Linearly polarized gluons TMDPDFs at NNLO in QCD

Sergio Leal Gómez (UCM)
In collaboration with: Daniel Gutiérrez-Reyes (UCM), Ignazio Scimemi (UCM) and Alexey Vladimirov (Universität Regensburg)

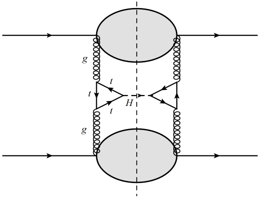
DIS 2019 TORINO

Outline

Higgs production

arXiv:1109.1444

- Gluons trigger most of the scattering processes at high energies.
- When two linearly polarized gluons, one from each hadron, participate in the scattering could give a term independent of the azimutal angle.
- In this way they can contribute to production of scalar or pseudoscalar particle.
- Linearly polarized gluon leads to a modulation of the Higgs transverse momentum.
- The contribution of linearly polarized gluons is 1 %.



Hadronic part of Higgs production cross section

$$\frac{d\sigma^{H(A)}}{d^3\vec{q}} \sim \left(f_1^g \otimes f_1^g \pm w_H \otimes h_1^{\perp g} \otimes h_1^{\perp g} \right)$$

For two photons decay we have

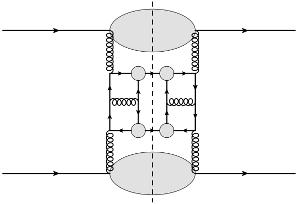
$$\frac{d\sigma^{gg}}{dq d\cos\theta} = F_1\left(\theta, Q=\sqrt{s}\right)\left(f_1^g \otimes f_1^g\right) + F_2\left(\theta, Q=\sqrt{s}\right)\left(w_H \otimes h_1^{\perp g} \otimes h_1^{\perp g}\right)$$

when $Q = m_H$ we have $F_1 \sim \pm F_2$

Quarkonium-pair production

arXiv:1710.01684

- Linearly polarized gluons can also generate a $\cos 2\phi (\cos 4\phi)$ modulation in azimuthal angle in gluon-fusuion scattering where a single(double) gluon-helicity flips occur.
- Among the quarkonium-associated-production di-J/ψ production has been the object of the largest number of experimental studies at the LHC and Tevatron: arXiv: 1109.0963, arXiv:1406.2380, arXiv:1406.2380, arXiv:1612.07451.
- For TMD factorization to apply, di-QQ production should result from a Single Parton Scattering and Final State Interaction should be negligible, which is satisfy when quarkonia are produced via CST.
- In the limet $M_Q << M_{QQ}$ we have $F_1 \sim F_4$.
- $\langle \cos 4\phi \rangle$ gets close to 50 % in the P_{QQ_T} region probed by CMS and ATLAS.



Cross section

$$\frac{d\sigma}{dM_{QQ}dY_{QQ}d\vec{P}^2_{QQ_T}d\Omega} \sim F_1\left(f_1^g \otimes f_1^g\right) + F_2\left(w_H \otimes h_1^{\perp g} \otimes h_1^{\perp g}\right) +$$
$$\cos 2\phi\left(F_3\left(w_3 \otimes f_1^g \otimes h_1^{\perp g}\right) + F_3'\left(w_3' \otimes h_1^{\perp g} \otimes f_1^g\right)\right) +$$
$$\cos 4\phi F_4\left(w_4 \otimes h_1^{\perp g} \otimes h_1^{\perp g}\right)$$

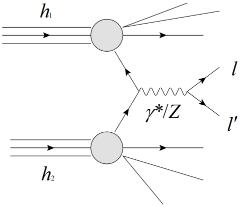
$\langle \cos n\phi \rangle$ is defined as

$$\langle \cos n\phi \rangle = \frac{\int d\phi \cos n\phi \frac{d\sigma}{dM_{QQ}dY_{QQ}d\vec{P}^2_{QQ_T}d\Omega}}{\int d\phi \frac{d\sigma}{dM_{QQ}dY_{QQ}d\vec{P}^2_{QQ_T}d\Omega}}$$

for n =2, 4.

Example: Drell-Yan

arXiv:1111.4996



- The cross section can be factorized as the product of TMDPDFs
- The hard component hard is process dependent and can be calculated by perturbation theory.

Differential cross section for a Drell-Yan process

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi}{3N_c}\frac{P}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{FF'}^{GG'}\left|C_V(q,\mu)\right|^2$$

$$\int \frac{d^2\vec{b}}{4\pi} e^{i\vec{b}\cdot\vec{q}} F_{f\leftarrow h_1}\left(x_1,\vec{b};\mu,\zeta\right) F_{f'\leftarrow h_2}\left(x_2,\vec{b};\mu,\zeta\right) + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

Factorization Theorems and TMDs

arXiv:1111.4996

Hadronic part of Drell-Yan process

$$d\sigma \sim \int d^4 y e^{-iq\cdot y} \langle h_1(p,s_1)\, h_2(\bar{p},s_2) \left| J^{\dagger\mu}(y)\, J^{\nu}(0) \right| h_1(p,s_1)\, h_2(\bar{p},s_2) \rangle$$

Factorized hadronic part

$$d\sigma \sim \int db_T e^{-iq_T b_T} \left| C_v\left(Q^2\right)\right|^2 \Phi_{f\leftarrow h_1}(x_1, b_T)\, \Phi_{f'\leftarrow h_2}(x_2, b_T) S(b_T)$$

In this form, we have a double counting of the soft modes. We need to subtract them from the TMDPDFs.

Factorization Theorems and TMDs

arXiv:1111.4996

Factorized hadronic part

$$d\sigma \sim \int db_T e^{-iq_T b_T} \left|C_v\left(Q^2\right)\right|^2 \frac{\Phi_{f\leftarrow h_1}\left(x_1, b_T\right)}{Z_{zero-bin}} \frac{\Phi_{f\prime\leftarrow h_2}\left(x_2, b_T\right)}{Z_{zero-bin}} S\left(b_T\right)$$

The $Z_{zero-bin}$ depends of the regularization scheme.

Soft function

$$S\left(b_T\right) = \frac{Tr_c}{N_c} \langle 0 \left| T\left[S_n^{T\dagger} \tilde{S}_{\bar{n}}^T\right]\left(0^+, 0^-, b_T\right) \bar{T}\left[\tilde{S}_{\bar{n}}^{T\dagger} S_n^T\right](0) \right| 0 \rangle$$

Soft function is a colourless object. arXiv: 1511.05590

TMD

We use their definition in impact parameter space. arXiv: 1702.06558

Gluon TMD

$$\Phi_{\mu\nu}\left(x,\vec{b}\right)=\frac{1}{xp^{+}}\int\frac{\mathrm{d}\lambda}{2\pi}e^{-ixp^{+}\lambda}F_{+\mu}\left(\lambda n+b_T\right)\mathcal{W}\left(\lambda,b_T\right)F_{+\nu}\left(0\right)$$

Quark TMD

$$\Phi_{ij}\left(x,\vec{b}\right)=\int\frac{\mathrm{d}\lambda}{2\pi}e^{-ixp^{+}\lambda}\bar{q}_i\left(\lambda n+b_T\right)\mathcal{W}\left(\lambda,b_T\right)q_j\left(0\right)$$

$$\mathcal{W}\left(\lambda,b_T\right)=\tilde{W}_n^T\left(\lambda n+b_T\right)\sum_X|X\rangle\langle X|\,\tilde{W}_n^T\left(0\right)$$

Wilson lines make the operator gauge invariant.

δ-scheme and Soft function

arXiv: 1604.07869

Soft Wilson Lines

$$\tilde{S}_{\bar{n}}(y) = P\exp\left[-ig\int_0^\infty d\sigma\,\bar{n}\cdot A(y+\bar{n}\sigma)\right] \to \tilde{S}_{\bar{n}}(y) = P\exp\left[-ig\int_0^\infty d\sigma\,\bar{n}\cdot A(y+\bar{n}\sigma)\,e^{-\delta^+\sigma}\right]$$

$$S_n(y) = P\exp\left[ig\int_{-\infty}^0 d\sigma\,n\cdot A(y+n\sigma)\right] \to S_n(y) = P\exp\left[ig\int_{-\infty}^0 d\sigma\,n\cdot A(y+n\sigma)\,e^{+\delta^-\sigma}\right]$$

Wilson Lines

$$\tilde{W}_{\bar{n}}(y) = P\exp\left[-ig\int_0^\infty d\sigma\,\bar{n}\cdot A(y+\bar{n}\sigma)\right] \to \tilde{S}_{\bar{n}}(y) = P\exp\left[-ig\int_0^\infty d\sigma\,\bar{n}\cdot A(y+\bar{n}\sigma)\,e^{-\delta^+\sigma x}\right]$$

$$W_n(y) = P\exp\left[ig\int_{-\infty}^0 d\sigma\,n\cdot A(y+n\sigma)\right] \to S_n(y) = P\exp\left[ig\int_{-\infty}^0 d\sigma\,n\cdot A(y+n\sigma)\,e^{+\delta^-\sigma x}\right]$$

Only the limit $\delta \to 0$ is relevant (and gauge invariant). The different definition of the regularization in the W and S Wilson lines solve the zero-bin problem.

δ-scheme and Soft function

With the δ-scheme we get

$$\boxed{Z_{zero-bin} = S(b_T)}$$

Factorized and regularizes hadronic part

$$d\sigma \sim \int db_T e^{-iq_T b_T} \left| C_v\left(Q^2, \mu\right) \right|^2 \frac{\Phi_{f\leftarrow h_1}\left(x_1, b_T, \epsilon, \delta; \mu, \zeta\right)}{\sqrt{S\left(b_T, \epsilon, \delta; \mu, \zeta\right)}} \frac{\Phi_{f'\leftarrow h_2}\left(x_2, b_T, \epsilon, \delta; \mu, \zeta\right)}{\sqrt{S\left(b_T, \epsilon, \delta; \mu, \zeta\right)}}$$

- The term $\dfrac{1}{\sqrt{S\left(b_T, \epsilon, \delta; \mu, \zeta\right)}}$ regularized the rapidity divergences.
- We have a new scale ζ associated with rapidity divergences.

Factorization Theorems and TMDs

- The cross section for Drell-Yan and SIDIS process can be written as the product of TMDs. This allows to manage in a consistent way the *rapidity* divergences.
- We have a nonperturbative well-defined TMD. We provide a perturbative calculation of TMD to extract its matching coefficient with unintegrated distribution in the asymptotic large-q_T limit without any reference to particular scattering process.

Polarized TMDs and small-b OPE

Decomposition over Lorentz invariants arXiv:0009343, arXiv:1502.05354

$$\Phi^{\mu\nu}_{g\leftarrow h}\left(x,\vec{b}\right)=\langle h\left|\Phi^{\mu\nu}\left(x,\vec{b}\right)\right|h\rangle$$
$$=\frac{1}{2}\left(-g_T^{\mu\nu}f_1^g-i\epsilon_T^{\mu\nu}S_L g_{1L}^g+2h_1^{\perp g}\left(\frac{g_T^{\mu\nu}}{2}+\frac{b^\mu b^\nu}{\vec{b}^2}\right)+\cdots\right)$$

- f_1^g TMDPDFs of unpolarized gluons up to NNLO. arXiv:1604.07869
- g_{1L}^g TMDPDFs of helicity gluons up to NLO. arXiv:1502.05354, arXiv: 1702.06558
- $h_1^{\perp g}$ TMDPDFs of linearly polarized gluons.

OPE a twist-2

$$\Phi_{\mu\nu}\left(x,\vec{b}\right)=\left[\left(C_{g\leftarrow q}\left(x,\vec{b}\right)\right)^{ab}_{\mu\nu}\otimes\phi_{ab}\left(x\right)\right]+\left[\left(C_{g\leftarrow g}\left(x,\vec{b}\right)\right)^{\alpha\beta}_{\mu\nu}\otimes\phi_{\alpha\beta}\left(x\right)\right]+\cdots$$

Unpolarized PDFs

$$\phi_{q\leftarrow h,ij}=\langle h\left|\phi_{ij}\left(x\right)\right|h\rangle=\frac{1}{2}f_q\left(x\right)\gamma^-_{ij}\cdots$$
$$\phi_{g\leftarrow h,\mu\nu}\left(x\right)=\langle h\left|\phi_{\mu\nu}\right|h\rangle=-\frac{1}{2}g^T_{\mu\nu}f_g\left(x\right)+\cdots$$

TMD

arXiv:1604.07869

Regularized TMD

Gluon TMD

$$\Gamma^{\mu\nu}\Phi^{ren}_{\mu\nu} = Z_g(\epsilon;\mu,\zeta)\, Z_3(\epsilon;\mu)^{-1}\, S(b_T,\epsilon,\delta;\mu,\zeta)^{-1/2}\,\Gamma^{\mu\nu}\Phi^{bare}_{\mu\nu}$$

Coefficient C

$$C^{[2]}_{f\to f'} = \phi^{[2]} - \sum_r C^{[1]}_{f\to r}\otimes f^{[1]}_{r\leftarrow f'} - f^{[2]}_{f\leftarrow f'}$$

Lorentz structures allow up to twist-2

$$\Gamma^g = \{g_T^{\mu\nu}, \epsilon_T^{\mu\nu}, b_T^\mu b_T^\nu / b_T^2\}$$

Matching coefficient

Coefficient $C^{[2]}_{g\leftarrow g}$

$$C^{[2]}_{g\leftarrow g}(x,L_\mu)=C_A^2 D^{[2]}_{C_A^2}(x,L_\mu)+C_A\,TrN_f\,D^{[2]}_{C_A\,TrN_f}(x,L_\mu)+C_F\,TrN_f\,D^{[2]}_{C_F\,TrN_f}(x,L_\mu)$$

with $L_X=\log\left(\frac{X^2 b_T^2}{4e^{-2\gamma_E}}\right)$. The functions $D_i(x,L_\mu)$ can be written as

$$D^{[n]}_i(x,L_\mu)=\sum_{k=0}^{2n} D^{[n,k]}_i(x)\,L^k_\mu$$

where $i=C_A\,TrN_f,\,C_A^2,\,C_F\,TrN_f$

Matching coefficients

Coefficient $C_A TrN_f$

$$D^{[2,0]}_{C_A TrN_f} = -\frac{8\left(-17 + 16x3x^2 + x^3 - 6x\mathrm{Log}\,(x)\right)}{9x}$$

Coefficient $C_F TrN_f$

$$D^{[2,0]}_{C_F TrN_f} = \frac{8\left(2\,(-1 + x)^3 + x\mathrm{Log}\,(x)^2\right)}{x}$$

Summary

- Linearly polarized gluons have phenomenological interest and still there is a lack of knowledge about them.
- Factorization theorems allow identify universal independent object: $\rightarrow$ TMDPDFs.
- Using δ-scheme for regularized rapidity divergences we are able to identified $Z_{\text{zero-bin}} = S\left(b_T, \epsilon, \delta; \mu, \zeta\right)$.
- Rapidity divergences have their own renormalization scale ζ.
- Coefficient $C^{[2]}\left(x, L_\mu\right)$ is well defined and only depends of x and logaritmicaly of b_T.

Thanks for your attention!!!