Energy and Atomic Number Scan in Electron-Ion collisions

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Outline

- NICA@JINR: EIC option
- QCD factorization and hadron spin structure: types of spin-dependent NPQCD functions
- TMDs vs GPDs
- Hadronic vs Heavy-Ion physics (SSA)
- Conclusions
NICA Complex

Baryonic Matter at Nuclotron (BM@N)

SPD (Spin Physics Detector)

MultiPurpose Detector (MPD)
Main targets of “NICA Complex”:

- *study of hot and dense baryonic matter*

- investigation of nucleon spin structure, *polarization phenomena*

- development of accelerator facility for HEP @ JINR providing *intensive beams of relativistic ions from* \( p \) to \( Au \) *polarized protons and deuterons with energy up to* 
  
  \[
  \sqrt{s_{NN}} = 11 \text{ GeV} \ (Au^{79+}, L \sim 10^{32} \text{ cm}^{-2} \text{ c}^{-1})
  \]
  
  \[
  \sqrt{s} = 27 \text{ GeV} \ (p, L \sim 10^{32} \text{ cm}^{-2} \text{ c}^{-1})
  \]
The NICA Beams

Heavy ion colliding beams up to $^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$

at $\sqrt{s_{NN}} = 4 \div 11$ GeV, $L_{\text{average}} = 1 \times 10^{27} \text{ cm}^{-2} \cdot \text{s}^{-1}$

Light-Heavy ion colliding beams of the same $\sqrt{s_{NN}}$ and the same or higher $L_{\text{average}}$

Polarized beams of protons and deuterons in collider mode:

$p \uparrow p \uparrow \sqrt{s_{pp}} = 12 \div 26$ GeV $L_{\text{max}} \approx 1 \times 10^{32} \text{ cm}^{-2} \cdot \text{s}^{-1}$

$d \uparrow d \uparrow \sqrt{s_{NN}} = 4 \div 13.8$ GeV

Extracted beams of light ions and polarized protons and deuterons for fixed target experiments:

$Li \div Au = 1 \div 4.5$ GeV/u ion kinetic energy

$p \uparrow, p \uparrow = 5 \div 12.6$ GeV kinetic energy

$d \uparrow, d \uparrow = 2 \div 5.9$ GeV/u ion kinetic energy

Applied research on ion beams at kinetic energy above 3 MeV/u
The kick-off meeting on formation of the MPD and BM@N Collaborations took place in Dubna on 11-13 April, 2018. Next one on April 17-18 2019.

detailed information about the meeting can be found at: https://indico.jinr.ru/conferenceDisplay.py?confId=385
NICA: heavy ions and hadrons

International Workshop on Spin Physics at NICA (SPIN-Praha-2018), July 9-13 2018

EIC as a possible future option is discussed. Which energy/beams to select?!
EIC: low x/heavy ions

- Energy scan: transitions between different values of thermodynamical parameters (especially chemical potential) and types of QCD factorization

- Atomic number scan: transition from small to large systems
Factorization (lh-> DIS, DVCS)

- Short and hard distances separated (JINR – Efremov, Radyushkin; Higher twist – Efremov,OT; DVCS-Anikin,OT)
Types of parton distributions

- Most general – Wigner function: non-symmetric partonic and hadronic momenta with transverse components
- The spin of both hadrons and partons fixed
Measurement of Wigner (GTMD) function

- Small $x - I_p$ (Hatta, Xiao, Yuan’16) or $A_p$ UP (Hagiwara, Hatta, Pasechnik, Tasevsky, OT’17) collisions

- Larger $x -$ UPC at SPD !?
Ellipticity: from Wigner function to flow

- Wigner Function: most detailed description of hadronic structure
- Elliptic WF is related to elliptic flow in in pp and pA!

Elliptic Flow in Small Systems due to Elliptic Gluon Distributions?
Yoshikazu Hagiwara, Yoshitaka Hatta, Bo-Wen Xiao, and Feng Yuan

Elliptic flow from color-dipole orientation in pp and pA collisions
Edmond Iancu and Amir H. Rezaeian

- Interference of L=0,2 quantum states
- How to explore this relation?
Types of parton distributions - II

- Too rich structure of Wigner function
- Simplifications – Putting some (transverse) momenta to zero or average over some variables
- Hadronic moments equal - inclusive
- Allow for proof of QCD factorization is some cases (perturbative corrections are taken into account by some kind of evolution)
Collinear vs $k_T$ factorization

- Collinear: NP longitudinal and pQCD transverse (GLAPD) evolution
- BFKL (also perturbative origin!-but differ by $P_{<->NP}$) NP transverse and pQCD longitudinal evolution
- GI for off-shell partons? $(x_{P+} k_T)^2 < 0$
- Special BFKL vertices, effective action
Single Spin Asymmetries: simplest example

Simplest example - (non-relativistic) elastic pion-nucleon scattering \( \pi \vec{N} \rightarrow \pi N \)

\[ M = a + ib(\vec{\sigma}\vec{n}) \vec{n} \] is the normal to the scattering plane.

Density matrix: \( \rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P}) \),

Differential cross-section: \( d\sigma \sim 1 + A(\vec{P}\vec{n}), A = \frac{2Im(ab^*)}{|a|^2 + |b|^2} \)
Single Spin Asymmetries

Main properties:
– Parity: transverse polarization
– Imaginary phase – can be seen from $T$-invariance or technically - from the imaginary $i$ in the (quark) density matrix

Various mechanisms – various sources of phases
Phases in QCD

- QCD factorization – soft and hard parts-
- Phases form soft, hard and overlap
- Assume (generalized) optical theorem – phase due to on-shell intermediate states – positive kinematic variable (= their invariant mass)
- Hard: Perturbative (a la QED: Barut, Fronsdal (1960):
Kane, Pumplin, Repko (78) Efremov (78)
QCD factorization: where to borrow imaginary parts?
Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q - e$ scattering in DIS):

$$A \sim \frac{\alpha_s m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"
Short+ large overlap–twist 3

- Quarks – only from hadrons
- Various options for factorization – shift of SH separation (prototype of duality)

- New option for SSA: Instead of 1-loop twist 2 – Born twist 3: Efremov, OT (85, Fermionic poles); Qiu, Sterman (91, GLUONIC poles)
Phases in QCD-Large distances in distributions

- Distributions: Sivers, Boer and Mulders – no positive kinematic variable producing phase
- QCD: Emerge only due to (initial of final state) interaction between hard and soft parts of the process
- Brodsky-Hwang-Schmidt model: the same SH interactions as twist 3 but non-suppressed by Q: Sivers function – leading (twist 2).
- Related in various complementary ways
Kinematical domains for SSA’s

- Sivers
- PT
- Twist 3
- FF’s
- X

Diagram shows relationships between kinematical domains.
Λ-polarisation

- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: ’87): Randomization – smearing – no direction normal to the scattering plane
Global polarization

- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- How to transform rotation to spin?
- Mechanisms – reflected in kinematical dependencies of polarization
Anomalous mechanism – polarization similar to CM(V)E

- 4-Velocity is also a **GAUGE FIELD** (V.I. Zakharov)
  \[ e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha \]

- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT ’10)

- Analogous to anomalous gluon contribution to nucleon spin (Efremov, OT’88)

- 4-velocity instead of gluon field!
Energy dependence

- Coupling -> chemical potential
  \[ Q_5^s = \frac{N_c}{2\pi^2} \int d^3x \frac{\mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k}{m} \]

- Field -> velocity; (Color) magnetic field strength -> vorticity;

- Topological current -> hydrodynamical helicity

- Large chemical potential: appropriate for NICA/FAIR energies

- Cf Regge theory (recall “Ridge”)
One might compare the prediction below with the right panel figures

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortain effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

One would expect that polarization is proportional to the anomalously induced axial current [7]

\[ j_A^\mu \sim \mu^2 \left( 1 - \frac{2\mu n}{3(\varepsilon + P)} \right) \varepsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \] (6)

where \( n \) and \( \varepsilon \) are the corresponding charge and energy densities and \( P \) is the pressure. Therefore, the \( \mu \) dependence of polarization must be stronger than that of the CVE, leading to the effect’s increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.
Energy dependence (Baznat, Gudima, Sorin, OT)

- Growth at low energy
- Close to STAR data!
- Baryon-antibaryon successfully described
Λ vs Anti Λ

![Graph showing the comparison between Λ and Anti Λ in Au + Au collisions with center-of-mass energy as a function of s_{NN}^{1/2}.]
Matching with hadronic approach? Phases?!

- Suggestion: DISSIPATION

- Quantized vortices in pionic superfluid

\[
 j_5^\mu = \frac{1}{4\pi^2 f^2_{\pi}} \epsilon^{\mu\nu\rho\sigma} (\partial_\nu \pi^0)(\partial_\rho \partial_\sigma \pi^0) \quad \frac{\pi^0}{f_{\pi}} = \mathbf{\mu} \cdot \mathbf{t} + \varphi(x_i) \quad \oint \partial_i \varphi dx_i = 2\pi n \\
\partial_i \varphi = \mu v_i
\]

- Transition of angular momentum to heavy baryonic degrees of freedom at vortex core is accompanied by dissipation (Zakharov, OT’17)
Core of quantized vortex

- Constant circulation – velocity increases when core is approached

- Helium ($v < v_{\text{sound}}$) bounded by intermolecular distances

- Pions ($v < c$) $\rightarrow$ (baryon) spin in the center
Thermal Wigner function

- Induced axial current

\[ W(x,k)_{AB} = -\frac{1}{(2\pi)^4} \int d^4y e^{-ik\cdot y} (\Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : ) \]

\[ \alpha_\mu = \frac{1}{T} u^\nu \partial_\nu u_\mu = \frac{a_\mu}{T}, \quad w_\mu = \frac{1}{2T} \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha u^\beta = \frac{\omega_\mu}{T} \]

\[ W(x,k) = \frac{1}{2} \int \frac{d^3p}{\varepsilon} \left( \delta^4(k-p)U(p)f(x,p)\overline{U}(p) - \delta^4(k+p)V(p)f^*(x,p)V(p) \right) \]

\[ \langle : j^5_\mu : \rangle = \left( \frac{1}{6} \left[ T^2 + \frac{a^2 - \omega^2}{4\pi^2} \right] + \frac{\mu^2}{2\pi} \varphi_\mu \right) + \frac{1}{2\pi} (\omega \cdot a) \cdot a_\mu \]

\[ \langle : j^5_\mu : \rangle = 2\pi \text{Im} \left[ \left( \frac{1}{6} (T^2 + \varphi^2) + \frac{\mu^2}{2\pi^2} \right) \varphi_\mu \right] \]

- “Imaginary acceleration” \( H + iE \sim \Omega + ia \)

- Irregularities below Unruh temperature \( T = a/2\pi \)
Experimental separation of rotation and acceleration effects?

- Rotation – revealed via large orbital momentum in AA(pA) collisions
- Acceleration (in final state) is manifested even in e+e- annihilation explaining the thermal spectra (Castorina, Kharzeev, Satz)
- Comparing eA and AA – disentangling rotation and acceleration effects?
CONCLUSIONS

- EIC at lower energy: Quark/gluon, valence/sea, UGDF/TMD, growth of chemical potential, “Hydro” SSA

- EIC for light ion/proton: transition from “Hydro” to TMD/twist 3 SSA, Anisotropic flows to anisotropic Wigner functions,…

- May be addressed at EIC@NICA plans
Problems for NICA

- SPD LoI: TMDs@DY
- TMDs – $J/\Psi$, $\gamma$
- GPDs: Exclusive DY-type (smaller x-section but lower background)
- GPDs from TMDs (pressure?!)
- Relation of HIC/hadronic spin (MPD/SPD) – polarization for hadrons, light and heavy ions
- China is actively participating in MPD&BMN; SPD – much welcomed
BACKUP
Fracture functions

- Common NP ingredient for FRAgmentation and struCTURE
- Structure functions – parton distributions
- Fracture functions – fractural (conditional, correlational, entangling?) parton distributions
- May be T-odd (Collins’95 – polarized beam jets; OT’01-T-odd Diffractive Distributions)
- Related by crossing to dihadron fragmentation functions
(T-odd) Fractalural (conditional) parton distributions
HT parton distributions
T-odd fracture function for hyperons polarization

- May be formally obtained from spin-dependent T-odd DIS (cf OT’99 for pions SSA-work in progress)
- Transverse spin in DIS – either transverse spin or transverse momentum of hyperon in SIDIS
- Both longitudinal and transverse polarizations appear
- SPD – extra hadrons (pions) with low TM
GPDs in exclusive limit of fractured distributions
Fractural PD

- Frac´tur`al
  a.1. Pertaining to, or consequent on, a fracture.
Twist 3 partonic subprocesses for SIDVCS
Real and virtual photons - most clean tests of QCD

- Both initial and final – real: Efremov, O.T. (85)
- Initial – quark/gluon, final – real: Efremov, OT (86, fermionic poles); Qui, Sterman (91, GLUONIC poles)
- Initial - real, final-virtual (or quark/gluon) – Korotkiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05; smooth transition from fermionic via hard to GLUONIC poles).
Sivers function and formfactors

- Relation between Sivers and AMM known on the level of matrix elements (Brodsky, Schmidt, Burkardt)
- Phase?
- Duality for observables?
- Solution: SSA in DY
SSA in exclusive limit

- Proton-antiproton – valence annihilation - cross section is described by Dirac FF squared
- The same SSA due to interference of Dirac and Pauli FF’s with a phase shift
- Exclusive large energy limit; $x \rightarrow 1$ : 
  $$\frac{d}{dx}T(x,x)/q(x) \rightarrow \text{Im } F_2/F_1$$
- No suppression of large $x$ – large E704 SSA
- Positivity: Twist 4 correction to $q(x)$ may be important
mechanisms for exclusive amplitudes (Anikin, Cherednikov, Stefanis, OT, 08)

- 2 pion production: GDA (small s) vs TDA+DA (small t)

- Scalar model - asymptotics (Efremov, Ginzburg, Radyushkin...)

[Diagrams showing Feynman diagrams for pion production and scalar model]
Duality in scalar model

- “Right” (TDA, red) and “wrong” (GDA, blue) asymptotics / exact result (>1- negative “Higher Twist”)

[Graph showing ratio of amplitudes vs. x/Q^2 with markers for Ration R_1 and Ration R_2]
Duality in QCD

- Qualitatively- surprisingly good, quantitatively - model-dependent
Duality and helicity amplitudes

- Holds if different mechanisms contribute to SAME helicity amplitudes
- Scalar- only one; QCD – L and T photons
- Other option : Different mechanisms – different helicity amplitudes (“unmatching”) 
- Example -> transition from perturbative phase to twist 3 (m -> M)
Twist 3 factorization (Efremov, OT ’84, Ratcliffe, Qiu, Sterman)

- Convolution of soft (S) and hard (T) parts

\[ d\sigma_s = \int dx_1 dx_2 \frac{1}{4} S p [S_\mu(x_1, x_2) T_\mu(x_1, x_2)] \]

- Vector and axial correlators: define hard process for both double \( g_2 \) and single asymmetries

\[ T_\mu(x_1, x_2) = \frac{M}{2\pi} (\hat{p}_1 \gamma^5 s_\mu b_A(x_1, x_2) - i\gamma_\rho \epsilon^{\rho\mu \rho\mu \rho} b_V(x_1, x_2)) \]
Twist 3 factorization -II

- Non-local operators for quark-gluon correlators

\[ b_A(x_1, x_2) = \frac{1}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \langle p_1, s | \bar{\psi}(0) \gamma^5 (D(\lambda_1) s) \psi(\lambda_2) | p_1, s \rangle, \]

\[ b_V(x_1, x_2) = \frac{i}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \epsilon^{\mu \nu_{12}} \langle p_1, s | \bar{\psi}(0) \gamma_5 D_\mu(\lambda_1) \psi(\lambda_2) | p_1, s \rangle \]

- Symmetry properties (from T-invariance)

\[ b_A(x_1, x_2) = b_A(x_2, x_1), \quad b_V(x_1, x_2) = -b_V(x_2, x_1) \]
Twist-3 factorization - III

- **Singularities**

\[
b_A(x_1, x_2) = \varphi_A(x_1)\delta(x_1 - x_2) + b^r_A(x_2, x_1).
\]

\[
b_V(x_1, x_2) = \frac{\varphi_V(x_1)}{x_1 - x_2} + b^r_V(x_1, x_2).
\]

- Very different: for axial – Wandzura-Wilczek term due to intrinsic transverse momentum

- For vector-GLUONIC POLE (Qiu, Sterman ’91) – large distance background
Sum rules

- EOM + n-independence (GI + rotational invariance) – relation to (genuine twist 3) DIS structure functions

\[
\int_0^1 x^n \bar{g}_2(x) \, dx = \int_0^1 x^n \left( \frac{n}{n+1} g_1(x) + g_2(x) \right) \, dx = \\
- \frac{1}{\pi(n+1)} \int_{|x_1, x_2, x_1 - x_2| \leq 1} dx_1 dx_2 \sum_f e_f^2 \left[ \frac{n}{2} b_V(x_1, x_2)(x_1^{n-1} - x_2^{n-1}) + b_A^r(x_1, x_2) \phi_n(x_1, x_2) \right], \quad \phi_n(x, y) = \frac{x^n - y^n}{x - y} - \frac{n}{2}(x^{n-1} - y^{n-1}), \quad n = 0, 2 \ldots
\]
Sum rules -II

- To simplify – low moments

\[ \int_0^1 x^2 \hat{g}_2(x) dx = \]

\(- \frac{1}{3\pi} \int_{|x_1, x_2, x_1 - x_2| \leq 1} dx_1 dx_2 \sum_f e_f^2 b_V(x_1, x_2) (x_1 - x_2) \]

- Especially simple – if only gluonic pole kept:

\[ \int_0^1 x^2 \bar{g}_2(x) dx = -\frac{1}{3\pi} \int_{|x_1, x_2, x_1 - x_2| \leq 1} dx_1 dx_2 \sum_f e_f^2 \varphi_V(x_1) \]

\[ = -\frac{1}{3\pi} \int_{-1}^1 dx_1 \sum_f e_f^2 \varphi_V(x_1) (2 - |x_1|) \]
Gluonic poles and Sivers function

- Gluonic poles – effective Sivers functions. Hard and Soft parts talk, but SOFTLY

- Implies the sum rule for effective Sivers function (soft=gluonic pole dominance assumed in the whole allowed x’s region of quark-gluon correlator)

\[ x \int_{T} f_{T}(x) = \frac{1}{2M} T(x, x) = \frac{1}{4} \phi_{v}(x) \]

\[ \int_{0}^{1} dxx^{2-}g_{2}(x) = \frac{4}{3\pi} \int_{0}^{1} dxx f_{T}(x)(2-x) \]
Compatibility of SSA and DIS

- Extractions of and modeling of Sivers function: – “mirror” u and d
- Second moment at % level
- Twist -3 \(g_2\) - similar for neutron and proton and of the same sign – no mirror picture seen – but supported by colour ordering!
- Scale of Sivers function reasonable, but flavor dependence differs qualitatively.
- Inclusion of pp data, global analysis including gluonic (=Sivers) and fermionic poles
- HERMES, RHIC, E704 – like phonons and rotons in liquid helium; small moment and large E704 SSA imply oscillations
- JLAB – measure SF and \(g_2\) in the same run
CONCLUSIONS

- 3rd way from SF to GP – proof of Torino recipe supplemented by colour correlations
- Effective SF – small in pp - factorization in terms of twist 3 only
- Large x – E704 region - relation between SF, GP and time-like FF’s
Outlook (high energies)

- TMD vs UGPD
- T-odd UGPD?
- T-odd (P/O) diffractive distributions (analogs - also at small energies)
- Quark-hadron duality: description of gluon coupling to “exotic” objects in diffractive production via their decay widths
Relation of Sivers function to GPDs

- Qualitatively similar to Anomalous Magnetic Moment (Brodsky et al)
- Quantification: weighted TM moment of Sivers PROPORTIONAL to GPD $E$ ($\text{hep-ph/0612205}$):
  $$x \int_T f_T(x) : xE(x)$$
- Burkardt SR for Sivers functions is now related to Ji SR for $E$ and, in turn, to Equivalence Principle
  $$\sum_{q,G} \int dxx f_T(x) = \sum_{q,G} \int dxxE(x) = 0$$
How gravity is coupled to nucleons?

- Current or constituent quark masses? – neither!
- Energy momentum tensor-like electromagnetic current describes the coupling to photons
Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not yet checked
- Anomalous gravitomagnetic moment is zero
  or
- Classical and QUANTUM rotators behave in the SAME way
Gravitational formfactors

\[ \langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)} \right] \Delta \alpha / 2M u(p) \]

- Conservation laws - zero Anomalous Gravitomagnetic Moment: \( \mu_G = J \) (g=2)

- \( P_{q,g} = A_{q,g}(0) \)
- \( A_q(0) + A_g(0) = 1 \)
- \( J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \)
- \( A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1 \)

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity – similar t-dependence to EM FF
Electromagnetism vs Gravity

- Interaction – field vs metric deviation

\[ M = \langle P' | J^\mu_q | P \rangle A_\mu(q) \]
\[ M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q) \]

- Static limit

\[ \langle P | J^\mu_q | P \rangle = 2e_q P^\mu \]
\[ \sum_{q,G} \langle P | T_{q,G}^{\mu\nu} | P \rangle = 2P^\mu P^\nu \]
\[ h_{00} = 2\phi(x) \]

\[ M_0 = \langle P | J^\mu_q | P \rangle A_\mu = 2e_q M \phi(q) \]
\[ M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q) \]

- Mass as charge – equivalence principle
Gravitomagnetism

- Gravitomagnetic field – action on spin – $1/2$
  from

$$ M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q) $$

$$ \vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i} \quad \text{spin dragging twice smaller than EM} $$

- Lorentz force – similar to EM case: factor $1/2$
  cancelled with 2 from
  Larmor frequency same as EM

$$ h_{00} = 2\phi(x) $$

$$ \vec{H}_L = \text{rot} \vec{g} $$

- Orbital and Spin momenta dragging – the same - Equivalence principle

$$ \omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L $$
Sivers function and Extended Equivalence principle

- Second moment of E – zero SEPARATELY for quarks and gluons –only in QCD beyond PT (OT, 2001) - supported by lattice simulations etc. -
- Gluon Sivers function is small! (COMPASS, STAR, Brodsky&Gardner)
- BUT: gluon orbital momentum is NOT small: total – about 1/2, if small spin – large (longitudinal) orbital momentum
- Gluon Sivers function should result from twist 3 correlator of 3 gluons: remains to be proved!
Generalization of Equivalence principle

- Various arguments: AGM $\Theta$ separately for quarks and gluons – most clear from the lattice (LHPC/SESAM, confirmed recently)