

Sivers Asymmetry in πN Drell-Yan process at COMPASS within TMD factorization

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Unpolarized Process

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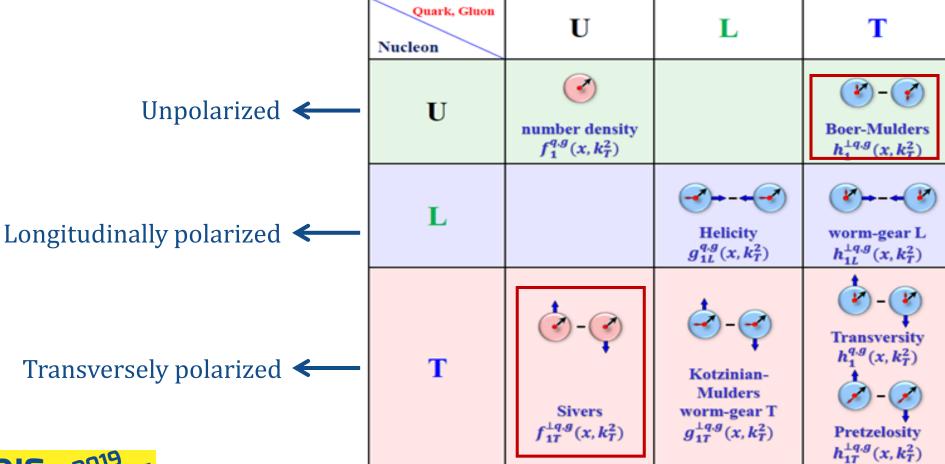


- Parton Distribution Functions (PDFs)
 - Leading twist: $f_1(x)$, $g_1(x)$, $h_1(x)$ describe the quark structure of hadrons
 - Only have one longitudinal freedom x, *i.e.*, quarks are perfectly collinear
- Transverse Momentum Dependent (TMD) PDFs
 - Admit a finite quark transverse momentum k_T
 - Provide 3D image of hadrons in momentum space: $f(x, k_T)$
 - Correlation between parton momentum and hadron spin





Leading twist TMD PDFs



TMD-PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.

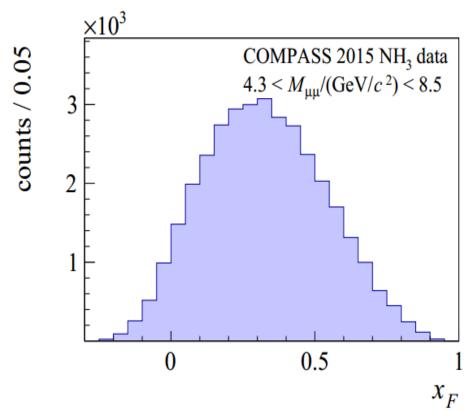


- ◆ Test of the sign change of the T-odd TMD PDFs is a fundamental quest in QCD dynamics. Measurement by COMPASS provides some hint on this sign change.
- ◆ It is important to have precise measurement and extraction of the proton Sivers function/Boer-Mulders function and pion Boer-Mulders function.
- ◆ It is also important to acquire the differential cross-section of the unpolarized process with high accuracy, as it always appears in the denominator of various asymmetries.

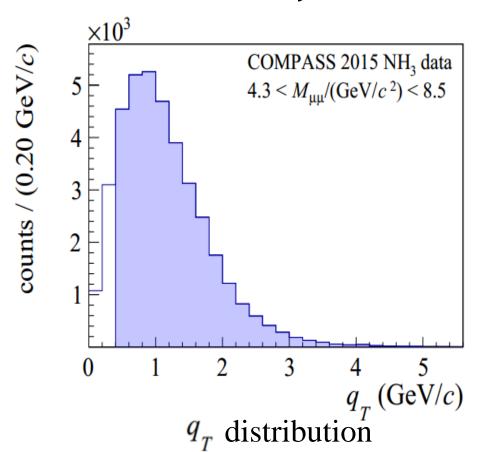




Recent COMPASS measurements (Unpolarized distribution)



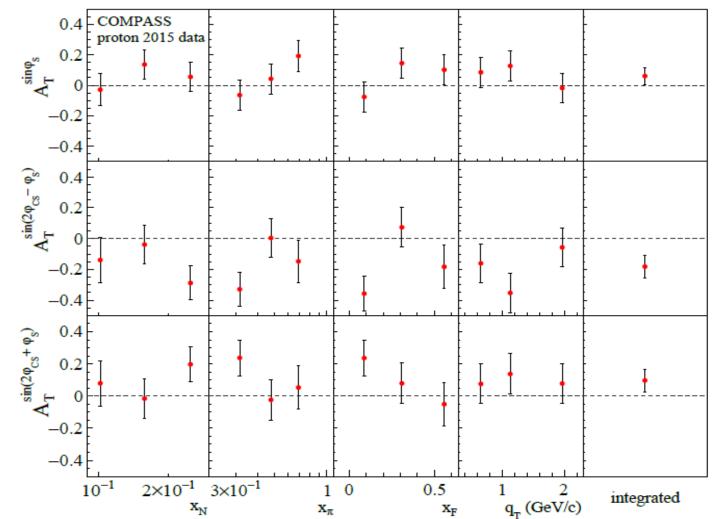
 x_F distribution of dilepton events







Recent COMPASS measurements (Single Spin Asymmetries)







- ◆ **Drell-Yan process** $A(P_1) + B(P_2) \rightarrow l^+(\ell) + l^-(\ell') + X$,
 - > The asymmetries

$$\begin{array}{lllll} A_{UU}^{\cos{(2\phi)}} & \propto h_{1,\pi}^{\perp q} & \otimes & h_{1,p}^{\perp q} \\ A_{UT}^{\sin{(\phi_S)}} & \propto f_{1,\pi}^q & \otimes & f_{1T,p}^{\perp q} \\ A_{UT}^{\sin{(2\phi-\phi_S)}} & \propto h_{1,\pi}^{\perp q} & \otimes & h_{1,p}^q \\ A_{UT}^{\sin{(2\phi+\phi_S)}} & \propto h_{1,\pi}^{\perp q} & \otimes & h_{1T,p}^{\perp q} \end{array}$$

Beam Target Boer-Mulders Boer-Mulders $f_{1,\pi}^{q}$ Sivers Boer-Mulders Transversity Boer-Mulders Pretzelosity







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lackloain The differential cross section for the unpolarized π^- -proton Drell-Yan process has the form

J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B 250 (1985) 199

$$\frac{d^4\sigma}{dQ^2dyd^2\boldsymbol{q}_{\perp}} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{q}_{\perp}\cdot\boldsymbol{b}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q,q_{\perp}),$$

- \bullet $\sigma_0 = \frac{4\pi\alpha_{em}^2}{3N_C sQ^2}$ is the cross section at tree level with $N_c = 3$
- ◆ The structure function in the first term with $\widetilde{W}_{UU}(Q;b)$ is dominant at the low q_{\perp} ≪ Q value
- Y_{UU} term provides necessary correction at moderate $q_{\perp} \sim Q$ value, which was neglected in this work





lacktriangle The structure function \widetilde{W}_{UU} can be written as

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \widetilde{f}_{1\bar{q}/\pi}^{\mathrm{sub}}(x_{\pi},b;\mu,\zeta_F) \widetilde{f}_{1q/p}^{\mathrm{sub}}(x_p,b;\mu,\zeta_F),$$

- $igoplus ilde{f}_{1\,q/H}^{
 m sub}$ is the subtracted distribution function in the b-space and universal.
- lacktriangle $H_{UU}(Q;\mu)$ is the factor associated with hard scattering and scheme-dependent.
- ◆ The way to subtract the soft factor in the distribution function depends on the scheme to regulate the light-cone singularity in the TMD definition.





• The TMD evolution equation for the ζ_F dependence is encoded in a Collins-Soper (CS) equation through

$$\frac{\partial \ln \tilde{f}^{\text{sub}}(x, b; \mu, \zeta_F)}{\partial \sqrt{\zeta_F}} = \tilde{K}(b; \mu)$$

Collins, Soper 81' Idilbi, Ji, Ma, Yuan 04'

lackloain The TMD evolution equation for the μ dependence is encoded in a RG equation through

$$\frac{d \tilde{K}}{d \ln \mu} = -\gamma_K(\alpha_s(\mu)),$$

$$\frac{d \ln \tilde{f}^{\text{sub}}(x, b; \mu, \zeta_F)}{d \ln \mu} = \gamma_F(\alpha_s(\mu); \frac{\zeta_F^2}{\mu^2}),$$





◆ The overall solution structure is the same as that for the Sudakov form factor.

• The energy evolution of TMDs from initial energy μ_b to another energy Q is encoded in the Sudakov-like form factor S by the exponential form $\exp(-S)$

$$f(x, b, Q) = \mathcal{F} \times e^{-S} \times f(x, b, \mu_b)$$





When Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation

$$F(x, k_{\perp}; Q) = \frac{1}{(2\pi)^2} \int d^2b e^{ik_{\perp} \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^{\infty} db \, b J_0(k_{\perp} b) F(x, b; Q)$$

Many different methods/proposals to model this non-perturbative part

$$db \, bJ_0(k_{\perp}b)F(x,b;Q)$$

To combine the information at small b (perturbative region) with that at large b(non-perturbative region), a matching procedure must be introduced.

$$b_* = b/\sqrt{1 + b^2/b_{\rm max}^2}$$
 $b_* \approx b$ at low values of b $b_* \approx b_{\rm max}$ at large b values.





The Sudakov-like form factor in can be separated into a perturbatively calculable part and a nonperturbative part

$$S = S_{pert} + S_{NP}$$
.

The perturbative part of S being

$$S_{\text{pert}}(Q, b) = \int_{\mu_{\tau}^{2}}^{Q^{2}} \frac{d\bar{\mu}^{2}}{\bar{\mu}^{2}} \left[A(\alpha_{s}(\bar{\mu})) \ln \frac{Q^{2}}{\bar{\mu}^{2}} + B(\alpha_{s}(\bar{\mu})) \right].$$

Non-perturbative form factor of S from pp DY (SIYY parameterization)

$$S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_2 \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_3 b^2 \left((x_0/x_1)^{\lambda} + (x_0/x_2)^{\lambda} \right) \\ S_{\mathrm{NP}} = g_1 b^2 + g_1 b^2 + g_2 b^2 + g_2 b^2 + g_3 b^2 + g_3$$



$$S_{\text{NP}}^{f_1^{q/p}}(Q,b) = \frac{g_2}{2} \ln \frac{b}{b_*} \ln \frac{Q}{Q_0} + \frac{g_1}{2} b^2,$$



In the small b region,

$$F(x,b;\mu,\zeta_F) = \sum_i C_{q \leftarrow i} \otimes f_i(x,\mu),$$

$$F(x,b;Q) \approx C \otimes F(x,c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

longitudinal/collinear part

perturbative Sudakov part

Non-perturbative Sudakov: fitted from data



Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14,



 With all the ingredients above, we can obtain the TMD distribution for proton

$$\tilde{f}_{1}^{u/p}(x,b;Q) = e^{-\frac{1}{2}S_{\text{pert}}(Q,b_{*}) - S_{\text{NP}}^{f_{1}^{q/p}}(Q,b)} \mathcal{F}(\alpha_{s}(Q)) \sum_{i} C_{q \leftarrow i}^{f_{1}} \otimes f_{1}^{i/p}(x,\mu_{b})$$

$$C_{q \leftarrow q'}(x,b;\mu,\zeta_{F}) = \delta_{qq'} \left[\delta(1-x) + \frac{\alpha_{s}}{\pi} \left(\frac{C_{F}}{2}(1-x) \right) \right],$$

$$C_{q \leftarrow g}(x,b;\mu,\zeta_{F}) = \frac{\alpha_{s}}{\pi} T_{R} x(1-x),$$

• If we perform a Fourier Transformation on $\tilde{f}_{1q/p}^{\mathrm{sub}}(x,b;Q)$

$$f_{1q/p}(x, k_{\perp}; Q) = \int_0^{\infty} \frac{dbb}{2\pi} J_0(k_{\perp}b) \tilde{f}_{1q/p}^{\text{sub}}(x, b; Q),$$



lacktriangle Assuming the non-perturbative Sudakov form factor $S_{\mathrm{NP}}^{f_1^{q/\pi}}(Q,b)$ for quark distribution function of π meson as

X. Wang, Z. Lu, I. Schmidt, JHEP 1708 (2017) 137

$$S_{\text{NP}}^{f_1^{q/\pi}} = g_1^{\pi} b^2 + g_2^{\pi} \ln \frac{b}{b_*} \ln \frac{Q}{Q_0}.$$

♦ With the assumption above, we can obtain the TMD distribution for pion

$$f_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\text{pert}}(Q,b_*) - S_{\text{NP}}^{f_1^{q/\pi}}(Q,b)} \mathcal{F}(\alpha_s(Q)) \sum_i C_{q \leftarrow i}^{f_1} \otimes f_1^{i/\pi}(x,\mu_b)$$

$$f_{1q/\pi}(x, k_{\perp}; Q) = \int_0^{\infty} \frac{dbb}{2\pi} J_0(k_{\perp}b) \tilde{f}_{1q/\pi}^{\text{sub}}(x, b; Q).$$





◆ The structure function is as follows

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \widetilde{f}_{q/\pi}^{\text{sub}}(x_1,b;\mu,\zeta_F) \widetilde{f}_{q/p}^{\text{sub}}(x_2,b;\mu,\zeta_F),$$

- If we absorb the hard factors H_{UU} and $\mathcal{F}(\alpha_s(Q))$ into the definition of C-coefficients, the C-coefficients become process-dependent
 - S. Catani, D. de Florian and M. Grazzini, Nucl. Phys. B 596 (2001) 299

$$C_{q \leftarrow q'}(x, b; \mu_b) = \delta_{qq'} \left[\delta(1 - x) + \frac{\alpha_s}{\pi} \left(\frac{C_F}{2} (1 - x) + \frac{C_F}{4} (\pi^2 - 8) \delta(1 - x) \right) \right],$$

$$C_{q \leftarrow g}(x, b; \mu_b) = \frac{\alpha_s}{\pi} T_R x (1 - x).$$





lacklost The structure function W_{UU} in b-space can be written as

$$\widetilde{W}_{UU}(Q;b) = e^{-S(Q^2,b)} \times \sum_{q,\bar{q}} e_q^2 C_{q\leftarrow i} \otimes f_{i/\pi^-}(x_1,\mu_b) C_{\bar{q}\leftarrow j} \otimes f_{j/p}(x_2,\mu_b)$$

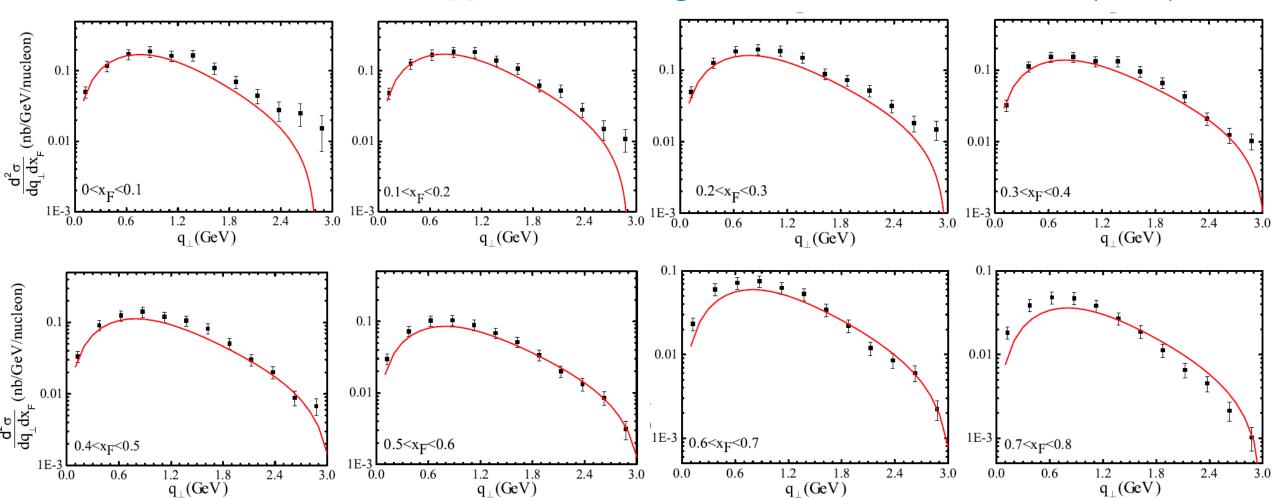
The differential cross section is

$$\frac{d^4\sigma}{dQ^2dyd^2\boldsymbol{q}_{\perp}} = \sigma_0 \int_0^{\infty} \frac{dbb}{2\pi} J_0(q_{\perp}b) \times \widetilde{W}_{UU}(Q;b),$$





• Fit the theoretical estimate with the experimental data from E615, we can obtain the parameters in $S_{\mathrm{NP}}^{f_1^{q/\pi}}(Q,b)$ X. Wang, Z. Lu, I. Schmidt, JHEP 1708 (2017) 137





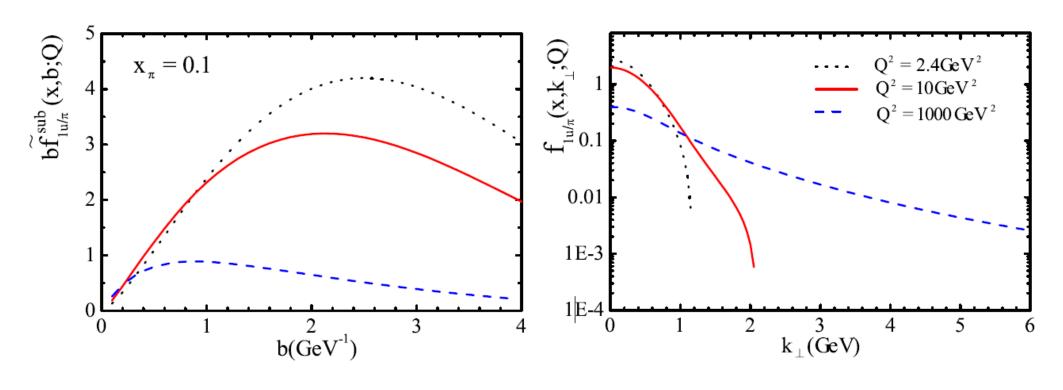
The fitting parameters are as

$$g_1^{\pi} = 0.082 \pm 0.022, \quad g_2^{\pi} = 0.394 \pm 0.103,$$

- lacklost The fit coincides with the experimental data well when $0 < x_F < 0.8$.
- lacklosh The fit breaks down when x_F is above 0.8 since at that region the TMD factorization is invalid and the higher twist effects dominant.
- lacklosh For the pion-induced Drell-Yan process in fixed-target scattering, the NLL threshold resummation effects are also important in the kinematic higher x_F .



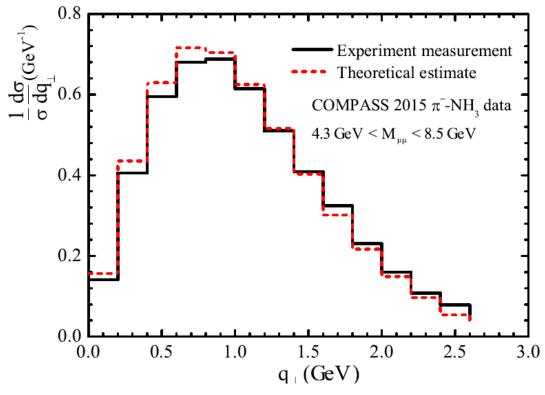




Subtracted unpolarized TMD distribution of the pion meson for valence quarks in b-space (left panel) and k_{\perp} -space (right panel), at energies: $Q^2 = 2.4 \text{ GeV}^2$ (dotted lines), $Q^2 = 10 \text{ GeV}^2$ (solid lines) and $Q^2 = 1000 \text{ GeV}^2$ (dashed lines).







The transverse spectrum of lepton pair production in the unpolarized pion-nucleon Drell-Yan process, with an NH₃ target at COMPASS. The dashed line is our theoretical calculation using the extracted Sudakov form factor for the pion TMD PDF. The solid line shows the experimental measurement at COMPASS.





- ♦ The theoretical is compatible with the COMPASS measurement at small q_{\perp} region with $q_{\perp} \ll Q$, indicating that our approach can be used as a first step to study the Drell-Yan process at COMPASS.
- Our study may provide a better understanding on the pion TMD distribution as well as its role in Drell-Yan process.
- The framework applied in this work can also be extended to the study of the azimuthal asymmetries in the π^-N Drell-Yan process.







01 Introduction

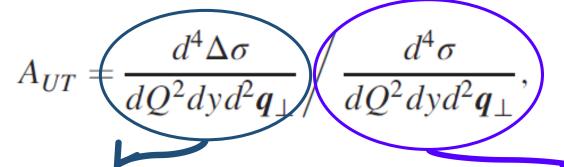
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◆ The transverse single spin asymmetry can be defined as

M. G. Echevarria, A. Idilbi, Z. B. Kang, and I. Vitev, Phys. Rev. D 89, 074013(2014)



Spin-dependent

Spin-independent(Unpolarized)

$$\frac{d^4\sigma}{dQ^2dyd^2\boldsymbol{q}_{\perp}} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{\boldsymbol{q}}_{\perp}\cdot\vec{\boldsymbol{b}}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q,q_{\perp}).$$

$$\frac{d^4 \Delta \sigma}{dQ^2 dy d^2 \mathbf{q}_{\perp}} = \sigma_0 \epsilon_{\perp}^{\alpha \beta} S_{\perp}^{\alpha} \int \frac{d^2 b}{(2\pi)^2} e^{i \vec{\mathbf{q}}_{\perp} \cdot \vec{b}} \widetilde{W}_{UT}^{\beta}(Q; b) + Y_{UT}^{\beta}(Q, q_{\perp}).$$
Since some





Spin-dependent structure function

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = H_{UT}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \widetilde{f}_{1\bar{q}/\pi}(x_{\pi},b;\mu,\zeta_F) \widetilde{f}_{1T\,q/p}^{\perp\alpha(DY)}(x_p,b;\mu,\zeta_F).$$



$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x,b;\mu,\zeta_F) = \int d^2\boldsymbol{k}_{\perp} e^{-i\vec{\boldsymbol{k}}_{\perp}\cdot\vec{\boldsymbol{b}}} \frac{k_{\perp}^{\alpha}}{M_p} f_{1T,q/p}^{\perp(\mathrm{DY})}(x,\boldsymbol{k}_{\perp};\mu),$$

Follows the same evolution equations and the solution structure can be written in the same form





- Perturbative Sudakov form factor has the same form as unpolarized PDF
- Nonperturbative Sudakov form factor has the parameterization as

M. G. Echevarria, A. Idilbi, Z. B. Kang, and I. Vitev, Phys. Rev. D 89, 074013(2014)

$$S_{\text{NP}}^{\text{Siv}} = \left(g_1^{\text{Siv}} + g_2^{\text{Siv}} \ln \frac{Q}{Q_0}\right) b^2,$$

$$g_1^{\text{Siv}} = \langle k_{s\perp}^2 \rangle_{Q_0} / 4 = 0.071 \text{GeV}^2$$
 $g_2^{\text{Siv}} = \frac{1}{2} g_2 = 0.08 \text{GeV}^2$





◆ In the small b region, the Sivers function can be also expressed as the convolution of perturbatively calculable hard coefficients and the corresponding collinear correlation functions as

$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x,b;\mu) = \left(\frac{-ib^{\alpha}}{2}\right) \sum_{i} \Delta C_{q\leftarrow i}^{T} \otimes \left(f_{i/p}^{(3)}(x',x'';\mu)\right).$$

Qiu-Sterman matrix element $T_{q,F}(x,x)$ is the most relevant one

$$T_{q,F}(x,x) = \int d^2k_{\perp} \frac{|k_{\perp}^2|}{M_p} f_{1T\,q/p}^{\perp DY}(x,k_{\perp}) = 2M_p f_{1T\,q/p}^{\perp (1)DY}(x),$$





Sivers function in the b space

$$\tilde{f}_{1T,q/p}^{\perp}(x,b;Q) = \frac{b^2}{2\pi} \sum_{i} \Delta C_{q\leftarrow i}^T \otimes T_{i,F}(x,x;\mu_b) e^{-S_{NP}^{\text{siv}} - \frac{1}{2}S_P},$$

Sivers function in the transverse momentum space

$$\frac{k_{\perp}}{M_{p}} f_{1T,q/p}^{\perp}(x,k_{\perp};Q) = \int_{0}^{\infty} db \frac{b^{2}}{2\pi} J_{1}(k_{\perp}b) \sum_{i} \Delta C_{q\leftarrow i}^{T} \otimes f_{1T,i/p}^{\perp(1)}(x,\mu_{b}) e^{-S_{NP}^{\text{siv}} - \frac{1}{2}S_{P}}.$$





◆ The spin-dependent differential cross section

$$\frac{d^4 \Delta \sigma}{dQ^2 dy d^2 \mathbf{q}_{\perp}} = \sigma_0 \epsilon^{\alpha \beta} S_{\perp}^{\alpha} \int \frac{d^2 b}{(2\pi)^2} e^{i \mathbf{\vec{q}}_{\perp} \cdot \mathbf{\vec{b}}} \widetilde{W}_{UT}^{\beta}(Q; b)$$

$$= \frac{\sigma_0}{4\pi} \int_0^\infty db b^2 J_1(q_\perp b) \sum_{q,i,j} e_q^2 \Delta C_{q\leftarrow i}^T T_{i,F}(x_p, x_p; \mu_b)$$

$$\times C_{\bar{q}\leftarrow j}\otimes f_{1,j/\pi}(x_{\pi},\mu_b)e^{-\left(S_{\rm NP}^{\rm Siv}+S_{\rm NP}^{f_{1q/\pi}}+S_{\rm P}\right)}.$$

$$\Delta C_{q \leftarrow q'}^{T}(x, b; \mu_b) = \delta_{qq'} \left[\delta(1 - x) + \frac{\alpha_s}{\pi} \left(-\frac{1}{4N_c} (1 - x) + \frac{C_F}{4} (\pi^2 - 8) \delta(1 - x) \right) \right].$$





Qiu-Sterman function parameterization

M. G. Echevarria, A. Idilbi, Z. B. Kang, and I. Vitev, Phys. Rev. D 89, 074013(2014)

$$T_{q,F}(x,x;\mu) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q^{\alpha_q} + \beta_q^{\beta_q})}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1-x)^{\beta_q} f_{q/p}(x,\mu),$$

Energy dependence

Set 1:propotional to unpolarized PDF

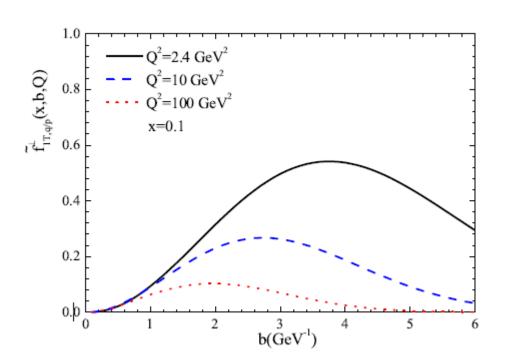
Set 2:adopt approximate evolution kernel

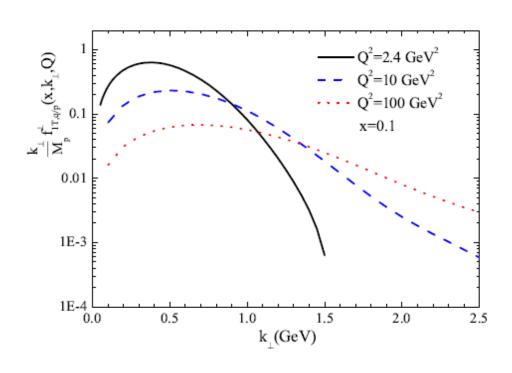
$$P_{qq}^{\rm QS} \approx P_{qq}^{f_1} - \frac{N_c}{2} \frac{1+z^2}{1-z} - N_c \delta(1-z), \qquad P_{qq}^{f_1} = \frac{4}{3} \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right).$$









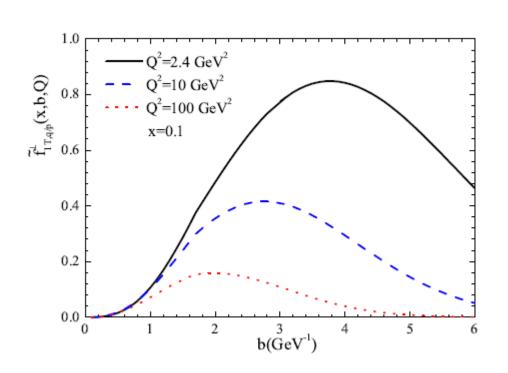


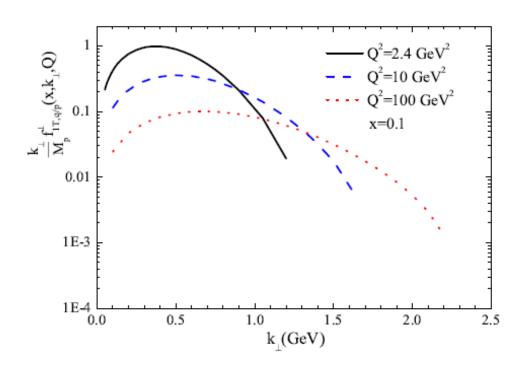


X. Wang and Z. Lu, Phys. Rev. D 97, 054005 (2018)









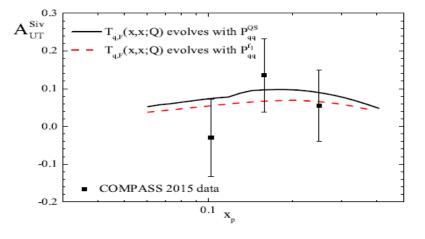


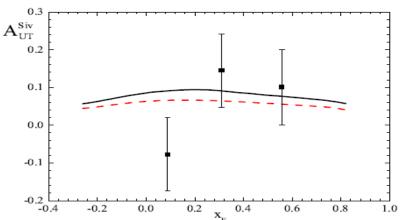
X. Wang and Z. Lu, Phys. Rev. D 97, 054005 (2018)

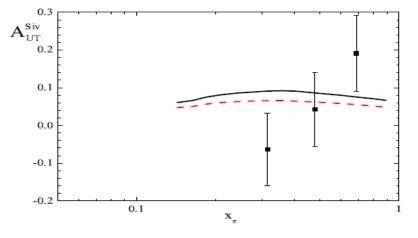


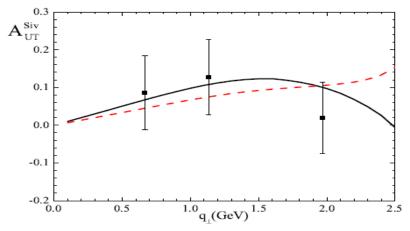
Sivers asymmetry with the COMPASS measurement















The Sivers asymmetry calculated from the TMD evolution formalism is consistent with the COMPASS measurement.

The scale dependence of the Qiu-Sterman function will play a role in the interpretation of the experimental data, and it should also be considered in the phenomenological studies.







01 Introduction

02 Unpolarized Process

- 03 Sivers Asymmetry
- 04 Summary



SUMMARY



- ◆ The Non-perturbative Sudakov form factor for the unpolairzed TMD of the pion is extracted for the first time from the E615 DY data within the TMD factorization incorporating TMD evolution.
- ♦ The transvere momentum spectrum of the dilepton agrees with the COMPASS measurement at small q_T region, indicating that our approach can be used a first step for precision study of pion-nucleon DY.
- ◆ The Sivers asymmetry calculated from the TMD evolution formalism is qualitatityely consistent with the data at COMPASS.



SUMMARY



- ◆ The results might be improved by including higher order calcuatoin of the hard coefficients, and with more flexible parameterization on the nonperturbative part.
- ◆ The framework applied here can be also extended to the study of the other azimuthal asymmetries in the pion-nucleon DY.







THANK YOU!





- **◆ Drell-Yan process** $A(P_1) + B(P_2) \rightarrow l^+(\ell) + l^-(\ell') + X$,
 - General form of the cross section (Beam: unpolarized Target: unpolarized/transverselv polarized)

$$\begin{split} &\frac{d\sigma}{d^4qd\Omega} \stackrel{\mathsf{LO}}{=} \frac{\alpha^2}{Fq^2} \hat{\sigma_U} \left\{ \begin{array}{l} \left(1 + \cos^2(\theta) + \sin^2(\theta) A_{UU}^{\cos(2\phi)} \cos(2\phi) \right) \\ + S_T \left[(1 + \cos^2(\theta)) A_{UT}^{\sin(\phi_S)} \sin(\phi_S) \\ + \sin^2(\theta) \left(A_{UT}^{\sin(2\phi + \phi_S)} \sin(2\phi + \phi_S) + A_{UT}^{\sin(2\phi - \phi_S)} \sin(2\phi - \phi_S) \right) \right] \right\} \end{split}$$





The kinematical variables defined as

$$s = (P_a + P_b)^2,$$

$$x_{a(b)} = q^2/(2P_{a(b)} \cdot q),$$

$$x_F = x_a - x_b,$$

$$M_{\mu\mu}^2 = Q^2 = q^2 = s \ x_a \ x_b,$$

the total centre-of-mass energy squared,

the momentum fraction carried by a parton from $H_{a(b)}$,

the Feynman variable,

the invariant mass squared of the dimuon.

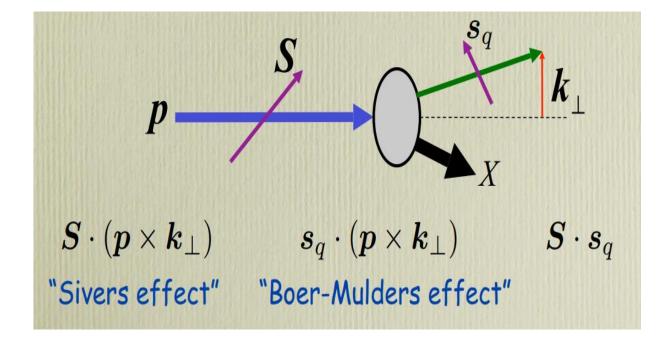
$$\tau = Q^2/s = x_{\pi}x_p,$$
 $y = \frac{1}{2}\ln\frac{q^+}{q^-} = \frac{1}{2}\ln\frac{x_{\pi}}{x_p},$ $x_{\pi/p} = \frac{\pm x_F + \sqrt{x_F^2 + 4\tau}}{2},$ $x_{\pi/p} = \sqrt{\tau}e^{\pm y}.$





◆ T-odd TMD PDFs

- Sivers function: correlation between the transverse spin of the nucleon and parton transverse momentum
- ➤ Boer-Mulders function: correlation between the transverse spin of the quark and quark transverse momentum

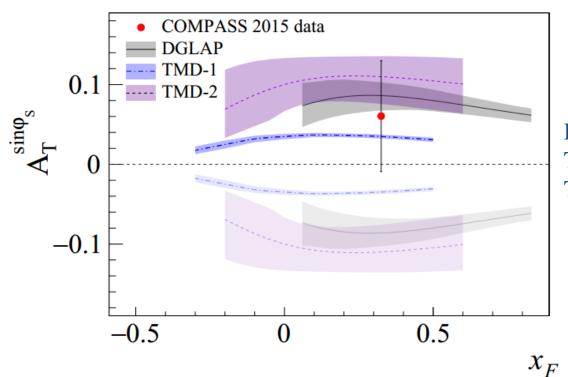






Test of sign change at COMPASS

PRL119, 112002 (2017)



DGLAP: Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP04,046

TMD-1: Echevarria, Idilbi, Kang, Vitev, PRD89,074013

TMD-2: Sun, Yuan, PRD88,114012

