Hard photoproduction of a photon pair with a large invariant mass

Aleksandra Pędrak[1], B. Pire[2], L. Szymanowski[1], J. Wagner[1]

[1] National Centre for Nuclear Research (NCBJ), Warsaw, Poland
[2] Centre de Physique Theorique, Ecole Polytechnique, CNRS, Palaiseau, France

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Plan of presentation

- Hard photoproduction of a diphoton with a large invariant mass;
  - Kinematics;
  - Differential cross section;
- Further research;
  - Electroproduction (Bethe–Heitler process with two–photon emission);
  - Photoproduction of a diphoton in NLO.

Hard photoproduction of a diphoton with a large invariant mass

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Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,[1] B. Pire,[2] L. Szymanowski,[1] and J. Wagner[1]
[1]*National Centre for Nuclear Research (NCBJ), 00681 Warsaw, Poland*
[2]*Centre de Physique Théorique, École Polytechnique, CNRS, 91128 Palaiseau, France*
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The electromagnetic probe has proven to be a very efficient way to access the three-dimensional structure of the nucleon, particularly thanks to the exclusive Compton processes. We explore the hard photoproduction of a large invariant mass diphoton in the kinematical regime where the diphoton is nearly forward and its invariant mass is the hard scale enabling to factorize the scattering amplitude in terms of generalized parton distributions. We calculate unpolarized cross sections and the angular asymmetry triggered by a linearly polarized photon beam.

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I. INTRODUCTION

The last twenty years have witnessed a tremendous progress in the understanding of hard exclusive scattering in the framework of the QCD collinear factorization of hard amplitudes in specific kinematics in terms of generalized parton distributions (GPDs) and hard perturbatively calculable coefficient functions [1,2].

In this paper, we study the exclusive photoproduction of two photons on a unpolarized proton or neutron target

$$v^\mu = an^\mu + bp^\mu + v_\perp^\mu, \tag{2}$$

with p and n the light-cone vectors

$$p^\mu = \frac{\sqrt{s}}{2}(1,0,0,1), \quad n^\mu = \frac{\sqrt{s}}{2}(1,0,0,-1), \quad p \cdot n = \frac{s}{2}, \tag{3}$$

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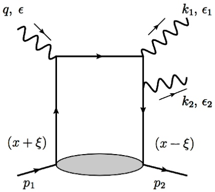
$$\gamma(q,\epsilon) + N(p_1,\lambda) \rightarrow \gamma_1(k_1,\epsilon_1) + \gamma_2(k_2,\epsilon_2) + N'(p_2,\lambda')$$



Motivation:

- the hard part is a pure electromagnetic process;
- is the $2 \rightarrow 3$ process which gives independent information about the GPDs;
- this process (because of the additionnal photon) gives access to C-odd part of GPDs $(H(x,\xi,t) + H(-x,\xi,t))$ and receives no contribution from gluon GPDs;
- lets us give an answer if factorization exists (or not) in this kind of $2 \rightarrow 3$ processes;

Large invariant mass $M_{\gamma\gamma}$: $2.10\, GeV^2 \leq M^2_{\gamma\gamma} \leq 9.47\, GeV^2$
Small momentum transfer $t = (p_2 - p_1)^2$: $-0.5\, GeV^2 \leq t_{min}(M^2_{\gamma\gamma}) \leq 0$

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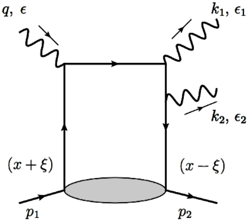

Sudakov basis

$$v^\mu = an^\mu + bp^\mu + v_\perp^m u;$$
$$p = \tfrac{\sqrt{s}}{2}(1,0,0,1),$$
$$n = \tfrac{\sqrt{s}}{2}(1,0,0,-1),$$
$$p \cdot n = \tfrac{s}{2}$$

Mandelstam invariants:

$$t = (p_2 - p_1)^2,$$
$$M_{\gamma\gamma}^2 = (k_1 + k_2)^2$$
$$-u' = -(k_2 - q)^2$$

$$p_1^\mu = (1+\xi)p^\mu + \frac{M^2}{s(1+\xi)}n^\mu, \qquad p_2^\mu = (1-\xi)p^\mu + \frac{M^2-\Delta_\perp^2}{s(1-\xi)}n^\mu + \Delta_\perp^\mu, \qquad q^\mu = n^\mu$$

$$k_1^\mu = \alpha_1 n^\mu + \frac{\left(p_\perp - \frac{\Delta_\perp}{2}\right)^2}{\alpha_1 s}p^\mu + p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}$$

$$k_2^\mu = \alpha_2 n^\mu + \frac{\left(p_\perp + \frac{\Delta_\perp}{2}\right)^2}{\alpha_2 s}p^\mu - p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}$$

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Differential Cros section

$$\frac{d\sigma}{dM^2_{\gamma\gamma}dtd(-u')} = \frac{1}{2}\frac{1}{(2\pi)^3 32 S^2_{\gamma N} M^2_{\gamma\gamma}} \sum_{\lambda_i,\lambda_f\lambda'_f,s_1,s_2} \frac{|\tau|^2}{4},$$

$S_{\gamma N} = (q + p_1)^2, \quad -u' = -(k_2 - q)^2$
Independent kinematical variables $\{t, u', M^2_{\gamma\gamma}\}$,
where

$$\tau = \frac{1}{4}\int_{-1}^{1} dx \sum_q \int dz^- e^{ixz^- P^+} \left\langle P_2 \left| \overline{\psi}_q \left(-\frac{z}{2}\right) \left[CF_q^V \not{n} + CF_q^A \not{n}\gamma^5 \right] \psi_q \left(\frac{z}{2}\right) \right| P_1 \right\rangle =$$

$$= \frac{1}{2}\frac{1}{2P^+}\int_{-1}^{1} dx \sum_q \left[CF_q^V(x,\xi) \left(H^q(x,\xi)\bar{U}(p_2) \not{n} U(p_1) + \right.\right.$$

$$\left. + E^q(x,\xi)\bar{U}(p_2)\frac{i\sigma^{\mu\nu}\Delta_\nu n_\mu}{2M} U(p_1) \right) +$$

$$\left. + CF_q^A(x,\xi) \left(\tilde{H}^q(x,\xi)\bar{U}(p_2) \not{n}\gamma^5 U(p_1) + \tilde{E}^q(x,\xi)\bar{U}(p_2)\frac{i\gamma_5(\Delta\cdot n)}{2M} U(p_1) \right) \right]$$

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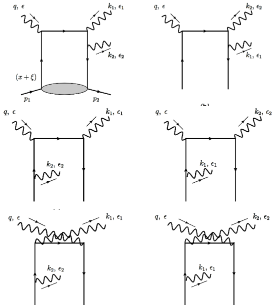
Vector Coefficient Function:

$$iCF_q^V = -ie_q^3\left[A^V\left(\frac{1}{D_1(x)D_2(x)}+\frac{1}{D_1(-x)D_2(-x)}\right)+\right.$$
$$B^V\left(\frac{1}{D_1(x)D_3(x)}+\frac{1}{D_1(-x)D_3(-x)}\right)+$$
$$\left.C^V\left(\frac{1}{D_2(x)D_3(-x)}+\frac{1}{D_2(-x)D_3(x)}\right)\right]$$



(e) (f)

Where:

$$D_1(x) = s(x+\xi+i\varepsilon)$$
$$D_2(x) = s\alpha_2(x-\xi+i\varepsilon)$$
$$D_3(x) = s\alpha_1(x-\xi+i\varepsilon)$$

$$A^V = 2s(V_{k_1} - V_p + \frac{1}{\alpha_1}V_{k2})$$
$$B^V = 2s(-V_{k2} + V_p - \frac{1}{\alpha_2}V_{k_1})$$
$$C^V = 2s((\alpha_2-\alpha_1)V_p + V_{k_2} - V_{k1})$$

$$V_{k_1} = (\epsilon_\perp(q)\cdot\epsilon_\perp^*(k_1))(p_\perp\cdot\epsilon_\perp^*(k_2))$$
$$V_{k_2} = (\epsilon_\perp(q)\cdot\epsilon_\perp^*(k_2))(p_\perp\cdot\epsilon_\perp^*(k_1))$$
$$V_p = (\epsilon_\perp^*(k_1)\cdot\epsilon_\perp^*(k_2))(p_\perp\cdot\epsilon_\perp(q))$$

Hard photoproduction of a diphoton with a large invariant mass

Axial Coefficient Function:

$$iCF_q^A = -ie_q^3 \left[A^A \left(\frac{1}{D_1(x)D_2(x)} - \frac{1}{D_1(-x)D_2(-x)} \right) + B^A \left(\frac{1}{D_1(x)D_3(x)} - \frac{1}{D_1(-x)D_3(-x)} \right) \right]$$

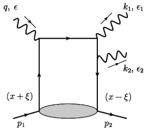
Where:

$$D_1(x) = s(x + \xi + i\varepsilon)$$
$$D_2(x) = s\alpha_2(x - \xi + i\varepsilon)$$
$$D_3(x) = s\alpha_1(x - \xi + i\varepsilon)$$

$$A^A = 4i \left(A_{k_1} + \frac{1}{\alpha_1} A_{k_2} - A_p \right)$$
$$B^A = 4i \left(-\frac{1}{\alpha_2} A_{k_1} - A_{k_2} + A_p \right)$$

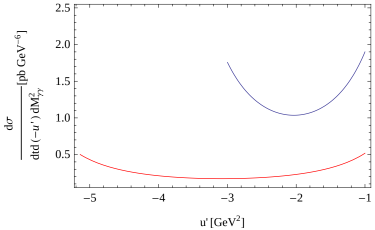
$$A_{k_1} = (p_\perp \cdot \epsilon_\perp^*(k_2)) \epsilon^{pn\epsilon_\perp(q)\epsilon_\perp^*(k_1)}$$
$$A_{k_2} = (p_\perp \cdot \epsilon_\perp^*(k_1)) \epsilon^{pn\epsilon_\perp(q)\epsilon_\perp^*(k_2)}$$
$$A_p = (\epsilon_\perp^*(k_1) \cdot \epsilon_\perp^*(k_2)) \epsilon^{pn\epsilon_\perp(q)p_\perp}$$

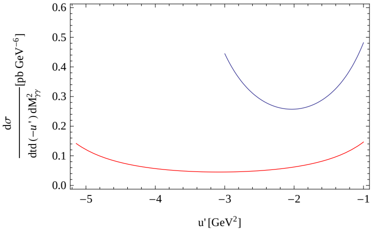
Hard photoproduction of a diphoton with a large invariant mass



$$t = (p_2 - p_1)^2,$$
$$M^2_{\gamma\gamma} = (k_1 + k_2)^2$$
$$-u' = -(k_2 - q)^2$$

Unpolarized differential cross section





The u' dependence of the unpolarized differential cross section $\frac{d\sigma}{dM^2_{\gamma\gamma}du'dt}$ at $t = t_{min}$ and $S_{\gamma N} = 20\,GeV^2$ for $M^2_{\gamma\gamma} = 4\,GeV^2$ (blue upper curve) and for $M^2_{\gamma\gamma} = 6\,GeV^2$ (red lower curve), for a proton target (left panel) and neutron target (right panel).

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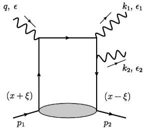
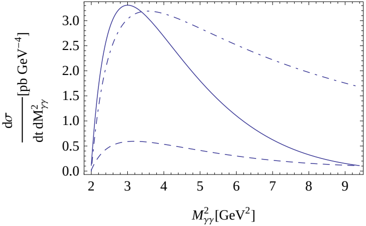
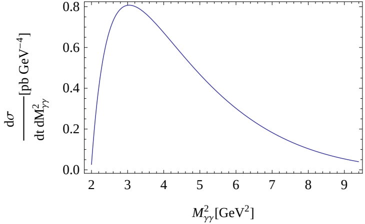
Hard photoproduction of a diphoton with a large invariant mass



$$t = (p_2 - p_1)^2,$$
$$M^2_{\gamma\gamma} = (k_1 + k_2)^2$$
$$-u' = -(k_2 - q)^2$$

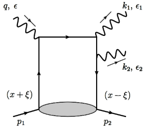
Unpolarized differential cross section





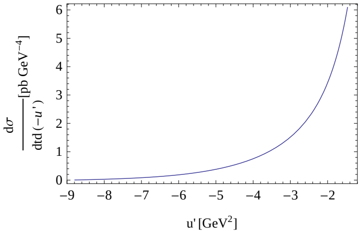
The $M^2_{\gamma\gamma}$ dependence of the unpolarized differential cross section on a proton (left panel) and on a neutron(right panel) at $t = t_{min}$ and $S_{\gamma N} = 20\ GeV^2$ (full curves), $S_{\gamma N} = 100\ GeV^2$ (dashed curve) and $S_{\gamma N} = 10^6\ GeV^2$ (dash-dotted curve, multiplied by 10^5).

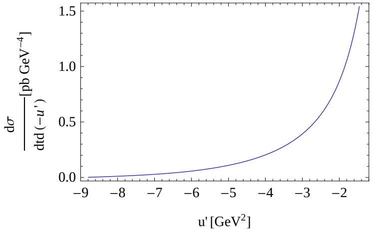
Hard photoproduction of a diphoton with a large invariant mass



$$t = (p_2 - p_1)^2,$$
$$M^2_{\gamma\gamma} = (k_1 + k_2)^2$$
$$-u' = -(k_2 - q)^2$$

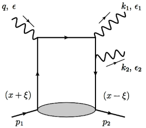
Unpolarized differential cross section





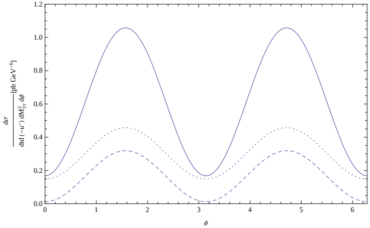
The u' dependence of the unpolarized differential cross section on a proton (left panel) and on a neutron(right panel) at $t = t_{min}$ and $S_{\gamma N} = 20\ GeV^2$ integrated over $M^2_{\gamma\gamma}$.

Hard photoproduction of a diphoton with a large invariant mass



$$t = (p_2 - p_1)^2,$$
$$M_{\gamma\gamma}^2 = (k_1 + k_2)^2$$
$$-u' = -(k_2 - q)^2$$
$$k_1^\mu = \alpha_1 n^\mu + \frac{(p_\perp)^2}{\alpha_1 s} p^\mu + p_\perp^\mu$$
$$k_2^\mu = \alpha_2 n^\mu + \frac{(p_\perp)^2}{\alpha_2 s} p^\mu - p_\perp^\mu$$

Polarized differential cross section



$$\epsilon(q) = (0, 1, 0, 0)$$
$$p_\perp = (0, p_T \cos\phi, p_T \sin\phi, 0)$$
$$\frac{d\sigma_l}{dM_{\gamma\gamma}^2 dt d(-u') d\phi} =$$
$$\frac{1}{2}\frac{1}{(2\pi)^4\, 32\, S_{\gamma M}^2 M_{\gamma\gamma}^2} \sum_{\lambda_1,\lambda_2,s_1,s_2} \frac{|\tau|^2}{2}$$

Azimuthal dependence of the differential cross section at $t = t_{min}$ and $S_{\gamma N} = 20 GeV^2$. $(M_{\gamma\gamma}^2, u') = (3, -2) GeV^2$ (solid line), $(M_{\gamma\gamma}^2, u') = (4, -1) GeV^2$ (dotted line), $(M_{\gamma\gamma}^2, u') = (4, -2) GeV^2$. ϕ is the angle between the initial photon polarization and one of the final photon momentum in the transverse plane

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Hard photoproduction of a diphoton with a large invariant mass. Further research.

- Electroproduction (Bethe–Heitler processes of two photon production)

$$\mathcal{T} = |\mathcal{T}_{\gamma\gamma} + \mathcal{T}_{BH}|^2 = |\mathcal{T}_{\gamma\gamma}|^2 + |\mathcal{T}_{BH}|^2 + \mathcal{I}$$

- Next to Leading Order calculations (QCD factorization)

Conclusions

- The unpolarized cross sections and the angular asymmetry triggered by a linearly polarized photon beam for a hard photoproduction of diphoton with large invariant mass have been calculated (the new observables);
- This is interesting process from experimental point of view (planned experiment in JLab: CLAS or GlueX);
- We plan to extend analysis to electroproduction (Bethe–Heitler processes of two photon production);
- We plan to calculate the NLO (the QCD factorizations in this kind of processes).

Conclusions

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Thank You!

The Goloskokov–Kroll model of the GPDs

Let $F = \{H, E, \tilde{H}, \tilde{E}\}$

$$F^i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x) f_i(\rho,\eta,t) + D_i(x,t)\theta(\xi^2 - x^2)$$

f_i is a double distribution, D_i is a D–term, i = gluon, sea, val.

$$f_i(\rho,\eta,t) = F^i(\rho,\xi=0,t)\omega_i(\rho,\eta)$$

where

$$\omega_i(\rho,\eta) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left((1-|\rho|^2)-\eta^2\right)^{n_i}}{(1-|\rho|)^{2n_i+1}}, \quad \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \omega_i(\rho,\eta) = 1.$$

$$F^i(\rho,\xi=0,t) = F^i(\rho,\xi=0,t=0)\exp(tp_{fi}(\rho))$$

where $p_{fi}(\rho) = -\alpha'_{fi}\ln\rho + b_{fi}$.

$D_i(x,t) = 0$

$$n_i = \begin{cases} 1 & \textit{for } i = val \\ 2 & \textit{for } i = sea,\ g \end{cases}$$

P. Kroll, H. Moutarde and F. Sabatié, Eur. Phys. J. C 73 , 2278(2013) .
The parameters have been obtained from a fit to the CTEQ6M PDFs
(J. Pumplin et al. J. High Energy Phys. 0207, 012 (2002))

The Goloskokov–Kroll model of the GPDs

Parametrization of the GPD H

$$F^i(\rho, \xi = 0, t = 0) = H^i(\rho, \xi = 0, t = 0) = \rho^{-\delta_i}(1 - \rho)^{2n_i+1} \sum_{j=0}^{3} c_{ij}\rho^{j/2}$$

	gluon	*strange*	u_{val}	d_{val}
δ	$0.10 + 0.06L - 0.0027L^2$	$1.10 + 0.06L - 0.0027L^2$	0.48	0.48
α'	$0.15\,GeV^{-2}$	$0.15\,GeV^{-2}$	$0.9\,GeV^{-2}$	$0.9\,GeV^{-2}$
c_0	$2.23 + 0.362L$	$0.123 + 0.0003L$	$1.52 + 0.248L$	$0.76 + 0.248L$
c_1	$5.43 - 7.00L$	$-0.327 - 0.004L$	$2.88 - 0.940L$	$3.11 - 1.36L$
c_2	$-34.0 + 22.5L$	$0.692 - 0.068L$	$-0.095L$	$-3.99 + 1.15L$
c_3	$40.6 - 21.6L$	$-0.486 + 0.038L$	0	0

$L = ln\frac{Q^2}{Q_0^2}$ where $Q_0^2 = 4\,GeV^2$

$$b_g = b_{sea} = 2.58\,GeV^{-2} + 0.25\,GeV^{-2} ln\frac{m^2}{Q^2 + m^2}, \quad b_{val} = 0$$

where m is a proton mass.

$$f^q_{val}(\rho, \eta, t) = (f^q(\rho, \eta, t) + f^q(-\rho, \eta, t))\,\Theta(\rho)$$
$$f^q_{sea}(\rho, \eta, t) = f^q(\rho, \eta, t)\Theta(\rho) - f^q(-\rho, \eta, t)\Theta(-\rho)$$

$$H^u_{sea} = H^d_{sea} = \kappa_s H^s_{sea}, \quad \kappa_s = 1 + 0.68/(1 + 0.52 ln Q^2/Q_0^2)$$

The parameters have been obtained from a fit to the CTEQ6M PDFs (J. Pumplin et al. J. High Energy Phys. 0207, 012 (2002))

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The Goloskokov–Kroll model of the GPDs

Parametrization of the GPD E

$$E^q_{val}(\rho,\xi=0,t=0) = B^{-1}(1-\alpha_{val},1+\beta^q_{val})\kappa_q\rho^{-\alpha_{val}}(1-\rho)^{\beta^q_v al}$$
$$E^s(\rho,\xi=0,t=0) = N_s\rho^{-1-\delta_g}(1-\rho)^{\beta_{Es}}$$
$$E^g(\rho,\xi=0,t=0) = N_g\rho^{-\delta_g}(1-\rho)^{\beta_{Eg}}$$

where κ_q is the flavour–q contribution to the nucleon anomalous magnetic moment ($\kappa_u = 1.67,\ \kappa_d = -2.03$)
$\beta^u_{val} = 4,\ \beta^d_{val} = 5.6,\ \alpha'_{eval} = \alpha'_{hval}\ \ b_{eval} = 0$
$\beta_{E_s} = 7,\ \beta_{E_g} = 6,\ b_{eg} = b_{es} = 0.9 b_{hg}$

The Goloskokov–Kroll model of the GPDs

Parametrization of the GPD $\tilde{H}$

$$\tilde{H}^q_{val}(\rho, \xi = 0, t = 0) = n_q A_q \rho^{-\alpha_{\tilde{h}q}} (1-\rho)^3 \sum_{j=0}^{2} \tilde{c}_{qj} \rho^j$$

($\tilde{H}^q_{sea}$, $\tilde{H}^q_g$ are neglected) where $n_u = 0.926 \pm 0.014$, $n_d = 0.341 \pm 0.018$.
A_q is the normalization factor:

$$A_q^{-1} = B(1-\alpha_{\tilde{h}q}, 4) \left(\tilde{c}_{q0} + \tilde{c}_{q1} \frac{1-\alpha_{\tilde{h}q}}{5-\alpha_{\tilde{h}q}} + \tilde{c}_{q2} \frac{(2-\alpha_{\tilde{h}q})(1-\alpha_{\tilde{h}q})}{(6-\alpha_{\tilde{h}q})(5-\alpha_{\tilde{h}q})} \right)$$

	u_{val}	d_{val}
$\alpha_{\tilde{h}}$	0.48	0.48
$b_{\tilde{h}}$	0	0
$\tilde{c}_0$	$0.170 + 0.03L$	$-0.320 - 0.040L$
$\tilde{c}_1$	$1.340 - 0.02L$	$-1.427 - 0.176L$
$\tilde{c}_2$	$0.120 - 0.40L$	$0.692 - 0.068L$

Parametrization of the GPD $\tilde{E}$

$$\tilde{E}^q_{val}(\rho, \xi = t = 0) = N^q_{\tilde{e}} \rho^{\alpha_{\tilde{e}}} (1-\rho)^5$$

$\alpha_{\tilde{e}}$	$b_{\tilde{e}}$	$N^u_{\tilde{e}}$	$N^d_{\tilde{e}}$
0.48	$0.9 GeV^{-1}$	14.0	4

Parameters

Definition of t:

$$t = (p_2 - p_1)^2 = -\frac{4M^2\xi^2}{1-\xi^2} - \Delta_t^2 \frac{1+\xi}{1-\xi}$$

The following calculations are done for $(-t)_{min} = t|_{\Delta_\perp = 0}$
Integration:
Integration is taken by using trapezoidal rule.

$$2.10\,GeV^2 \leq M_{\gamma\gamma}^2 \leq 9.47\,GeV^2$$
$$(-u')_{min} \leq (-u') \leq (-u')_{maxMax}$$
$$(-u')_{min} = 1\ GeV^2$$
$$(-u')_{maxMax} = (-t) + M_{\gamma\gamma}^2 - (-t')_{min}$$
$$(-t')_{min} = 1\ GeV^2$$

The domain of integration is taken in analogy with the paper: *R. Boussarie, B. Pire, L. Szymanowski, S. Wallon "Exclusive photoproduction of a $\gamma\rho$ pair with a large invariant mass"* arXiv:1609.03830v1.

GPDs contribution

$$\frac{d\sigma}{dM^2_{\gamma\gamma}\,dt\,d(-u')} = \underbrace{\mathbb{A}(1-\xi^2)\mathcal{H}\mathcal{H}^*}_{d\sigma_I} + \underbrace{\mathbb{A}(-\xi^2)(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^*)}_{d\sigma_{II}} + \underbrace{\mathbb{A}\left(\frac{\xi^4}{1-\xi^2}\right)\mathcal{E}\mathcal{E}^*}_{d\sigma_{III}}$$

$$+ \underbrace{\mathbb{A}(1-\xi^2)\tilde{\mathcal{H}}\tilde{\mathcal{H}}^*}_{d\sigma_{IV}} + \underbrace{\mathbb{A}(-\xi^2)(\tilde{\mathcal{H}}\tilde{\mathcal{E}}^* + \tilde{\mathcal{E}}\tilde{\mathcal{H}}^*)}_{d\sigma_V} + \underbrace{\mathbb{A}\left(\frac{\xi^4}{1-\xi^2}\right)\tilde{\mathcal{E}}\tilde{\mathcal{E}}^*}_{d\sigma_{VI}}$$

where $\mathbb{A} = \frac{1}{16(2\pi)^3 32\, S^2_{\gamma N} M^2_{\gamma\gamma}}$ and $\mathcal{F}(\xi) = \sum_q \int_{-1}^{1} dx C F_q(x,\xi) F^q(x,\xi)$

$(-u')$	$d\sigma$	$d\sigma_I$	$d\sigma_{II}$	$d\sigma_{III}$	$d\sigma_{IV}$	$d\sigma_V$	$d\sigma_{VI}$
1.	1.901	1.874	−0.06313	0.0005506	0.12311	−0.03683	0.002755
1.186	1.557	1.539	−0.05149	0.0004455	0.09506	−0.02844	0.002127
1.336	1.367	1.354	−0.04506	0.0003875	0.07957	−0.02380	0.001781
1.485	1.231	1.221	−0.04044	0.0003457	0.06842	−0.02047	0.001531
1.635	1.135	1.128	−0.03721	0.0003166	0.06064	−0.01814	0.001357
1.785	1.074	1.068	−0.03513	0.0002978	0.05563	−0.01664	0.001245
1.935	1.042	1.037	−0.03405	0.0002881	0.05303	−0.01587	0.001187
2.085	1.038	1.033	−0.03391	0.0002868	0.05269	−0.01576	0.001179
2.235	1.061	1.056	−0.03470	0.0002939	0.05459	−0.01633	0.001222
2.386	1.113	1.107	−0.03647	0.0003099	0.05886	−0.01761	0.001317
2.537	1.199	1.190	−0.03936	0.0003360	0.06582	−0.01969	0.001473
2.688	1.324	1.312	−0.04359	0.0003742	0.07601	−0.02274	0.001701
2.839	1.499	1.483	−0.04953	0.0004279	0.09034	−0.02703	0.002022
2.990	1.744	1.721	−0.05781	0.0005026	0.11030	−0.03301	0.002469

Properties of gluons and sea GPDs

$$F^g(-x,\xi,t) = F^g(x,\xi,t)$$
$$\tilde{F}^g(-x,\xi,t) = -\tilde{F}^g(x,\xi,t)$$

$$F^{sea}(-x,\xi,t) = -F^{sea}(x,\xi,t)$$
$$\tilde{F}^{sea}(-x,\xi,t) = \tilde{F}^{sea}(x,\xi,t)$$

Kleiss–Stirling techniques

$$\varepsilon(k,\lambda)^{\mu} = \frac{1}{\sqrt{4(p\cdot k)}}\bar{u}_{\lambda}(k)\gamma^{\mu}u_{\lambda}(p), \qquad (1)$$

where p^{μ} is a lightlike vector not collinear to k^{μ}.

$$u(q,+) = \frac{s(p_1,p_2)}{m}u_{+}(p_1) + u_{-}(p_2),$$
$$u(q,-) = \frac{t(p_1,p_2)}{m}u_{-}(p_1) + u_{+}(p_2),$$
$$v(q,+) = \frac{s(p_1,p_2)}{m}u_{+}(p_1) - u_{-}(p_2),$$
$$v(q,-) = \frac{t(p_1,p_2)}{m}u_{-}(p_1) - u_{+}(p_2),$$

where $p_1^2 = p_2^2 = 0$ and $p_1^{\mu} + p_2^{\mu} = q^m u$

$$s(p_1,p_2) = \bar{u}_{+}(p_1)u_{-}(p_2), \qquad t(p_1,p_2) = \bar{u}_{-}(p_1)u_{+}(p_2)$$

R.Kleiss, W.J. Stirling (1985) Nuclear Physics B262 235-262