The impact of the errors of collinear functions in describing unintegrated SIDIS data

Andrea Simonelli
SIDIS in the LARGE $q_T$ regime

Collinear Factorization:

\[
\frac{d\sigma}{dx_{Bj} \, dy \, dz_h \, dP_T^2} = \left( \frac{\alpha_S}{\pi} \right) \sum_{ij} \int_{x_{Bj}}^{x_{MAX}} \frac{dx}{x} \int_{z_h}^{z_{MAX}} \frac{dz}{z} \times \\
\times f_i \left( \frac{x_{Bj}}{x}, Q^2 \right) \left[ \frac{d\hat{\sigma}_{ij}}{dx \, dy \, dz \, dq_T^2} \delta \left( z^2 \frac{P_T^2}{z_h^2} - \frac{1-x}{x} \frac{1-z}{z} \right) \right] D_j \left( \frac{z_h}{z}, Q^2 \right)
\]

In general:

**MEASURED** → $O = H \otimes \sum_i F_i$  
**EXTRACTED** from experimental data

**COMPUTED** at FO in perturbation theory

Depending on the $\alpha_s$ order the collinear functions will be labeled by: LO, NLO, NNLO...

M. Anselmino, M. Boglione, A. Prokudin, and C. Türk,  
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In this case NLO collinear functions should be used:

- **PDFs:** CT10 NLO
- **FFs:** DSS NLO

\[
\frac{d\sigma}{dx\,dQ^2\,dz\,dP_T^2} / \frac{d\sigma^{DIS}}{dx\,dQ^2}
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Comparison with COMPASS data

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\[ \frac{d\sigma}{dx\,dQ^2\,dz\,dP_T^2} / \frac{d\sigma^{DIS}}{dx\,dQ^2} \]

\[ Q^2 = 2.3 \text{ GeV}^2 \]
\[ x = 0.0254 \]
\[ y = 0.309 \]
A different choice for FFs

\[ Q^2 = 10 \text{ GeV}^2 \]
A different choice for FFs

Look at the gluon!
Comparison with COMPASS DATA

Basically, the only change is in the fragmenting gluon contribution:

- PDFs: CT10 NLO
- FFs: DSS LO
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Comparison with COMPASS DATA

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- PDFs: **CT10 NLO**
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Errors and Collinear Functions

How much does the error associated to the extraction of the collinear functions affect the SIDIS cross section at large $q_T$?

$Q^2 = 2.1 \text{ GeV}^2$

$x = 0.0106$

$y = 0.674$

NEURAL NETWORK SETS

NN PDFs NLO

NN FFs NLO
The error associated to the PDFs is **negligible** in comparison with that of FFs.
Neural Network FFs

LO
NLO

Favored

Disfavored

$g$
Neural Network FFs

$Q^2 = 10 \text{ GeV}^2$

LO

NLO
WARNING!

The FFs are extracted from $e^+ e^-$ data, at values of $Q^2$ much larger than in COMPASS.

$Q^2 = 100 \text{ GeV}^2$
NN FF at NLO vs NN FFs at LO

...This is what happens using a COMPASS-like $Q^2$:
NN FFs: Comparison in the SIDIS cross section

The error bars associated to NLO FFs are on average larger:

\[ Q^2 = 2.1 \text{ GeV}^2 \]
\[ x = 0.0106 \]
\[ y = 0.674 \]
SIDIS in the LOW $q_T$ regime

TMD Factorization:

$$\frac{d\sigma}{dx B_j \ dy \ dz_1 \ dq_T^2} = \pi z^2 H (Q; \mu) \int \frac{d^2q_T}{(2\pi)^2} e^{i q_T \cdot \vec{b}_T} \sum_j e_j^2 \tilde{F} (x, b_T, \mu, \zeta_F) \tilde{D} (z, b_T, \mu, \zeta_D)$$

The TMD PDF is a complex object...

$$\tilde{F} (x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\tilde{x}}{\tilde{x}} \ C_{f,j} \left( \frac{x}{\tilde{x}} \right) f_j (\tilde{x}, \mu_b) \times$$

$$\times \ exp \left\{ \frac{1}{2} \log \left( \frac{\zeta_F}{\mu^2} \right) \tilde{K} (b_*, \mu_b) + \int_{m_{u_b}}^Q \frac{d\mu}{\mu} \gamma_F \left( \frac{\mu, \zeta_F}{\mu^2} \right) \right\} \times M_F (x, b_T)$$

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“A study on the interplay between perturbative QCD and CSS/TMD formalism in SIDIS processes”
The **TMD PDF** is a complex object...

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\tilde{F}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\tilde{x}}{\tilde{x}} \, C_f \, f_j \left( \frac{x}{\tilde{x}}, \mu_b \right) \times \\
\times \exp \left\{ \frac{1}{2} \log \left( \frac{\zeta_F}{\mu^2} \right) \tilde{K}(b_* \mu_b) + \int_{\mu \gamma F}^{Q} \frac{d\mu}{\mu} \gamma F \left( \mu, \frac{\zeta_F}{\mu^2} \right) \right\} \times M_F(x, b_T)
\]

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**Errors of Collinear Functions in the W-TERM**

**ERRORS:**
- Factor of ~ 5
- Very large band in low $q_T$ region for the right FFs.

**CENTRAL LINES:**
Effect of a different choice of the FF set.
BACK UP SLIDES
A different choice for FFs

Let’s consider again the collinear factorization theorem:

\[ \mathcal{O} = H \otimes \sum_i F_i \]

As the order of \( \alpha_s \) increases, the **HARD** part grows since the **phase space enlarges** more and more.

For SIDIS:

As a consequence the **COLLINEAR FUNCTIONS** contribution decreases.
Central lines for NN FFs:

\[ \frac{d\sigma}{dx dQ^2 dz dP_T^2} \]

- \( Q^2 = 2.1 \ GeV^2 \)
- \( x = 0.0106 \)
- \( y = 0.674 \)