Lorentz violation in Hadronic Physics

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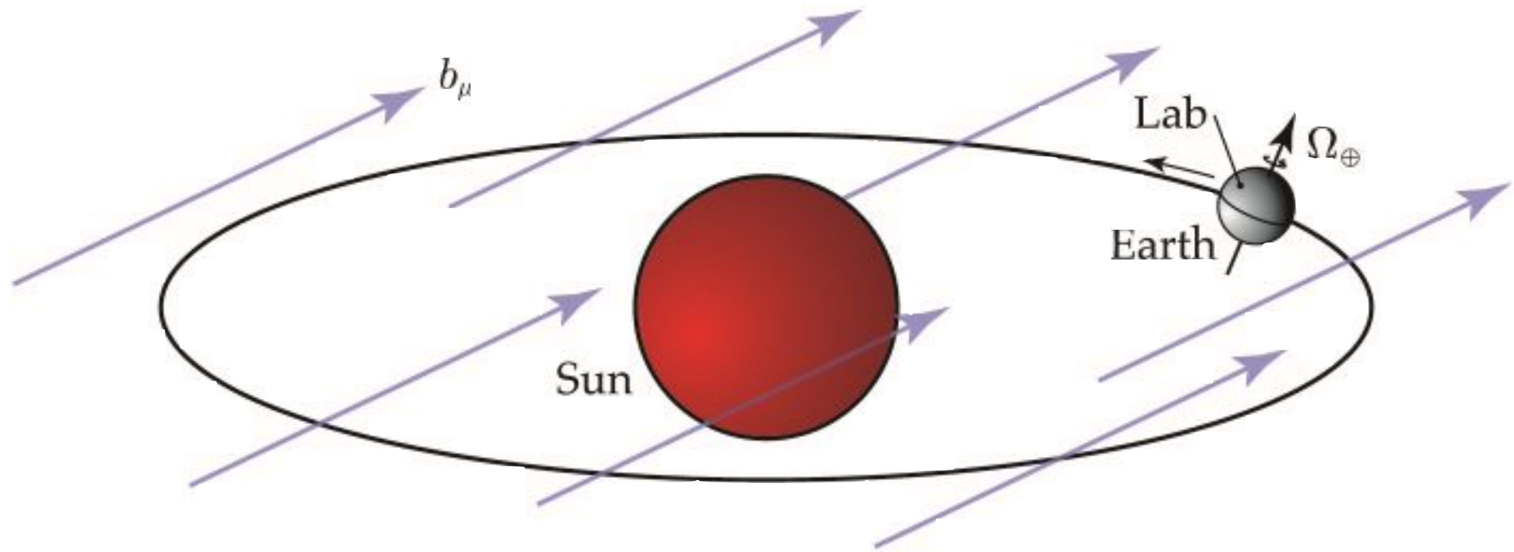


DIS 2019 TORINO 8-12 April
XXVII International Workshop on Deep Inelastic Scattering and Related Subjects

A. Kostelecky, E.L. and A. Vieira [1610.09318]
E.L. and N. Sherrill [1805.11684]
A. Kostelecky, E.L., N. Sherrill and A. Vieira [1904.xxxx]

Outline

- Standard Model Extension: an effective description of Lorentz Violating effects
- Existing experimental constraints and challenges
- The special role of deep inelastic e-p scattering (dual description in terms of parton-model and Operator Produce Expansion)
- Expected upper bounds from HERA data (Zeus and H1) and from the planned electron-ion collider in the JLAB (JLEIC) and Brookhaven (eRHIC) configurations
- Expected bounds from Drell-Yan at LHC

The Standard Model Extension (SME)

- Lorentz invariance is perhaps the most tested symmetry in nature
- Consistent theories in which the Lorentz symmetry is explicitly broken are extremely difficult to construct
- A much easier path (inspired by vacua found in superstring theories) is to spontaneously break Lorentz invariance with the introduction of constant background fields whose vacuum expectation values are not Lorentz invariant (i.e. have intrinsic directions)
- Instead of considering explicit models (in which the expectation values of background fields are obtained explicitly from a potential) we focus on a general parametrization of all possible effects:
  the Standard Model Extension [hep-ph/9703464; Colladay, Kostelecky]
  [hep-ph/9809521; Colladay, Kostelecky]
  [hep-th/0312310; Kostelecky]

The Standard Model Extension (SME)

- Let's choose a reference frame (a set of coordinates) and consider for instance

$$\mathscr{L} = \mathscr{L}_{\rm SM}(x, \phi, \psi, V^\mu, g_i) + a_\mu \, \bar\psi \gamma^\mu \psi$$

  where the SM Lagrangian density is a function of scalar (ϕ), spinor (ψ), vector(V^μ) fields and couplings (g_i), and a^μ are 4 new coupling constants.

- Under a particle Lorentz transformation: $x^\mu \rightarrow \Lambda^\mu{}_\nu \, x^\nu$, (ϕ,ψ,V^μ) transform as usual and (g_i, a^μ) are invariant. The presence of a^μ implies that amplitudes between states connected by this transformation are not connected as in the SM

- Under an observer Lorentz transformation (by which we mean just a change of coordinates), the physics obviously must be invariant. In the new frame we see a Lagrangian which is identical in form to the original one and in which $a^\mu \rightarrow \Lambda^\mu{}_\nu \, a^\nu$

- This implies that the principle of relativity is violated:
  the lifetime of a boosted muon measured in the original frame and the lifetime of a muon at rest in the original frame but measured in a boosted frame will differ

The Standard Model Extension (SME)

- The fact that observer invariance is preserved implies
  - Standard Quantization
  - Microcausality
  - Spin-Statistic Theorem
  - Observer Lorentz covariance
  - Hermiticity
  - Positivity of the Energy
  - Power counting renormalizability
  - Conservation of Energy-Momentum for constant Lorentz Violating vacuum expectations values

- For simplicity we focus on renormalizable interactions: the resulting theory is known as ***minimal SME***

The QCD sector of the SME

- The $SU(3)\times U(1)$ gauge, lepton and quark sectors are ($\psi = u,d,e$):

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \bar\psi(\gamma^\mu i D_\mu - m_\psi)\psi$$

$$\delta\mathcal{L}_{\rm SME} = -\frac{1}{4}\kappa_F^{\kappa\lambda\mu\nu}F_{\kappa\lambda}F_{\mu\nu} - \frac{1}{4}\kappa_G^{\kappa\lambda\mu\nu}G^a_{\kappa\lambda}G^a_{\mu\nu} + \bar\psi(\Gamma^\mu i D_\mu - M)\psi$$

where $\Gamma^\mu = c^{\mu\nu}\gamma_\nu + d^{\mu\nu}\gamma_\nu + e^\mu + i f^\mu\gamma^5 + \frac{1}{2}g^{\alpha\beta\mu}\sigma_{\alpha\beta}$

$$M = a_\mu\gamma^\mu + b_\mu\gamma^5\gamma^\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

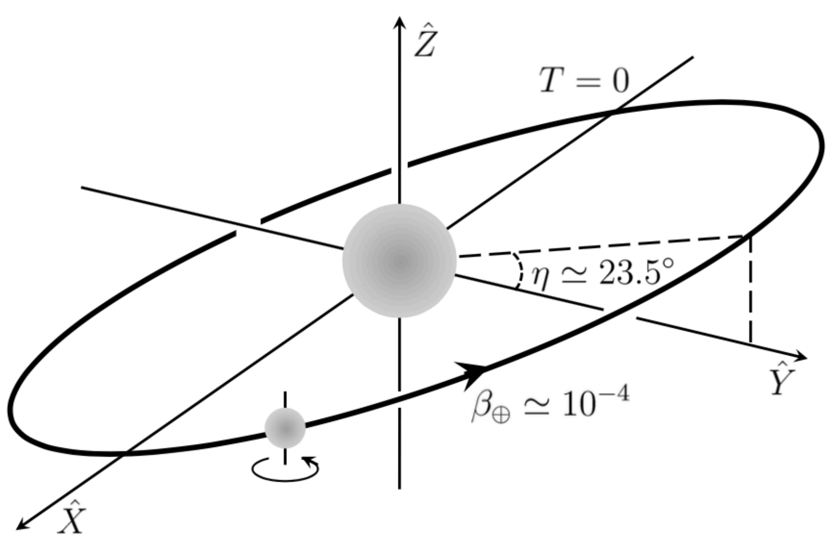
D_μ is the standard QCD & QED covariant derivative

- The various coefficients (e.g. $c^{\mu\nu}$) have fixed values in some reference inertial frame. Their values in the lab frame are given by a standard Lorentz transformation.
- Under observer transformations the SME Lagrangian is a scalar: for example $c^{\mu\nu}$ and $\bar\psi\gamma_\mu i D_\nu\psi$ are both tensors
- Under particle transformations $\bar\psi\gamma_\mu i D_\nu\psi$ is a tensor while $c^{\mu\nu}$ are 16 scalars
- One can include non-renormalizable interactions. For instance:

$$\delta\mathscr{L}^{(5)} = -\frac{1}{2}a^{(5)\mu\alpha\beta}\bar\psi i D_{(\alpha} i D_{\beta)}\psi + \cdots$$

Existing Constraints

- All coefficients for Lorentz-Violation are defined with respect to a Sun-centered celestial-equatorial frame (which is the "most" inertial frame that is accessible [0801.0287; Kostelecky, Russell]



- Experiments that focus on the properties of stable particles (electrons, muons, protons, neutrons, photons) yield very strong constraints:

$$
\begin{aligned}
\kappa_F^{\alpha\beta\mu\nu} &< \left[10^{-14} - 10^{-32}\right] \\
c^{\mu\nu}_{\text{electron}} &< \left[10^{-17} - 10^{-21}\right] \\
c^{\mu\nu}_{\text{proton}} &< \left[10^{-20} - 10^{-28}\right] \\
c^{\mu\nu}_{\text{neutron}} &< \left[10^{-13} - 10^{-29}\right] \\
c^{\mu\nu}_{\text{muon}} &< 10^{-11}
\end{aligned}
$$

Bounds come from *O(200)* experiments

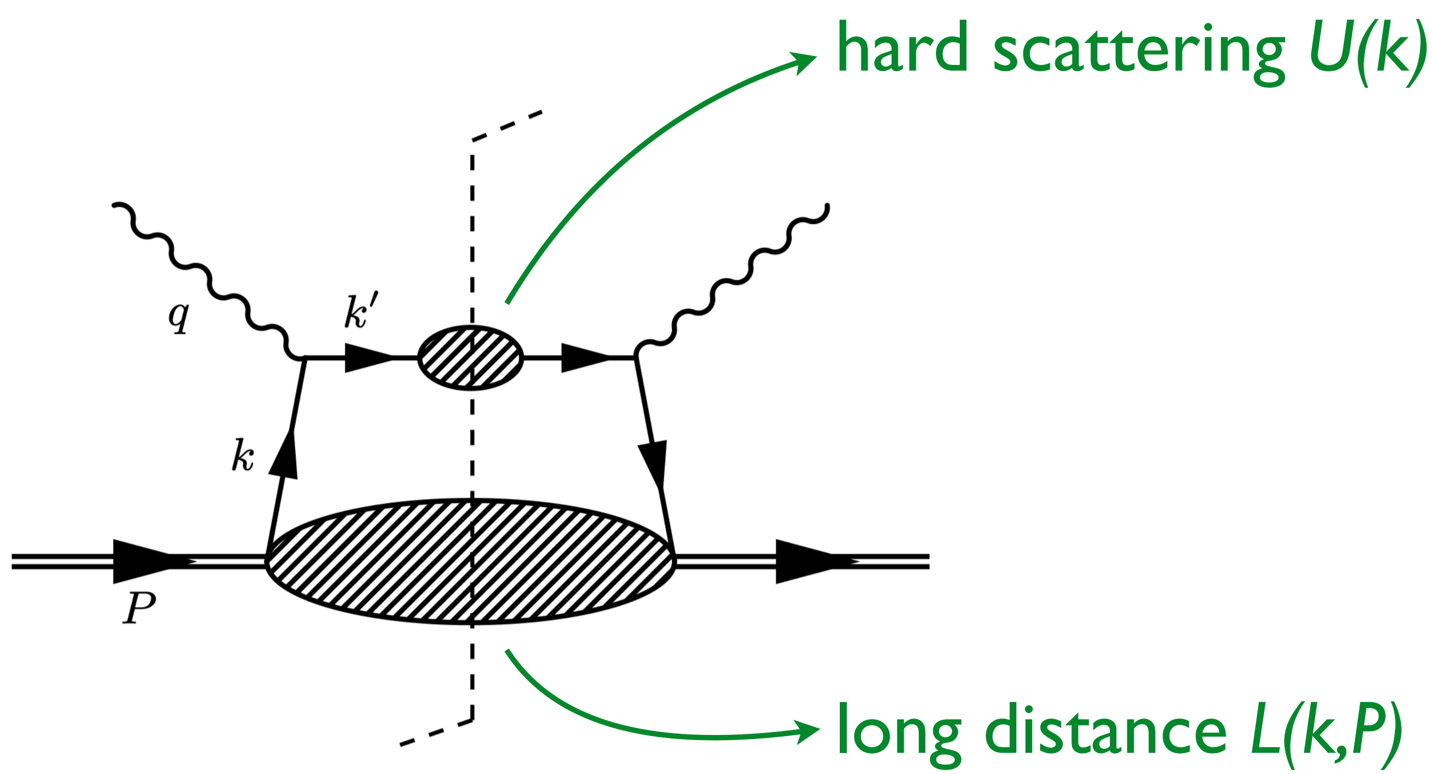
- Note that extreme care has to be used in interpreting these bounds. Experiments can only place limits on "physical" combinations of coefficients.
- Coefficients in the quark sectors are almost completely unconstrained due to the difficulty of accessing quark level transitions directly.

Existing Constraints: the QCD sector

- The main problem is connecting coefficients in the quark sectors to observable hadronic properties
- There are several avenues that one can pursue:
  - Low energy processes sensitive to short distance physics
    e.g.: meson-antimeson mixing [Kostelecky, Berger; Di Domenico, Van Kooten, van Tilburg]
  - Impact on hadron properties
    - Connecting the effective LV coefficients of proton, neutron and pion to the fundamental quark LV coefficients [Kamand, Altschul, Schindler]
    - Impact of the quark LV coefficients on various proton PDFs
  - High energy hadronic interactions where, using factorization, it is possible to (partially) bypass non-perturbative problems and directly relate observables to the underlying quark dynamics:
    - Deep Inelastic Scattering at HERA [Kostelecky, EL, Vieira]
    - Reach of DIS at the Electron Ion Collider [EL, Sherrill]
    - LHC phenomenology, e.g. Drell-Yan [Kostelecky, EL, Sherrill, Vieira]

Deep Inelastic Scattering: Standard Model (factorization)

- The parton model picture emerges from an all-orders proof of factorization:

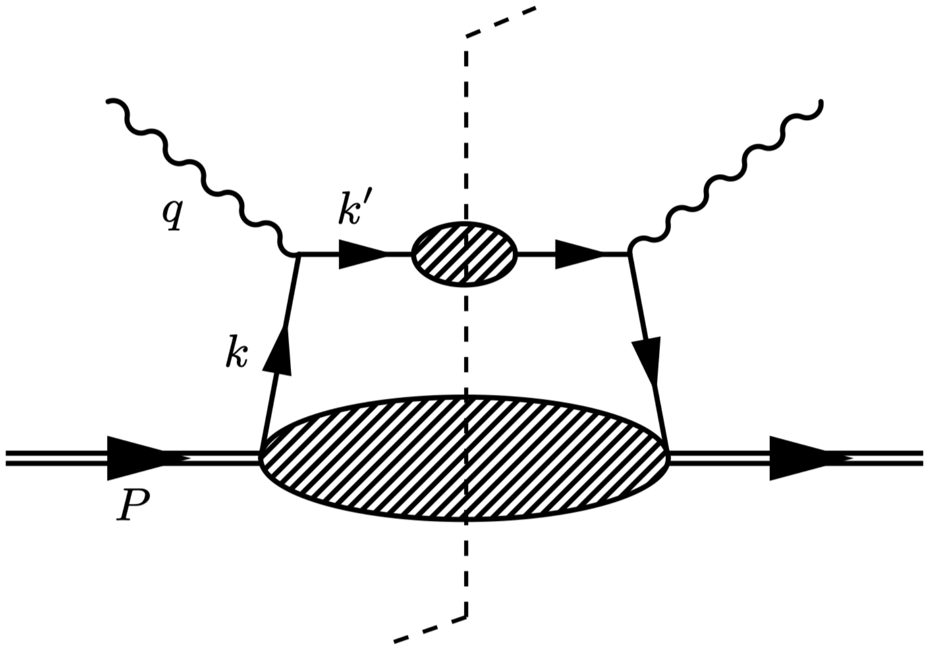


- The dominant region comes from almost on-shell momenta ($k^2 \sim \Lambda^2, m_p^2$)
- It is convenient to use the γ^*-proton center of momentum frame (Breit frame).
- In light-cone coordinates:

$$\begin{cases} P^\mu \simeq (\bar{n} \cdot P)\, n^\mu = (Q{,}0{,}0_\perp) \\ q^\mu \simeq (-Q, Q{,}0_\perp) \end{cases} \Longrightarrow \quad k^\mu = \xi P^\mu + \underbrace{\delta k^\mu}^{\text{small}} \quad \Longrightarrow \quad k + q \simeq (0{,}Q{,}0_\perp)$$

on shell after taking the imaginary part of the propagator

Deep Inelastic Scattering: Standard Model (factorization)

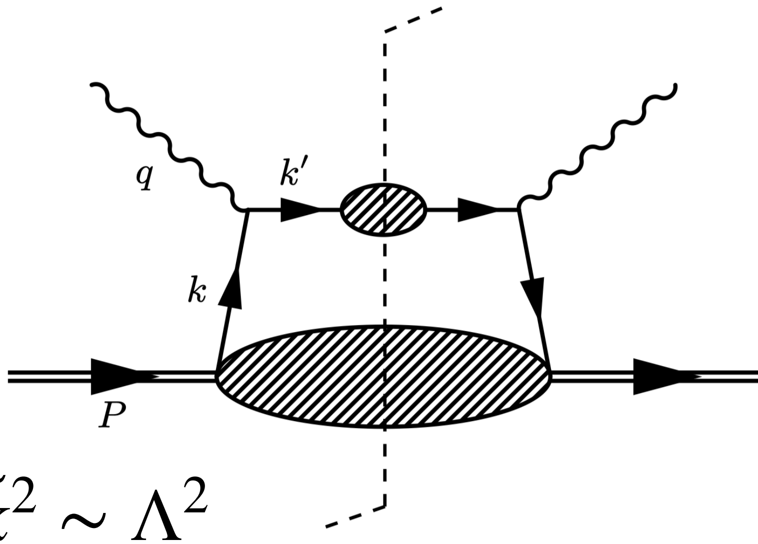


- In the Breit frame it can be shown that only one component of k enters the hard scattering (i.e. the other components are small and there are no singularities): starting from $k^\mu = \xi P^\mu + \delta k^\mu$, the $\delta k^\mu \to 0$ limit is smooth.
- The amplitude becomes a one dimensional convolution of universal parton distribution functions and hard scatterings.
- The parton distribution functions admit a covariant expression:

$$f(n \cdot k, P^\mu) = \int \frac{d\lambda}{2\pi} e^{-i(n\cdot k)\lambda} \langle P | \bar{\psi}(\lambda n) \frac{\not{n}}{2} \psi(0) | P \rangle$$

- Reparameterization invariance (rescaling of n) and covariance imply that the PDF can only depend on the ratio $\xi \equiv n \cdot k / n \cdot P$: $f(n \cdot k, P^\mu) \to f(\xi)$

Deep Inelastic Scattering: SME (factorization)



- In our case: $\mathscr{L} = \frac{1}{2}\bar{q}\gamma_\mu(g^{\mu\nu} + c^{\mu\nu})i\overleftrightarrow{D}_\nu q$
- The quark dispersion relation is modified:

$$k_\mu(\eta^{\mu\nu} + c^{\mu\nu})(\eta_{\nu\lambda} + c_{\nu\lambda})k^\lambda = \tilde{k}^\mu\tilde{k}_\mu = 0$$

- In the proof of factorization we need to take k such that $\tilde{k}^2 \sim \Lambda^2$
- Covariance forces the choice: $\tilde{k}^\mu = \xi P^\mu + \delta\tilde{k}^\mu$
- Taking the imaginary part of the internal propagator ($k'^\mu = k + q$) forces $\tilde{k}'^2 = (\tilde{k} + \tilde{q})^2 \sim \Lambda^2$
- The proof of factorization is almost identical to the SM case after transforming to a modified Breit frame defined as the $P - \tilde{q}$ center of mass frame
- The parton distribution functions become:

$$f(\underbrace{n\cdot\tilde{k}, P^\mu, c^{\mu\nu}}) = \int \frac{d\lambda}{2\pi} e^{-i(n\cdot\tilde{k})\lambda}\langle P|\bar{\psi}(\lambda\tilde{n})\frac{\not{n}}{2}\psi(0)|P\rangle$$

$$\left(\frac{n\cdot\tilde{k}}{n\cdot P}, \frac{c_{\mu\nu}n^\mu P^\nu}{n\cdot P}, \frac{c_{\mu\nu}P^\mu P^\nu}{\Lambda^2}\right) = \left(\xi, c_{\mu\nu}n^\mu\bar{n}^\nu, c_{\mu\nu}\bar{n}^\mu\bar{n}^\mu\frac{(n\cdot P)^2}{\Lambda^2}\right)$$

Deep Inelastic Scattering: SME (factorization)

- The DIS cross section is $d\sigma \sim L^{\mu\nu} W_{\mu\nu}$, where $L^{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic tensors (the latter is expressed in terms of W_1 and W_2)
- In the SM: $T^{\mu\nu} \sim \int_0^1 \frac{f_i(\xi)}{\xi} Q_i^2 \xi P_\alpha (\xi P_\beta + q_\beta) \frac{\mathrm{Tr}[\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu]}{(\xi P + q)^2 + i\varepsilon} + (\mu \leftrightarrow \nu, q \leftrightarrow -q)$
- In the SME: $T^{\mu\nu} \sim \int_0^1 \frac{f_i(\xi, \cdots)}{\xi} Q_i^2 \xi P_\alpha (\xi P_\beta + q_\beta) \frac{\mathrm{Tr}[\Gamma^\alpha \Gamma^\mu \Gamma^\beta \Gamma^\nu]}{(\xi P + \tilde{q})^2 + i\varepsilon} + (\mu \leftrightarrow \nu, q \leftrightarrow -q)$

  where $\Gamma^\mu = \gamma^\mu + c^{\mu\nu}\gamma_\nu$ and $W^{\mu\nu} = \mathrm{Im} T^{\mu\nu}$

  - The trace in the numerator is simply expanded keeping only linear terms in $c^{\mu\nu}$
  - We need the imaginary part of the denominator:

$$\frac{1}{\pi}\mathrm{Im}\frac{1}{(\xi P + \tilde{q})^2 + i\varepsilon} = \delta(\tilde{q}^2 + 2\xi P \cdot \tilde{q}) = \frac{1}{2P\cdot q}\left[\delta(\xi - x) + \underset{\uparrow}{\delta'}(\xi - x) c^{\mu\nu} H_{\mu\nu}\right]$$

Yields terms proportional to the derivative of the PDFs

Deep Inelastic Scattering: Standard Model (OPE)

- e-P DIS admits a rigorous description using the OPE and dispersion relations
- Using the optical theorem one can write (J_μ is the electromagnetic current):

$$d\sigma \sim |M(eP \to eX)|^2 \sim \mathrm{Im}[M(eP \to eP)] \sim \mathrm{Im}\langle P | T J_\mu(z) J_\nu(0) | P \rangle$$

- We need the product of currents at small z^2 but the OPE is only valid at small z^μ
- The two regions (which correspond to $\frac{2P\cdot q}{-q^2} = \frac{1}{x} > 1$ and $\frac{2P\cdot q}{-q^2} \sim 0$) are connected by dispersion relations (ITEP sum rules)
- The OPE reads: $T J_\mu(z) J_\nu(0) \sim C^{\mu\nu\mu_1\cdots\mu_n} O_{\mu_1\cdots\mu_n}$

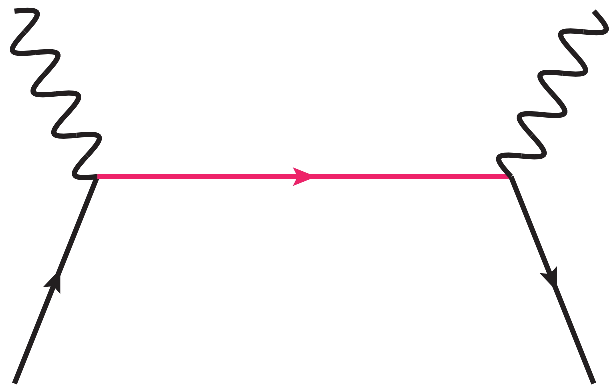
$$C^{\mu\nu\mu_1\cdots\mu_n} \sim \frac{q^{\mu_1}}{Q^2}\cdots\frac{q^{\mu_n}}{Q^2}\left(\frac{q^\mu q^\nu}{Q^2} - g^{\mu\nu}\right) + \cdots$$

$$O_{\mu_1\cdots\mu_n} = \bar{q}\gamma_{\mu_1} iD_{\mu_2}\cdots iD_{\mu_n} q + \text{symmetrizations} - \text{traces}$$

$$\boxed{\langle P | O_{\mu_1\cdots\mu_n} | P \rangle = A_n P_{\mu_1}\cdots P_{\mu_n}}$$

Deep Inelastic Scattering: SME (OPE)

- We seek the OPE for the product of two electromagnetic currents:



$$\bar\psi_f(x)\Gamma^\mu_f \frac{i(i\tilde{\not\partial} + \tilde{\not q})}{(i\tilde\partial + \tilde q)^2}\Gamma^\nu_f \psi_f(0)$$

$$\boxed{\begin{aligned}\tilde q^\mu &= (g^{\mu\nu} + c^{\mu\nu}) q_\nu \\ \tilde\gamma^\mu &= (g^{\mu\nu} + c^{\mu\nu})\gamma_\nu\end{aligned}}$$

- Expand: $\displaystyle \frac{1}{(i\tilde\partial + \tilde q)^2} = \frac{1}{\tilde q^2}\sum_{n=0}^{\infty}\left(-\frac{2i\tilde q\cdot\tilde\partial}{\tilde q^2}\right)^n + O(\tilde\partial^2/\tilde q^2)$

- Operator basis: $\hat O_{\mu_1\cdots\mu_n} = \bar q \gamma_{\mu_1} i\tilde D_{\mu_2}\cdots i\tilde D_{\mu_n} q$ + symmetrizations − traces

  * Why symmetric? Why are traces suppressed?

In the SM this follows directly from the fact that the matrix elements of the operators are functions of the sole proton momentum:
only matrix elements of symmetric operator are non-vanishing and traces are proportional to $P^2 = m_p^2 \ll Q^2$

Deep Inelastic Scattering: SME (OPE)

- The perturbative evaluation of the matrix elements of the SME operators between on-shell SME quark states with momentum k ($\tilde{k}^2 = 0$) yields:
  $\langle k | \bar{q}\gamma^{\mu_1} i\tilde{\partial}^{\mu_2} \cdots i\tilde{\partial}^{\mu_n} q | k \rangle \propto \tilde{k}^{\mu_1} \cdots \tilde{k}^{\mu_n}$ $\Rightarrow$ totally symmetric and traceless
- For n=2: $\hat{O}^{\mu_1\mu_2} = \bar{q}\gamma_\alpha i\tilde{D}_\beta q \ \left(g^{\alpha\mu_1}g^{\beta\mu_2} + g^{\alpha\mu_2}g^{\beta\mu_1} - 2g^{\alpha\beta}g^{\mu_1\mu_2}\right)$

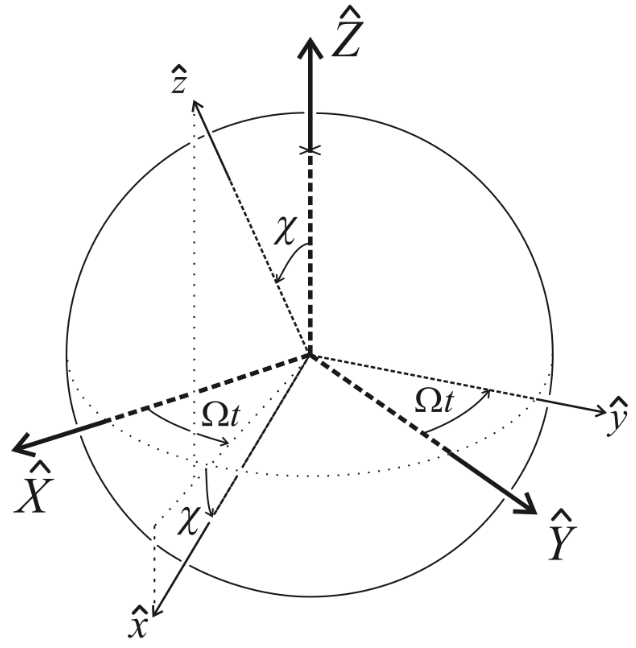
$$= \underbrace{\boxed{\bar{q}\tilde{\gamma}_\alpha i D_\beta q}}_{= T_{\alpha\beta}} \left(g^{\alpha\mu_1}g^{\beta\mu_2} + g^{\alpha\mu_2}g^{\beta\mu_1} - 2g^{\alpha\beta}g^{\mu_1\mu_2} + \text{antisymm in } \alpha, \beta\right)$$

$= T_{\alpha\beta}$ the SME energy-momentum tensor $\Rightarrow \langle P | T_{\alpha\beta} | P \rangle \propto P_\alpha P_\beta$

$$\langle P | \hat{O}^{\mu_1\mu_2} | P \rangle = \langle P | T_{\alpha\beta} | P \rangle \left(g^{\alpha\mu_1}g^{\beta\mu_2} + g^{\alpha\mu_2}g^{\beta\mu_1} - 2g^{\alpha\beta}g^{\mu_1\mu_2} + \text{antisymm in } \alpha, \beta\right)$$
$$\propto P^{\mu_1}P^{\mu_2}$$

- All of this strongly suggests: $\langle P | \hat{O}^{\mu_1\cdots\mu_n} | P \rangle = 2A_n P^{\mu_1} \cdots P^{\mu_n}$
- The matrix elements A_n are the moments of the quarks PDFs and can depend on scalar quantities like $c_{\mu\nu}P^\mu P^\nu / \Lambda^2$
- Putting everything together reproduces exactly the factorization result

Sun-centered vs lab frames



- The tensor $c_{\mu\nu}$ as it appears in our equations is related to the corresponding tensor in the non-rotating inertial frame by a spatial rotation:

$$\mathcal{R} = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \mp 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos\chi\cos\omega_\oplus T_\oplus & \cos\chi\sin\omega_\oplus T_\oplus & -\sin\chi \\ -\sin\omega_\oplus T_\oplus & \cos\omega_\oplus T_\oplus & 0 \\ \sin\chi\cos\omega_\oplus T_\oplus & \sin\chi\sin\omega_\oplus T_\oplus & \cos\chi \end{pmatrix}$$

  where χ is the colatitude of the collider (HERA, JLEIC, eRHIC), $\omega_\oplus = 2\pi/(23h56m)$ is the sidereal frequency, $T_\oplus$ is the local sidereal time, φ is the orientation of the beams at the colliding points (usually two per collider)

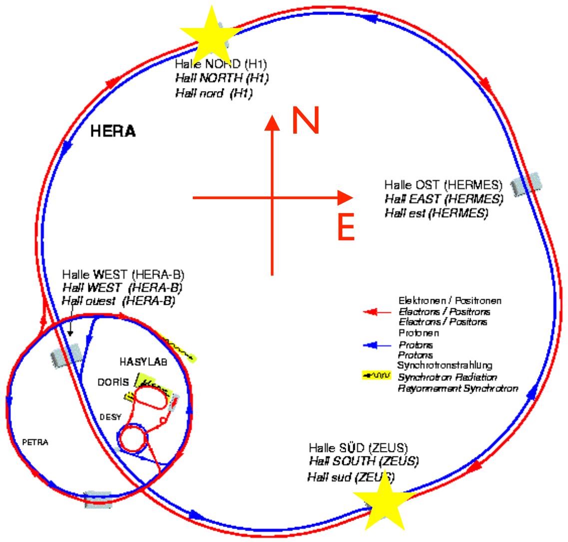
- The c_f^{ij} and c_f^{0i} components of the $c_f^{\mu\nu}$ tensor are given by $c_f^{KL} R_{Ki} R_{Lj}$ and $c_f^{TK} R_{iK}$, where c_f^{AB} $(A, B = T, X, Y, Z)$ is the tensor in the Sun-centered frame.

- The structure of the time dependent DIS cross section is:

$$\sigma(T_\oplus) = \sigma_{\mathrm{SM}} \Big[1 + (c_f^{TT},\ c_f^{TZ},\ c_f^{ZZ}) + (c_f^{TX},\ c_f^{TY},\ c_f^{YZ},\ c_f^{XZ})(\cos\omega_\oplus T_\oplus,\ \sin\omega_\oplus T_\oplus) + (c_f^{XY},\ c_f^{XX} - c_f^{YY})(\cos 2\omega_\oplus T_\oplus,\ \sin 2\omega_\oplus T_\oplus) \Big]$$

Sun-centered vs lab frames

HERA

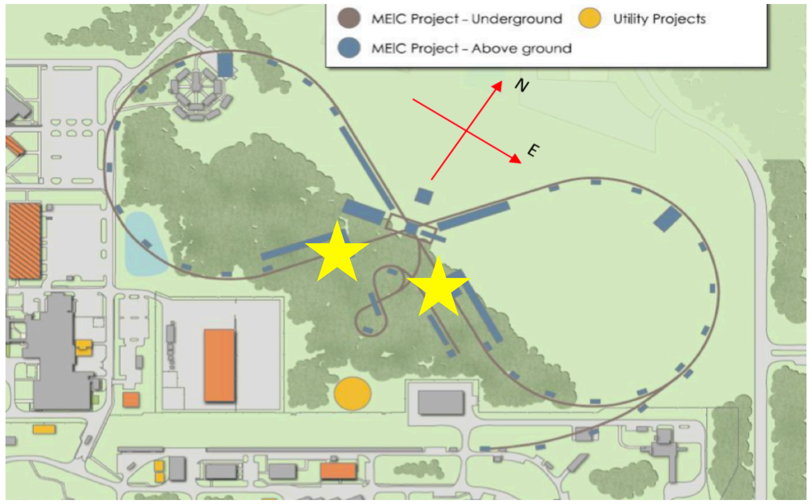


$\chi=36.4^o$

$\varphi_{ZEUS}=20^o$ NoE

$\varphi_{H1}=-20^o$ NoE

JLEIC

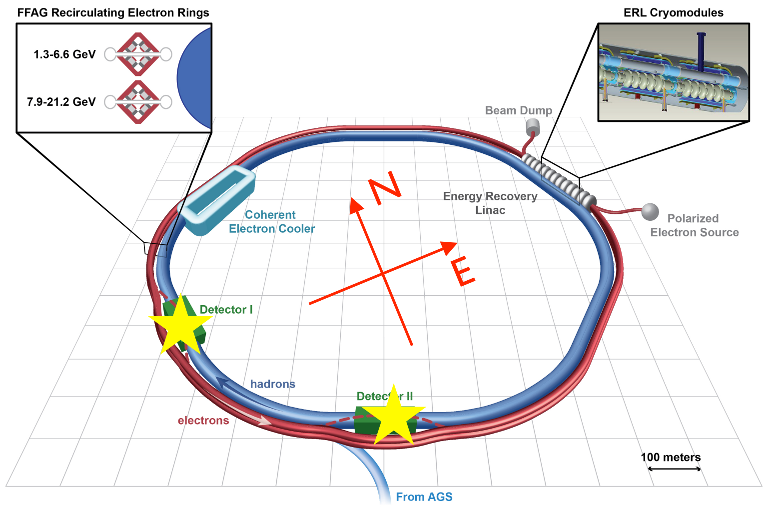


$\chi=52.9^o$

$\varphi_{JLEIC1}=+47.6^o$ NoE

$\varphi_{JLEIC1}=-35.0^o$ NoE

eRHIC



$\chi=49.1^o$

$\varphi_{eRHIC1}=-78.5^o$ NoE

$\varphi_{eRHIC2}=-16.8^o$ NoE

- For instance, the LV part of total integrated cross section is schematically: $\sigma_{LV} \sim \# \, c_{TZ} \sin(\varphi) + \# \, c_{ZZ} \cos(2\varphi)$ where # is of order 1. This implies that $\varphi{\sim}90^o$ (eRHIC1) and $\varphi{\sim}45^o$(JLEIC1) loose sensitivity to c_{TZ} and c_{ZZ}, respectively!

Expected constraints on LV couplings: DIS

- For HERA, we consider 644 neutral current measurements performed by *ZEUS* and *H1* [arXiv:1506.06042]
- Parameters of the two proposed EIC designs and simulated data-set that we use

	JLEIC	eRHIC	HERA
Location	Jefferson Lab	Brookhaven	DESY
Luminosity	10^{34} cm^{-2} s^{-1}	10^{33} cm^{-2} s^{-1}	4×10^{31} cm^{-2} s^{-1}
E_e (GeV)	[3,12]	[5,20]	27.5
E_p (GeV)	[20,100]	[50,250]	920
simulated [E_e,E_p]	$E_e = 10$ $E_p = (20,60,80,100)$	$E_e = (5,10,15,20)$ $E_p = (50,100,250)$	existing data sets

The expected bounds we present are calculated considering 10 years running (which correspond to $1\ \mathrm{ab}^{-1}$ for JLEIC and $100\ \mathrm{fb}^{-1}$ for eRHIC)

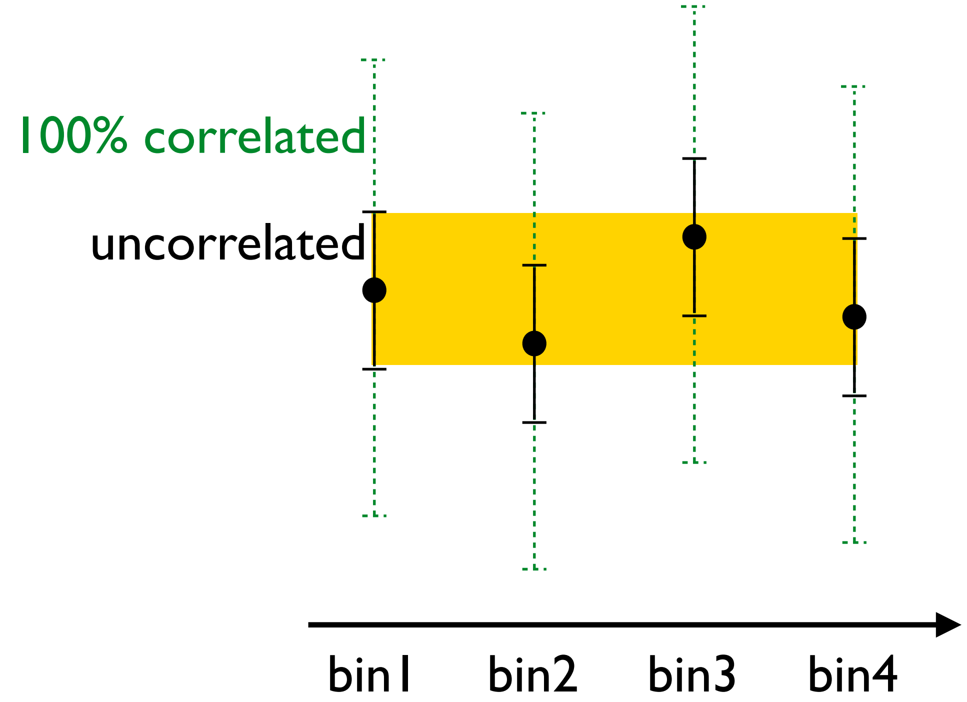
Expected constraints on LV couplings from DIS

- For HERA, we consider 644 neutral current measurements performed by *ZEUS* and *H1* [arXiv:1506.06042]
- For the two EIC configurations, we use Monte Carlo simulations corresponding to an integrated luminosity of 100 fb^{-1} [A. Accardi, Y. Furletova, E. Aschenauer, B. Page]

- ***For each measurement (i.e. each x and Q^2 bin):***
  - We estimate how the uncertainty increases due to a sidereal binning (4 bins)
  - We calculate the binned integrals of the SME DIS reduced cross section
  - We generate a set of 10^3 possible experimental results assuming a normal distribution and the absence of LV effects. Systematic and luminosity uncertainties among the sidereal time bins are expected to be highly correlated. We consider both the extreme cases of 0% (aggressive) and 100% (conservative) correlation.
  - For each set we extract the frequentist 95% C.L. upper limit using a standard chi-squared.
  - The expected upper limit is the median of the upper limits over the set

Expected constraints on LV couplings from DIS

- In the calculation of these expected constraints we have assumed very conservatively that the total uncertainties on the cross sections measured in the sidereal time bins are **uncorrelated**.
- The constraints on coefficients which induce sidereal time variation are sensitive only to uncorrelated uncertainties:



Note that each sidereal time bin collects several months worth of data

- We need to quantify the correlation of systematic uncertainties between the various sidereal time bins
- Note that day/nights effects are diluted by the sidereal time binning if data are taken over a long enough period

Expected constraints on LV couplings from DIS

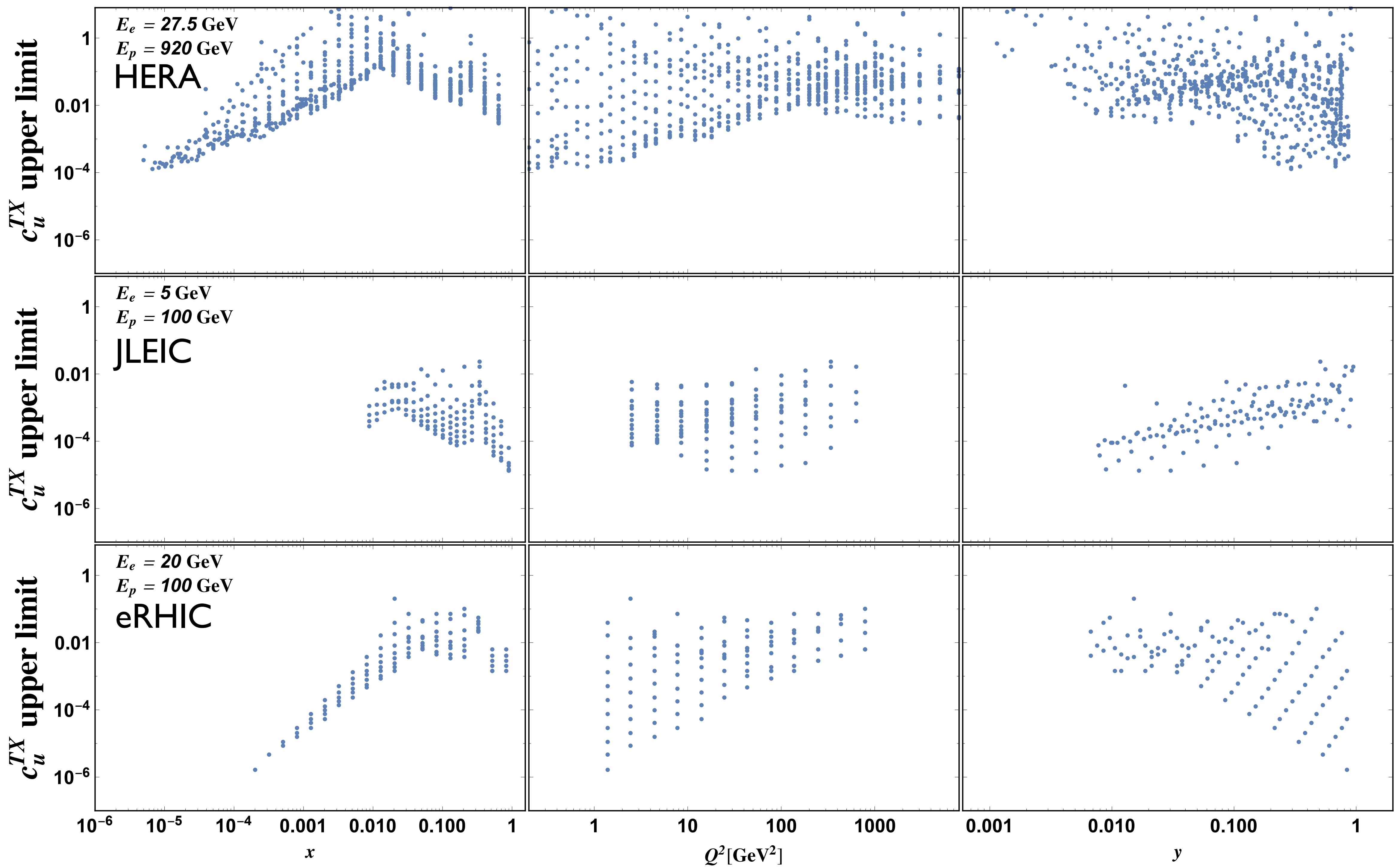
- Expected bounds in units of 10^{-5}

	HERA Individual	HERA Global	JLEIC Individual	JLEIC Global	eRHIC Individual	eRHIC Global
$\lvert c_u^{TX}\rvert$	13. [13.]	4.0 [4.4]	1.3 [9.3]	0.57 [4.3]	0.17 [22.]	0.096 [9.7]
	13. [13.]	4.0 [4.4]	1.2 [8.5]	0.51 [3.9]	0.12 [15.]	0.067 [6.6]
$\lvert c_u^{TY}\rvert$	13. [13.]	4.0 [4.3]	1.3 [9.1]	0.55 [4.3]	0.17 [22.]	0.096 [9.6]
	13. [13.]	4.0 [4.3]	1.1 [8.4]	0.50 [3.9]	0.12 [15.]	0.067 [6.6]
$\lvert c_u^{XZ}\rvert$	63. [66.]	20. [21.]	1.9 [13.]	0.79 [6.0]	0.23 [30.]	0.13 [12.]
	63. [66.]	20. [21.]	2.1 [16.]	0.91 [7.0]	0.53 [70.]	0.30 [28.]
$\lvert c_u^{YZ}\rvert$	65. [65.]	20. [21.]	1.8 [13.]	0.76 [6.1]	0.22 [30.]	0.13 [12.]
	65. [65.]	20. [21.]	2.1 [16.]	0.90 [7.1]	0.53 [70.]	0.30 [28.]
$\lvert c_u^{XY}\rvert$	31. [33.]	9.8 [10.]	6.9 [50.]	2.8 [23.]	0.61 [82.]	0.35 [32.]
	31. [33.]	9.8 [10.]	3.2 [23.]	1.3 [10.]	0.26 [34.]	0.15 [13.]
$\lvert c_u^{XX} - c_u^{YY}\rvert$	98. [100.]	31. [33.]	5.9 [43.]	2.4 [19.]	1.8 [240.]	1.0 [92.]
	98. [100.]	31. [33.]	6.3 [45.]	2.6 [20.]	1.2 [170.]	0.72 [65.]

- For each coefficient and collider we show bounds corresponding to the two experiments. In square bracket we show the uncorrelated bounds.
- Bounds in the down sector are an order of magnitude weaker (electric charge, PDFs)

Expected constraints on LV couplings from DIS

- The best bounds tend to come from small x, large y and moderate Q²



Drell-Yan: SME

- Factorization for Drell-Yan in the SME is achieved following the same steps as in the DIS case. In particular, the PDF's are identical to those obtained for DIS.

- The cross section is particularly simple:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9Q^4} \sum_{j=u,d,s,\cdots} e_j^2 \left[\left(1 - c_j^{00} + c_j^{33}\right) \int_\tau^1 \frac{\tau dx}{x} \left(f_j(x) f_{\bar{j}}(\tau/x) + f_j(\tau/x) f_{\bar{j}}(x)\right) \right.$$
$$- \int_\tau^1 \frac{\tau dx}{x^2} \left[\left(x - \frac{\tau}{x}\right) \left(c_j^{00} + c_j^{33}\right) \right] \left(f_j(x) f_{\bar{j}}(\tau/x) + f_j(\tau/x) f_{\bar{j}}(x)\right)$$
$$\left. + \int_\tau^1 \frac{\tau dx}{2x^2} \left[\left(x - \frac{\tau}{x}\right)^2 \left(c_j^{00} + c_j^{33}\right) \right] \left(f_j(x) f'_{\bar{j}}(\tau/x) + f'_j(\tau/x) f_{\bar{j}}(x)\right) \right]$$

- Using Drell-Yan data from CMS for $Q^2 < 60\ GeV^2$, we obtain (in units of 10^{-5}):

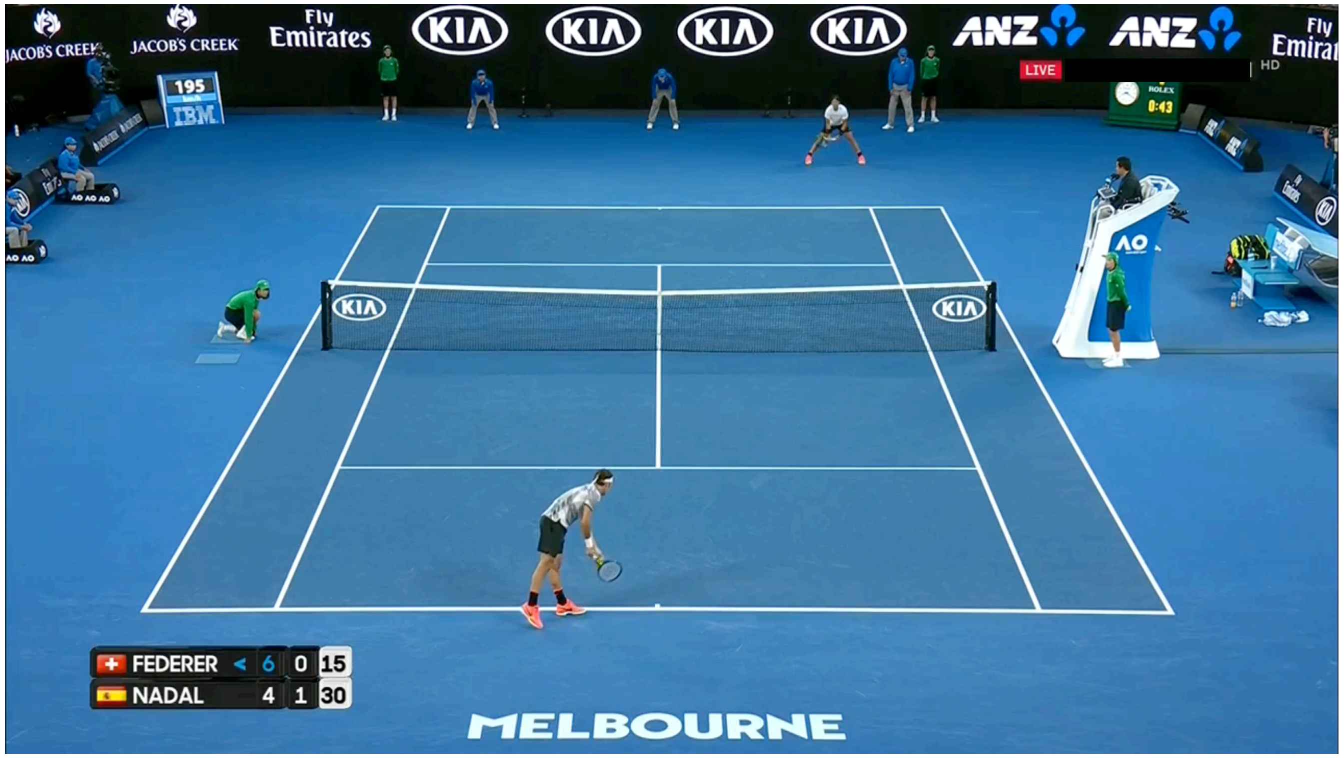
	Individual	**Global**
$\lvert c_u^{XZ} \rvert$	7.3 [19]	6.1 [15]
$\lvert c_u^{YZ} \rvert$	7.1 [19]	6.1 [15]
$\lvert c_u^{XY} \rvert$	2.7 [7.0]	2.3 [5.7]
$\lvert c_u^{XX} - c_u^{YY} \rvert$	15 [39]	12.9 [32]

Conclusions

- The Standard Model Extension is a generic extension of the SM that incorporates particle Lorentz Violation while preserving Lorentz covariance
- Coefficients in the photon, electron, muon, proton and neutron sectors are strongly constrained.
- The quark sector is much harder to constraint because of the nature of QCD
- We focused on electron-proton Deep Inelastic Scattering and Drell-Yan for which high statistics measurements exist (and are possible in the future) and found that bounds in the $10^{-5,6}$ range are attainable using existing HERA/LHC and future EIC data.
- Analysis of a subset of Zeus data is undergoing
- Future studies include
  - Impact on PDFs (standard and polarization dependent)
  - Inclusion of weak effects (Z-pole observables, …)

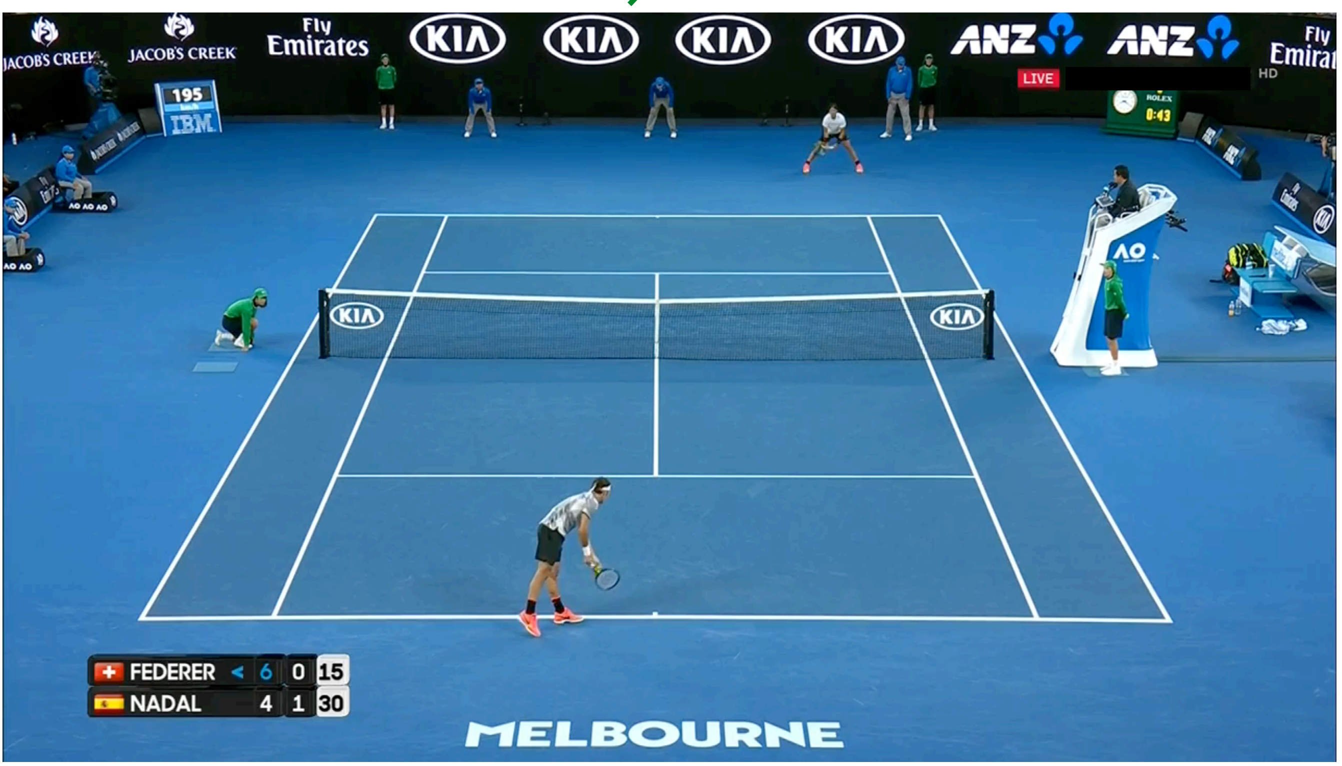
back-up slides

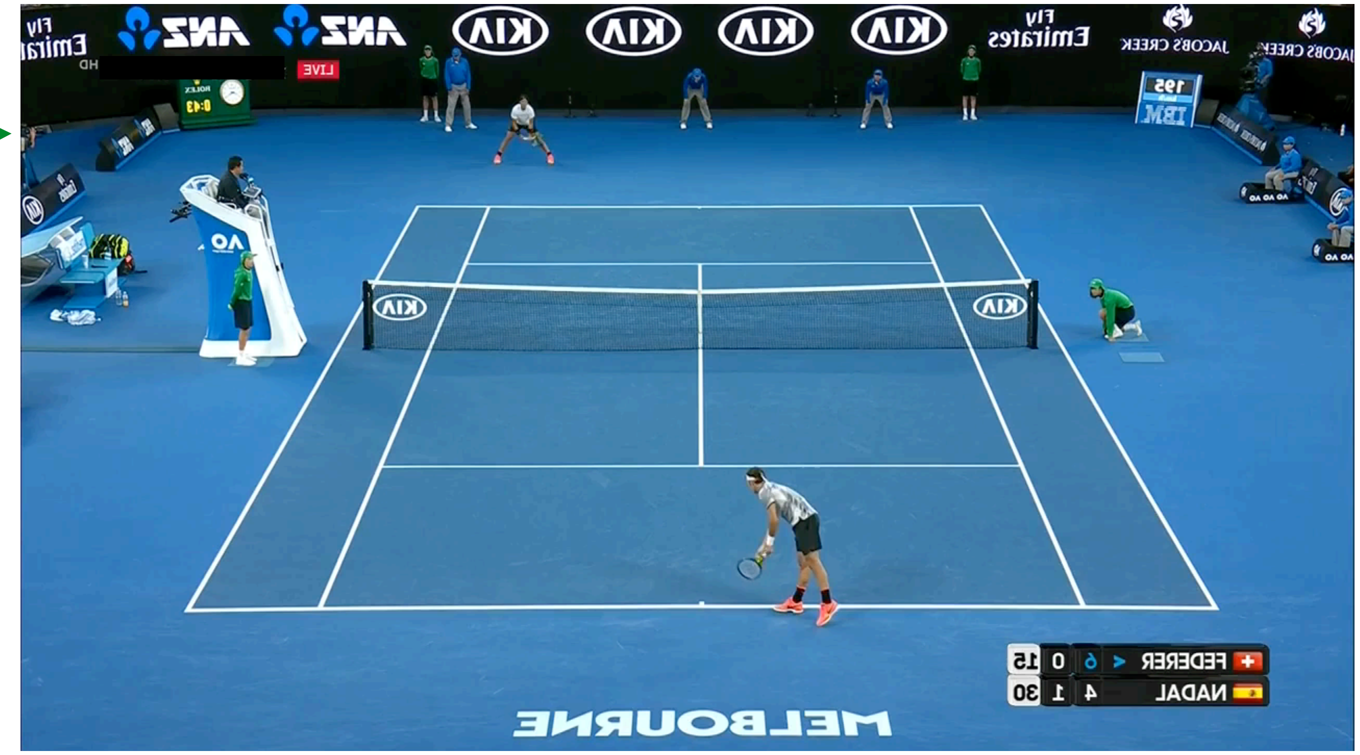
Symmetry Transformations: Parity



Symmetry Transformations: Parity

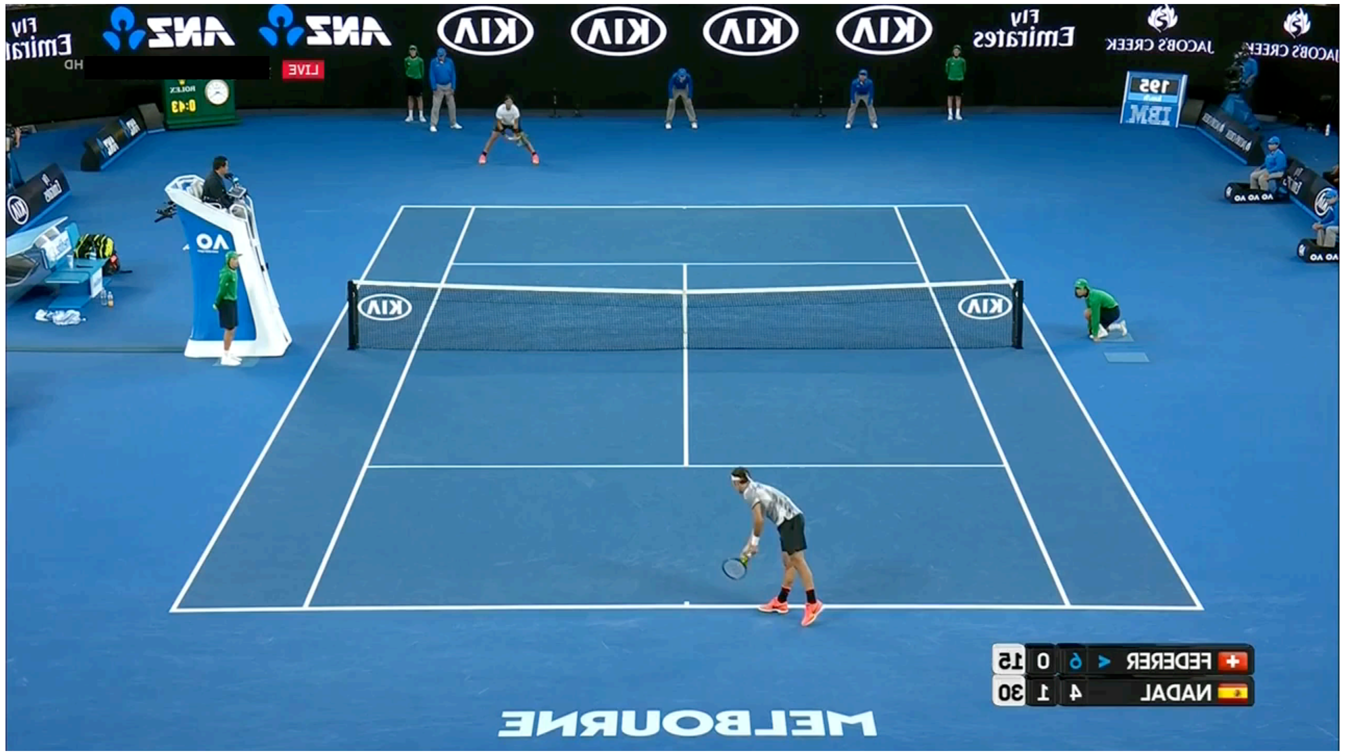
Observer transformation

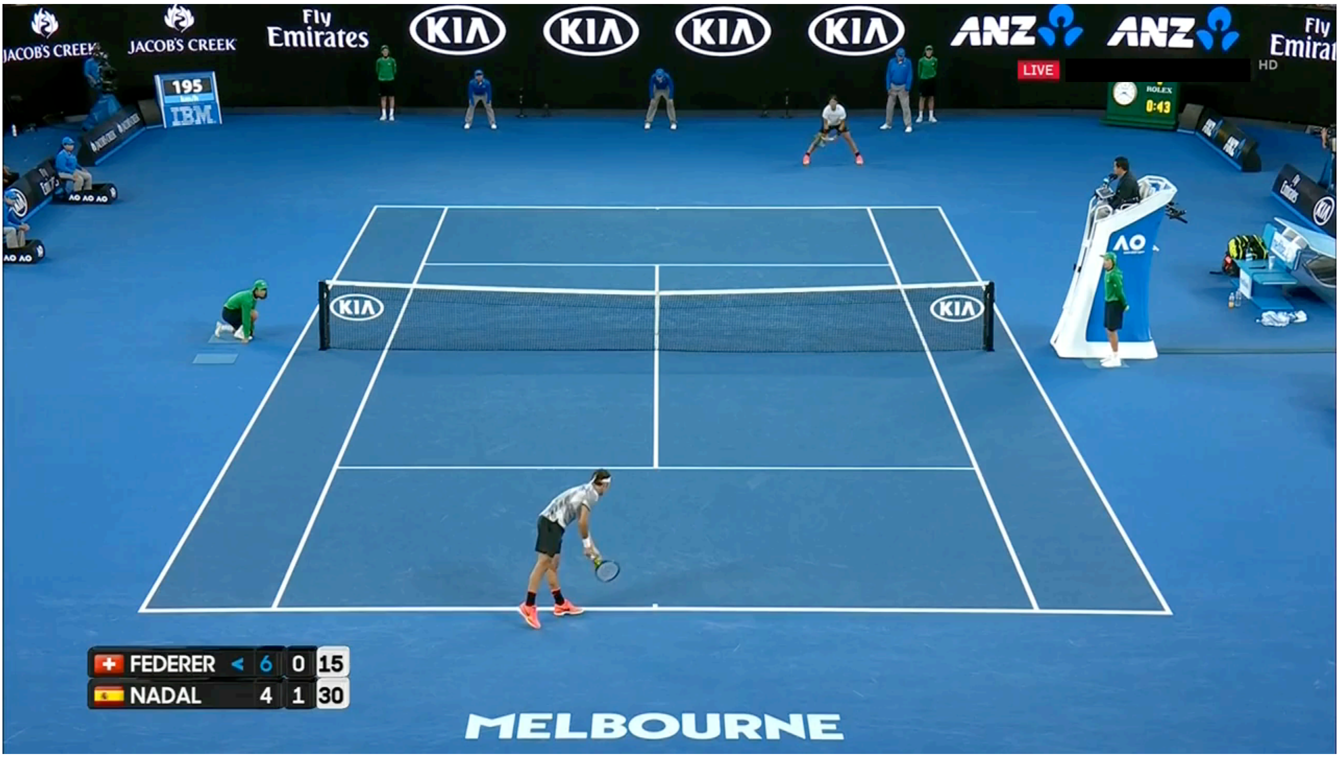




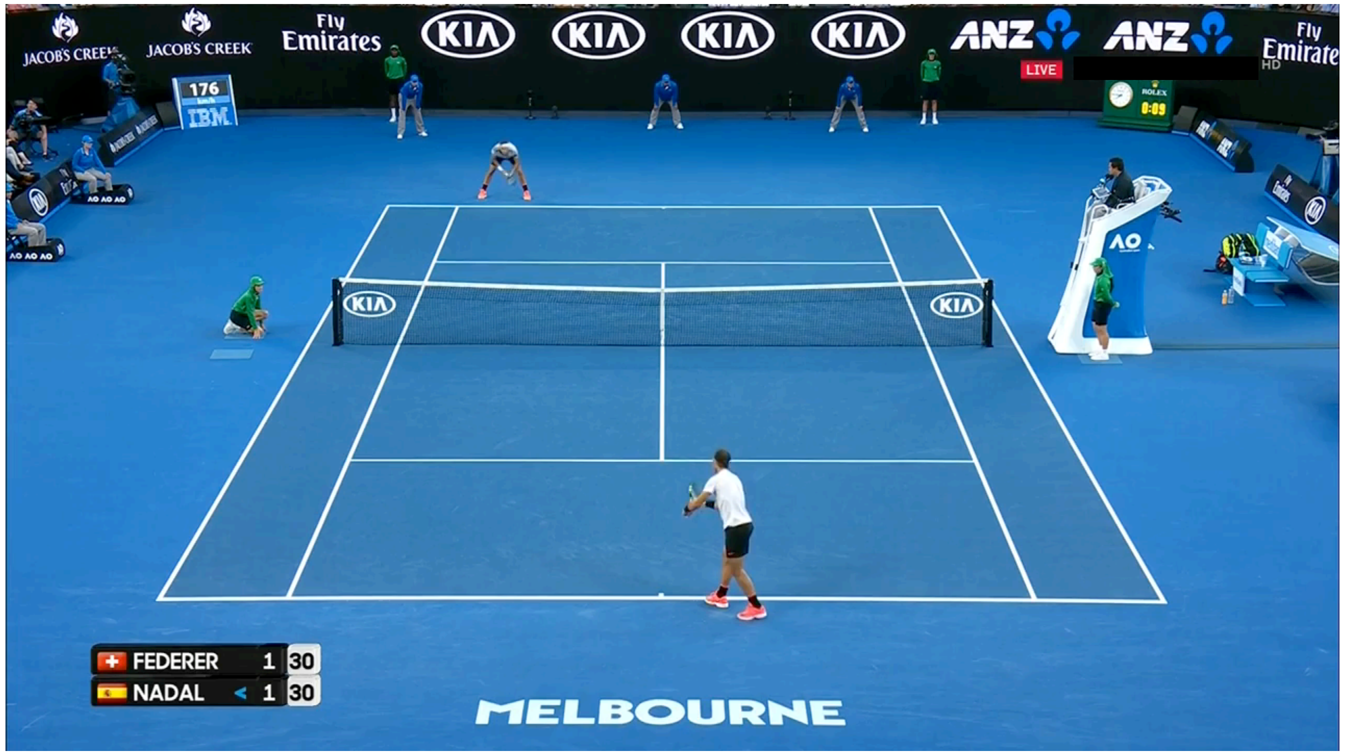
Symmetry Transformations: Parity

Observer transformation





Particle transformation



Using χ_{PT} to connect LV in quarks and hadrons

[Kamand, Altschul, Schindler; 1608.06503]

- The SME terms we are considering, can be written as:

$$\delta\mathcal{L}_{\rm SME} = i\bar{Q}_L C_L^{\mu\nu}\gamma_\mu D_\nu Q_L + i\bar{Q}_R C_R^{\mu\nu}\gamma_\mu D_\nu Q_R$$

where $C^{\mu\nu}_{L/R} = \begin{bmatrix} c^{\mu\nu}_{u_{L/R}} & 0 \\ 0 & c^{\mu\nu}_{d_{L/R}} \end{bmatrix}$ and $c_q^{\mu\nu} = (c^{\mu\nu}_{q_L} + c^{\mu\nu}_{q_R})/2$

- Strong Isospin invariance ($Q_L \to L\, Q_L$, $Q_R \to R\, Q_R$) can be restored by assigning:

$$C_L^{\mu\nu} \to L C_L^{\mu\nu} L^\dagger, \quad C_R^{\mu\nu} \to R C_R^{\mu\nu} R^\dagger$$

- One can then use these spurions to add LV terms to the Chiral Lagrangian:

$$\begin{aligned}
\mathcal{L}^{\rm LO}_\pi &= \beta^{(1)}\frac{F^2}{4}\left({}^1C_{R\mu\nu} + {}^1C_{L\mu\nu}\right)\mathrm{Tr}[(\partial^\mu U)^\dagger\partial^\nu U] \\
\mathcal{L}^{\rm LO}_{\pi N} &= \Big\{\alpha^{(1)}\bar{\Psi}[(u^\dagger\, {}^3C_R^{\mu\nu}u + u\, {}^3C_L^{\mu\nu}u^\dagger)(\gamma_\nu iD_\mu + \gamma_\mu iD_\nu)]\Psi \\
&\quad + \alpha^{(2)}\left({}^1C_R^{\mu\nu} + {}^1C_L^{\mu\nu}\right)\bar{\Psi}(\gamma_\nu iD_\mu + \gamma_\mu iD_\nu)]\Psi \\
&\quad + \alpha^{(3)}\bar{\Psi}[(u^\dagger\, {}^3C_R^{\mu\nu}u - u\, {}^3C_L^{\mu\nu}u^\dagger)(\gamma_\nu\gamma^5 iD_\mu + \gamma_\mu\gamma^5 iD_\nu)]\Psi \\
&\quad + \alpha^{(4)}\left({}^1C_R^{\mu\nu} - {}^1C_L^{\mu\nu}\right)\bar{\Psi}(\gamma_\nu\gamma^5 iD_\mu + \gamma_\mu\gamma^5 iD_\nu)\Psi\Big\},
\end{aligned}$$

pions: $U = exp(i\sum\phi_a\tau_a/F)$

nucleon doublet

where $u^2 = U$, ${}^1C^{\mu\nu}_{L,R}$ and ${}^3C^{\mu\nu}_{L,R}$ are the trace and traceless parts of $C^{\mu\nu}_{L,R}$ and transform as

$$\begin{aligned}
{}^1C_L^{\mu\nu} &\to {}^1C_L^{\mu\nu}, \quad {}^3C_L^{\mu\nu} \to L\,{}^3C_L^{\mu\nu}L^\dagger, \\
{}^1C_R^{\mu\nu} &\to {}^1C_R^{\mu\nu}, \quad {}^3C_R^{\mu\nu} \to R\,{}^3C_R^{\mu\nu}R^\dagger
\end{aligned}$$

Using χ_{PT} to connect LV in quarks and hadrons

[Kamand, Altschul, Schindler; 1608.06503]

- Relevant two and four pion interactions (no three pion vertices):

$$\mathcal{L}_\pi^{\mathrm{LO},2\phi} = \frac{\beta^{(1)}}{2}(c^{\mu\nu}_{u_L} + c^{\mu\nu}_{d_L} + c^{\mu\nu}_{u_R} + c^{\mu\nu}_{d_L})\partial_\mu\phi_a\partial_\nu\phi_a$$

$$\mathcal{L}_\pi^{\mathrm{LO},4\phi} = \frac{\beta^{(1)}}{6F^2}(c^{\mu\nu}_{u_L} + c^{\mu\nu}_{d_L} + c^{\mu\nu}_{u_R} + c^{\mu\nu}_{d_L})(\phi_a\phi_b\partial_\mu\phi_a\partial_\nu\phi_b - \phi_b\phi_b\partial_\mu\phi_a\partial_\nu\phi_a)$$

- The proton free Lagrangian becomes

$$\delta L_{\mathrm{SME}} = \bar{\psi}_p\left[(\eta^{\mu\nu} + c^{\mu\nu}_p)\gamma_\nu iD_\mu - m_p\right]\psi_p$$

with

$$c^{\mu\nu}_p = \left[\frac{1}{2}\alpha^{(1)} + \alpha^{(2)}\right](c^{\mu\nu}_{u_L} + c^{\mu\nu}_{u_R}) + \left[-\frac{1}{2}\alpha^{(1)} + \alpha^{(2)}\right](c^{\mu\nu}_{d_L} + c^{\mu\nu}_{d_R})$$

- The $\alpha^{(1,2)}$ and $\beta^{(1)}$ coefficients are non-perturbative and expected to be $O(1)$
- If this is accurate the bounds on these coefficients are of order $O(10^{-25}\div 10^{-20})$

Potential problem

- There are questions related to the role of LV in the gluon sector: it is possible to move $c^{\mu\nu}_q$ ($q=u$ or d) into a $\kappa^\alpha{}_{\mu\alpha\nu}$. It is not clear how to assign spurion transformation properties to the latter.

Physical coefficients

- Not all coefficients introduced above are physical
- Some coefficients can be eliminated via a field redefinitions like:

  $\psi(x) \to e^{if(x)}\psi(x)$

  $\psi(x) \to [1 + v(x) \cdot \Gamma]\psi(x) \quad \text{with} \quad \Gamma = \gamma^\alpha,\ \gamma_5\gamma^\alpha,\ \sigma^{\alpha\beta}$

  $\Rightarrow$ in this way a_μ and the antisymmetric part of $c_{\mu\nu}$ can be eliminated

- Some parts of the coefficients are not LV. For instance, even after removing its antisymmetric part we have: $c_{\mu\nu} = \underbrace{[c_{\mu\nu}]_{traceless\ \&\ symmetric}}_{\text{L.V.}} + \underbrace{\alpha\ \eta_{\mu\nu}}_{\text{L.I.}}$

- Some coefficients can be eliminated via a choice of coordinates:

$$\mathcal{L} = -\frac{1}{4}\left(\kappa^{\kappa\lambda\mu\nu} + \eta^{\kappa\mu}\eta^{\lambda\nu}\right)F_{\kappa\lambda}F_{\mu\nu} + \left(\eta^{\mu\nu} + c^{\mu\nu}\right)\bar{\psi}\gamma_\mu iD_\nu\psi$$

$$\Big\downarrow\ x^\mu \to x^\mu - \frac{1}{2}\kappa^{\alpha\mu}{}_{\alpha\nu}x^\nu$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left(\eta^{\mu\nu} + c^{\mu\nu} + \frac{1}{2}\kappa^{\alpha}{}_{\mu\alpha\nu}\right)\bar{\psi}\gamma_\mu iD_\nu\psi$$

We can choose one sector of the SME to define the scales of the four coordinates

LV in the quark sector: our setup

- We will focus on Deep Inelastic electron-proton Scattering: we are affected by coefficients that appear in the electron ($c_e^{\mu\nu}$), photon ($\kappa_F^{\alpha\beta\mu\nu}$), quark ($c_{u,d}^{\mu\nu}$) and gluon ($\kappa_G^{\alpha\beta\mu\nu}$) sectors.
  - We adopt coordinates in which the photon does not show any spin-independent Lorentz violation ($\kappa_F{}^{\alpha}{}_{\mu\alpha\nu} = 0$)
  - All coefficients in the electron sector are strongly constrained and do not contribute appreciably to electron-proton DIS
  - We set the gluon coefficients to zero and focus on the quarks
- Assuming spontaneous Lorentz violation at scales of order M_{Planck}, we expect "natural" size for most coefficients to be given by the ratio of some low energy mass to M_{Planck}. Nevertheless the need to perform direct experimental searches should not be understated.
- Note that the quark coefficients contribute via divergent loops to LV in the photon and electron sector. This does not change our fundamental set up but might rise issues of fine tuning/naturalness in the electron sector. We leave a detailed investigation of this point to a forthcoming analysis.

LV in the quark sector: general considerations

- In our case: $\mathcal{L} = \bar{q}\left[\frac{1}{2}\gamma_\mu (g^{\mu\nu} + c^{\mu\nu}) i\overleftrightarrow{D_\nu} - m\right] q$
- The quark dispersion relation is modified:

$$0 = \tilde{p}^\mu \tilde{p}_\mu - m^2 = p_\mu (\eta^{\mu\nu} + c^{\mu\nu})(\eta_{\nu\lambda} + c_{\nu\lambda}) p^\lambda - m^2$$

  - Velocity ($\vec{v} = \vec{\nabla}_p E(\vec{p})$) and momentum are not parallel anymore
  - Sums and averages over quark spins are affected
  - The cross section flux factor is also affected (must use velocities of colliding particles!)
  - In our case the relevant flux factor involves the electron and proton, both of which receive negligible Lorentz violating effects (given the kind of constraints that we will be able to achieve)

- We treat the traceless tensor $c_{\mu\nu}$ as a small perturbation resulting in a standard Feynman diagram expansion
- The modified dispersion relation (momentum and velocity are not parallel anymore) creates difficulties for a straightforward parton model implementation: one reason for focusing on DIS is the dual parton model and OPE approach

Planned HERA analysis

- We (Nathan Sherrill and I) proposed to re-analyze ZEUS data to place bounds on Lorentz Violating coefficients and have been allowed to join the collaboration
- This entails binning all data in sidereal time and to perform independent measurements in each bin
- The main obstacle is understanding the time-dependence of systematic uncertainties. E.g. at HERA we have a measurement of the average instantaneous luminosity but it is difficult to estimate the induced uncorrelated uncertainties in the various sidereal time bins
- The main idea is to use the fact that different (x,Q) bins have very different dependence on LV coefficients. By considering the following double ratio (i indexes the sidereal time bin, the reference point has poor sensitivity on the LV coefficients):

$$\left(\frac{\sigma_i(x,Q)}{\sigma_1(x,Q)} \right) / \left(\frac{\sigma_i(\bar{x},\bar{Q})}{\sigma_1(\bar{x},\bar{Q})} \right)$$

- $i/1$ ratios eliminates point-to-point systematic uncertainties
- Normalization to reference point eliminates luminosity uncertainties