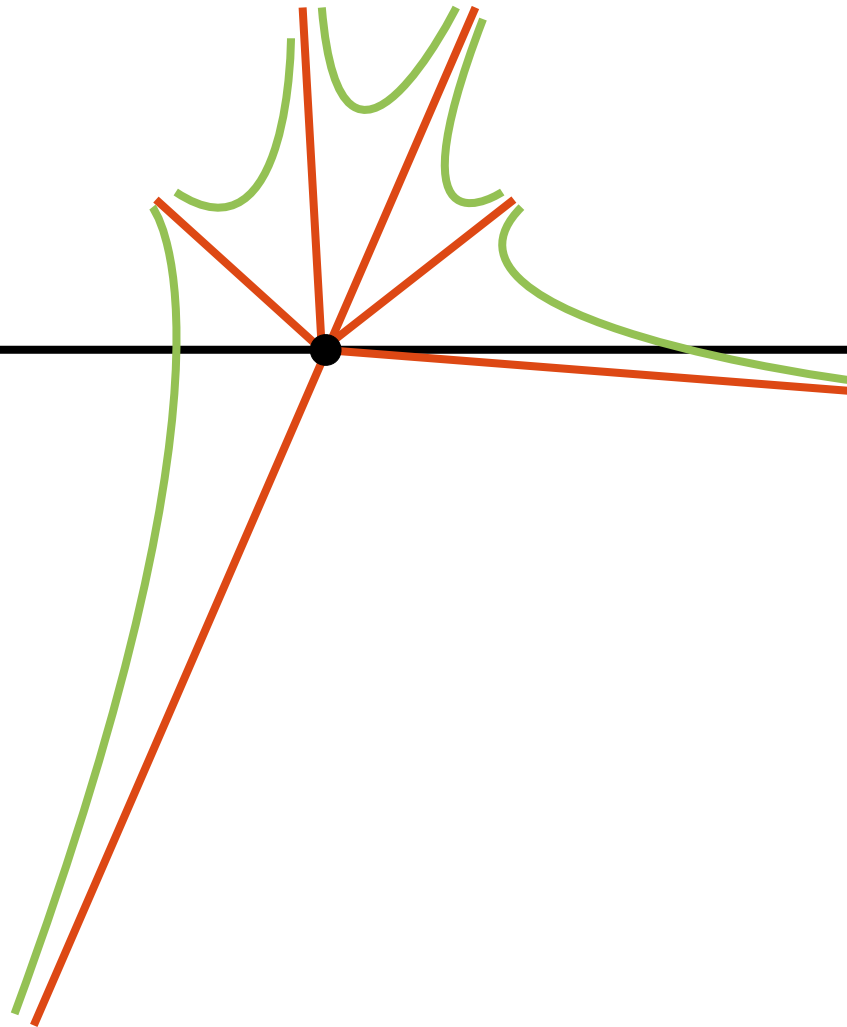


1/N breakdowns possible, scales up to several 10s of emissions for $d=2$.

The cluster model is based on a single colour flow after the shower has stopped.

Essentially no evolution is considered, just decays.

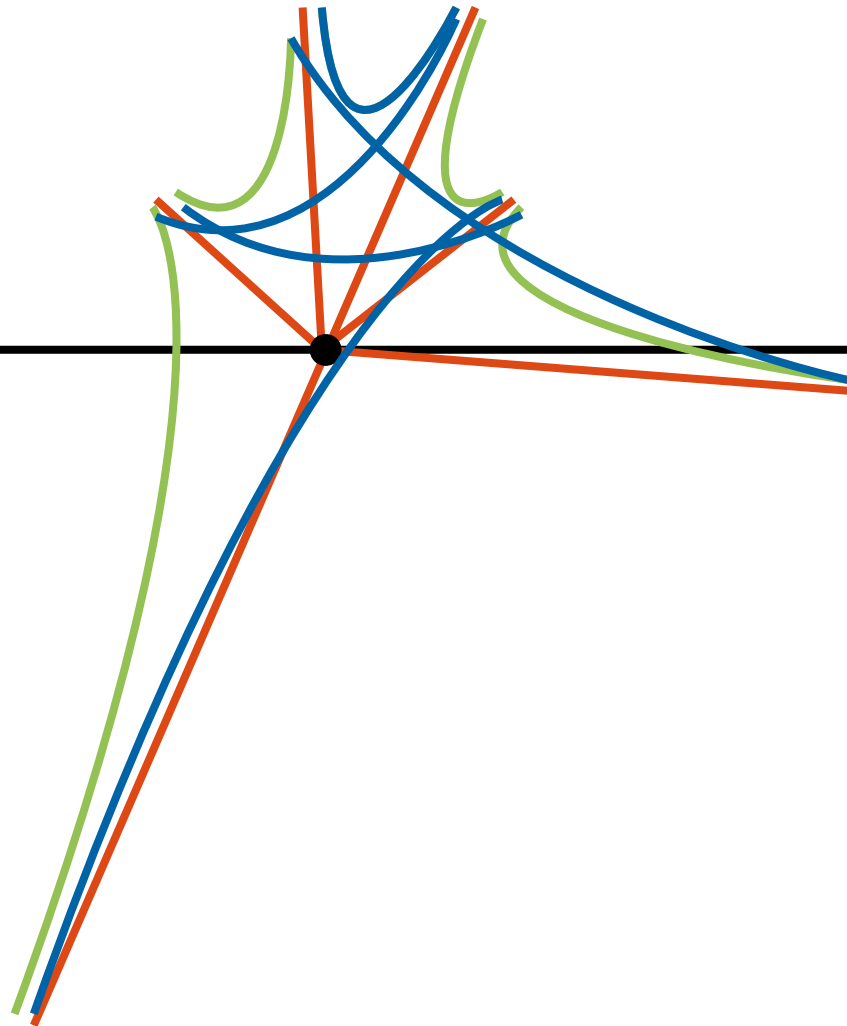
[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]



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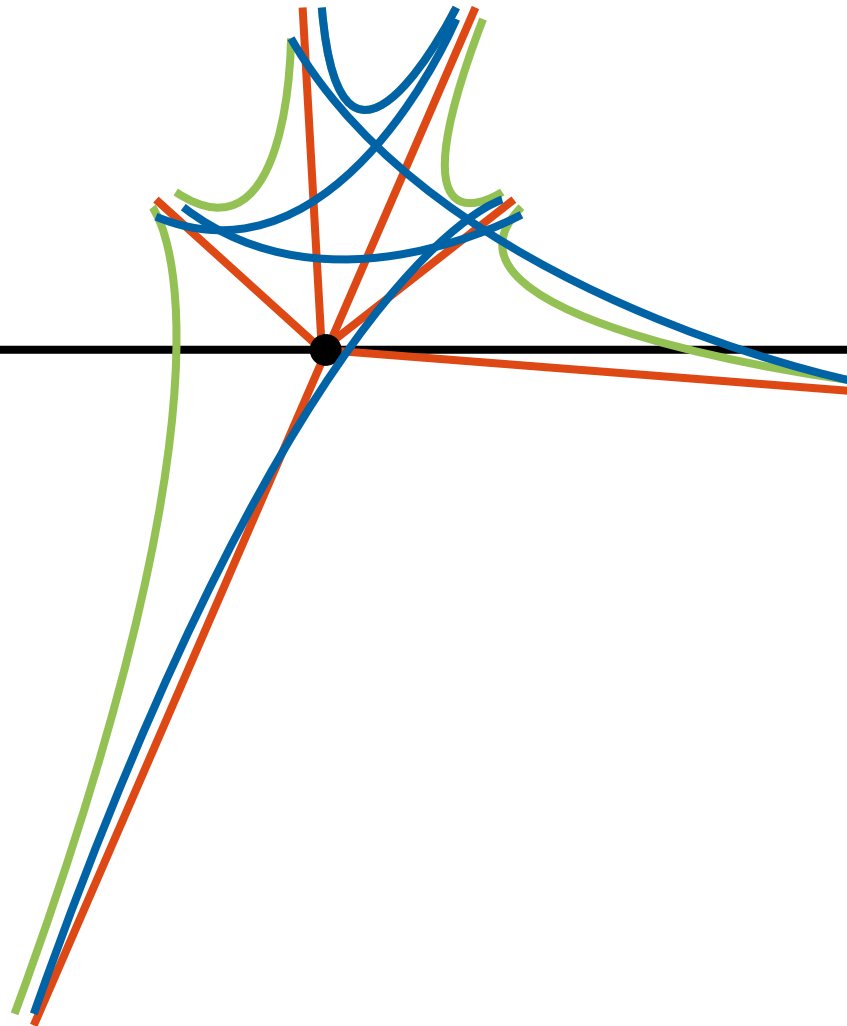
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View as an evolving amplitude, driven by a single initial colour flow:

$$|\mathcal{M}\rangle = e^{\Gamma} |\text{clusters}\rangle$$

$$P_{\text{reco}} \sim |\langle \text{clusters}' | \mathcal{M} \rangle|^2$$



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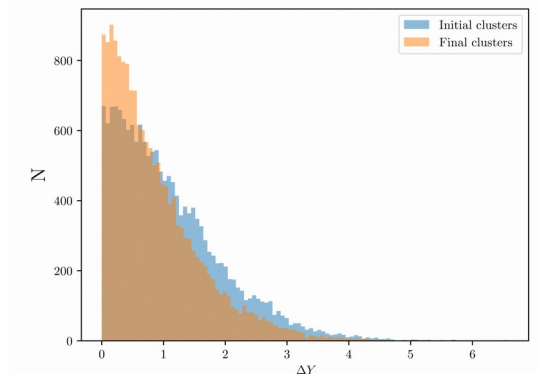
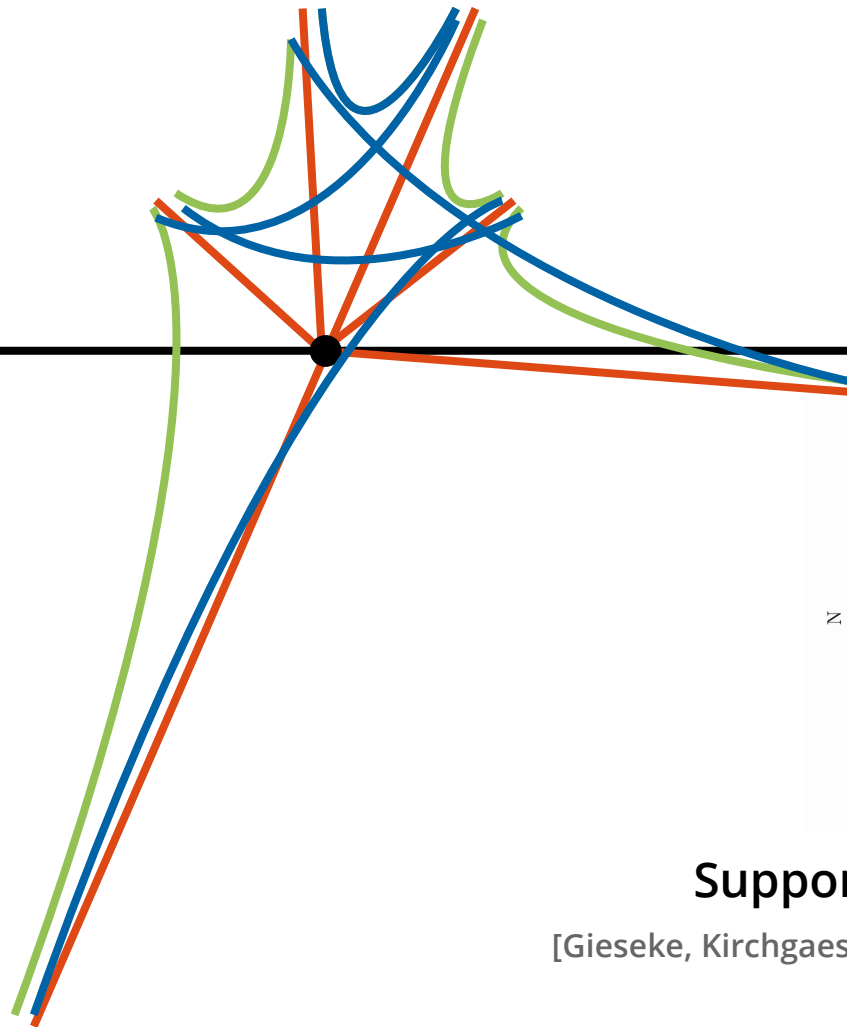
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[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]



Supports geometric models

[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]

Non-global observables require the by most complex resummation formalism, formulated as evolution at the amplitude level.

In an endeavor to formulate more precise parton shower algorithms this complexity sets the level of fundamental formulation.

It is possible to build Monte Carlo evolution codes, and a full parton shower application is in reach, as well as higher orders in the evolution.

Perturbative aspects of colour reconnection can be disentangled from genuine non-perturbative effects.

Thank you!