LINEARLY POLARIZED GLUONS TMDPDFS AT NNLO IN QCD

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DIS 2019 TORINO
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Higgs production

Gluons trigger most of the scattering processes at high energies.

When two linearly polarized gluons, one from each hadron, participate in the scattering could give a term independent of the azimuthal angle.

In this way they can contribute to production of scalar or pseudoscalar particle.

Linearly polarized gluon leads to a modulation of the Higgs transverse momentum.

The contribution of linearly polarized gluons is 1%.

\[
\frac{d\sigma^{H(A)}}{d^3q} \sim \left( f_1^g \otimes f_1^g \pm w_H \otimes h_{1\perp}^g \otimes h_{1\perp}^g \right)
\]

For two photons decay we have

\[
\frac{d\sigma^{gg}}{dq d\cos \theta} = F_1 (\theta, Q = \sqrt{s}) \left( f_1^g \otimes f_1^g \right) + F_2 (\theta, Q = \sqrt{s}) \left( w_H \otimes h_{1\perp}^g \otimes h_{1\perp}^g \right)
\]

when \( Q = m_H \) we have \( F_1 \sim \pm F_2 \)
Linearly polarized gluons can also generate a $\cos 2\phi (\cos 4\phi)$ modulation in azimuthal angle in gluon-fusion scattering where a single (double) gluon-helicity flips occur.


For TMD factorization to apply, di-$QQ$ production should result from a Single Parton Scattering and Final State Interaction should be negligible, which is satisfy when quarkonia are produced via CST.

In the limit $M_Q << M_{QQ}$ we have $F_1 \sim F_4$.

$\langle \cos 4\phi \rangle$ gets close to 50% in the $P_{QQ_T}$ region probed by CMS and ATLAS.

Cross section

$$\frac{d\sigma}{dM_{QQ}dY_{QQ}dP_{QQ_T}^2 d\Omega} \sim F_1 (f^g_1 \otimes f^g_1) + F_2 \left( w_H \otimes h_{1}^{\perp g} \otimes h_{1}^{\perp g} \right) +$$

$$\cos 2\phi \left( F_3 \left( w_3 \otimes f^g_1 \otimes h_{1}^{\perp g} \right) + F'_3 \left( w'_3 \otimes h_{1}^{\perp g} \otimes f^g_1 \right) \right) +$$

$$\cos 4\phi F_4 \left( w_4 \otimes h_{1}^{\perp g} \otimes h_{1}^{\perp g} \right)$$

$\langle \cos n\phi \rangle$ is defined as

$$\langle \cos n\phi \rangle = \frac{\int d\phi \cos n\phi \frac{d\sigma}{dM_{QQ}dY_{QQ}dP_{QQ_T}^2 d\Omega}}{\int d\phi \frac{d\sigma}{dM_{QQ}dY_{QQ}dP_{QQ_T}^2 d\Omega}}$$

for $n = 2, 4$.  

\[ (UCM) \]
**Example: Drell-Yan**

The cross section can be factorized as the product of TMDPDFs

The hard component hard is process dependent and can be calculated by perturbation theory.

**Differential cross section for a Drell-Yan process**

\[
\frac{d\sigma}{dQ^2dydq_T^2} = \frac{4\pi}{3N_c} \frac{P}{sQ^2} \sum_{GG'} z_{GG'}^{l'l'}(q) \sum_{ff'} z_{FF'}^{GG'} \left| C_V(q, \mu) \right|^2
\]

\[
\int \frac{d^2\vec{b}}{4\pi} e^{i\vec{b} \cdot \vec{q}} F_f \left( x_1, \vec{b}, \mu, \zeta \right) F_{f'} \left( x_2, \vec{b}, \mu, \zeta \right) + \mathcal{O} \left( \frac{q_T}{Q} \right)
\]
Factorization Theorems and TMDs

Hadronic part of Drell-Yan process

\[ d\sigma \sim \int d^4 ye^{-i q \cdot y} \langle h_1 (p, s_1) h_2 (\bar{p}, s_2) | J^{\dagger \mu} (y) J^{\nu} (0) | h_1 (p, s_1) h_2 (\bar{p}, s_2) \rangle \]

Factorized hadronic part

\[ d\sigma \sim \int db_T e^{-i q T b_T} | C_v (Q^2) |^2 \Phi_f \leftarrow h_1 (x_1, b_T) \Phi_{f, \leftarrow h_2} (x_2, b_T) S (b_T) \]

In this form, we have a double counting of the soft modes. We need to subtract them from the TMDPDFs.
Factorization Theorems and TMDs

Factorized hadronic part

\[ d\sigma \sim \int d b_T e^{-i q_T b_T} |C_V(Q^2)|^2 \frac{\Phi_{f \leftarrow h_1}(x_1, b_T)}{Z_{\text{zero-bin}}} \frac{\Phi_{f \leftarrow h_2}(x_2, b_T)}{Z_{\text{zero-bin}}} S(b_T) \]

The \( Z_{\text{zero-bin}} \) depends of the regularization scheme.

Soft function

\[ S(b_T) = \frac{Tr_c}{N_c} \langle 0 | T \left[ S_n^{T \dagger} \bar{S}_n^{T} \right] (0^+, 0^-, b_T) \bar{T} \left[ \bar{S}_n^{T \dagger} S_n^{T} \right] (0) | 0 \rangle \]

Soft function is a colourless object. arXiv: 1511.05590
We use their definition in impact parameter space. arXiv: 1702.06558

**Gluon TMD**

\[
\phi_{\mu\nu}(x, \vec{b}) = \frac{1}{xp^+} \int \frac{d\lambda}{2\pi} e^{-ixp^+\lambda} F_{+\mu}(\lambda n + b_T) \mathcal{W}(\lambda, b_T) F_{+\nu}(0)
\]

**Quark TMD**

\[
\phi_{ij}(x, \vec{b}) = \int \frac{d\lambda}{2\pi} e^{-ixp^+\lambda} \bar{q}_i(\lambda n + b_T) \mathcal{W}(\lambda, b_T) q_j(0)
\]

\[
\mathcal{W}(\lambda, b_T) = \tilde{\mathcal{W}}_n^T(\lambda n + b_T) \sum_X |X\rangle\langle X| \tilde{\mathcal{W}}_n^T(0)
\]

Wilson lines make the operator gauge invariant.
**δ-scheme and Soft function**

**Soft Wilson Lines**

\[
\tilde{S}_n(y) = P \exp \left[ -ig \int_0^\infty d\sigma \tilde{n} \cdot A(y + \tilde{n}\sigma) \right] \rightarrow \tilde{S}_n(y) = P \exp \left[ -ig \int_0^\infty d\sigma \tilde{n} \cdot A(y + \tilde{n}\sigma) e^{-\delta^+ \sigma} \right]
\]

\[
S_n(y) = P \exp \left[ ig \int_{-\infty}^0 d\sigma n \cdot A(y + n\sigma) \right] \rightarrow S_n(y) = P \exp \left[ ig \int_{-\infty}^0 d\sigma n \cdot A(y + n\sigma) e^{+\delta^- \sigma} \right]
\]

**Wilson Lines**

\[
\tilde{W}_n(y) = P \exp \left[ -ig \int_0^\infty d\sigma \tilde{n} \cdot A(y + \tilde{n}\sigma) \right] \rightarrow \tilde{S}_n(y) = P \exp \left[ -ig \int_0^\infty d\sigma \tilde{n} \cdot A(y + \tilde{n}\sigma) e^{-\delta^+ \sigma x} \right]
\]

\[
W_n(y) = P \exp \left[ ig \int_{-\infty}^0 d\sigma n \cdot A(y + n\sigma) \right] \rightarrow S_n(y) = P \exp \left[ ig \int_{-\infty}^0 d\sigma n \cdot A(y + n\sigma) e^{+\delta^- \sigma x} \right]
\]

Only the limit \( \delta \rightarrow 0 \) is relevant (and gauge invariant). The different definition of the regularization in the \( W \) and \( S \) Wilson lines solve the zero-bin problem.
With the $\delta$-scheme we get

$$Z_{\text{zero-bin}} = S(b_T)$$

**Factorized and regularizes hadronic part**

$$d\sigma \sim \int db_T e^{-iq_T b_T} \left| C_v (Q^2, \mu) \right|^2 \frac{\Phi_{f \leftarrow h_1} (x_1, b_T, \epsilon, \delta; \mu, \zeta)}{\sqrt{S (b_T, \epsilon, \delta; \mu, \zeta)}} \frac{\Phi_{f' \leftarrow h_2} (x_2, b_T, \epsilon, \delta; \mu, \zeta)}{\sqrt{S (b_T, \epsilon, \delta; \mu, \zeta)}}$$

- The term $\frac{1}{\sqrt{S (b_T, \epsilon, \delta; \mu, \zeta)}}$ regularized the rapidity divergences.
- We have a new scale $\zeta$ associated with rapidity divergences.
The cross section for Drell-Yan and SIDIS process can be written as the product of TMDs. This allows to manage in a consistent way the *rapidity* divergences.

We have a nonperturbative well-defined TMD. We provide a perturbative calculation of TMD to extract its matching coefficient with unintegrated distribution in the asymptotic large-$q_T$ limit without any reference to particular scattering process.
Polarized TMDs and small-$b$ OPE

Decomposition over Lorentz invariants

$$
\Phi_{g \leftarrow h}^{\mu\nu}(x, \vec{b}) = \langle h \big| \Phi^{\mu\nu}(x, \vec{b}) \big| h \rangle \\
= \frac{1}{2} \left( -g_T^{\mu\nu} f_1^g - i\epsilon_T^{\mu\nu} S_L g_{1L}^g + 2 h_{1L}^g \left( \frac{g_T^{\mu\nu}}{2} + \frac{b^\mu b^\nu}{\vec{b}^2} \right) + \cdots \right)
$$

- $f_1^g$ TMDPDFs of unpolarized gluons up to NNLO. arXiv:1604.07869
- $g_{1L}^g$ TMDPDFs of helicity gluons up to NLO. arXiv:1502.05354, arXiv: 1702.06558
- $h_{1L}^g$ TMDPDFs of linearly polarized gluons.

OPE a twist-2

$$
\Phi_{\mu\nu}(x, \vec{b}) = \left[ \left( C_{g \leftarrow q}(x, \vec{b}) \right)_{\mu\nu}^{ab} \otimes \Phi_{ab}(x) \right] + \left[ \left( C_{g \leftarrow g}(x, \vec{b}) \right)_{\mu\nu}^{\alpha\beta} \otimes \Phi_{\alpha\beta}(x) \right] + \cdots
$$

Unpolarized PDFs

$$
\phi_{q \leftarrow h, ij} = \langle h \big| \phi_{ij}(x) \big| h \rangle = \frac{1}{2} f_q(x) \gamma_{ij} \cdots
$$
$$
\phi_{g \leftarrow h, \mu\nu}(x) = \langle h \big| \phi_{\mu\nu} \big| h \rangle = -\frac{1}{2} g_T^{\mu\nu} f_g(x) + \cdots
$$
**Regularized TMD**

**Gluon TMD**

\[
\Gamma^{\mu \nu} \Phi^\text{ren}_{\mu \nu} = Z_g (\epsilon; \mu, \zeta) \ Z_3 (\epsilon; \mu)^{-1} \ S (b_T, \epsilon, \delta; \mu, \zeta)^{-1/2} \ \Gamma^{\mu \nu} \Phi^\text{bare}_{\mu \nu}
\]

**Coefficient C**

\[
C^{[2]}_{f \rightarrow f'} = \phi^{[2]} - \sum_r C^{[1]}_{f \rightarrow r} \otimes f^{[1]}_{r \leftarrow f'} - f^{[2]}_{f \leftarrow f'}
\]

Lorentz structures allow up to twist-2

\[
\Gamma^g = \{ g_T^{\mu \nu}, \epsilon_T^{\mu \nu}, b_T^\mu b_T^\nu / b_T^2 \}
\]
Matching coefficient

Coefficient $C_{g \leftarrow g}^{[2]}$

$$C_{g \leftarrow g}^{[2]}(x, L_\mu) = C_A^2 D_{C_A}^{[2]}(x, L_\mu) + C_A TrN_f D_{C_A TrN_f}^{[2]}(x, L_\mu) + C_F TrN_f D_{C_F TrN_f}^{[2]}(x, L_\mu)$$

with $L_x = \log \left( \frac{X^2 b_T^2}{4 e^{-2\gamma_E}} \right)$. The functions $D_i(x, L_\mu)$ can be written as

$$D_i^{[n]}(x, L_\mu) = \sum_{k=0}^{2n} D_i^{[n,k]}(x) L_\mu^k$$

where $i = C_A TrN_f, C_A^2, C_F TrN_f$
**Matching coefficients**

**Coefficient $C_A \, TrN_f$**

\[
D^{[2,0]}_{C_A \, TrN_f} = - \frac{8 \left( -17 + 16x^3 + x^3 - 6x\log(x) \right)}{9x}
\]

**Coefficient $C_F \, TrN_f$**

\[
D^{[2,0]}_{C_F \, TrN_f} = \frac{8 \left( 2(-1 + x)^3 + x\log(x)^2 \right)}{x}
\]
Linearly polarized gluons have phenomenological interest and still there is a lack of knowledge about them.

Factorization theorems allow identify universal independent object: $\rightarrow$ TMDPDFs.

Using $\delta$-scheme for regularized rapidity divergences we are able to identified $Z_{\text{zero-bin}} = S(b_T, \epsilon, \delta; \mu, \zeta)$.

Rapidity divergences have their own renormalization scale $\zeta$.

Coefficient $C^{[2]}(x, L_\mu)$ is well defined and only depends of $x$ and logaritmicaly of $b_T$. 
Thanks for your attention!!!